Indian Institute of Technology Delhi
Center for Applied Research in Electronics (CARE)
Sensor Array Signal Processing (CRL 704)
Instructor: Dr. Neel Kanth Kundu
Spring 2023-24 Assignment 1

Due Date: 2-Feb-2024 (11.59 pm) Total Points: 55

Submission Instructions: Submit a report with the plots and your explanations for the questions. Additionally, submit the MATLAB codes for the questions. Ensure that all the dependencies of the MATLAB codes are included such that the plots are reproduced by running the corresponding `.m` files. Name the MATLAB files such that it is easy to identify which question it refer to. Upload a single zip file containing the pdf report and MATLAB codes to Moodle.

1. **DTFT Computations using Two–Sided Sequences** In this exercise, we consider the DTFT of two–sided sequences (including autocovariance sequences) and understand some basic properties of autocovariance sequences.

   (a) We first consider how to use the DTFT to determine $\phi(\omega)$ from $r(k)$ on a computer. We are given an ACS:

   $$r(k) = \begin{cases} \frac{M-|k|}{M}, & |k| \leq M \\ 0, & \text{otherwise} \end{cases}$$

   Generate and plot $r(k)$ for $M = 10$. Now, in MATLAB form a vector x of length $L = 256$ as:

   $$\mathrm{x} = [r(0), r(1), \ldots, r(M), 0 \ldots, 0, r(-M), \ldots, r(-1)]$$

   Verify that `xf = fft(x)` gives $\phi(\omega_k)$ for $\omega_k = 2\pi k/L$. (Note that the elements of `xf` should be nonnegative and real.). Explain why this particular choice of `x` is needed, citing appropriate circular shift and zero padding properties of the DTFT.
   Note that `xf` often contains a very small imaginary part due to computer round-off error; replacing `xf` by `real(xf)` truncates this imaginary component and leads to more expected results when plotting. **(5)**

   (b) Alternatively, since we can readily derive the analytical expression for $\phi(\omega)$, we can instead work backwards. Form a vector

   $$\mathrm{yf} = [\phi(0), \phi(2\pi/L), \phi(4\pi/L), \ldots, \phi((L-1)2\pi/L)]$$

   and find `y=ifft(yf)`. Verify that `y` closely approximates the ACS. **(5)**

   Hint: The `fft` command results in spectral estimates from 0 to 2π instead of the more common range of $-\pi$ to π. The MATLAB command `fftshift` can be used to exchange the first and second halves of the `fft` output to correspond to a frequency range of $-\pi$ to π. Similarly, `fftshift` can be used on the output of the `ifft` operation to "center" the zero-lag of an ACS. Experiment with `fftshift` to achieve both of these results.

Use the command `plot (w, fftshift (fft(x)))` with frequency vector `w` = `pi` $*$ `(-(L-1)/L : 2/L : (L-1)/L)` (for odd series length), and `w` = `pi` $*$ `(-1 : 2/L : (L-1)/L)` (for even series length) in order to get the spectral values at the proper frequencies. Similarly, plot the ACS with `stem(t,fftshift(ifft(xf)))` with time vector `t=-(L-1)/2:(L-1)/2` (for odd series length), and `t=-L/2:L/2-1` to get a proper plot of ACS.

Tools for Periodogram Spectral Estimation: The following MATLAB functions are provided for computing periodogram-based spectral estimates. In each case, `y` is the input data vector, `L` controls the frequency sample spacing of the output, and the output vector `phi` $= \phi(\omega_k)$ where $\omega_k = \frac{2\pi k}{L}$. MATLAB functions that generate the Correlogram, Blackman-Tukey, Windowed Periodogram, Bartlett, Welch, and Daniell spectral estimates are as follows:

- `phi` = `correlogramse(y, L)` Implements the correlogram spectral estimate.
- `phi` = `btse(y, w, L)` Implements the Blackman-Tukey spectral estimate; `w` is the vector $[w(0), \ldots, w(M-1)]^T$.
- `phi=periodogramse (y,v,L)` Implements the windowed periodogram spectral estimate; `v` is a vector of window function elements $[v(1), \ldots, v(N)]^T$, and should be the same size as `y`. If `v` is a vector of ones, this function implements the unwindowed periodogram spectral estimate.
- `phi=bartlettse (y,M,L)` Implements the Bartlett spectral estimate; `M` is the size of each subsequence.
- `phi` = `welchse(y, v, K, L)` Implements the Welch spectral estimate; `M` is the size of each subsequence, `v` is the window function $[v(1), \ldots, v(M)]^T$ applied to each subsequence, and `K` is the overlap parameter.
- `phi= daniellse (y,J,Ntilde)` Implements the Daniell spectral; `J` and `Ntilde` correspond to J and $\tilde{N}$.

