Control Systems Lab

Experiment 4

Second order Dynamic systems

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Date: January 2021

Contents

4.1 Objective:

(a) The objective of this exercise will be to study the time domain parameters of second order dynamic system under step input signal in simulation environment.

(b) To compare output response of un-damped, under damped, critically damped and over damped systems.

4.2 Background:

Second-order system under Step response:

The standard second-order system transfer function has the form of,

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2} \qquad 4.1$$

Where ω_n is the natural frequency of oscillation and ζ is the damping ratio. The properties of the system response depend on the values of ω_n and ζ parameters.

The time response of control system consists of two parts. Transient response and steady state response.

$$C(t) = C_{tr}(t) + C_{ss}(t). \qquad 4.2$$

Most of the dynamic systems use time as its independent variable. Analysis of response means to see the variation of output with respect to time. The output of the system takes some finite time to reach to its final value. Every system has a tendency to oppose the oscillatory behavior of the system which is called damping. The damping is measured by a factor called damping ratio of the system. If the damping is very high, then there will not be any oscillations in the output. The output is purely exponential. Such system is called an over damped system.

$1<\xi<\infty$ --- Over damped system.

$\xi=1$ --- Critically damped system.

$0<\xi<1$ --- Under damped system.

$\xi=0$ --- Undamped system.

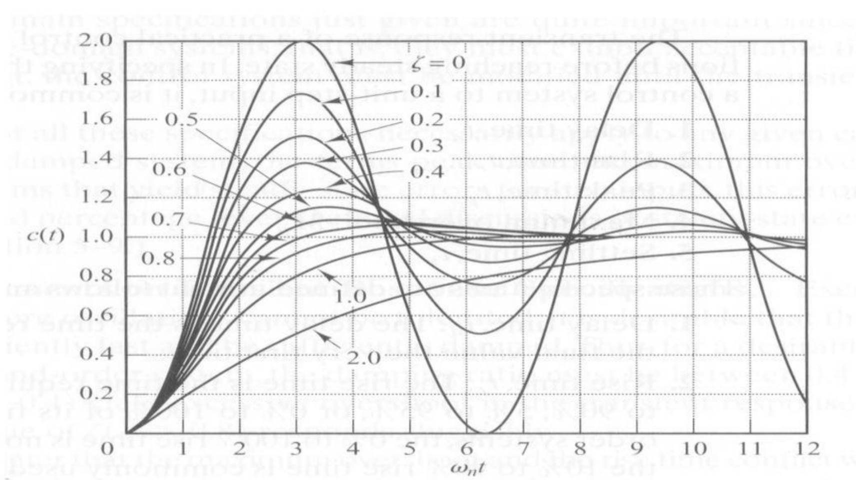

Figure- 4.1 Effect of damping ratio on the output response of a second order system Second-order system

Time domain Parameters:

