Secant through its endpoints.

PART-A

6) If $u = x+3y^2+z^3$, $v = 4x^2yz$, $w = 2z^2-xy$ then apply $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$

A:- Given $u = x+3y^2+z^3$

$v = 4x^2yz$; $w = 2z^2-xy$.

Three eqn's partial diff w.r.to x, y & z.

$\frac{\partial u}{\partial x}(x+3y^2+z^3) = 1$

$\frac{\partial u}{\partial y}(x+3y^2+z^3) = 6y$

$\frac{\partial u}{\partial z}(x+3y^2+z^3) = 3z^2$

$\frac{\partial v}{\partial x}(4x^2yz) = 8xyz$

$\frac{\partial v}{\partial y}(4x^2yz) = 4x^2z$

$\frac{\partial v}{\partial z}(4x^2yz) = 4x^2y$

$\frac{\partial w}{\partial x}(2z^2-xy) = -y$

$\frac{\partial w}{\partial y}(2z^2-xy) = -x$

$\frac{\partial w}{\partial z}(2z^2-xy) = 4z$

then $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

$$= \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 6y & 3z^2 \\ 8xyz & 4x^2z & 4x^2y \\ -y & -x & 4z \end{vmatrix}$$

$= 1(16x^2z^2 + 4x^3y)$

$-6y(32xyz^2 + 4x^2y^2)$

$+3z^2(-8x^2yz + 4x^2yz)$

$= 16x^2z^2 + 4x^3y - 192xy^2z^2$

$-24x^2y^3 + 12x^2yz^3$

32
×6
192

7) Identify whether the following functions are functionally dependent or not. If functionally dependent, find the relation b/w them $u = \frac{x+y}{1-xy}$ $v = \tan^{-1}x + \tan^{-1}y$.

A:- Relation between x and y.

we have $u = \frac{x+y}{1-xy}$.

$v = \tan^{-1}x + \tan^{-1}y$.

$= \tan^{-1}\frac{x+y}{1-xy}$

$= \tan^{-1}u$.

$\therefore \tan v = u$ //

8) A rectangular box open at the top is to have volume of 32 cubic. Find the dimensions of the box requiring least material for its construction.

A: We have a open rectangular box with capacity $32 cm^3$.

volume (V) = 32 = (xyz)

Area (A) = $xy + 2yz + 2zx$.

$f(x,y,z) = xyz - 32$.

Construct the new function is

$F(x,y,z) = f(x,y,z) + \lambda\phi(x,y,z)$

$= xyz - 32 + \lambda(xy + 2yz + 2zx)$

$\frac{\partial F}{\partial x} = y$.

$= xy + 2yz + 2zx + \lambda(xyz - 32)$

$\frac{\partial F}{\partial x} = y + 2z + \lambda yz = 0$.

$\lambda = \frac{-(y+2z)}{yz}$ —— (1)

$\frac{\partial F}{\partial y} = x + 2z + \lambda(xz) = 0$.

$\lambda = \frac{-(x+2z)}{xz}$ —— (2).

$\frac{\partial F}{\partial z} = 2y + 2x + \lambda(xy) = 0$.

$\lambda = \frac{-2(x+y)}{xy}$ —— (3).

$xyz = 32$ —— (4).

from (1) & (2).

$\frac{-(y+2z)}{yz} = \frac{-(x+2z)}{xz}$

$xy + 2zx = xy + 2zy$

$x = y$.

from (2) & (3).

$\frac{-(x+2z)}{xz} = \frac{-2(x+y)}{xy}$

$xy + 2yz = 2xz + 2yz$

$z = y/2$.

from eqn (4)

$y \times y \times \frac{y}{2} = 32$

$y^3 = 64$.

$y = 4$; $x = 4$; $z = 2$

Dimension $(x,y,z) = (4,4,2)$

Q) Identify whether the following functions are functionally dependent or not. If functionally dependent, find the relation b/w them $X = u\sqrt{1-v^2} + v\sqrt{1-u^2}$ & $y = \sin^{-1}u + \sin^{-1}v$. doubt

A: Relation b/w x and y.

