Single and Multiple Relay Selection Schemes and their Achievable Diversity Orders

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Abstract—This paper is on relay selection schemes for wireless relay networks. First, we derive the diversity of many single-relay selection schemes in the literature. Then, we generalize the idea of relay selection by allowing more than one relay to cooperate. The SNR-optimal multiple relay selection scheme can be achieved by exhaustive search, whose complexity increases exponentially in the network size. To reduce the complexity, several SNR-suboptimal multiple relay selection schemes are proposed, whose complexity is linear in the number of relays. They are proved to achieve full diversity. Simulation shows that they perform much better than the corresponding single relay selection methods and very close to the SNR-optimal multiple relay selection scheme. In addition, for large networks, these multiple relay selection schemes require the same amount of feedback bits from the receiver as single relay selection schemes.

Index Terms—Wireless relay networks, cooperative diversity, relay selection.

I. INTRODUCTION AND NETWORK MODEL

COOPERATIVE diversity is basically the improvement in capacity or reliability of a wireless network by having the mobile nodes in the network help each other's communication task. It is one of the most effective ways to mitigate the fading effect of wireless channels in a network. Recently, with the increasing interests in wireless network communication, research in cooperative diversity attracts considerable attention.

A variety of cooperative schemes have been proposed in the literature with different design issues and channel information assumptions [1]–[20]. Among these schemes, two of the most famous ones are amplify-and-forward (AF) and decode-and-forward (DF). However, for most pioneer AF schemes, every relay cooperates and each relay uses its maximum transmit power (e.g., [2], [6], [7], [10]). For most DF schemes, whether a relay decodes only depends on the quality of the channel between that relay and the transmitter, and the decoding relays always use their maximum power.

On one hand, it is conceivable that better performance can be achieved if the relays adaptively adjust their transmit power according to the quality of the channels. There have been

some recent work on relay networks with adaptive power control [21]–[35]. In [24], [25], performance of networks with multiple parallel AF relays using the same channel and a short-term aggregate relay power constraint is analyzed. A more practical assumption is that each node has its own power constraint. In [36], [37], the idea of beamforming in networks with parallel AF relays and a separate short-term power constraint on each node is proposed. The optimal relay power control problem is solved analytically. Then, network beamforming schemes with partial channel information at relays through limited feedback are analyzed in [38], [39].

On the other hand, we can improve the network performance by selecting the cooperating relays globally. This is especially useful for networks with simple nodes and complexity constraints, in which the arbitrary power adjustment may not be practical or desirable. Compared to networks whose nodes are allowed arbitrary power adjustment, networks whose nodes are allowed a finite number of power levels only are more feasible and manageable considering the amount of overhead and feedback, the network life time and performance analysis, etc.. Each node in the network can choose one of its power levels to cooperate. In this paper, we focus on the simplest case. We assume that each relay has two power levels: zero and its maximum power, i.e., a relay either cooperates with its full power or does not cooperate at all. Thus, the problem is how to select the cooperating relays.

There exists a rich body of literature on relay selection [40]–[53]. While several relay selection schemes are proposed for both DF and AF networks, most of these schemes are on single relay selection, i.e., only one of the relays cooperates.

In this paper, we generalize the idea of single relay selection to multiple relay selection. We work on networks with parallel AF relays. It is assumed that each relay only knows its own channels but the receiver knows all channel values through training. Also, we make the practical assumption that each node has its own power constraint. We first derive the achievable diversity (in the sense of error rate) of some existing single relay selection schemes. Then, we work on multiple relay selection schemes. The SNR-maximizing (and thus error-rate-minimizing) multiple relay selection can be achieved through an exhaustive search over all the relays. However, its complexity is exponential in the network size. Thus, we propose multiple SNR-suboptimal (and thus error-rate-suboptimal) relay selection schemes based on some relay ordering functions. These schemes achieve full diversity and low error rates. In addition, they have linear complexity and perform close to the optimal scheme. The number of cooperating relays of our schemes varies with the channel
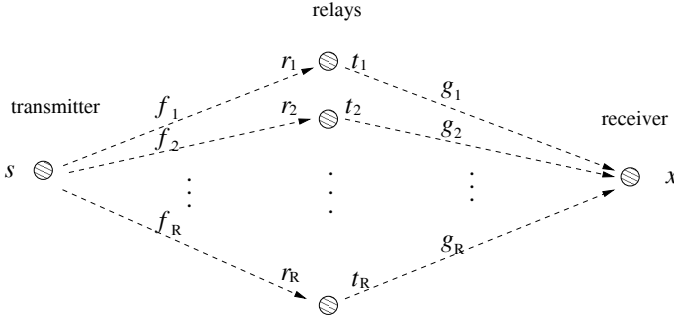
relays

Fig. 1. Wireless relay network.

values. However, unlike the selection DF in [10], in our schemes, whether a relay cooperates depends on not only its own channels but also all others. Also, unlike the scheme in [48], all relays share the same communication channel.

The paper is organized as follows. In the following subsection, the network model is described in details. In Section II, we review some existing single relay selection schemes and show their diversity orders. Then in Section III, the idea of multiple relay selection is proposed and several suboptimal schemes are introduced. Section IV contains the simulation results. Conclusions of this paper are highlighted in Section V. Some useful results in random process and technical proofs are given in the appendixes.

A. Network Model and Problem Statement

Consider a relay network with one transmit-and-receive pair and R relays as depicted in Fig. 1. Every relay has only one antenna which can be used for both transmission and reception. Denote the channel from the transmitter to the ith relay as f_i and the channel from the ith relay to the receiver as g_i. There is no direct link between the transmitter and the receiver. However, our results can be applied to the case with a direct link straightforwardly. Assume that the ith relay knows its own channels f_i and g_i, and the receiver knows all channels $f_1, \ldots, f_R$ and $g_1, \ldots, g_R$. Although the schemes proposed in this paper are valid for any channel statistics, for the diversity analysis, it is assumed that all channels are normalized i.i.d. Rayleigh random variables.

For each transmission, the power used at the transmitter and the power used at the ith relay are no larger than P and P_i, respectively. Note that in this paper, only short-term power constraint is considered, that is, there is an upper bound on the average transmit power of each node for each transmission. A node cannot save its power to favor transmissions with better channel realizations. When there is a transmission task between the transmitter and receiver, a relay either cooperates with its full power or does not cooperate at all.

