Control Systems Lab
Experiment 3
Bump Test Modeling

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Contents

3.1 Objective:

To perform a bump test on a first order dynamic system.

3.2 Theory:

Linear time-invariant dynamical systems are categorized under first-order systems, second- order systems further higher-order systems. The transfer function of all first-order systems has a standard form. This enables us to investigate the response of first- order systems collectively, for any specific input. The response of a first-order system depends on its DC gain *K,* and time constant τ. Both *K* and τ are functions of system parameters. The bump test is a simple test based on the **step response** of a stable first order system.

3.3 Background:

The standard form of transfer function of a first-order dynamic system is:

$$\mathbf{G(S)} = \frac{\mathbf{Y(S)}}{\mathbf{U(S)}} = \frac{\mathbf{K}}{\mathbf{\tau S+1}} \qquad 3.1$$

Where *Y(s)* and *U(s)* are the Laplace transform of the output and input variables, respectively, *K* is the DC gain, and τ is the time constant. For a step input U(S) =1/S*,* the output response of the system is:

$$Y(S) = \frac{Y(S)}{U(S)} * U(S) = \frac{K}{\tau S + 1} * \frac{1}{S} = \frac{K}{S(\tau S + 1)} \qquad 3.2$$

$$y(t) = L^{-1}\left[\frac{K}{S(\tau S + 1)}\right] = K\left(1 - e^{\frac{-t}{\tau}}\right) \qquad 3.3$$

It is clear from (3.3) that $y \to K$ as $t \to \infty$. The DC gain can therefore be interpreted as the final value of the output for a unit step input. The time constant is the time required for *y(t)* to reach 63.2% of its final value. Indeed, at t=τ, *y(t)*=0.632*K* for a unit step input. For a unit step input, the change in input is one. In general, for a step input of magnitude A at t=τ, *y(t)*=0.632*K.*

3.4 Waveform Analysis:

The figure shows the input and output waveforms obtained in the bump test method.

