MODULE 5

PART B

① $I = \int_3^7 x^2 \log x \, dx = \int_3^7 x^2 \ln x \, dx = 177.4816$

$\boxed{y = x^2 \ln x = f(x)}$

$x_0 = 3$	$y_0 = 9.8875$
$x_1 = 4$	$y_1 = 22.1807$
$x_2 = 5$	$y_2 = 40.2359$
$x_3 = 6$	$y_3 = 64.5033$
$x_4 = 7$	$y_4 = 95.3495$

Given $h = 1$

* simpson's $\frac{1}{3}$ rule :

$$\boxed{I = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots) + 2(y_2 + y_4 + \dots)\right]}$$

$$I = \frac{1}{3}\left[532.4408\right] = 177.4802667 //$$

$$= \frac{1}{3}\left[((9.8875) + (95.3495)) + 4(22.1807 + 64.5033) + 2(40.2359)\right]$$

$$= \frac{532.4408}{3} = 177.4802667$$

shift + Mod setup
Radian

(2) $I = \int_0^{\pi/2} \sin x \, dx$

Given $n = 4$; $a = 0, b = \pi/2$; $f(x) = \sin x$

Therefore $h = \frac{b-a}{n} = \frac{\pi/2 - 0}{4} = \pi/8$

$\boxed{h = \pi/8}$

Radian values

$x_0 = 0$	$y_0 = 0$
$x_1 = \frac{\pi}{8}$	$y_1 = 0.3826$
$x_2 = \frac{\pi}{4}$	$y_2 = 0.7071$
$x_3 = \frac{3\pi}{8}$	$y_3 = 0.9238$
$x_4 = \frac{\pi}{2}$	$y_4 = 1$

* By using Trapezoidal rule ; —

$I_T = \frac{h}{2}[(y_0+y_4) + (y_1+y_2+y_3)] = \frac{\pi}{16}[5.027] = 0.987049$

$\boxed{I_T = 0.9870491418}$ //

* By using simpson 1/3 rule :-

$I_{S_{1/3}} = \frac{h}{3}[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)] = \frac{\pi/8}{3}[7.629]$

$= \frac{\pi}{24}[7.629] = 0.9986337$

$\boxed{I_{S_{1/3}} = 0.9986337648}$ //

(3) $\int_{-2}^{2}\int_{0}^{4}(x^2-xy+y^2)\,dx\,dy$

Using double integral trazpizodal rule, $\boxed{h=1, K=1}$

x/y	$x_0=0$	$x_1=1$	$x_2=2$	$x_3=3$	$x_4=4$
$y_0=-2$	4	7	12	19	28
$y_1=-1$	1	3	7	13	21
$y_2=0$	0	1	4	9	16
$y_3=1$	1	1	3	7	13
$y_4=2$	4	3	4	7	12
8					

$$\therefore \boxed{\int_{x_1}^{x_2}\int_{y_1}^{y_2} f(x,y)\,dx\,dy = \int_{x_1}^{x_2}\left[\int_{y_1}^{y_2} f(x,y)\,dy\right]dx = I}$$

$\Rightarrow I_T$ double integral trapzoidal rule

$$\boxed{I_T = \frac{hK}{4}\Big[\text{(sum of the values of corners)} + 2\text{(sum of the values of } f^n \text{ at remaining nodes of the boundary)} + 4\text{(sum of the values of } f^n \text{ at interior values)}\Big]}$$

According to table

$h=1, K=1$

$$I_T = \frac{1}{4}\Big[(4+28+12+4)+2(7+12+19+21+16+13+7+4+3+1+0+1)$$
$$+4(3+7+13+1+4+9+1+3+7)\cancel{+8}\Big]$$

$$I_T = \frac{1}{4}\big[448\big]$$

$$I_T = 112$$

$$\boxed{I_T = \int_{-2}^{2}\int_{0}^{4} (x^2 - xy + y^2)\,dx\,dy = 112}$$

(4) $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$

we know,

$$\boxed{y' = \frac{dy}{dx} = f(x,y)} \to (1)$$

$$\boxed{y(x_0) = y_0} \to (2)$$

$\therefore$ Therefor the Taylor series is

$$\boxed{y(x) = y_0 + \frac{h}{1!}y_0' + \frac{h}{2!}y_0'' + \frac{h}{3!}y_0''' + \frac{h}{4!}y_0^{iv} + \ldots}$$

