

19. PERCENTAGES

Fraction	$\frac{1}{2}$	2^2							
Equivalent percentage	50%	100%							
Fraction	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$						
Equivalent Percentage	33.33%	66.66%	100%						
Fraction	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$					
Equivalent Percentage	25%	50%	75%	100%					
Fraction	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$				
Equivalent percentage	20%	40%	60%	80%	100%				
Fraction	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$			
Equivalent Percentage	$16\frac{2}{3}\%$	33%	50%	$66\frac{2}{3}\%$	$83\frac{1}{3}\%$	100%			
Fraction	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{7}{7}$		
Equivalent percentage	14.2857%	28.5714%	42.8571%	57.1428%	71.4285%	85.7142%	100%		
Fraction	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$	
Equivalent percentage	12.5%	25%	37.5%	50%	62.5%	75%	87.5%	100%	
Fraction	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	9 9
Equivalent percentage	11.11%	22.22%	33.33%	44.44%	55.55%	66.66%	77.77%	88.88%	100%
Fraction	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{3}{11}$	$\frac{4}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{7}{11}$	$\frac{8}{11}$	$\frac{9}{11}$
Equivalent percentage	9.09%	18.18%	27.27%	36.36%	45.45%	54.54%	63.63%	72.72%	81.81%
									90.9%

- | Expressing any quantity by taking 100 as reference is called "PER CENT".
- | To convert a percentage value to a fraction, one should divide it with 100.
Eg: $75\% = \frac{75}{100} = \frac{3}{4}$
- | To convert a fraction value to a percentage, one should multiply it with 100.
Eg: $\frac{3}{5} = (\frac{3}{5}) \times 100 = 60\%$
Eg: Rani scored 450 out of 600 marks in her annual examinations. Her percentage of marks = $(450/600) \times 100 = 75\%$
- | Eg: Raju secured 68% marks in his annual examinations held for 600. Marks secured by Raju = $(68/100) \times 600 = 408$
- | If a particular number is increased by X% and to get the original number, **it should not be decreased by X%**, it should be decreased by a different percentage value.
Eg: If 100 is increased by 20%, then the value is 120. Now to get 100 again, 120 should be decreased by 20, i.e., $(20/120) \times 100 = 16.66\%$.
- | If a particular number is decreased by X% and to

get the original number, **it should not be increased by X%**.

Eg: If 100 is decreased by 25%, then the value is 75. Now to get 100 again, 75 should be increased by 25, i.e., $(25/75) \times 100 = 33.33\%$.

Majorly questions from percentages can be classified into four different types. They are:

Type 1:

- | If two variables are inversely related and if one of them increases by $1/x$, then the other one has to be decreased by $1/(x+1)$, so that their product remain same.

Eg 1: If length of a rectangle increases by 25%, then in order to remain the area same, by how much percentage its breadth has to be decreased?

Sol: Here length of a rectangle is inversely proportional to its breadth. As length is increased by $25\% = 1/4$, its breadth has to be decreased by $1/5 = 20\%$ so that area remains same.

Eg 2: If the speed of journey increases by 33.33%, then by how much percentage the time consumption be decreased in covering the same distance?

Sol: Here Speed is inversely proportional to time. As speed is increased by $33.33\% = 1/3$, time consumption decreases by $1/4 = 25\%$, in covering the same distance.

Type 2:

If Z is a quantity which is the product of two other quantities X and Y and if there is a change of $x\%$ in X and $y\%$ in Y , then the percentage change in Z , which is given by z is, $x \pm y \pm (xy/100)$. Here + for increase and - for decrease.

Eg 1: If the length and breadth of rectangle are increased by 20% and 30% respectively, then what is the percentage change in the area of the rectangle?

Sol: percentage change in the area of the rectangle $= 20+30+(20*30/100) = 56\%$. So area increases by 56%.

Eg 2: If the price of a commodity increases by 40% and its consumption decreases by 30%, then what is the percentage change in the expenditure?

Sol: Percentage change in the expenditure $= 40-30-(40*30/100) = -2$. So expenditure decreases by 2%.

If a particular number is increased by $X\%$ and then decreased by $X\%$ (OR) decreased by $X\%$ and then increased by $X\%$, then the net percentage change in the number is $\left(\frac{x^2}{100}\right)\%$ decrease.

