

$$i = \frac{dq}{dt} \Rightarrow q = \int i(t) dt$$

$$V_{ab} = \frac{dw}{dq}$$

$$P = \frac{dw}{dt} = V_i$$

$$R = \frac{P}{A}$$

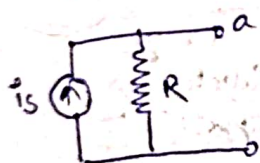
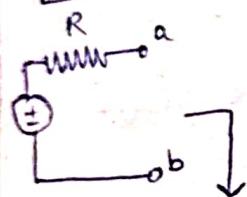
$$KCL \Rightarrow \sum i_n = 0$$

$$KVL \Rightarrow \sum V_m = 0$$

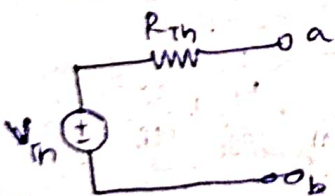
Superposition principle

Volt/Current across an element is sum of Voltages/Currents due to each independent source acting alone

Source Transformation

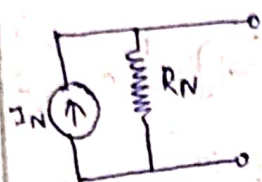
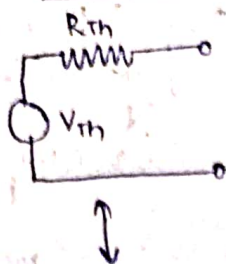


Thevenin Theorem



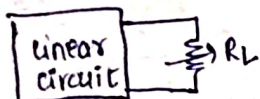
To find R_{Th} short circuit all independent voltage sources and open all independent current sources

Norton Theorem



$$I_N = \frac{V_{Th}}{R_{Th}} \quad R_N = R_{Th}$$

Maximum power Transfer principle



Max power with-drawn by R_L when $R_L = R_{Th}$ from linear circuit

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

phasors

$$z = x + jy = re^{j\phi}$$

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

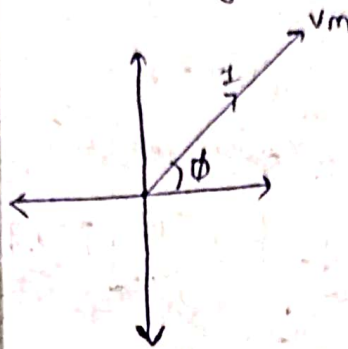
$$v(t) = V_m \cos(\omega t + \phi)$$

$$V = V_m e^{j\phi} \rightarrow \text{phasor}$$

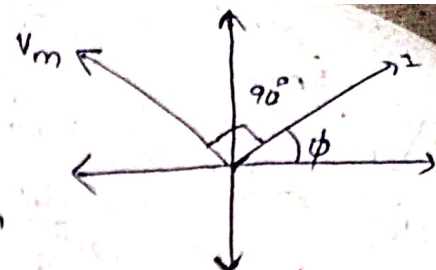
$$R \Rightarrow V = IR$$

$$L \Rightarrow V = j\omega LI$$

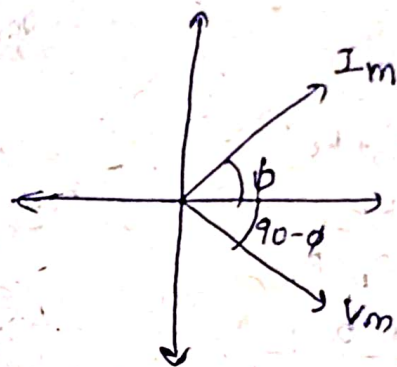
$$C \Rightarrow V = \frac{1}{j\omega C} I$$



