Control Systems Lab

Experiment 6

Root Locus for Stability Analysis

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Contents

6.1 Objective:

To plot the root locus for a given dynamic system and comment on its stability.

6.2 Root Locus for Control Systems

The root locus of an open-loop transfer function H(s) is a plot of the locations (locus) of all possible closed loop poles with proportional gain k and unity feedback:

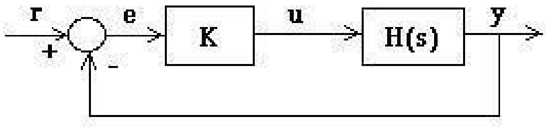

Figure 1: Closed-Loop System

Consider a closed-loop system transfer function;

$$\frac{Y(S)}{R(S)} = \frac{KH(S)}{1+KH(S)} \qquad (1)$$

Thus, the poles of the closed loop system are values of s such that 1 + K H(S) = 0, where K is the forward path gain.

If we use the relation H(S) = b(S)/a(S), then the equation (1) can have the form;

$$a(S) + Kb(S) = 0 \qquad (2)$$

$$\frac{a(S)}{K} + b(S) = 0 \qquad (3)$$

Let n = order of a(S) and m = order of b(S) [Order of a polynomial is the highest power of S that appears in it].

We will consider all positive values of k. **In the limit as k -> 0, the poles of the closed-loop system are a(S) = 0 or the poles of H(S). In the limit as k -> infinity, the poles of the closed-loop system are b(S) = 0 or the zeros of H(S).**

No matter what we pick 'k' to be, the closed-loop system must always have n poles, where n is the number of poles of H(S). The root locus must have n branches; **each branch starts at a pole of H(S) and goes to a zero of H(S). If H(S) has more poles than zeros (as is often the case), m < n and we say that H(S) has zeros at infinity.** In this case, the limit of H(S) as S-> **infinity is zero. The number of zeros at infinity is n-m, the number of poles minus the number of zeros, and is the number of branches of the root locus that go to infinity (asymptotes).**

Since the root locus is actually the locations of all possible closed loop poles, from the root locus we can select a gain such that our closed-loop system will perform the way we want. If any of the selected poles are on the right half plane, the closed-loop system will be unstable.

The poles that are closest to the imaginary axis have the greatest influence on the closed-loop response, so even though the system has three or four poles, it may still act like a second or even first order system depending on the location(s) of the dominant pole(s).

6.3 Various terminologies involved in Root Locus Technique of stability analysis:

Root locus technique is used to find the roots of the characteristics equation. This technique provides a graphical method of plotting the locus of the roots in the s plane as a given parameter usually gain is varied over the complete range of values. This method brings in to focus the complete dynamic response of the system. By using root locus method, the designer can predict the effects location of closed loop poles by varying the gain value or adding open loop poles and/or open loop zeroes. The closed loop poles are the roots of the characteristic equation.

Various terms related to root locus technique that we will use frequently in this article.

1. Characteristic Equation Related to Root Locus Technique: 1 + G(s)H(s) = 0 is known as characteristic equation. Now on differentiating the characteristic equation and on equating dk/ds equals to zero, we can get break away points.
2. Break away Points: Suppose two root loci which start from pole and moves in opposite direction collide with each other such that after collision they start moving in different directions in the symmetrical way. Or the breakaway points at which multiple roots of the characteristic equation 1 + G(s)H(s)= 0 occur. The value of K is maximum at the points where the branches of root loci break away. Break away points may be real, imaginary or complex.
3. Break in Point: Condition of break in to be there on the plot is written below: Root locus must be present between two adjacent zeros on the real axis.
4. Centre of Gravity: It is also known centroid and is defined as the point on the plot from

$$\sigma_A = \frac{(Sum\ of\ real\ parts\ of\ poles) - (Sum\ of\ real\ parts\ of\ zeros)}{N - M} \quad (4)$$

5. where all the asymptotes start. Mathematically, it is calculated by the difference of summation of poles and zeros in the transfer function when divided by the difference of total number of poles and total number of zeros. Centre of gravity is always real & it is denoted by σ_A. Where N is number of poles & M is number of zeros.
6. Asymptotes of Root Loci: Asymptote originates from the centre of gravity or centroid and goes to infinity at definite some angle. Asymptotes provide direction to the root locus when they depart break away points.
7. Angle of Asymptotes: Asymptotes makes some angle with the real axis and this angle can be calculated from the given formula, Where p = 0, 1, 2…………………(N-M-1)

$$Angle\ of\ asymptotes = \frac{(2p+1) \times 180}{N - M} \quad (5)$$

8. <u>Angle of Arrival or Departure:</u> We calculate angle of departure when there exist complex poles in the system. Angle of departure can be calculated as [180- {(sum of angles to a complex pole from the other poles) - (sum of angle to a complex pole from the zeros)}].

9. <u>Intersection of Root Locus with the Imaginary Axis:</u> In order to find out the point of intersection root locus with imaginary axis, we have to use Routh Hurwitz criterion. First, we find the auxiliary equation then the corresponding value of K will give the value of the point of intersection.
10. <u>Symmetry of Root Locus:</u> Root locus is symmetric about the 'x' axis or the real axis.

6.3 Plotting the Root Locus of a Transfer Function using MATLAB Simulation:

Consider the control system with the following open loop transfer function:

$$\mathrm{G(S)H(S)} = \frac{80(S+5)}{S^2\,(S+50)} \qquad (6)$$

MAT LAB code for Root Locus plot:

```
num= [80 400];
den= [1 50 0 0];
sys=tf(num,den)
rlocus (sys);
pole(sys)
zero(sys)
```

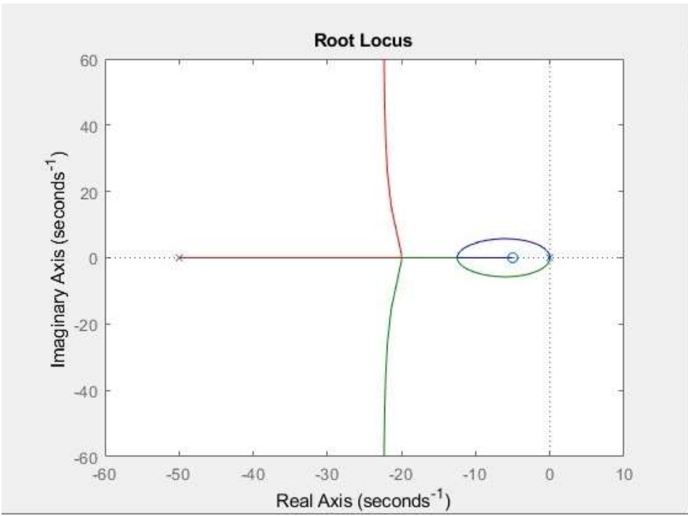

Figure 2: Root locus plotted in MATLAB

6.4 Exercise to be performed and submitted:

1. Execute the above programming using MATLAB and obtain the root locus plot. Verify

the same with the plot shown in figure 2.

2. Determine the number of asymptotes and the angle of asymptotes analytically and tally with the simulation result.
3. Comment on the stability of the system.