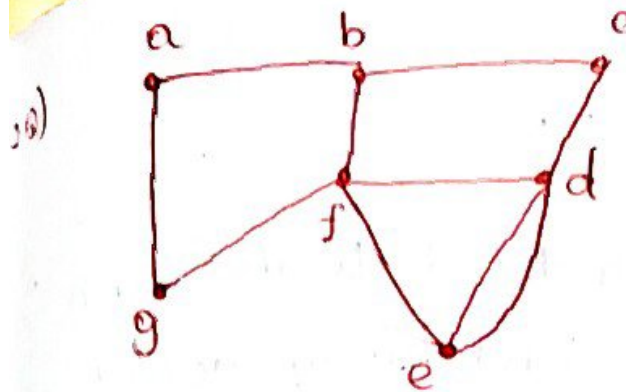


## Module-II

(Q) Let  $G$  be the graph. define the terms :

- a) walk
- b) Trail
- c) Path
- d) closed walk

- 1) Walk: A walk in a graph is a sequence of vertices where each adjacent pair of vertices is connected by an edge. The walk can revisit the vertices and edges.
- 2) Trail: Trail in a graph is a walk where no edges are repeated.
- 3) Path: Path in a graph is a trail where no vertex are repeated.
- 4) closed walk: closed walk is a walk where the walk start and end at the same end.



i) (Find a trail that is not a path) from a to c :

Trail from a to c :  $a-g-f-e-d-c$

ii) a path from a to c :

path a to c :  $a-b-c$

iii) a circuit starting at b :

circuit : A circuit is a trail with closed  
(or)

A closed trail is a circuit

circuit :  $b-f-d-c-b$

iv) a cycle starting at b :

cycle : A closed path is called cycle

cycle at b :  $b-f-a-b$

3Q) define Eulerian circuit?

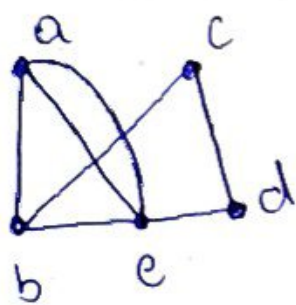
Eulerian circuit:

Eulerian circuit is a closed trail with all edges covered ~~at~~ <sup>only</sup> once and every vertex of the have even degree

§

A graph is called a Euler graph if it contains Eulerian circuit.

Graph  $(G)$   $V(G) = \{a, b, c, d, e\}$   
 $E(G) = \{ab, ae, bc, cd, de, ea, eb\}$



a) Walk:  $a-b-e-a-e-d-c-b-a$

b) Trail:  $a-b-c-d$

c) Path:  $a-b-e-d-c$



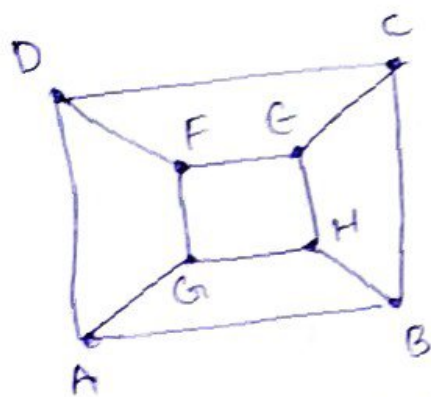
Q1) Example of graph having hamilton cycle  
but not a Eulerian circuit?

Hamilton cycle: (or) Hamilton circuit

\* each vertex of the graph will be visited exactly once except start and end vertex

\* start & end vertex must be same.

Q2)



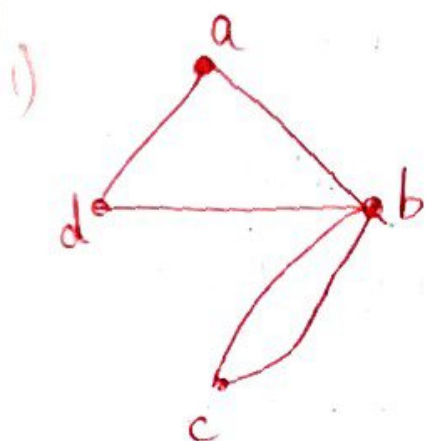
Not Euler graph degrees are odd

H cycle: A-B-C-D-F-E-H-G-A

Q3) Hamilton cycle (or) Graph?