$X_u = \sqrt{1-v^2} + \frac{1}{2}v(1-u^2)\frac{v}{2\sqrt{1-u^2}}(-2u)$

$X_v = \frac{-2uv}{2\sqrt{1-v^2}} + \sqrt{1-u^2}$

$y_u = \frac{1}{\sqrt{1-u^2}}$; $y_v = \frac{1}{\sqrt{1-v^2}}$

$$= \begin{vmatrix} \sqrt{1-v^2} - \frac{uv}{\sqrt{1-u^2}} & \sqrt{1-u^2} - \frac{uv}{\sqrt{1-v^2}} \\ \frac{1}{\sqrt{1-u^2}} & \frac{1}{\sqrt{1-v^2}} \end{vmatrix}$$

$= \cancel{\sqrt{1-v^2}\sqrt{1-u^2} - uv}$

$y = \sin^{-1}(u\sqrt{1-v^2} + v\sqrt{1-u^2})$

$y = \sin^{-1}x$

10) Find the minimum value of $x^2+y^2+z^2$ given that $xyz=a^3$

A:- $f(x,y,z) = x^2+y^2+z^2$ &

$\phi(x,y,z) = xyz - a^3 = 0$.

$F(x,y,z) = f(x,y,z) + \lambda\phi(x,y,z)$

$= x^2+y^2+z^2+\lambda(xyz-a^3)$

$\frac{\partial F}{\partial x} = 2x + \lambda yz$.

$\lambda = \frac{-2x}{yz}$ — (1).

$\frac{\partial F}{\partial y} = 2y + \lambda xz$

$\lambda = \frac{-2y}{xz}$ — (2)

$\frac{\partial F}{\partial z} = 2z + \lambda(xy)$

$\lambda = \frac{-2z}{xy}$ — (3).

$xyz = a^3$ — (4).

from (1) & (2).

$\frac{+2x}{yz} = \frac{+2y}{xz}$.

$x^2 \cancel{z} = y^2 \cancel{z}$

$x^2 = y^2$

$x = \pm y$.

from (2) & (3).

$\frac{+2y}{xz} = \frac{+2z}{xy}$

$\cancel{x}y^2 = \cancel{x}z^2$

$y = \pm z$.

$x = y = z$ in (4).

$x \times x \times x = a^3$.

$x^3 = a^3 \quad x = \pm a$

$y = \pm a$; $z = \pm a$.

min value is $f(a,a,a)$

$= a^2+a^2+a^2$

$= 3a^2$ //—

PART-B:-

11) If $x = u(1-v)$; $y = uv$, then show that $JJ' = 1$, if $x+y^2=u$, $y+z^2=v$, $z+x^2=w$, find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

A:- $x = u(1-v)$, $y = uv$.

$J = \frac{\partial(u,v)}{\partial(x,y)}$ $\quad J' = \frac{\partial(x,y)}{\partial(u,v)}$

$J' = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1-v & -u \\ v & u \end{vmatrix}$

$J' = u(1-v) + uv$

$J = \frac{\partial(u,v)}{\partial(x,y)} = \frac{\frac{\partial(f_1,f_2)}{\partial(x,y)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$

$f_1(x - u(1-v)) = 0$.

$f_2(y - uv) = 0$

$\frac{(-1)^2 \begin{vmatrix} f_{1x} & f_{1y} \\ f_{2x} & f_{2y} \end{vmatrix}}{\begin{vmatrix} f_{1u} & f_{1v} \\ f_{2u} & f_{2v} \end{vmatrix}} = (-1)^2 \frac{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} -(1-v) & u \\ -v & -u \end{vmatrix}}$

$J = \frac{1}{u(1-v)+uv}$.

