**Department of Electrical, Computer, & Biomedical Engineering**
Faculty of Engineering & Architectural Science

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| **Course Title:** | Signals and Systems 1 |
| **Course Number:** | ELE 532 |
| **Semester/Year (e.g.F2016)** | F2021 |

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|---|---|
| **Instructor:** | Soosan Behesti |

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| *Assignment/Lab Number:* | 03 |
| *Assignment/Lab Title:* | 03 |

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| *Submission Date:* | 2021-11-21 |
| *Due Date:* | 2021-11-21 |

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$$x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$$

$$x(t+T_0) = x(t)$$

$$\cos\frac{3\pi}{10}(t+T_0) + \frac{1}{2}\cos\frac{\pi}{10}(t+T_0) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$$

① $\frac{3\pi}{10}(t+T_0) = \frac{3\pi}{10}t + 2\pi k_1$ ② $\frac{\pi}{10}(t+T_0) = \frac{\pi}{10}t + 2\pi k_2$

$$\cancel{\frac{3\pi}{10}t} + \frac{3\pi}{10}T_0 = \cancel{\frac{3\pi}{10}t} + 2\pi k_1 \qquad \frac{\pi}{10}T_0 = 2\pi k_2$$

$$\frac{3\pi}{10}T_0 = 2\pi k_1 \qquad T_0 = \frac{2\pi}{\pi/10}k_2$$

$$3\pi T_0 = 20\pi k_1 \qquad = \frac{20\pi}{\pi}k_2$$

$$T_0 = \frac{20\pi k_1}{3\pi} \qquad = 20k_2$$

$$= \frac{20}{3}k_1$$

LCM of $\frac{20}{3}$ and 20 is 20

$$\frac{20}{3}\times 3 = 20, \quad T_0 = 20$$

$$w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$$

$$\cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$$

$$= \frac{e^{j\frac{3\pi}{10}t} + e^{-j\frac{3\pi}{10}t}}{2} + \frac{1}{2}\left(\frac{e^{j\pi/10\,t} + e^{-j\pi/10\,t}}{2}\right)$$

$$= \frac{e^{j3\pi/10\,t} + e^{-j3\pi/10\,t}}{2} + \frac{e^{j\pi/10\,t} + e^{-j\pi/10\,t}}{4}$$

$$= \frac{1}{2}e^{j3\pi/10\,t} + \frac{1}{2}e^{-j3\pi/10\,t} + \frac{1}{4}e^{j\pi/10\,t} + \frac{1}{4}e^{-j\pi/10\,t}$$

$$D_3 = \frac{1}{2}, \; D_{-3} = \frac{1}{2}, \; D_1 = \frac{1}{4}, \; D_1 = \frac{1}{4}$$

$$D_n = \frac{1}{T_0}\int_{<T_0>} x(t)\, e^{-j w_0 n t}\, dt$$

$$= \frac{1}{20}\int_{<T_0>} \left(\tfrac{1}{2} e^{j 3\pi/10\, t} + \tfrac{1}{2} e^{-j 3\pi/10\, t} + \tfrac{1}{4} e^{j \pi/10\, t} + \tfrac{1}{4} e^{-j \pi/10\, t}\right)$$

$$\cdot \left(e^{-j \pi/10\, n t}\right) dt$$

$$= \frac{1}{20}\left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\pi/10} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\pi/10}\right.$$

$$\left. + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\pi/10} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\pi/10}\right]$$

$$D_n = \tfrac{1}{2}\left[\operatorname{sinc}[(3-n)\pi] + \operatorname{sinc}[(3+n)\pi]\right.$$

$$\left. + \tfrac{1}{2}\operatorname{sinc}[(1+n)\pi] + \tfrac{1}{2}\operatorname{sinc}[(1-n)\pi]\right]$$

A2)

$$x_2(t) = \quad T_0 = 20,\ w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$$

$$D_n = \frac{1}{T_0}\int_{<T_0>} x(t)\, e^{-j w_0 n t}\, dt$$

$$= \frac{1}{20}\int_{-5}^{5} (1)\, e^{-j w_0 n t}\, dt$$

$$= \frac{1}{20}\int_{-5}^{5} e^{-j \pi/10\, n t}\, dt$$

$$= \frac{1}{20}\left[\frac{e^{-j n \pi/10\, t}}{-j n \pi/10}\right]_{-5}^{5}$$

$$D_n = \frac{1}{20}\left[\frac{-10}{j n\pi} e^{-j n\pi/2} + \frac{10}{j n \pi} e^{j n \pi/2}\right]$$

$$= \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$x_3(t) = T_0 = 40$$

$$w_0 = \frac{2\pi}{40} = \frac{\pi}{20}$$

$$D_n = \frac{1}{T_0} \int_{-5}^{5} x(t)\, e^{-jw_0 n t}\, dt$$

$$= \frac{1}{40} \int_{-5}^{5} e^{-jw_0 n t}\, dt$$

$$= \frac{1}{40} \int_{-5}^{5} e^{-j \pi/20\, n t}\, dt$$

$$= \frac{1}{40} \left[ \frac{e^{-j\pi/20\, nt}}{-j\pi/20\, nt} \right]_{-5}^{5}$$

