

6. CLOCKS

The hour hand and the minute hand of a clock move in relation to each other continuously and at any given point of time, they make an angle between 0° and 180° with each other.

If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

Minutes hand:

One full revolution = one hour = 360°

One minute = $360^\circ/60 = 6^\circ$ (one hour = 60 min)

One second = $6^\circ/60 = 1/10^\circ$

Hours hand:

One full revolution = 12 hours = 360°

One hour = $360^\circ/12 = 30^\circ$

One minute = $30^\circ/60 = 1/2^\circ$ (one hour = 60 min)

In 1 minute, Minute Hand covers 6° and Hour hand covers $1/2^\circ$.

Therefore, Relative angle = $6 - 1/2 = 5\frac{1}{2}^\circ$ per minute
Alternately, in 1 hour, the minute hand covers 60 minute divisions whereas the hour hand covers 5 minute divisions.

\therefore Relative Speed = $60 - 5 = 55$ minutes per hour.

Points to Note:

- | Any angle is made 22 times in a period of 12 hours.
- | In a period of 12 hours, the two hands coincide 11 times and oppose each other 11 times.
- | The time gap between any two coincidences is $12/11$ hours or $65\frac{5}{11}$ minutes.
- | If the hands of a clock (which do not show the correct time) coincide every m minutes, then If $m > 65\frac{5}{11}$, then the watch is going slow or losing time. If $m < 65\frac{5}{11}$, then the watch is going fast or gaining time.

To calculate the angle ' θ ' between the hands of a clock, we use the following formula

$$(i) \quad \theta = 30h - \frac{11}{2}m \quad \left(\text{when } 30h > \frac{11}{2}m \right)$$

$$(ii) \quad \theta = \frac{11}{2}m - 30h \quad \left(\text{when } \frac{11}{2}m > 30h \right)$$

(Where m = minutes and h = hours)

Worked out examples

1. What is the angle between the minute hand and the hour hand of a clock at 5 hours 40 minutes?

Sol: The angle between the hands can be calculated by

$$\theta = \left| \frac{11}{2}m - 30h \right|, \text{ where } m \text{ is minutes and } h \text{ is hours.}$$

Here, $m = 40$ and $h = 5$

$$\therefore \theta = \left| \frac{11}{2} \times 40 - 30 \times 5 \right|$$

$$= |220 - 150| = 70^\circ$$

The angle between the two hands is 70° .

2. Find the angle between the hands of a clock when the time is 30 min past 2.

Sol: The angle between the hands can be calculated

$$\text{by } \theta = \left| \frac{11}{2}m - 30h \right|, \text{ where } m \text{ is minutes and } h \text{ is hours.}$$

Here, $m = 30$ and $h = 2$

$$\therefore \theta = \left| \frac{11}{2} \times 30 - 30 \times 2 \right|$$

$$= |165 - 60| = 105^\circ$$

The angle between the two hands is 105° .

3. Find the angle between the hands of a clock when the time is 20 min to 8.

Sol: The given time is 7: 40

The angle between the hands can be calculated

$$\text{by } \theta = \left| \frac{11}{2}m - 30h \right|, \text{ where } m \text{ is minutes and } h \text{ is hours.}$$

Here, $m = 40$ and $h = 7$

$$\therefore \theta = \left| \frac{11}{2} \times 40 - 30 \times 7 \right|$$

$$= |220 - 210| = 10^\circ$$

The angle between the two hands is 10° .

4. Find the angle between the hands of a clock when the time is 10 min past 12.

Sol: Here we need to consider the time as 00: 10

The angle between the hands can be calculated

$$\text{by } \theta = \left| \frac{11}{2}m - 30h \right|, \text{ where } m \text{ is minutes and } h \text{ is hours.}$$

Here, $m = 10$ and $h = 0$

$$\therefore \theta = \left| \frac{11}{2} \times 10 - 30 \times 0 \right|$$

$$= |55 - 0| = 55^\circ$$

The angle between the two hands is 55° .

5. Find the time between 2 and 3 O'clock at which the minute hand and the hour hand make an angle of 70° with each other.

Sol: In the formula $\theta = \left| \frac{11}{2}m - 30h \right|$

$$\theta = 70^\circ \text{ and } h = 2$$

$$\therefore 70 = \frac{11}{2}m - 30 \times 2$$

$$\frac{11}{2}m = 130$$

$$m = \frac{260}{11} = 23 \frac{7}{11} \text{ min}$$

or

$$\therefore \frac{11}{2}m = 10$$

$$m = \frac{20}{11} \text{ min} = 1 \frac{9}{11}$$

Therefore, the angle between the hour hand and the minute hand is 70° at $1 \frac{9}{11}$ minutes past 2 O'clock and at $23 \frac{7}{11}$ minutes past 2 O'clock.

6. Find the time between 3 and 4 O'clock at which the minute hand and the hour hand overlap.

When the two hands overlap, the angle between them is 0° .

$$\theta = \left| \frac{11}{2}m - 30h \right|$$

$$\theta = 0^\circ \text{ and } h = 3$$

$$\frac{11}{2}m = 30 \times 3$$

$$m = \frac{180}{11} = 16 \frac{4}{11} \text{ min past 3.}$$

7. Find the time between 4 and 5 O'clock at which the minute hand and the hour hand are perpendicular to each other.

When two hands are perpendicular, $\theta = 90^\circ$ & $h = 4$

$$\therefore \theta = \left[\frac{11}{2}m - 30h \right] \text{ or } \left[30h - \frac{11}{2}m \right]$$

$$90 = \frac{11}{2}m - 30 \times 4$$

$$\frac{11}{2}m = 210$$

$$m = 420/11 = 38 \frac{2}{11} \text{ minutes past 4.}$$

or

$$90 = 30 \times 4 - \frac{11}{2}m$$

$$\frac{11}{2}m = 30$$

$$m = \frac{60}{11} = 5 \frac{5}{11} \text{ minutes past 4.}$$

8. Find the time between 2 and 3 O'clock at which the minute hand and the hour hand are on the same straight line but are facing opposite directions.

$$180^\circ = \frac{11}{2}m - 30h$$

$$\frac{11}{2}m = 180 + 60 = 240$$

$$m = \frac{480}{11} = 43 \frac{7}{11}$$

So, at $43 \frac{7}{11}$ minutes past 2 O'clock the hands will be at 180° .