$JJ' = \frac{1}{\cancel{u(1-v)+uv}} \cancel{(u(1-v)+uv)}$

$JJ' = 1$

given:- $u = x+y^2$ $\quad f_1(u-x-y^2)=0$

$v = y+z^2$ $\quad f_2(v-y-z^2)=0$

$w = z+x^2$ $\quad f_3(w-z-x^2)=0$

$\frac{\partial(x,y,z)}{\partial(u,v,w)} = (-1)^3 \times \frac{\frac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}}{\frac{\partial(f_1,f_2,f_3)}{\partial(x,y,z)}}$

$= (-1)\times \frac{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} -1 & -2y & 0 \\ 0 & -1 & -2z \\ -2x & 0 & -1 \end{vmatrix}}$

$= \frac{-1}{-1-8xyz} = \frac{1}{1+8xyz}$

12) If $u = x^2 - y^2$; $v = 2xy$ where $x = r\cos\theta$, $y = r\sin\theta$ then show that $\frac{\partial(u,v)}{\partial(x,y)} = 4r^3$.

A:- $u = r^2(\cos^2\theta - \sin^2\theta)$

$= r^2\cos 2\theta$.

$v = 2.r^2\sin\theta\cos\theta$.

$= r^2\sin 2\theta$.

$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_r & u_\theta \\ v_r & v_\theta \end{vmatrix}$

$u_r = 2r\cos 2\theta$, $u_\theta = -2r^2\sin 2\theta$

$v_r = 2r\sin 2\theta$, $v_\theta = 2r^2\cos 2\theta$.

$\begin{vmatrix} 2r\cos 2\theta & -2r^2\sin 2\theta \\ 2r\sin 2\theta & 2r^2\cos 2\theta \end{vmatrix}$

$= 4r^3\cos^2 2\theta + 4r^3\sin^2 2\theta$

$= 4r^3(\cos^2 2\theta + \sin^2 2\theta)$

$= 4r^3$.

13) If $x = e^r\sec\theta$, $y = e^r\tan\theta$ then prove that

$\frac{\partial(x,y)}{\partial(r,\theta)} \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

A: $x_r = e^r\sec\theta$, $x_\theta = e^r\tan\theta\sec\theta$

$y_r = e^r\tan\theta$, $y_\theta = -e^r\sec^2\theta$.

$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} e^r\sec\theta & e^r\sec\theta\tan\theta \\ e^r\tan\theta & e^r\sec^2\theta \end{vmatrix}$

$= e^{2r}\sec^3\theta - e^{2r}\sec\theta\tan^2\theta$

$= e^{2r}\sec\theta = J$

$x = e^r\sec\theta$, $y = e^r\tan\theta$

$x^2 - y^2 = e^{2r}\sec^2\theta - e^{2r}\tan^2\theta$.

$= e^{2r}$.

$2r = \log(x^2-y^2)$

$r = \frac{1}{2}\log(x^2-y^2)$

$\boxed{r = \log\sqrt{x^2-y^2}}$ — (1)

$\frac{x}{y} = \frac{e^r\sec\theta}{e^r\tan\theta}$.

$= \operatorname{cosec}\theta$.

$\sin\theta = y/x$.

$\theta = \sin^{-1}(y/x)$

$r_x = \frac{1}{\sqrt{x^2-y^2}} \cdot \frac{1}{2\sqrt{x^2-y^2}} (2x)$

$= \frac{x}{x^2-y^2}$

$r_y = \frac{-2y}{2(x^2-y^2)} = \frac{-y}{x^2-y^2}$

$\theta_x = \frac{1}{\sqrt{1-\frac{y^2}{x^2}}} (-y/x^2)$

$$= \frac{x}{\sqrt{x^2-y^2}} \cdot \left(\frac{-y}{x^2}\right)$$

$$= \frac{-y}{x\sqrt{x^2-y^2}}$$

$$\theta_y = \frac{1}{\sqrt{x^2-y^2}}$$

$$\frac{\partial(r,\theta)}{\partial(x,y)} = \begin{vmatrix} \frac{x}{x^2-y^2} & \frac{-y}{x^2-y^2} \\ \frac{-y}{x\sqrt{x^2-y^2}} & \frac{1}{\sqrt{x^2-y^2}} \end{vmatrix}$$

$$= \frac{x}{(x^2-y^2)\sqrt{(x^2-y^2)}} - \frac{y^2}{x(x^2-y^2)\sqrt{x^2-y^2}}$$

$$= \frac{\cancel{x^2-y^2}}{x\cancel{(x^2-y^2)}\sqrt{x^2-y^2}}$$

$$= \frac{1}{x\sqrt{x^2-y^2}}$$

$$= \frac{1}{e^r\sec\theta\sqrt{e^{2r}}}$$

$$= \frac{1}{e^{2r}\sec\theta}$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$$

15) Make use of lagrange's multiplies method to find the minimum value of the function x^2+y^2, subject to the condition $ax+by=c$.

