

Heat Transfer and Fluid Flow Calculations of Industrial Shell Boilers and Evaluation of Operation Conditions – Draft

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Abstract

This thesis develops a coupled combustion–heat transfer–hydraulics model for a three pass fire tube industrial shell boiler and evaluates its performance under realistic operating conditions. Implemented in Python, the modelling framework integrates (i) detailed fuel-air combustion, (ii) six sequential gas side heat exchange stages representing furnace radiation, convective tube banks, reversal chambers, and economizer, and (iii) a water/steam circuit governed by saturated boiling in the pressure parts and single phase heating in the economizer. The gas–water energy balance is solved using a one dimensional marching algorithm, which updates local heat transfer coefficients, wall temperatures, and segmental duties based on a full resistance network combining convection, radiation, fouling, and conduction.

Combustion calculations provide the adiabatic flame temperature, the fully burnt flue gas composition, and the total heat release from the lower heating value of the supplied natural gas fuel. Hydraulic losses are resolved concurrently using friction factor and minor loss correlations applied to each stage, yielding a complete gas side pressure drop profile. Boiler level performance metrics, including useful heat transfer, direct and indirect efficiencies, stack temperature, and the decomposition of radiative and convective duties, obtained after convergence of a fixed point iteration that links assumed efficiency to the resulting steam mass flow.

Sensitivity studies quantify the influence of excess air ratio, drum pressure, and firing rate on thermal performance, heat transfer distribution, pressure drop, and steam capacity. The results demonstrate that efficiency exhibits a shallow optimum near the design excess air setting; that pressure chiefly affects steam quantity rather than boiler efficiency; and that firing rate scales heat duties approximately linearly within the practical load range. The modelling framework provides a physics based tool suitable for analyzing industrial shell boiler behavior, supporting performance evaluation, operational optimization, and design exploration.

Chapter 1

Introduction

Industrial shell boilers remain one of the most widely deployed technologies for producing saturated steam and hot water in small to medium industrial plants. Their popularity arises from their compact construction, robust heat transfer surfaces, straightforward operation, and comparatively low installation and maintenance requirements. Typical applications span food and beverage processing, chemicals and pharmaceuticals, textiles, healthcare, and general manufacturing sectors where steady, reliable steam generation is essential for heating, processing, and auxiliary services.

Despite their apparent simplicity, the thermal behaviour of shell boilers is governed by tightly coupled processes: multi stage radiative and convective heat transfer, natural circulation boiling inside the pressure parts, complex flue gas property variations, and geometry dependent hydraulic losses. Modern operation demands higher efficiency, reduced emissions, increased reliability, and improved control.

This thesis develops a physics based model for a three pass fire tube shell boiler that integrates combustion calculations, detailed flue gas thermophysical properties, multi stage heat transfer modelling, and hydraulic loss estimation. The model is implemented as a one dimensional marching solver applied to six sequential heat exchange stages:

$$\text{HX}_1 \rightarrow \text{HX}_2 \rightarrow \text{HX}_3 \rightarrow \text{HX}_4 \rightarrow \text{HX}_5 \rightarrow \text{HX}_6,$$

representing the furnace, reversal chambers, convective tube banks, and the economiser, see figure 1.1. On the water side, the boiler drum provides a saturated interface for nucleate boiling in the pressure parts, while the economiser section is treated as a single phase internal flow. Gas side properties are supplied by Cantera, enabling temperature dependent transport, specific heat, thermal conductivity, and radiative behaviour to be modelled.

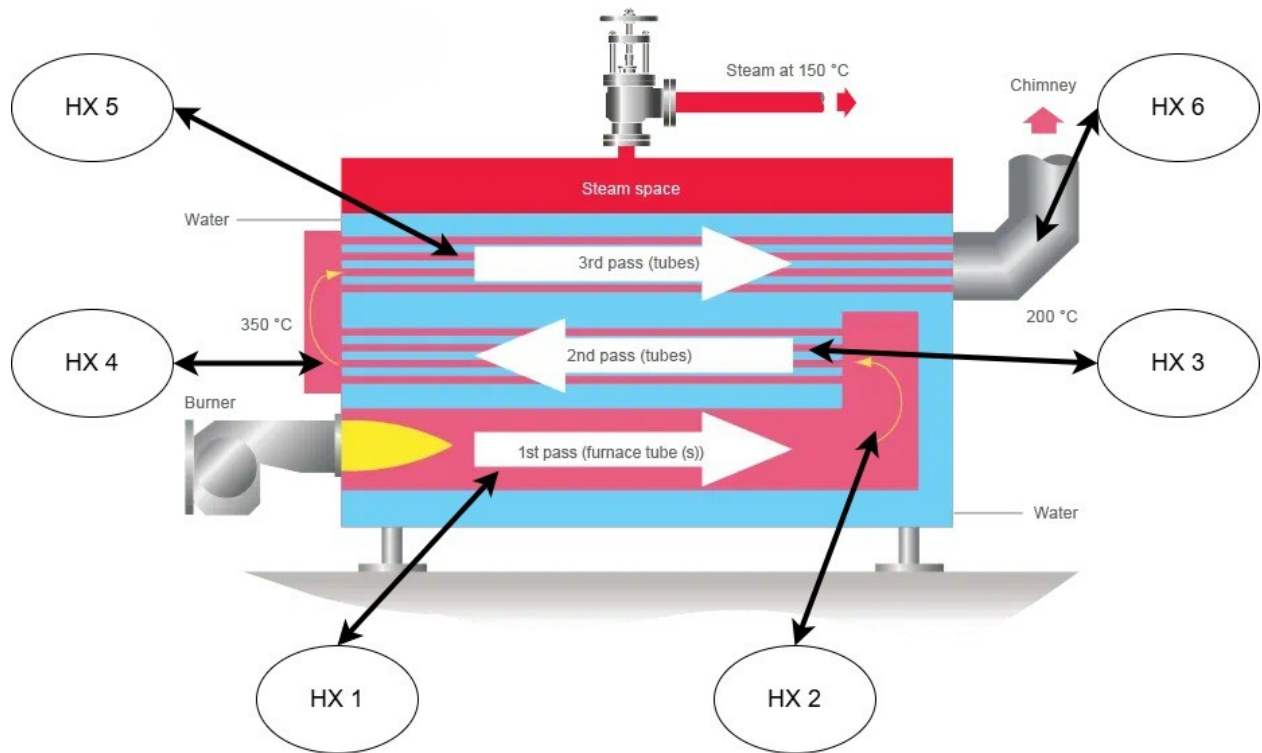


Figure 1.1: Shell boiler labeled stages.

The overall objectives of the study are:

1. To construct a unified combustion-boiler model capable of predicting flue gas temperature, composition, adiabatic flame temperature, and total heat input based on fuel composition and excess air settings.
2. To resolve heat transfer processes along the boiler using stage specific geometries, convection correlations, and a spectral based gas radiation model.
3. To quantify hydraulic losses across each pass using friction factor relations and minor loss coefficients, yielding the total boiler gas side pressure drop.
4. To compute boiler level performance, including useful heat transfer, direct and indirect efficiencies, stack temperature, and stage wise duties.
5. To evaluate sensitivity of boiler performance to key operating parameters, excess air ratio, drum pressure, and fuel mass flow rate.

The numerical framework is structured such that the water/steam mass flow is determined iteratively from the global energy balance. For each operating condition, a fixed point loop between assumed efficiency and resulting steam flow is solved until convergence, ensuring consistency between combustion input, heat transfer output, and steam generation.

The remainder of this thesis is organised as follows. Chapter 2 identifies typical industrial applications of shell boilers and introduces key design features. Chapter 3 describes the

boiler geometry and outlines the six heat transfer stages. Chapter 4 develops the combustion and flue gas model, including stoichiometry and adiabatic flame temperature prediction. Chapter 5 covers the heat transfer framework, combining convection and radiation on the gas side with pool boiling and single phase correlations on the water side. Chapter 6 presents the hydraulic model. Chapter 7 reports the resulting boiler performance, while Chapter 8 examines the sensitivity of the system to variations in λ , pressure, and firing rate. Chapter 9 concludes with a summary of findings.

Chapter 2

Industrial Application of Shell Boilers

Fire tube boilers- shell boilers

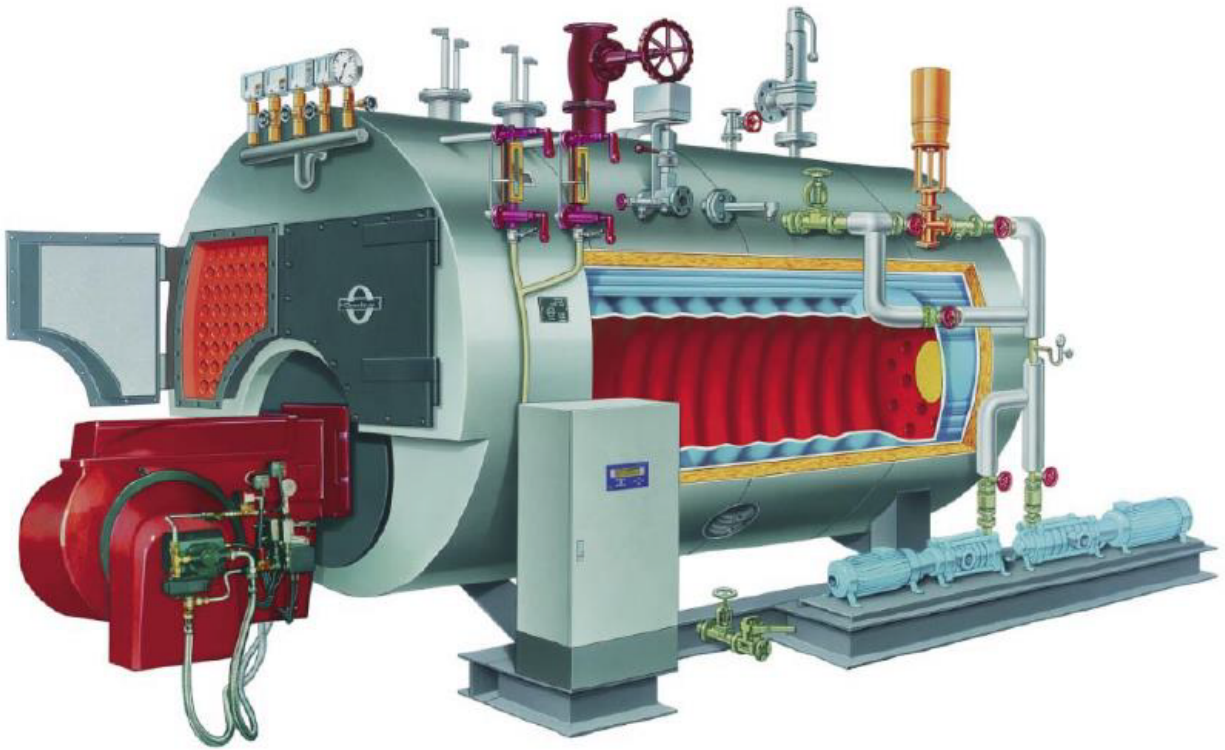


Figure 2.1: Example of a packaged three-pass fire-tube shell boiler in industrial service.

2.1 Typical Industries

Shell (fire-tube) boilers are widely used in small to medium steam and hot water duties where compactness, robustness, and simple operation are prioritized over very high pressure or very large throughput. Typical sectors include:

- Food and beverage
 - Breweries, dairies, sugar refineries
 - Canneries, bakeries, confectionery plants
 - CIP (clean-in-place) systems and sterilization
- Chemical and pharmaceutical
 - Fine chemicals, specialty chemicals
 - Active pharmaceutical ingredient (API) and formulation plants
 - Steam for reactors, jacket heating, and clean steam generators
- Textiles and paper
 - Dyeing, washing, drying, and calendaring operations
 - Small paper mills and converting facilities
- Healthcare and institutional
 - Hospitals, clinics, and laboratories (space heating, humidification, sterilizers, autoclaves)
 - Universities, office complexes, district heating sub-plants
- Light manufacturing and general industry
 - Metal finishing, surface treatment, and cleaning
 - Rubber and plastics processing
 - Laundry services and commercial dry-cleaning

2.2 Standard Steam Duties

Shell boilers are normally applied in low to medium pressure ranges and moderate steam capacities:

- Typical operating pressure range:
 - Saturated steam: 6–25 bar, occasionally up to 30 bar
 - Hot-water service: 10–16 bar
- Steam-generation rates (order of magnitude):
 - Small units: 0.5–5 t/h
 - Medium units: 5–20 t/h
 - Large shell boilers (upper practical range): 20–40 t/h, beyond which water-tube designs are usually preferred

2.3 Advantages and Limitations

Advantages

- Compact and integrated construction
 - Furnace, passes, and steam/water space are combined in a single pressure body.
 - Relatively small footprint and simple installation.
- Operational simplicity
 - Straightforward start-up and shutdown procedures.
 - Typically tolerant of moderate load swings and cycling (within design limits).
 - Often delivered as packaged units with burner, controls, and safety devices pre-engineered.
- Low-to-moderate capital cost
 - Attractive for small and medium plants, boiler houses, and decentralized steam supply.
- Good part-load performance
 - Large water content provides thermal buffer, reducing short-cycling of the burner.
 - Reasonable efficiency across a wide load range, especially with economizers.
- Maintenance and inspection
 - Accessible gas passes and tube bundles (depending on design) for cleaning and inspection.
 - Long-established technology with wide service and parts availability.

Limitations

- Pressure and capacity limits
 - Practical upper bounds on shell diameter and plate thickness limit maximum pressure and steam rate.
 - For very high pressure (e.g., >40–60 bar) or very large capacities, water-tube boilers are more suitable.
- Response time
 - Large water inventory slows thermal response to rapid, large load changes compared with water-tube boilers.
- Efficiency ceiling
 - Radiative and convective heat-transfer surfaces are constrained by geometry.

- Very high efficiencies often require additional heat-recovery equipment (economizers, condensing stages, air preheaters).
- Transport and installation constraints
 - Shell diameter and weight can be limited by route and lifting capacity.
 - Retrofitting within existing boiler houses may be constrained by overall envelope.

2.4 Multi-Pass Layout

Industrial shell boilers typically adopt multi-pass fire-tube configurations to enhance convective heat transfer and maintain acceptable gas-side velocities:

- Two-pass layout
 - First pass: large diameter furnace tube running from burner front to rear tubeplate.
 - Second pass: return of flue gas through banks of small-diameter fire-tubes back to the front tubeplate and flue outlet.
 - Simpler construction but lower total heat-transfer surface compared with three-pass designs.
- Three-pass layout (most common for industrial shell boilers)
 - Pass 1: large diameter furnace tube running from burner front to rear tubeplate.
 - Pass 2: First bank of smoke-tubes (typically reversing at the rear turnaround chamber).
 - Pass 3: Second bank of smoke-tubes.
 - Provides higher overall heat-transfer surface, more uniform gas cooling, and lower exit-gas temperatures.
- Extended heat-recovery sections
 - Economizer: additional convective heat exchanger in the flue-gas path downstream of the boiler to preheat feedwater.
 - Air preheater / condensing sections: for high-efficiency systems using suitable fuels and materials.
- Flow arrangement
 - Gas-side: burner → furnace (Pass 1) → turnaround chamber → tube bank(s) (Passes 2 and 3) → stack.
 - Water/steam side: natural circulation between heated tube surfaces and the upper steam space within the drum/shell; feedwater introduced at cooler regions (often via economizer), steam drawn from the top of the shell.

This multi-pass concept underpins the subsequent detailed modelling of each convective and radiative heat-transfer stage HX_1-HX_6 in the simulation.

Chapter 3

Configuration

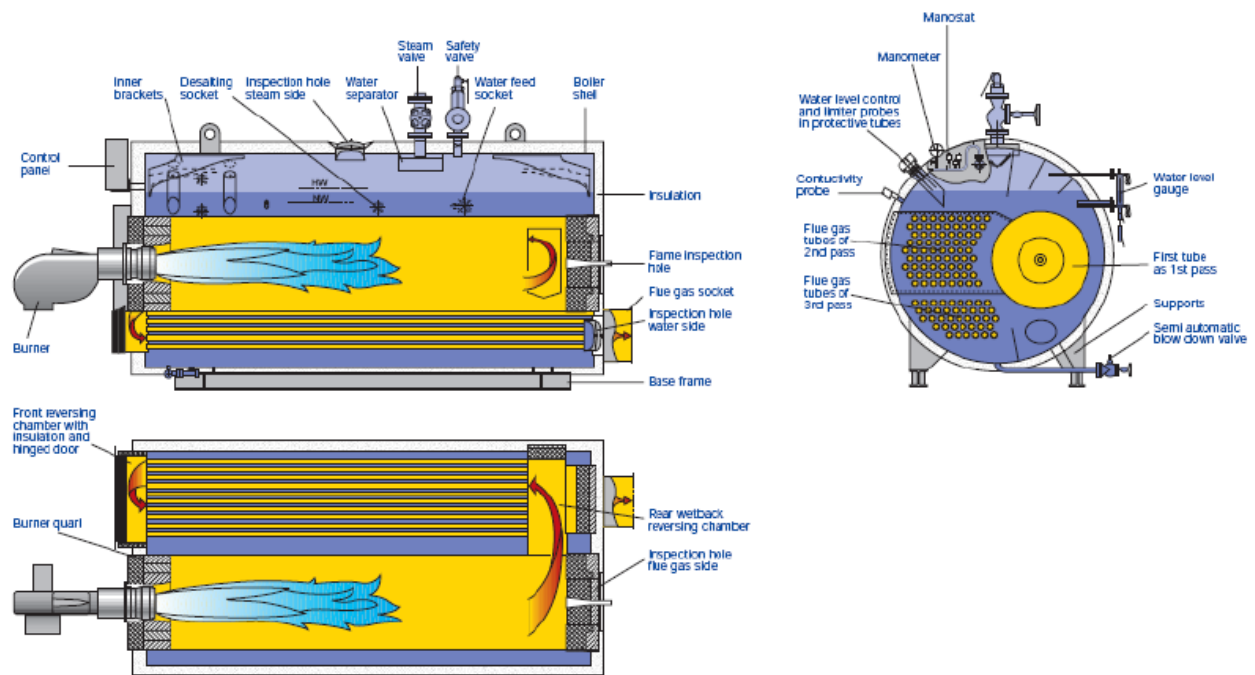


Figure 3.1: Example of shell boiler setup components

The simulated unit is a three pass fire tube shell boiler with six distinct gas side heat transfer stages and a single common steam drum on the water/steam side. Hot flue gas from the burner traverses a radiative furnace, two reversal chambers, two convective tube banks, and a final economizer before leaving to the stack.

