Minimal Cover for a Set of FDs

Minimal Cover for a Set of FDs

- ***Minimal cover*** **G for a set of FDs F:**
  - Closure of F = Closure of G.
  - Right hand side of each FD in G is a single attribute.
  - If we modify G by deleting an FD or by deleting attributes from an FD in G, the closure changes.
- **Intuitively, every FD in G is needed, and ``*as small as possible*'' in order to get the same closure as F.**
- **e.g., $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$ has the following minimal cover:**
  - $A \rightarrow B$, $ACD \rightarrow E$, $EF \rightarrow G$ and $EF \rightarrow H$
- **Minimal Cover implies Lossless-Join, Defendency Preserving Decomposition.**
  - Start with M.C. of F, do the decomposition from Minimal Cover of F

Functional dependencies

Our goal:

given a set of FD set, F, find an alternative FD set, G that is:

- smaller
- equivalent

Bad news:

Testing F=G (F+ = G+) is computationally expensive

Good news:

Canonical Cover algorithm:

given a set of FD, F, finds minimal FD set equivalent to F

Minimal: can't find another equivalent FD set w/ fewer FD's

Minimal cover

F = { AB -> C, C -> A, BC -> D, ACD -> B, D -> E, D -> G, BE -> C, CG -> B,

CG -> D, CE -> A, CE -> G}

Notice that we have single attribute on the RHS in all FDs, we need to look for extraneous (redundant) attributes on the LHS and also look for FDs that are redundant.

Canonical Cover Algorithm

Given:

$$F = \{ A \rightarrow BC, \\ B \rightarrow CE, \\ A \rightarrow E, \\ AC \rightarrow H, \\ D \rightarrow B\}$$

- Fc = F
- No G that is equivalent to F and is smaller than Fc

Determines canonical cover of F:

$$Fc = \{ A \rightarrow BH, \\ B \rightarrow CE, \\ D \rightarrow B\}$$

Another example:

$$F = \{ A \rightarrow BC, \\ B \rightarrow C, \\ A \rightarrow B, \\ AB \rightarrow C, \\ AC \rightarrow D\}$$ $\longrightarrow$ CC Algorithm $\longrightarrow$ $$Fc = \{ A \rightarrow BD, \\ B \rightarrow C\}$$

Canonical Cover Algorithm

Basic Algorithm

ALGORITHM CanonicalCover (X: FD set)
BEGIN
REPEAT UNTIL STABLE
(1) Where possible, apply Additivity rule (A's axioms)
(e.g., A →BC, A→CD becomes A→BCD)
(2) remove "extraneous attributes" from each FD
(e.g., AB→C, A→B becomes
A→B, B→C
i.e., A is extraneous in AB→C)

Extraneous Attributes

(1) Extraneous is RHS?

e.g.: can we replace $A \rightarrow BC$ with $A \rightarrow C$?

(i.e. Is B extraneous in $A \rightarrow BC$?)

(2) Extraneous in LHS ?

e.g.: can we replace $AB \rightarrow C$ with $A \rightarrow C$?

(i.e. Is B extraneous in $AB \rightarrow C$?)

Simple but expensive test:

1. Replace $A \rightarrow BC$ (or $AB \rightarrow C$) with $A \rightarrow C$ in F

$$F2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$$

or

$$F - \{AB \rightarrow C\} \cup \{A \rightarrow C\}$$

2. Test if $F2+ = F+$?

   if yes, then B extraneous

Extraneous Attributes

A. RHS: Is B extraneous in $A \rightarrow BC$?

step 1: $F2 = F - \{A \rightarrow BC\} \cup \{A \rightarrow C\}$
step 2: $F+ = F2+$?

To simplify step 2, observe that $F2+ \subseteq F+$

Why? Have effectively removed $A \rightarrow B$ from F i.e., not new FD's in F2+)
When is $F+ = F2+$?

Ans. When $(A \rightarrow B)$ in $F2+$

Idea: if $F2+$ includes: $A \rightarrow B$ and $A \rightarrow C$,
then it includes $A \rightarrow BC$

Extraneous Attributes

B. LHS: Is B extraneous in A B→C ?

step 1: F2 = F - {AB →C} U {A →C}
step 2: F+ = F2+ ?