A two-step AF protocol is used to send information. During the first step, the transmitter sends $\sqrt{P}s$ where the information symbol s is selected randomly from a codebook. For a complex number a, we use $|a|$ and $\arg a$ to denote its magnitude and phase, respectively. If we normalize s as $\mathrm{E}|s|^2 = 1$, the average power used at the transmitter is P. During the second step, the ith relay scales its received signal by $\frac{\alpha_i \sqrt{P_i} e^{j\theta_i}}{\sqrt{1+|f_i|^2 P}}$ (so that its transmit power is $\alpha_i^2 P_i$) and forwards it to the receiver.

It is assumed that the relays are perfectly synchronized and transmit at the same time. The angle θ_i is used to adjust the phase of the signal. It is obvious that an optimal choice is $\theta_i = -(\arg f_i + \arg g_i)$. Thus, the receiver gets

$$x = \sqrt{P} \sum_{i=1}^{R} \frac{\alpha_i |f_i g_i| \sqrt{P_i}}{\sqrt{1+|f_i|^2 P}} s + \sum_{i=1}^{R} \frac{\alpha_i |g_i| \sqrt{P_i}}{\sqrt{1+|f_i|^2 P}} v_i + w, \quad (1)$$

where w is the noise at the receiver and $v_i = w_i e^{-\arg f_i}$ with w_i the noise at the ith relay. The noises are assumed to be i.i.d. complex Gaussian random variables with zero-mean and unit-variance. It is easy to see that v_i and w_i have the same distribution. The average receive SNR can be calculated to be

$$SNR_{general} = \frac{P \left(\sum_{i=1}^{R} \frac{\alpha_i |f_i g_i| \sqrt{P_i}}{\sqrt{1+|f_i|^2 P}} \right)^2}{1 + \sum_{i=1}^{R} \frac{\alpha_i^2 |g_i|^2 P_i}{1+|f_i|^2 P}}. \quad (2)$$

Our multiple relay selection problem is thus the following:

$$\max_{\alpha_1, \ldots, \alpha_R} SNR_{general} \quad \text{s.t.} \quad \alpha_i \in \{0, 1\}. \quad (3)$$

Since the receiver has full knowledge of all channels, this maximization problem is equivalent to the maximization of the capacity and the minimization of the error rate. This problem formulation is similar to our work in [36]. But, here, the relays are not allowed to adjust their transmit powers arbitrarily. Instead, each relay has only two choices: to cooperate with full power or not to cooperate at all.

In (3), we have generalized the idea of relay selection to allow more than one relay to cooperate. Although most pioneer cooperative diversity protocols require all relays to cooperate and most relay selection schemes allow only one relay to cooperate, there are a couple of papers on multiple relay selection [10], [48]. The selection DF protocol proposed in [10] allows a relay to cooperate if the magnitude of its backward channel (the channel with the transmitter) exceeds a fixed threshold. This paper focuses on multiple relay selection schemes that maximize the receive SNR. In our case, unlike selection DF, whether a relay cooperates depends on not only its backward channel but the channels of all relays. Our work is different from [48] in two ways. First, the relays in our network model share the same channel while the relays in [48] use orthogonal channels. Second, [48] considers the receive SNR maximization under a total transmit power constraint and the minimization of the total transmit power under a receive SNR constraint. In our paper, we work on the receive SNR maximization while each relay is given an individual power constraint. Note that with our schemes, the set of cooperative relays varies and thus the total transmit power varies with the channel realizations.

II. SINGLE RELAY SELECTION SCHEMES AND THEIR DIVERSITY ORDERS

In this section, we review some existing single relay selection schemes in the literature. As the diversity orders of many of these schemes is unknown, our main contribution in this section is the diversity derivation. In the diversity derivation, for simplicity of the presentation, we assume that all nodes in the network have the same power constraint, i.e.,

$P = P_1 = \cdots = P_R$, and analyze how the error rate scales with P. For the heterogeneous case, the same diversity result can be obtained while $P, P_1, \ldots, P_R$ have the same scaling, i.e., P_i/P is a constant while P increases.

Denote the receive SNR given that only relays in a set A cooperate as SNR_A. If only the ith relay cooperates, the receive SNR of the network can be calculated to be

$$SNR_{\{i\}} = \frac{|f_i g_i|^2 P P_i}{1 + |f_i|^2 P + |g_i|^2 P_i}. \tag{4}$$

A. Best Relay Selection

In [39], [50]–[52], the relay whose path has the maximum SNR is selected. This is obviously the optimal single relay selection scheme. The error rate of this scheme is first discussed in [52], in which an approximation on the cumulative density function of the receive SNR is used. Then, a rigorous upper bound on the error rate of this scheme is given in [39]. Both papers show that this scheme achieves a diversity order of R.

B. Nearest Neighbor Selection

In [40], [47], the nearest neighbor selection is proposed, in which the relay that is the nearest to the base station cooperates. In both papers, DF is used and node spatial positions are considered. In our AF network with perfect channel information, this scheme means choosing the relay with the largest $P|f_i|^2$ or $P_i|g_i|^2$. Note that we have slightly modified the original scheme to incorporate different relay power constraints. "The nearest relay" is not necessarily the *spatially* nearest relay to the transmitter or receiver, but the relay with the strongest channel to the transmitter or receiver.

Theorem 1. *The diversity order of the nearest neighbor relay selection is 1.*

Proof: See Appendix B.　■

Theorem 1 indicates that for full diversity, we need channel information of both transmission steps to select relays.

C. Best Worse Channel Selection

For dual-hop protocols, each relay has two channels, the channel from the transmitter to the relay and the channel from the relay to the receiver. In [54] and [47], the best worse channel selection is used, in which the relay whose worse channel, $\min\{|f_i|, |g_i|\}$, is the best is selected. The diversity-multiplexing trade-off of this scheme is analyzed in [54], in which the diversity analysis is based on the outage probability. To incorporate different relay power constraints, we modify the selection function to be $\min\{P|f_i|^2, P_i|g_i|^2\}$. We work on the error rate and prove that the best worse channel selection achieves full diversity.

Theorem 2. *The best worse channel selection achieves the full diversity order of R.*

Proof: See Appendix C.　■

D. Best Harmonic Mean Selection

In [54] and [53], the best harmonic mean selection is proposed, in which the relay selection function is chosen as the harmonic mean of the two channels' magnitudes: $\left(|f_i|^{-2} + |g_i|^{-2}\right)^{-1}$. The relay with the largest harmonic mean cooperates. Symbol error rate of this scheme is analyzed in [53]. However, the derivation is not rigorous since an upper bound on the receive SNR is used. In this paper, we first modify the selection function to be $\left(\frac{1}{P|f_i|^2} + \frac{1}{P_i|g_i|^2}\right)^{-1}$ to incorporate the different relay powers, then give a rigorous upper bound on the symbol error rate.