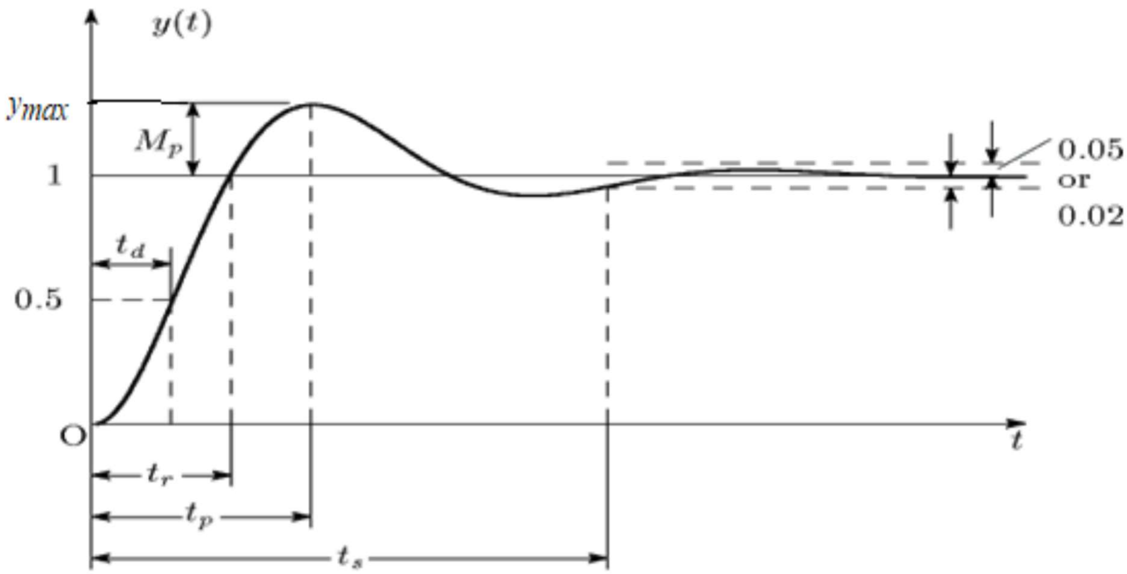

Figure- 4.2 Standard output response of Second-order system

Maximum (percentage) overshoot (Mp):

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_p. For a response similar to Figure 4.2, the percent overshoot is found using

$$M_P = 100\,(y_{max} - 1) \qquad 4.3$$

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$M_P = 100 \times e^{\left(\frac{-\Pi\zeta}{\sqrt{1-\zeta^2}}\right)} \qquad 4.4$$

Peak time (t_p):

The time taken by the response to reach its maximum value is the peak time of the system.

It is dented as t_p.

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as:

$$t_p = \frac{\Pi}{\omega_n\sqrt{1-\zeta^2}} \qquad 4.5$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

Delay time (t_d):

The delay time is the time required for the response to reach half the final value the very first time.

Rise time (t_r):

The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped second-order systems, the 0% to 100% rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.

The peak time is the time required for the response to reach the first peak of the overshoot.

Settling time (t_s):

The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%). The settling time is related to the largest time constant of the control system.

4.3 MATLAB code for second order systems with different types of damping:

CASE 1: Under Damped Second Order System, $G(s)=10/(s^2+2s+10)$

```
num1=[10];
den1=[1 2 10];
disp('Transfer Function of the system');
G=tf(num1,den1)
 [Wn Z P] = damp(G);
Wn=Wn(1);
Z=Z(1);
t=0: 0.1: 20;
figure(1);
step(num1,den1,t)
title('Under Damped Second Order System Response for Step Input');
grid on;
```