	Half day	Full day
Coincide	11	22
Opposite	11	22
Any other except 0° and 180°	22	44

Exercise

Directions for questions 1 to 25: Select the correct alternative from the given choices.

- What is the angle covered by the minute hand in 32 minutes?
(1) 66° (2) 192° (3) 96° (4) 120°
- By how many degrees does an hour hand move in three quarter of an hour?
(1) 5° (2) 7.5° (3) 10° (4) 22.5°
- By how many degrees will the hour hand move, in the same time, in which the minute hand moves 60° ?
(1) 5° (2) 8° (3) 7° (4) 60°
- What is the angle between the hands of the clock, when it shows 20 minutes past 6?
(1) 40° (2) 70° (3) 80° (4) 100°
- When the clock shows 7 hours 08 minutes, what is the angle between the hands of the clock?
(1) 160° (2) 156° (3) 166° (4) 146°
- What is the angle between the hands of a clock when the time is 10:20?
(1) 160° (2) 170° (3) 175° (4) 165°
- When the clock shows 30 minutes past 11 O'clock, what is the angle between the two hands of the clock?
(1) 110° (2) 120° (3) 165° (4) 160°

8. At what time between 7 and 8 O'clock, will both the two hands of the clock coincide?
 (1) 7:38 $\frac{3}{11}$ (2) 7:35 $\frac{6}{11}$
 (3) 7:38 $\frac{2}{11}$ (4) 7:36 $\frac{6}{11}$
9. At what time between 4 and 5 O'clock are the hands of a clock in the opposite directions?
 (1) 4:52 $\frac{3}{11}$ (2) 4:54 $\frac{6}{11}$
 (3) 4:53 $\frac{9}{11}$ (4) 4:58 $\frac{2}{11}$
10. The angle between the hands of a clock is 20° and the hour hand is in between 5 and 6. What is the time shown by the clock?
 (1) 5: 17 $\frac{3}{11}$ (2) 5: 30 $\frac{10}{11}$
 (3) 5: 15 $\frac{5}{11}$ (4) 5: 23 $\frac{7}{11}$
 (5) Both (2) and (4)
11. How many times, the hands of a clock will be at 50° with each other in a day?
 (1) 22 (2) 40 (3) 44 (4) 48
12. How many times, the minute hand of a clock overlaps with the hour hand from 7 a.m. to 5 p.m. in a day?
 (1) 10 (2) 6 (3) 7 (4) 9
13. In a day,
 (i) For how many times will the hands be at the same position?
 (1) 10 (2) 11 (3) 12 (4) 22
 (ii) For how many times will the hands be pointing to opposite direction?
 (1) 11 (2) 12 (3) 22 (4) 24
14. In a day,
 (i) For how many times will the hands be perpendicular to each other?
 (1) 44 (2) 45 (3) 46 (4) 48
 (ii) For how many times will the angle between the hands be 150° ?
 (1) 48 (2) 46 (3) 44 (4) 42
15. A.T =actual time, M.T =mirror time
 (i) If A.T. = 8:20, find M.T.
 (1) 7:40 (2) 3:20 (3) 2:50 (4) 3:40
 (ii) If M.T. = 4:45, find A.T.
 (1) 7:40 (2) 8:20 (3) 7:15 (4) 5:15
16. A clock loses 5 seconds an hour and is set right Sunday at noon. What time will it indicate on the following Wednesday at noon?
 (1) 11:40 (2) 11:58 (3) 11:54 (4) 11:48
17. A clock strikes once at 1 O'clock, twice at 2 O'clock, three times at 3 O'clock and so on. If it takes 10 seconds to strike at 6 O'clock, find the time taken by it to strike at 12 O'clock.
 (1) 18 sec (2) 22 sec
 (3) 24 sec (4) 26 sec
18. A clock strikes once at 1 O' clock , twice at 2 O' clock, thrice at 3 O' clock and so on. If the clock takes 24 seconds to strike at 4 O' clock, how much time will it take to strike at 8 O' clock?
 (1) 40 (2) 44 (3) 48 (4) 56
19. a. If the hands of a clock coincide for every 66 minutes, then the clock loses how many minutes a day?
 (1) 10 $\frac{10}{11}$ (2) 11 $\frac{109}{121}$
 (3) 10 $\frac{109}{121}$ (4) 12
 b. If the hands of a clock coincide for every 65 minutes, then the clock gains how many minutes a day?
 (1) 10 $\frac{10}{143}$ (2) 11 $\frac{109}{121}$
 (3) 15 (4) 10
20. Find the time between 5 and 6 O'clock at which the minute hand and the hour hand are perpendicular to each other.
 (1) 5:10 $\frac{10}{11}$ (2) 5:14 $\frac{6}{11}$
 (3) 5:13 $\frac{9}{11}$ (4) 5:18 $\frac{2}{11}$

Clocks							
1	2	6	2	11	3	16	3
2	4	7	3	12	4	17	2
3	1	8	3	13	4,3	18	4
4	2	9	2	14	1,3	19	2,1
5	3	10	5	15	4,3	20	1