3.1 Layout

The gas path is represented as:

Burner \rightarrow HX₁ \rightarrow HX₂ \rightarrow HX₃ \rightarrow HX₄ \rightarrow HX₅ \rightarrow HX₆ \rightarrow stack

with the following interpretation:

- HX₁ – Furnace (first pass)
Large, single furnace tube where combustion products enter directly from the burner and transfer heat mainly by radiation and high-temperature convection to the surrounding water/steam.
- HX₂ – First reversal chamber
Short cylindrical wet back chamber that turns the flow from the furnace outlet into the first convective tube bank (gas direction change = 180°).
- HX₃ – First convective tube bank (second pass) Bank of small diameter fire tubes arranged in a staggered pattern inside the shell, to boost convection; flue gas flows inside of the tubes, water/steam outside.
- HX₄ – Second reversal chamber Second turning chamber redirecting gas from the first to the second tube bank.
- HX₅ – Second convective tube bank (third pass) Second fire-tube bundle, representing the last in-boiler convective pass.
- HX₆ – Economizer Separate, downstream tube bank used to preheat feedwater in single-phase operation before entering the drum/boiler circuit, recovering heat, and boosting efficiency of the boiler.

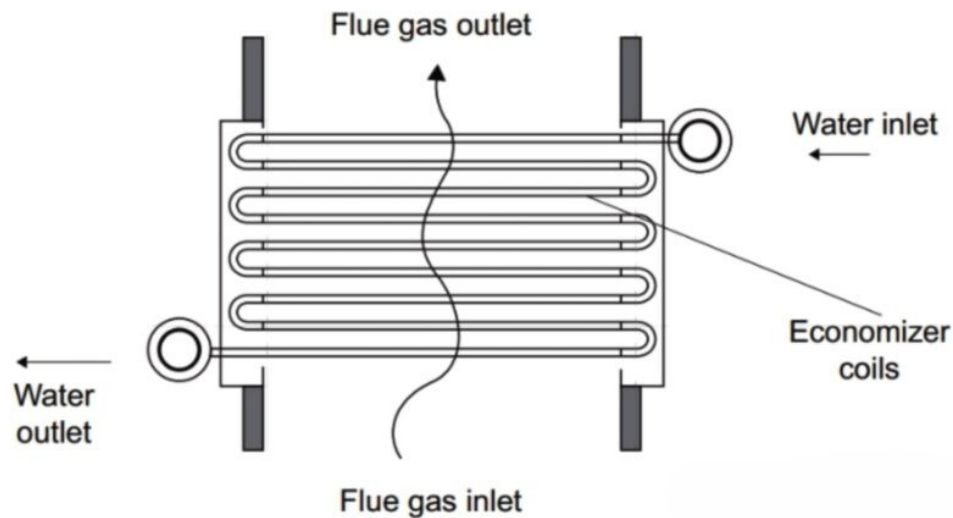


Figure 3.2: Cross-section of the economizer tube bundle HX₆, showing gas-side cross-flow and water-side internal flow.

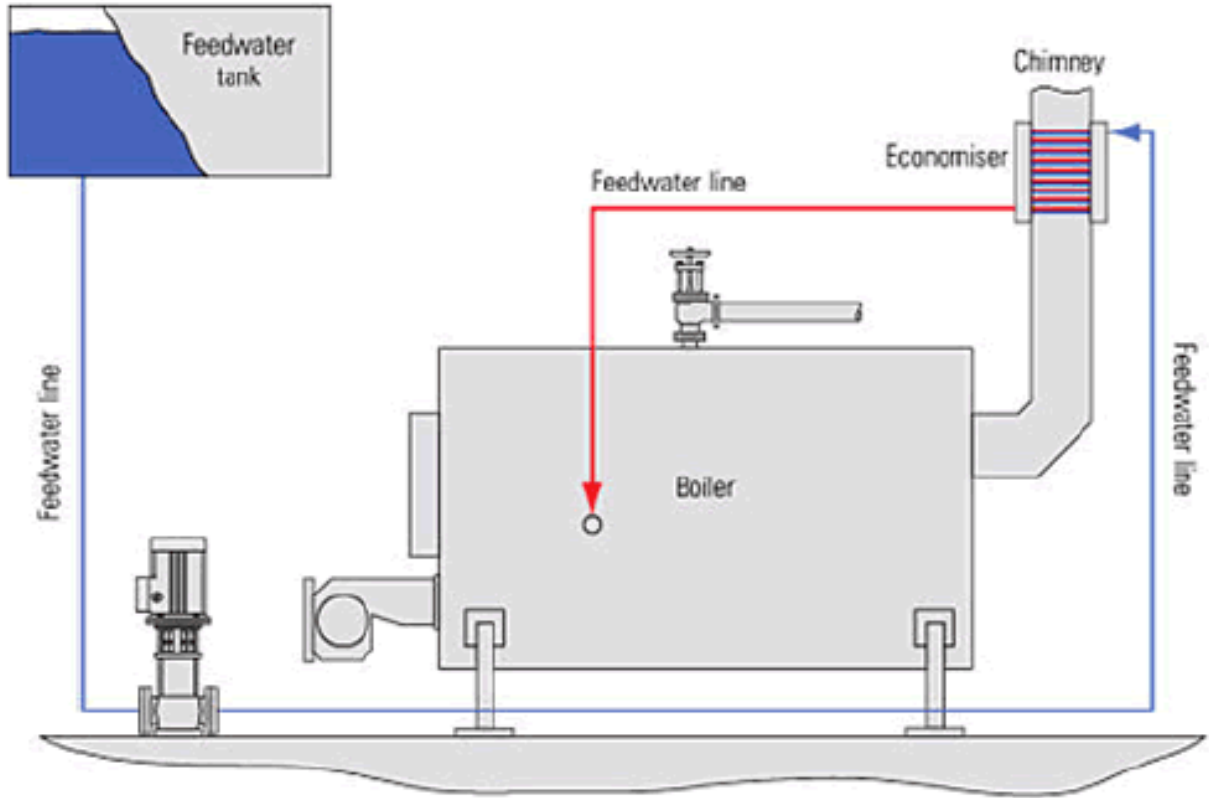


Figure 1: Economizer in Fire Tube Steam Boiler.

Figure 3.3: Three-pass shell boiler with rear-mounted economizer for feedwater preheating.

3.2 Geometry and surface specification

Drum configuration

The boiler has a single horizontal steam drum. Its inner diameter is

$$D_{i,\text{drum}} = 4.5 \text{ m}$$

and its length

$$L_{\text{drum}} = 5.0 \text{ m}$$

The drum is not modelled with internal separators or circulation hardware. It simply supplies the saturated water/steam state at boiler pressure, while all circulation effects are represented by the single 1-D water/steam stream used in the heat-transfer stages.

Flue gas passes

All six pressure part stages of the simulated boiler are represented with a consolidated geometric and surface specification.

Table 3.1: Flue gas stages key parameters

| Element | Kind | Di [m] | L [m] | N_tubes [-] | Wall t [mm] | Roughness [μm] | Pool boiling [-] |
|-----------------|--------------|-----------|----------|----------------|----------------|--------------------------------|---------------------|
| HX ₁ | single_tube | 1.40 | 5.276 | 1 | 2.9 | 0.5 | true |
| HX ₂ | reversal_ch. | 1.60 | 0.80 | 1 | 2.9 | 0.5 | true |
| HX ₃ | tube_bank | 0.076 | 4.975 | 118 | 2.9 | 0.5 | true |
| HX ₄ | reversal_ch. | 1.60 | 0.80 | 1 | 2.9 | 0.5 | true |
| HX ₅ | tube_bank | 0.076 | 5.620 | 100 | 2.9 | 0.5 | true |
| HX ₆ | economizer | 0.076 | 7.50 | 160 | 2.5 | 0.5 | false |

The input file `stages.yaml`, provided in Annex A, contain the complete detailed specifications and is parsed at runtime by the configuration loader (`new_loader.py`). This separates numerical solution algorithms from geometry and surface data, and allows different boiler variants to be simulated by simply modifying the YAML files.

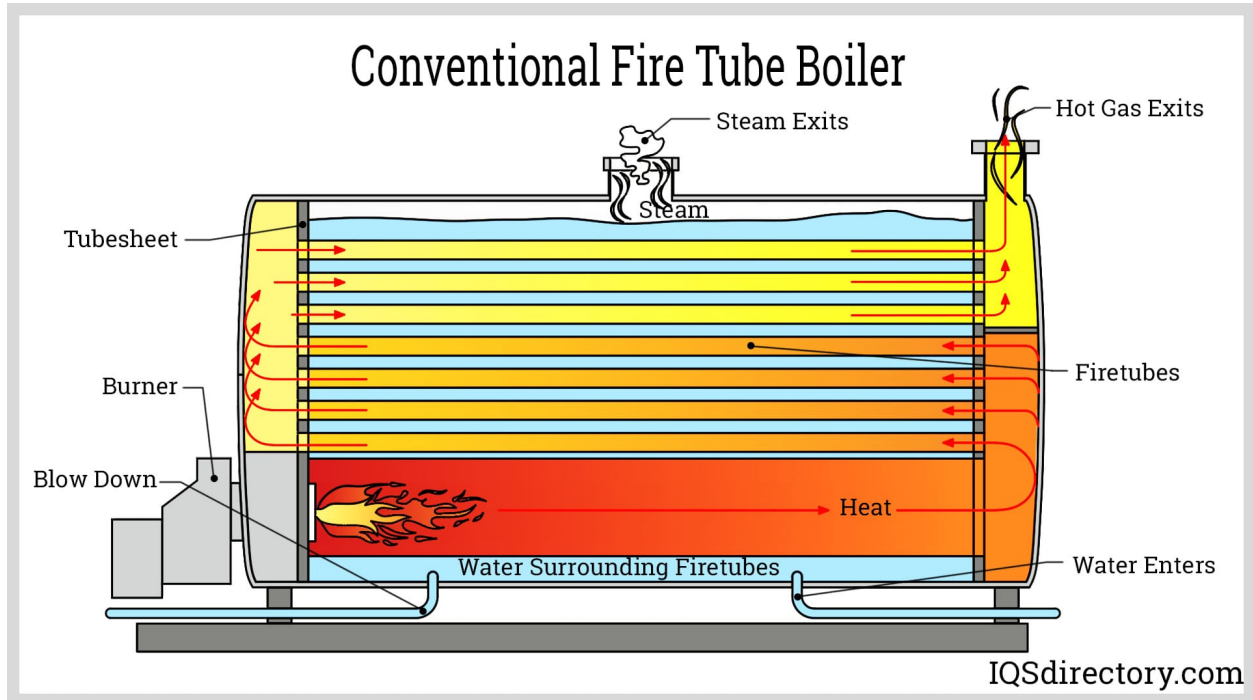


Figure 3.4: Detailed cross-section of the simulated boiler, showing drum, furnace, tube banks and reversal chambers.

All pressure-part stages (HX₁–HX₅) share the same steel wall thermal conductivity of

$k_{\text{wall}} = 16 \text{ W/m/K}$. The economizer (HX_6) is modelled with a higher wall conductivity $k_{\text{wall}} = 30 \text{ W/m/K}$ and a clean surface (zero fouling thickness) to represent a best-case heat-recovery configuration.

The YAML configuration supplies wall, surface, and hydraulic properties not captured in the tabulated geometry. Each pressure-part exchanger defines wall thickness, wall conductivity, surface roughness, emissivity, and optional fouling layers with specified thickness and conductivity. Most stages use a uniform carbon-steel wall with smooth surfaces and thin fouling layers, while the economizer uses a thinner, higher-conductivity wall and no fouling to reflect a cleaned section.

The steam drum defines diameter, length, and internal surface properties with its own roughness and fouling settings.

Reversal chambers specify curvature radius and nozzle minor-loss coefficients used in pressure-drop calculations.

Tube-bank stages define full shell-side layout: shell diameter, tube count and pitch, tube-row arrangement (staggered or inline), baffle spacing and cut, and bundle clearances. Evaporator banks use tighter pitch and spacing to enhance shell-side transfer, whereas the economizer uses a more open inline layout with a larger tube count and longer tubes.

These YAML entries are translated by the loader into the geometric and hydraulic quantities required for cross-flow areas, Reynolds numbers, and shell-side heat-transfer evaluation.

3.3 Assumptions and limitations

1. Combustion and flue gas

- Ideal complete combustion, with fixed excess air,
- Adiabatic flame temperature from equilibrium chemistry, using NASA polynomials.
- Ideal gas mixture $p = \rho RT$, with transport properties $\mu(T)$ $k(T)$ $c_p(T)$ from polynomial data.
- Steady state boiler operation, with fixed fuel air and feedwater.
- Boiler efficiency computed on HHV or LHV basis, using standard energy balance equations.

2. Heat transfer

- One dimensional steady heat transfer per stage.
- Uniform wall conductivity and thickness, radial conduction only.
- Gas side HTC from standard correlations properties **vary** with temperature pressure and composition.
- Gas radiation via band averaged grey model for CO_2 and H_2O , no spectral resolution, and no soot formation.
- Water side HTC uses IAPWS-IF97 properties, homogenized two phase model.
- Drum at fixed pressure, and perfect steam water separation (no carryover).

3. Hydraulic and thermal performance

- 1D, steady, single phase flow.
- Constant mass flow along each stage.
- Compressibility effects appear only through property variations $\rho(T, P)$ and $\mu(T, P)$ in Re and $\rho V^2/2$.
- Stage level minor loss coefficients are lumped, and uniformly distributed along the stage.
- Gas side ΔP in economizer stage is neglected.

Chapter 4

Combustion Model

The combustion module forms the first stage of the design workflow. It reads all run-specific configurations and parameters from YAML files and simulates the combustion process for the specified fuel and air streams at the given excess air ratio. Its outputs are the adiabatic flame temperature, used as the inlet temperature for the flue gas entering the first pass, and the fully combusted flue gas composition. Together, these results define the flue-gas stream supplied to the heat-transfer model.

4.1 Fuel Stream

The boiler is fired with a natural-gas-type fuel defined in the simulation input (`config/fuel.yaml`).

The fuel is supplied at $300K$ and $1.013 \times 10^5 Pa$ with a mass flow rate of $0.1 kg/s$. Its composition is specified on a mass-fraction basis and converted internally to mole fractions for all stoichiometric and thermodynamic calculations.

Table 4-1 summarizes the fuel composition in both mass and mole fraction form.

| Component | Formula | Mass fraction w_i [-] | Mole fraction x_i [-] | Comment |
|-----------|-------------|----------------------------|----------------------------|---|
| Methane | CH_4 | 0.80 | 0.8895 | Main combustible, dominant contributor to LHV |
| Ethane | C_2H_6 | 0.10 | 0.0593 | Heavier hydrocarbon, increases LHV and required O_2 |
| Propane | C_3H_8 | 0.04 | 0.0162 | Heavier hydrocarbon, raises flame temperature |
| n-Butane | C_4H_{10} | 0.01 | 0.00307 | Minor heavy hydrocarbon fraction |

| Component | Formula | Mass fraction w_i [-] | Mole fraction x_i [-] | Comment |
|------------------|----------------------|----------------------------|----------------------------|---|
| Hydrogen sulfide | H_2S | 0.01 | 0.00523 | Sulfur-bearing contaminant → SO_2 in flue gas |
| Nitrogen | N_2 | 0.02 | 0.0127 | Inert ballast in the fuel stream |
| Carbon dioxide | CO_2 | 0.01 | 0.00405 | Inert (already fully oxidized) |
| Water vapour | H_2O | 0.01 | 0.00990 | Moisture carried with the fuel |

The mass fractions sum to 1.0 by definition. The mole fractions x_i are obtained from

$$x_i = \frac{\frac{w_i}{M_i}}{\sum_j \frac{w_j}{M_j}}$$

which is provided by the function `to_mol` in `combustion/mass_mole.py`, where M_i is the molar mass of species i from `molar_masses` in `common/constants.py`.

4.2 Air Stream

4.3 Model flow

The purpose of the combustion model is to determine combustion conditions inside the furnace (1st pass), resulting in a fully burnt flue gas stream entering the heat transfer model at adiabatic temperature.

