Estimation of Random Channel Gain for SISO Visible Light Communications System Estimation du gain aléatoire d'un canal pour un système de communication SISO à lumière visible

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***Abstract*— In this article, the estimation of random channel gain is studied for a single-input single-output (SISO) visible light communication (VLC) system. Five different estimators, namely maximum likelihood (ML), least square (LS), maximum posteriori probability (MAP), linear minimum mean square error (LMMSE), and minimum mean square error (MMSE), are proposed. The performances of these estimators are compared with the derived Bayesian Cramér–Rao lower bound (BCRLB), which can be used as a benchmark to evaluate the efficiency of the unbiased estimators. The presented analytical results, corroborated with Monte Carlo simulations, indicate that the MMSE estimator provides the best results. Additionally, the increasing number of pilot symbols as well as the ascending transmitted power improve the system performance. On the other hand, the noise variance has a negative effect on the channel estimation in terms of mean square error (MSE), and thus, it can dramatically reduce the performance of the estimators.**

***Résumé*— Dans cet article, l'estimation du gain aléatoire du canal est étudiée pour un système de communication à lumière visible (VLC) à une seule entrée et une seule sortie (SISO). Cinq estimateurs différents, à savoir le maximum de vraisemblance (ML), les moindres carrés (LS), la probabilité maximale a posteriori (MAP), l'erreur quadratique moyenne minimale linéaire (LMMSE) et l'erreur quadratique moyenne minimale (MMSE), sont proposés. Les performances de ces estimateurs sont comparées à la borne inférieure bayésienne de Cramér-Rao (BCRLB), qui peut être utilisée comme référence pour évaluer l'efficacité des estimateurs sans biais. Les résultats analytiques présentés, corroborés par des simulations de Monte Carlo, indiquent que l'estimateur MMSE fournit les meilleurs résultats. En outre, l'augmentation du nombre de symboles pilotes ainsi que l'augmentation de la puissance transmise améliorent les performances du système. D'autre part, la variance du bruit a un effet négatif sur l'estimation du canal en termes d'erreur quadratique moyenne (MSE) et peut donc réduire considérablement les performances des estimateurs.**

***Index Terms*— Bayesian Cramér–Rao lower bound (BCRLB), channel estimation, least square (LS), linear minimum mean square error (LMMSE), maximum likelihood (ML), maximum posteriori probability (MAP), visible light communication (VLC).**

I. Introduction

VISIBLE light communication (VLC) is promising on two counts: illumination and communication. Lately, VLC has been attracting the attention of many researchers owing to its unique properties [1]. It offers not only high security and wide license-free bandwidth (400–800 THz) at high frequencies, but also an impressive data rate with no electromagnetic interference (EMI) as well as high power efficiency [2]. Therefore, it can be safely utilized in a variety of settings, including but not limited to hospitals, petrochemical factories, and aircraft cabins [3], [4], [5]. It was, thanks to the recent developments in indoor illumination, namely the widespread deployment of light-emitting diodes (LEDs), which can ensure rapid light intensity switching rates (thousands of times per second) and enable high-speed data transfer, that gave rise to VLC technologies [6].

A. Importance of the Study

The evolution of the wireless communication industry has triggered some radical changes in modern life. Today, the consumption of many high-data-rate services, such as virtual reality gaming and video streaming, has become commonplace. Moreover, the increase in social media users (and users of other diverse smartphone applications) instigates strong competition between network operators. As a result, network providers face difficulties in meeting the users' needs as the radio-frequency (RF) spectrum becomes more and more congested.

To solve the RF spectrum crunch problem, it is imperative we find alternative wireless communication technologies. VLC is one such technology that can be used to actually replace RF communication. It is receiving considerable support because the deployment of VLC access points is a very straightforward process since existing LEDs can be reused.

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Additionally, replacing the incandescent and fluorescent lighting components with LEDs is cost-effective, energy-saving, and environmentally friendly.

B. Related Works

Although channel estimation is a key point of interest, especially when it comes to the equalization, synchronization, and implementation of precoding and detecting functions, there are conspicuously limited studies discussing these estimation scenarios in VLC systems. In one of these studies [7], channel estimation was applied for a multiple-input multiple-output (MIMO) VLC system to reduce the cochannel interference between the transmitters with low computational complexity. In another [8], the symbol error probability (SEP) of an indoor single-input single-output (SISO) VLC system with a static receiver, using the least-square (LS) estimate of the utilized statistical channel, was investigated.

Assuming a fixed channel state at each transmission slot, the bit error rate (BER) performance of a SISO-VLC system subject to signal-dependent shot noise was investigated in [9]. The channel estimation was realized by not only LS, but also maximum likelihood (ML) estimators in [9].

In [10], the performance of LS and minimum mean square error (MMSE) methods was compared in terms of both BER and mean square error (MSE), and this was done by extending the orthogonal frequency division multiplexing (OFDM) technique to VLC following certain adjustments. Novel channel estimation methods were also proposed based on a variable statistic window mechanism and a neural network solution in [11] and [12], respectively.

