

⑨

$$[\forall x, P(x) \rightarrow Q(x)] \wedge [\forall x, Q(x) \rightarrow \neg \neg Q(x)]$$

$$[\forall x, P(x) \rightarrow Q(x)] \wedge [\forall x, Q(x) \rightarrow \neg \neg Q(x)].$$

contrapositive $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$

$$[\forall x, P(x) \rightarrow \neg \neg Q(x)]$$

again using contrapositive property

$$P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$$

$$\neg P(x) \rightarrow \neg(\neg \neg Q(x))$$

$$\neg P(x) \rightarrow \neg Q(x)$$

④ Do this using Truth Table

P	Q	$\neg Q$	$\neg P$	$\neg P \rightarrow Q$	$Q \rightarrow P$	$(\neg P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T	F	F	T	T	T
T	F	T	F	T	F	F
F	T	F	T	T	F	F
T	F	T	F	T	F	F
F	T	F	T	T	F	F
F	F	T	T	F	T	T
F	F	F	T	F	T	F

pdnf: sum of min. terms

To write min terms take only True value in last value.

Means Final Value is True in $(\sim P \Rightarrow q) \wedge (q \Rightarrow P)$

Now check P, q, & value

$P(T) \quad (q \vee T) \quad \&(T)$ Means

			$(\sim P \Rightarrow q) \wedge (q \Rightarrow P)$ Minterm
P	q	$\&$	
T	T	T	T
T	T	F	T
F	F	T	T

$P \wedge q \wedge \&$
 $P \wedge q \wedge \sim r$.
 $\sim P \wedge \sim q \wedge \&$.

pdnf = sum of min. terms

$$= (P \wedge q \wedge \&) \cup (P \wedge q \wedge \sim r) \cup (\sim P \wedge \sim q \wedge \&).$$

pcnf = sum of max. terms
To write max. terms take false in last term

			$(\sim P \Rightarrow q) \wedge (q \Rightarrow P)$ Max-term
P	q	$\&$	
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	F	F

$P \vee \sim q \vee \&$
 $P \vee \sim q \vee \sim r$.
 $\sim P \vee q \vee \&$
 $\sim P \vee q \vee \sim r$.
 $\sim P \vee \sim q \vee \&$
 $\sim P \vee \sim q \vee \sim r$.

Max. terms $(P \vee \sim q \vee \&)$, $(P \vee \sim q \vee \sim r)$, $(\sim P \vee q \vee \&)$,

$(\sim P \vee q \vee \sim r)$ $(\sim P \vee \sim q \vee \&)$ $(\sim P \vee \sim q \vee \sim r)$

$(P \vee q \vee \&) \wedge (P \vee q \vee \sim r) \wedge (\sim P \vee q \vee \&)$.

$\wedge (\sim P \vee \sim q \vee \sim r)$.
 \therefore product of Max. terms = pcnf.