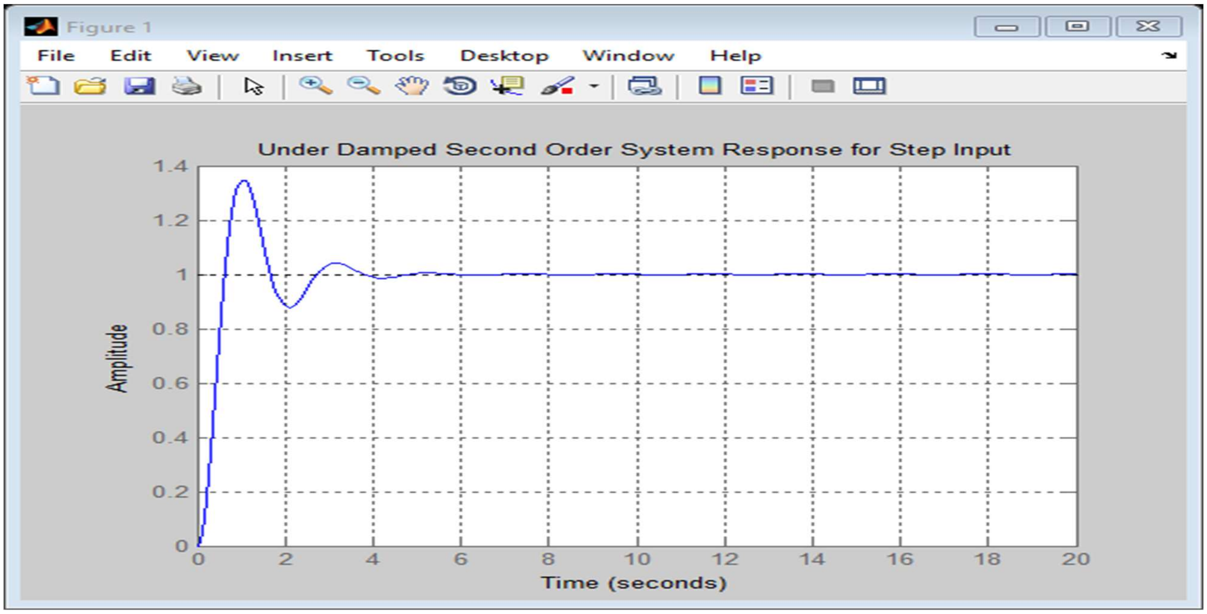

Figure- 4.3 output response of under damped second order system with step input

CASE 2: Un-damped Second Order System, G(s) = $\frac{10}{S^2+10}$

```
num2=[10];
den2=[1 0 10];
figure(2);
step(num2,den2,t)
title('Undamped Second Order System Response for Step Input');
grid on;
```