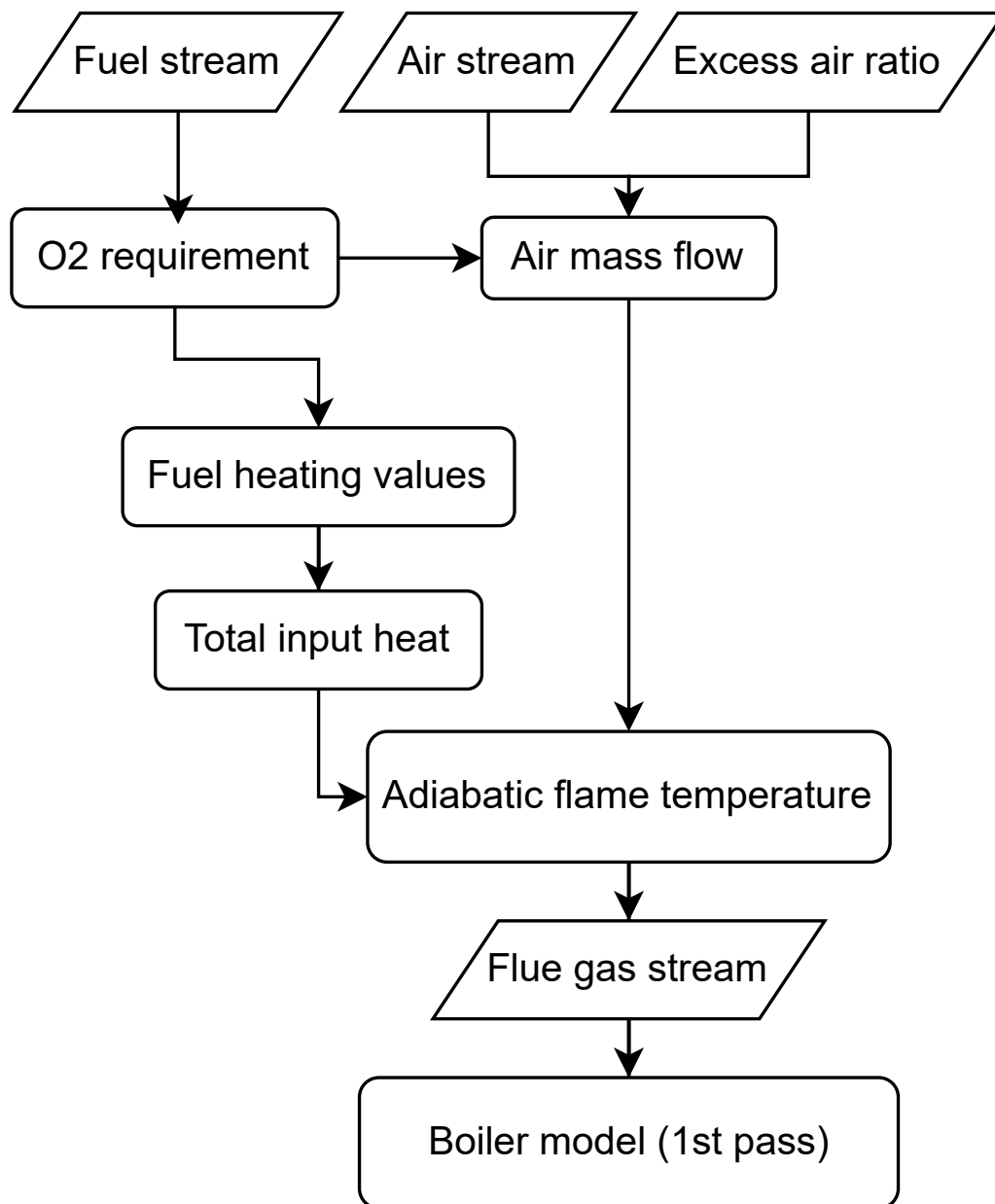


Figure 4.1: Combustion flow

4.4 Stoichiometric O₂ requirement

Evaluated the stoichiometric oxygen requirement via `stoich_o2_required_per_mol_fuel` in `combustion/flue.py`. The algorithm is:

1. Use per mole of species stoichiometric O₂ factors $\nu_{O_2,i}$ from `o2_per_mol` in `common/constants.py`:

| Species | Global reaction (complete combustion) | $\nu_{O_2,i}$ [mol O ₂ / mol species] |
|---|--|--|
| CH ₄ | CH ₄ + 2 O ₂ → CO ₂ + 2 H ₂ O | 2.0 |
| C ₂ H ₆ | C ₂ H ₆ + 3.5 O ₂ → 2 CO ₂ + 3 H ₂ O | 3.5 |
| C ₃ H ₈ | C ₃ H ₈ + 5 O ₂ → 3 CO ₂ + 4 H ₂ O | 5.0 |
| C ₄ H ₁₀ | C ₄ H ₁₀ + 6.5 O ₂ → 4 CO ₂ + 5 H ₂ O | 6.5 |
| H ₂ S | H ₂ S + 1 O ₂ → SO ₂ + H ₂ O | 1.0 |
| N ₂ , CO ₂ , H ₂ O | Inert/fully oxidized → no additional O ₂ | 0.0 |

2. Compute the stoichiometric O₂ requirement per mole of fuel mixture as

$$\nu_{O_2,\text{stoich}} = \sum_i x_i \nu_{O_2,i}$$

Using the mole fractions from Section 4.1 for the present fuel:

- $x_{\text{CH}_4} = 0.8895$
- $x_{\text{C}_2\text{H}_6} = 0.0593$
- $x_{\text{C}_3\text{H}_8} = 0.0162$
- $x_{\text{C}_4\text{H}_{10}} = 0.00307$
- $x_{\text{H}_2\text{S}} = 0.00523$
- remaining species: $x_{\text{N}_2}, x_{\text{CO}_2}, x_{\text{H}_2\text{O}}$ are inert in the stoichiometric balance.

Hence

$$\nu_{O_2,\text{stoich}} = 2.09 \text{ mol O}_2 \text{ per mol fuel mixture}$$

This is exactly what `stoich_o2_required_per_mol_fuel` returns:

```
def stoich_o2_required_per_mol_fuel(fuel: GasStream) -> Q:
    fuel_x = to_mole(fuel.comp)
    total = sum(fuel_x[k] * o2_per_mol.get(k, 0.0) for k in fuel_x)
    return Q(total, "dimensionless")
```

3. For later hydraulic and performance interpretation, it is also useful to express this on a mass basis.

For 1 kg of fuel, the total fuel moles are

$$n_{\text{fuel,total}} = \sum_i \frac{w_i}{M_i} = 56.1 \text{ mol fuel/kg}$$

Thus the stoichiometric O_2 requirement per unit fuel mass is

$$n_{\text{O}_2,\text{stoich}}^{(m)} = \nu_{\text{O}_2,\text{stoich}} n_{\text{fuel,total}} = 1.17 \times 10^2 \text{ mol O}_2/\text{kg fuel}$$

Converting to mass of O_2 per kg of fuel:

$$\dot{m}_{\text{O}_2,\text{stoich}} = n_{\text{O}_2,\text{stoich}}^{(m)} M_{\text{O}_2} = 3.75 \text{ kg O}_2/\text{kg fuel}$$

So, for this fuel:

- Stoichiometric oxygen requirement:

$$\nu_{\text{O}_2,\text{stoich}} = 2.09 \text{ mol O}_2 \text{ per mol fuel mixture}$$

- Equivalent mass requirement:

$$\dot{m}_{\text{O}_2,\text{stoich}} = 3.75 \text{ kg O}_2 \text{ per kg fuel}$$

4.5 Air–fuel ratio and excess air λ

The simulation specifies an excess air ratio

$$\lambda = 1.1$$

in `config/operation.yaml`. This value enters the calculation through `air_flow_rates(air, fuel, excess)` in `combustion/flue.py`.

4.5.1 Actual O_2 supplied

Using:

$$\dot{n}_{\text{O}_2,\text{actual}} = \lambda \dot{n}_{\text{O}_2,\text{stoich}}$$

Thus:

$$\dot{n}_{\text{O}_2,\text{actual}} = 1.1 \nu_{\text{O}_2,\text{stoich}} \dot{n}_{\text{fuel}}$$

The molar fuel flow is determined from the mass-flow rate:

- Fuel mass flow:

$$\dot{m}_f = 0.5 \text{ kg/s}$$

- Total moles per unit mass of fuel mixture (from the mixture molar mass calculation):

$$n_{\text{fuel, total}} = 56.1 \text{ mol/kg}$$

- Therefore the total molar fuel flow:

$$\dot{n}_f = 56.1 \times 0.5 = 28.05 \text{ mol/s}$$

Hence the stoichiometric and actual O_2 flows are:

$$\dot{n}_{\text{O}_2, \text{stoich}} = 2.09 \times 28.05 = 58.7 \text{ mol/s}$$

$$\dot{n}_{\text{O}_2, \text{actual}} = 1.1 \times 58.7 = 64.6 \text{ mol/s}$$

4.5.2 Air required

Air O_2 mole fraction (from `air.yaml`):

$$x_{\text{O}_2, \text{air}} = 0.2095$$

Thus:

$$\dot{n}_{\text{air}} = \frac{\dot{n}_{\text{O}_2, \text{actual}}}{x_{\text{O}_2, \text{air}}} = \frac{64.6}{0.2095} = 308 \text{ mol/s}$$

The air molar mass (mixture weighted) is:

$$M_{\text{air}} = 0.02897 \text{ kg/mol}$$

Therefore the mass-based air flow rate:

$$\dot{m}_{\text{air}} = \dot{n}_{\text{air}} M_{\text{air}} = 308 \times 0.02897 = 8.93 \text{ kg/s}$$

4.5.3 Air–fuel ratio

Mass-based air–fuel ratio:

$$\text{AFR} = \frac{\dot{m}_{\text{air}}}{\dot{m}_f} = \frac{8.93}{0.5} = 17.9$$

4.6 Lower heating value (LHV) and heat release

The fuel lower and higher heating values, and the corresponding firing rate, are evaluated in `combustion/heat.py` by the function `compute_LHV_HHV(fuel)` and then used by `total_input_heat(fuel, air)`.

4.6.1 Method

4.6.1.1 Latent heat of water

Obtain the latent heat of vaporization of water at the reference pressure $P_{\text{ref}} = 101,325$ Pa from the IAPWS-97 correlation:

```
latent_H2O = WaterProps.h_g(P_ref) - WaterProps.h_f(P_ref)
```

where:

- h_g is the saturated vapour enthalpy,
- h_f is the saturated liquid enthalpy.

4.6.1.2 Reference formation enthalpies

Standard formation enthalpies Δh_f° (at 298.15 K, 1 bar) are taken from `common/constants.py` in kJ/mol:

Table 4.1: Standard enthalpy of formation of selected species (Johnson et al. 2024)

| Species | Δh_f° (kJ mol ⁻¹) |
|--------------------------------|--|
| CH ₄ | -74.8 |
| C ₂ H ₆ | -84.7 |
| C ₃ H ₈ | -103.8 |
| C ₄ H ₁₀ | -126.1 |
| SO ₂ | -296.8 |
| CO ₂ | -393.5 |
| H ₂ O(l) | -285.5 |

4.6.1.3 Products for HHV and LHV

For each fuel species, complete combustion is considered:

- $\text{CH}_4 + 2 \text{O}_2 \rightarrow \text{CO}_2 + 2 \text{H}_2\text{O}$
- $\text{C}_2\text{H}_6 + 3.5 \text{O}_2 \rightarrow 2 \text{CO}_2 + 3 \text{H}_2\text{O}$

Builds product formation enthalpies for:

- HHV assumption: water as liquid (condensed)
- LHV assumption: water as vapour (no condensation heat recovered)

```
H2O_liq = _dHf["H2O"] # kJ/mol
H2O_vap = _dHf["H2O"] + latent_H2O * M_H2O # (kJ/kg)*(kg/mol) = kJ/mol
```

Then, looping over the *molar* fuel composition `mol_comp = to_mole(fuel.comp)`:

```
react = 0
HHV_p = 0
LHV_p = 0

for comp, x in mol_comp.items():
    dh = _dHf.get(comp, 0)
    react += x * dh

    C, H = parse_CH(comp)
    if C is not None:
        HHV_p += x * (C * _dHf["CO2"] + (H/2) * H2O_liq)
        LHV_p += x * (C * _dHf["CO2"] + (H/2) * H2O_vap)
    elif comp == "H2S":
        HHV_p += x * (_dHf["SO2"] + H2O_liq)
        LHV_p += x * (_dHf["SO2"] + H2O_vap)
    else:
        HHV_p += x * dh
        LHV_p += x * dh
```

Here:

- `react` represents the mixture-averaged formation enthalpy of the fuel (kJ/mol),
- `HHV_p`, `LHV_p` represent the mixture-averaged formation enthalpy of the ideal products for HHV and LHV definitions.

4.6.1.4 Mixture HHV and LHV (molar, then mass-based)

The mixture molar higher and lower heating values are:

$$\text{HHV}_{\text{mol}} = h_{\text{react}} - h_{\text{prod,HHV}}, \quad \text{LHV}_{\text{mol}} = h_{\text{react}} - h_{\text{prod,LHV}}$$

```
HHV_mol = react - HHV_p      # kJ/mol
LHV_mol = react - LHV_p      # kJ/mol
```

These are converted to mass-based heating values using the mixture molar mass M_{mix} from `mix_molar_mass(mol_comp)`:

```
HHV_kg = HHV_mol / M_mix     # kJ/kg
LHV_kg = LHV_mol / M_mix     # kJ/kg
```

The function returns these, together with the corresponding firing powers:

```
P_HHV = (HHV_kg * fuel.mass_flow).to("kW")
P_LHV = (LHV_kg * fuel.mass_flow).to("kW")
```

4.6.2 Numerical results for the present fuel

For the fuel specified above, the mixture heating values are:

- Higher heating value (HHV, mass-based):

$$\text{HHV}_{\text{mix}} = 52 \text{ MJ/kg}$$

- Lower heating value (LHV, mass-based):

$$\text{LHV}_{\text{mix}} = 47 \text{ MJ/kg}$$

For the specified fuel mass flow rate:

$$\dot{m}_f = 0.5 \text{ kg/s}$$

the resulting firing rates are:

- On an HHV basis:

$$P_{\text{HHV}} = \dot{m}_f \text{HHV}_{\text{mix}} = 0.5 \times 52 \text{ MJ/s} = 26 \text{ MW}$$

- On an LHV basis (used consistently in the simulation):

$$P_{\text{LHV}} = \dot{m}_f \text{LHV}_{\text{mix}} = 0.5 \times 47 \text{ MJ/s} = 23.6 \text{ MW}$$

These correspond directly to `P_HHV` and `P_LHV` returned by `compute_LHV_HHV`.

4.6.3 Total heat input to the boiler Q_{in}

The function `total_input_heat(fuel, air)` combines chemical and sensible contributions:

```
def total_input_heat(fuel, air):
    _, _, _, power_LHV = compute_LHV_HHV(fuel)
    fuel_sens = sensible_heat(fuel)
    air_sens = sensible_heat(air)
    Q_in = (power_LHV + fuel_sens + air_sens).to("kW")
    return power_LHV, Q_in
```

where `sensible_heat(stream)` uses:

$$Q_{\text{sens}} = \dot{m} c_p (T - T_{\text{ref}})$$

Both fuel and air enter at 300 K, while the reference is 298.15 K; the resulting sensible contributions are small compared with the chemical term P_{LHV} (on the order of tens of kW versus tens of MW). Therefore, numerically:

- LHV-based chemical heat input:

$$P_{\text{LHV}} = 23.6 \text{ MW}$$

- Total heat input including sensible:

$$Q_{\text{in}} = P_{\text{LHV}} + Q_{\text{sens,fuel}} + Q_{\text{sens,air}} = 23.6 \text{ MW} \quad (\text{increase} < 0.1\%)$$

The quantity `Q_in` in the `CombustionResult` object is thus interpreted in the rest of the boiler model as the total LHV-based heat release available to be transferred to the water/steam side.

4.7 Adiabatic flame temperature

The adiabatic flame temperature T_{ad} is evaluated in the model by the function `adiabatic_flame_T(air, fuel)` in `combustion/adiabatic_flame_temperature.py`. This routine uses Cantera and an enthalpy–pressure equilibrium (HP) calculation to determine the final equilibrium temperature and composition of the flue gas, assuming:

- complete mixing of fuel and air,
- no heat losses to the surroundings (adiabatic),
- constant system pressure (equal to the air/fuel inlet pressure),
- chemical equilibrium among all gas species in `config/flue_cantera.yaml`.

4.7.1 Thermodynamic formulation

Let the fuel and air streams be characterized by:

- mass flows \dot{m}_{fuel} , \dot{m}_{air} ,
- inlet temperatures T_{fuel} , T_{air} ,

- pressure P ,
- compositions (mole fractions) X_{fuel} , X_{air} .

The total inlet enthalpy rate of the unmixed reactants is

$$\dot{H}_{\text{react}} = \dot{m}_{\text{air}} h_{\text{air}}(T_{\text{air}}, P, X_{\text{air}}) + \dot{m}_{\text{fuel}} h_{\text{fuel}}(T_{\text{fuel}}, P, X_{\text{fuel}})$$

The total mass flow is

$$\dot{m}_{\text{tot}} = \dot{m}_{\text{air}} + \dot{m}_{\text{fuel}}$$

so the mixture-averaged specific enthalpy of the reactants is

$$h_{\text{target}} = \frac{\dot{H}_{\text{react}}}{\dot{m}_{\text{tot}}}$$

The adiabatic, constant-pressure equilibrium state is then defined by the constraints:

$$\begin{aligned} h_{\text{products}}(T_{\text{ad}}, P, \mathbf{X}_{\text{eq}}) &= h_{\text{target}} \\ P_{\text{out}} &= P \\ \mathbf{X}_{\text{eq}} &\text{ satisfies chemical equilibrium at } (T_{\text{ad}}, P) \end{aligned}$$

Cantera is used to enforce this condition via its HP equilibrium mode.

