* Numericals

1. Calculate th velocity and kinetic energy of an electron of wavelength 0.21 nm.

sol Given,

$\lambda = 0.21 \times 10^{-9}$ m

$m = 9.1 \times 10^{-31}$ kg

$v = ?$

$KE = ?$

de Broglie wavelength, $\lambda = h/mv$

$$v = h/m\lambda$$

$$= \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.21 \times 10^{-9}}$$

$$= 3.46729 \times 10^{6} \text{ m/s}$$

Kinetic energy, $KE = \frac{1}{2}mv^2$

$$KE = \frac{1}{2} \cdot 9.1 \times 10^{-31} \times (3.467 \times 10^{6})^2$$

$$= 54.691 \times 10^{-31+12}$$

$$= 54.691 \times 10^{-19} \text{ J}$$

2. Calculate the de Broglie wavelength associated with a proton moving with a velocity of 1/10 of velocity of light. (Mass of proton = 1.674×10^{-27} kg).

Sol de Broglie wavelength, $\lambda = h/mv$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.674 \times 10^{-27} \times 3 \times 10^{7}}$$

$$= 1.31 \times 10^{-14} \text{ m}$$

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3. Calculate the wavelength of an electron raised to a potential 15 kV.

Sol de Broglie wavelength, $\lambda = h/mv$

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å} = \frac{12.26}{\sqrt{15000}} = \frac{12.26}{122.47} = 0.1 \text{ Å}$$

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4. If the kinetic energy of the neutron is 0.025 eV calculate its de Broglie wavelength (mass of neutron = 1.674×10^{-27} kg)

Sol kinetic energy, KE = 0.025 eV = $0.025 \times 1.6 \times 10^{-19}$ J

$$KE = \frac{1}{2} mv^2$$

$$v = \left(\frac{2 \times 0.025 \times 1.6 \times 10^{-19}}{1.674 \times 10^{-27}}\right)^{1/2} = (0.04779 \times 10^{8})^{1/2}$$

$$= 0.2186 \times 10^{4} \text{ m/s}$$

de Broglie wavelength $\lambda = h/mv$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.674 \times 10^{-27} \times 0.2186 \times 10^{4}} = 0.181 \text{ nm}$$

5. Calculate the velocity and KE of an electron of wavelength 1.66×10^{-10} m.

Sol de Broglie wavelength $\lambda = h/mv$

$v = h/m\lambda$

$$v = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.66 \times 10^{-10}}$$

$v = 43.86 \times 10^{5}$ m/s

kinetic energy of electron

$$E = \frac{1}{2} m v^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times 43.86 \times 43.86 \times 10^{10}$$

$$= 8752.83 \times 10^{-21} \text{ J} = 0.875 \times 10^{-17} \text{ J}$$

$$= \frac{0.8752 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} = 54.68 \text{ eV}$$

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6. Calculate the wavelength of an electron raised to a potential 1600 V.

Sol de Broglie wavelength $\lambda = \frac{12.26}{\sqrt{V}}$ Å

$$\lambda = \frac{12.26}{\sqrt{1600}} = \frac{12.26}{40} = 0.3065 \text{ Å}$$

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7. Calculate the energies that can be passed by a particle of mass 8.50×10^{-31} kg which is placed in an infinite potential box of width 10^{-9} cm.

Sol The possible energy of particle in an infinite potential box of width L is given by

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

$m = 8.50 \times 10^{-31}$ kg, $L = 1 \times 10^{-11}$ m, $h = 6.626 \times 10^{-31}$ JS for ground state $n=1$

$$E = \frac{(6.626 \times 10^{-34})^2}{8(8.5 \times 10^{-31})(1 \times 10^{-11})^2} = 6.456 \times 10^{-16} \text{ Joule}$$

for first excited state, $E_2 = 4 \times 6.4456 \times 10^{-16}$

$= 25.8268 \times 10^{-16}$ Joule

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8. Find the lowest energy of an electron confined in a square box of side 0.1 nm.

Sol The possible energies of a particle in an infinite potential box of width L is given by

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$m = 9.1 \times 10^{-31}$ kg, $L = 0.1 \times 10^{-9}$ m, $h = 6.626 \times 10^{-34}$ JS

For lowest energy $n=1$

$$E_1 = \frac{(6.626 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(0.1 \times 10^{-9})^2} = 60.307 \times 10^{-19} \text{ Joule}$$

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9. An electron is bound in 1-dimensional infinite well of width 1×10^{-10} m. Find the energy values of ground state and first 2 excited states.

Sol The possible energies of a particle in an infinite potential bar of width L is given by $E_n = \frac{n^2 h^2}{8mL^2}$

$m = 9.1 \times 10^{-31}$ kg, $L = 1 \times 10^{-10}$ m, $h = 6.626 \times 10^{-34}$ J.s

for ground state $n=1$,

$$E_1 = \frac{(6.626 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(10^{-10})^2} = 0.6031 \times 10^{-17} \text{ Joule}$$

For first excited level $E_2 = 4 \times 0.6031 \times 10^{-17}$

$= 2.412 \times 10^{-17}$ Joule

For second excited level $E_3 = 9 \times 0.6031 \times 10^{-17}$

$= 5.428 \times 10^{-17}$ Joule

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10. An electron is bound in one-dimensional box of size 4×10^{-10} m. What will be its minimum energy?

Sol The possible energies of a particle in an infinite potential box of width L is given by $E_n = \frac{n^2 h^2}{8mL^2}$

$m = 9.1 \times 10^{-31}$ kg, $L = 4 \times 10^{-10}$ m, $h = 6.626 \times 10^{-34}$ Js

For minimum energy $n = 1$

$$E_1 = \frac{(6.626 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(4 \times 10^{-10})^2} = 0.346 \times 10^{-18} \text{ Joule}$$

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11. Calculate the wave length of matter wave associated with a neutron whose kinetic energy is 1.5 times the rest mass of electron [Given that mass of neutron $= 1.676 \times 10^{-27}$ kg, mass of electron $= 9.1 \times 10^{-31}$ kg, planck's constant $= 6.626 \times 10^{-34}$ Jsec, velocity of light $= 3 \times 10^{8}$ m/s].

Sol For neutron $\frac{1}{2}mv^2 = 1.5 \times 9.1 \times 10^{-31}$

$$v^2 = \frac{2(1.5 \times 9.1 \times 10^{-31})}{1.676 \times 10^{-27}}$$

$$v = \sqrt{16.289 \times 10^{-4}}$$

$$v = 4.040 \times 10^{-2} \text{ m/s}$$

The de Broglie wavelength expression is $\lambda = h/mv$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.676 \times 10^{-27} \times 4.046 \times 10^{-2}}$$

$$\lambda = 9.76 \times 10^{-6} \, m$$

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12. Electrons are accelerated by 344 volts and are reflected from a crystal, the first reflection maximum occurs when the glancing angle is $60°$. Determine the spacing of crystal.

Sol Given,

$V = 344 \, V$

$\theta = 60°$

We have $\lambda = \frac{12.27}{\sqrt{344}} = \frac{12.27}{18.547} = 0.6615 \, Å$

From Bragg's law we have $2d\sin\theta = n\lambda$

First reflection max. i.e $n = 1$, spacing $= d = ?$

So, $d = \frac{n\lambda}{2\sin\theta} = \frac{1 \times 0.6615}{2 \times \sin 60°} = \frac{1 \times 0.6615}{2 \times 0.8660} = 0.3819 \, Å$