Theorem 3. *The best harmonic mean selection achieves the full diversity order of R.*

Proof: See Appendix D.　■

III. MULTIPLE RELAY SELECTION SCHEMES

In this section, we work on the multiple relay selection problem in (3). Due to the relay coupling, the SNR function is a nonlinear function of α_i. Thus, our problem is a general nonlinear 0-1 programming.[1] Linear integer programming is known to be NP-complete. Nonlinear integer programming is certainly harder than the linear case.

As there are R relays and each relay has two choices, there are $2^R - 1$ possibilities (the case that no relay cooperates is obviously not optimal). Since the receiver knows all the channels, it can always find the optimal solution by exhaustive search. But the computational complexity of this exhaustive scheme is exponential in R, and the number of required feedback bits is R since one bit information for each relay is needed. For networks with a large number of relays, having the amount of feedback bits linear in the network size is undesirable. The real challenge of the problem is to find multiple relay selection schemes with low complexity (for example, linear in R), good performance, and, at the same time, a small amount of feedback bits.

A. The Idea of Relay Ordering

In our network beamforming paper [36], a relay ordering is proposed, based on which we can find the relays that use their full power sequentially. Similarly, in this work, if there is a relay ordering associated with the issue of to cooperate or not, the multiple relay selection problem can be solved with linear complexity. Actually, with such an ordering, we only need to find the number of cooperating relays.

Definition 1 (Optimal relay ordering and optimal relay ordering function)**.**

- *An ordering $(\tau_1, \ldots, \tau_R)$ of $(1, \ldots, R)$ is called an* optimal relay ordering *associated with a relay action if for any $i < j$ Relay τ_i has a higher priority to Relay τ_j for this action. In other words, if Relay τ_i should not do the associated relay action, then Relay τ_j should not do it either.*
- *A function $h(f, g, P)$ is called an* optimal relay ordering function *if it can induce the optimal relay ordering, i.e.,*

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[1]If the relays use orthogonal channels as in [48], the optimization problem becomes a linear 0-1 knapsack problem.

if we order $h_i = h(f_i, g_i, P_i)$ as $h_{\tau_1} > \cdots > h_{\tau_R}$, then $(\tau_1, \ldots, \tau_R)$ is the optimal relay ordering.

In [36], for the relay action: to cooperate with full power given that any fractional power is allowed, an optimal relay ordering function exists and is proved to be

$$h(f_i, g_i, P_i) = \frac{|f_i|\sqrt{1 + |f_i|^2 P}}{|g_i|\sqrt{P_i}}. \tag{5}$$

In this paper, the relay action is to cooperate with full power given that the alternative is not to cooperate. Assume that an optimal relay ordering exists. Without loss of generality, assume it is $(1, \ldots, R)$. To solve the multiple relay selection problem, we only need to find the largest value among $SNR_{\{1\}}, SNR_{\{1,2\}}, \ldots, SNR_{\{1,\ldots,R\}}$, whose complexity is linear. Actually for any relay ordering, this selection scheme can be used and the complexity is linear. But the network performance may not be optimal.

The relay ordering idea we proposed here to solve the problem in (3) is similar to the greedy approximation algorithm for knapsack problems [55]. But, our ordering definition is more general.

To use multiple relay selection schemes in a distributive network, feedback from the receiver is needed as each relay only knows its own channels. Another attractive feature of relay ordering is that they allow the required number of feedback bits kept fixed while the number of relays increases. To show this, without loss of generality, assume that $(1, \ldots, R)$ is a relay ordering and h is its ordering function. For simplicity of presentation, let $h_{R+1} = -\infty$. Our relay selection scheme based on h is as follows:

1) The receiver finds the k such that $SNR_{\{1,\ldots,k\}}$ is the largest among $SNR_{\{1\}}, \ldots, SNR_{\{1,\ldots,R\}}$.
2) The receiver sends h_{th} whose value is between h_k and h_{k+1}.
3) Relay i compares its h_i with h_{th}. If $h_i > h_{th}$, it cooperates with full power. Otherwise, it does not cooperate.

The number of feedback bits is thus fixed as one real number is broadcasted. Another scheme is to feed back one bit for each relay indicating whether it cooperates or not. The number of feedback bits is thus R. For small values of R, the second scheme is preferred while when R is large, the first scheme is more efficient. The amount of feedback bits is thus $\min\{B, R\}$, with B the required number of bits to represent a real number.

Single relay selections can use similar feedback schemes. For the second scheme, since it is known that only one relay cooperates, the receiver can feed back the index of that relay only and $\log R$ bits are enough. Thus, the required amount of feedback bits is $\min\{B, \log R\}$, which is the same as multiple relay selection schemes when $\log R \geq B$.

The feedback real number h_{th} works as a threshold. Note that it varies for each channel realization and depends on all channel values. In selection DF [10], a threshold on the quality of relays' backward channels is set to decide whether a relay cooperates. But that threshold is a constant for all channel realizations and is only a function of the transmission rate.

TABLE I
RELAY SELECTION RESULTS OF THE COUNTER EXAMPLE OF THEOREM 4

Number of cooperating relays i	S_i	Receive SNR
1	{3}	1.7229
2	{1,2}	5.2786
3	{1,2,3}	6.4663

B. Multiple Relay Selection Schemes with Linear Complexity

Theorem 4. *For networks with more than 2 relays, for the relay action, to cooperate with full power while the alternative is not to cooperate, no optimal relay ordering exists.*

Proof: It is obvious that if there is no optimal relay ordering for 3-relay networks, there is no optimal relay ordering for networks with more than 3 relays. Thus, to prove this theorem, we only need to show that for 3-relay networks, there exists a specific channel realization such that the optimal relay ordering property cannot be satisfied. Denote S_i as the set of cooperating relays that results in the maximum receive SNR given that i relays cooperate. To prove this theorem, it is thus suffice to find an example such that for some $1 \leq i < j \leq 3$, S_i is not a subset of S_j. We find such an example by computer search. Let $P = P_1 = P_2 = P_3 = 10$dB and

$$\begin{aligned} f_1 &= 1.1139 + 0.5491i & g_1 &= -0.3987 - 0.0170i \\ f_2 &= 0.9367 + 0.0212i & g_2 &= -0.3851 - 0.2791i \\ f_3 &= 0.2923 + 0.3814i & g_3 &= -0.6378 - 0.7522i. \end{aligned}$$

Table I shows the relay selection results. S_1 is not a subset of S_2. ∎

In the following, we propose several relay ordering functions. Since no optimal relay ordering function exists, these relay ordering functions are suboptimal in the sense that the multiple relay selection schemes based on them are suboptimal in the receive SNR with respect to the SNR-optimal multiple relay selection scheme obtained by exhaustive search. We also give discussions on the diversity orders and array gains of the suboptimal multiple relay selection schemes.

