CS6015 : LINEAR ALGEBRA AND RANDOM PROCESSES

PROGRAMMING ASSIGNMENT

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Group No : 12

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0.1 Problem-1

Q1. a) Given matrix is

$$A = \begin{bmatrix} 3 & 6 & 6 \\ 4 & 8 & 8 \end{bmatrix}$$

The column space of A

$$C(A) = span\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Projection matrix onto C(A)

$$Pc = C(A) * inv(C(A)' * C(A)) * C(A)' \tag{1}$$

Therefore we get,

$$Pc = \begin{bmatrix} 0.3600 & 0.4800 \\ 0.4800 & 0.6400 \end{bmatrix}$$

using equation 1.

Q1. b) The row space of A

$$C(A') = span\begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix}$$

Projection matrix onto C(A')

$$Pr = C(A') * inv(C(A')' * C(A')) * C(A')' \tag{2}$$

Therefore we get,

$$Pr = \begin{bmatrix} 0.1111 & 0.2222 & 0.2222 \\ 0.2222 & 0.4444 & 0.4444 \\ 0.2222 & 0.4444 & 0.4444 \end{bmatrix}$$

using equation 2.

Now by multiplying Pc, A and Pr

$$B = Pc * A * Pr \tag{3}$$

We get,

$$B = \begin{bmatrix} 0.3600 & 0.4800 \\ 0.4800 & 0.6400 \end{bmatrix} \begin{bmatrix} 3 & 6 & 6 \\ 4 & 8 & 8 \end{bmatrix} \begin{bmatrix} 0.1111 & 0.2222 & 0.2222 \\ 0.2222 & 0.4444 & 0.4444 \\ 0.2222 & 0.4444 & 0.4444 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 6 \\ 4 & 8 & 8 \end{bmatrix} \begin{bmatrix} 0.1111 & 0.2222 & 0.2222 \\ 0.2222 & 0.4444 & 0.4444 \\ 0.2222 & 0.4444 & 0.4444 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 6 \\ 4 & 8 & 8 \end{bmatrix}$$

Here, we get matrix B same as matrix A.

Explanation:
We observed that

$$Pc * A = A \tag{4}$$

$$A * Pr = A \tag{5}$$

which means multiplying the projection matrix Pc on the left side of the matrix A is equivalent to projecting each column of matrix A onto the column space or subspace of A which gives back the matrix A itself.

Similarly, multiplying the projection matrix Pr on the right side of the matrix A is equivalent to projecting each row of matrix A onto the row space or subspace of A which also gives back the matrix A itself.

0.2 PROBLEM-2

Q2. a)

The intensity histogram of an image is a histogram that shows the number of pixels in an image at each different intensity value found in that image.

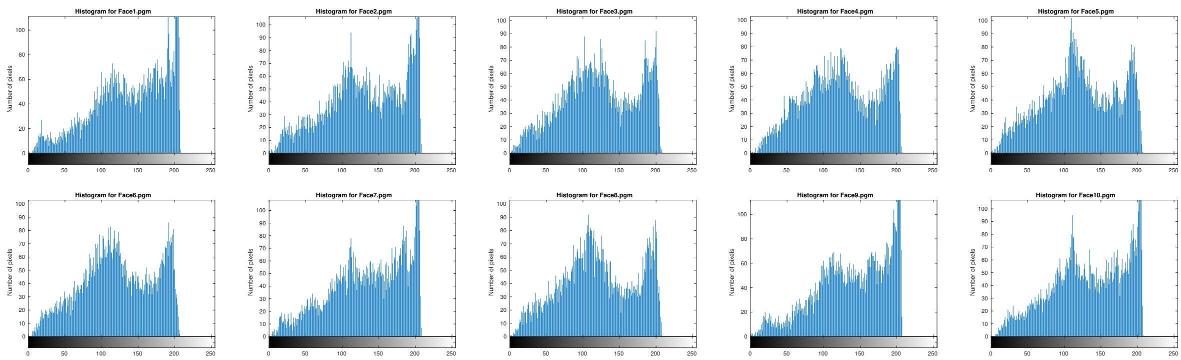

Figure 1: Intensity histograms for each image (face-1 to face 10).