Additionally, the problem of imperfect CSI was studied for MIMO-VLC systems and their performance is extensively studied in the literature [13], [14], [15], [16]. Specifically, to estimate the deterministic channel in a massive MIMO system, a fast and flexible denoising convolutional neural network (FFDNet)-based channel estimation scheme was proposed in [13]. Okumuş and Panayirci [14] proposed a new channel estimation technique for the generalized LED to estimate the channel of the MIMO-OFDM-based VLC system. The BER was obtained to analyze the performance of the proposed estimation method of the fixed channel gain for the considered system. Moreover, the CRLB was derived to compare it with the derived MSE of the proposed algorithm. Furthermore, in [15], a new channel estimation technique based on compressive sensing for the MIMO-OFDM VLC system was proposed. The authors studied the performance of the proposed estimation method in terms of BER, MSE, and pilot overhead experimentally.

The channel estimation methods for OFDM-based VLC were studied in [10], [17], [18], and [19]. In particular, Hussein et al. [10] investigated the channel estimation of the direct current-biased optical OFDM (DCO-OFDM) VLC system using LS and MMSE estimation methods. The performance of the two estimation methods was analyzed and compared. Afterward, the channel estimation of the DCO-OFDM VLC system was investigated considering the effect of the clipping noise in [17] and [18].

C. Motivation

A VLC transmitter is characterized by its use of LEDs and the way it modulates data, specifically using the intensity modulation (IM) technique. While the receiver does have the photodiodes (PDs) to detect the light beams, the modulation process in VLC is not similar to that in RF. In RF, the amplitude, frequency, and phase modulation techniques are used, while in VLC, the light intensity is modified according to the data in a process referred to as the IM/DD modulation technique.

It is important to mention that VLC channels have some properties that set them apart from RF. The VLC transmitted signal should be nonnegative and have a real value since light is used for data modulation (and it is well known that a light signal cannot be negative or have a complex value).

Accurate channel modeling is vital to ensure a reliable and efficient design of the VLC system. Many works have considered the constraints of the VLC channel gain. However, only a few studied the random channel gain of a VLC system with perfect CSI [9], [20], and an even scarcer number of works considered imperfect CSI for the VLC channel gain [21].

Considering the lack of studies concerning VLC systems with random channels and imperfect CSI, this work aims to estimate the random channel gain of a SISO-VLC scheme. To the authors' best knowledge, this is the first study concerning the random channel gain estimation of VLC systems.

D. Contributions

In response to the aforementioned motivations, the main contributions of this work can be summarized as follows.

1) The random channel of the SISO-VLC system is estimated using LS, ML, MAP, LMMSE, and MMSE estimators.
2) To get a bound to evaluate the performance of the estimators, the Bayesian Cramér–Rao lower bound (BCRLB) for the random VLC channel is derived.
3) The achievements of the considered estimators in terms of both MSE and BER are derived, plotted, and compared to each other.
4) The effect of two substantial factors, that is, the number of symbols and the noise variance that affects the MSE of the estimators, are studied.
5) Compared to [9], this work considers a more realistic system where a random channel gain is assumed rather than the deterministic channel gain assumed in [9]. Consequently, the Bayesian estimation approach is used to derive three more estimators (LMMSE, MMSE, and MAP). In addition, BCRLB is derived to obtain a benchmark for the performance of all proposed estimators.

Article Organization: The rest of the article is organized as follows. The considered system and the random channel model are defined in Section II. Then, the proposed estimation methods and the derived BCRLB are detailed in Section III. The obtained simulation results are presented and discussed in Section IV, and finally the article is concluded with Section V.

Notations: Vectors are denoted by bold-face lower-case letters, while the transpose operation is indicated by $[.]^T$.

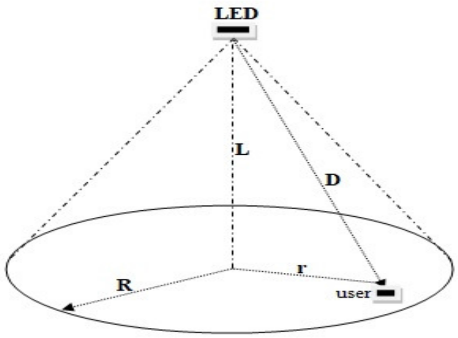

Fig. 1. SISO VLC system model.

$\mathbf{E}\{\cdot\}$ is used for statistical expectations, and $\mathcal{N}(\mu, \sigma^2)$ represents a Gaussian random variable with mean μ and variance σ^2.

