Module - II

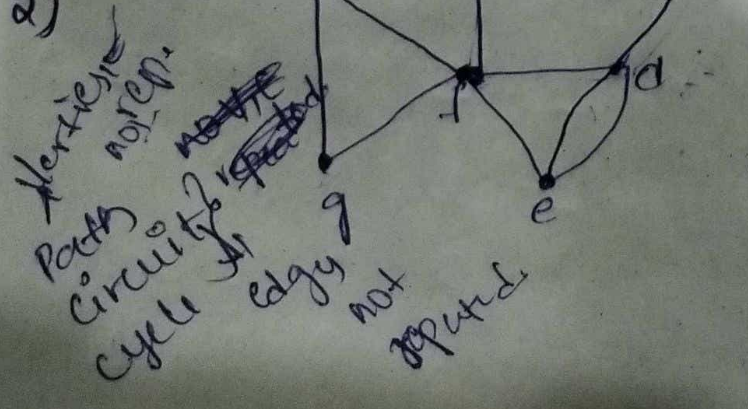
1) **Walk:-** In a graph G, a walk is sequence of vertices and edges where each edge is connected to the vertex. The sequence of ~~$\$$~~ vertices $V_1, V_2 \ldots V_k$ The sequ of edges $e_1, e_2 \ldots e_{k-1}$. edge e_i connects vertices V_i and V_{i+1} such that $1 \leq i \leq k-1$.

ii) **Trail.** A graph is a walk in which no edge is repeated. a trail sequence of vertices and edges are each edge is unique.

c) **Path:-** A path in a graph is a trail in which no vertex is repeated. It's a sequence of vertices and edges where each vertex is unique. which may be the same if the path is closed.

d) **Closed walk:-** A closed walk is a walk in which the start vertex and the end vertex are the same

2)



Path.
~~Trails~~:-

$a \to b \to c$

$a \to f \to d \to c$

$a \to g \to f \to d \to c$.

$a \to f \to e \to d \to c$

$a \to f \to b \to c$.

~~Path~~ :- Trail :-

a → b → c

a → f → d → c.

a → g → f → a → b → c.

a → f → b → c.

a → f → e → d → c.

Circuit :- b → f → d → c → b

b → f → e → d → c → b

b → a → ~~b~~g → f → b.

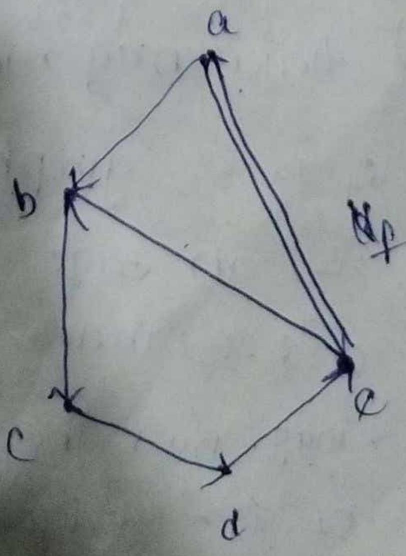
cycle :- b → f → d → c → b

b → f → e → d → c → b

b → a → g → f → b.

3) Eulerian Circuit :- A circuit that tranverse every edges of the graph exactly one, starting and ending at the same vertex. every vertex has an even degree

4) V(G) = {a, b, c, d, e} E(G) = {ab, ae, bc, cd, de, ~~ea~~, eb}



Walk :- a → b → c → d

a → e → d → c

Trail :- e → b → c → d → e.

a → b → c → d → e → a.

Path:-

a → b → c → d.

a → e → b → c.

5)

```
A – B – C – D – E
|       |       |
F   G – H – I – J
```

The graph has a hamitonian cycle, which is a cycle that visits every ~~es~~ vertex exactly one. one such hamitonian cycle in this ~~cycle~~graph could be A-B-C-D-E-J-I-H-G-F-A. However, this graph does not have an Eulerian circuit because it contain vertices with odd vertices.

6) 1) Hamiltonian Graph

2) Vertex Degree

3) Necessary but not Sufficient

4) connectivity

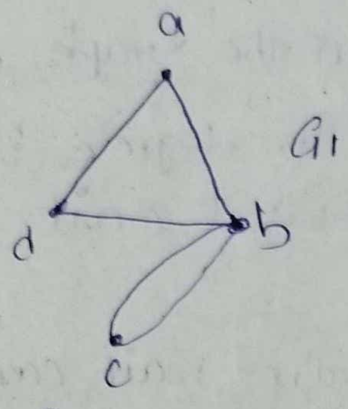
5) Dirac's Theorem

6) Ore's Theorem.