4.7.2 Implementation

Key steps from `adiabatic_flame_T`:

1. Convert the mass-based composition of fuel and air to mole fractions using `to_mole(...)` (from `combustion/mass_mole.py`).
2. Create three Cantera `Solution` objects using the mechanism `config/flue_cantera.yaml`:

```
gas_air = ct.Solution("config/flue_cantera.yaml", "gas_mix")
gas_fuel = ct.Solution("config/flue_cantera.yaml", "gas_mix")
gas_mix = ct.Solution("config/flue_cantera.yaml", "gas_mix")
```

3. Set the inlet states of the separate streams:

```
gas_air.TPX = T_air, P_Pa, X_air
gas_fuel.TPX = T_fuel, P_Pa, X_fuel
```

4. Compute reactant enthalpy rate and target specific enthalpy:

```
Hdot_react = m_air * gas_air.enthalpy_mass + m_fuel * gas_fuel.enthalpy_mass
h_target = Hdot_react / m_tot # J/kg of mixture
```

5. Build the overall reactant composition X_{react} from the molar flow rates of each component in each stream:

```
n_air = molar_flow(air.comp, air.mass_flow)
n_fuel = molar_flow(fuel.comp, fuel.mass_flow)

# Accumulate species molar flow rates
n_dot_sp = {...}
X_react = {k: v / n_sum for k, v in n_dot_sp.items()}
```

6. Initialize the mixture and perform HP equilibrium:

```
gas_mix.TPX = 300.0, P_Pa, X_react # initial guess for T
gas_mix.HP = h_target, P_Pa        # enforce (H,P)
gas_mix.equilibrate("HP")          # chemical equilibrium
```

7. Construct the resulting flue-gas stream:

```
Y_eq = gas_mix.Y # equilibrium mass fractions
comp_eq = {sp: Q_(float(Y_eq[i]), "'") for i, sp in enumerate(gas_mix.species_names)
           if Y_eq[i] > 1e-15}

flue = GasStream(
    mass_flow = Q_(m_tot, "kg/s"),
    T          = Q_(gas_mix.T, "K"),
    P          = air.P,
    comp       = comp_eq,
)
```

The adiabatic flame temperature is then available as `flue.T` and is also stored in `CombustionResult.T_ad`.

4.7.3 Numerical result for the present case

For the given conditions:

- Fuel: natural-gas-type mixture from Section 4.1, $\dot{m}_{\text{fuel}} = 0.5 \text{ kg/s}$, $T_{\text{fuel}} = 300 \text{ K}$, $1.013 \times 10^5 \text{ Pa}$.
- Air: dry air at 300 K and $1.013 \times 10^5 \text{ Pa}$, composition from `config/air.yaml`.
- Excess air: $\lambda = 1.1$ (10 % excess air).

the HP-equilibrium calculation yields an adiabatic flame temperature on the order of:

$$T_{\text{ad}} = 2,050 \text{ K} \quad (= 1,780^\circ\text{C})$$

This value is consistent with typical adiabatic flame temperatures for natural gas with around 10 % excess air and confirms that the combustion zone (furnace) operates at

very high gas temperatures, driving strong radiative and convective heat transfer to the shell-side water/steam.

The scalar τ_{ad} is passed forward and written into the boiler summary CSV (`*_boiler_summary.csv`) for reference and later comparison with non-adiabatic stack temperatures obtained from the full boiler simulation.

4.8 Flue gas composition

In the combustion model two different flue-gas streams are distinguished:

1. An **equilibrium flue gas at adiabatic flame conditions** (`flue_ad`), obtained from high-temperature HP equilibrium in Cantera.
2. A **fully burnt boiler flue gas** (`flue`), obtained from pure stoichiometry with excess air and no dissociation, used throughout the heat-exchanger network.

Both are represented as `GasStream` objects and stored in the `CombustionResult`, but they serve different purposes in the boiler calculation.

4.8.1 Definitions and distinction

- **Equilibrium flue gas** (`flue_ad`)

- Thermodynamic state: high-temperature HP equilibrium at the adiabatic flame temperature.
- Contains all equilibrium species allowed by the mechanism (major products + dissociation products + radicals).
- Used only to:
 - * determine the adiabatic flame temperature T_{ad} ,
 - * report equilibrium composition in diagnostics/CSV.

- **Fully burnt flue gas** (`flue`)

- Thermodynamic state: chemically frozen, fully burnt mixture at the same temperature and pressure as the equilibrium gas at burner exit.
- Contains only “engineering” products (CO_2 , H_2O , SO_2 , O_2 , N_2 , Ar) with no CO , H_2 , NO_x or radicals.
- Used as the hot-side gas in all boiler heat-transfer and pressure-drop calculations.

Hence, equilibrium chemistry is confined to the flame-temperature calculation, while the boiler itself is solved with a simplified, fully burnt flue gas consistent with complete combustion and 10 % excess air.

4.8.2 Equilibrium flue gas at adiabatic conditions

The adiabatic flame calculation is performed in `combustion/adiabatic_flame_temperature.py` via the function `adiabatic_flame_T(air, fuel):`

- The inlet **air** and **fuel** streams are:
 - represented as `GasStream` objects (mass flow, T , P , mass fractions),
 - converted to mole fractions (`to_mole`) and set into separate Cantera `Solution` objects (`gas_air`, `gas_fuel`) based on `config/flue_cantera.yaml`.

- A mixed-reactant state is constructed at constant pressure:

- Total enthalpy flow of reactants:

$$\dot{H}_{\text{react}} = \dot{m}_{\text{air}} h_{\text{air}} + \dot{m}_{\text{fuel}} h_{\text{fuel}}$$

- Target specific enthalpy:

$$h_{\text{target}} = \dot{H}_{\text{react}} / \dot{m}_{\text{tot}}$$

- Overall reactant mole fractions are built from molar flow rates of air and fuel.

- The mixture is then set in Cantera (`gas_mix`) with:

- composition X_{react} ,
 - pressure $P = P_{\text{air}}$,
 - specific enthalpy $h = h_{\text{target}}$,
 - and equilibrated under HP constraints:

```
gas_mix.TPX = 300.0, P_Pa, X_react    # T placeholder
gas_mix.HP = h_target, P_Pa
gas_mix.equilibrate("HP")
```

- After equilibrium:

- The **adiabatic flame temperature** is `gas_mix.T`.
 - The **equilibrium mass fractions** are read from `gas_mix.Y`:

```
Y_eq = gas_mix.Y
comp_eq = {
    sp: Q_(float(Y_eq[i]), "")
    for i, sp in enumerate(gas_mix.species_names)
    if Y_eq[i] > 1e-15
}
```

- These are stored in the equilibrium flue-gas stream:


```

flue_ad = GasStream(
    mass_flow = Q_(m_tot, "kg/s"),
    T          = Q_(gas_mix.T, "K"),
    P          = air.P,
    comp       = comp_eq,
)

```

Typical equilibrium composition ($\lambda = 1.1$, natural gas, $T_{ad} = 2050$ K) is:

- Major species:
 - $CO_2 \approx 0.085\text{--}0.095$
 - $H_2O \approx 0.075\text{--}0.085$
 - $O_2 \approx 0.020\text{--}0.030$ (excess air)
 - $N_2 \approx 0.78\text{--}0.80$
- Dissociation / minor species:
 - $CO \approx 10^{-3}$
 - $H_2 \approx 10^{-4}$
 - $NO \approx 10^{-4}\text{--}10^{-5}$
 - OH, O , radicals $< 10^{-6}$
 - $SO_2 = 10^{-4}$ (from fuel H_2S)

This composition is physically consistent with high-temperature equilibrium at $2000K$ and slight dissociation.

The object `flue_ad` is stored in `CombustionResult` and is only used to:

- provide T_{ad} and equilibrium composition to the boiler summary CSV,
- support diagnostic post-processing.

It is **not** used directly in the heat-exchanger network.

4.8.3 Fully burnt boiler flue gas

The boiler thermal model requires a chemically simple flue-gas mixture to compute heat transfer and pressure drop. For that purpose a **fully burnt** flue gas is constructed in `combustion/flue.py` and `combustion/combustor.py`:

1. In `Combustor.run()` the air mass flow is first set from stoichiometry plus excess air:

```
air.mass_flow = air_flow_rates(air, fuel, self.excess_air_ratio)
```

2. The fully burnt flue-gas composition is then computed from pure stoichiometry:

```
mass_comp_burnt, m_dot_flue = from_fuel_and_air(fuel, air)
```

- `from_fuel_and_air` assumes complete oxidation of:
 - C-containing species $\rightarrow CO_2$,
 - $H \rightarrow H_2O$,
 - $S \rightarrow SO_2$,
- including CO_2 and H_2O already present in the inlet fuel and air.

- The allowed product set is:
 - CO_2 , H_2O , SO_2 , O_2 , N_2 , Ar .
- Residual O_2 is determined by the imposed excess air ratio λ ; there is no CO , H_2 , NO_x , or radicals in this stream.

Internally, `from_fuel_and_air` works with molar balances:

- determines stoichiometric O_2 demand per mole of fuel (`stoich_O2_required_per_mol_fuel`),
- combines fuel and air mole fractions to get:

$$\dot{n}_{CO_2}, \dot{n}_{H_2O}, \dot{n}_{SO_2}, \dot{n}_{O_2}, \dot{n}_{N_2}, \dot{n}_{Ar}$$

- normalizes by total moles to obtain mole fractions, converts to mass fractions (`to_mass`), and returns both:
 - `mass_comp` (mass fractions),
 - `m_dot` (total mass flow of flue gas).

3. The fully burnt flue-gas stream is then created as:

```
flue_boiler = GasStream(
    mass_flow = Q_(m_dot_flue, "kg/s"),
    T          = T_ad,          # assume recombination to near Tad at burner exit
    P          = air.P,
    comp       = {sp: Q_(y, "") for sp, y in mass_comp_burnt.items()},
)
```

4. `CombustionResult` is populated with both flue streams:

```
return CombustionResult(
    LHV        = power_LHV,
    Q_in       = Q_in,
    T_ad       = T_ad,
    flue       = flue_boiler, # fully burnt flue used in boiler model
    flue_ad    = flue_ad,    # equilibrium flue at Tad (diagnostics)
    fuel_LHV_mass = LHV_mass,
    fuel_P_LHV  = P_LHV,
)
```

The **boiler solver** (`run_hx`) always receives `combustion.flue` (i.e. `flue_boiler`) as its gas inlet, and this fully burnt composition is used for:

- gas properties (c_p , ρ , μ , k),
- heat-transfer coefficients,
- radiative heat transfer (emissivity based on $CO_2/H_2O/SO_2$),
- pressure-drop estimates and stack temperature.

Thus, the equilibrium flue gas provides a physically consistent high-temperature reference, while the fully burnt flue gas represents the practical working fluid in the convective–radiative sections of the boiler.

4.8.4 Output fields

The flue-gas information exposed to the rest of the model and to the post-processing is encapsulated in `CombustionResult`:

```
@dataclass(frozen=True)
class CombustionResult:
    LHV: Q_
    Q_in: Q_
    T_ad: Q_
    flue: GasStream          # fully-burnt flue used in boiler
    flue_ad: GasStream | None = None  # equilibrium flue at T_ad (optional)
    fuel_LHV_mass: Q_ | None = None
    fuel_P_LHV: Q_ | None = None
```

The relevant report/CSV entries are:

| Field | Meaning |
|---------|--|
| T_ad | Adiabatic flame temperature from HP equilibrium |
| flue_ad | GasStream of equilibrium flue gas (adiabatic composition, diagnostics) |
| flue | GasStream of fully burnt flue gas used in all boiler HX calculations |

This completes the description of how flue-gas composition is defined, distinguished, and used in the boiler model.

Chapter 5

Heat Transfer Model

5.1 Fundamental heat-balance equations

The boiler is modelled as a one-dimensional counter-current heat exchanger composed of six stages (HX₁–HX₅). Heat transfer is resolved along the gas flow direction x , while water flows in the opposite direction. Each stage is discretized into segments of length dx ; all local quantities are defined per unit length.

- Notation (per segment)
- x – axial coordinate along the gas flow [m]
- dx – marching step in x [m]
- \dot{m}_g, \dot{m}_w – gas and water mass flow rates [kg/s]
- $T_g(x), T_w(x)$ – bulk gas and water temperatures [K]
- $T_{gw}(x), T_{ww}(x)$ – gas-side and water-side wall temperatures [K]
- $h_g(x), h_w(x)$ – total gas-side and water-side heat-transfer coefficients [W/m²·K]
- P_g, P_w – gas-side and water-side wetted perimeters [m]
- $q'(x)$ – linear heat flux (heat per unit length) [W/m]
- $UA'(x)$ – overall conductance per unit length [W/K/m]

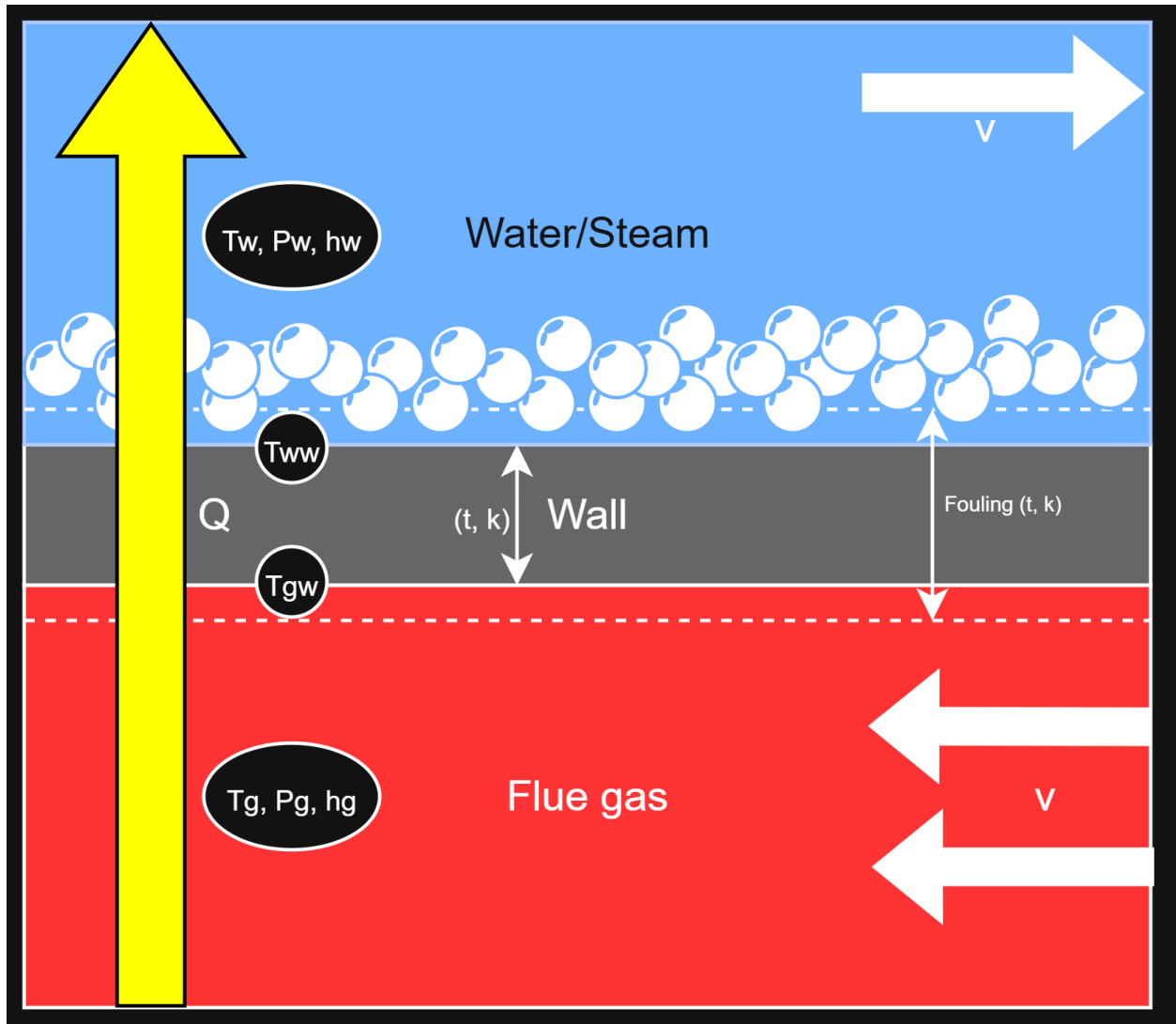


Figure 5.1: Cross section of heat transfer network from gas to water/steam

5.2 Local energy balance

For each differential segment of length dx , the model enforces a one-dimensional steady-state energy balance between the gas, the water and the tube wall:

- Heat transferred across the wall:

$$q'(x) = U A'(x) [T_g(x) - T_w(x)]$$

- Relation to the segment duty:

$$dQ(x) = q'(x) dx$$

- Gas stream:

$$dQ(x) = -\dot{m}_g dh_g(x) \Rightarrow \frac{dh_g}{dx} = -\frac{q'(x)}{\dot{m}_g}$$

- Water stream:

$$dQ(x) = +\dot{m}_w dh_w(x) \Rightarrow \frac{dh_w}{dx} = +\frac{q'(x)}{\dot{m}_w}$$

In the numerical implementation these equations are applied in finite-difference form over each marching step:

$$Q_{\text{step}} = q'(x) \Delta x$$

$$\Delta h_g = -\frac{Q_{\text{step}}}{\dot{m}_g}, \quad \Delta h_w = +\frac{Q_{\text{step}}}{\dot{m}_w}$$

5.3 Overall conductance and resistance network

The overall conductance per unit length $UA'(x)$ is obtained from a radial series of thermal resistances per unit length:

- Gas-side convection:

$$R'_g = \frac{1}{h_g(x) P_g}$$

- Gas-side fouling:

$$R'_{fg} = R'_{fi}(P_g) \quad (\text{from specified fouling thickness and conductivity})$$

- Tube wall:

$$R'_w = \frac{\ln(D_o/D_i)}{2\pi k_w}$$

- Water-side fouling:

$$R'_{fc} = R'_{fo}(P_w)$$

- Water-side convection:

$$R'_c = \frac{1}{h_w(x) P_w}$$

where D_i and D_o are the tube inner and outer diameters and k_w is the tube wall thermal conductivity. Combining these contributions:

$$\frac{1}{UA'(x)} = R'_g + R'_{fg} + R'_w + R'_{fc} + R'_c$$

or equivalently,

$$UA'(x) = \left[\frac{1}{h_g P_g} + R'_{fg} + R'_w + R'_{fc} + \frac{1}{h_w P_w} \right]^{-1}$$

The linear heat flux then follows directly:

$$q'(x) = UA'(x) [T_g(x) - T_w(x)]$$

5.4 Wall temperature update and thermal convergence

The tube wall temperatures on the gas and water sides, T_{gw} and T_{ww} , are updated using a two node wall model in each marching step.