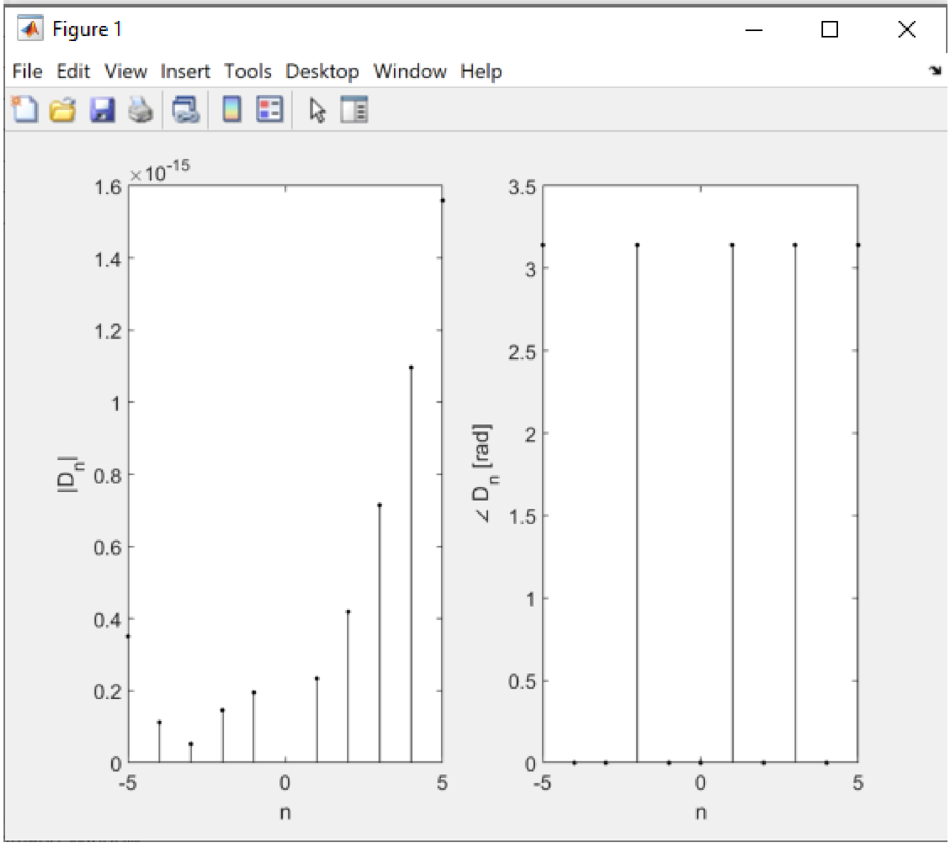
$$= \frac{1}{40} \left[ \frac{-20\, e^{-jn\pi/4}}{jn\pi} + \frac{20\, e^{j\pi/4}}{jn\pi} \right]$$

$$D_n = \frac{1}{n\pi} \sin\left(\frac{n\pi}{4}\right)$$

A4)

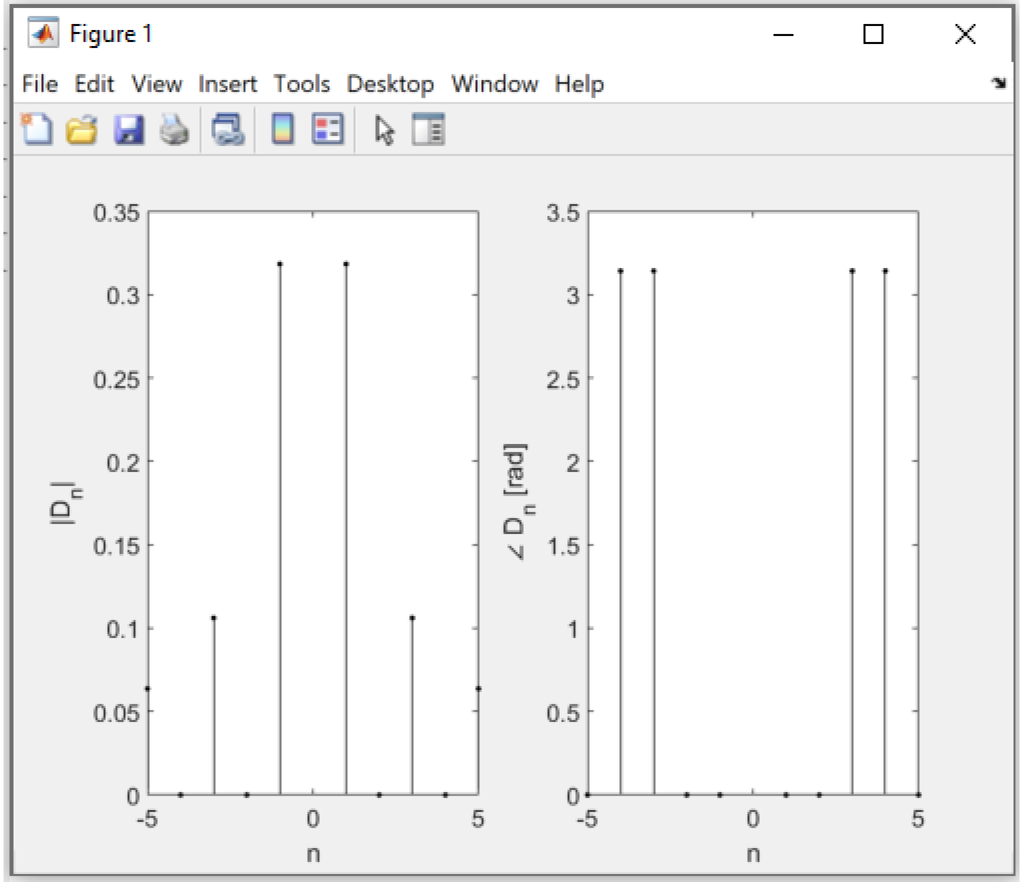
```
%A4, part a
%x1(t)

clf;
n = (-5:5);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi))
+(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