II. System and Channel Models

We consider a SISO-VLC downlink transmission model in an indoor environment as shown in Fig. 1. In this model, a single LED is hung on the L meters high ceiling to serve a single user on the floor allowed to be located anywhere inside a circular area with a radius R. Assuming that the receiver is mobile, and thus, the channel gain may change randomly, the received signal can be defined by

$$y = hx + n \tag{1}$$

where $x = s + d$ denotes the optical signal transmitted from the LED to the receiver, which performs a symbol-by-symbol detection, while s is the transmitted symbol with a dc bias level d ensuring a positive value. As can be seen from (1), the signal experiences additive noise n, which follows a zero-mean Gaussian distribution with a variance value of σ_n^2, that is, $n \sim \mathcal{N}(0, \sigma_n^2)$. Moreover, h represents the channel gain with the probability density function (pdf) given as [22]

$$f_h(h) = \begin{cases} \Upsilon\, h^{\frac{-2}{m+3}-1}, & h_{\min} \leq h \leq h_{\max} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where Υ can be expanded as

$$\Upsilon = \frac{2C^{\frac{2}{m+3}}\left((m+1)L^{(m+1)}\right)^{\frac{2}{m+3}}}{(m+3)R^2}. \tag{3}$$

Here, C being a transceiver-dependent constant, L is the vertical distance from the LED to the receiver plane, and m denotes the order of the Lambertian radiation pattern followed by the LED, which is given by

$$m = \frac{-1}{\log_2(\cos(\Phi_{1/2}))} \tag{4}$$

where $\Phi_{1/2}$ is the LED transmitter emission semiangle at half power level.

For the condition of the channel power to be unity, that is, $\mathbf{E}\{h^2\} = 1$, it can be expressed as [8]

$$\left(C(m+1)L^{m+1}\right)^2 = \frac{(m+2)R^2\left(L^2\left(R^2+L^2\right)\right)^{m+2}}{\left(R^2+L^2\right)^{m+2} - L^{2(m+2)}} \tag{5}$$

which leads to obtain $h_{\min}$ and $h_{\max}$ as follows:

$$h_{\min} = \frac{RL^{(m+2)}(m+2)^{1/2}}{\left(R^2+L^2\right)^{1/2}\left(\left(R^2+L^2\right)^{m+2} - L^{2(m+2)}\right)^{1/2}} \tag{6}$$

and

$$h_{\max} = \frac{RL^{(m+2)}(m+2)^{1/2}}{L\left(\left(R^2+L^2\right)^{m+2} - L^{2(m+2)}\right)^{1/2}}. \tag{7}$$

Consequently, noting that $\sigma_h^2 = \mathbf{E}\{h^2\} - \mu_h^2$, the mean value of the random channel gain h can be calculated from

$$\mu_h = \frac{(m+3)\Upsilon \times \left(RL^{(m+2)}(m+2)^{1/2}\right)^{\frac{m+1}{m+3}}}{(m+1) \times \left(\left(R^2+L^2\right)^{m+2} - L^{2(m+2)}\right)^{\frac{m+1}{2(m+3)}}} \times \left(\frac{1}{L^{\frac{m+1}{m+3}}} - \frac{1}{(L^2+R^2)^{\frac{m+1}{2(m+3)}}}\right). \tag{8}$$

III. BCRLB and Channel Estimation

In this section, we first present the derivation of a BCRLB as a benchmark to evaluate the efficiency of the proposed estimators. Then, four different estimators (ML, LS, maximum posteriori probability (MAP), linear MMSE (LMMSE), and MMSE) are introduced to solve the random channel gain estimation problem of the VLC system.

A. Bayesian Cramér–Rao Lower Bound

Assume that a sequence of N pilot symbols, $\mathbf{x}$, is conveyed from the transmitter of the VLC system to the receiver to estimate the random channel gain, h, before the realization of data transmission. Denoting the pilot symbols as $\mathbf{x} = [x_1, x_2, \ldots, x_{\mathrm{N}}]^T$ and defining the various additive noises with $\mathbf{n} = [n_1, n_2, \ldots, n_{\mathrm{N}}]^T$, the received signal vector $\mathbf{y} = [y_1, y_2, \ldots, y_{\mathrm{N}}]^T$ can be expressed by

$$\mathbf{y} = h\mathbf{x} + \mathbf{n}. \tag{9}$$

Note that the elements of $\mathbf{n}$ are independent and identically distributed (i.i.d.) zero mean Gaussian random variables, that is, $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 I_N)$, where $\mathbf{0}$ and I_N denote the $N \times 1$ vector of zeros and identity matrix of size N, respectively. Now, suppose that the prior information of the channel gain pdf, $f_h(h)$, is available at the receiver, the Bayesian information is given as [23]

$$J = \mathbf{E}\left\{I_f(h)\right\} + I_R \tag{10}$$

where I_R represents the contribution of the prior information and it can be calculated from

$$I_R = \mathbf{E}\left\{\left(\frac{\partial}{\partial h}\ln f_h(h)\right)^2\right\} \tag{11}$$

where $\ln f_h(h)$ is the log-pdf of the random variable h, the first derivative with respect to h of which is obtained as

$$\frac{\partial}{\partial h}\ln f_h(h) = \left(\frac{-2}{m+3} - 1\right)\frac{1}{h}. \tag{12}$$

Therefore,

$$I_R = \left(\frac{2}{m+3}+1\right)^2 \mathbf{E}\left\{\frac{1}{h^2}\right\}. \tag{13}$$

