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CS3233
Competitive Programming

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Week 12 – Harder Stuffs


Jan-Apr 2009


Jan-Apr 2010


Jan-Apr 2011


Jan-Apr 2012

Outline

- Mini Contest #10 (the last one) + Discussion + Break
- CLASS PHOTO!!
- Admins
- Last Lecture (let’s not get too ambitious):
  - Problem Decomposition (Section 8.2)
  - Meet in the Middle/Bidirectional Search

The harder ones

(you have seen some of these before; now, let's demistify some of them)

Soft skills needed:

Ability to spot the individual components and break them apart!

This is based on what I know from ~ 1422 UVa problems

PROBLEM DECOMPOSITION

Problem Decomposition (1)

Binary Search the Answers + X

- We have seen this form earlier (Chapter 3.3)
- But the “X” component of this ‘classical’ combination can be “many thing”, not just simulation problem
- So far, I have seen that X can be:
  - Greedy algorithm: UVa 714, 11516
  - MCBM: UVa 10804, 11262
  - SSSP: UVa 10186
  - Max Flow: UVa 10983
- Tips to spot this type: If you guess the answer, will the problem turn into a True/False problem?

Problem Decomposition (2)

Involving DP 1D Range Sum/Max/Min

- This one can be easily decomposed
- Tips to spot this type:
  The problem ask you for **static range queries**
  - Especially the 1D one
  - Usually range sum, but can also be max/min queries, how?
- **Range Sum Query**: Pre-process the answers in O(n)
  - dp[0] = ans[0]
  - dp[i] = dp[i-1] + ans[i] $\forall$ i $\in$[1..n-1]
- So that each **RSQ** can be answered in O(1)
  - rsq(i, j) =dp[j] if i == 0, or dp[j] - dp[i - 1] if j > 0

Problem Decomposition (3)

SSSP/APSP/SCC contraction + DP/Something else

- Another 'classical' combination is to use shortest path (or SCC contraction) as one sub problem to transform the original problem into a shortest path table (or a DAG) and then pass this table (or DAG) to a DP/other solution
  - BFS/Dijsktra's to build shortest path matrix → DP-TSP (UVa 10937, 10944, 10405, 11813, NOI 2011)
  - Run Dijkstra's algorithm → build DAG from SP information → Counting paths on DAG (UVa 10917)
  - Run Floyd Warshall's algorithm → do something else (UVa 1233, 10793, 11463)
  - Run Tarjan's SCC algorithm to contract SCC → Longest Path in DAG (UVa 11324)
- Tips to spot this type: Shortest path (or SCC) is one of the component, but not the only one…

Problem Decomposition (4)

X + Y

- Here, X is the "main issue"
  - But that problem is written in Y flavour
- Tips to spot this type: Usually,
  - X is either: BFS, Complete Search, Binary Search, (mostly Chapter 3 stuffs), and
  - Y is either: Graph, Mathematics, or Geometry (mostly Chapter 4-5-7 stuffs)
- Example: UVa 11730
  - Actually a BFS (SSSP on unweighted graph) problem
  - But the graph is implicitly derived via Mathematical rules

Problem Decomposition (5)

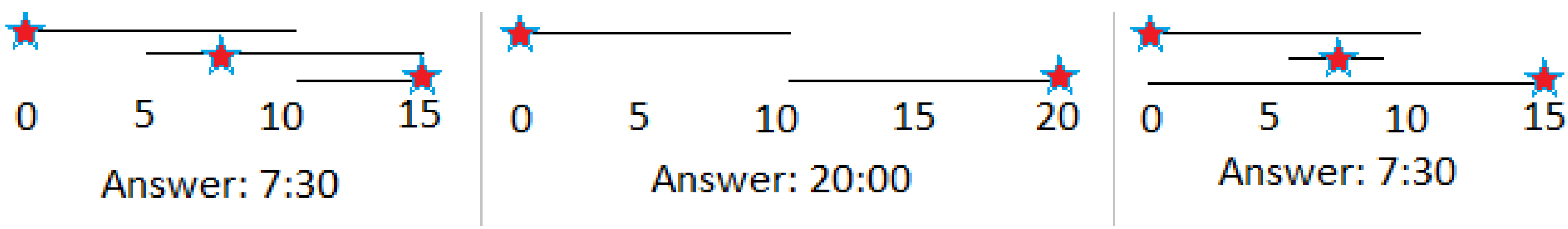
Involving (Advanced) Data Structures/DS

- Tips to spot this type: If you got a problem "AC" but very slow (TLE)
- Consider the possibility that some operations in your algorithm can be optimized by using a better DS
  - This better DS are usually harder to implement though
- These DSes are usually:
  - Balanced BST: map/set,
    or the self-coded one due to the need to *augment data*
  - Binary Indexed (Fenwick) Tree
  - Segment Tree, etc

Problem Decomposition (6)

Three (or More?) Components

- UVa 1079 – A Careful Approach
  - http://uva.onlinejudge.org/external/10/1079.html
  - ACM ICPC World Finals 2009 problem
- Solution:
  - Complete Search + Binary Search the Answer + Greedy :O



