

# Dynamic Programming

# Maximum Sum Sub-Array

Given an array

31	-41	59	26	-53	58	97	-93	-23	84
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Each sub-array has a sum. What is the maximum such sum possible.

Maximum Sum for any sub Array ending at  $i^{\text{th}}$  location

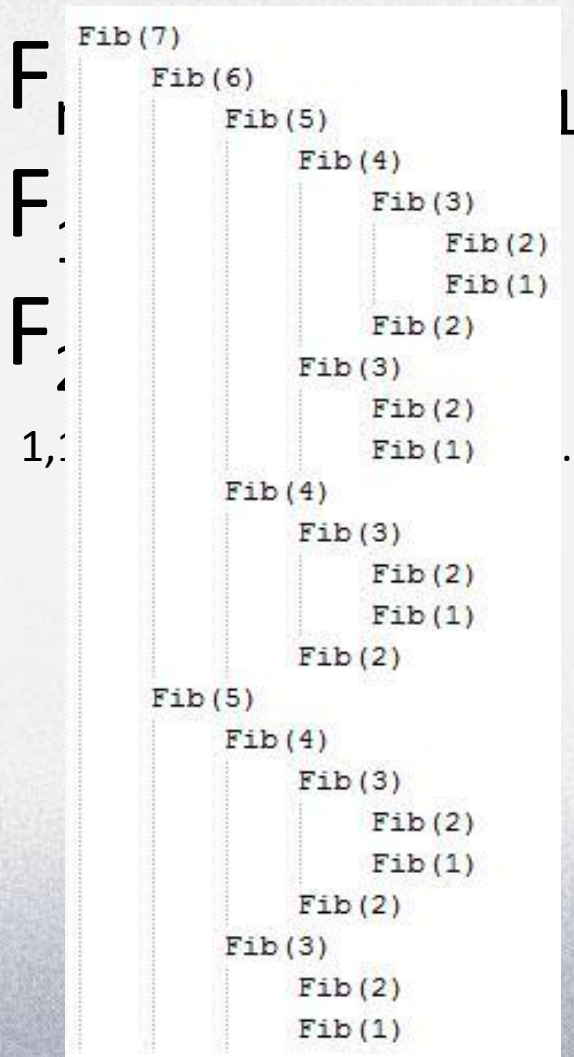
31	0	59	85	32	90	187	94	71	155
----	---	----	----	----	----	-----	----	----	-----

Maximum so far

31	31	59	85	85	90	187	187	187	187
----	----	----	----	----	----	-----	-----	-----	-----

# Fibonacci Sequence

Execution Trace:



Naïve Recursive Function

```
int Fib(int n){  
    if(n==1 || n==2)  
        return 1;  
    return Fib(n-1)+Fib(n-2);  
}
```

