Least Square fit for a Qudratic curve

Compiled and Designed by Sai Satyapriya

$$E = \sum_{i=1}^{n} \left[y_i - f(x_i) \right]^2$$

Here,

$$f(x_i) = ax_i^2 + bx_i + c$$

$$E = \sum_{i=1}^{n} \left[y_i - (ax_i^2 + bx_i + c) \right]^2$$

or

$$E = \sum_{i=1}^{n} \left[y_i - ax_i^2 - bx_i - c \right]^2$$

$$\frac{\partial E}{\partial a} = 2 \sum_{i=1}^{n} \left[y_i - ax_i^2 - bx_i - c \right] (-x_i^2) = 0$$

$$\Rightarrow \sum y_i x_i^2 - a \sum x_i^4 - b \sum x_i^3 - c \sum x_i^2 = 0$$

$$\boxed{a \sum x_i^4 + b \sum x_i^3 + c \sum x_i^2 = \sum y_i x_i^2}$$

—

$$\frac{\partial E}{\partial b} = 2 \sum_{i=1}^{n} \left[y_i - ax_i^2 - bx_i - c \right] (-x_i) = 0$$

$$\Rightarrow \sum y_i x_i - a \sum x_i^3 - b \sum x_i^2 - c \sum x_i = 0$$

$$\boxed{a \sum x_i^3 + b \sum x_i^2 + c \sum x_i = \sum y_i x_i}$$

—

$$\frac{\partial E}{\partial c} = 2 \sum_{i=1}^{n} \left[y_i - ax_i^2 - bx_i - c \right] (-1) = 0$$

$$\Rightarrow \sum y_i - a \sum x_i^2 - b \sum x_i - c \sum 1 = 0$$

$$\boxed{a \sum x_i^2 + b \sum x_i + cN = \sum y_i}$$

—

Matrix form

$$\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & N \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum y_i x_i^2 \\ \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

—

Solution

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & N \end{bmatrix}^{-1} \begin{bmatrix} \sum y_i x_i^2 \\ \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

—

Compact notation

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} X_a & X_b & X_c \\ X_b & X_c & X \\ X_c & X & N \end{bmatrix}^{-1} \begin{bmatrix} Y_a \\ Y_b \\ Y \end{bmatrix}$$