Problem Decomposition (7)

- There are many other possible combinations…
- Note: If there are X basic types of contest problems…
  - There can be ${}_{X}C_{2}$ possible pairs of combinations
  - And there can be ${}_{X}C_{3}$ triples…
- You will get more familiar to spot the individual components as you master them
  - All the best

a.k.a. Bidirectional Search

MEET IN THE MIDDLE

UVa 11212 – Editing a Book

Rujia Liu's Problem

- Given *n* equal-length paragraphs numbered from 1 to *n*
- Arrange them in the order of 1, 2, ..., *n*
- With the help of a clipboard,
  you can press Ctrl-X (cut) and Ctrl-V (paste) several times
  - You cannot cut twice before pasting, but you can cut several contiguous paragraphs at the same time - they'll be pasted in order
- The question: What is the minimum number of steps required?
- Example 1: In order to make {2, 4, (1), 5, 3, 6} sorted,
  you can cut 1 and paste it before 2 → {**1,** 2, 4, 5, (3), 6}
  then cut 3 and paste before 4 → {1, 2, **3,** 4, 5, 6} → done √
- Example 2: In order to make {(3, 4, 5), 1, 2} sorted,
  you can cut {3, 4, 5} and paste it after {1, 2} → {1, 2, **3, 4, 5**} √
  or cut {1, 2} and paste it before {3, 4, 5} → {**1, 2**, 3, 4, 5} √

Loose Upper Bound

- Answer: *k*-1
  - Where *k* is the number of paragraph in the wrong position
- Trivial but wrong algorithm:
  - Cut a paragraph that is in the wrong position
  - Paste that paragraph in the correct position
  - After *k*-1 such cut-paste, we will have a sorted paragraph
    - The last wrong position will be in the correct position at this stage
  - But this may not be the shortest way
- Examples:
  - {(3), 2, 1} → {(2), 1, **3**} → {1, **2**, 3} → 2 steps
  - {(5), 4, 3, 2, 1} → {(4), 3, 2, 1, **5**} → {(3), 2, 1, **4**, 5} → {(2), 1, **3**, 4, 5} → {1, **2**, 3, 4, 5} → 4 steps

The Actual Answers

- {3, 2, 1}
  - Answer: 2 steps, e.g.
    - {(3), 2, 1} → {(2), 1, **3**} → {1, **2**, 3}, or
    - {3, 2, (1)} → {**1**, (3), 2,} → {1, 2, **3**}
- {5, 4, 3, 2, 1}
  - Answer: Only **3** steps, e.g.
    - {5, 4, (3, 2), 1} → {**3, (2**, 5), 4, 1} → {3, 4, (1, **2), 5**} → {**1, 2**, 3, 4, 5}
- How about {5, 4, 9, 8, 7, 3, 2, 1, 6}?
  - Answer: 4, but very hard to compute manually
- How about {9, 8, 7, 6, 5, 4, 3, 2, 1}?
  - Answer: 5, but very hard to compute manually

Some Analysis

- There are at most n! permutations of paragraphs
  - With maximum n = 9, this is 9! or 362880
  - The number of vertices is not that big actually
- Given a permutation of length n (a vertex)
  - There are ${}_nC_2$ possible cutting points (index i, j $\in$ [1..n])
  - There are n possible pasting points (index k $\in$ [1..(n-(j-i+1))])
  - Therefore, for each vertex, there are about $O(n^3)$ branches
- The worst case behavior if we run single BFS on this search space graph: $O(V+E) = O(n! + n!*n^3) = O(n!*n^3)$
  - With n = 9, this is $9! * 9^3 = 264539520$ ~ 265 M, TLE (or maybe MLE…)

All other details are hidden for NUS ACM ICPC/Singapore IOI teams only :)

Actually we will still meet again next week for final contest :D

SOME PARTING WORDS

What You Have Been Exposed To
(as of Tonight, Wed 04 Apr 2012)

- Competitive Coding Style
- Extensive usage of libraries
- Bitmask
- BIT/FT
- Iterative BF Techniques: Subset, Permutation
- Recursive backtracking
- Some classical Greedy problems
- Binary Search the Answer
- The thinking process to get DP states and transitions
- Graph DS, Traversal: DFS/BFS, MST (briefly), SSSP: Dijkstra's, Bellman Ford's, APSP: Floyd Warshall's
- Tarjan's SCC algorithm
- More DP techniques
- Network Flow: Edmonds Karps'
- Bipartite Graph: MCBM++
- Mathematics-related problems: Log techniques, Big Integer, Prime Factor techniques, Modulo arithmetic
- Various string processing skills
- Suffix Tree/Array: String Matching, Longest Repeated Substring, Longest Common Substring
- Basic geometry routines
- Algorithms on polygon
- Problem decomposition
- Meet in the middle/bidirectional search

What You Have NOT Been Exposed To
(as of Tonight, Wed 04 Apr 2012)

- Many more cool and exotic algorithms out there :O
- Maybe read CP3 in the future ☺
- Or join NUS ACM ICPC trainings
- Or do self study