Pure Resistive circuit



Inductive
 Z_{PL}
Inductor potential leads



Capacitor
 Z_{CL}
Capacitor current leads

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$\text{average power} = \frac{1}{2} I_m V_m \cos(\theta_v - \theta_i)$$

$$P_{max} = \frac{|V_{Th}|^2}{4R_{Th}} = \left(\frac{1}{\sqrt{2}} I_m\right) \left(\frac{1}{\sqrt{2}} V_m\right) \cos(\theta_v - \theta_i) = I_{rms} V_{rms} \cos(\theta_v - \theta_i)$$

phasors or

$$P = S \cos(\theta_v - \theta_i)$$

$S \Rightarrow$ apparent power

$\cos(\theta_v - \theta_i) \rightarrow$ power factor

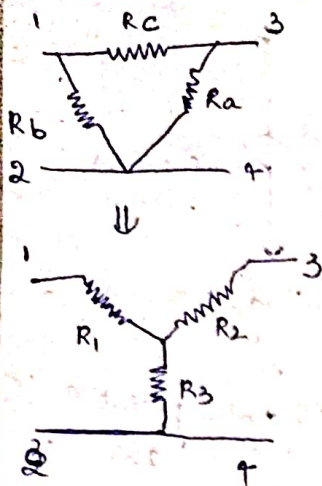
$$S = P + jQ$$

$$P = I_{rms} V_{rms} \cos \theta \rightarrow \text{real power}$$

$$Q = I_{rms} V_{rms} \sin \theta \rightarrow \text{Reactive power}$$

$$P = I_{rms}^2 R$$

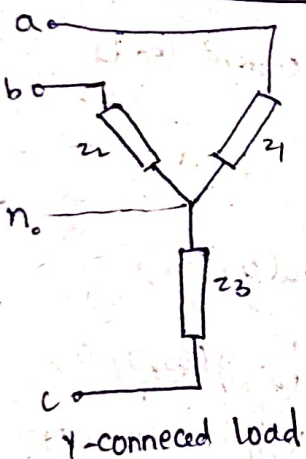
$$Q = I_{rms}^2 X$$



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

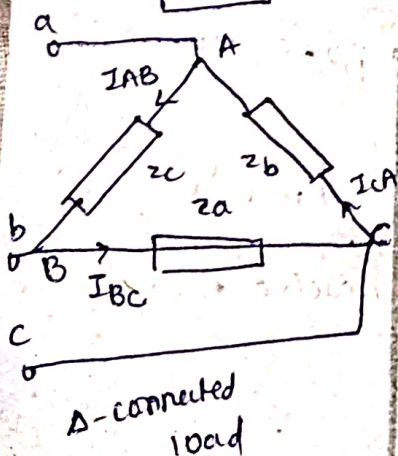
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

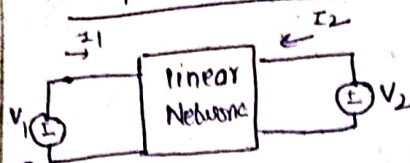


$$Z_D = 3Z_Y$$

$$D = 3Y$$



Two-port Networks



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Open-circuit admittance parameters

g

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Short circuit admittance parameters

Transmission parameters

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

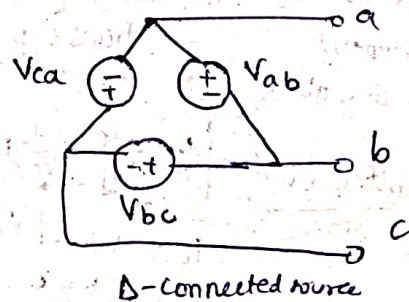
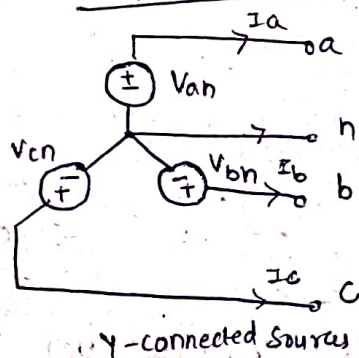
Hybrid

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

g-parameters

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

Three phase circuits



Power in 3-phase systems

per phase

$$P = V_p I_p \cos \theta$$

$$Q = V_p I_p \sin \theta$$

$$S_p = V_p I_p$$

Total

$$P = 3V_p I_p \cos \theta$$

$$Q = 3V_p I_p \sin \theta$$

$$S = 3V_p I_p$$

$$S_p = P_p + jQ_p$$

P → average

Q → Reactive

S → apparent

$$P = \sqrt{3} V_L I_L \cos \theta$$

$$Q = \sqrt{3} V_L I_L \sin \theta$$

$$S = \sqrt{3} V_L I_L$$

units P → kW
S → kVA
Q → kVAR

$$X(s) = \frac{a_0}{(s+\lambda)^r} + \frac{a_1}{(s+\lambda)^{r+1}} + \dots + \frac{a_{r-1}}{(s+\lambda)^r} + \frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + \dots + \frac{k_n}{s+p_n}$$