The expected value of a random variable z can be calculated by $\mathbf{E}\{g(z)\} = \int_{z_{\min}}^{z_{\max}} g(z) f_z(z)dz$, and accordingly applying this rule to the term $\mathbf{E}\{(1/h^2)\}$, (13) can be rewritten as follows:

$$I_R = -\Upsilon \frac{(m+5)^2}{2(m+3)(m+4)}\left(h_{\max}^{\left(\frac{-2m-8}{m+3}\right)} - h_{\min}^{\left(\frac{-2m-8}{m+3}\right)}\right). \tag{14}$$

Note that the term $\mathbf{E}\{I_f(h)\}$ in (10) represents the contribution of the data and it should be obtained to find the Bayesian information. Therefore, $I_f(h)$ can be calculated first as

$$I_f(h) = -\mathbf{E}\left\{\frac{\partial^2}{\partial h^2}\ln f(\mathbf{y}|h)\right\} \tag{15}$$

where $\ln f(\mathbf{y}|h)$ is the log-likelihood function of the unknown channel h. Since we assume that all the samples are i.i.d., the joint pdf of N observations, which is also equal to the likelihood function of h, $f(\mathbf{y}|h)$, can be written as

$$f(\mathbf{y}|h) = \left(\frac{1}{\sqrt{2\pi\sigma_n^2}}\right)^N \exp\left(\frac{-1}{2\sigma_n^2}\sum_{i=1}^{N}(y_i - hx_i)^2\right). \tag{16}$$

Then, taking the natural logarithm of $f(\mathbf{y}|h)$ ends up with

$$\ln f(\mathbf{y}|h) = -\frac{N}{2}\ln\left(2\pi\sigma_n^2\right) - \frac{1}{2\sigma_n^2}\sum_{i=1}^{N}(y_i - hx_i)^2. \tag{17}$$

Following that, the second partial derivative of $\ln f(\mathbf{y}|h)$ with respect to h is obtained as

$$\frac{\partial^2}{\partial h^2}\ln f(\mathbf{y}|h) = -\frac{1}{\sigma_n^2}\sum_{i=1}^{N}x_i^2. \tag{18}$$

Finally, $I_f(h)$ can be written by utilizing (18) as follows:

$$I_f(h) = -\mathbf{E}\left\{\frac{\partial^2}{\partial h^2}\ln f(\mathbf{y}|h)\right\} = \frac{1}{\sigma_n^2}\sum_{i=1}^{N}x_i^2. \tag{19}$$

Now, assuming that the transmitted pilot $x_i = p$ for all i, $I_f(h)$ can be written as

$$I_f(h) = \frac{Np^2}{\sigma_n^2}. \tag{20}$$

In this case, it can be easily noticed from (20) that $I_f(h)$ does not depend on h. Thus, $\mathbf{E}\{I_f(h)\} = I_f(h)$. Therefore, utilizing that and substituting (14) in (10), the Fisher information can be achieved by

$$J = \frac{\Upsilon(m+5)}{2(m+4)}\left(h_{\max}^{\left(\frac{-2m-8}{m+3}\right)} - h_{\min}^{\left(\frac{-2m-8}{m+3}\right)}\right) + \frac{Np^2}{\sigma_n^2}. \tag{21}$$

Substituting (6) and (7) into (21), the Fisher information can be written as

$$J = \frac{\Upsilon(m+5)}{2(m+4)}\left(\frac{\left(\left(R^2+L^2\right)^{m+2} - L^{2(m+2)}\right)}{(m+2)R^2L^{2(m+2)}}\right)^{\left(\frac{m+4}{m+3}\right)} \times \left(L^{2\left(\frac{m+4}{m+3}\right)} - (L^2+R^2)^{\left(\frac{m+4}{m+3}\right)}\right) + \frac{Np^2}{\sigma_n^2}. \tag{22}$$

Consequently, the BCRLB can be defined by

$$\sigma_\epsilon^2 \geq \frac{1}{J}. \tag{23}$$

Suppose that no prior information of channel gain is available at the receiver, then the Fisher information expression reduces to $J = (Np^2/\sigma_n^2)$, which is equal to the Fisher information of a VLC system with constant channel gain. Comparing the Fisher information for both the cases of existence and nonexistence of prior channel information, it is obvious that the value of the Fisher information is higher in the presence of channel information, which yields a smaller bound meaning better system performance.

B. ML Estimator

The ML estimator can be used to estimate an unknown parameter by maximizing its likelihood function. Therefore, we can apply this principle for the estimation of the random channel gain of the VLC system by maximizing the log-likelihood function of h, which is given in (17), as follows:

$$\frac{\partial}{\partial h}\ln f(\mathbf{y}|h) = \frac{1}{\sigma_n^2}\sum_{i=1}^{N}x_i(y_i - hx_i) = 0. \tag{24}$$

Assume that the same pilot signals are sent for each time, that is, $x_i = p$ for all i, then the estimated value of h can be calculated by using ML estimator from

$$\hat{h}_{\mathrm{ML}} = \frac{1}{Np}\sum_{i=1}^{N}y_i. \tag{25}$$

Defining that $\hat{h}_{\mathrm{ML}} = h - \epsilon_{\mathrm{ML}}$, where the estimation error ϵ_{ML} has zero mean and variance σ_{ML}^2, it can be written that

$$\mathbf{E}\{\hat{h}_{\mathrm{ML}}\} = \frac{1}{Np}\sum_{i=1}^{N}\mathbf{E}\{y_i\} = \mathbf{E}\{h\} = \mu_h. \tag{26}$$

Here, it is clear that the ML estimator is unbiased.[1] Furthermore, the variance of the estimation error is defined by

$$\sigma_{\mathrm{ML}}^2 = \mathbf{E}\left\{(\epsilon_{\mathrm{ML}})^2\right\} = \frac{\sigma_n^2}{Np^2}. \tag{27}$$

From the above equation, it can be easily noted that the ML estimation error does not depend on the channel gain h.