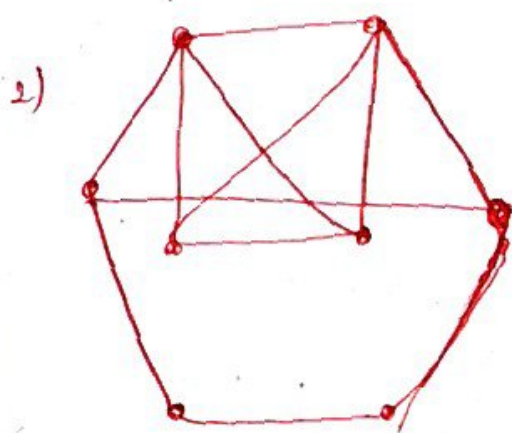
Q4)

26) do they have hamilton cycle & Eulerian circuit?



hamilton: No, because there is no way to visit every vertex exactly once

Eulerian: No, because all vertex no even degree



hamilton: Yes, can cover all vertex at once

Eulerian: not all vertex have even degree

94)

11Q). 2 applications of Eulerian circuit?

1) Circuit designing and electrical engineering.  
 • used in designing circuits where every component connection (such as wire, resistor, capacitor) is visited exactly once  
 \* It reduces error & minimize redundancy

2) Genomics and DNA sequencing.  
 \* The use to identify overlaps and connections b/w DNA segments

12Q) define Hamilton path, circuit?

1) Hamilton path:

Hamilton path in a graph  $G$  is a path that visits every vertex exactly at once but unlikely cycle. It don't need to return to the starting vertex.

Hamilton cycle:

also known as circuit that visit every vertex of  $G$  exactly once and return to starting vertex



Q) Eulerian circuit focus on traversing edges exactly once?

14Q) define the following for graph  $G$ ?

- 1) cycle
- 2) circuit
- 3) length

Cycle: A cycle in a graph  $G$  is a closed path that start and ends at the same vertex  
(01)

A cycle in a graph is a sequence of vertices where first and last vertices are same, and each pair of consecutive vertices are connected by an edge

circuit: A circuit in a graph is defined as a closed trail that starts and ends at the same vertex.

(or)

A circuit is a sequence of vertices where the first and last vertices are same.

length: The length of a path or a cycle in a graph refers to the total no of edges taken along the path.

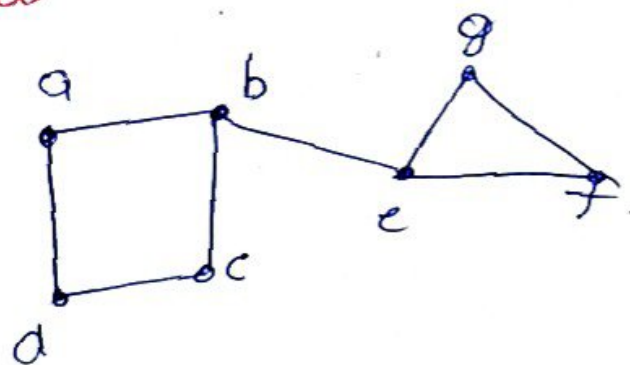
53) define a connected graph?

connected graph: A graph  $G$  is said to be a connected graph if there exists a path b/w every pair of vertices.

disconnected graph?

A graph  $G$  is said to be a disconnected graph, if it does not contain at least two connected vertices.

Ex:

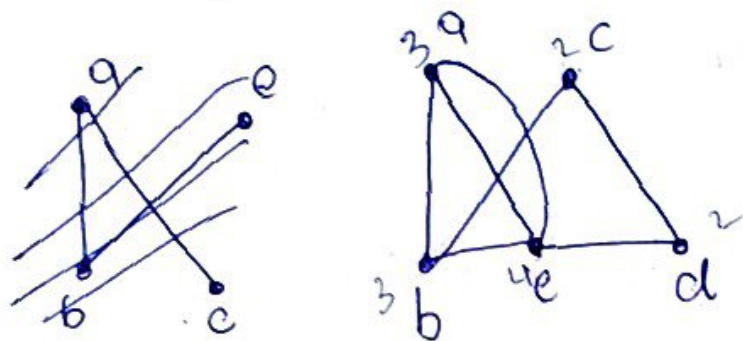




## Part-B

10) Graph  $G_1$  :  $V(G_1) = \{a, b, c, d, e\}$   
and edge  $E(G_1) = \{ab, ae, bc, cd, de, ca, eb\}$

a) Draw Graph.



b) Is  $G_1$  connected?

Yes

c) Is  $G_1$  simple?

Yes No

d) degree of each vertex?

vertex  $a = 3$

$b = 3$

$c = 2$

$d = 2$

$e = 4$

c) edges incident to b?