- Function 1-The worse channel ordering:

$$h(f_i, g_i, P_i) = \min\{P|f_i|^2, P_i|g_i|^2\}.$$

- Function 2-The harmonic mean ordering:

$$h(f_i, g_i, P_i) = \left(\frac{1}{P|f_i|^2} + \frac{1}{P_i|g_i|^2} \right)^{-1}.$$

- Function 3-The SNR ordering:

$$h(f_i, g_i, P_i) = \frac{PP_i|f_i g_i|^2}{1 + P|f_i|^2 + P_i|g_i|^2}.$$

The receive SNRs of the above multiple relay selection schemes are always no smaller than their corresponding single relay selection schemes. We have shown in the previous section that the single relay selection schemes based on these relay ordering functions achieve full diversity. Therefore, all the three schemes provide full diversity and a larger coding gain compared to the corresponding single relay selection schemes.

C. A Multiple Relay Selection Scheme with Quadratic Complexity

There is a trade-off between the performance and computational complexity. In this subsection, we propose a relay selection scheme with a complexity that is quadratic in the number of relays but provides a performance better than those of the suboptimal schemes with linear complexity.

The scheme is based on recursion. Let $S_0 = \emptyset$. For a given number of cooperating relays i, denote S_i the set of i relays that we choose to cooperate. Let

$$S_{i+1} = S_i \cup \{j_i\}, \text{ where } j_i = \arg \max_{j \in \{1,\dots,R\} \setminus S_i} SNR_{S_i \cup \{j\}}.$$

Thus, the set of cooperating relays is

$$S_{qua} = \arg \max_{S_i, 1 \leq i \leq R} SNR_{S_i}.$$

In other words, for each step, we add one relay to the previous set. This relay is the one in $\{1, \dots, R\} \setminus S_i$ that results in the best receive SNR. Finally we search over the R sets obtained from the R steps and choose the one with the largest receive SNR. Note that S_i is always a subset of S_{i+1}. The number of computations in the ith step is $R - i + 1$. Thus the total complexity scales as

$$R + \sum_{i=1}^{R} (R - i + 1) = \frac{R(R+1)}{2} \sim R^2.$$

Note that S_1 contains the relay whose path has the best receive SNR. Thus, this scheme is at least as good as the best relay selection, which achieves full diversity. So, this scheme also achieves full diversity and has a larger array gain than the best relay selection. Simulation shows that it performs almost the same as the optimal multiple relay selection.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we show the simulated block error rates of the single and multiple relay selection schemes mentioned in this paper. In all simulations, the horizontal axis indicates the power of the transmitter P and the power of Relay i scales with P according to some constant. The information symbol s is modulated by BPSK. Thus, the shown block error rates are also bit error rates. All channels are generated as i.i.d. complex Gaussian random variables with zero-mean and unit-variance. The noises at the relays and receiver are also i.i.d. complex Gaussian random variables with zero-mean and unit-variance.

In Fig. 2, we show performance of single relay selection schemes discussed in Section II in 2-relay and 3-relay networks. All nodes have the same power constraint. We can see that the best relay selection, the best harmonic mean selection, and the best worse selection all achieve the full diversity order of R, while the nearest neighbor selection has the diversity order of 1 only. These are consistent with our analytical diversity results in Theorems 2 and 3. Note that since all relays have the same power constraint, due to symmetry, the two nearest neighbor selection schemes perform the same. Unsurprisingly, the best relay selection performs the best but its superiority to the best harmonic mean selection and the best worse selection is negligible. For comparison, performance of network beamforming in [36] is also shown. Network
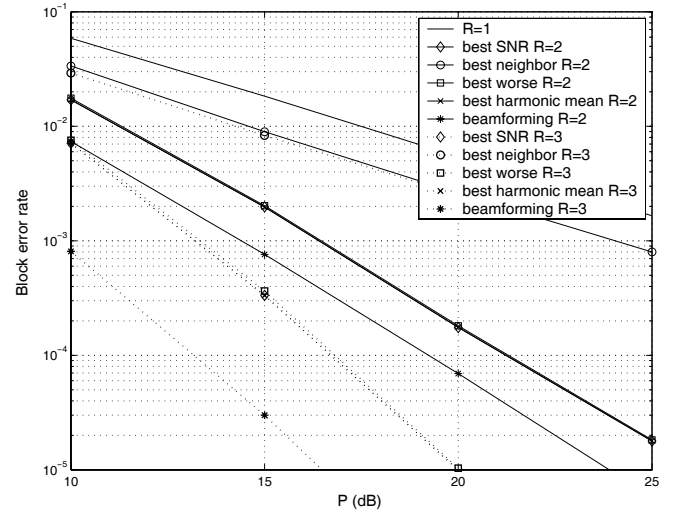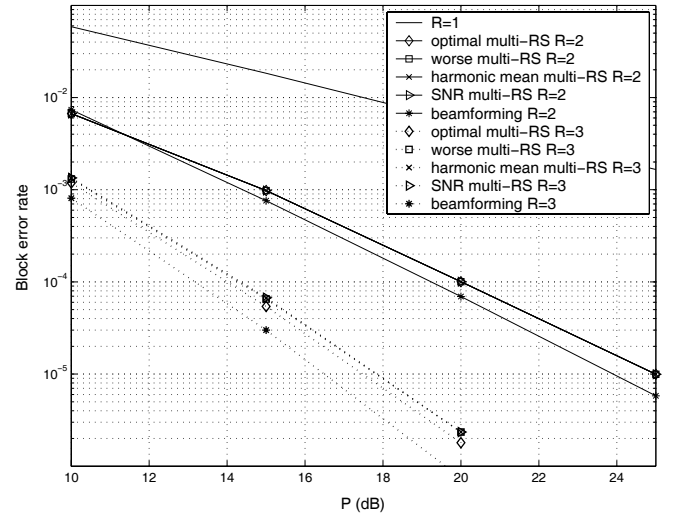

Fig. 2.   Single relay selection schemes.