C. LS Estimator

The LS estimator is actually a special case of the ML estimator. In this technique, the squared distinctions between the observations and their expected values are minimized. Therefore, the LS estimated channel gain can be defined by

$$\hat{h}_{\mathrm{LS}} = \frac{\mathbf{x}^T\mathbf{y}}{\|\mathbf{x}\|^2} \tag{28}$$

which is obviously equal to $\hat{h}_{\mathrm{ML}}$ under the assumption of that $x_i = p$ for all i, that is, $\hat{h}_{\mathrm{LS}} = (1/Np)\sum_{i=1}^{N} y_i$. Thus, it can be easily concluded that σ_{LS}^2 and σ_{ML}^2 are equal.

[1] An estimator of a given parameter, for example, h, is said to be unbiased if its expected value is equal to the true value of the parameter that should be estimated.

D. MAP Estimator

The MAP estimator uses the statistical prior information of an unknown parameter and its likelihood function in the estimation process. Hence, using the MAP estimator, the random channel gain h can be estimated from

$$\hat{h}_{\text{MAP}} = \arg\max_h f(h|\mathbf{y}) = \arg\max_h \ln f(h|\mathbf{y}) \tag{29}$$

where $f(h|\mathbf{y})$ is the posteriori function, which can be given by using the Bayes theorem as follows:

$$f(h|\mathbf{y}) = \frac{f(\mathbf{y}|h) f_h(h)}{f_{\mathbf{Y}}(\mathbf{y})}. \tag{30}$$

Here, $f_{\mathbf{Y}}(\mathbf{y})$ is the marginal pdf of the observations, which is a constant value depending on h.

Now, according to the MAP estimation process, the log-posteriori function should be maximized by finding the roots of the first derivative of it from

$$\frac{\partial}{\partial h} \ln f(\mathbf{y}|h) f_h(h) = 0. \tag{31}$$

Utilizing (2) and (16) under the assumption of that $x_i = p$ for all i, the above expression can be rewritten as

$$\frac{1}{\sigma_n^2}\sum_{i=1}^{N} p(y_i - hp) + \left(\frac{-2}{m+3} - 1\right)\frac{1}{h} = 0. \tag{32}$$

Then, the quadratic expression of the above equation can be given as follows:

$$h^2 - \frac{1}{Np}\left(\sum_{i=1}^{N} y_i\right)h + \frac{\sigma_n^2}{Np^2}\left(\frac{m+5}{m+3}\right) = 0 \tag{33}$$

the solution of which can be given by

$$\hat{h}_{\text{MAP}} = \frac{1}{2}\left[\frac{\sum_{i=1}^{N} y_i}{Np} + \sqrt{\left(\frac{\sum_{i=1}^{N} y_i}{Np}\right)^2 - \frac{4\sigma_n^2(m+5)}{Np^2(m+3)}}\right]. \tag{34}$$

Note that in the case where the terms under the square root are negative, meaning that they do not satisfy the conditions of the VLC channel, we set $\hat{h}_{\text{MAP}}$ equal to $h_{\min}$. It is difficult to find a closed form for the MSE of the ML estimator, so the simulations are used to study and compare the performance of the ML estimator. Besides that, the MAP estimator is clearly biased, that is, $\mathbf{E}\{\hat{h}_{\text{MAP}}\} \neq \mathbf{E}\{h\}$. However, for a sufficiently large value of the signal-to-noise ratio (Np/σ^2), the MAP estimation tends to be unbiased and it approaches the ML estimator, that is,

$$\lim_{\frac{Np}{\sigma^2}\to\infty} \hat{h}_{\text{MAP}} = \frac{1}{NP}\sum_{i=1}^{N} y_i = \hat{h}_{\text{ML}}. \tag{35}$$

Thus, since the estimation is biased, the estimation error $\epsilon_{\text{MAP}} = h - \hat{h}_{\text{MAP}}$ cannot have a zero mean. It is extremely difficult to analytically study the MSE of the MAP estimation, and therefore, the performance of the MAP estimator will be investigated by using Monte Carlo simulations.

