Indian Institute of Technology Delhi
Center for Applied Research in Electronics (CARE)
Sensor Array Signal Processing (CRL 704)
Instructor: Dr. Neel Kanth Kundu
Spring 2023-24 Assignment 2

Due Date: 22-Apr-2024 (11.59 pm) Total Points: 45

Submission Instructions: Submit a report with the plots and your explanations for the questions. Additionally, submit the MATLAB codes for the questions. Ensure that all the dependencies of the MATLAB codes are included such that the plots are reproduced by running the corresponding `.m` files. Name the MATLAB files such that it is easy to identify which question it refer to. Upload a single zip file containing the pdf report and MATLAB codes to Moodle.

Tools for Periodogram Spectral Estimation: The following MATLAB functions are provided for use in DOA estimation:

- Y = `uladata(theta,` P, N, `sig2, m, d)` Generates an $m \times N$ data matrix $Y = [y(1), \ldots, y(N)]$ for a ULA with n sources arriving at angles (in degrees from -90° to 90°) given by the elements of the $n \times 1$ vector theta. The source signals are zero mean Gaussian with covariance matrix $P = E\{s(t)s^*(t)\}$. The noise component is spatially white Gaussian with covariance $\sigma^2 I$, where $\sigma^2 =$ `sig2`. The element spacing is equal to `d` in wavelengths.

- `phi=beamform (Y,L,d)` Implements the beamforming spatial spectral estimate in equation (6.3.18) of the textbook for an m-element ULA with sensor spacing d in wavelengths. The $m \times N$ matrix Y is as defined above. The parameter L controls the DOA sampling, and phi is the spatial spectral estimate phi $= \left[\hat{\phi}(\theta_1), \ldots, \hat{\phi}(\theta_L)\right]$ where $\theta_k = -\frac{\pi}{2} + \frac{\pi k}{L}$.

- `phi=capon_sp(Y,L,d)` Implements the Capon spatial spectral estimator in equation (6.3.26) of the textbook; the input and output parameters are defined as those in `beamform`.

- `theta=root_music_doa(Y,n,d)` Implements the Root MUSIC method in Section 4.5 of the textbook, adapted for spatial spectral estimation using a ULA. The parameters `Y` and `d` are as in `beamform`, and theta is the vector containing the `n` DOA estimates $\left[\hat{\theta}_1, \ldots, \hat{\theta}_n\right]^T$.

- `theta=esprit_doa(Y,n,d)` Implements the ESPRIT method for a ULA. The parameters `Y` and `d` are as in `beamform`, and theta is the vector containing the n DOA estimates $\left[\hat{\theta}_1, \ldots, \hat{\theta}_n\right]^T$. The two subarrays for ESPRIT are made from the first $m-1$ and last $m-1$ elements of the array.

1. **Comparison of Spatial Spectral Estimators**: Simulate the following scenario. Two signals with wavelength λ impinge on an array of sensors from DOAs $\theta_1 = 0^\circ$ and a θ_2 that will be varied. The signals are mutually uncorrelated complex Gaussian with unit power, so that $P = E\{s(t)s^*(t)\} = I$. The array is a 10-element ULA with element spacing $d = \lambda/2$. The measurements are corrupted by additive complex Gaussian white noise with unit power. A total of $N = 100$ snapshots are collected.

(a) Let $\theta_2 = 15°$. Compare the results of the beamforming, Capon, Root MUSIC, and ESPRIT methods for this example. The results can be shown by plotting the spatial spectrum estimates from beamforming and Capon for 50 Monte-Carlo experiments; for Root MUSIC and ESPRIT, plot vertical lines of equal height located at the DOA estimates from the 50 Monte-Carlo experiments. How do the methods compare? Are the properties of the various estimators analogous to the time series case for two sinusoids in noise? **(10)**

(b) Repeat for $\theta = 7.5°$ **(10)**

2. **Performance of Spatial Spectral Estimators for Coherent Source Signals**: **(10)**

In this exercise, we will see what happens when the source signals are fully correlated (or coherent). Use the same parameters and estimation methods as in Q1 with $\theta_2 = 15°$, but with

$$P = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Note that the sources are coherent as $\text{rank}(P) = 1$. Compare the results of the four methods for this case, again by plotting the spatial spectrum and "DOA line spectrum" estimates (as in Q1) for 50 Monte-Carlo experiments from each estimator. Which method appears to be the best in this case?

3. **Spatial Spectral Estimators applied to Measured Data**: **(15)**

Apply the four DOA estimators from Q1 to the real data in the file `submarine.mat`. These data are underwater measurements collected by the Swedish Defense Agency in the Baltic Sea. The 6–element array of hydrophones used in the experiment can be assumed to be a ULA with inter-element spacing equal to 0.9 m. The wavelength of the signal is approximately 5.32 m. Can you find the "submarine(s)"?