where

$h = x - x_0$

$x_0 = 0$

$\boxed{h = x}$

$$\boxed{f(x,y) = x - y^2}$$

$\Rightarrow x_0 = 0,\ y_0 = 1$

$\Rightarrow y' = x - y^2$

$\Rightarrow y'' = 1 - 2yy'$

$\Rightarrow y''' = 0 - 2((y')^2 + yy'')$

$\Rightarrow y^{iv} = \cancel{-2[2(y')(y'') + (y'' + yy''')]}$

$= -2[2y'(y'') + y(y''') + y''(y')]$

put, $x_0 = 0;\ y_0 = 1$

$\Rightarrow \boxed{y_0' = -1}$

$\Rightarrow y_0'' = 1 - 2(1)(-1) = 3 \Rightarrow \boxed{y_0'' = 3}$

$\Rightarrow y_0''' = -2((-1)^2 + (1)(3)) = -2(-1-3) = -2(-4) = 8$

$= -8$

$\Rightarrow \boxed{y_0''' = -8}$

$\Rightarrow y_0^{iv} = -2[2(-1)(3) + \overset{(1)(-8)+(3)(-1)}{\cancel{(3)(1)(4)}}] = -2(\overset{-6-8-3}{\cancel{-6-4}}) = 34$

$$\boxed{y_0^{iv} = 34}$$

Sub $y_0, y_0', y_0'', y_0''', y_0^{iv}, h$ in series

$$y(x) = 1 + \frac{x}{1!}(-1) + \frac{x^2}{2!}(3) + \frac{x^3}{3!}(4) + \frac{x^4}{4!}(34)$$

$$y(x) = 1 - x + \frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{12}x^4$$

$$\boxed{y(x) = 1 - x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{34}x^4}$$

$$\boxed{y(0.1) = 0.91566}$$

The value of $y(0.1) = 0.9156696078$

$$\boxed{y(0.1) = 0.91566}$$

(5) Given,

$\boxed{y(0.1) = ?}$

$y'' - xy' - y = 0$

$y(0) = 1 \Rightarrow x_0 = 0, \boxed{y_0 = 1}$

$y'(0) \Rightarrow \boxed{y'_0 = 0}$

$\Rightarrow \boxed{y'' = y + xy'} \longrightarrow (1)$

$\Rightarrow y''_0 = y_0 + x_0 y'_0 = 1 + (0)$

$\Rightarrow \boxed{y''_0 = 1}$

$\Rightarrow y''' = 1y' + y' + x\,y'' = 2y' + xy''$

$\Rightarrow \boxed{y'''_0 = 0}$

$\Rightarrow y^{iv} = 2y'' + y'' + xy'''$

$\Rightarrow \boxed{y^{iv}_0 = 3}$

* Taylor series :-

$$\boxed{y(x) = y_0 + \frac{h^1}{1!} y'_0 + \frac{h^2}{2!} y''_0 + \frac{h^3}{3!} y'''_0 + \frac{h^4}{4!} y^{iv}_0 + \dots}$$

where $h = x - x_0 \Rightarrow \boxed{h = x}$

$\therefore \boxed{x_0 = 0}$

$$y(x) = 1 + x(0) + \frac{x^2}{2}(1) + \frac{x^3}{6}(0) + \frac{x^4}{24}(3)$$

$$\boxed{y(x) = 1 + \frac{x^2}{2} + \frac{x^4}{8}} \Rightarrow \boxed{y(0.1) = 1.0051}$$

⑥ Given,

$$\boxed{\frac{dy}{dx} = f(x, y) = y - x \text{ and } y(x_0) = y(0) = y_0 = 2}$$

$f(x, y) = y - x$, $x_0 = 0$, $y_0 = 2$ | $\left.\begin{matrix} y(0.1) \\ y(0.2) \end{matrix}\right\} ?$

* Range kutta method : (2nd order)

$\therefore$ $y(0.1)$, $y(0.2)$, $y(0)$

$\boxed{h = 0.1}$

$\Rightarrow$ $\boxed{h = 0.1}$

$\Rightarrow$ $\boxed{y_1 = y(0.1) = y_0 + \frac{1}{2}(K_1 + K_2)}$

$\Rightarrow$ $K_1 = 0.1 \times (f(x_0, y_0))$

$= 0.1 \times (2)$

$\boxed{K_1 = 0.2}$

Formula, —

$\boxed{y_1 = y_0 + \frac{1}{2}(K_1 + K_2)}$

$\boxed{K_1 = h \times f(x_0, y_0)}$

$\boxed{K_2 = h \times f(x_0 + h, y_0 + K_1)}$

$\Rightarrow$ $K_2 = 0.1 \times f(x_0 + h, y_0 + K_1) = (0.1)(f(0.1, 2.2)) = (0.1)(2.1$