E. MMSE Estimator

MMSE estimator is an estimator that minimizes the mean squared error. Using the MMSE estimator, estimated h is the mean of the posterior function $f(h|\mathbf{y})$ given as

$$\hat{h}_{\text{MMSE}} = \mathbf{E}\{h|\mathbf{y}\} \tag{36}$$

where $f(h|\mathbf{y})$ is the posterior function which is given in (30). Using Bayes' theorem, the MMSE estimator can be given as

$$\hat{h}_{\text{MMSE}} = \int_{h_{\min}}^{h_{\max}} \frac{h f(\mathbf{y}|h)\ f_h(h)}{f_{\mathbf{Y}}(\mathbf{y})}\, dh = \frac{\Upsilon}{f_{\mathbf{Y}}(\mathbf{y})(2\pi\sigma_n^2)^{N/2}} \times \int_{h_{\min}}^{h_{\max}} h^{\frac{-2}{m+3}} \exp\left(-\frac{1}{2\sigma_n^2}\sum_{i=1}^{N}(y_i - hx_i)^2\right) dh. \tag{37}$$

Assuming $x_i = p, \forall i = 1, \ldots, N$, therefore, $\hat{h}_{\text{MMSE}}$ can be given as

$$\hat{h}_{\text{MMSE}} = \frac{\Upsilon}{f_{\mathbf{Y}}(\mathbf{y})(2\pi\sigma_n^2)^{N/2}} \times \int_{h_{\min}}^{h_{\max}} h^{\frac{-2}{m+3}} \exp\left(-\frac{1}{2\sigma_n^2}\sum_{i=1}^{N}(y_i - hp)^2\right) dh. \tag{38}$$

The marginal pdf $f_{\mathbf{Y}}(\mathbf{y})$ can be given as

$$f_{\mathbf{Y}}(\mathbf{y}) = \prod_{i=1}^{N} f_{Y_i}(y_i) = \prod_{i=1}^{N}\int_{h_{\min}}^{h_{\max}} f(y_i|h) f_h(h)\, dh = \frac{\Upsilon^N}{(2\pi\sigma_n^2)^{N/2}} \prod_{i=1}^{N}\int_{h_{\min}}^{h_{\max}} \exp\left(-\frac{(y_i - hx_i)^2}{2\sigma_n^2}\right) h^{\frac{-2}{m+3}-1}\, dh. \tag{39}$$

Following the previous assumption that $x_i = p, \forall i = 1, \ldots, N$, so

$$f_{\mathbf{Y}}(\mathbf{y}) = \frac{\Upsilon^N}{(2\pi\sigma_n^2)^{N/2}} \times \prod_{i=1}^{N}\int_{h_{\min}}^{h_{\max}} \exp\left(-\frac{(y_i - hp)^2}{2\sigma_n^2}\right) h^{\frac{-2}{m+3}-1}\, dh. \tag{40}$$

Therefore,

$$\hat{h}_{\text{MMSE}} = \frac{\Upsilon^{(-N+1)} \int_{h_{\min}}^{h_{\max}} h^{\frac{-2}{m+3}} \exp\left(-\frac{1}{2\sigma_n^2}\sum_{i=1}^{N}(y_i - hp)^2\right) dh}{\prod_{i=1}^{N}\int_{h_{\min}}^{h_{\max}} \exp\left(-\frac{1}{2\sigma_n^2}(y_i - hp)^2\right) h^{\frac{-2}{m+3}-1}\, dh}. \tag{41}$$

It can be noted from the previous equation that it is extremely difficult to analytically study the MSE of the MMSE estimation, so we resort to the Monte Carlo simulations to study the performance of the MMSE estimator.

F. LMMSE Estimator

Due to the complexity of computing the MMSE estimator, a less complex estimator called LMMSE, which requires the mean and variance values of an unknown parameter as the prior information, is employed to estimate h. The LMMSE estimated random channel gain, h_{LMMSE}, can be defined by

$$\hat{h}_{\text{LMMSE}} = \frac{\sigma_h^2}{\sigma_h^2\|\mathbf{x}\|^2 + \sigma_n^2}\mathbf{x}^T(\mathbf{y} - \mathbf{x}\mu_h) + \mu_h \tag{42}$$

where the mean of the channel is calculated from $\mu_h = \mathbf{E}\{h\}$, while the variance is obtained from $\sigma_h^2 = \mathbf{E}\{h^2\} - (\mu_h)^2$.

Defining that $\hat{h}_{\text{LMMSE}} = h - \epsilon_{\text{LMMSE}}$, where the estimation error ϵ_{LMMSE} is a Gaussian random variable that follows $\mathcal{N}(0, \sigma_{\text{LMMSE}}^2)$, the variance of ϵ_{LMMSE} can be written as

$$\sigma_{\text{LMMSE}}^2 = \frac{\sigma_h^2 \sigma_n^2}{\sigma_h^2 N p^2 + \sigma_n^2} = \left(\frac{N p^2}{\sigma_n^2} + \frac{1}{\sigma_h^2} \right)^{-1}. \quad (43)$$

Many observations can be made from (43), and they can all be summarized as follows:

