$$\dot{N} = \mu(1-p)T - \frac{\beta SN}{T} - \mu N$$

$$\dot{S} = \frac{\beta SN}{T} - \frac{\rho SA}{T} + \sigma\phi S - \mu S$$

$$\dot{\phi} = \frac{\rho SA}{T} - \sigma\phi S - \gamma\phi - \mu\phi$$

$$\dot{A} = \mu pT + \gamma\phi - \mu A$$

Total adult pop. is
N - Non-smokers
S - Smokers
ϕ - Quitters
A - Aware individuals

T constant

$$T = N + S + \phi + A$$

$$\frac{d}{dt}(N+S+\phi+A) = \mu(1-p)T - \frac{\beta SN}{T} - \mu N + \frac{\beta SN}{T} - \frac{\rho SA}{T} + \sigma\phi S - \mu S + \frac{\rho SA}{T} - \sigma\phi S - \gamma\phi - \mu\phi + \mu pT + \gamma\phi - \mu A$$

$$\Rightarrow \frac{d(T)}{dt} = \mu T - \mu pT - \mu N - \mu S - \mu\phi + \mu pT - \mu A$$

$$\Rightarrow \frac{d}{dt}T = \mu(T - N - S - \phi - A) = 0$$

$$\Rightarrow \boxed{\frac{dT}{dt} = 0} \Rightarrow \boxed{T \text{ is constant}}$$

$$\Rightarrow \frac{N}{T} + \frac{S}{T} + \frac{\phi}{T} + \frac{A}{T} = 1$$

$$\Rightarrow \boxed{n + s + q + a = 1}$$

$n = \frac{N}{T}$, $s = \frac{S}{T}$, $q = \frac{\phi}{T}$, $a = \frac{A}{T}$ → For adults

Reduced System (using substitutions).

$$\frac{dr}{dt} = \mu(1-p) - \beta rs - \mu r \quad (F) \qquad (1)$$

$$\frac{ds}{dt} = \beta rs - psa + \sigma_s qsT - \mu s \quad (G) \qquad (2)$$

$$\frac{dq}{dt} = psa - \sigma_q qsT - \gamma q - \mu q \quad (H) \qquad (3)$$

$$\frac{da}{dt} = \mu p + \gamma q - \mu a \quad (I) \qquad (4)$$

Eq. points

One is trivial. Put (2) = 0.

$\Rightarrow (1-p, 0, 0, p)$ — (I)

Other fixed pts?

Jacobian

$$H(r,s,q,a) = \begin{bmatrix} -\beta s - \mu & -\beta r & 0 & 0 \\ \beta s & \beta r - pa + \sigma_q qT - \mu & \sigma_s sT & -ps \\ 0 & pa - \sigma_q qT & -\sigma sT - \gamma - \mu & ps \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

Evaluating at (I), we get,

$$\begin{bmatrix} -\mu & \beta(1-p) & 0 & 0 \\ 0 & \beta(1-p) - pa - \mu & 0 & 0 \\ 0 & p^2 & -\gamma - \mu & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

The corresponding eigenvalues: $-\mu$, $-\gamma-\mu$, $\beta-\beta p-p\alpha-\mu$.

Schematic Diagram