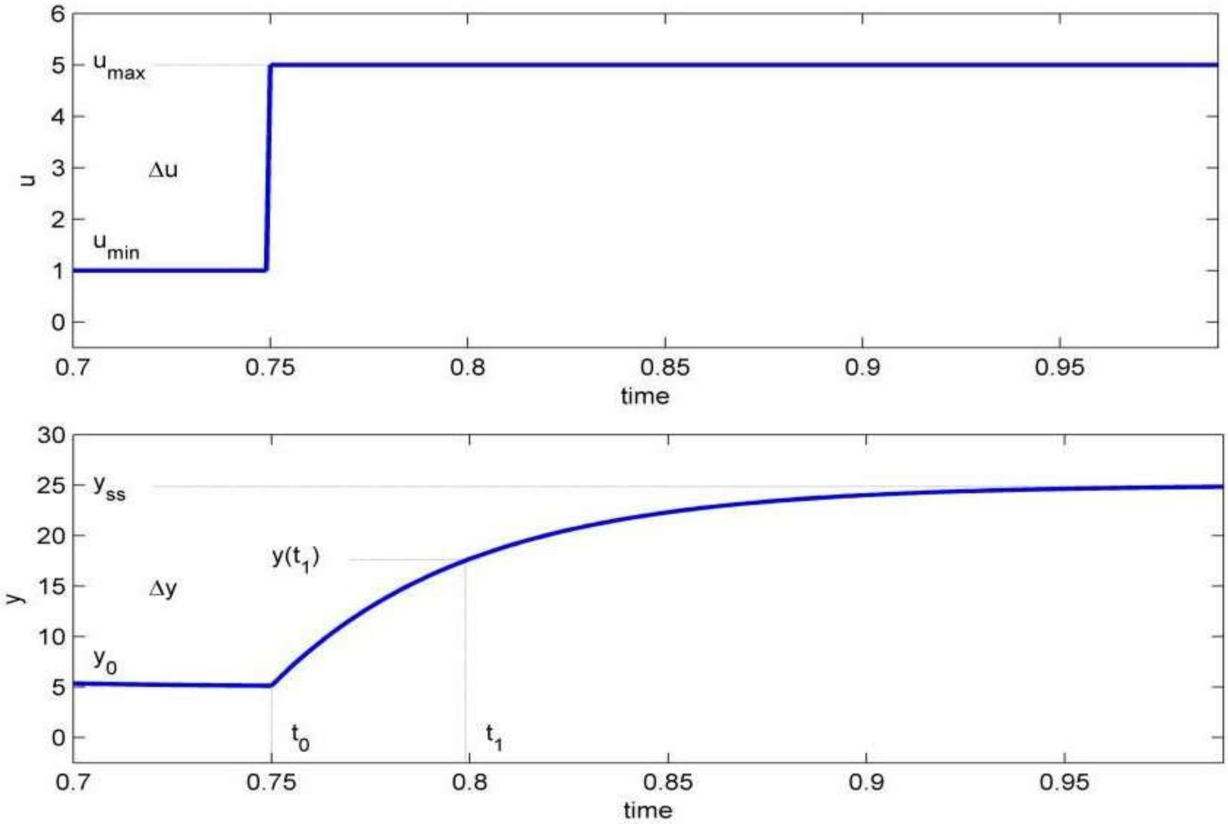

Figure 1: Input and output waveforms obtained in the bump test method

The step input begins at time t_0. The input signal has a minimum value of u_{min} and a maximum value of u_{max}. The resulting output signal is initially at y_0. Once the step is applied, the output tries to follow it and eventually settles at its steady-state value y_{ss}. From the output and input signals, the steady-state gain is:

$$K = \frac{\Delta y}{\Delta u} \tag{3.4}$$

Where $\Delta y = y_{ss} - y_0$ and $\Delta u = u_{max} - u_{min}$. In order to find the model time constant, τ, we can first calculate where the output is supposed to be at the time constant from the equation:

$$y(t_1) = 0.632\Delta y + y_0 \tag{3.5}$$

Then, we can read the time t_1 corresponding to $y(t_1)$ from the response data in Figure 1. From the figure we can see that the time t_1 is equal to:

$$t_1 = t_0 + \tau$$

From this, the model time constant can be found as:

$$\tau = t_1 - t_0 \tag{3.6}$$

3.5 MATLAB code based exercise:

Based on equation 3.1, write a transfer function of the first order system with the given specifications; K = 5 rad/V-s and τ = 0.05 s. Write the MATLAB code to obtain the output response for the above equation with a step input.

CODE:

```
num = [5];
den = [0.05 1];
g = tf (num,den)
step(g,'r')
```

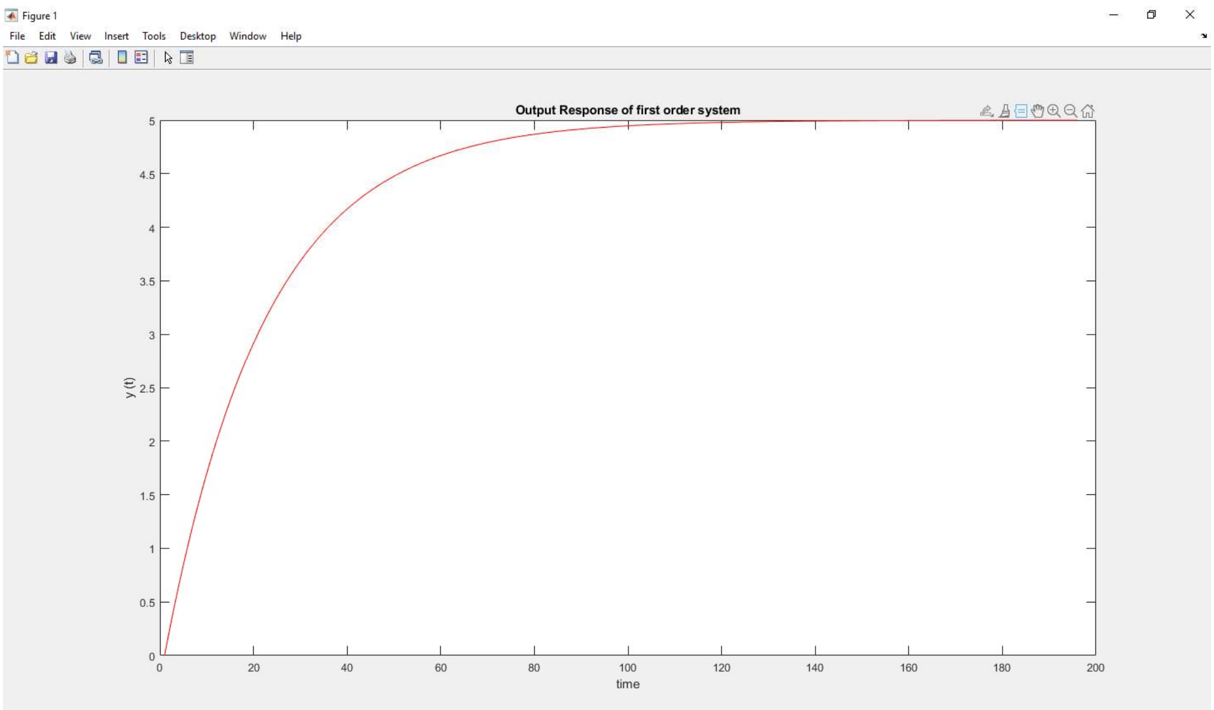

Figure 2: Expected Output from MATLAB code

3.6 Simulink Exercise to be performed and submitted:

Construct a Simulink block diagram with a transfer function described in Equation 3.1. A step input as shown in Figure 2 is given to it and the response of the system is observed in a scope. The step response shown in Figure 2 is generated using this transfer function with $K = 5$ rad/V-s and $\tau = 0.05$ s.