Given $q'(x)$, the wall side energy balances yield:

$$T_{gw} = T_g - \frac{q'}{h_{g,\text{tot}}}$$

$$T_{ww} = T_w + \frac{q'}{h_w}$$

The wall conduction temperature drop is:

$$\Delta T_{\text{wall}} = T_{gw} - T_{ww}$$

which is also equal to:

$$\Delta T_{\text{wall}} = q' [R'_{fg} + R'_w + R'_{fc}]$$

A consistency check is applied; if the implied wall temperature difference from conduction differs from the one implied by convection, the marching solver iterates the HTC evaluation once with relaxed updates (default under-relaxation factor 0.35).

In the actual implementation this consistency check is performed by iterating on T_{gw} , T_{ww} , and q' using the full resistance network (gas convection, gas fouling, wall, water fouling, water convection), with an under-relaxation factor applied to both wall temperatures and the linear heat flux.

If temperature overshoot (negative film coefficient, reversed driving force) is detected within a step, the step is automatically halved and recomputed.

5.5 Stage and boiler level duties

For a stage of length L_j , the stage heat duty and stage level conductance are obtained by integrating the local quantities along x :

$$Q_{\text{stage},j} = \int_0^{L_j} q'(x) dx \approx \sum_i q'_i \Delta x_i$$

$$(UA)_j = \int_0^{L_j} UA'(x) dx \approx \sum_i UA'_i \Delta x_i$$

The total useful boiler duty is the sum of all stage duties:

$$Q_{\text{useful}} = \sum_{j=1}^6 Q_{\text{stage},j}$$

These integrated quantities are later used in the performance and efficiency evaluation (Section 7) and for constructing stage-wise summary tables.

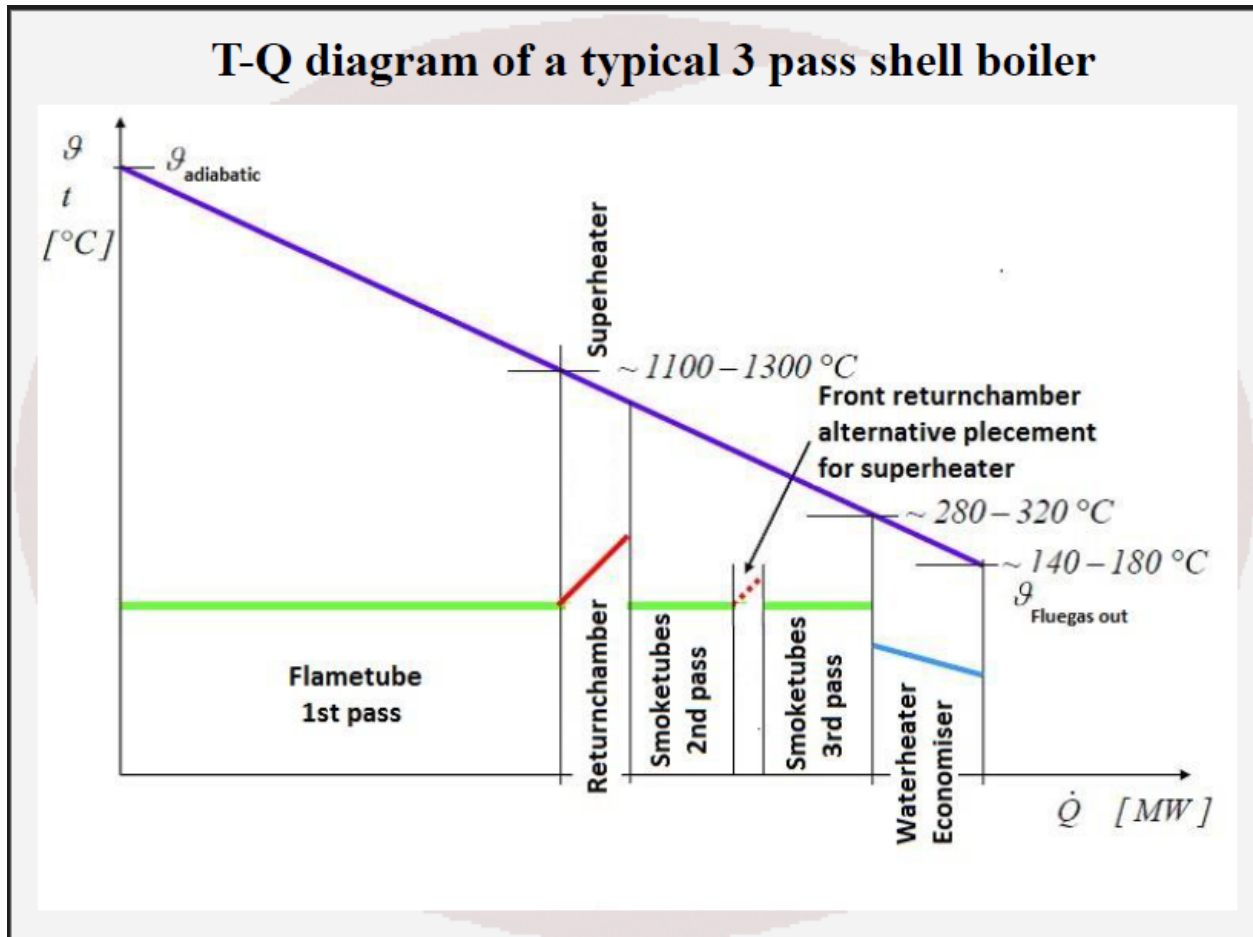


Figure 5.2: Representative T - Q diagram for the three-pass boiler, showing gas and water/steam temperature evolution and stage heat duties HX_1 - HX_6 .

5.6 Gas side

Gas side heat transfer is computed with geometry aware correlations based on local gas properties from Cantera (GasProps) and stage specific geometry from the GeometryBuilder. For each marching step, the total gas side HTC is split into a convective and a radiative contribution:

$$h_{g,\text{tot}} = h_{g,\text{conv}} + h_{g,\text{rad}}$$

The implementation uses the helper `gas_htc_parts(g, spec, T_{gw})`, which returns $(h_{g,\text{conv}}, h_{g,\text{rad}})$ in $\text{W}/\text{m}^2\cdot\text{K}$, and then sums them in `gas_htc`.

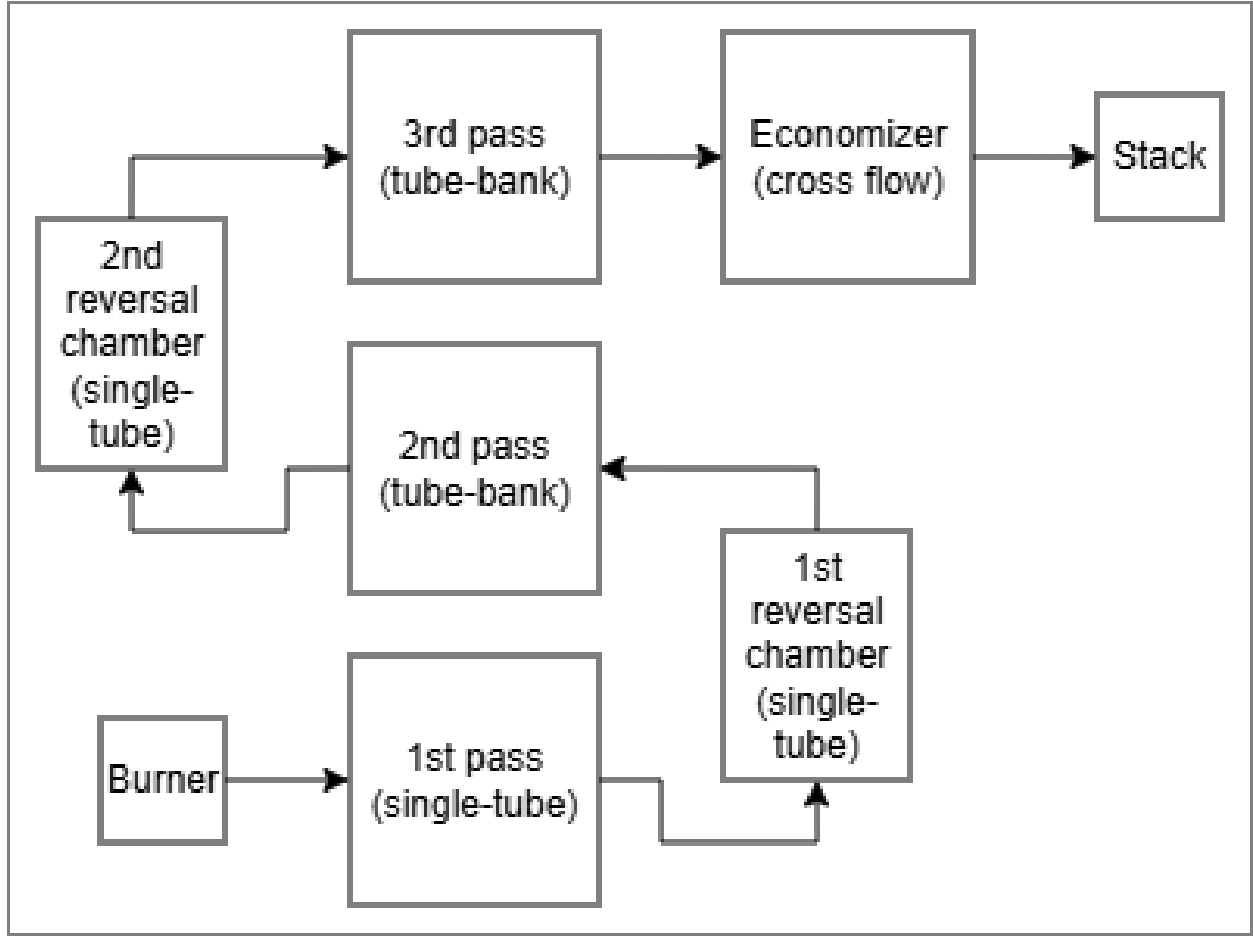


Figure 5.3: Path of flue gas through the 6 stages

5.6.1 Single tube and reversal chamber

Stages of kind `single_tube` and `reversal_chamber`, corresponding to furnace (first pass), and both reversal chambers, are treated as internal forced convection in a circular duct. The characteristic quantities are:

- Diameter: D (supplied by `stages.yaml`)
- Length: L (supplied by `stages.yaml`)
- Flow area: $A = \frac{1}{4} \pi D^2$ (calculated by geometry builder)
- Velocity:

$$V = \frac{\dot{m}_g}{\rho_g A}$$

- Reynolds and Prandtl numbers:

$$\text{Re} = \frac{\rho_g V D}{\mu_g}, \quad \text{Pr} = \frac{c_{p,g} \mu_g}{k_g}$$

Local gas properties $\rho_g, \mu_g, k_g, c_{p,g}$ are obtained from the Cantera mixture via the functions defined in `common\props.py`, at the local gas temperature and pressure. (McBride et al. 1993)

Laminar/developing flow (Graetz-type)

For $Re < 2300$, uses a Graetz correlation for thermally developing laminar flow:

$$Gz = Re Pr \frac{D}{L}$$

$$Nu = 3.66 + \frac{0.0668 Gz}{1 + 0.04 Gz^{2/3}}$$

(Incropera et al. 2011)

Turbulent flow (Gnielinski with Petukhov friction factor)

For $Re \geq 2300$, the Gnielinski correlation is applied with a Petukhov friction factor:

$$f = (0.79 \ln Re - 1.64)^{-2}$$

(Munson et al. 2013)

$$Nu = \frac{\frac{f}{8}(Re - 1000) Pr}{1 + 12.7 \sqrt{\frac{f}{8}} (Pr^{2/3} - 1)}$$

(Incropera et al. 2011) The local convective heat-transfer coefficient is then:

$$h_{g,conv} = \frac{Nu k_g}{D}$$

(Incropera et al. 2011)

This same internal correlation is used for `single_tube`, `reversal_chamber` and `tube_bank` gas-side flow (see below).

5.6.2 Tube bank

Stages `tube_bank` correspond to tube bundles inside the shell, ie. first and second passes. In this model, the gas side is still treated as internal flow inside the tubes:

- Hot side (gas): inside tubes (inner diameter D_i), using the same internal forced convection model as in Section 5.2.1.

Thus the gas side convective HTC in tube-bank stages is:

$$h_{g,\text{conv}}^{(\text{HX3},5)} = \frac{\text{Nu}(\text{Re}, \text{Pr}) k_g}{D_i}$$

with Nu given by the Graetz/Gnielinski formulation above, and Re, Pr computed from the local gas properties and tube hydraulic diameter.

5.6.3 Economizer

The economizer `economiser` stage reverses the roles: gas flows outside the tubes in cross flow, while water flows inside. The gas side convection is then modelled as external cross flow over a tube bank.

Key geometry quantities (from `GeometryBuilder` for the economizer):

- Tube outer diameter: $D = D_o$
- Gas side cross flow area: $A_{\text{bulk}} = A_{\text{hot,flow}}$
- Optional maximum/mean velocity factor:

$$V_{\text{bulk}} = \frac{\dot{m}_g}{\rho_g A_{\text{bulk}}}, \quad V = u_{\text{max}} V_{\text{bulk}}$$

where u_{max} is calculated depending on the tube bank arrangement and spacing between tubes.

- Reynolds and Prandtl numbers:

$$\text{Re} = \frac{\rho_g V D}{\mu_g}, \quad \text{Pr} = \frac{c_{p,g} \mu_g}{k_g}$$

For "economiser" stages the primary correlation is a banded Zukauskas form for cross flow over tube banks:

$$\text{Nu} = C \text{Re}^m \text{Pr}^n$$

(Incropera et al. 2011)

where the coefficients C, m are selected from standard bands as a function of Reynolds number and tube arrangement (inline vs staggered), and the exponent n is:

$$n = \begin{cases} 0.36, & \text{Pr} \leq 10 \\ 0.25, & \text{Pr} > 10 \end{cases}$$

If Re falls outside the tabulated bands, the model falls back to the Churchill–Bernstein correlation for cross flow over a single cylinder:

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re}{282000} \right)^{5/8} \right]^{4/5}$$

(Incropera et al. 2011) The gas-side convective HTC in the economizer is then:

$$h_{g,\text{conv}}^{(\text{HX6})} = \frac{Nu k_g}{D_o}$$

(Incropera et al. 2011)

5.6.4 Radiation model

Radiative heat transfer from the flue gas to the furnace surfaces is explicitly accounted for by a participating medium model for the H_2O/CO_2 mixture. The implementation follows a simplified Smith–Shen–Friedman style four gray model.

For each step, the gas emissivity is computed as:

1. Partial pressures of participating species:

$$p_{H_2O} = y_{H_2O} P, \quad p_{CO_2} = y_{CO_2} P$$

(Modest 2013) where y_i are molar (or mass-fraction-equivalent) composition entries from the flue gas stream, and P is the local gas pressure.

2. Mean beam length:

$$L_b = \begin{cases} L_{\text{rad,override}}, & \text{if specified in the stage} \\ 0.9 D_{h,\text{gas}}, & \text{otherwise} \end{cases}$$

(Modest 2013) with $D_{h,\text{gas}}$ the gas-side hydraulic diameter.

3. Effective optical thickness in each gray band:

$$p_{\text{ratio}} = \frac{p_{\text{H}_2\text{O}} + p_{\text{CO}_2}}{P_{\text{atm}}}$$

(Modest 2013)

$$\tau_j = K_j \left(\frac{T}{1000 \text{ K}} \right)^{T_{\text{exp}}} p_{\text{ratio}} L_b$$

(Modest 2013)

where K_j and weighting factors A_j are fixed band coefficients, T is the gas temperature, and T_{exp} is a temperature exponent (default 0.65, configurable per stage via rad_Texp).