Eg: If a number 100 increased by 20% and then decreased by 20%, then the net percentage change in the number is 4% decrease.

Eg: If a number 100 is decreased by 30% and then increased by 30%, then the net percentage change in the number is 9% decrease.

Type 3:

Case 1: An article is sold at a loss of $x\%$. Had it been sold for Rs.R more, then a profit of $y\%$ is obtained. Then the cost price of the article is $\{R/(x+y)\}*100$.

Eg: An article is sold at a loss of 12%. Had it been sold for Rs.90 more, then a profit of 18% is obtained. Then the cost price of the article is $\{90/(12+18)\}*100 = 300$.

Case 2: An article is sold at a loss of $x\%$. Had it been sold for Rs.R more, then a loss of $y\%$ is obtained. Then the cost price of the article is $\{R/(x-y)\}*100$.

Eg: An article is sold at a loss of 12%. Had it been sold for Rs.90 more, then a loss of 8% is obtained. Then

the cost price of the article is $\{90/(12-8)\}*100 = 2250$.

Case 3: An article is sold at a profit of $x\%$. Had it been sold for Rs.R less, then a loss of $y\%$ is obtained. Then the cost price of the article is $\{R/(x+y)\}*100$.

Eg: An article is sold at a profit of 15%. Had it been sold for Rs.90 less, then a loss of 9% is obtained. Then the cost price of the article is $\{90/(15+9)\}*100 = 375$.

Case 4: An article is sold at a profit of $x\%$. Had it been sold for Rs.R less, then a profit of $y\%$ is obtained. Then the cost price of the article is $\{R/(x-y)\}*100$.

Eg: An article is sold at a profit of 15%. Had it been sold for Rs.90 less, then a profit of 9% is obtained. Then the cost price of the article is $\{90/(15-9)\}*100 = 1500$.

Type 4:

If the number of boys and girls in a class are x and y respectively ($x > y$), then

- The number of boys is what percentage of the number of girls?
- The number of girls is what percentage of the number of boys?
- The number of boys is what percentage more than the number of girls?
- The number of girls is what percentage less than the number of boys?
- The number of boys is what percentage of the total number of students of the class?
- The number of girls is what percentage of the total number of students of the class?

Sol: i) $(x/y)*100$

ii) $(y/x)*100$

iii) $\{(x-y)/y\}*100$

iv) $\{(y-x)/x\}*100$

v) $\{x/(x+y)\}*100$

vi) $\{y/(x+y)\}*100$

1: The population of India in the year 1990 was 80 crores and in the year 2010 it was 100 crore. What is the percentage increase in the population of India from 1990 to 2010?

Sol: There is an increase of 20 crore in the population of India from 1990 to 2010, when compared to the population in 1990.

∴ The percentage increase

$$= \frac{20 \text{ crores}}{80 \text{ crores}} \times 100 = 25\%.$$

a. Comparison of the change in the same variable i.e., Percentage change

$$= \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100$$

b. Comparison of two variables at the same time i.e., Percentage more (or less)

$$= \frac{\text{Difference}}{\text{Smaller (or Greater)}} \times 100$$

- 2: In an examination Anni scored 80 marks and Binni scored 60 marks.
- What percent of Anni's marks are Binni's marks?
 - What percent of Binni's marks are Anni's marks?
 - By how much percent are Anni's marks more than that of Binni?
 - By how much percent are Binni's marks less than that of Anni?
 - What percent of total marks of both Anni and Binni are Anni's marks?
 - What is the percent of Binni's marks of the total marks?

a. **Percentage increase** = $\frac{\text{Actual increase}}{\text{Smaller value}} \times 100$

b. **Percentage decrease** = $\frac{\text{Actual decrease}}{\text{Greater value}} \times 100$

- 3: In an examination if A scored 60% of the maximum marks and B scored 84% of the maximum marks, then by what percentage B marks are more than that of A?

Sol: $\frac{24\% \text{ of the maximum marks}}{60\% \text{ of the maximum marks}} \times 100$

i.e., 40% more than A.

- 4: If Anil's salary is 25% more than Binoy's salary, by what percent is Binoy's salary less than Anil's salary?

