Least Square fit for a straight line

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$$E = \sum_{i=1}^{n} \left[y_i - f(x_i)\right]^2$$

Here,

$$f(x_i) = mx_i + c$$

$$E = \sum_{i=1}^{n} \left[y_i - (mx_i + c)\right]^2$$

or

$$E = \sum_{i=1}^{n} \left[y_i - mx_i - c\right]^2$$

$$\frac{\partial E}{\partial m} = 2\sum_{i=1}^{n} \left[y_i - mx_i - c\right](-x_i) = 0$$

$$\Rightarrow \sum y_i x_i - m\sum x_i^2 - c\sum x_i = 0$$

$$\boxed{m\sum x_i^2 + c\sum x_i = \sum y_i x_i}$$

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$$\frac{\partial E}{\partial c} = 2\sum_{i=1}^{n} \left[y_i - mx_i - c\right](-1) = 0$$

$$\Rightarrow \sum y_i - m\sum x_i - c\sum 1 = 0$$

$$\boxed{m\sum x_i + cN = \sum y_i}$$

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Matrix form

$$\begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & N \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

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Solution

$$\begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & N \end{bmatrix}^{-1} \begin{bmatrix} \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

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