A)  $\{be, be, ba\}$

d) neighbours of a

$a = \{b, e\}$

g) walk, trail, path in  $G$  of even length 3

walk from a:  $a-b-c-d$

trail from a:  $a-c-d-c-b$

path from ~~b~~:  $b-c-d-e-a$



h) closed walk, circuit, and cycle in  $G$  start at 'e'

closed walk e:  $e-a-b-e$

circuit e:  $e-d-c-b-e$

cycle e:  $e-a-b-c-d-e$

i) Is  $G$  eulerian, semi eulerian, (or) neither

Graph  $G$  is not eulerian but it is semi eulerian because not all vertices are even degree and

There exist a trail

Semi eulerian: They have (2 vertices) with odd degree and it is open trail

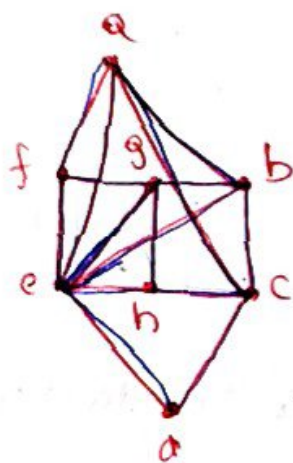
we don't get to where we start and must start at odd degree vertex

2Q)

- a) photographer: Eulerian circuit/trail  
 b) Salem: "  
 c) Flower: Hamilton cycle/path

3Q)

a)

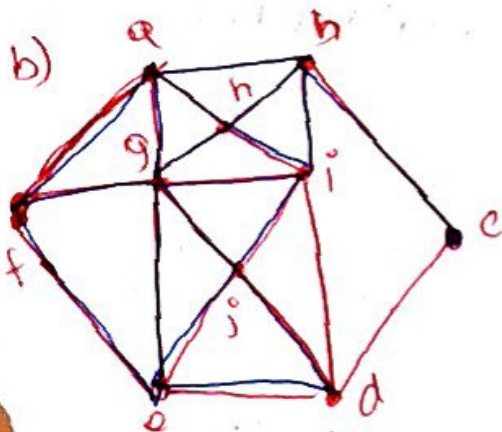


degree  $\rightarrow$   $a=4$   $f=3$   
 $b=4$   $g=4$   
 $c=4$   $h=3$   
 $d=2$   
 $e=6$

b) semi eulerian

c) a semi eulerian graph  
 doesn't contain Eulerian circuit  
 but it contains trail  
 eulerian trails:

$f-a-b-c-d-e-h-a-$   
 $e-b-g-f-e-g-h$



degree  $\rightarrow$   $a=4$   $f=3$   
 $b=4$   $g=6$   
 $c=3$   $h=4$   
 $d=4$   $i=2$   
 $e=4$   $j=4$

b) semi eulerian

c) eulerian trail:



## 10) properties of Hamiltonian Graph?

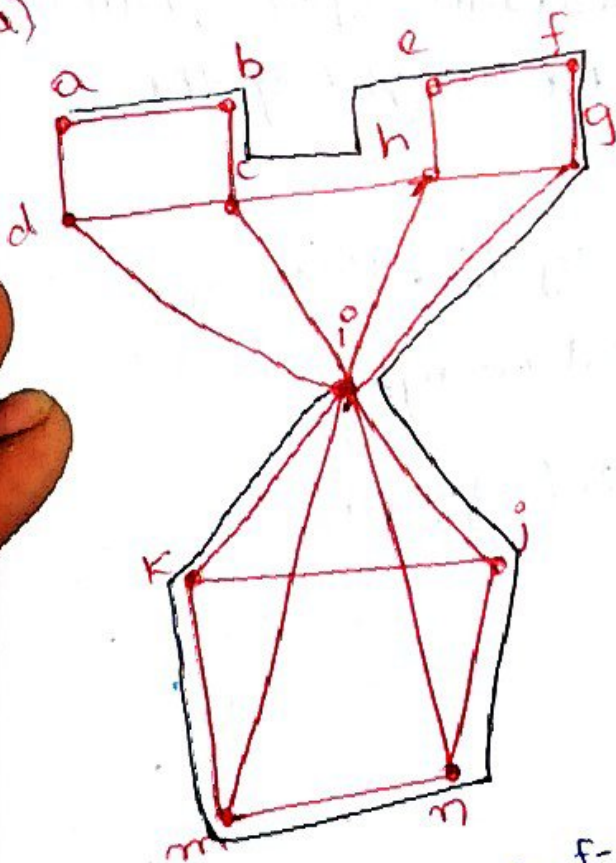
### A) Hamiltonian Graphs

A Graph  $G$  is said to be an Hamiltonian Graph if it contains a Hamiltonian cycle (or) circuit.