```
%A4, part a
%x1(t)
```

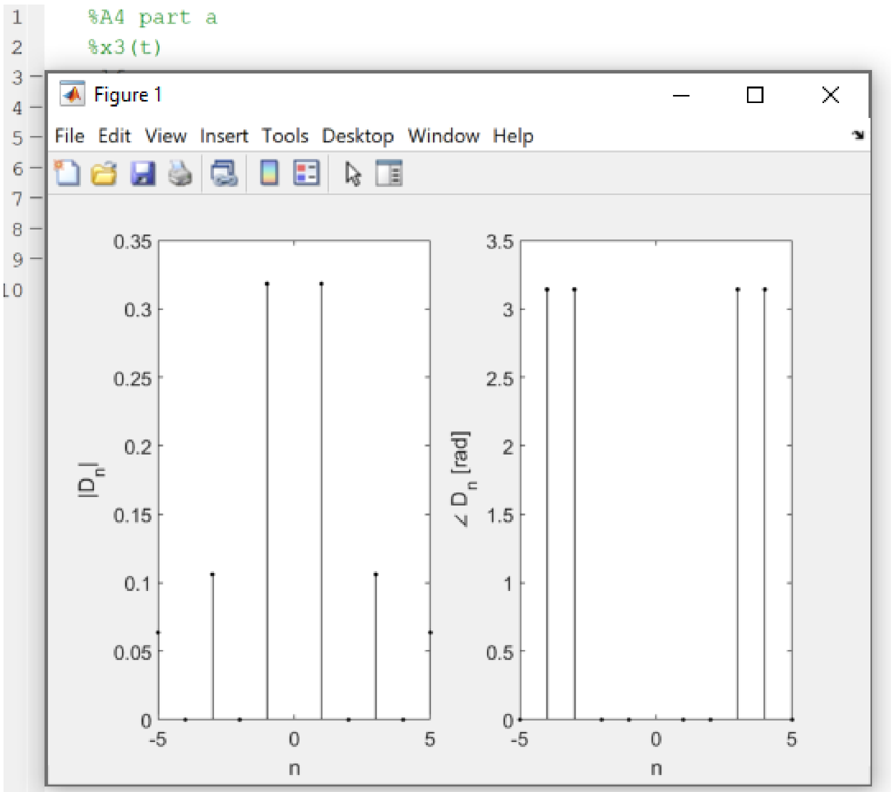



```
%A4 part a
%x2(t)

clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

```
%A4 part a
%x2(t)
```



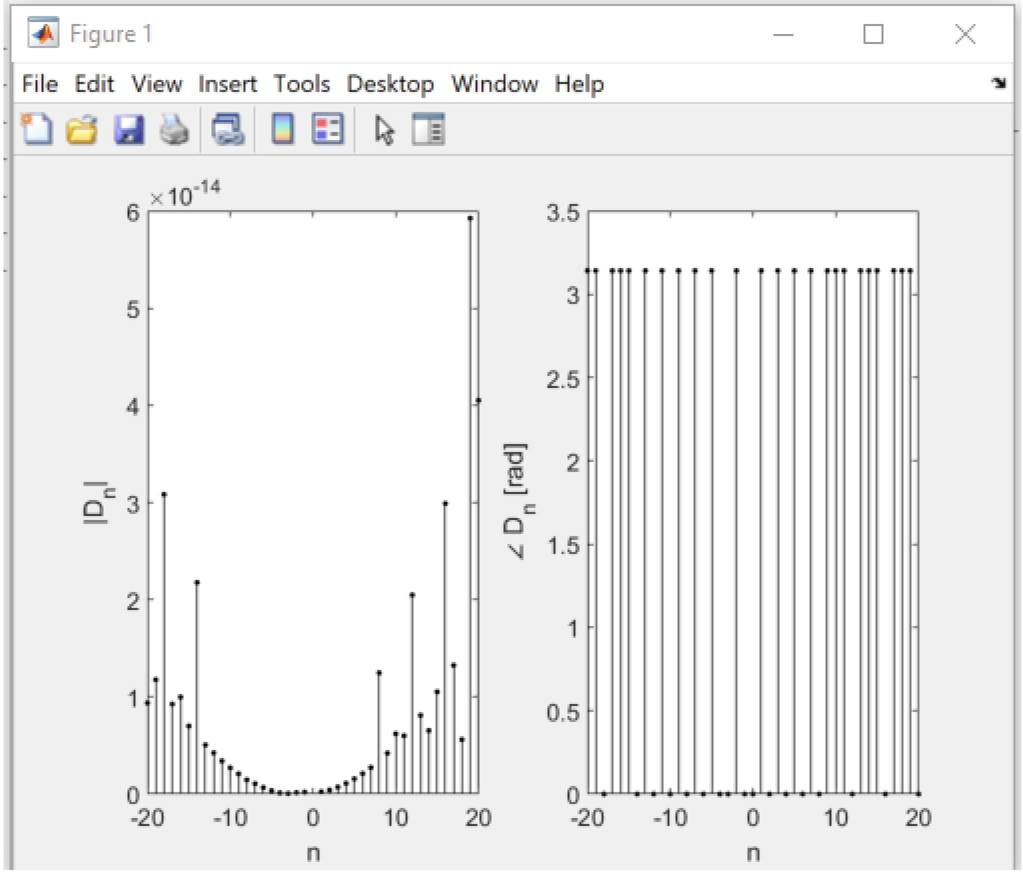

```
%A4 part a
%x3(t)
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```




b)

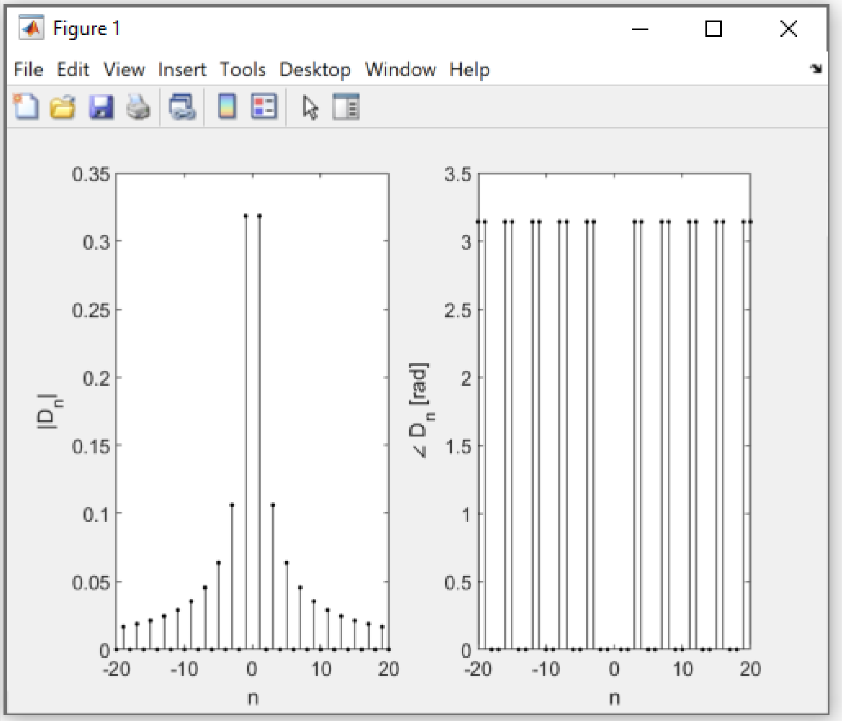
```
%A4
%part b, x1(t)