7) Adjacency matrix:- In an adjacency matrix A is a graph, the entry a_{ij} is 1 if there is an edge between vertex i and vertex j and 0 otherwise. If the graph is directed, a_{ij} may not be to a_{ji}, indicating the direction of the edge

vertex-edge Incidence matrix II:- In a vertex-edge incidence matrix B, the rows represent vertices and the columns represent edges.

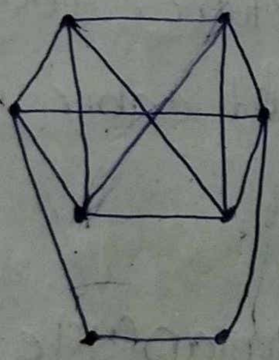
8)



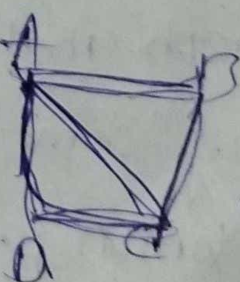
G1

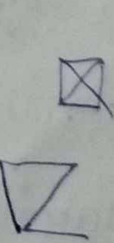
a-d-b-c-b→a.

→ Hamiltonian cycle (not).
→ Eulerian circuit.



G2
-> Hamitonian cycle
-> Eulerian circuit

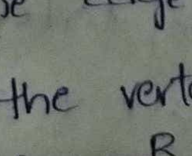




9) Konigsberg ~~tester~~ bridge:-

There must be one edge enter the vertex.

The order of the vertex is even degree.

ex:- 

(not a Konigsberg bridge

10) Connectedness:- The graph must be connected, meaning that there is a path between every pair of vertices.

even degree:- Every vertex in the graph must have an even degree. The degree of a vertex is the No. of edge incident on it.

11) 1) mail Delivery:- Imagine you're mail carrier trying to deliever mail to every house in a neighborhood without retracing your steps. An Eulerian circuit would help you plan the most efficient route.

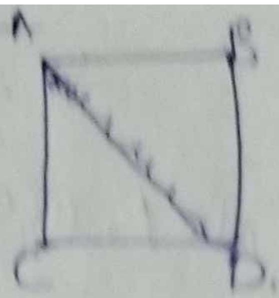
2) Cable wiring: when installing cables in a building or network infrastructure, you want to minimize the amount of cable used and avoid overlapping paths..

) Hamiltonian cycle:- A Hamiltonian cycle in a graph that visits vertex exactly once. starting and ending same.

A-B-C-D-A.

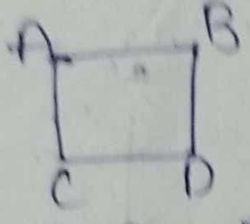
Hamiltonian path:- A hamiltonian path in a graph that visits vertex exactly once, but it doesn't have to end at the starting vertex like a cycle

13) Yes, Question ex:-



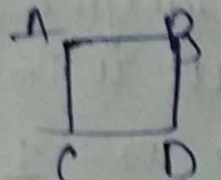
A-B-C-D-A.

14) cycle:- The Graph which the vertices are repeated with closed path. it starts and end at the same vertex.



A-B-~~B~~D-C-A

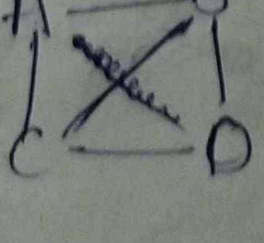
Circuit:- A circuit in a graph is a cycle in which no vertex (except the starting and ending vertex) is reapeated.



A-B-C-D-A.

length:- The length of a cycle or circuit in a graph is the number of edges it contains.

15) A connected graph is a graph in which there is a path between every pair of vertices.

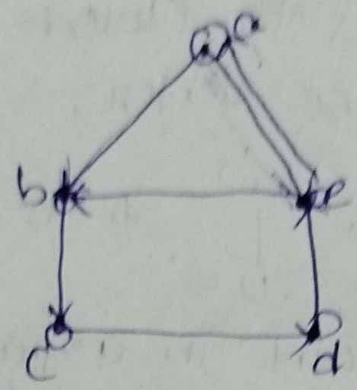


You can reach vertex B from vertex A by following the edge A-B.

PART - B

1) V(G) = {a, b, c, d, e}, E(G) = {ab, ae, bc, cd, de, ea, eb}

a)



b) Yes, the G is connected.

c) No

d) deg of a:

indegree :- 1
out " :- 2

d → in - 1
out - 1

b → in - 1
out - 1.

e → in - 2
out - 2.

c → in - 1
c → 1

e) a → b, e → b.

f) e → a.

g) walk:-

a → b → c
a → b → c → d → e.
a → b → c → d.
a → b → c → d → e → a.

length of has 3.

a → b → c.
b → c → d
c → d → e.
d → e → b.