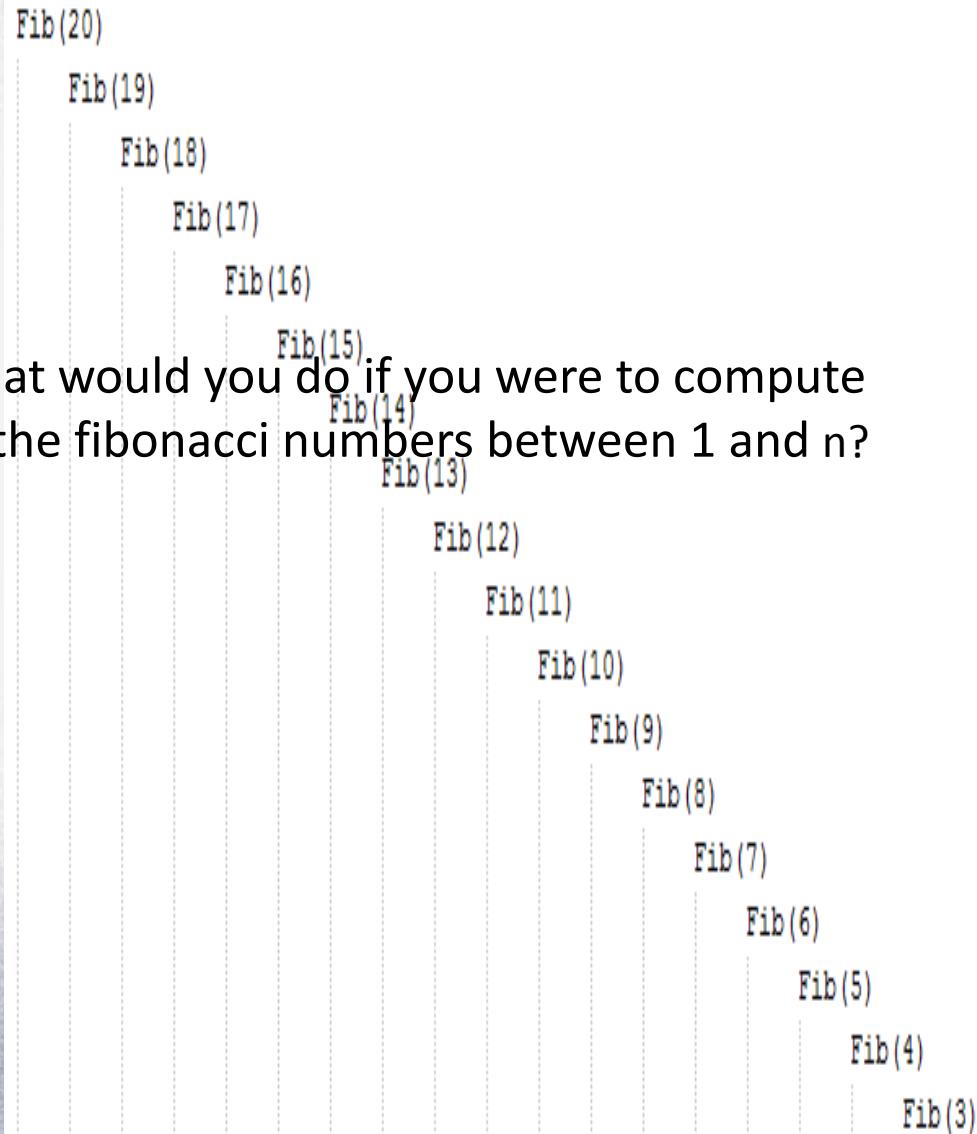


# Fibonacci Sequence (Doing it Cleverly)

```
int fib[100];
memset(fib,0,sizeof(fib));
fib[1]=fib[2]=1;

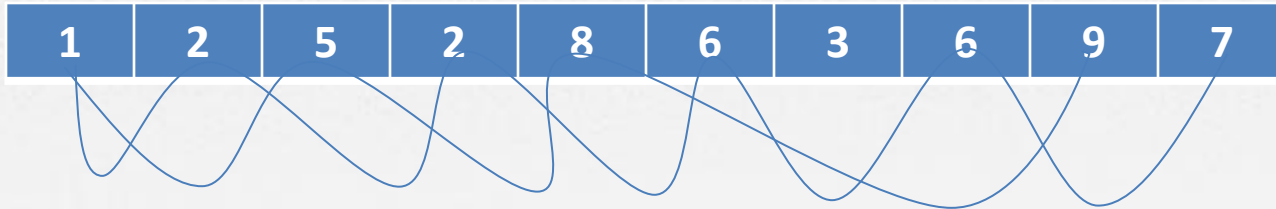
int Fib(int n){
    int x,y;
    if(n==1 || n==2)
        return 1;
    if(fib[n-1])
        x = fib[n-1];
    else
        x = fib[n-1] = Fib(n-1);
    if(fib[n-2])
        y = fib[n-2];
    else
        y = fib[n-2] = Fib(n-2);
    return x+y;
}
```

What would you do if you were to compute all the fibonacci numbers between 1 and n?



# Longest Non Decreasing Subsequence

Given an array



Find a subsequence which is non decreasing and of maximum length

1-5-8-9 Forms a non decreasing subsequence

So does 1-2-2-6-6-7 but it is longer

Our aim is to find the longest such subsequence

# Longest Non Decreasing Subsequence

1	2	5	2	8	6	3	6	9	7
---	---	---	---	---	---	---	---	---	---

Length of LNDS ending at  $i^{\text{th}}$  Loc

1	2	3	3	4	4	4	5	6	6
---	---	---	---	---	---	---	---	---	---

```
for(i=0;i<100;i++){
    max= 0;
    for(j=0;j<i;j++){
        if(A[i] >= A[j] && L[j] > max)
            max = L[j];
    }
    L[i] = max+1;
}
```



# Longest Common Subsequence

A	B	C	B	D	A	B
B	D	C	A	B	A	

Find a string such that it is a subsequence in both of the arrays and its length is longest

CAB is a subsequence of both the arrays

BDAB is also a subsequence of both the arrays, but is longer

# Practice Problems

<http://www.spoj.pl/problems/COINS/>

<http://www.spoj.pl/problems/AIBOHP/>

<http://www.spoj.pl/problems/IOIPALIN/>

<http://www.spoj.pl/problems/ADFRUITS/>

<http://www.spoj.pl/problems/NY10E/>

<http://www.spoj.pl/problems/EDIST/>

<http://www.spoj.pl/problems/RENT/>

<http://www.spoj.pl/problems/BABTWR/>