$\Rightarrow$ $\boxed{K_2 = 0.21}$

Hence

$$y_1 = y(0.1) = y_0 + \frac{1}{2}(K_1 + K_2)$$

$$= 2 + \frac{1}{2}(0.2 + 0.21) = 2 + \frac{1}{2}(0.41)$$

$$= 2 + 0.2050$$

$$\boxed{y(0.1) = 2.2050}$$

Now

$$\boxed{y_2 = y(0.2) = y_1 + \frac{1}{2}(K_1 + K_2)}$$

$$K_1 = (0.1) f(x_1, y_1) = 0.1(f(0.1, 2.2050)) = 0.1(2.1050)$$

$$\boxed{K_1 = 0.21050}$$

$$K_2 = (0.1) f(x_1 + h, y_1 + K_1) = (0.1)(f(0.2, 2.4155))$$

$$= (0.1)(2.2155)$$

$$\boxed{K_2 = 0.22155}$$

i.e,

$$\boxed{\begin{aligned} K_1 &= 0.21050 \\ K_2 &= 0.22155 \end{aligned}}$$

$$y_2 = y(0.2) = 2.2050 + \frac{1}{2}\left[0.21050 + 0.22155\right]$$

$$\boxed{y(0.2) = 2.416025}$$

similarly $y_3 = y(0.3) = 2.6492$, $y_4 = y(0.4) = 2.8909$

$$\boxed{\begin{aligned} y(0.1) &= 2.2050 \\ y(0.2) &= 2.416025 \end{aligned}}$$

$\frac{dy}{dx} = 1+y^2, \quad y_0 = 0, \; x_0 = 0$

$\therefore y(0.2), y(0.4), y(0.6)$ $\Rightarrow \boxed{f(x,y) = 1+y^2}$

$\boxed{h = 0.2}$

Range Kutta Method: (2nd order)

$$\boxed{\begin{aligned} y_1 &= y_0 + \frac{1}{2}(K_1 + K_2) \\ K_1 &= h \times f(x_0, y_0) \\ K_2 &= h \times f(x_0 + h, y_0 + K_1) \end{aligned}}$$

$y_1 = 0 + \frac{1}{2}(K_1 + K_2) \Rightarrow \boxed{y(0.2) = \frac{1}{2} K_1 + K_2}$