1) Raising σ_h^2 or σ_n^2 increases the value of σ_{LMMSE}^2, while increasing p or N has the opposite effect.
2) LMMSE estimator provides better performance compared to the ML for the VLC system discussed in this study. However, LMMSE is more complex, as it requires more information about the channel gain, h, that is, σ_h^2 and σ_n^2. This can be proven by

$$\frac{\sigma_{\text{ML}}^2}{\sigma_{\text{LMMSE}}^2} = \frac{\frac{\sigma_n^2}{Np^2}}{\left[\frac{Np^2}{\sigma_n^2} + \frac{1}{\sigma_h^2} \right]^{-1}} = 1 + \frac{\sigma_n^2}{N P^2 \sigma_h^2} > 1. \quad (44)$$

3) It can be noticed that $\lim_{\sigma_h^2 \to \infty} \sigma_{\text{LMMSE}}^2 = (\sigma_n^2 / Np^2) = \sigma_{\text{ML}}^2$. The interpretation of this result is that when σ_h^2 is very high, the case equals to have no prior information, so LMMSE has the same performance as ML. This note is compatible with the former one.
4) It can be easily seen that $\lim_{\sigma_n^2 \to \infty} \sigma_{\text{LMMSE}}^2 = \sigma_h^2$. This is expected because, at extremely high σ_n^2, the noise is distributed flatly on the entire range, which yields the MSE of LMMSE estimator too dependent on the prior information of channel gain, that is, σ_h^2.

G. Computational Complexity Analysis

In this section, the computational complexity of all studied estimators in this work is compared. Naturally for the LS and ML estimators, the estimated channel gain can be obtained only using the observations (i.e., there is no need for any other information related to the channel or noise). This means that the LS and ML are the simplest estimators, while in the MAP estimator, the channel gain estimation requires information on the noise variance in addition to the data observations. Comparing all estimators together, it can be observed that LMMS and MMSE are the most computationally complex since they require the statistical information of the noise and the channel gain, that is, σ_n^2 and σ_h^2, in addition to the data observations $\boldsymbol{y}$.

IV. Discussions and Simulation Results

In this section, computer simulation results are presented and discussed. Numerical results are obtained by utilizing Monte Carlo simulations assuming 10^7 transmitted symbols and supported by analytical derivations. Additionally, the source power is defined by $p \in [0.1 - 0.5]w$. All the simulation parameters are listed in Table I. Note that since some parameters such as the noise variance σ_n^2 and the number of symbol pilots are assigned to different values to study the

TABLE I
Simulation Parameters

Parameters	Values
Channel gain mean μ_h	0.82
Channel gain variance σ_h^2	0.3276
Minimum value of channel gain h_{min}	0.25
Maximum value of channel gain h_{min}	2.636
Order of the Lambertian radiation pattern m	1
Maximum cell radius covered by the LED R	3 m
Vertical distance from the LED to the receiver plane L	2 m
Signal power p	$[0.1 - 0.5]w$

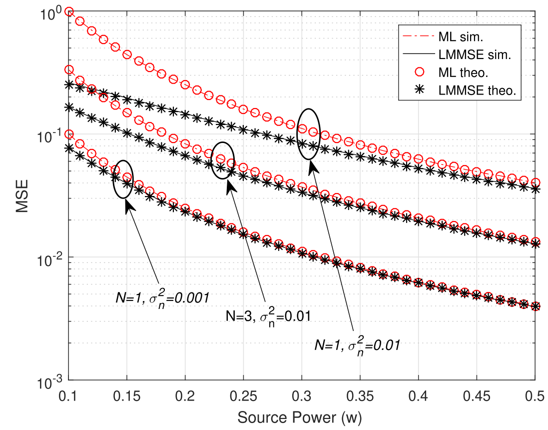

Fig. 2. Simulation and theoretical MSE of the ML and LMMSE estimators with respect to the source power for different values of σ_n^2 and N.

effect of them on the VLC system performance, they are not listed in the table but they are illustrated on the curves.

First, the variations of MSE with the signal power for the VLC system with random channel gain employing ML and LMMSE estimation methods separately at the receivers are plotted for different values of pilot symbols and the noise variance in Fig. 2. The computation plots obtained from the analytical results in (27) and (43) are compared to the plots obtained by simulation. The curves show a perfect match between the simulation and the analytical results at different numbers of pilot symbols, σ_n^2 and a range of transmitted signal power which implying the validity of the derived expressions for all cases.

Additionally, the MSE performance of the ML and LMMSE estimators is compared for different values of pilot symbols and the noise variance in Fig. 2. As seen from this figure, accelerating the number of pilots for a fixed value of $\sigma_n^2 = 0.01$ enhances the MSE performance for both the ML and LMMSE estimators, the results of which get closer to higher N. It is also clear, as expected, in Fig. 2 that the performance of the VLC system with random channel gain degrades with the increase in the noise variance, that is, the lower the noise variance σ_n^2 is, the lower MSE the estimator provides.