Sol: Let Binoy's salary be Rs.100. Then Anil's salary = Rs.125.

The percentage by which Binoy's salary is less than

$$\text{Anil's salary} = \frac{25}{125} \times 100 = 20\%.$$

- 5: The length and breadth of a rectangle are increased by 10% and 20%. What is the percentage increase in its area?

Sol: Let the length be 100 units and breadth be 100 units. Area = $100 \times 100 = 10000$

When the length is increased by 10%, the new length = $100 + 10\% \text{ of } 100 = 110$

Similarly, the new breadth = 120

$$\text{New area} = 110 \times 120 = 13200$$

$$\text{The percent increase} = \frac{32}{100} \times 100 = 32\%$$

Growth and Depreciation:

If a quantity x increases by r% each for n times, then the

$$\text{final value} = x \left(1 + \frac{r}{100}\right)^n. \text{ For successive decrease of}$$

$$r\% \text{ for } n \text{ times, final value} = x \left(1 - \frac{r}{100}\right)^n.$$

If the quantity x increases by $r_1\%$, $r_2\%$, $r_3\%$, ... successively, then the final value

$$= \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \left(1 + \frac{r_3}{100}\right) \dots \dots$$

If it decreases, then the final value

$$= x \left(1 - \frac{r_1}{100}\right) \left(1 - \frac{r_2}{100}\right) \left(1 - \frac{r_3}{100}\right) \dots \dots$$

If x changes by $r_1\%$, $r_2\%$, $r_3\%$, ... successively, the final value

$$= x \left(1 \pm \frac{r_1}{100}\right) \left(1 \pm \frac{r_2}{100}\right) \left(1 \pm \frac{r_3}{100}\right) \dots \dots$$

(+) sign indicates increase or growth and the (-) sign indicates decrease or depreciation.

In some cases, we may be given the present value of the variable as x, to calculate the past value

$$\frac{x}{\left(1 + \frac{r}{100}\right)^n}$$

- 6: The present cost of a mobile is Rs.8000 and every year its value decreases by 10%. Find the cost of the mobile at the end of second year.

(1) 5380 (2) 6120 (3) 6480 (4) 6840

- 7: The population of a country was 12 crore in 2010. The population increased by 20% from 2005 to 2010 and by 25% from 2000 to 2005. What was the population in 2000?

(1) 6 cr (2) 7 cr (3) 8 cr (4) 9 cr

PRACTICE EXERCISE - 1

Directions for questions: Select the correct alternative from the given choices.

1. A. If A spends 25% more than B, then by what percent did B spend less than A?

(1) 25 (2) 33.33 (3) 20 (4) 30

- B. A number is increased by 25%, in order to get the original number, by what percentage it has to be decreased?

(1) 16.66 (2) 25 (3) 33.33 (4) 20

- C. A number is increased by 15% and then decreased by 15%, then what is the percentage change in the number?

(1) 2.25% increase (2) 2.25% decrease

(3) 22.5% increase (4) no change

- D. The ratio of number of girls to that of boys is 5:3, then,

a) What percent of total students are girls?