clf;
n = (-20:20);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

```
%A4
%part b, x1(t)
```



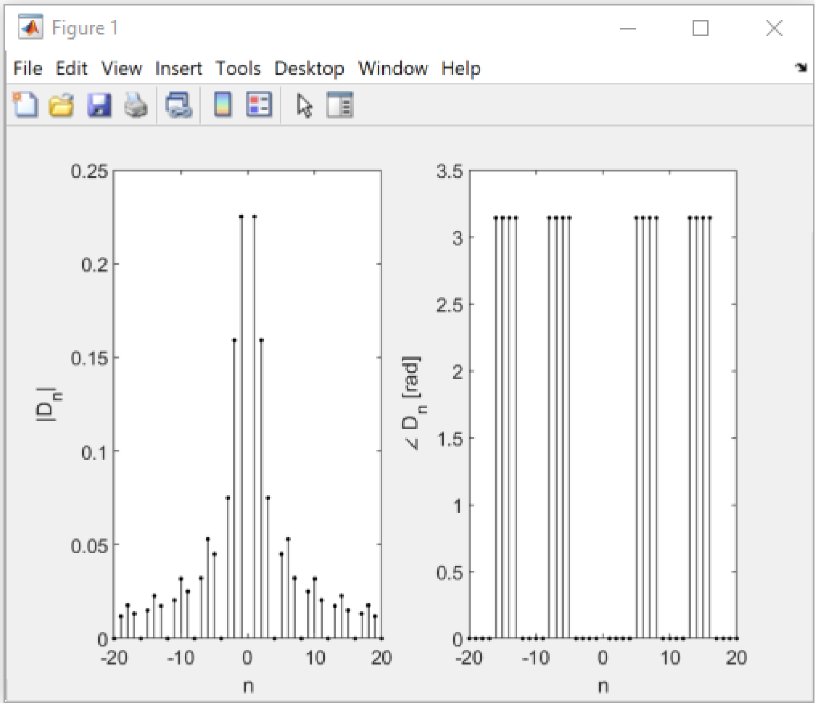

```
%A4
%part b, x2(t)

clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

```
%A4
%part b, x2(t)
```




```
%A4
%part b, x3(t)

clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]')
```

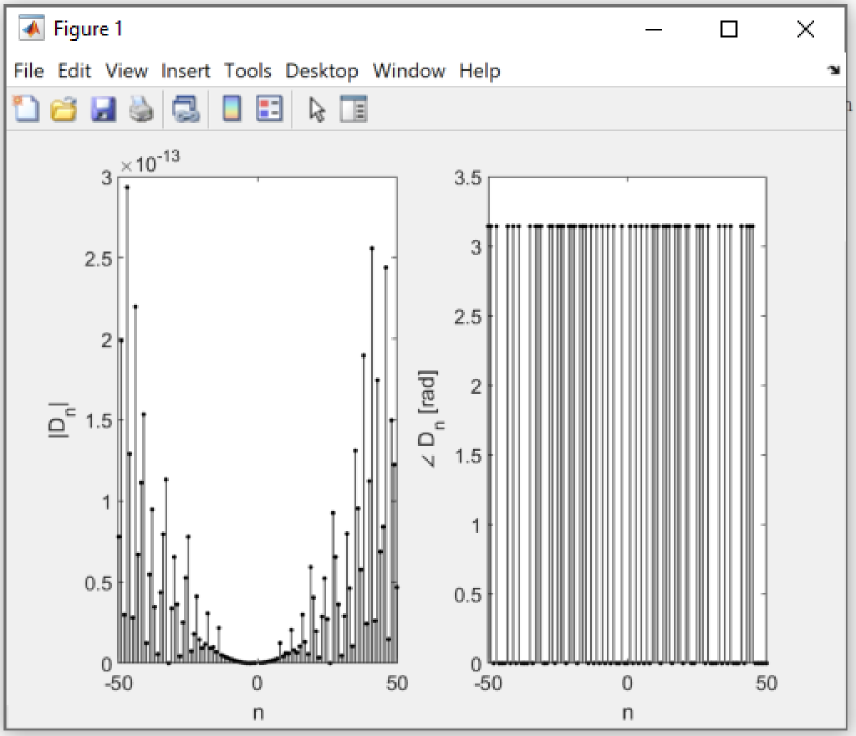
```
%A4
%part b, x3(t)
```



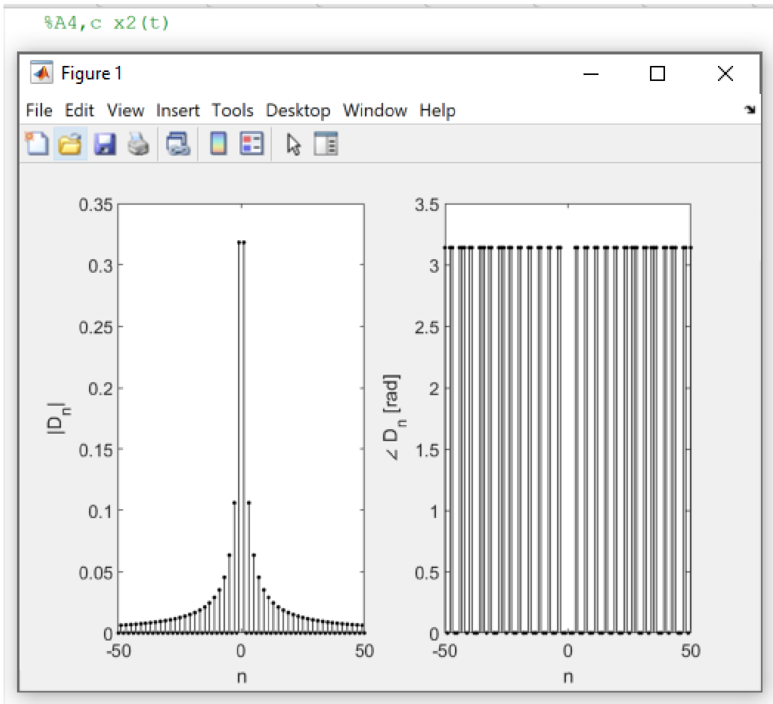

c)