Q2. b

The basis given in EigenVector.mat file plotted as images is shown below. Each column of the matrix represents an image. The dimension of the matrix is 8464 x 8464, hence we are getting 8464 images each of size 92 x 92. Here, we have plotted top 50 eigen basis vectors as images.

Figure 2: Images corresponding to 50 eigen basis

Q2. c To project the given images onto the eigen space, there is a need to find those eigen vectors which can appropriately describe the images. To accomplish this, the dot product of each eigen vector in the eigen space and the images are calculated, which in turn serves as a metric to help us analyze the importance of those eigen vectors in our images. The reconstructed and the error images for top K different values where K = 1,1000, 5900 are shown in Figure 3.

As observed from the graphs in Figure 4 and 5, all of the 10 images can be reconstructed with less than 1 % relative error using Top K values where K is approximately greater than or equal to 5700, for each of the images.

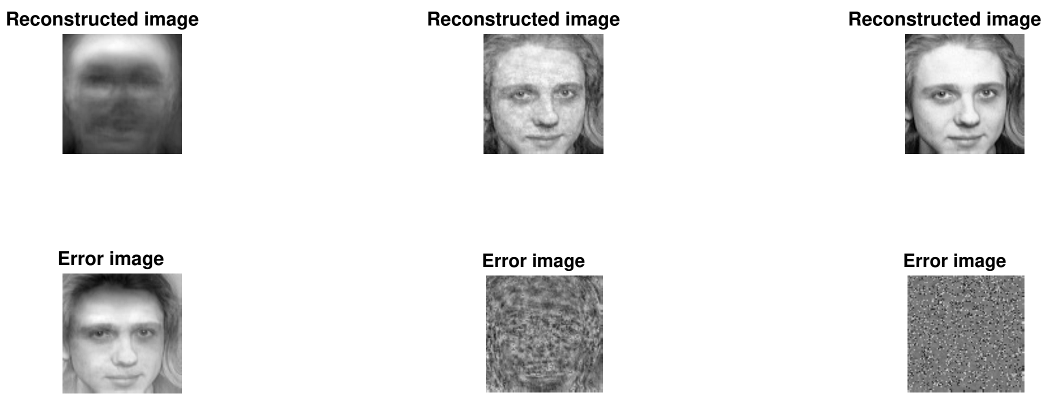

Figure 3: Reconstructed and error images for K = 1,1000,5900 for face10

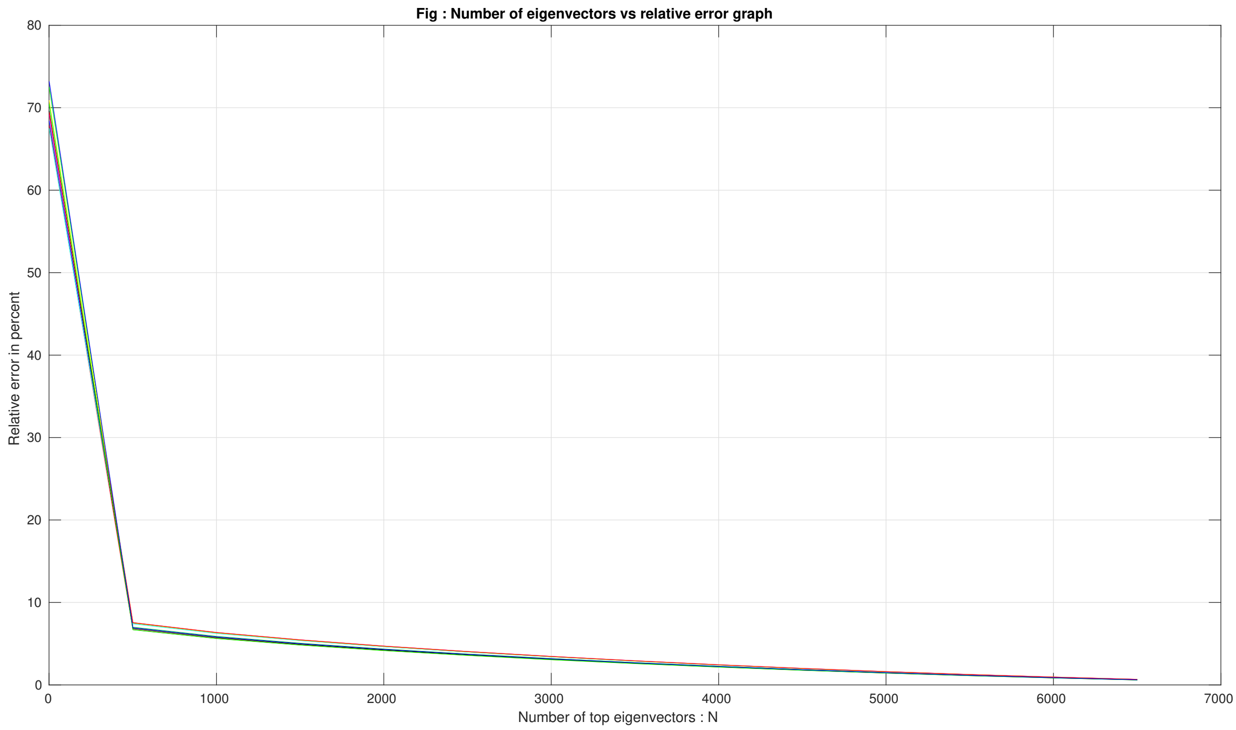

Figure 4: Relative Error V/S Top K eigen vectors plot for all 10 faces

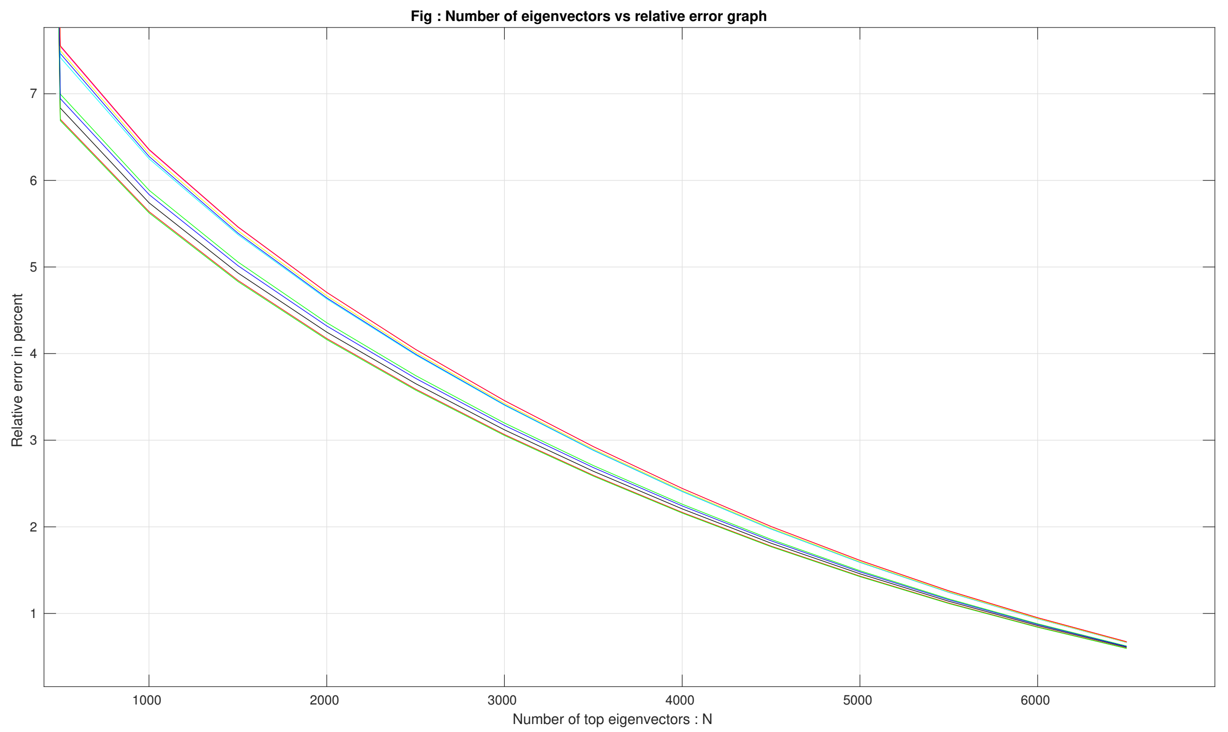

Figure 5: Zoomed image of Figure No. 5