2. **Zero Padding Effects on Periodogram Estimators**: Consider the sequence

$$y(t) = 10\sin(0.2 \cdot 2\pi t + \phi_1) + 5\sin((0.2 + 1/N)2\pi t + \phi_2) + e(t),$$

where $t = 0, \ldots, N-1$, and $e(t)$ is white Gaussian noise with variance 1. Let $N = 64$ and $\phi_1 = \phi_2 = 0$.

   (a) Plot the periodogram for the sequence $\{y(t)\}$, and the sequence $\{y(t)\}$ zero padded with $N, 3N, 5N$, and $7N$ zeroes. **(5)**

   (b) Explain the difference between the five periodograms. Why does the first periodogram not give a good description of the spectral content of the signal? **(5)**

3. **Resolution and leakage properties of periodogram**

   (a) Plot the frequency response (magnitude in dB) of Bartlett and rectangular window for different values of $M = 64, 128, 256, 512$. Comment on the 3 dB bandwidth of the windows. Hint: Use `freqz` function of MATLAB. **(5)**

   (b) Consider

$$y(t) = a_1 \sin(f_0 \cdot 2\pi t + \phi_1) + a_2 \sin((f_0 + \alpha/N)\, 2\pi t + \phi_2) + e(t)$$

where $e(t)$ is real-valued Gaussian white noise with zero mean and variance σ^2. Let $f_0 = 0.2$, $N = 256$, $\sigma^2 = 0$. Plot the unwindowed periodogram and Blackman-Tukey estimate with Bartlett window with window size $M = N$ for different values of $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9$. (Plot the spectral estimates in dB on the y-axis as a function of frequency f in Hz on the x-axis). Determine the threshold α for which the two frequencies of the signal can be resolved for different PSD estimators. Hint: For better visualization, zoom the PSD estimate plots around f_0. **(5)**

(c) Generate the sinusoidal sequence given in 3(b) for $\alpha = 4, \sigma^2 = 0$, and $\phi_1 = \phi_2 = 0$. Set $a_1 = 1$ and vary a_2 (choose $a_2 = 1, 0.1, 0.01$). As before, plot the unwindowed periodogram and Blackman-Tukey estimate with Bartlett window with window size $M = N$ for different values of $a_2 = 1, 0.1, 0.01$. Comment on the ability to identify the second sinusoidal term from the spectral estimate. Qualitatively, which PSD estimator is better and why? **(5)**

4. **Periodogram–Based Estimators applied to Measured Data**:

Consider the data sets in the files `sunspotdata.mat` and `lynxdata.mat`. Apply periodogram–based estimation techniques to estimate the spectral content of these data. Use the following estimators: (i) Periodogram with rectangular temporal window , (ii) Periodogram with Bartlett temporal window, (iii) Blackman-Tukey with Bartlett window ($M = N/4$), (iv) Bartlett method ($M = N/4$), (v) Welch method with rectangular temporal window ($M = N/2,\ K = N/2$) and (vi) Daniell method ($J = 4,\ \tilde{N} = 16N$). Plot the PSD estimates for all the estimators in dB on the y-axis as a function of frequency f on the x-axis (show the plots for $0 \leq f \leq 0.5$) **(10)**

Now answer the following questions:

(a) Are there sinusoidal components (or periodic structure) in the data? If so, how many components and at what frequencies? **(5)**

(b) Nonlinear transformations and linear or polynomial trend removal are often applied before spectral analysis of a time series. For the lynx data, compare your spectral analysis results from the original data, and the data transformed first by taking the logarithm of each sample and then by subtracting the sample mean of this logarithmic data. Does the logarithmic transformation make the data more sinusoidal in nature? **(5)**