A:- $\phi(x,y) = ax+by-c=0$

$f(x,y) = x^2+y^2$

$F(x,y) = f(x,y) + \lambda\phi(x,y)$

$= x^2+y^2+\lambda(ax+by-c)$

$$\frac{\partial F}{\partial x} = 2x+a\lambda=0.$$

$$\lambda = \frac{-2x}{a} \quad \text{--- (1)}$$

$$\frac{\partial F}{\partial y} = 2y+b\lambda=0.$$

$$\lambda = \frac{-2y}{b} \quad \text{--- (2)}$$

$$ax+by=c \quad \text{--- (3)}$$

from (1) & (2).

$$\frac{-2x}{a} = \frac{-2y}{b}$$

$$x = \frac{ay}{b}.$$

$$a\left(\frac{ay}{b}\right)+by = c.$$

$$a^2y+b^2y = bc$$

$$y = \frac{bc}{a^2+b^2}$$

$$x = \frac{ac}{a^2+b^2}$$

$$f(x,y) = x^2+y^2.$$

$$= \frac{a^2c^2}{(a^2+b^2)^2} + \frac{b^2c^2}{(a^2+b^2)^2}$$

$$= \frac{c^2\cancel{(a^2+b^2)}}{(a^2+b^2)^{\cancel{2}}}$$

$$= \frac{c^2}{a^2+b^2} \quad \text{(minimum value)}$$

16) Find the max volume of a rectangular parallelopipped that can be inscribed in a sphere using lagrange multipli-ers method.

$V = 8xyz$

$f(x,y,z) = 8xyz$

$\phi(x,y,z) = x^2+y^2+z^2-r^2$

$F(x,y,z) = f(x,y,z) + \lambda\phi(x,y,z)$

$= 8xyz + \lambda(x^2+y^2+z^2-r^2)$

$\frac{\partial F}{\partial x} = 8yz + 2\lambda x = 0.$

$\lambda = -\frac{4yz}{x}$ — ①

$\frac{\partial F}{\partial y} = 8zx + 2\lambda y = 0.$

$\lambda = -\frac{4zx}{y}$ — ②

$\frac{\partial F}{\partial z} = 8xy + 2\lambda z = 0.$

$\lambda = -\frac{4xy}{z}$ — ③

$x^2+y^2+z^2 = r^2$ — ④

from ① & ②

$-\frac{4yz}{x} = -\frac{4zx}{y}$ $\quad x^2 = y^2$, $x = \pm y$

from ② & ③

$-\frac{4zx}{y} = -\frac{4xy}{z}$ $\quad z^2 = y^2$, $z = \pm y$

$x^2+y^2+z^2 = r^2$

$3y^2 = r^2$

$y = \frac{\pm r}{\sqrt{3}}$; $x = \frac{\pm r}{\sqrt{3}}$; $z = \frac{\pm r}{\sqrt{3}}$.

$\max f(x,y,z) = \frac{8r^3}{3\sqrt{3}}$

17) Find the volume of the largest (maxima) rectangular parallelopipped that can be inscribed in the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$

A:- $f(x,y,z) = 8xyz.$

$\phi(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0.$

$F(x,y,z) = f(x,y,z) + \lambda\phi(x,y,z)$

$= 8xyz + \lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1\right)$

$\frac{\partial F}{\partial x} = 8yz + \frac{2\lambda x}{a^2} = 0.$

$\lambda = -\frac{4a^2yz}{x}$ — ①

$\frac{\partial F}{\partial y} = 8xz + \frac{2\lambda y}{b^2} = 0.$

$\lambda = -\frac{4b^2xz}{y}$ — ②

$\frac{\partial F}{\partial z} = 8xy + \frac{2\lambda z}{c^2} = 0.$

$\lambda = -\frac{4c^2xy}{z}$ — ③

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ — ④

from ① & ②

$-\frac{4a^2yz}{x} = -\frac{4b^2xz}{y}$

$a^2y^2 = b^2x^2$

$x = \pm\frac{ay}{b}$

from ② & ③

$-\frac{4b^2xz}{y} = -\frac{4c^2xy}{z}$

$b^2z^2 = c^2y^2$

$z = \pm\frac{cy}{b}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$\frac{a^2y^2}{a^2b^2} + \frac{y^2}{b^2} + \frac{c^2y^2}{c^2b^2} = 1$; $\frac{3y^2}{b^2} = 1$