Hint: The block diagram should be constructed using a step input, transfer function block and a scope. Transfer function block could be located under "continuous" in Simulink library. Set the time of the step input block to get the desired output.

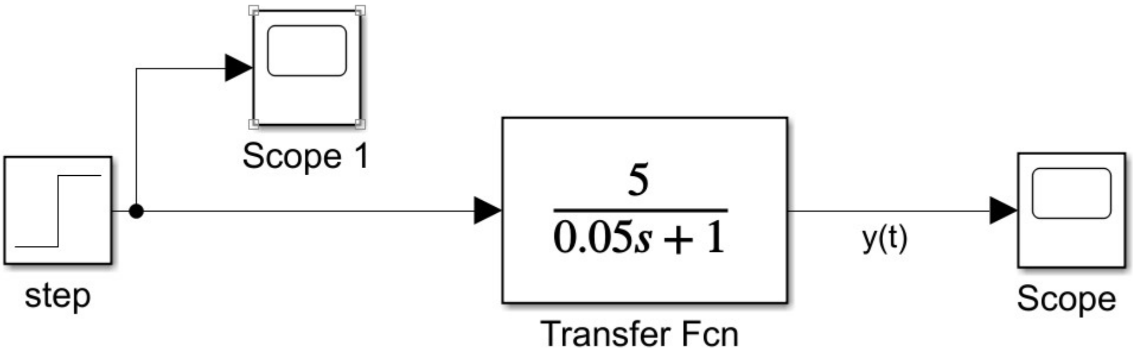

Figure 3: Simulink model