Fig. 3.   Multiple relay selection schemes.

beamforming requires arbitrary relay power adjustment. We can see that compared to network beamforming single relay selection schemes are about 2dB worse for 2-relay networks and 4dB worse for 3-relay networks.

Fig. 3 shows the performance of the multiple relay selection schemes we proposed in Section III for 2-relay and 3-relay networks. The suboptimal schemes we propose are less than 0.5dB worse than the optimal multiple relay selection. However, they are far superior in complexity. Also, multiple relay selections are only about 1-1.5dB worse than network beamforming. Thus, allowing multiple relay selection reduces the gap between relay selection and network beamforming by about 1dB in 2-relay networks and 2.5dB in 3-relay networks.

We also show performance of a 5-relay network with different relay power constraints in Fig. 4. We have $P_1 = 2P, P_2 = \frac{3}{2}P, P_3 = \frac{1}{2}P, P_4 = \frac{2}{3}P$, and $P_5 = \frac{1}{3}P$. Error rates of both single and multiple relay selection schemes are shown. Since the multiple relay selection schemes based on the harmonic mean ordering and the best worse channel ordering have about the same performance as the multiple relay selection scheme based on the SNR ordering, their bit error rate curves are
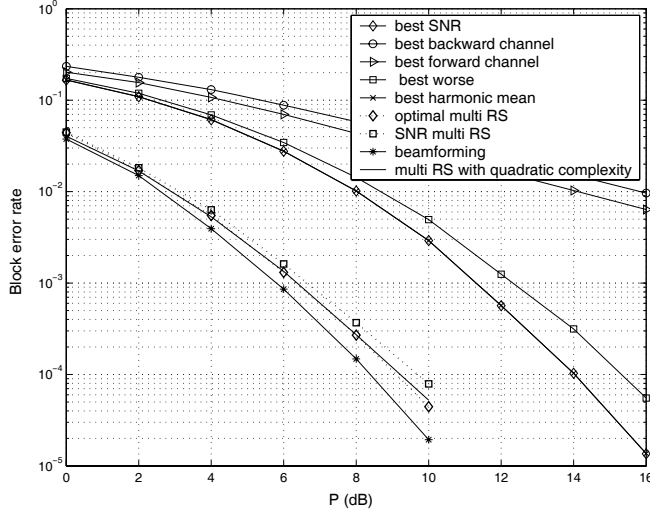
Fig. 4.   Single and multiple relay selections in 5-relay networks.

omitted to make the figure less busy. Similar diversity order and array gain behavior can be observed. The suboptimal multiple relay selection schemes are less than 1 dB worse than the optimal multiple relay selection scheme and 1.5dB worse than network beamforming. Allowing multiple relay selection induces an improvement of around 5dB. The performance of the multiple relay selection scheme with quadratic complexity proposed in Section III-C is also shown in this figure. We can see that it performs almost the same as the optimal multiple relay selection.

In this work, each relay has its own power constraint and the total power used for the transmission is not considered. Assume that the ordering function h is used and $h_1 > \cdots > h_R$. It is obvious that the total transmit power in the whole network, $P_{total} = \sum_{i=1}^{k} P_i$, increases as k the number of cooperating relays increases. Thus, multiple relay selection schemes may not be power efficient. However, since the main focus of this paper is to maximize the receive SNR given a separate power constraint on each node, the power efficiency of the whole network is not an issue and will be left for future work. One simple modification we can make toward better power efficiency is as follows. Instead of choosing the k resulting in the maximum receive SNR, we can choose the smallest k such that $SNR_{\{1,...,k\}} > SNR_{\{1,...,k+1\}}$. In other words, we examine $SNR_{\{1\}}, \ldots, SNR_{\{1,...,R\}}$ one by one till the SNR values stop increasing. This modification ensures that "bad" relays do not cooperate, although there may be some overall SNR loss.

To understand this a little more, we propose the following power efficiency problem:

$$\max_{\alpha_1,...,\alpha_R} \frac{SNR_{general}}{P + \sum_{i=1}^{R} \alpha_i^2 P_i} \quad \text{s.t.} \quad \alpha_i \in \{0, 1\}. \quad (6)$$

This problem is also a general nonlinear 0-1 programming. It can be solved by exhaustively searching over all relays. Suboptimal algorithms for nonlinear integer programming can be found in [57].

In Fig. 5, we show the bit error rates, power efficiency, and total transmit power of the following four schemes: the SNR-optimal multiple relay selection, the efficiency-optimal
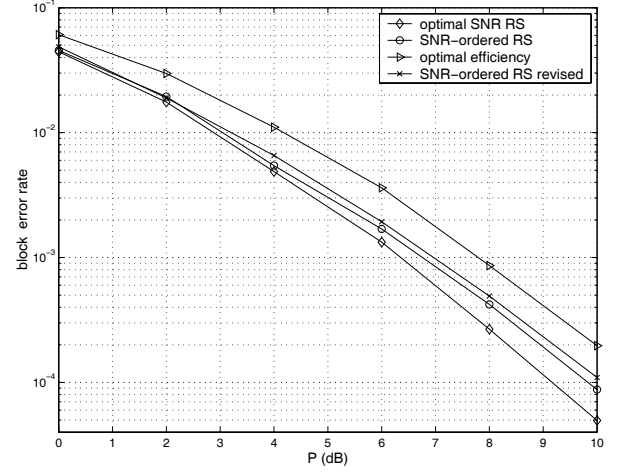
(a) Block error rate
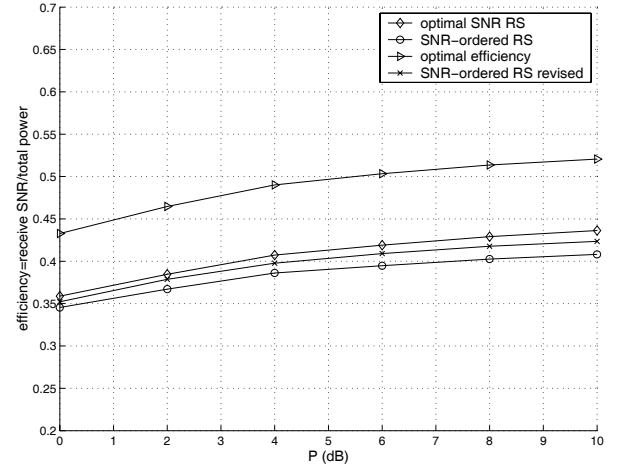
(b) Power efficiency
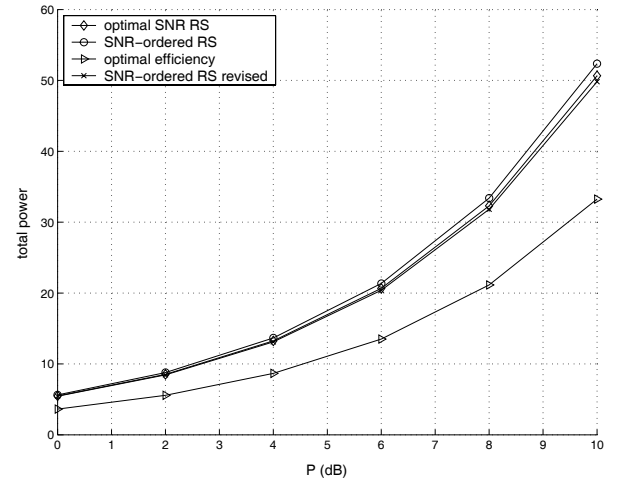
(c) Total transmit power