Trail:-

a → b → c.
a → b → c → d → e → a.
a → e → b.

Path:-

a → b → c → d → e
a → b → c

closed walk:-

h) $A \to b \to c \to d \to e \to a$.

circuit:-

$a \to b \to c \to d \to e \to a$.

$a \to d \to e \to b \to c \to d$.

cycle:- $a \to b \to c \to d \to e \to a$.

$d \to e \to b \to c \to d$

i)

2) a) Photographer visiting bridges:-

* Eulerian circuit or Trail:- Not applicable because an Eulerian circuit required visiting every bridge exactly once and returning to the starting point. since the photographer needs to return to his hotel which isn't a bridge, it doesn't fit.

Hamiltonian cycle or path:- Not applicable because an Hamiltonian cycle every bridge exactly once without the need to return to the starting point.

ii) Repairing streets:-

i) Yes, applicable. if every street segment connects with every other street segment. because it visits each street exactly once and cover all streets.

ii) Not applicable because it doesn't matter if very vertex is visited exactly once.

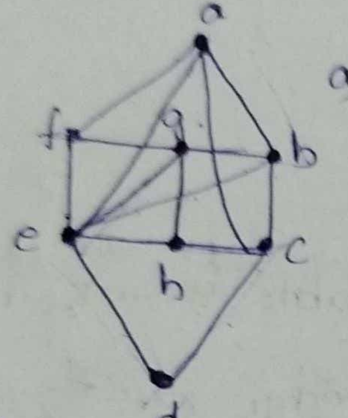
c) Delivering flowers:-

i) Not applicable because it doesn't need to visit each customer exactly once.

ii) Not applicable because it doesn't require visiting each ~~local~~ customer exactly once.

3)

a)



a) a = 4, b = 4, c = 4, d = 2, e = 6, f = 3, g = 4, h = 3.

b) Semi-Eulerian; ~~bec~~..

c) ~~a - b f - e - d - c - b - a~~ . a → f → e → d → c → h → g → b →

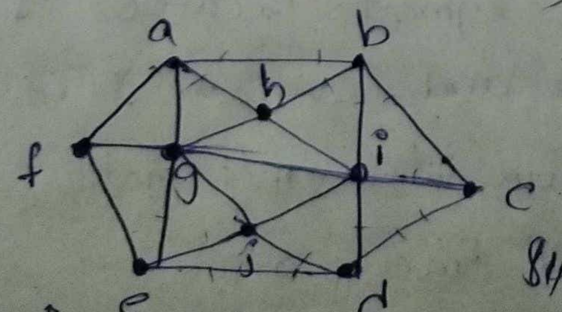
~~a - e - d - c - b - a~~ . a → b → c → d → e → h → g → f → a.

~~a - c - b - a~~.

~~a - f - e - b - c - b - a~~.

It is a not a Eulerian Circuit because the edges are not repeated. And starting and ending are the same vertex, but it doesn't have even degree.

b).

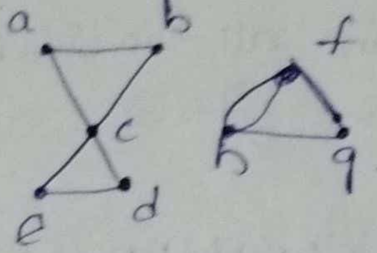


a) a = 4, b = 4, c = 3, d = 4, e = 4, f = 3, g = 4, h = 4, i = 4, j = 4

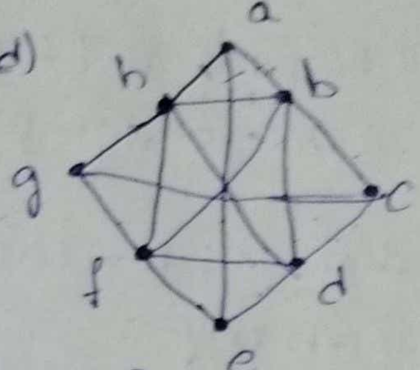
b) Semi-eulerian

c) Not a eulerian Circuit.

c)



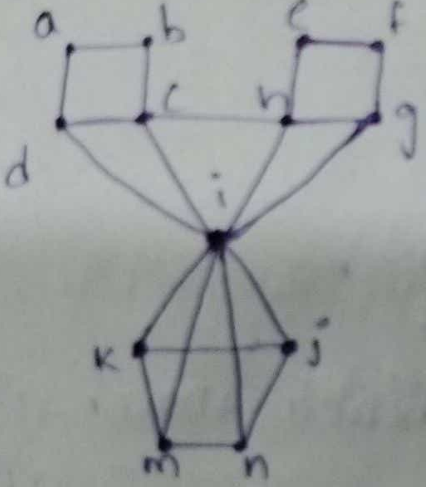
d)



a) $a = 3, b = 5, c = 3, d = 5$

$f = 3$ $f = 5$, $g = 3, h = 5$

4) Sme as 6th answer.