4. Total gas emissivity:

$$\varepsilon_g = 1 - \sum_{j=1}^4 A_j \exp(-\tau_j)$$

(Modest 2013) with ε_g constrained to $[0, 1]$.

A mean-film temperature is used for the linearized radiative HTC:

$$T_{\text{film}} = \frac{T_g + T_{gw}}{2}$$

$$h_{g,\text{rad}} = 4 \sigma F \varepsilon_g T_{\text{film}}^3$$

(Modest 2013)

where:

- σ is the Stefan–Boltzmann constant,
- F is an effective view factor (default 1.0 or stage-specific rad_F).

The gas-side total HTC reported and used in the resistance network is then:

$$h_{g,\text{tot}} = h_{g,\text{conv}} + h_{g,\text{rad}}$$

and the corresponding convective/radiative contributions to the linear heat flux are tracked via:

$$q'_{\text{conv}} = q' \frac{h_{g,\text{conv}}}{h_{g,\text{tot}}}, \quad q'_{\text{rad}} = q' - q'_{\text{conv}}$$

These diagnostics are later integrated on a per-stage basis to quantify the share of convective vs radiative heat transfer in each section of the boiler.

5.7 Water side

Water side heat transfer is computed with geometry dependent correlations using local water properties from IAPWS97 (`WaterProps`), with stage specific geometry from the `GeometryBuilder`. The solver always works with a single effective water side heat transfer coefficient $h_w(x)$ per marching step, which may represent:

- pure pool boiling at a saturated surface,
- a Chen type combination of forced convection and nucleate boiling, or
- single phase forced convection.

In the implementation this logic is encapsulated in `water_htc`, which returns (h_w) for each step.

5.7.1 General formulation and boiling treatment

The six stages of the boiler are divided, from the water-side point of view, into:

- $\text{HX}_1\text{--}\text{HX}_5$: pool-boiling stages
(`pool_boiling = true` in the stage specification)
- HX_6 : economizer stage
(`pool_boiling = false`)

Process in boilers drawn in T-S chart

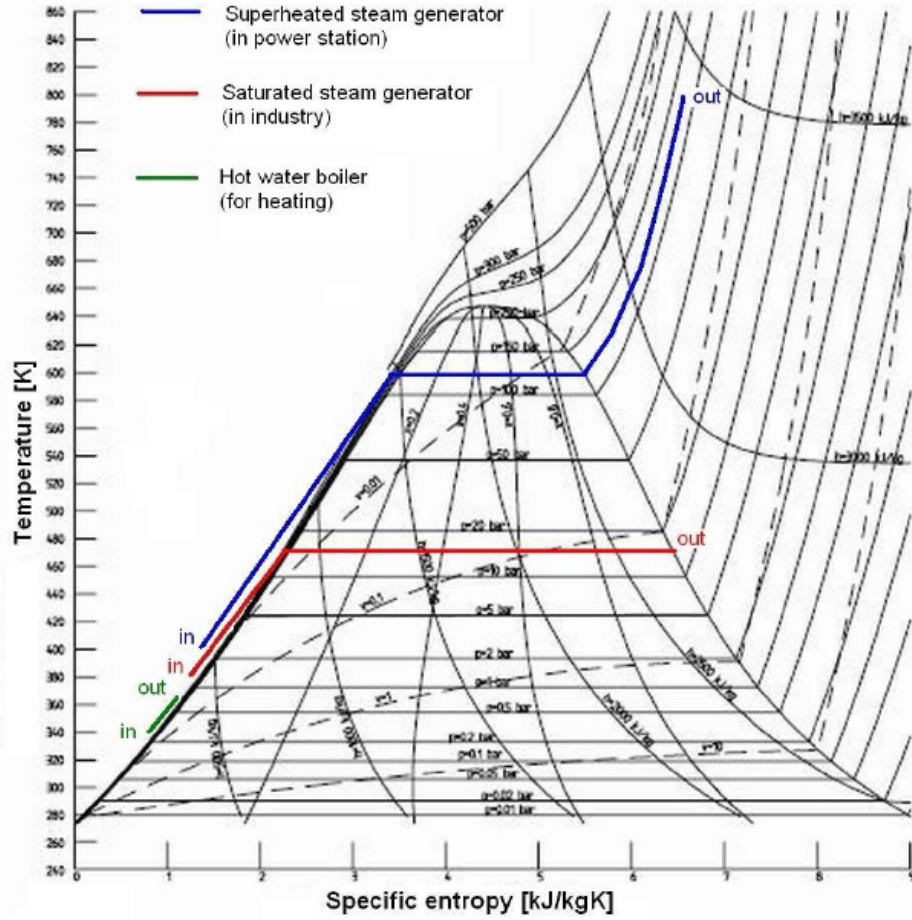


Figure 5.4: Temperature–entropy ($T-s$) representation of the feedwater heating and evaporation process across economiser and boiler at the operating pressure.

The solver applies the following decision tree at each marching step:

1. Pool-boiling stages (HX_1-HX_5)

If the stage is flagged as `pool_boiling = true`, the bulk water temperature entering the wall-energy balance is fixed at the saturation temperature at the local pressure:

$$T_w = T_{\text{sat}}(p_w),$$

and the water-side HTC is computed from the Cooper pool boiling correlation for nucleate boiling correlation:

$$h_{\text{Cooper}} = 55 p_r^{0.12-0.2 \log_{10}(R_p)} [-\log_{10}(p_r)]^{-0.55} q''^{0.67}$$

where

$$p_r = \frac{p}{p_{\text{crit}}} = \text{reduced pressure}, \quad R_p = \text{surface roughness } (\mu\text{m}), \quad q'' = \text{heat flux.}$$

(Incropera et al. 2011)

This nucleate-boiling HTC is then used directly:

$$h_w = h_{w,\text{nb}},$$

and the step is always marked as boiling in the post processing.

In other words, the main boiling surfaces of the boiler (furnace, passes, reversal chambers) are represented as heated surfaces in a saturated pool, with the HTC governed by the local heat flux and surface roughness rather than by a detailed prediction of liquid velocity. This matches the natural circulation character of these sections.

2. Non pool boiling stages (HX₆, economizer)

For stages with `pool_boiling = false`, the model can represent both single phase convection and flow boiling via a Chen type formulation.

a. Boiling detection

A helper determines whether the local state is boiling based on the bulk enthalpy h and, when needed, the wall temperature T_{wall} :

- if

$$h_f(p_w) \leq h \leq h_g(p_w)$$

the state is inside the saturation interval and is treated as two phase;

- if $h < h_f(p_w)$ (slightly subcooled liquid) but the wall superheat is sufficiently high,

$$T_{\text{wall}} > T_{\text{sat}}(p_w) + \Delta T_{\text{crit}},$$

the state is also treated as boiling;

- otherwise the flow is treated as single-phase liquid.

Here h_f and h_g are saturated-liquid and saturated vapor enthalpies at the local pressure, obtained via IAPWS97.

b. Single-phase regime

If boiling is not detected, the water side HTC is purely convective:

$$h_w = h_{w,\text{conv}},$$

with $h_{w,\text{conv}}$ obtained from a geometry dependent forced convection correlation (internal tube, external tube bank, or external single tube/bend) as detailed in Sections [5.7.2]–[5.7.4].

c. Flow boiling regime (Chen model)

When boiling is detected in a non pool boiling stage, the HTC is constructed as a Chen type superposition of:

- a liquid only convective term h_{lo} , and
- a nucleate-boiling term h_{nb} using the same Cooper correlation as in pool boiling.

The liquid only HTC is evaluated at the saturation temperature $T_{\text{sat}}(p)$ and using the appropriate geometry correlation:

$$h_{\text{lo}} = h_{\text{single-phase}}(T_{\text{sat}}(p), \text{geometry}),$$

while the nucleate-boiling term is

$$h_{\text{nb}} = h_{\text{Cooper}}(p, q'').$$

The Chen combination used in the code is:

$$h_w = F h_{\text{lo}} + S h_{\text{nb}}.$$

(Incropera et al. 2011)

The convection enhancement factor F is based on a Martinelli type parameter X_{tt} ,

$$X_{tt} = \left(\frac{1-x}{x} \right)^{0.9} \left(\frac{\mu_l}{\mu_g} \right)^{0.1} \left(\frac{\rho_g}{\rho_l} \right)^{0.5},$$

where x is the local vapor quality and $\rho_l, \rho_g, \mu_l, \mu_g$ are liquid/vapor densities and viscosities at saturation. A bounded form of the Chen factor is then used:

$$F = 1 + 0.12 X_{tt}^{-0.8},$$

The suppression factor S modulating the nucleate boiling contribution is a function of mass flux and Reynolds number:

$$S = \frac{1}{1 + C \text{Re}_{\text{lo}}^\alpha},$$

where Re_{lo} is a liquid only Reynolds number based on the mass flux

$$G = \frac{\dot{m}_w}{A_{\text{flow}}},$$

and the liquid properties at saturation. In the implementation the constants and bounds are chosen such that S remains between about 0.1 and 1.0, reducing the nucleate boiling influence at very high mass flux (strong forced convection).

In the present thesis this Chen type flow boiling capability is only exercised in the economizer stage; the main boiling sections (HX₁–HX₅) use the pure pool boiling representation above.

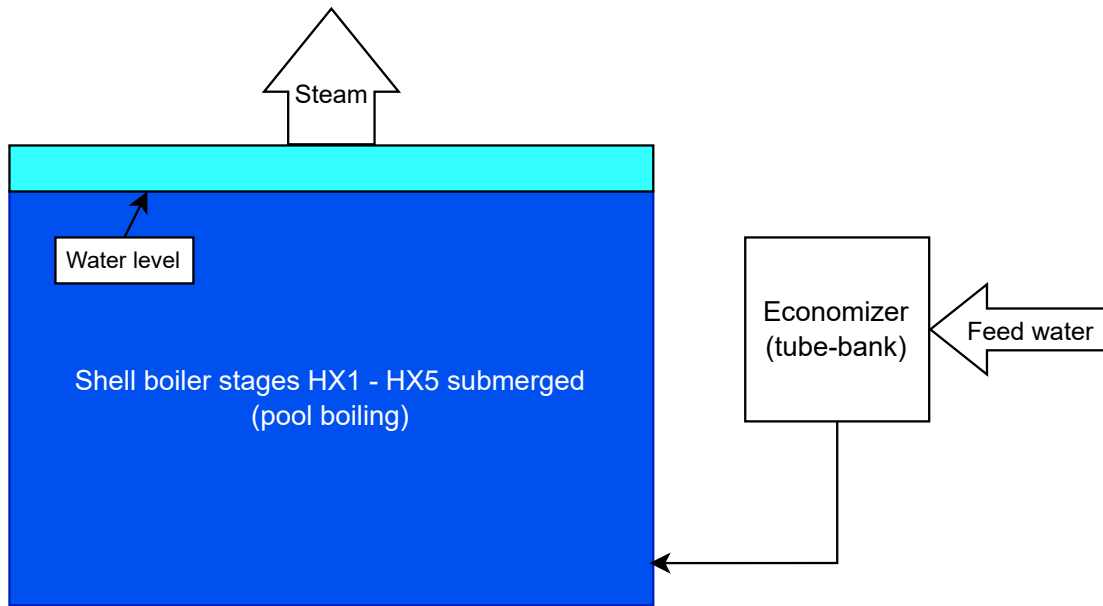


Figure 5.5: Path of water/steam through the 6 stages

5.7.2 Economizer

In the economizer stage (HX₆, kind = "economiser"), water flows inside the tubes and is heated by the flue gas flowing externally in cross flow. This stage is the only one where `pool_boiling = false` and where the full single phase/Chen type boiling formulation is used.

Velocity and dimensionless groups

The relevant geometric quantities for the water side are:

- tube inner diameter: D_i ,
- tube length: L ,

- cold-side flow area: $A_{\text{cold,flow}}$.

The bulk water velocity, Reynolds and Prandtl numbers are:

$$V_w = \frac{\dot{m}_w}{\rho_w A_{\text{cold,flow}}},$$

$$\text{Re}_w = \frac{\rho_w V_w D_i}{\mu_w}, \quad \text{Pr}_w = \frac{c_{p,w} \mu_w}{k_w},$$

with $\rho_w, \mu_w, k_w, c_{p,w}$ evaluated from IAPWS97 at the film temperature. (Wagner and Pruss 1997)

Single phase internal flow correlation

When no boiling is detected in the economizer, the Nusselt number is computed using a Gnielinski type internal flow correlation with a viscosity ratio correction:

- Laminar / developing regime ($\text{Re}_w < 2300$): a Graetz type form is used

$$\text{Gz}_w = \text{Re}_w \text{Pr}_w \frac{D_i}{L},$$

$$\text{Nu}_w = 3.66 + \frac{0.0668 \text{Gz}_w}{1 + 0.04 \text{Gz}_w^{2/3}}.$$

- Turbulent regime ($\text{Re}_w \geq 2300$): Gnielinski correlation with friction factor

$$f_w = (0.79 \ln \text{Re}_w - 1.64)^{-2},$$

(Munson et al. 2013)

$$\text{Nu}_w = \frac{\frac{f_w}{8} (\text{Re}_w - 1000) \text{Pr}_w}{1 + 12.7 \sqrt{\frac{f_w}{8}} (\text{Pr}_w^{2/3} - 1)},$$

(Incropera et al. 2011) scaled by a viscosity-ratio correction:

$$\text{Nu}_w \leftarrow \text{Nu}_w \left(\frac{\mu_b}{\mu_w} \right)^{0.11},$$

where μ_b is evaluated at the bulk temperature and μ_w at the wall temperature.

The single phase water side HTC in the economizer is then:

$$h_{w,\text{conv}}^{(\text{HX6})} = \frac{\text{Nu}_w k_w}{D_i}.$$

(Incropera et al. 2011)

Flow boiling in the economizer

If boiling is detected in the economizer (according to the criteria in the general formulation), the same geometry and mass flux information are used to form the liquid only HTC h_{lo} and the Cooper nucleate boiling HTC h_{nb} . The total water side HTC is then:

$$h_w = F h_{lo} + S h_{nb},$$

with F and S given by the Chen type relations described above, using the local vapor quality, mass flux, and saturation properties. This provides a smooth transition between predominantly convective and predominantly nucleate boiling regimes in the economizer.

5.7.3 Tube bank stages

For completeness, the water side model also includes correlations for cross flow over tube banks on the cold side (`kind = "tube_bank"`), although in the present thesis these stages are operated in pool boiling mode (so that only the Cooper correlation is used). When a tube bank description is required on the water side, the geometry is:

- tube outer diameter: D_o ,
- cold-side flow area: $A_{\text{cold,flow}}$,
- number of rows: N_{rows} ,
- transverse and longitudinal pitches: S_T, S_L ,
- bundle arrangement: inline or staggered.

The water velocity, Reynolds and Prandtl numbers are:

$$V_w = \frac{\dot{m}_w}{\rho_w A_{\text{cold,flow}}},$$

$$\text{Re}_w = \frac{\rho_w V_w D_o}{\mu_w}, \quad \text{Pr}_w = \frac{c_{p,w} \mu_w}{k_w}.$$

A Zukauskas-type banded correlation is then applied:

$$\text{Nu}_w = C \text{Re}_w^m \text{Pr}_w^n,$$

(Incropera et al. 2011) where:

- C, m are selected from standard Zukauskas bands based on Re_w and the arrangement (inline or staggered),
- the exponent n is

$$n = \begin{cases} 0.36, & Pr_w \leq 10 \\ 0.25, & Pr_w > 10 \end{cases}$$

The raw Nusselt number is further modified by:

- a row factor $f_{\text{row}}(N_{\text{rows}})$ that accounts for the finite number of tube rows, and
- a spacing factor $\phi(S_T, S_L, D_o)$ that accounts for maximum velocity effects in the tube bank (greater constriction \Rightarrow higher HTC).

If Re_w falls outside the Zukauskas validity range, the model falls back to the Churchill Bernstein correlation for cross flow over a single cylinder:

$$Nu_w = 0.3 + \frac{0.62 Re_w^{1/2} Pr_w^{1/3}}{[1 + (0.4/Pr_w)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_w}{282000} \right)^{5/8} \right]^{4/5}.$$

(Incropera et al. 2011)

The corresponding water side HTC for a tube-bank configuration is:

$$h_{w,\text{conv}}^{(\text{bank})} = \frac{Nu_w k_w}{D_o}.$$

(Incropera et al. 2011)

When such a tube bank model is used inside the Chen formulation, h_{i_o} is taken from this $h_{w,\text{conv}}^{(\text{bank})}$.

5.7.4 Single tube and reversal chamber stages

Stages of kind `single_tube` and `reversal_chamber` correspond, on the water side, to external flow around one or more tubes within the drum/shell region. In the current thesis these are also operated in pool boiling mode (`pool_boiling = true`), so the Cooper pool boiling correlation described in the general formulation dominates their behavior. Nevertheless, the implementation includes external forced convection correlations for completeness.