Fig. 5.   Block error rate, power efficiency, and total transmit power of 5-relay networks.

multiple relay selection, the multiple relay selection based on SNR ordering, and its revised version toward power efficiency discussed in the above. The SNR-optimal multiple relay selection is the solution to (3) and is obtained by an exhaustive search over all the relays. The network has 5 relays with the same power constraints to the previous case. We can see that

the optimal efficiency relay selection scheme trades off 1.5dB reliability against a 35% saving in the total power. The power efficiency of the former is about 1.2 times the later one, which may result in a considerable increase in the lifetime of the network. The SNR ordered multiple relay selection is worse than the optimal SNR relay selection in both the error rate and power efficiency. However, it has a lower complexity. Compared to the SNR ordered multiple relay selection scheme, the revised scheme trades off a little performance against the power efficiency. However, the bit error rate and efficiency difference of the two schemes is very small.

Synchronization and training are also important practical issues of cooperative networks. In this work, it is assumed that each relay knows its own channels, including both the channel magnitudes and phases, perfectly. Also, during the second step, the relays are assumed to transmit simultaneously. These require the relays to be synchronized at both the symbol and the carrier levels, which can be difficult to achieve in practical networks with a lot of distributed relays. At the receiver, it is assumed that all channels are known perfectly through a training process and there is no estimation errors. For networks with imperfect synchronization and channel estimation errors, the SNR expression and the suboptimal relay ordering functions will be affected. However, these are out of the main focus of this paper and are left for future work. Some asynchronous network design results can be found in [33], [39], [58]. Training procedures to reveal the entire channel state to the receiver can be found in [59], [60].

V. CONCLUSIONS

In this paper, we work on network relay selection schemes. First, achievable diversity orders of single relay selection schemes are discussed. We show that single relay selection schemes based on SNR, harmonic mean, and the worse link quality achieve full diversity, while the nearest neighbor single relay selection can only achieve diversity 1. Then, we generalize the idea by allowing more than one relay to cooperate. Although we cannot find linear algorithms for the optimal multiple relay selection, by introducing the idea of relay ordering, we propose several suboptimal schemes with linear complexity. We also propose a scheme with quadratic complexity. These suboptimal schemes perform close to the optimal and much better than their corresponding single relay selection schemes.

APPENDIX A
USEFUL RANDOM PROCESS RESULTS

In this section, we derive pdfs of two random variables that are useful for later diversity derivations.

Claim 1. *Assume that $X_1, \ldots, X_R$ are i.i.d. random variables with the exponential distribution: e^{-x}. Let $X_{\min} = \min\{X_1, \ldots, X_R\}$ and $X_{\max} = \max\{X_1, \ldots, X_R\}$.*
1) *The pdf of $X_{\min}$ is Re^{-Rx}.*
2) *The pdf of $X_{\max}$ is $R\left(1 - e^{-x}\right)^{R-1} e^{-x}$.*

   Proof:

$$\mathrm{P}\left(X_{\min} \geq x\right) = \prod_{i=1}^{R} \mathrm{P}\left(X_i \geq x\right) = \left(\int_x^\infty e^{-x} dx\right)^R = e^{-Rx}.$$

Thus, the pdf of $X_{\min}$ is

$$-\frac{d\mathrm{P}\left(X_{\min} \geq x\right)}{dx} = Re^{-Rx}.$$

Also,

$$\mathrm{P}\left(X_{\max} \leq x\right) = \prod_{i=1}^{R} \mathrm{P}\left(X_i \leq x\right) = \left(1 - e^{-x}\right)^R.$$

Thus, the pdf of $X_{\max}$ is

$$\frac{d\mathrm{P}\left(X_{\max} \leq x\right)}{dx} = R\left(1 - e^{-x}\right)^{R-1} e^{-x}.$$

$\blacksquare$

APPENDIX B
PROOF OF THEOREM 1

If $R = 1$, the diversity order of the network is obviously 1. Thus, we only need to consider the case of $R \geq 2$. Since $SNR_{\{i\}}$ is an increasing function of $|f_i|^2$ and $|g_i|^2$, the receive SNR of the nearest neighbor selection is no less than that of single relay networks. Thus, its diversity order is at least 1. We only need to prove that it is also no more than 1.

Assume that the ith relay has the largest $|f_i|^2$. Thus, from Claim 1, the pdfs of $|f_i|^2$ and $|g_i|^2$ are $R\left(1 - e^{-x}\right)^{R-1} e^{-x}$ and e^{-x}, respectively. From (4), we can prove that

$$SNR_{\{i\}} \leq \min\left(|f_i|^2, |g_i|^2\right) P.$$

Let $Y = \min\left(|f_i|^2, |g_i|^2\right)$. We have

$$\mathrm{P}(Y \geq y) = \mathrm{P}(|f_i|^2 \geq y)\mathrm{P}(|g_i|^2 \geq y).$$

Thus, the pdf of Y is

$$-\frac{d\mathrm{P}(Y \geq y)}{dy}$$
$$= -\mathrm{P}(|f_i|^2 \geq y)\frac{d\mathrm{P}(|g_i|^2 \geq y)}{dy} - \frac{d\mathrm{P}(|f_i|^2 \geq y)}{dy}\mathrm{P}(|g_i|^2 \geq y)$$
$$\geq -\mathrm{P}(|f_i|^2 \geq y)\frac{d\mathrm{P}(|g_i|^2 \geq y)}{dy}$$
$$= e^{-y}\int_y^\infty R\left(1 - e^{-x}\right)^{R-1} e^{-x} dx$$
$$\triangleq h(y).$$

