Eliminated n

$$\frac{ds}{dt} = \beta s(1-s-q-a) - \alpha sa + \sigma qs - \mu s \quad \text{— (1)}$$

$$\frac{dq}{dt} = \alpha sa - \sigma qs - \gamma q - \mu q \quad \text{— (2)}$$

$$\frac{da}{dt} = \mu p + \gamma q - \mu a \quad \text{— (3)}$$

Nispace

From (3),

$$a(q) = p + \left(\frac{\gamma}{\mu}\right) q \quad \text{— (i)}$$

From (2),

$$q(a,s) = \frac{\alpha s a}{\sigma s + \gamma + \mu} \quad \text{— (ii)}$$

Using $q(a,s)$ in (i),

$$a = p + \left(\frac{\gamma}{\mu}\right)\frac{(\alpha s a)}{(\sigma s + \gamma + \mu)}$$

$$\Rightarrow a\left[1 - \frac{\gamma \alpha s}{\sigma \mu s + \mu(\gamma+\mu)}\right] = p$$

$$\Rightarrow a = \frac{p\left[\sigma \mu s + \mu(\gamma+\mu)\right]}{s(\sigma\mu - \gamma\alpha) + \mu(\gamma+\mu)} = \boxed{\frac{p\mu\,(\sigma s + \gamma + \mu)}{s(\sigma\mu - \gamma d) + \mu(\gamma+\mu)}}$$

Using this result in (ii),

$$q = \frac{\alpha s p\mu \cancel{(\sigma s + \gamma + \mu)}}{\cancel{(\sigma s + \gamma + \mu)}\left[s(\sigma\mu - \gamma d) + \mu(\gamma+\mu)\right]} = \boxed{\frac{(\alpha p\mu)s}{s(\sigma a - \gamma d) + \mu(\gamma+\mu)}}$$

Using (1),

$$\beta(1-s-q-a) - \alpha a + \sigma q = \mu \longrightarrow \text{Doubt here}$$

Eigenvalues:

$-\mu$, $-\gamma-\mu$, $-\alpha p - \beta p + \beta - \mu$ < 0

$$\Rightarrow \beta(1-p) < \mu + \alpha p$$

$$\Rightarrow \boxed{\frac{\beta(1-p)}{\mu + \alpha p}} \rightarrow R_0$$

$$\boxed{R_0 = \frac{\beta(1-p)}{\mu + \alpha p}}$$

$$d = \mu(\gamma + \mu)$$

$$K = \sigma\mu - \gamma d$$

$$B\left[1 - s - \frac{(\alpha p\mu)s}{sK+\phi} - \frac{(p\mu\sigma)s + p\phi}{sK+\phi}\right]$$

$$- \frac{\alpha\left[(p\mu\sigma)s + p\phi\right]}{sK+\phi} + \frac{\sigma(\alpha p\mu)s}{sK+\phi} = \mu$$

$$\Rightarrow B\left[sK+\phi - s(sK+\phi) - (\alpha p\mu)s - (p\mu\sigma)s - p\phi - s\phi\right]$$

$$- \alpha\left[(p\mu\sigma)s + p\phi\right] + \sigma(\alpha p\mu)s = \mu sK + \mu\phi$$

$$-s^2KB + s\left(KB - B\phi - B\alpha p\mu - Bp\mu\sigma - \phi B - \alpha p\mu\sigma + \sigma\alpha p\mu - \mu K\right)$$

$$+ B\phi - p\phi B - \alpha p\phi - \mu\phi = 0$$

a coeff:

$-KB$

b coeff:

$KB - B\phi - B\alpha p\mu - Bp\mu\sigma - \phi B - \mu K$

c coeff:

$\phi(B - pB - \alpha p - \mu)$

If $c > 0$, $B(1-p) > \mu + \alpha p$

$\Rightarrow \boxed{R > 1}$

$$B\left[\gamma K + \phi - s(\gamma K + \phi) - (\alpha p \mu)s - p\mu\sigma s - p\phi\right]$$

$$-\alpha p\mu\sigma s - \alpha p\phi + \sigma\alpha p\mu s = \mu s K + \mu\phi$$

$$\cancel{B\gamma K} + B\phi - \cancel{B\gamma K} - \cancel{B\gamma\phi} - \alpha p\mu s B - B p\mu\sigma\gamma - Bp\phi$$

$$\cancel{-\alpha p\mu\sigma s} - \alpha p\phi + \cancel{\sigma\alpha p\mu s} = \cancel{\mu s K} + \mu\phi$$

$$\Rightarrow \quad -(BK)s^2 + s\left(BK - B\phi - \alpha p\mu B - Bp\mu\sigma - \mu K\right)$$
$$+ B\phi - Bp\phi - \alpha p\phi - \mu\phi = 0$$

$$a = B(\gamma\alpha - \sigma\mu)$$
$$b = BK - B\phi - p\mu B(\alpha+\sigma) - \mu K$$
$$c = \phi(B - Bp - \alpha p - \mu)$$
$$= \phi\left(B(1-p) - \alpha p - \mu\right)$$
$$= \phi\left[R(\mu + \alpha p) - (\alpha p + \mu)\right]$$
$$= \boxed{\phi(\alpha p + \mu)(R-1)}$$

$$= \boxed{\mu(\gamma + \mu)(\alpha p + \mu)(R-1)}$$

$$R = \frac{B(1-p)}{\mu + \alpha p}$$

$$B(1-p) = R(\mu + \alpha p)$$

If $R > 1$, $c > 0$
$R < 1$, $c < 0$

Working with:

$\beta K - \beta\phi - \alpha p\mu\beta - \beta p\mu\sigma - \mu K$

$K = \sigma\mu - \gamma\alpha$
$\phi = \mu\gamma + \mu^2$

$\beta(\sigma\mu - \gamma\alpha) - \beta(\mu\gamma + \mu^2) - \alpha p\mu\beta - \beta p\mu\sigma$
$- \mu(\sigma\mu - \gamma\alpha)$

$= \cancel{\beta\sigma\mu} - \cancel{\beta\gamma\alpha} - \cancel{\beta\mu\gamma} - \cancel{\beta\mu^2} - \alpha p\mu\beta - \beta p\mu\sigma$
$- \sigma\mu^2 + \cancel{\mu\gamma\alpha}$

$= \beta\mu(\sigma - \gamma) + \gamma\alpha(\mu - \beta) - \mu^2(\sigma + \beta) - \beta p\mu(\sigma + \alpha)$

$- \mu[\mu\beta + \mu\sigma - \beta p\sigma - \beta p\alpha]$

b simplified

$\mu(\gamma\alpha + \beta\sigma) - \mu^2(\beta + \sigma) - \beta p\mu(\sigma + \alpha) - \beta\gamma(\alpha + \gamma)$