In Fig. 3, a comparison between the estimators considered in this study (ML, LS, MAP, LMMSE, and MMSE estimators) and BCRLB is provided. The comparison is held between all estimators at different values of the noise variance, $\sigma_n^2 = 0.001, 0.005, 0.01$. It should be mentioned that the curves are plotted at diverse values of noise variance while maintaining

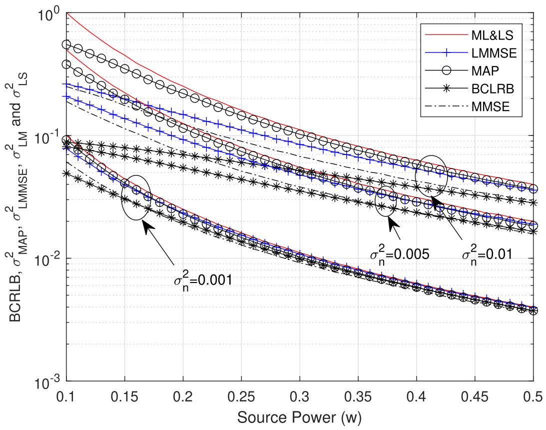

Fig. 3. Comparison of the BCRLB, σ^2_{MAP}, σ^2_{LMMSE}, σ^2_{LM}, and σ^2_{LS} for different values of σ_n^2.

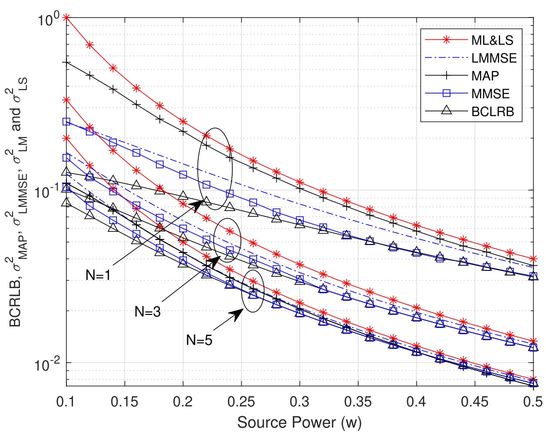

Fig. 4. Comparison of the BCRLB, σ^2_{MAP}, σ^2_{LMMSE}, σ^2_{LM}, and σ^2_{LS} for different values of N.

constant values of the source power p and the number of pilots N. It can be observed from the figure that an increase in noise variance σ_n^2 causes a significant reduction in the MSE of BCRB and all considered estimators. In general, MMSE has the least MSE among all estimators, while both LS and ML estimators have the highest MSE. In addition, by boosting the power, the performance of the estimators approaches that of BCLB.

The effect of the number of pilots, N, on the estimation error variances of all the estimators as well as the BCRLB are presented in Fig. 4. This figure demonstrates that with fixing σ_n^2 and p, transmitting more pilots enhances the performance of the estimation process. Therefore, we can conclude that improving the estimation quality can be possible by increasing the number of pilots or transmitting the signals with more power, or both. Moreover, the obtained results are in parallel with the ones presented in Fig. 3, where the MAP estimator attains worse performance than the LMMSE, while it is better than the ML and LS. LMMSE is better than MAP and worse than the MMSE in terms of MSE. In addition to that, the gap between the performance of all these estimators and the BCRLB shrinks with higher power and/or more pilot symbols.

Lastly, in Fig. 5, the BER of all the considered estimators (MAP, LMMSE, and MMSE) are plotted on the y-axis versus

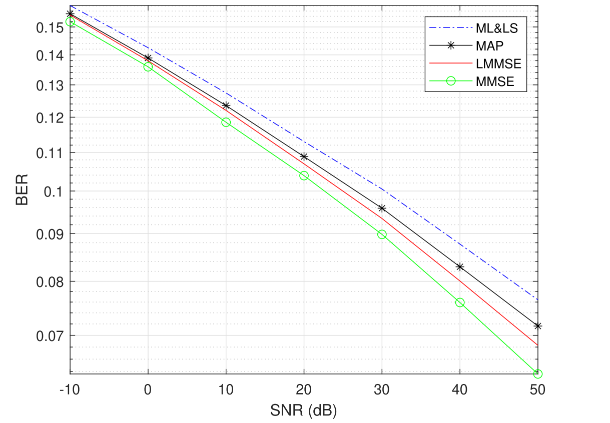

Fig. 5. Comparison of the BER for ML, LS, MAP, LMMSE, and MMSE.

the SNR on the x-axis. The SNR is in (dB) calculated at $\sigma_n^2 = 0.1$. It can be noted that both ML and LS estimators have the highest BER among all other estimators, which emphasizes that they have the worst performance, but they have a preference when it comes to the simplicity of calculations. Oppositely, MMSE has the lowest BER but at the expense of more deploying complexity, which can be figured out obviously from (41). It is also worth highlighting the fact that the results of the BER curves are in line with those of MSE for the same estimators.

V. Conclusion

In this work, a comprehensive study of channel estimation methods for a SISO-VLC system has been conducted. The performance of five different estimators is compared to each other and also the derived BCLRB. Furthermore, the closed-form expressions are derived for the BCLRB as well as the MSE of LMMSE, ML, and LS estimators. The obtained results show that the performance of the LS and ML estimators is the same for the proposed system. It is also proved that the LMMSE estimator achieves the best results in terms of MSE, while the ML and LS have the worst. Additionally, the effects of the pilot symbols and the noise variance on the performance of the estimators are discussed.

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