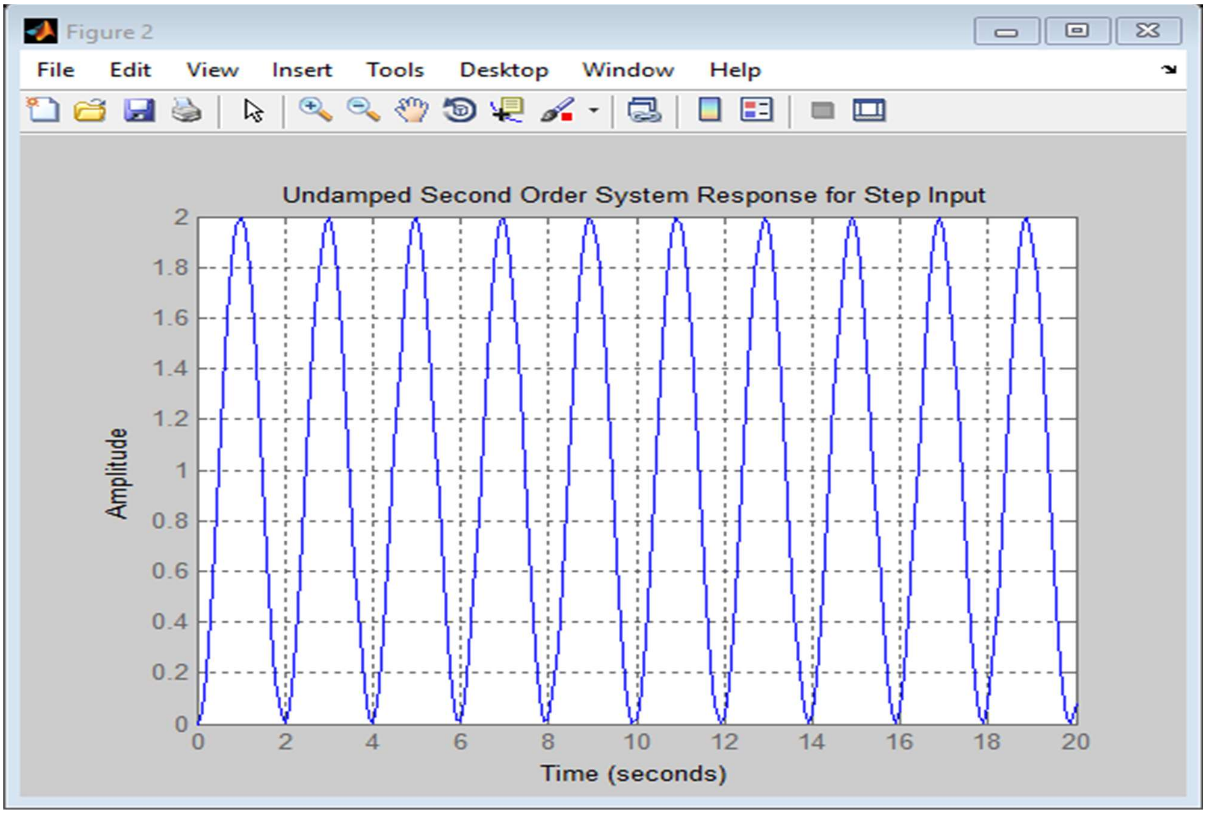

Figure- 4.4 output response of un-damped second order system with step input

CASE 3: Critically Damped Second Order System G(s)=10/(s^2+7.32s+10)

```
num3= [10];
den3=[1  7.32  10];
figure(3);
step(num3, den3,t)
title('Critically Damped Second Order System Response for Step Input');
grid on;
```

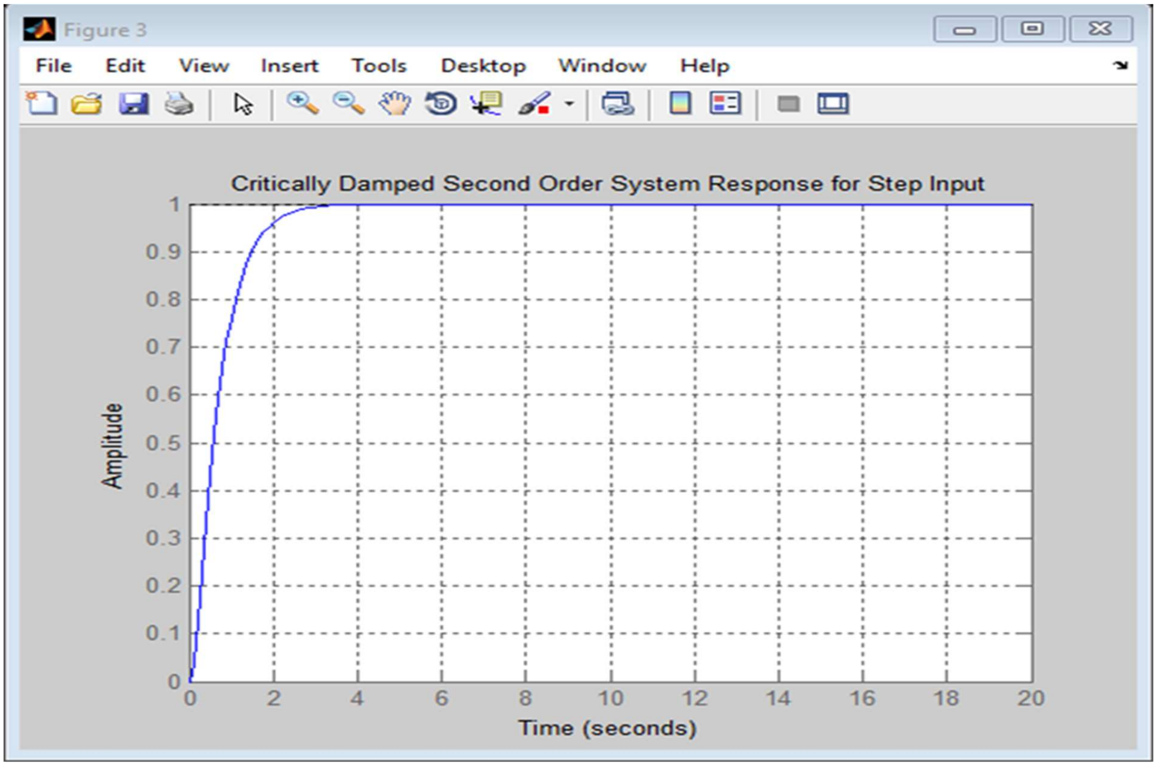

Figure- 4.5 output response of critically damped second order system with step input