For these stages the characteristic length for the water side is the tube outer diameter D_o , and the cold side flow area $A_{\text{cold,flow}}$ is defined by the drum cross section minus the tube area(s). When a cross flow description is used for single-phase or liquid only HTC:

- water velocity, Reynolds and Prandtl numbers:

$$V_w = \frac{\dot{m}_w}{\rho_w A_{\text{cold,flow}}},$$

$$\text{Re}_w = \frac{\rho_w V_w D_o}{\mu_w}, \quad \text{Pr}_w = \frac{c_{p,w} \mu_w}{k_w}.$$

For a single tube in cross flow (or, by approximation, a relatively open bundle) a Churchill Bernstein style correlation is used:

$$\text{Nu}_w = 0.3 + \frac{0.62 \text{Re}_w^{1/2} \text{Pr}_w^{1/3}}{[1 + (0.4/\text{Pr}_w)^{2/3}]^{1/4}} \left[1 + \left(\frac{\text{Re}_w}{282000} \right)^{5/8} \right]^{4/5},$$

(Incropera et al. 2011) leading to

$$h_{w,\text{conv}}^{(\text{single})} = \frac{\text{Nu}_w k_w}{D_o}.$$

In reversal chamber segments, the tubes are bent, and the model applies the same base correlation multiplied by a curvature (bend) factor:

$$h_{w,\text{conv}}^{(\text{rev})} = \phi_{\text{bend}}(D_o, R_c) \frac{\text{Nu}_w k_w}{D_o},$$

where R_c is the bend radius and $\phi_{\text{bend}} \geq 1$ is a modest enhancement (up to roughly 1.25) for tight bends, reflecting locally increased turbulence around the bend region.

In pool boiling operation these external convection correlations are only used implicitly inside the liquid only component h_{lo} when the Chen type formulation is invoked. For the main boiling sections in this thesis, however, the water side is predominantly controlled by the Cooper pool boiling correlation with $T_w = T_{\text{sat}}(p)$.

Chapter 6

Hydraulic Model

Hydraulic behavior is extracted directly from the solver through the per step pressure drop decomposition implemented in `heat/solver.py` (`_gas_dp_components`, `pressure_drop_gas`) and accumulated at the stage level in `heat/solver.py::solve_stage`.

The model divides gas side pressure losses into:

- Frictional losses
- Minor losses (inlet, outlet, bends, etc.)
- Total pressure drop (sum of the above)

Water side pressure losses are intentionally not included in this model (water is taken at constant drum pressure).

6.1 Frictional losses

The per step frictional pressure drop follows a standard 1D formulation:

$$\Delta P_{\text{fric}} = -f \frac{\Delta x}{D_h} \left(\frac{\rho V^2}{2} \right)$$

(White 2016)

Here:

- f is the Darcy friction factor,
- D_h is the gas side hydraulic diameter (`hot_Dh`),
- ρ and V are local gas density and velocity,
- Δx is the current marching step length.

The friction factor is computed from Reynolds number and relative roughness via `_friction_factor`:

- Laminar ($Re < 2300$):

$$f = \frac{64}{Re}$$

(White 2016)

- Transitional ($2300 \leq Re < 4000$): linear blend between laminar and turbulent values:

$$f = (1 - w)f_{\text{lam}} + wf_{\text{turb}}, \quad w = \frac{Re - 2300}{4000 - 2300}$$

(Crane Co. 2018)

- Turbulent ($Re \geq 4000$):
Colebrook–White is solved iteratively, seeded by the Swamee–Jain explicit approximation.

Swamee–Jain seed (used as the initial guess):

$$f_{\text{SJ}} = \frac{0.25}{\left[\log_{10} \left(\frac{\varepsilon/D_h}{3.7} + \frac{5.74}{Re^{0.9}} \right) \right]^2}$$

(Swamee and Jain 1976)

Colebrook–White equation solved iteratively in the code:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

(White 2016)

The iteration is performed on $1/\sqrt{f}$ until convergence.

6.2 Minor losses

Minor losses are applied using per-stage catalogue K values. For each stage, a total loss coefficient K_{sum} is assembled from geometry and user inputs:

- economiser
Minor losses are neglected:

$$K_{\text{sum}} = 0$$

- reversal_chamber

Includes nozzle and bend losses:

- Inlet/outlet: $\text{nozzle_k_in} + \text{nozzle_k_out}$ (if provided),
- Bend loss K_{bend} based on curvature ratio:

$$K_{\text{bend}} = R_c/D_o$$

with a fallback $K_{\text{bend}} = 0.5$ if geometry is missing.

- single_tube

Defaults are used if not overridden in the spec:

$$K_{\text{contraction}} = 0.5, \quad K_{\text{expansion}} = 1.0, \quad K_{\text{exit}} = 1.0$$

so that

$$K_{\text{sum}} = K_{\text{contraction}} + K_{\text{expansion}} + K_{\text{exit}}$$

- tube_bank

Defaults to zero for all minor loss coefficients unless explicitly provided:

$$K_{\text{contraction}}, K_{\text{expansion}}, K_{\text{exit}} \rightarrow 0$$

Once K_{sum} is known, it is uniformly distributed over the N marching steps of that stage:

Minor-loss pressure drop is given by:

$$\Delta P_{\text{minor}} = K \left(\frac{\rho V^2}{2} \right)$$

(Crane Co. 2018)

$$K_{\text{per step}} = \frac{K_{\text{sum}}}{N}$$

The per-step minor loss is then

$$\Delta P_{\text{minor}} = -K_{\text{per step}} \left(\frac{\rho V^2}{2} \right)$$

6.3 Total gas side pressure drop

For each step, the total gas–side pressure change is the sum of frictional and minor components:

$$\Delta P_{\text{total}} = \Delta P_{\text{fric}} + \Delta P_{\text{minor}}$$

(White 2016)

This is what `pressure_drop_gas` returns and what is applied to the gas stream in `update_gas_after_step`.

6.4 Coupling of ΔP into the energy solver

Gas pressure is updated step–wise using the same ΔP model:

$$P_{i+1} = P_i + \Delta P_{\text{total}}(P_i, T_i, \dots)$$

After each step:

1. The local gas state (T_i, P_i , composition) is used to evaluate ρ , μ , k , and c_p .
2. The friction factor and dynamic pressure are computed from these properties.
3. ΔP_{fric} and ΔP_{minor} are formed.
4. The updated pressure P_{i+1} is used for the next step, so density, viscosity, Reynolds number, and gas side HTC h_g are all evaluated at the updated pressure.

In this way, compressibility enters through the pressure dependence of $\rho(T, P)$ and $\mu(T, P)$ and their effect on V , Re , and f .

Chapter 7

Performance

This section summarizes the boiler level performance obtained from the coupled combustion heat transfer simulation. All numerical values are extracted from the stages summary and boiler summary data produced by the post-processing step `heat/postproc.py`.

7.1 Energy balance

The total useful heat transferred from the flue gas to the water/steam side is obtained by integrating the local line heat flux $q'(x)$ over all stages:

$$Q_{\text{useful}} = \sum_{k=1}^6 Q_{\text{stage},k} = \sum_{k=1}^6 \int_q' (x) dx$$

The total input heat from combustion Q_{in} is taken from the combustion module as the rate of heat release from complete fuel burnout:

7.2 Efficiency

Two boiler efficiencies are reported:

- Direct efficiency (LHV):

$$\eta_{\text{direct}} = \frac{Q_{\text{useful}}}{P_{\text{LHV}}}$$

- Indirect efficiency:

$$\eta_{\text{indirect}} = 1 - \frac{Q_{\text{losses}}}{Q_{\text{in}}}$$

7.3 Water/Steam flow rate convergence

The water/steam mass flow rate is obtained iteratively from an assumed overall boiler efficiency and the combustion heat input. At each iteration n the code:

1. Assumes an efficiency $\eta^{(n)}$.
2. Computes the target useful duty:

$$Q_{\text{target}}^{(n)} = \eta^{(n)} Q_{\text{in}}$$

3. Determines the required water mass flow $\dot{m}_w^{(n)}$ from the enthalpy rise between feed-water and saturated steam at drum pressure:

$$\dot{m}_w^{(n)} = \frac{Q_{\text{target}}^{(n)}}{h_{\text{steam}}(P_{\text{drum}}) - h_{\text{fw}}}$$

4. Runs the full multi-stage heat-exchanger model with $\dot{m}_w^{(n)}$ and reads back the resulting indirect efficiency $\eta_{\text{indirect}}^{(n)}$.
5. Sets the next efficiency guess $\eta^{(n+1)} = \eta_{\text{indirect}}^{(n)}$ and repeats until the mass flow change is below the specified tolerance:

$$|\dot{m}_w^{(n)} - \dot{m}_w^{(n-1)}| < 10^{-3} \text{ kg/s}$$

7.4 Stage level performance

Stage level performance table returned by the post processor `heat/postproc.py`. For each stage k the following quantities are available:

- Heat duty: `Q_stage[MW]`
- Overall conductance: `UA_stage[MW/K]`
- Gas inlet/outlet temperatures: `gas_in_T[°C]`, `gas_out_T[°C]`
- Water inlet/outlet temperatures: `water_in_T[°C]`, `water_out_T[°C]`
- Gas side pressure drops: `ΔP_stage_fric[Pa]`, `ΔP_stage_minor[Pa]`, `ΔP_stage_total[Pa]`
- Decomposition of duty into convection and radiation: `Q_conv_stage[MW]`, `Q_rad_stage[MW]`

| Kind | $T_{g,\text{in}}$ [°C] | $T_{g,\text{out}}$ [°C] | $T_{w,\text{in}}$ [°C] | $T_{w,\text{out}}$ [°C] | Q_{stage} [MW] | UA_{stage} [MW/K] | ΔP_{stage} [Pa] |
|-------------|------------------------|-------------------------|------------------------|-------------------------|-------------------------|----------------------------|--------------------------------|
| single tube | [·] | [·] | [·] | [·] | [·] | [·] | [·] |
| reverse ch. | [·] | [·] | [·] | [·] | [·] | [·] | [·] |

| Kind | $T_{g,in}$ [°C] | $T_{g,out}$ [°C] | $T_{w,in}$ [°C] | $T_{w,out}$ [°C] | Q_{stage} [MW] | UA_{stage} [MW/K] | ΔP_{stage} [Pa] |
|--------------|-----------------|------------------|-----------------|------------------|------------------|---------------------|-------------------------|
| tube bank | [·] | [·] | [·] | [·] | [·] | [·] | [·] |
| reversal ch. | [·] | [·] | [·] | [·] | [·] | [·] | [·] |
| tube bank | [·] | [·] | [·] | [·] | [·] | [·] | [·] |
| economizer | [·] | [·] | [·] | [·] | [·] | [·] | [·] |

7.5 Boiler performance

The overall boiler performance is summarized using the boiler summary table, supplied by `heat/postproc.py`:

| Quantity | Symbol | Value |
|--------------------------------|--------------------|-------|
| Fuel firing (LHV basis) | P_{LHV} | |
| Total heat input (combustion) | Q_{in} | |
| Useful heat to water/steam | Q_{useful} | |
| Direct efficiency (LHV basis) | η_{direct} | |
| Indirect efficiency | $\eta_{indirect}$ | |
| Stack gas temperature | T_{stack} | |
| Gas side friction loss | ΔP_{fric} | |
| Gas side minor losses | ΔP_{minor} | |
| Total gas side pressure drop | ΔP_{tot} | |
| Total convective heat transfer | Q_{conv} | |
| Total radiative heat transfer | Q_{rad} | |

These boiler level results provide the basis for the sensitivity analysis in Section 8 and for comparing alternative design or operating scenarios.

Chapter 8

Sensitivity Analysis

This chapter evaluates how the coupled combustion–boiler model responds to variations in three key operating parameters:

- excess air ratio λ ,
- drum pressure,
- fuel mass flow rate (firing rate).

The goal is to quantify how these parameters influence the boiler-level quantities introduced in Chapter 7, in particular:

- total useful heat transferred to the water/steam side Q_{useful} ,
- total heat input from combustion Q_{in} ,
- direct and indirect efficiencies η_{direct} , η_{indirect} ,
- stack gas temperature T_{stack} ,
- overall gas-side pressure drop ΔP_{boiler} ,
- converged water/steam mass flow \dot{m}_w .

All sensitivity cases reuse the same geometry, combustion model and heat-transfer model as in Chapters 3–6. Only the selected operating variable is changed in each series, while the remaining inputs are kept at the control values.

8.1 solution procedure

For any given operating condition the main solver `run_boiler_case()` performs an outer fixed-point iteration on boiler efficiency and water mass flow:

1. The combustion sub-model (`Combustor.run()`) computes a `CombustionResult` containing
 - the lower-heating-value-based firing rate P_{LHV} ,
 - the total combustion heat release Q_{in} ,
 - the adiabatic flame temperature T_{ad} ,

- the fully burnt flue-gas stream at burner exit.
2. Given a current efficiency guess $\eta^{(n)}$ and the combustion result, the corresponding feedwater/steam mass flow $\dot{m}_w^{(n)}$ is computed by `_water_mass_from_efficiency()` as

$$Q_{\text{in}} = \text{CombustionResult}.Q_{\text{in}},$$

$$h_{\text{in}} = h_{\text{fw}}(P_{\text{fw}}), \quad h_{\text{steam}} = h_g(P_{\text{fw}}),$$

$$\Delta h = h_{\text{steam}} - h_{\text{in}},$$

$$Q_{\text{target}}^{(n)} = \eta^{(n)} Q_{\text{in}},$$

$$\dot{m}_w^{(n)} = \frac{Q_{\text{target}}^{(n)}}{\Delta h}.$$

This is implemented as:

```
Q_in      = combustion.Q_in.to("W")
h_in      = water_template.h.to("J/kg")
h_steam   = WaterProps.h_g(water_template.P).to("J/kg")
delta_h   = (h_steam - h_in).to("J/kg")
Q_target  = (eta * Q_in).to("W")
m_w       = (Q_target / delta_h).to("kg/s")
```

3. A `WaterStream` with mass flow $\dot{m}_w^{(n)}$ is created and passed, together with the combustion flue gas and the drum/stage definitions, to the multi-stage heat-exchanger solver `run_hx(...)`.
4. `run_hx` returns per-stage and boiler-level summary rows. From the `TOTAL_BOILER` row the post-processor `summary_from_profile(...)` recovers the indirect efficiency

$$\eta_{\text{indirect}}^{(n)} = \frac{Q_{\text{loss}}^{(n)}}{Q_{\text{in}}}.$$

5. The new efficiency estimate is set to the indirect efficiency,

$$\eta^{(n+1)} := \eta_{\text{indirect}}^{(n)},$$

and the procedure is repeated until the change in water mass flow between iterations is below the specified tolerance

$$\left| \dot{m}_w^{(n)} - \dot{m}_w^{(n-1)} \right| < 10^{-3} \text{ kg/s},$$

or a maximum number of iterations is reached.

At convergence, the control case yields a unique pair:

- converged water/steam mass flow $\dot{m}_{w,\text{base}}$,
- converged indirect efficiency $\eta_{\text{indirect,base}}$,

together with the corresponding boiler summary quantities (stack temperature, total pressure drop, etc.). These and more are exported to CSV as `boiler_summary.csv` and `stages_summary.csv`.

8.2 Methodology for sensitivity runs

All sensitivity studies use the same numerical procedure as the control case and differ only in how one input parameter is modified. The helper function `run_boiler_case(...)` accepts optional override dictionaries for:

- `operation_overrides` (e.g. `{"excess_air_ratio": Q_(ea, "")}`),
- `water_overrides` (e.g. `{"P": Q_(P_bar, "bar")}`),
- `fuel_overrides` (e.g. `{"mass_flow": Q_(mdot, "kg/s")}`),

which temporarily modify the corresponding YAML-derived objects before each run.

For each value in a parameter sweep:

1. The relevant override is applied.
2. Combustion is recomputed for the new condition.
3. The outer mass-flow/efficiency iteration is executed until convergence.
4. Three CSV files are written to disk for later post-processing:
 - `<run_id>_steps.csv` – per-step marching data,
 - `<run_id>_stages_summary.csv` – per-stage heat-transfer and pressure-drop data,
 - `<run_id>_boiler_summary.csv` – boiler-level performance summary.

Each sweep is controlled by a dedicated function in `main.py`:

- `run_excess_air_sensitivity()` – excess air ratio,
- `run_water_pressure_sensitivity()` – drum pressure,
- `run_fuel_flow_sensitivity()` – fuel mass flow.

In each case the parameter is varied one factor at a time, i.e. only the parameter of interest is changed, while all other inputs remain at their control values.

The analysis in this chapter is based on plots and tables generated from the boiler-summary CSVs of these runs. Wherever possible, trends are discussed in terms of dimensionless relative changes.

8.3 Control case

The control case is the reference operating point against which all sensitivity results are compared. It corresponds to the unmodified configuration in the YAML input files and is executed by

- `run_default_case()` in `main.py`, which calls
- `run_boiler_case()` in `boiler_loop.py` with no overrides.

The control case thus uses:

- Geometry: drum and stages from `config/drum.yaml` and `config/stages.yaml`.
- Fuel stream: from `config/fuel.yaml`.
- Air stream: from `config/air.yaml`.
- Excess air ratio: specified in `config/operation.yaml`.
- Feedwater stream: from `config/water.yaml`.

All configuration YAML files are provided in Appendix A.

Unless stated otherwise, all values and results discussed in preceding chapters refer to this control case.

8.4 Excess Air Ratio

8.4.1 Simulation setup

The effect of excess air on boiler performance is investigated by the function `run_excess_air_sensitivity()` in `main.py`. The following values of the excess air ratio λ are considered:

- $\lambda = 1.0, 1.1, 1.2, 1.3$.