```
%A4
%part c, x1(t)

clf;
n = (-50:50);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

```
%A4
%part c, x1(t)
```

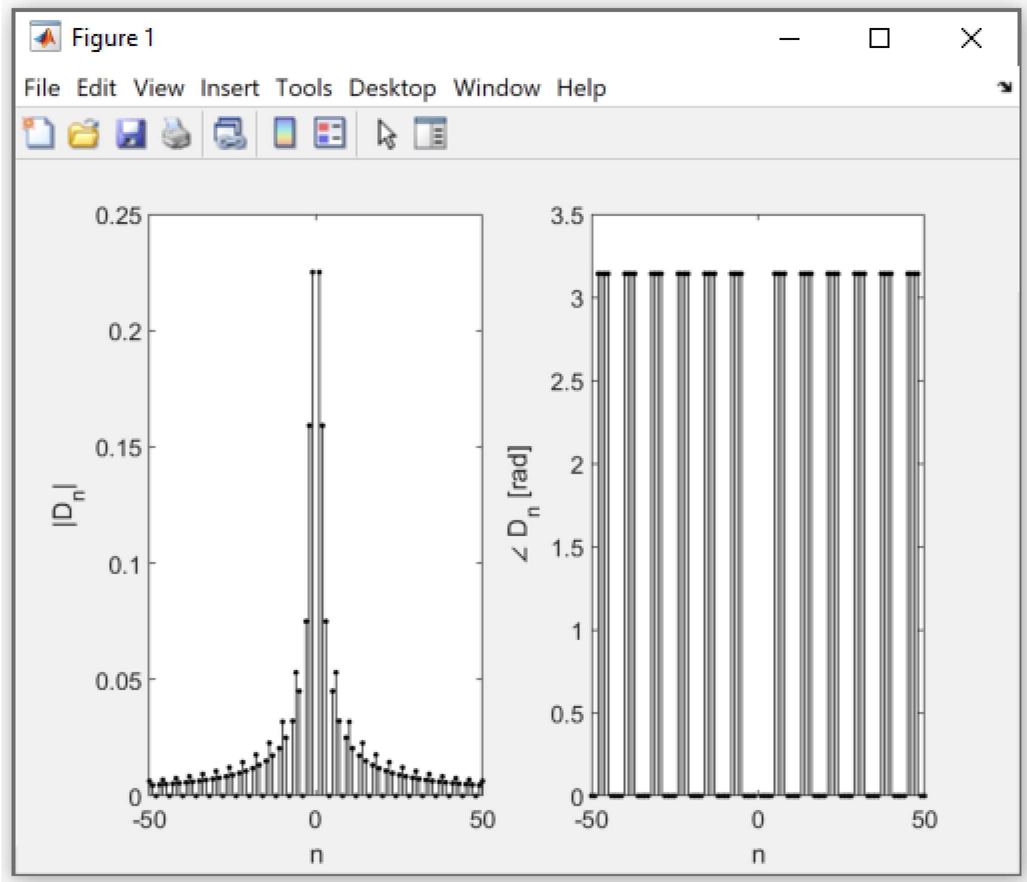



```
%A4,c x2(t)

clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```



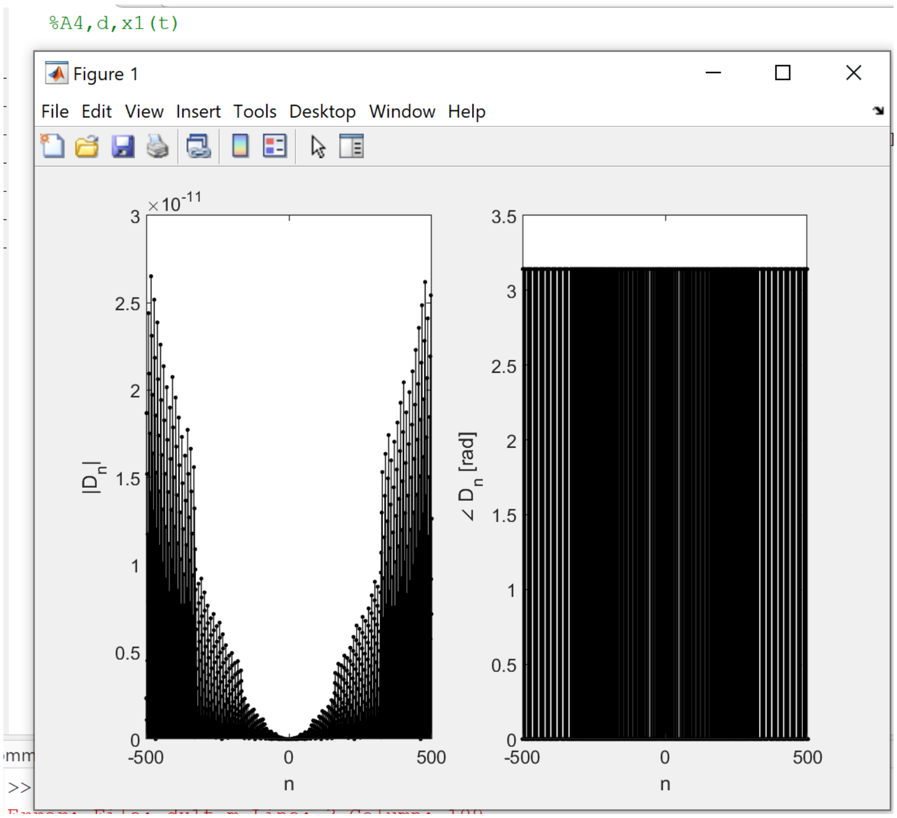

