$\boxed{\text{Vector Fields}}$

Parameters original:
$\beta, p, \gamma, \sigma, \mu$

SOP

Define $s = \dfrac{S}{T}$, $n = \dfrac{N}{T}$, $q = \dfrac{Q}{T}$, $a = \dfrac{A}{T}$.

The reduced system of eqn's will be:

$$\frac{dn}{dt} = \mu(1-p) - \beta s n - \mu n$$

$$\frac{ds}{dt} = \beta s n - p s a + \sigma q s - \mu s$$

$$\frac{dq}{dt} = p s a - \sigma q s - \gamma q - \mu q$$

$$\frac{da}{dt} = \mu p + \gamma q - \mu a$$

$\boxed{n + s + q + a = 1}$

Define,
$\omega = \gamma + \mu$

Eliminate the first eqn, we get,

$\beta s(1-s-q-a)$

$$\frac{ds}{dt} = \beta s n - p s a + \sigma q s - \mu s \quad (1)$$

$$\frac{dq}{dt} = p s a - \sigma q s - \gamma q - \mu q \quad (2)$$

$$\frac{da}{dt} = \mu p + \gamma q - \mu a \quad (3)$$

From (3), $a(q) = p + \left(\dfrac{\gamma}{\mu}\right) q \rightarrow$ a in terms of q.

$$q(a,s) = \frac{p s a}{\sigma s + \gamma + \mu} = \frac{p s a}{\sigma s + \omega}$$

Parameters

Define, $\omega = \gamma + \mu$, $\phi = p\mu\sigma$, $h = \omega p\mu$,

$K = \sigma\mu - \gamma p$, $\delta = \mu\omega$

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Using this in $a(q)$, we get,

$$a(q) = p + \left(\frac{\gamma}{\mu}\right)\frac{psa}{(\sigma s + \omega)}$$

$$\Rightarrow a\left[1 - \frac{\gamma p s}{\mu(\sigma s + \omega)}\right] = p$$

$\Rightarrow$

$$a = \frac{p\mu(\sigma s + \omega)}{\sigma\mu s + \mu\omega - \gamma p s}$$

$$\Rightarrow a(s) = \frac{(p\mu\sigma)s + \omega p\mu}{s(\sigma\mu - \gamma p) + \mu\omega}$$

$$\Rightarrow \boxed{a(s) = \frac{\phi s + h}{Ks + \delta}} \quad \text{I}$$

Substituting I in $q(a, s)$,

$$q(s) = \frac{ps(\phi s + h)}{(\sigma s + \omega)(Ks + \delta)}$$

$$q(s) = \frac{ps \cdot p\mu\cancel{(\sigma s + \omega)}}{(\sigma\mu s + \mu\omega - \gamma p s)\cancel{(\sigma s + \omega)}}$$

$$= \boxed{\frac{p^2 s\mu}{Ks + \delta}} \quad \text{II}$$

Using I and II in I, we get,

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$$B(1-s-q-a) - pa + \sigma q = \mu$$

$$\Rightarrow B\left(1 - s - \frac{p^2 s\mu - \phi s - h}{Ks+\delta}\right)$$

$$- \frac{p(\phi s + h)}{Ks+\delta} + \frac{\sigma p^2 s\mu}{Ks+\delta} = \mu$$

$$\Rightarrow B\left(Ks + \delta - Ks^2 - \delta s - p^2 s\mu - \phi s - h\right)$$

$$- p(\phi s + h) + \sigma p^2 s\mu = \mu(Ks+\delta)$$

$$\Rightarrow \cancel{BKs + B\delta - BKs^2 - B\delta s - Bp^2 s\mu}$$

$$\cancel{- B\phi s - Bh - p\phi s - ph}$$

$$\cancel{+ \sigma p^2 s\mu = \mu Ks + \mu\delta}$$

$$\Rightarrow (-KB)s^2 + s\left(BK - B\delta - Bp^2\mu - B\phi - p\phi + \sigma p^2\mu - \mu K\right)$$

$$\cancel{+ B(\delta - h) + ph = 0}$$

$$\cancel{-h(B+p) - \mu\delta = 0}$$

$$+ B(\delta - h) - ph - \mu\delta = 0$$

$$\boxed{\begin{aligned} a &= -KB \\ b &= \text{[redacted]} \\ c &= B(\delta - h) - ph - \mu\delta \end{aligned}}$$

doing symbolic computation $\Leftarrow$ {

For $c > 0$,

$$\cancel{\mu(-B\gamma p + B\gamma - B\mu p + B\mu - \gamma p^2 - \gamma\mu p^2 - \mu + 1) > 0}$$

$$\cancel{-B\gamma(p-1) - B\mu(p-1) - \gamma(p^2+1) - \mu(p^2+1) > -1}$$

$\cancel{\Rightarrow \beta(\gamma+\mu)(p-1) + (\gamma+\mu)(p^2+1) < 1}$

$\cancel{\Rightarrow (\gamma+\mu)\left(\beta(p-1)+p^2+1\right) < 1}$

For $\lambda > 0$,

$\cancel{\mu}(-\beta\gamma p + \beta\gamma - \beta\mu p + \beta\mu - \gamma\mu - \gamma p^2$
$- \mu^2 - \mu p^2) > 0$

$\Rightarrow -\beta\gamma(p-1) - \beta\mu(p-1) - \gamma(\mu+p^2)$
$- \mu(\mu+p^2) > 0$

$\Rightarrow -\beta\cancel{(\gamma+\mu)}(p-1) > \cancel{(\gamma+\mu)}(\mu+p^2)$

$\Rightarrow \boxed{\dfrac{\beta(1-p)}{\mu+p^2} > 1}$

From the Jacobian analysis, we get,

$\boxed{\dfrac{\mu+p^2}{\beta(1-p)} < 1}$

So, we are getting consistent results.

So, $\boxed{R = \dfrac{\beta(1-p)}{\mu+p^2}}$

Eigenvalues: $-\mu$, $-\mu-\gamma$,
$-p\beta - p^2 + \beta - \mu$

$-p\beta - p^2 + \beta - \mu < 0$

$\Rightarrow -p^2 - \mu < \beta(p-1)$

$\Rightarrow \boxed{\dfrac{p^2+\mu}{\beta(1-p)} < 1}$