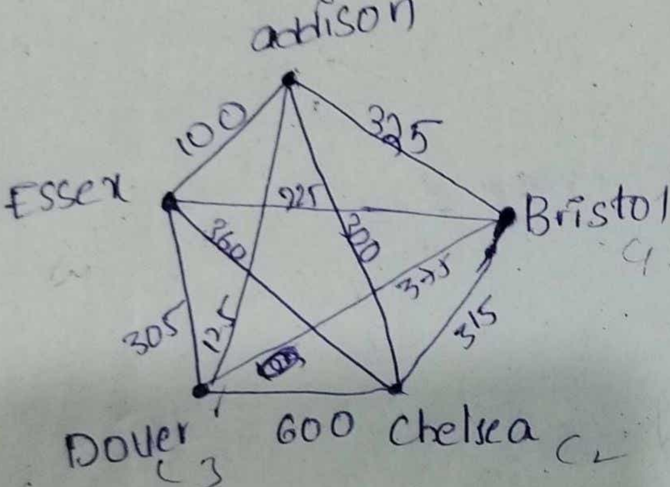


5) 1) List all possible routes: Imagine all the different ~~types~~ ways you can visit the cities.

2) calculate distance: for each route, add up the distance between each city.

3) Find the shortest route: look for the route with the smallest total distance.

6)



1) ~~find~~ list all possible routes:-

* addison → Bristol → Chelsea → Dover → Essex → addision
  $325 + 315 + \cancel{125}\ 600 + 305 + 100 =$ ~~1,170~~ 1,645

* addison → Bristol → Essex → Dover → Chelsea → addision = 1,255

* ad → Br → es – ch → Do – ad = 1,635

* ad → ~~Br~~ es → D → Br – ch – add = 1,395

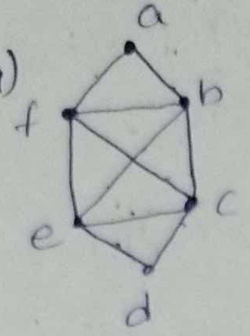
* ad → ch → Bri → Do → ES → ad = 1,395.

* ad → ch → ES → B → ad = 1,635.

* ad – Dov → ch → ES → B → ad

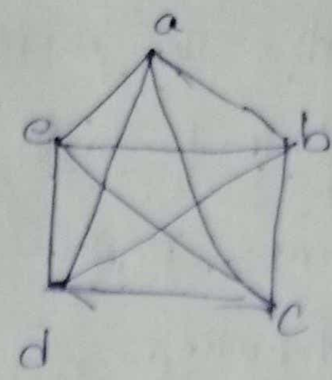
the cheapest route is 1,395

7) a)



Yes, it is a Eulerian.

b)



1) a → f → e → d → c → b → a.

2) a → f → e → d → c → f → b → a.

3) a → b → c → d → e → f → a.

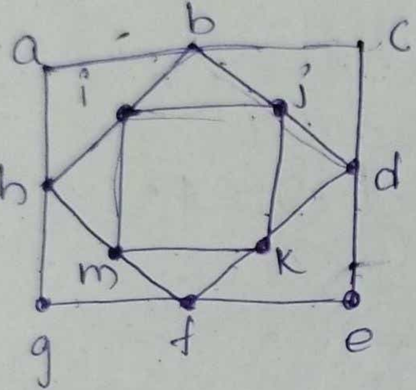
a → b → c → d → e → a.

a → e → d → c → b → a

a → b → e → d → c → a.

⇒ (yes)

c)



~~a → b → c → d → e → f → g → h~~ → a.

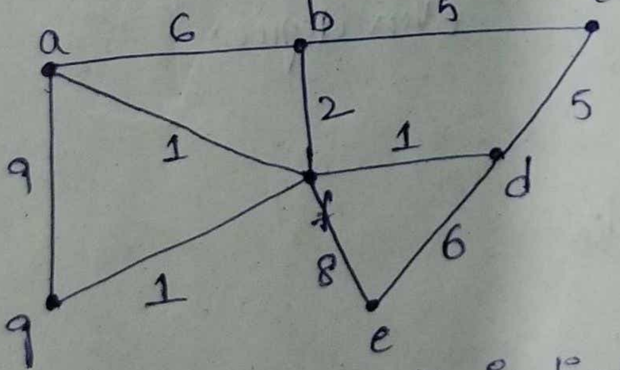
~~a → h → g → f → e →~~

a → b → c → d → j → i → h → m → k → f → e →

f → g → h → a.