$K_1 = 0.2 \times (1+0) = 0.2$

$K_2 = 0.2 \times (f(0.2, 0.2)) = (0.2)(1.004) = 0.2008$

$\Rightarrow \boxed{y(0.2) = 0.2004} \Rightarrow \boxed{y_1 = y(0.2) = 0.2004}$

Now, $y(0.4)$

~~$y_2 = y(0.4) = y_1 + f(x_1, y_1) = 0.2004 + f(0.2, 0$~~

$y_2 = y_1 + \frac{1}{2}(K_1 + K_2)$

$y_2 = (0.2004) + \frac{1}{2}(K_1 + K_2)$

$\boxed{y_2 = 0.4210976885}$

$\boxed{y(0.4)}$ //

$K_1 = h \times f(x_1, y_1)$

$= (0.2)(f(0.2, 0.2004))$

$= (0.2)(1 + (0.2004)^2)$

$\boxed{K_1 = 0.208032032}$

$K_2 = h \times f(x_1 + h, y_1 + K_1)$

$(0.2) f(0.4, 0.408432032)$

$\boxed{K_2 = 0.233363345}$

$y_3 = y_2 + \frac{1}{2}(K_1 + K_2)$

$K_1 = (0.2) f(x_2, y_2)$

$= (0.2) f(0.4, 0.4210976885)$

$\boxed{K_1 = 0.2354646527}$

$K_2 = h \times f(x_2 + h, y_2 + K_1)$

$K_2 = (0.2) f(0.6, 0.6565623412)$

$\boxed{K_2 = 0.2862148216}$

$y(0.6) = y_3 = y_2 + \frac{1}{2}(K_1 + K_2)$

$= 0.4210976855 + \frac{1}{2}(K_1 + K_2)$

$\boxed{y(0.6) = 0.6819374227}$

$\int \frac{dy}{1+y^2} = \int x$

$\Rightarrow \tan^{-1}(y) = x + c$

$x = 0, y = 0$

$c = 0$

$\tan^{-1}(y) = x$

$y = \tan x$

$y = \tan x$

$y(0.2) = 0.2027$

rad

$y(0.4) = 0.4227$

$y(0.6) = 0.6841$

* $y(0.2) = 0.2004$

$y(0.4) = 0.4210976885$

$y(0.6) = 0.6819374227$

$\int \frac{dy}{1+y^2} = \int dx$

$\tan^{-1} y - x = c$

$\tan^{-1} y - x = 0$

$x = 0, y = 0$

$c = 0$

$y = \tan x$

(8) same Q5

(9) $f(x,y) \Rightarrow \frac{dy}{dx} = xy + y^2$; $y(0) = 1$

$\boxed{\begin{matrix} y_0 = 1 \\ x_0 = 0 \end{matrix}}$

$y' = xy + y^2 \Rightarrow \boxed{y'_0 = 1}$

$y'' = xy' + y + 2yy' \Rightarrow \boxed{y''_0 = 3}$

$y''' = (xy'' + y') + (y') + 2(y'y' + yy'')$

$y_0''' = 1 + 1 + 2(1 + 3)$

$\boxed{y_0''' = 10}$

$y^{IV} = (xy''' + y'' + y'') + (y'') + 2(2y'y'' + y'y'' + yy''')$

$y_0^{IV} = 9 + 2(2(3) + (3) + (10)) = 9 + 2(6 + 13)$

$\boxed{y_0^{IV} = 47}$

Taylor series ; $h = x - x_0 \Rightarrow \boxed{h = x}$

$\Rightarrow$ ~~$y(x) = y_0 + \frac{y_0'}{1!}$~~

$$\boxed{y(x) = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0^{IV} + \cdots}$$

$$\boxed{y(x) = 1 + x + \frac{3}{2}x^2 + \frac{10}{6}x^3 + \frac{47}{24}x^4}$$

$\boxed{y(0.1) = 1.1168695,\ y(0.2) = 1.276466667}$ //

(10) $y' = 1 + \frac{2xy}{1+x^2}$, $y(0) = 0$

$f(x,y) = 1 + \frac{2xy}{1+x^2}$ | $\left.\begin{matrix} y(0.1) \\ y(0.2) \\ y(0.3) \end{matrix}\right\} h = 0.1$

*R-K (4th order) :-

$$\boxed{y_1 = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]}$$

where

$K_1 = h \times f(x_0, y_0)$

$K_2 = h \times f(x_0 + h/2, y_0 + K_1/2)$

$K_3 = h \times f(x_0 + h/2, y_0 + K_2/2)$

$K_4 = h \times f(x_0 + h, y_0 + K_3)$

$\Rightarrow K_1 = (0.1)(1 + 0) = 0.1$

$\Rightarrow K_2 = (0.1) f(0.05, 0.05) = 0.1004987531$

$\Rightarrow K_3 = (0.1) f(0.05, 0.0502493765\!6)$

$\boxed{K_3 = 0.1005012407}$

$\Rightarrow K_4 = (0.1)(f(1, 0.1005012407))$

$\boxed{K_4 = 0.1100501241}$

$\boxed{y_1 = y(0.1) = 0.102008352}$.

$\boxed{y(0.1) = 0.1006}$ ✓

Now, y_2

$$y_2 = y_1 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$x_1 + \frac{h}{2} = 0.1 + 0.05$

$K_1 = h \times f(x_1, y_1) = 0.0020199 67366$

$K_2 = h \times f(x_1 + h/2, y_1 + K_1/2) = 0.1030225287$

$K_3 = h \times f(x_1 + h/2, y_1 + K_2/2) = 0.1048042426$

$K_4 = h \times f(x_1 + h, y_1 + K_3) = 0.1097182397$

$\cancel{y_2 = 0.18898}$

$$\boxed{y_2 = y(0.2) = 0.1898078975}$$

$y(0.2) = 0.20529$

$$\frac{dy}{dx} = 1 + \frac{2xy}{1+x^2}$$

$$\frac{dy}{dx} + y\left(\frac{-2x}{1+x^2}\right) = 1$$

$$e^{-\int \frac{2x}{1+x^2}dx} = e^{-\log(1+x^2)} = e^{\log_e(1+x^2)^{-1}} = \frac{1}{1+x^2}$$

$$\frac{y}{1+x^2} = \int \frac{1}{1+x^2}\, dx$$

$$\frac{y}{1+x^2} = \tan^{-1}(x) + c$$

$\boxed{0 = c}$

$$\frac{y}{1+x^2} = \tan^{-1}(x)$$

$$y = (1+x^2)(\tan^{-1}(x))$$