CASE 4: Over Damped Second Order System G(s)=10/(s^2+3s+10)

```
num4= [10];
den4= [1 3 10];
figure (4);
step (num4, den4, t)
title ('Over Damped Second Order System Response for Step Input');
grid on;
```

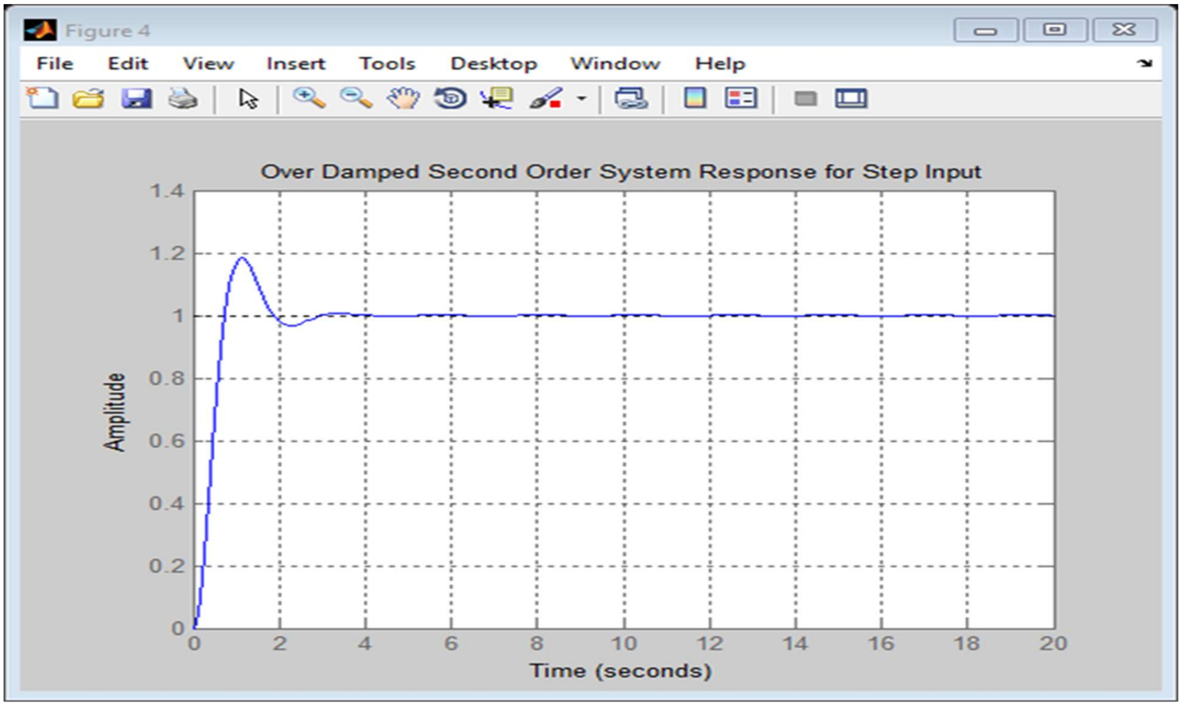

Figure- 4.6 output response of over damped second order system with step input

4.4 Exercise to be performed and submitted:

(a) Verify the time response of the system shown in case 1, 2, 3 and 4 using MATLAB coding and identify its transient, steady-state parameters for each case study from the generated response graphs.

(b) Consider a second order under-damped system with transfer function, $G(S) = \frac{10}{S^2+2S+1}$
Find out the **peak time** and **maximum overshoot (M_P)** of this system by both the **analytical calculation using equations 4.4 and 4.5** and **graphically** as shown in the **figure 4.2.**