- (1) 60% (2) 62.5% (3) 37.5% (4) 75%
- b) The number of boys is what percentage less than the number of girls?
- (1) 40% (2) 60 (3) 33.33% (4) 66.66%
2. A cricket team enjoys 60% of success in the first 15 matches played by it. What should be the success rate in the remaining matches to be played to have an overall success rate of 75%, if the total number of matches to be played is 40?
- (1) 84 (2) 80 (3) 90 (4) 75
3. In a village, 40% of the voters are males, 30% of the males are illiterates and 25% of females are literates. By what percentage is the male illiteracy less than the female literacy?
- (1) 20% (2) 66.66% (3) 33.33% (4) 36%
4. Two numbers are respectively 50% and 80% more than a third number. If the second number is 150 more than the first number, then the third number is...
- (1) 500 (2) 400 (3) 250 (4) 360
5. The length and breadth of a rectangle are increased by 20% and 10% respectively. What is the percentage increase in the area of the rectangle?
- (1) 25% (2) 32% (3) 40% (4) 20%
6. X's salary is 30% more than Y's salary. Y's salary is 20% less than Z's salary. What percentage of Z's salary is X's salary?
- (1) 104% (2) 96% (3) 125% (4) 80%
7. Pragnyan secured 25% of maximum marks in an examination and failed by 18 marks. If pass percentage is 40%, then find the maximum marks.
- (1) 100 (2) 120 (3) 150 (4) 135
8. In an election there are only two contestants Hulk and Bulk. Hulk secured 58% of the valid votes and won by 24000 votes. How many votes were polled in favour of Bulk?
- (1) 34000 (2) 45000 (3) 58000 (4) 63000
9. Manohar saves 20% of his salary. If Manohar's salary is increased by 10%, he increases his new savings by 20% over his existing savings. What is the percentage increase in Manohar's expenditure?
- (1) 7.5% (2) 8% (3) 6% (4) 9%
10. If the speed of journey is increases by 50%, then by what percentage the time taken to cover the same distance decreases?
- (1) 50% (2) 33.33% (3) 25% (4) 66.66%
11. If the manpower deployed in completing a work is decreases by 25%, then by what percentage the time taken to complete the increases?
- (1) 16.66% (2) 33.33% (3) 25% (4) 20%
12. If the length of a rectangle increases by 80%, by what percentage the breadth of the rectangle has to be decreased so that the area remains same?
- (1) 50% (2) 33.33% (3) 44.44% (4) 66.66%
13. If the breadth of the rectangle increases by 25%, by what percentage its area increases provided the length remains same?
- (1) 20% (2) 33.33% (3) 25% (4) 16.66%
14. If the speed and time of travel increases by 20% and 25% respectively, what is the percentage increase in the distance covered?
- (1) 50% (2) 45% (3) 22.5% (4) 40%
15. If the length of the rectangle increases by 30% and breadth decreases by 20%, what is the percentage increase in the area of the rectangle?
- (1) 5% (2) 10% (3) 4% (4) 8%
16. If the price of petrol decreases by 30% and its consumption increases by 40%, what is the percentage change in the expenditure on petrol?
- (1) 2% decrease (2) 2% increase (3) 5% increase (4) 5% decrease (5) None of these
17. If the length and breadth of a rectangle decreases by 30% and 20% respectively, what is the percentage decrease in the area of the rectangle?
- (1) 56% (2) 50% (3) 25% (4) 44%
18. When the price of a commodity is increased by 25%, in order to keep the expenditure remain same, by how much percentage the consumption should be decreased?
- (1) 30 (2) $33\frac{1}{3}$ (3) 25 (4) 20
19. When the price of a commodity is increased by 50%, then by how much percentage its consumption should be decreased such that the expenditure on the commodity increases by 20%?
- (1) 30 (2) $33\frac{1}{3}$ (3) 25 (4) 20
20. If the price of an article is increased by 25% and the consumption is increased by 20%, then by what percent the expenditure should be increased?
- (1) 45% (2) 40% (3) 50% (4) 22.5%
21. Bharat sold his cycle at 13.5% loss. Had he sold it for Rs.600 more, he would have made a profit of 24%. What is the cost price of the cycle (in Rs.)?
- (1) 1600 (2) 800 (3) 1000 (4) 1200
22. In an exam Ravi got 28% marks and failed by 14 marks, Sami got 48% marks and passed by 16 marks. Find the pass marks.
- (1) 35 (2) 56 (3) 62 (4) 48
23. Alekhya sold her mobile at 12% profit. Had she sold it for Rs.950 less, she would have got a profit of 7%. Find the cost price of the mobile (in Rs.)
- (1) 12000 (2) 15000 (3) 19000 (4) 5000

Directions for Q24 and Q25: Refer the following table.

Year	2013	2014	2015
Turnover in crores	120	160	150

24. What is the percentage increase in the turnover from the year 2013 to 2014?
- (1) 25% (2) 33.33% (3) 20% (4) 16.66%
25. What is the percentage change in turnover during the given years?
- (1) 25% (2) 33.33% (3) 20% (4) 16.66%

Percentages										
	A.3 B.4 C.2 D.2,1	6	1	11	2	16	1	21	1	
1										
2	1	7	2	12	3	17	4	22	2	
3	1	8	4	13	3	18	4	23	3	
4	1	9	1	14	1	19	4	24	2	
5	2	10	2	15	3	20	3	25	1	