$x = \pm\frac{a}{\sqrt{3}}$; $y = \pm\frac{b}{\sqrt{3}}$; $z = \pm\frac{c}{\sqrt{3}}$

$8xyz = 8abc$

$= 8\left(\frac{b}{\sqrt{3}}\right)\left(\frac{-a}{\sqrt{3}}\right)\left(\frac{-c}{\sqrt{3}}\right)$

$= \frac{8abc}{3\sqrt{3}}$ (max)

$8xyz = -8abc$

$= 8\left(\frac{a}{\sqrt{3}}\right)\left(\frac{-b}{\sqrt{3}}\right)\left(\frac{c}{\sqrt{3}}\right)$

$= -\frac{8abc}{3\sqrt{3}}$ (min)

18) Show the ~~vol of the largest~~ stationary points of $u(x,y) = \sin x . \sin y . \sin(x+y)$ where $0 < x < \pi$, $0 < y < \pi$ & find the max value of the function u.

A:- $u(x,y) = \sin x . \sin y . \sin(x+y)$

$\frac{\partial u}{\partial x} = \cos x . \sin y \sin(x+y) + \sin x \sin y \cos(x+y)$

$= \sin y (\cos x (\sin(x+y) + \sin x \cos(x+y))$

$= \sin(2x+y) \sin y$

$\frac{\partial u}{\partial y} = \cos y \sin x \sin(x+y) + \sin x \sin y \cos(x+y)$

$= \sin x (\sin y \cos(x+y) + \cos y \sin(x+y))$

$= \sin(x+2y) \sin x$

$r = \frac{\partial^2 u}{\partial x^2} = \sin y . (\cos(2x+y)) . 2$

$= 2 \sin y \cos(2x+y)$

$s = \frac{\partial^2 u}{\partial x \partial y} = \sin(x+2y) \cos x + \sin x \cos(x+2y)$

$= \sin(2x+2y) - \sin 2(x+y)$

$t = \frac{\partial^2 u}{\partial x^2} = 2 \sin x . \cos(x+2y)$

for stationary points

$\frac{\partial u}{\partial x} = 0$; $\frac{\partial u}{\partial y} = 0$,

$\sin(2x+y) \sin y = 0$ — (1)

$\sin(x+2y) \sin x = 0$ — (2)

from (1)

$\sin y = 0$.

$\sin(2x+y) = \sin \pi$

$2x+y = \pi$

from (2)

$\sin x = 0$.

$\sin(x+2y) = \sin\pi$

$x+2y = \pi$

$2x+y = \pi$.

$2x+4y = \pi$

$-3y = -\pi$.

$x = \pi/3$; $y = \pi/3$. ; $z = \pi/3$

$r = 2 \times \frac{\sqrt{3}}{2} (-1) = -\sqrt{3}$.

$s = -\frac{\sqrt{3}}{2}$ $t = -\sqrt{3}$.

$rt - s^2 = 3 - 3/4$

$9/4 > 0$.

~~r~~, $r < 0$.

max of $f(\pi/3, \pi/3) = \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2}$ //

20) Find the points on the plane $3x+2y+z-12=0$ nearest to the origin.

A:- $\phi(x,y,z) = 3x+2y+z-12 = 0$ from origin

$D = \sqrt{x^2+y^2+z^2}$

$= x^2+y^2+z^2$

$f(x,y,z) = x^2+y^2+z^2$

$F(x,y,z) = f(x,y,z) + \lambda\phi(x,y,z)$

$= x^2+y^2+z^2+\lambda(3x+2y+z-12)$

$\frac{\partial F}{\partial x} = 2x+3\lambda = 0;\ \lambda = \frac{-2x}{3}$ — (1)

$\frac{\partial F}{\partial y} = 2y+2\lambda = 0;\ \lambda = -y$ — (2)

$\frac{\partial F}{\partial z} = 2z+\lambda = 0;\ \lambda = -2z$ — (3)

~~fro~~

$3x+2y+z=12$ — (4).

from (1) & (2).