```
%A4,c x3(t)
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

%A4,c x3(t)




d)
%A4,d,x1(t)

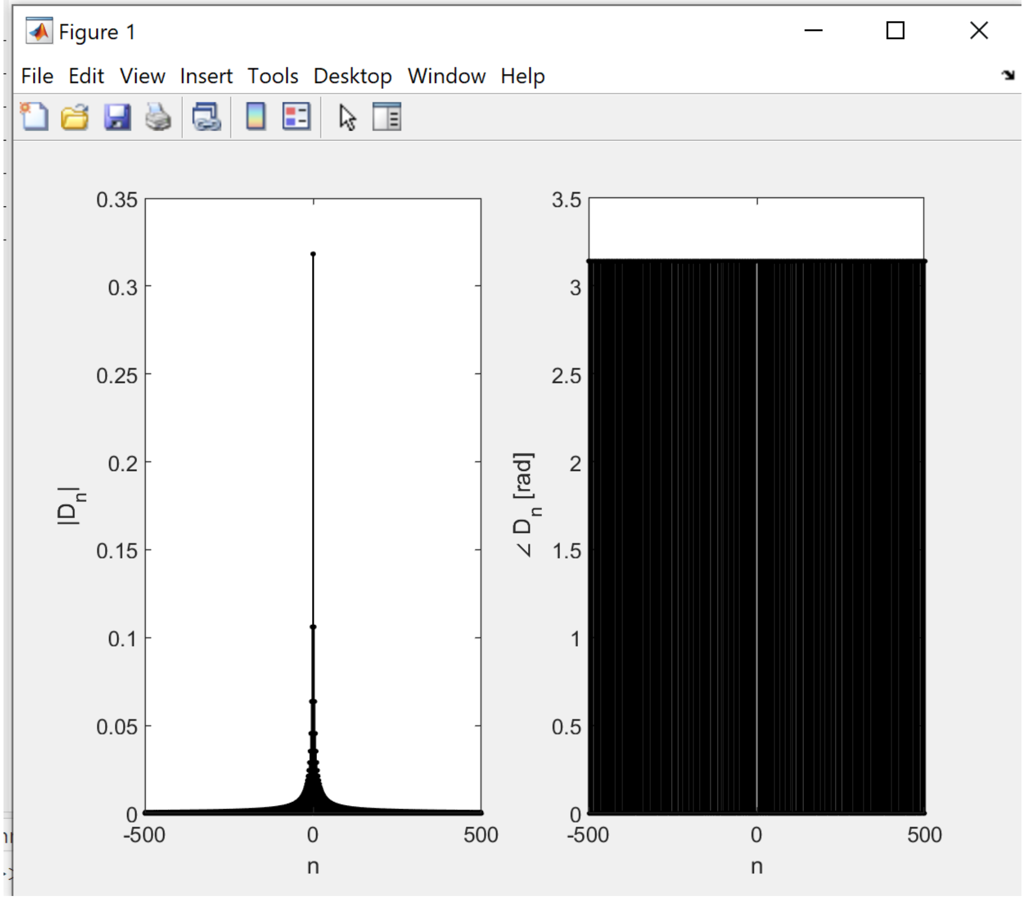
```
clf;
n = (-500:500);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi))
+(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```




%A4,d,x2(t)

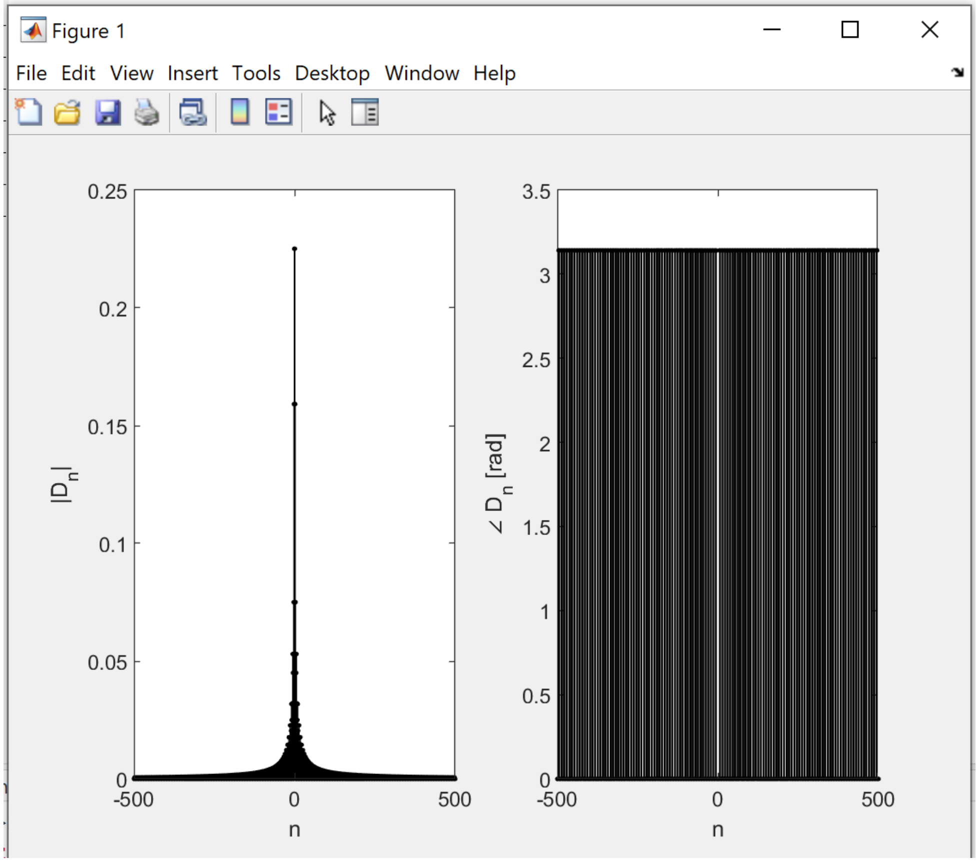
```
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

```
%A4,d,x2(t)
```




```
%A4,d, x3(t)

clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