laplace $X(s) = \int_{-\infty}^{\infty} x(t) e^{st} dt$

$$X(s) = \frac{k(s+z_1)(s+z_2) \dots (s+z_m)}{(s+p_1)(s+p_2) \dots (s+p_n)}$$

$$= \frac{k_1}{(s+p_1)} + \frac{k_2}{(s+p_2)} + \dots + \frac{k_n}{(s+p_n)}$$

$$k_i = \left[(s+p_i) X(s) \right]_{s=-p_i}$$

for repeated roots

$$a_j = \left[\frac{1}{j!} \frac{d^j}{ds^j} (s+\lambda)^r X(s) \right]_{s=-\lambda}$$

Initial Value Theorem

$$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$$

Final Value Theorem

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

$$x(t) = 0 \text{ for } t < 0$$

Y-Y

phase voltage

$$\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -120^\circ \\ V_{cn} &= V_p \angle +120^\circ \end{aligned}$$

line voltage

$$\begin{aligned} V_{ab} &= \sqrt{3} V_p \angle 30^\circ \\ V_{bc} &= V_{ab} \angle -120^\circ \\ V_{ca} &= V_{ab} \angle +120^\circ \end{aligned}$$

phase current

Same as line currents

line current

$$\begin{aligned} I_a &= V_{an} / Z_Y \\ I_b &= I_a \angle -120^\circ \\ I_c &= I_a \angle +120^\circ \end{aligned}$$

Y-Δ

$$\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_{an} \angle -120^\circ \\ V_{cn} &= V_{an} \angle +120^\circ \end{aligned}$$

$$\begin{aligned} V_{AB} &= \sqrt{3} V_p \angle 30^\circ \\ V_{BC} &= V_{AB} \angle -120^\circ \\ V_{CA} &= V_{AB} \angle +120^\circ \end{aligned}$$

$$\begin{aligned} I_{AB} &= \frac{V_{AB}}{Z_\Delta} \\ I_{BC} &= \frac{V_{BC}}{Z_\Delta} \\ I_{CA} &= \frac{V_{CA}}{Z_\Delta} \end{aligned}$$

$$\begin{aligned} I_a &= \sqrt{3} I_{AB} \angle -30^\circ \\ I_b &= I_a \angle -120^\circ \\ I_c &= I_a \angle +120^\circ \end{aligned}$$

Δ-Δ

$$\begin{aligned} V_{ab} &= V_p \angle 0^\circ \\ V_{bc} &= V_p \angle -120^\circ \\ V_{ca} &= V_p \angle +120^\circ \end{aligned}$$

Same as line voltages

$$\begin{aligned} I_{AB} &= \frac{V_{ab}}{Z_\Delta} \\ I_{BC} &= \frac{V_{bc}}{Z_\Delta} \\ I_{CA} &= \frac{V_{ca}}{Z_\Delta} \end{aligned}$$

$$\begin{aligned} I_a &= I_{AB} \sqrt{3} \angle -30^\circ \\ I_b &= I_a \angle -120^\circ \\ I_c &= I_a \angle +120^\circ \end{aligned}$$

Δ-Y

$$\begin{aligned} V_{ab} &= V_p \angle 0^\circ \\ V_{bc} &= V_p \angle -120^\circ \\ V_{ca} &= V_p \angle +120^\circ \end{aligned}$$

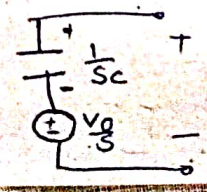
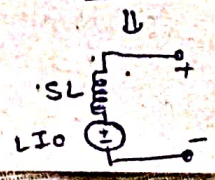
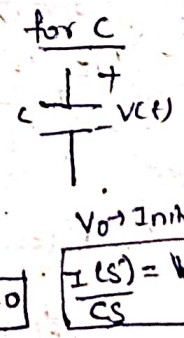
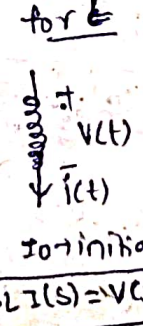
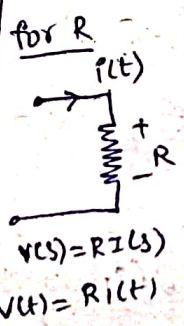
Same as phase voltages