The average error rate of nearest neighbor selection is

$$BER = \mathrm{E}_{f_i, g_i} Q\left(\sqrt{SNR_{\{i\}}}\right)$$
$$\geq \mathrm{E}_Y Q(\sqrt{YP}) \geq \int_0^\infty h(y)Q(\sqrt{yP}) dy.$$

Now let us prove a lower bound for Q-function. A well-known inequality of the Q-function is

$$Q(x) \geq \frac{2}{\sqrt{\pi}} \frac{1}{x + \sqrt{x^2 + 4}} e^{-\frac{x^2}{2}}.$$

Since $x + \sqrt{x^2 + 4} \leq (\sqrt{2}+1)x$ for $x \geq 2$ and $x + \sqrt{x^2 + 4} \leq (\sqrt{2} + 1)2$ for $x \leq 2$, we have $x + \sqrt{x^2 + 4} \leq (\sqrt{2} + 1)\max\{2, x\}$ and thus $Q(x) \geq \min\{h_1(x), h_2(x)\}$, where $h_1(x) = \frac{1}{3x}e^{-\frac{x^2}{2}}$ and $h_2(x) = \frac{1}{5}e^{-\frac{x^2}{2}}$. Thus,

$$BER \geq \min\left\{\int_0^\infty h(y)h_1(\sqrt{yP}) dy, \int_0^\infty h(y)h_2(\sqrt{yP}) dy\right\}.$$

We have

$$\int_0^\infty h(y)h_1(\sqrt{yP})dy$$

$$=\int_0^\infty \frac{1}{3\sqrt{P}}\frac{e^{-\frac{Py}{2}}}{\sqrt{y}}e^{-y}\int_y^\infty R\left(1-e^{-x}\right)^{R-1}e^{-x}dxdy$$

$$=\frac{R}{3\sqrt{P}}\int\int_{x\geq y}\frac{e^{-(\frac{P}{2}+1)y}}{\sqrt{y}}\left(1-e^{-x}\right)^{R-1}e^{-x}dxdy$$

$$=\frac{R}{3\sqrt{P}}\int_0^\infty\left(1-e^{-x}\right)^{R-1}e^{-x}dx\int_0^x\frac{e^{-(\frac{P}{2}+1)y}}{\sqrt{y}}dy$$

$$=\frac{R\sqrt{2\pi}}{3\sqrt{P(P+2)}}\int_0^\infty\left(1-e^{-x}\right)^{R-1}e^{-x}\Phi\left(\left(\frac{P}{2}+1\right)x\right)dx$$

$$=\frac{R\sqrt{2\pi}}{3\sqrt{P(P+2)}}\sum_{i=0}^{R-1}(-1)^i\binom{R-1}{i}$$
$$\int_0^\infty e^{-(i+1)x}\Phi\left(\left(\frac{P}{2}+1\right)x\right)dx$$

$$=\frac{R\sqrt{2\pi}}{3\sqrt{P(P+2)}}\sum_{i=0}^{R-1}(-1)^i\binom{R-1}{i}\frac{1}{i+1}\frac{\sqrt{\frac{P}{2}+1}}{\sqrt{\frac{P}{2}+i+2}},$$

where $\Phi(x)$ is the error function. We can prove that $\sum_{i=0}^{R-1}(-1)^i\binom{R-1}{i}\frac{1}{i+1}>0$ for any R. Thus,

$$\int_0^\infty h(y)h_1(\sqrt{yP})dy \sim O\left(P^{-1}\right)$$

for $P\gg 1$. Also

$$\int_0^\infty h(y)h_2(\sqrt{yP})dy$$

$$=\frac{1}{5}\int_0^\infty\int_y^\infty e^{-\frac{Py}{2}}e^{-y}R\left(1-e^{-x}\right)^{R-1}e^{-x}dxdy$$

$$=\frac{R}{5}\int_0^\infty\left(1-e^{-x}\right)^{R-1}e^{-x}dx\int_0^x e^{-(\frac{P}{2}+1)y}dy$$

$$=\frac{R}{5\left(1+\frac{P}{2}\right)}\int_0^\infty\left(1-e^{-(\frac{P}{2}+1)x}\right)\left(1-e^{-x}\right)^{R-1}e^{-x}dx$$

$$=\frac{R}{5\left(1+\frac{P}{2}\right)}\left[\int_0^\infty\left(1-e^{-x}\right)^{R-1}e^{-x}dx\right.$$
$$\left.-\int_0^\infty\left(1-e^{-x}\right)^{R-1}e^{-(\frac{P}{2}+2)x}dx\right]$$

$$=\frac{R}{5\left(1+\frac{P}{2}\right)}\left[B(1,R)-B\left(2+\frac{P}{2},R\right)\right]$$

$$=\frac{R}{5\left(1+\frac{P}{2}\right)}\left[\frac{\Gamma(R)\Gamma(1)}{\Gamma(R+1)}-\frac{\Gamma(R)\Gamma(2+P/2)}{\Gamma(R+2+P/2)}\right]$$

$$=\frac{1}{5}\left(\frac{1}{1+P/2}-\frac{(R-1)!}{\prod_{i=1}^R(i+1+P/2)}\right)$$

$$=O\left(P^{-1}\right)\quad\text{for }R\geq 2,$$

where $B(x,y)$ is the Beta function and $\Gamma(x)$ is the Gamma function. Therefore, $BER\geq C\cdot\left(P^{-1}\right)$ for some constant C.

APPENDIX C
PROOF OF THEOREM 2

Assume that the ith relay has the best worse channel. From Claim 1, the pdf for $Y_j=\min\{|f_j|^2,|g_j|^2\}$ is $2e^{-2x}$ for any j. Thus, $Y_i=\max\{Y_1,\ldots,Y_R\}$, and

$$P\left(Y_i\leq x\right)=\prod_{j=1}^R P\left(Y_j\leq x\right)$$

$$=\left(\int_0^x 2e^{-2x}dx\right)^R=\left(1-e^{-2x}\right)^R.$$

The pdf of Y is

$$\frac{dP\left(Y\leq x\right)}{dx}=2R\left(1-e^{-2x}\right)^{R-1}e^{-2x}.$$

When $Y_i=\min\{|f_i|^2,|g_i|^2\}\geq 1/P$, from (4), we have

$$SNR_{\{i\}}\geq\frac{|f_ig_i|^2 P}{2(|f_i|^2+|g_i|^2)}\geq\frac{1}{4}\min\{|f_i|^2,|g_i|^2\}=\frac{YP}{4}.$$