$\frac{2x}{3} = y \quad x = \frac{3y}{2}$.

from (2) & (3).

$-y = -2z.\ z = y/2$

sub in (4).

$3\left(\frac{3y}{2}\right)+2y+y/2 = 12$

$\frac{9y}{2}+2y+\frac{y}{2} = 12$

$\frac{14y}{2} = 12 \quad y = \frac{12}{7}$

$x = \frac{18}{7};\ z = 6/7$

point $(18/7, 12/7, 6/7)$

19) Calculate the stationary points of $f(x,y) = \sin x + \sin y + \sin(x+y)$ where $0 < x < \pi$, $0 < y < \pi$ & find the max value of the function f

A: $f(x,y) = \sin x + \sin y + \sin(x+y)$

$\frac{\partial f}{\partial x} = \cos x + \cos(x+y)$

$\frac{\partial f}{\partial y} = \cos y + \cos(x+y)$

$r = -\sin x - \sin(x+y)$

$s = -\sin(x+y)$

$t = -\sin y - \sin(x+y)$

stationary points

$\frac{\partial F}{\partial x} = 0;\ \frac{\partial F}{\partial y} = 0.$

$\cos x + \cos(x+y) = 0$. $\quad \cos y + \cos(x+y) = 0$.

$\cos x = \cos y \quad x = y$.

$x = y$.

$\cos(x+y) = -\cos x$

$\cos x + \cos(x+y) + \cos y + \cos(x+y) = 0$

$2\cos(x+y) + \cos x + \cos y = 0$.

$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\therefore \cancel{2}\cos 2x = -\cancel{2}\cos x$.

$\cos 2x = \cos(\pi - x)$

$2x = \pi - x$.

$x = \pi/3,\ y = \pi/3$

$r = -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = \frac{-\cancel{2}\sqrt{3}}{\cancel{2}} = -\sqrt{3}$

$s = -\frac{\sqrt{3}}{2}$

$t = -\sqrt{3}$

$rt - s^2 = 3 - 3/4 = 9/4 > 0.\ r < 0.$

max val $f(x,y)$ at $(\pi/3, \pi/3)$ is $\frac{3\sqrt{3}}{2}$ //

14) If $ux = yz,\ vy = zx,\ wz = xy$ then find the Jacobian

$$\frac{\partial(x,y,z)}{\partial(u,v,w)}$$

A:- $$\frac{\partial(x,y,z)}{\partial(u,v,w)} = (-1)^3 \frac{\dfrac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}}{\dfrac{\partial(f_1,f_2,f_3)}{\partial(x,y,z)}}$$

$f_1\ (ux - yz) = 0$

$f_2\ (vy - zx) = 0$

$f_3\ (wz - xy) = 0$

$$= (-1)^3 \frac{\begin{vmatrix} f_{1u} & f_{1v} & f_{1w} \\ f_{2u} & f_{2v} & f_{2w} \\ f_{3u} & f_{3v} & f_{3w} \end{vmatrix}}{\begin{vmatrix} f_{1x} & f_{1y} & f_{1z} \\ f_{2x} & f_{2y} & f_{2z} \\ f_{3x} & f_{3y} & f_{3z} \end{vmatrix}}$$

$$= (-1)^3 \frac{\begin{vmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{vmatrix}}{\begin{vmatrix} u & -z & -y \\ -z & v & -x \\ -y & -x & w \end{vmatrix}}$$

$$= \frac{-xyz}{u(vw - x^2) + z(-zw - xy) - y(xz + vy)}$$

$$= \frac{-xyz}{uvw - ux^2 - z^2w - xyz - xyz - vy^2}$$

$$= \frac{-xyz}{uvw - (ux^2 + vy^2 + wz^2) - 2xyz}$$