⇒ Yes

8)



Dijkstra alogorithm means it is finding the shortest path from one place to another place

1) Start : You begin a point.

2) Neighbours :- you look all the places you can reach directly from the starting point.

3) coasts:- How much coast get to each place.

4) Choose the best:- Pick the lowest cost.

5) Repeat:- Repeat 2 to 4 for the place you just picked.

6) stop:- Keep doing until you reach your distination.

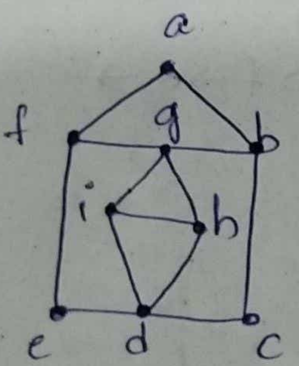
* $g \to a = 9$.

$\boxed{g \to f = 1}$

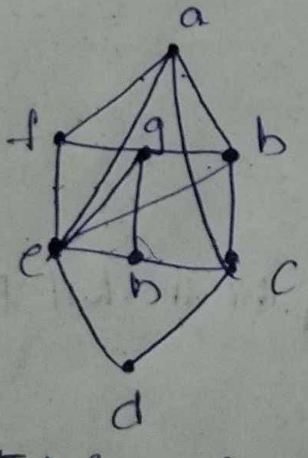
$g \to f \to a \to b \to c \to d \to e \to f \to g = 43$

$g \to f \to a \to g = 11$

9)

i) 

No, it is not a Hamiltonian
every ~~degree~~ Vertex does not have even degree.

ii) 

No.

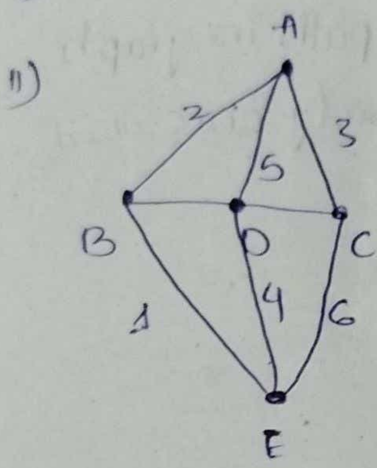
10) Eulerian Circuit:- An eulerian circuit is a path graph which travel each edge once and returns to the starting point.

ex:- for ex:- A,B,C,D are vertices and edges connecting like A-B,B-C, C-D and D-A.

Hamiltonian cycle:- A Hamiltonian cycle is path in a graph that visists each vertex exaclty once and starting vertex

ex:- A,B,C,D are vertices edges are A-B,B-C,C-D,~~D-A~~.

11)



A-B = 2

A-D = 5

A-C = 3

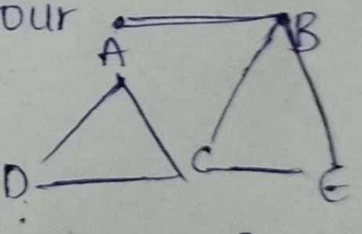
A → B → E → 3

A → D → E → 9.

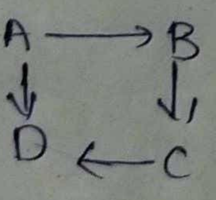
The shortest path is 2.

A → B → E → C → D

12) No ~~yes~~, we can't travese each bridge exactly once and return to your starting point becauing every ~~de~~ vertex doesn't have even degree.



13)



$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 1 \\ B & 0 & 0 & 1 & 0 \\ C & 0 & 0 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array}$$

Walk:- for length 3

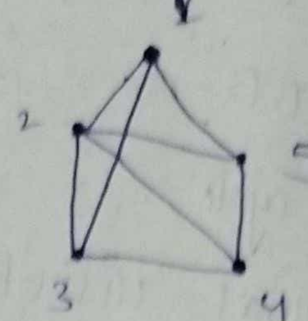
A → B → C.

for length 2

A → B

A → D.

14) $\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 0 & 1 & 0 \end{array}$



No, it is not eulerian circuit. every vertex do have even degree.

15) a) Eulerian Tours: An eulerian Tour is path in graph that travels through each edge extacly once and returns at the same point.

b) 12 answer.

c) 8 answer.