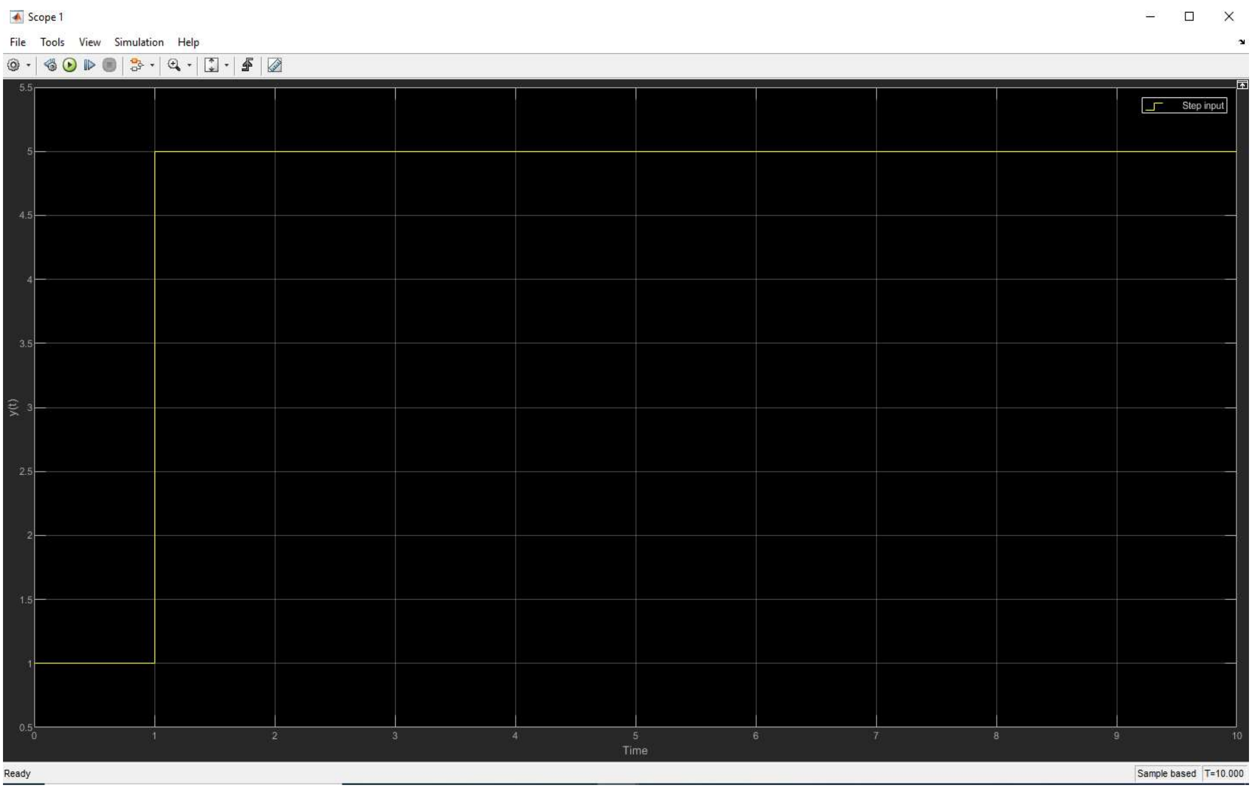

Figure 4: Expected **Input** waveform of Simulink exercise

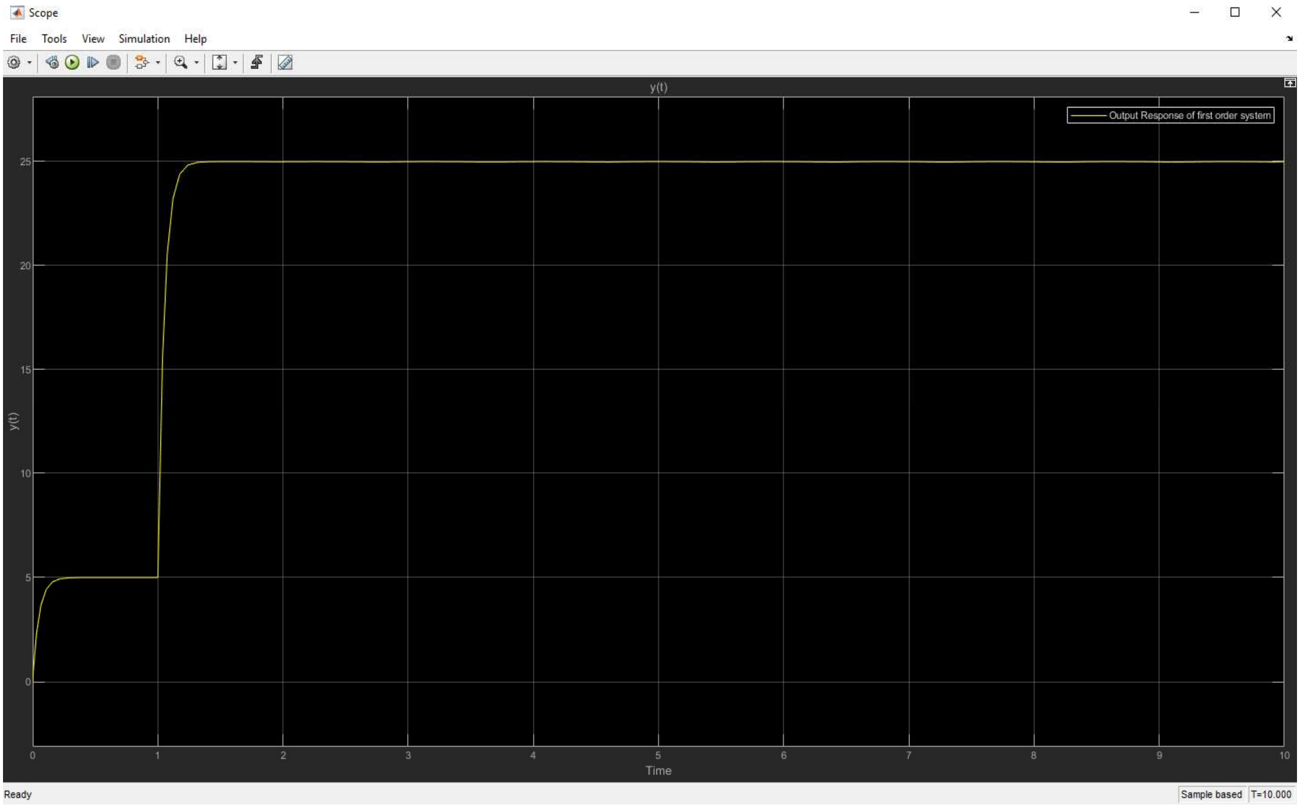

Figure 5: Expected **Output** waveform of Simulink exercise

Note : Figure 1 response is validated by the Input and output waveforms obtained from Simulink model as shown in figure 4 and figure 5.