(or)

A Graph with closed path and covering every vertex is called Hamiltonian

a)



b) also not Hamiltonian

cycles: a-b-c-h-e-f-g

not a Hamiltonian graph

## 5Q) Brute force Algorithm?

A) Brute force Algorithm is used in travelling salesman Problem to find the shortest path and evaluate every possible route.

steps involved in finding path using Brute

→ Input : weighted graph  $G$

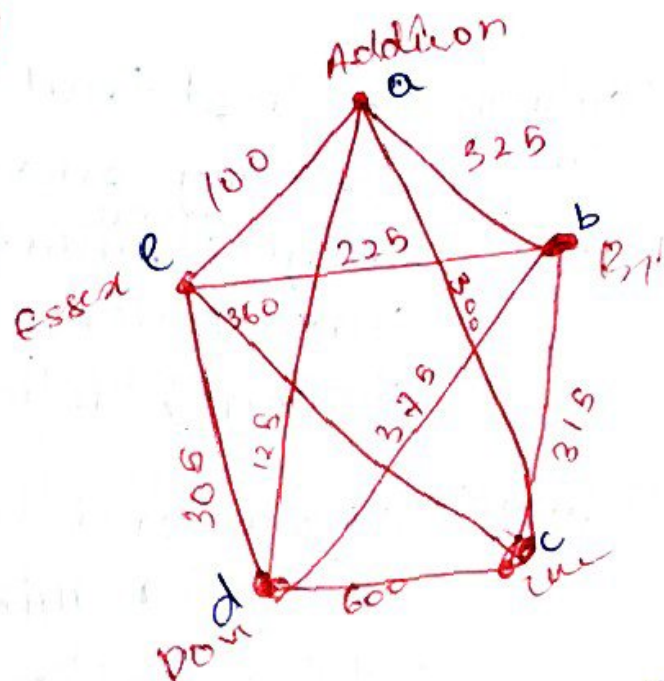
Step 1 : choose a starting vertex call it  $v$

Step 2 : find all the hamiltonian cycle from  $v$ .  
and calculate the total weight of the cycle

Step 3 : compare all  $(n-1)!$  cycles. Pick one with the least total weight

Output : Minimum weighted graph

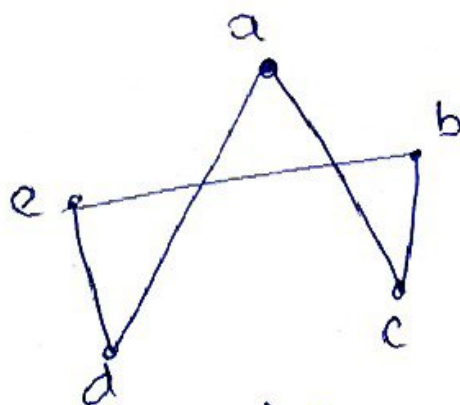
6a)



Step 1: let a better starting vertex

① cycle: ab cdea  
a e d c b a  
1645

② cycle:



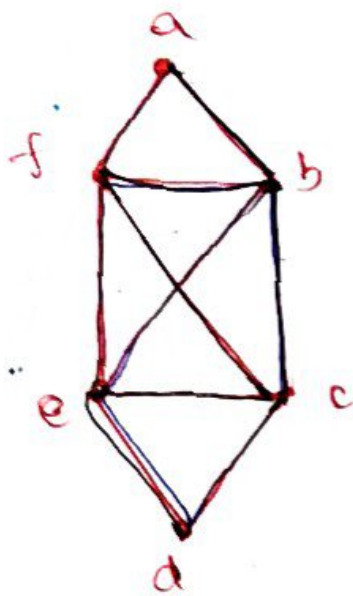
a c b e d a  
a d e b c a

1270





7Q)



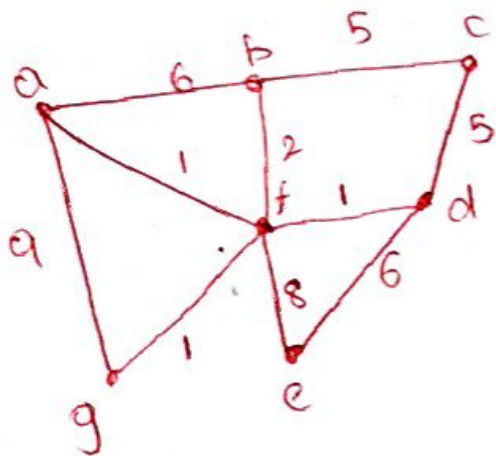
Eulerian circuit : closed trail covering every edge <sup>only once</sup> & having same vertex at starting & ending