To simplify step 2, observe that F+ $\subseteq$ F2+

Why? A→C "implies" AB→C. therefore all FD's in F+ also in F2+. i.e., there may be new FD's in F2+)
But AB→C does not "imply" A→C

When is F+ = F2+ ?

Ans. When (A→C) in F+

Idea: if F+ includes: A→C then it will include all the FD's of F2+.

Extraneous attributes

A. RHS :

Given F = {A→BC, B→C} is C extraneous in A →BC?

why or why not?

Ans: yes, because

A→C in { A→B, B→C}+

Proof. 1. A→ B

2. B →C

3. A→C transitivity using Armstrong's axioms

Extraneous attributes

B. LHS :

Given $F = \{A \rightarrow B, AB \rightarrow C\}$ is B extraneous in $AB \rightarrow C$?

why or why not?

Ans: yes, because

$A \rightarrow C$ in $F+$

Proof. 1. $A \rightarrow B$

2. $AB \rightarrow C$

3. $A \rightarrow C$ using pseudotransitivity on 1 and 2

Actually, we have $AA \rightarrow C$ but $\{A, A\} = \{A\}$

Canonical Cover Algorithm

ALGORITHM CanonicalCover (F: set of FD's)
BEGIN
REPEAT UNTIL STABLE
(1) Where possible, apply Additivity rule (A's axioms)

(2) Remove all extraneous attributes:
a. Test if B extraneous in $A \rightarrow BC$
(B extraneous if
$(A \rightarrow B)$ in $(F - \{A \rightarrow BC\} \cup \{A \rightarrow C\})^+$)
b. Test if B extraneous in $AB \rightarrow C$
(B extraneous in $AB \rightarrow C$ if
$(A \rightarrow C)$ in F^+)

Canonical Cover Algorithm

Example: determine the canonical cover of

$F = \{A \rightarrow BC, B \rightarrow CE, A \rightarrow E\}$

Iteration 1:

a. $F = \{ A \rightarrow BCE, B \rightarrow CE\}$
b. Must check for upto 5 extraneous attributes

- B extraneous in $A \rightarrow BCE$? No
- C extraneous in $A \rightarrow BCE$?
  yes: $(A \rightarrow C)$ in $\{ A \rightarrow BE, B \rightarrow CE\}$
  1. $A \rightarrow BE$ -> 2. $A \rightarrow B$ -> 3. $A \rightarrow CE$ -> 4. $A \rightarrow C$
- E extraneous in $A \rightarrow BE$?

Canonical Cover Algorithm

Example: determine the canonical cover of

$F = \{A \rightarrow BC, B \rightarrow CE, A \rightarrow E\}$

Iteration 1:

a. $F = \{ A \rightarrow BCE, B \rightarrow CE\}$

b. Must check for upto 5 extraneous attributes

- B extraneous in $A \rightarrow BCE$? No
- C extraneous in $A \rightarrow BCE$? Yes
- E extraneous in $A \rightarrow BE$?

  1. $A \rightarrow B$ -> 2. $A \rightarrow CE$ -> $A \rightarrow E$
- E extraneous in $B \rightarrow CE$ No
- C extraneous in $B \rightarrow CE$ No

Iteration 2:

a. $F = \{ A \rightarrow B, B \rightarrow CE\}$

b. Extraneous attributes:

- C extraneous in $B \rightarrow CE$ No
- E extraneous in $B \rightarrow CE$ No

DONE

Canonical Cover Algorithm

Find the canonical cover of

$$F = \{\ A \rightarrow BC,\ B \rightarrow CE,\ A \rightarrow E,\ AC \rightarrow H,\ D \rightarrow B\}$$

Ans: $F_c = \{\ A \rightarrow BH,\ B \rightarrow CE,\ D \rightarrow B\}$

Canonical Cover Algorithm

Find two different canonical covers of:

$F = \{ A \rightarrow BC, \quad B \rightarrow CA, \quad C \rightarrow AB \}$

Ans:

$Fc1 = \{ A \rightarrow B, B \rightarrow C, C \rightarrow A \}$

and

$Fc2 = \{ A \rightarrow C, B \rightarrow A, C \rightarrow B \}$