Same as line currents

$$\begin{aligned} I_a &= \frac{V_p \angle -30^\circ}{\sqrt{3} Z_Y} \\ I_b &= I_a \angle -120^\circ \\ I_c &= I_a \angle +120^\circ \end{aligned}$$

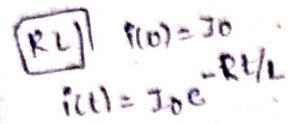
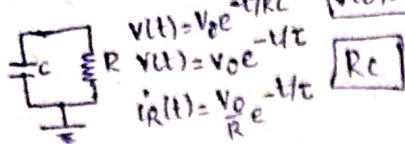
signal	Laplace Transform	ROC	unilateral Laplace Transform
$a x(t) + b y(t)$	$a X(s) + b Y(s)$	$R_1 \cap R_2$	$\mathcal{L}\left(\frac{dx}{dt}\right) = sX(s) - x(0^-)$
$x(t-t_0)$	$e^{-st_0} X(s)$	R	$\mathcal{L}\left(\frac{d^2x}{dt^2}\right) = s^2 X(s) - s(x(0^-)) - \left.\frac{dx}{dt}\right _{t=0^-}$
$e^{s_0 t} x(t)$	$X(s-s_0)$	$R + \text{Re}(s_0)$	
$x^*(t)$	$X^*(s^*)$	R	
$\frac{dx(t)}{dt}$	$sX(s)$	at least R	
$-tx(t)$	$\frac{dX(s)}{ds}$	R	
$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{s} X(s)$		

Transfer Impedance

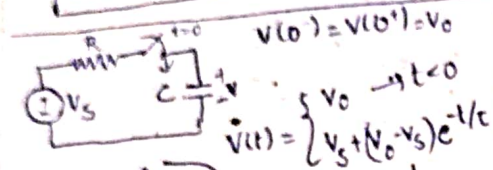


First-order circuit

(1) Natural response / Source-free



$\tau = L/R$
 $i(t) = I_0 e^{-t/\tau}$

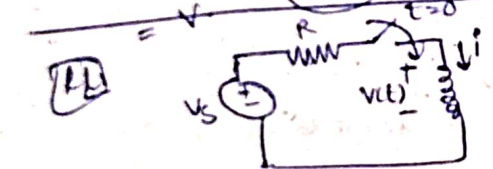


Voltage of capacitance cannot change instantaneously

complete response

$i(t) = \frac{V_s}{R} e^{-t/RC} u(t)$

= Natural + forced response



$\tau = \frac{L}{R}$
 $v(t) = V_s e^{-t/\tau} u(t)$

Transfer function for two port

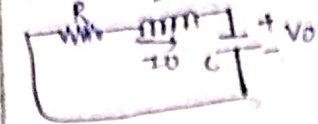
Numerator

$V_2(s)$ $Z_2(s)$
 $V_1(s)$ $G_{12}(s)$ $Y_{12}(s)$
 $Z_{12}(s)$ $d_{12}(s)$

denominator

Second order

(1) Source free



$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$
 $i = A e^{st}$
 $s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$

$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$\alpha = \frac{R}{2L}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

$\alpha > \omega_0$

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$\alpha = \omega_0$

$i(t) = (A_2 + A_1 t) e^{-\alpha t}$

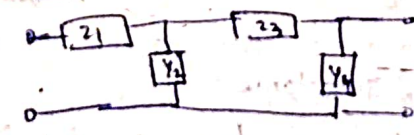
$\alpha < \omega_0$

$s_1 = -\alpha + j\omega_d$

$s_2 = -\alpha - j\omega_d$

$i(t) = e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

Ladder Network



$Z = Z_1 + \frac{1}{Y_2 + \frac{1}{Z_2 + \frac{1}{Y_3 + \frac{1}{Z_3 + \frac{1}{Y_4 + \frac{1}{Z_4}}}}}}$

Poles and zeroes

$F(s) = \frac{H(s-z_1)(s-z_2)(s-z_3) \dots (s-z_n) \rightarrow \text{zeros}}{(s-p_1)(s-p_2) \dots (s-p_m) \rightarrow \text{poles}}$