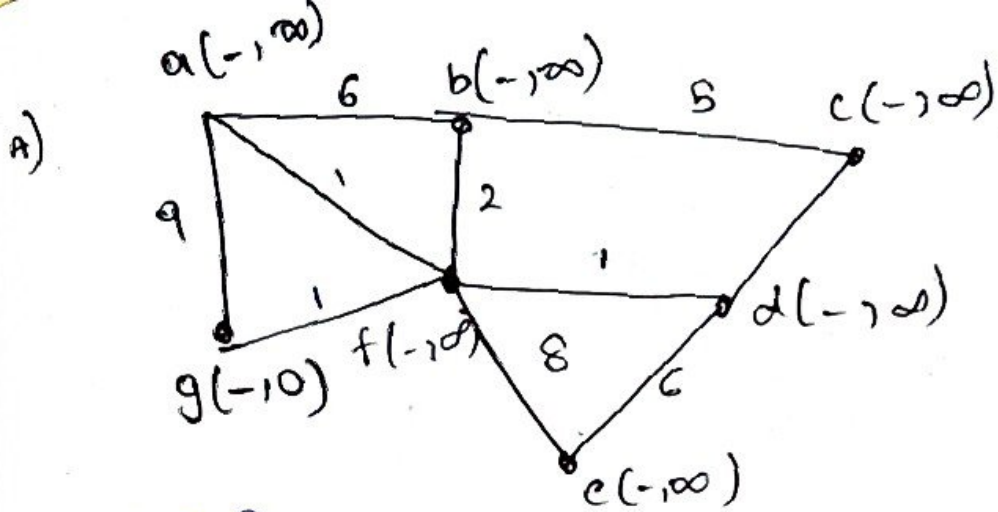
Eulerian trail : same as circuit but no requirement of ending & start

degree  $\rightarrow$   $a=2$   $d=2$   
 $b=4$   $e=4$   
 $c=4$   $f=4$

eulerian circuit :  $a-b-c-d-e-b-f-c-e-f-a$

ii) Dijkstra's Algorithm:





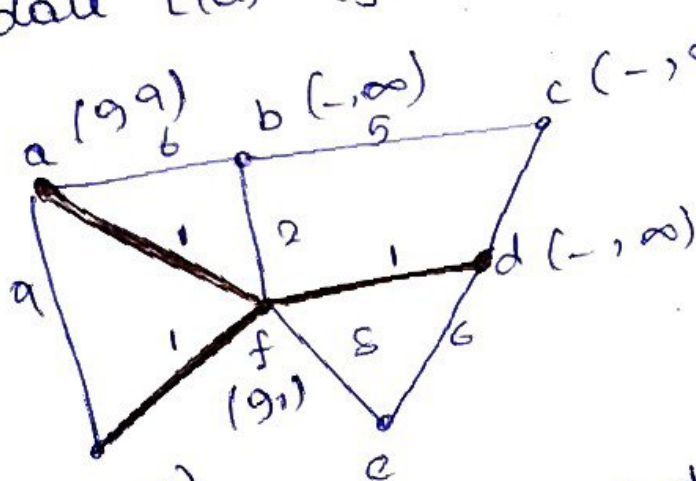
step - 1 :

$$F = \{a, f\}$$

$$w(g) + w(ga) = 0 + 9 < \infty = w(a)$$

$$w(g) + w(gf) = 0 + 1 < \infty = w(f)$$

update  $L(a) = (g, 9)$  &  $L(f) = (g, 1)$



step - 4  
 $u = f$

$$F = \{a, b, e, d\}$$

$$w(f) + w(fa) = 1 + 9 < \infty \Rightarrow w(a) = 2$$

$$w(f) + w(fb) = 1 + 2 < \infty = w(b) = 3$$

$$w(f) + w(fe) = 1 + 8 < \infty = w(e) = 9$$

$$w(f) + w(fd) = 1 + 1 < \infty = w(d) = 2$$

step - 3  
 $u = a$   
 $w(b) + w(ba) = 2 + 6 = 8$   
 $< 2 = w(b)$   
we don't update