```
%A4,d, x3(t)
```



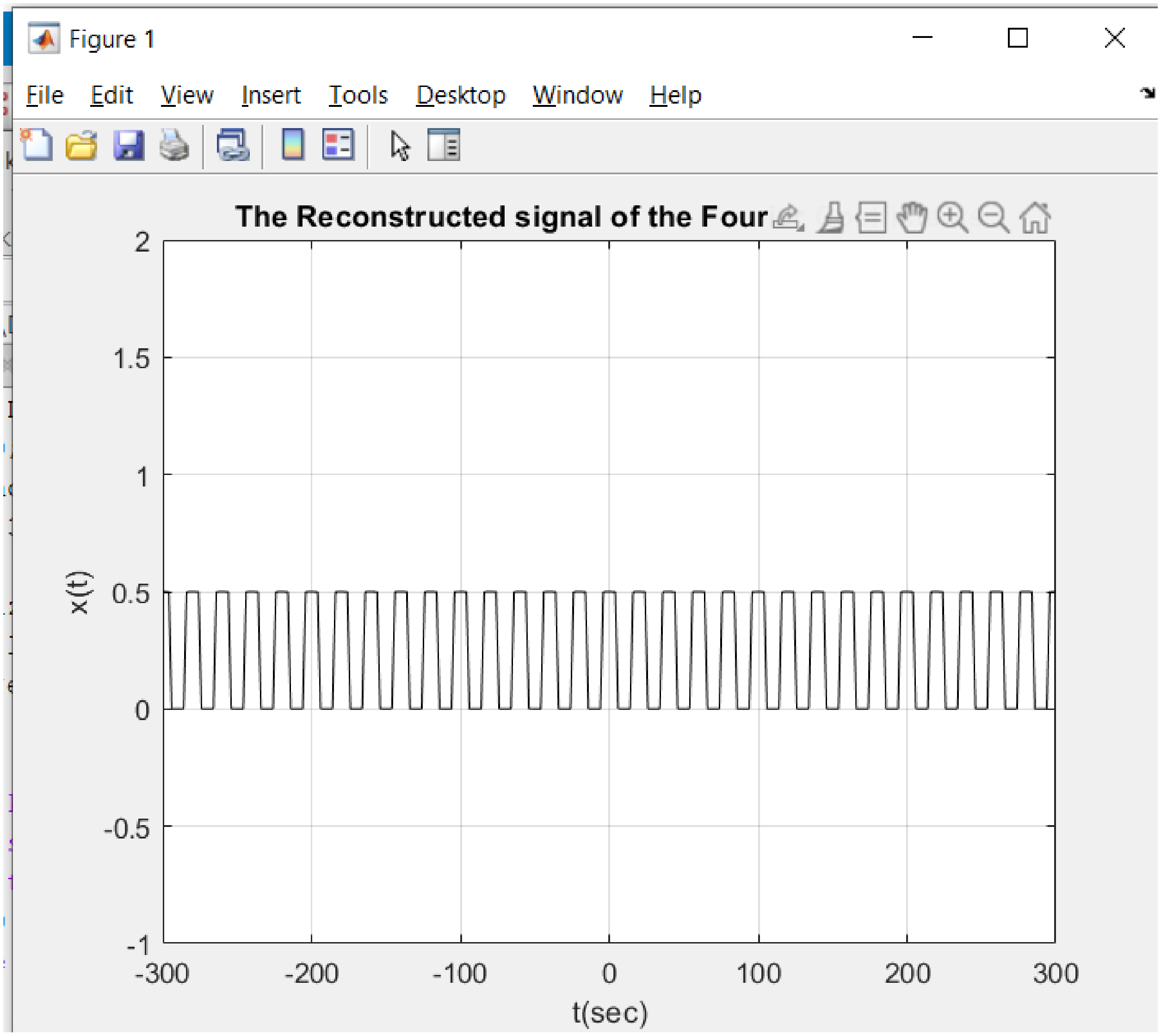

A.5

Sample for x3(t) D in line 3 is D_n

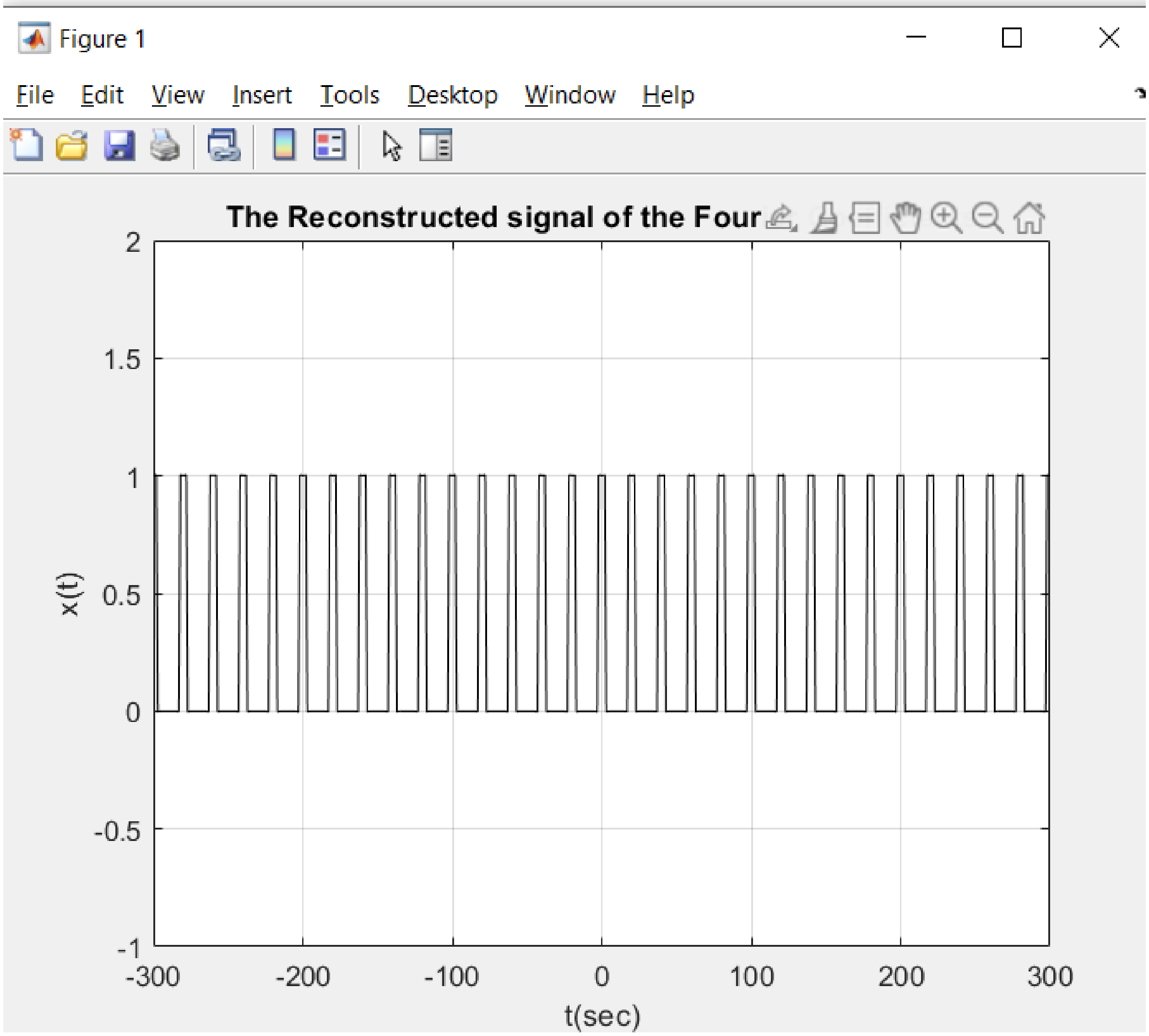
```
function [D] = a5
n=-500:500;
D=0.25*sinc(n/4);
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
 x=x+D(i)*exp(1i*n(i)*w*t);
 't';
end
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('The Reconstructed signal of the Fourier Coefficients');
Grid;
```

A.6
x1(t)

x2(t)




x3(t)




B.1

$$x_1(t) = cos\left(\frac{3\pi}{10}\right)t + \frac{1}{2}cos\left(\frac{\pi}{10}\right)t$$

$$w_{01} = \frac{3\pi}{10},\ w_{02} = \frac{\pi}{10}$$

$$w_0 = \frac{greatest\ common\ factor\ of\ 3\ and\ 1}{least\ common\ factor\ of\ 10\ and\ 10} = \frac{1}{10}$$

$$w_0 = \frac{\pi}{10}$$

For $x_2(t) => \ T_0 = \ 20sec$

$w_0 = \ \frac{\pi}{10}$ = 0.314rad/sec

For $x_3(t) => \ T_0 = \ 40sec$

$w_0 = \ \frac{\pi}{20}$ = 0.157rad/sec

B.2

The main difference between $x_1(t)$ and $x_2(t)$ is that one function has sinc function and has only four distinct fourier coefficient and the other has regular sin function and has infinite fourier coefficients for $D_n$.

B.3

Signal $x_2(t)$ has bigger fundamental frequency values when compared to $x_3(t)$ in terms of fourier coefficients.

B.4

$x_1(t) = \ cos\left(\frac{3\pi}{10}\right) t \ + \frac{1}{2}cos\left(\frac{\pi}{10}\right) t$

For $x_4(t) => \ 60sec$

$D_0$ for $x_4(t)$is $\frac{1}{2}$

$D_0 = \frac{1}{2}$ for every $x_n(t)$where n = 1,2,3..... ∞. Derived form $x_2(t)$

B.5

In $x_1(t)$ the reconstructed signal will not change because it has only four fourier coefficients and for $x_2(t)$ and $x_3(t)$ have an infinite number of fourier coefficients, So the more you increase the number of coefficients the perfect the signal becomes.

B.6

As we know that for $x_1(t)$ has only four fourier coefficients, these four coefficients are enough to perfectly reconstruct the signal. But for $x_2(t)$ and $x_3(t)$These Functions have infinite fourier coefficients so we need infinite coefficients to perfectly reconstruct the signal.

B.7

In the signal produced there are infinite fourier coefficients and they can't be stored in matlab, So this is not a viable scenario. Having finite coefficients helps in reconstructing the signal and this is a viable scenario.