When $Y_i=\min\{|f_i|^2,|g_i|^2\}<1/P$, we have $SNR_{\{i\}}\geq 0$.
The error rate can be upper bounded as

$$EQ\left(\sqrt{SNR_{\{i\}}}\right)\leq\frac{1}{2}Ee^{-\frac{SNR_{\{i\}}}{2}}$$

$$\leq\frac{1}{2}\int_{1/P}^\infty e^{-\frac{Py}{8}}p(y)dy+\frac{1}{2}P\left(Y_i<\frac{1}{P}\right).$$

The first term equals

$$\frac{1}{2}\int_{\frac{1}{P}}^\infty e^{-\frac{Py}{8}}2R\left(1-e^{-2y}\right)^{R-1}e^{-2y}dy$$

$$=\frac{1}{2}\int_{\frac{1}{P}}^\infty 2R\left(1-e^{-2y}\right)^{R-1}e^{-\left(2+\frac{P}{8}\right)y}dy$$

$$\leq\frac{R}{2}\int_0^\infty\left(1-e^{-x}\right)^{R-1}e^{-\left(1+\frac{P}{16}\right)x}dx$$

$$=\frac{R}{2}B\left(1+\frac{P}{16},R\right)=\frac{(R-1)!}{\prod_{i=1}^R\left(1+\frac{P}{16}+i\right)}=O\left(P^{-R}\right).$$

Now we look at the second term.

$$\frac{1}{2}P\left(Y_i<\frac{1}{P}\right)$$

$$=\frac{1}{2}\int_0^{\frac{1}{P}}2R\left(1-e^{-2x}\right)^{R-1}e^{-2x}dx$$

$$\leq\frac{1}{2}\int_0^{\frac{1}{P}}2R\left(1-e^{-\frac{2}{P}}\right)^{R-1}dx$$

$$=\frac{R}{P}\left(1-e^{-\frac{2}{P}}\right)^{R-1}=O\left(P^{-R}\right).$$

Thus, the error rate of this scheme scales no slower than $O\left(P^{-R}\right)$.

APPENDIX D
PROOF OF THEOREM 3

Assume that Relay i has the maximum harmonic mean and Relay j has the best worse channel. Let $X_k=\frac{1}{|f_k|^{-2}+|g_k|^{-2}}$ and $Y_k=\min\{|f_k|^2,|g_k|^2\}$ for $k=1,\ldots,R$. When $Y_i\geq 1/P$, we have

$$SNR_{\{i\}}\geq\frac{|f_ig_i|^2 P}{2(|f_i|^2+|g_i|^2)}=\frac{X_i}{2}\geq\frac{X_j}{2}.$$

The error rate can be upper bounded as

$$
\begin{aligned}
\mathrm{E}Q\left(\sqrt{SNR_{\{i\}}}\right) &\leq \frac{1}{2}\mathrm{E}e^{-\frac{SNR_{\{i\}}}{2}}\\
&\leq \frac{1}{2}\int_{Y_i \geq 1/P} e^{-\frac{x_i}{4}}p_{X_i}(x)dx + \frac{1}{2}\mathrm{P}\left(Y_i < \frac{1}{P}\right)\\
&< \frac{1}{2}\int_0^\infty e^{-\frac{x_i}{4}}p_{X_i}(x)dx + \frac{1}{2}\mathrm{P}\left(Y_i < \frac{1}{P}\right)\\
&\leq \frac{1}{2}\int_0^\infty e^{-\frac{x_j}{4}}p_{X_j}(x)dx + \frac{1}{2}\mathrm{P}\left(Y_i < \frac{1}{P}\right),
\end{aligned}
$$

since $X_j \leq X_i$ for any channel values. From the calculation in Appendix C, we have

$$
\int_0^\infty e^{-\frac{x_j}{4}}p_{X_j}(x)dx = \frac{(R-1)!}{\prod_{i=1}^R \left(1+\frac{P}{8}+i\right)} = O\left(P^{-R}\right).
$$

To calculate $\mathrm{P}\left(Y_i < \frac{1}{P}\right)$, let us first introduce the following results. For any k, we have

$$
Y_k \leq \frac{1}{P} \Rightarrow X_k \leq \frac{1}{P} \Rightarrow Y_k \leq \frac{2}{P}. \tag{7}
$$

To prove this, from the definition of X_k and Y_k, we have

$$
\begin{aligned}
Y_k \leq \frac{1}{P} &\Rightarrow \min\{|x_i|^2, |y_i|^2\} \leq \frac{1}{P}\\
&\Rightarrow |x_i|^2 \leq \frac{1}{P} \text{ or } |y_i|^2 \leq \frac{1}{P}\\
&\Rightarrow |x_i|^{-2} \geq P \text{ or } |y_i|^{-2} \geq P\\
&\Rightarrow |x_i|^{-2} + |y_i|^{-2} \geq P\\
&\Rightarrow X_k \leq \frac{1}{P}
\end{aligned}
$$

and

$$
\begin{aligned}
X_k \leq \frac{1}{P} &\Rightarrow |x_i|^{-2} + |y_i|^{-2} \geq P\\
&\Rightarrow |x_i|^{-2} \geq \frac{P}{2} \text{ or } |y_i|^{-2} \geq \frac{P}{2}\\
&\Rightarrow |x_i|^2 \leq \frac{2}{P} \text{ or } |y_i|^2 \leq \frac{2}{P}\\
&\Rightarrow \min\{|x_i|^2, |y_i|^2\} \leq \frac{2}{P}\\
&\Rightarrow Y_k \leq \frac{2}{P}.
\end{aligned}
$$

Using (7) we have

$$
\begin{aligned}
\mathrm{P}\left(Y_i < \frac{1}{P}\right) &\leq \mathrm{P}\left(X_i \leq \frac{1}{P}\right)\\
&= \mathrm{P}\left(X_1, \ldots, X_R \leq \frac{1}{P}\right) = \prod_{k=1}^R \mathrm{P}\left(X_k \leq \frac{1}{P}\right).
\end{aligned}
$$

Again, using (7), we have

$$
\mathrm{P}\left(X_k \leq \frac{1}{P}\right) \leq \mathrm{P}\left(Y_k \leq \frac{2}{P}\right) = \int_0^{2/P} 2e^{-2x}dx = \left(1 - e^{-\frac{4}{P}}\right).
$$

Thus

$$
\mathrm{P}\left(Y_i < \frac{1}{P}\right) \leq \left(1 - e^{-\frac{4}{P}}\right)^R = O\left(P^{-R}\right)
$$

and the maximum harmonic mean selection achieves the full diversity order of R.

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