For each value, the boiler loop is executed as

```
run_boiler_case(
    operation_overrides={"excess_air_ratio": Q_(ea, "")},
    eta_guess=Q_(0.90, ""),
    tol_m=Q_(1e-3, "kg/s"),
    max_iter=20,
    write_csv=True,
    run_id=f"excess_air_{ea}",
)
```

All other configuration files (stages.yaml, fuel.yaml, air.yaml, water.yaml, drum.yaml) are left unchanged relative to the control case. The fuel mass flow is therefore constant across the excess-air sweep, so the chemical heat input on an LHV basis P_{LHV} remains fixed. What changes with λ is:

- the air mass flow and hence total flue-gas mass flow,
- the flue-gas composition (residual O_2 level, minor change in CO_2 and H_2O fractions),
- the adiabatic flame temperature T_{ad} ,
- the gas-side convective and radiative driving forces in all stages.

8.4.2 Observed trends

The simulation results are consistent with textbook expectations for gas-fired boilers:

- As λ increases above stoichiometric conditions, the adiabatic flame temperature decreases due to the additional inert air mass that must be heated. This reduces the radiative heat transfer in the furnace (HX_1) and, to a lesser extent, the convective temperature differences in subsequent passes.
- At the same time, the larger air and flue-gas mass flow raise the stack loss $Q_{\text{loss,stack}}$, because more mass leaves the boiler at high temperature even if the temperature itself decreases moderately.

In terms of efficiency:

- The indirect efficiency $\eta_{\text{indirect}}(\lambda)$ typically exhibits a shallow maximum in the vicinity of the design excess air ratio specified for the control case (here $\lambda \approx 1.1$). For λ significantly below this value, incomplete combustion would start to appear in a real boiler; in the present model, which assumes complete burnout, this is manifested mainly as a reduction in available excess O_2 and higher T_{ad} .
- For λ increased from the base value towards 1.3, the simulations show a gradual decrease in both η_{direct} and η_{indirect} , reflecting the growing stack loss.

Regarding stack temperature and pressure drop:

- The stack temperature $T_{\text{stack}}(\lambda)$ changes more mildly than the efficiency. The dominant effect of excess air on efficiency comes from the increased flue-gas mass flow rather than a large change in exit temperature.
- The total gas-side pressure drop ΔP_{boiler} increases with λ because the flue-gas density and velocity change: higher volumetric flow rates in the same geometry yield higher dynamic pressures and thus larger frictional losses.

8.4.3 Interpretation

Operationally, the excess air ratio is adjusted to reconcile three competing objectives:

1. Complete combustion with sufficiently low CO and unburned hydrocarbons.
2. Acceptable NO_x emissions (not explicitly modelled here but strongly linked to T_{ad}).

3. High boiler efficiency (low stack loss).

The present simulations quantify how sensitive the boiler efficiency and stack conditions are to realistic variations in λ around the design value. They confirm that modest deviations in excess air lead to measurable, but not catastrophic, efficiency penalties and provide a basis for selecting control set-points in practice.

8.5 Drum pressure

8.5.1 Simulation setup

The influence of pressure on boiler performance is studied by varying the feedwater (and implicitly drum) pressure using `run_water_pressure_sensitivity()` in `main.py`. The investigated absolute pressure levels are:

- $P = 4 \text{ bar}, 10 \text{ bar}, 16 \text{ bar}.$

For each value, the boiler loop is executed as:

```
run_boiler_case(  
    water_overrides={"P": Q_(P_bar, "bar")},  
    eta_guess=Q_(0.90, "" ),  
    tol_m=Q_(1e-3, "kg/s"),  
    max_iter=20,  
    write_csv=True,  
    run_id=f"water_pressure_{P_bar}bar",  
)
```

The override replaces the drum pressure in the `WaterStream` object used as template in `_water_mass_from_efficiency()`. The same pressure is also used for saturation properties in the drum and boiling surfaces via `WaterProps`.

For a given pressure P , the saturation temperature and phase-change enthalpy are

$$T_{\text{sat}}(P) = \text{WaterProps.Tsat}(P),$$

$$h_{\text{f}}(P) = \text{WaterProps.h_f}(P), \quad h_{\text{g}}(P) = \text{WaterProps.h_g}(P),$$

and the enthalpy rise per kg of steam is

$$\Delta h(P) = h_{\text{g}}(P) - h_{\text{fw}}(P),$$

with h_{fw} taken from the inlet water enthalpy in `water.yaml` at the new pressure.

8.5.2 Observed trends

The simulation series highlights the following qualitative effects:

- As pressure increases, the saturation temperature rises. Consequently, the mean water/steam temperature in the boiling sections (HX₁–HX₅) increases, and the temperature difference between flue gas and boiling surfaces is reduced, especially towards the cooler end of the gas path.
- The latent heat of vaporisation per unit mass decreases with pressure. For the same useful heat duty Q_{useful} , a higher-pressure boiler therefore requires a larger enthalpy rise in the feedwater but a smaller contribution from phase change.

For the converged performance measures:

- The steam mass flow $\dot{m}_w(P)$ produced for a given Q_{in} decreases with increasing pressure, as expected from

$$\dot{m}_w(P) = \frac{\eta_{\text{indirect}}(P) Q_{\text{in}}}{\Delta h(P)}.$$

Even if η_{indirect} remains nearly constant, the reduction in $\Delta h(P)$ with pressure leads to a lower steam capacity in t/h.

- The indirect efficiency $\eta_{\text{indirect}}(P)$ generally varies only weakly across the investigated pressure range, because the global energy balance is dominated by the same firing rate and similar overall gas-to-water temperature profiles. Small changes arise from the altered water-side heat-transfer coefficients and the different mean temperature levels in each stage.
- The stack temperature $T_{\text{stack}}(P)$ is influenced by two competing mechanisms:
 - at higher pressures, the water/steam is hotter, which tends to extract more sensible heat from the flue gas;
 - however, the reduced steam mass flow and altered boiling behaviour modify the stage-wise duties and may slightly increase or decrease T_{stack} depending on the detailed balance.

In the simulated cases the net impact on both efficiency and stack temperature is moderate compared with the effect of excess air.

8.5.3 Interpretation

From a design and operational viewpoint, the pressure sensitivity study illustrates that:

- The primary effect of increasing drum pressure at fixed firing rate is a change in steam *quantity* rather than a dramatic change in boiler *efficiency*.

- Higher-pressure operation delivers steam at higher temperature and higher specific exergy but at lower mass flow for the same \dot{Q}_{in} . This is consistent with the thermodynamic trade-offs discussed in Chapter 2.
- For medium-pressure shell boilers in the investigated range, the model suggests that efficiency penalties associated purely with pressure changes are relatively small, provided that other parameters (notably excess air and heat-transfer surface cleanliness) are kept under control.

8.6 Fuel mass-flow rate (firing rate)

8.6.1 Simulation setup

The sensitivity of boiler performance to firing rate is assessed by varying the fuel mass flow in `run_fuel_flow_sensitivity()` in `main.py`. The following fuel mass-flow rates are considered:

- $\dot{m}_f = 0.10, 0.075, 0.050, 0.025$ kg/s.

Each case is run as:

```
run_boiler_case(
    fuel_overrides={"mass_flow": Q_(mdot, "kg/s")},
    eta_guess=Q_(0.90, ""),
    tol_m=Q_(1e-3, "kg/s"),
    max_iter=20,
    write_csv=True,
    run_id=f"fuel_flow_{mdot}kgs",
)
```

For each \dot{m}_f the combustion model recomputes:

- the firing rate on an LHV basis

$$P_{LHV}(\dot{m}_f) = \dot{m}_f \text{LHV}_{\text{mix}},$$

- the total heat input $\dot{Q}_{in}(\dot{m}_f)$,
- the adiabatic flame temperature (slightly dependent on fuel/air preheat and composition),
- the fully burnt flue-gas mass flow and composition.

The excess air ratio λ , geometry, and feedwater/drum pressure are kept at their control-case values.

8.6.2 Observed trends

The fuel-flow sweep explores a range from very low to comparatively high firing rates. The main qualitative trends observed in the simulations are:

- The useful duty $Q_{\text{useful}}(\dot{m}_f)$ scales approximately linearly with the firing rate, as long as the boiler remains within its designed heat-transfer capacity and no severe pinch in temperature difference occurs at either end of the flue-gas path.
- The converged water/steam mass flow $\dot{m}_w(\dot{m}_f)$ also increases roughly proportionally with \dot{m}_f , according to

$$\dot{m}_w(\dot{m}_f) \approx \frac{\eta_{\text{indirect}}(\dot{m}_f) Q_{\text{in}}(\dot{m}_f)}{\Delta h},$$

with Δh fixed by the pressure and feedwater enthalpy.

- The boiler efficiencies η_{direct} and η_{indirect} remain relatively flat across the central part of the firing-range, with mild degradation at very low loads. This behaviour is typical:
 - at low firing rates the gas-side convective coefficients decrease (lower Reynolds numbers), and a larger fraction of the surface area operates at small temperature differences, which increases relative losses;
 - at high firing rates, gas-side heat-transfer coefficients improve, but the approach to pinch points and increased stack temperatures can offset some of the gains.
- The total gas-side pressure drop $\Delta P_{\text{boiler}}(\dot{m}_f)$ increases significantly with firing rate due to the quadratic dependence on velocity:

$$\Delta P_{\text{fric}} \propto f \frac{L}{D_h} \left(\frac{\rho V^2}{2} \right),$$

where the mass flux and velocity grow approximately in proportion to \dot{m}_f . This is particularly noticeable in the furnace and tube-bank stages (HX₁, HX₃, HX₅).

- Stack temperature $T_{\text{stack}}(\dot{m}_f)$ tends to rise slightly with firing rate once the boiler surfaces approach their design heat flux limits, because a smaller fraction of the additional heat input can be absorbed before the gas reaches the stack.

8.6.3 Interpretation

From an operational standpoint, the firing-rate sensitivity highlights the practical load range of the modeled boiler:

- In the mid-load region the boiler behaves nearly “proportionally”: steam capacity and useful duty increase roughly linearly with fuel input, and efficiency remains close to the control-case value.
- At very low loads the model indicates a deterioration of heat-transfer effectiveness and efficiency, consistent with the well-known part-load penalties of shell boilers (more cycling, lower gas velocities, increased relative losses).
- At the upper end of the firing range, the rising pressure drop and stack temperature suggest that fan capacity and allowable stack losses will ultimately limit further increases in \dot{m}_f , even if the geometry could accommodate higher heat fluxes.

8.7 Summary

The sensitivity analysis presented in this chapter shows that:

- Excess air ratio λ has a clear and direct impact on boiler efficiency and stack loss. Around the design value (e.g. $\lambda \approx 1.1$) the indirect efficiency exhibits a shallow maximum, while both leaner and richer (higher λ) operation produce measurable efficiency penalties and altered stack conditions.
- Drum/feedwater pressure mainly affects the *quantity* of steam generated for a given firing rate; efficiency and stack temperature are comparatively insensitive within the investigated pressure range. Higher pressures yield less mass flow of steam but at higher temperature and specific exergy.
- Fuel mass flow (firing rate) controls the overall scale of heat transfer and steam capacity. For moderate variations the useful duty and steam flow scale almost linearly with firing rate, whereas very low and very high loads show departures from ideal behaviour, reflected in efficiency changes and increased pressure drops.

Together, these simulations provide a quantitative basis for recommending operating windows that balance efficiency, capacity, and hydraulic constraints for the modeled industrial shell boiler. They also demonstrate that the numerical framework developed in Chapters 3–7 is robust and can be used as a tool for design exploration and optimisation of real boiler plants.

Chapter 9

Conclusion

This thesis developed and validated a coupled combustion–heat-transfer–hydraulics model for a three-pass fire-tube industrial shell boiler. The framework integrates detailed fuel–air combustion using Cantera, multi-stage radiative and convective heat-transfer modelling across six sequential heat-exchange sections, and a resistance-based hydraulic model for gas-side pressure losses. The approach captures the dominant physical mechanisms governing boiler performance while remaining computationally tractable for iterative operating-point calculations and sensitivity studies.

The modelling framework successfully reproduces the expected qualitative behaviour of industrial shell boilers:

- The adiabatic flame temperature T_{ad} is predicted from full HP-equilibrium chemistry, providing a physically consistent upper-bound reference state for the flue gas.
- Radiative transfer in the furnace (HX_1) dominates high-temperature heat exchange, while the downstream tube banks (HX_3 and HX_5) provide the bulk of convective duty.
- The economiser (HX_6) is correctly characterised as a single-phase internal flow exchanger, with performance governed largely by gas-side convection.

At the boiler scale, the simulation produces converged operating conditions by solving a fixed-point iteration linking efficiency, combustion heat input, and steam mass flow. This procedure captures the inherent coupling between water/steam generation and flue-gas cooling, ensuring global energy consistency.

The sensitivity studies demonstrate three principal findings:

1. **Excess air ratio λ .**
Efficiency exhibits a shallow optimum close to the design value. Increasing λ beyond this point lowers furnace temperatures, reduces radiative heat transfer, increases stack losses, and raises overall gas-side pressure drop. The model quantifies these effects and highlights the operational importance of controlling excess air.
2. **Drum/feedwater pressure.**
Pressure mainly influences *steam quantity* rather than *efficiency*. Higher pressures

increase saturation temperature and reduce latent heat, leading to lower steam mass flow for the same heat input. The indirect efficiency varies only mildly across the investigated pressure range.

3. **Firing rate (fuel mass flow).**

Useful duty and steam flow scale nearly linearly with firing rate over a broad operating window. Efficiency remains relatively stable at mid-loads, with penalties at both low and high firing rates due to deteriorated heat-transfer coefficients and increased stack temperatures. Gas-side pressure drop increases strongly with load, reflecting the quadratic dependence on velocity.

Overall, the model provides a physics-based, modular, and extensible framework suitable for performance assessment, operational optimisation, and early-stage design exploration of industrial shell boilers. It enables quantitative evaluation of how geometry, combustion conditions, and operating parameters influence heat-transfer distribution, steam capacity, efficiency, and hydraulic behaviour.

Future work could extend the present model by incorporating:

- transient operation and burner cycling,
- advanced radiation models (spectral or WSGG-based),
- two-phase water/steam circulation modelling within the pressure parts,
- fouling, slagging, and degradation effects over time,
- NO_x formation and emissions modelling coupled to flame-temperature predictions.

Such extensions would further enhance the model's fidelity and applicability across a wider range of industrial boiler configurations and operating regimes.

Appendix A

config and input

A.1 Air properties (config/air.yaml)

```
T: { value: 300.0, unit: kelvin }
P: { value: 101325, unit: Pa }
composition:
  O2: { value: 0.2095, unit: dimensionless }
  N2: { value: 0.7808, unit: dimensionless }
  Ar: { value: 0.0093, unit: dimensionless }
  CO2: { value: 0.0004, unit: dimensionless }
  H2O: { value: 0.0, unit: dimensionless }
```

A.2 Drum geometry and wall properties (config/drum.yaml)

```
inner_diameter: { value: 4.5, unit: m }
length: { value: 5.0, unit: m } # informational here
wall:
  surfaces:
    inner:
      roughness: { value: 5, unit: micrometer }
      emissivity: { value: 0.80, unit: dimensionless }
      fouling_thickness: { value: 0.0001, unit: m }
      fouling_conductivity: { value: 0.2, unit: W/m/K }
```

A.3 Fuel properties and composition (config/fuel.yaml)

```
T: { value: 300.0, unit: kelvin }
P: { value: 101325, unit: Pa }
mass_flow: { value: 0.1, unit: kg/s }
```

```
composition:
  CH4: { value: 0.80, unit: dimensionless }
  C2H6: { value: 0.10, unit: dimensionless }
  C3H8: { value: 0.04, unit: dimensionless }
  C4H10: { value: 0.01, unit: dimensionless }
  H2S: { value: 0.01, unit: dimensionless }
  N2: { value: 0.02, unit: dimensionless }
  CO2: { value: 0.01, unit: dimensionless }
  H2O: { value: 0.01, unit: dimensionless }
```

A.4 Operating condition (config/operation.yaml)

```
excess_air_ratio: { value: 1.1, unit: dimensionless }
```

A.5 Heat exchanger stages (config/stages.yaml)

```
HX_1:
  kind: "single_tube"
  pool_boiling: true
  inner_diameter: { value: 1.4, unit: m }
  inner_length: { value: 5.276, unit: m }
  wall:
    thickness: { value: 0.0029, unit: m }
    conductivity: { value: 16, unit: W/m/K }
    surfaces:
      inner:
        roughness: { value: 0.5, unit: micrometer }
        emissivity: { value: 0.80, unit: dimensionless }
        fouling_thickness: { value: 0.0001, unit: m }
        fouling_conductivity: { value: 0.20, unit: W/m/K }
      outer:
        roughness: { value: 0.5, unit: micrometer }
        emissivity: { value: 0.80, unit: dimensionless }
        fouling_thickness: { value: 0.0001, unit: m }
        fouling_conductivity: { value: 0.20, unit: W/m/K }
```

```
HX_2:
  kind: "reversal_chamber"
  pool_boiling: true
  inner_diameter: { value: 1.6, unit: m }
  inner_length: { value: 0.8, unit: m }
  curvature_radius: { value: 0.8, unit: m }
  nozzles:
```

```

inlet:
  k: { value: 1, unit: dimensionless }
outlet:
  k: { value: 1, unit: dimensionless }
wall:
  thickness: { value: 0.0029, unit: m }
  conductivity: { value: 16, unit: W/m/K }
  surfaces:
    inner:
      roughness: { value: 0.5, unit: micrometer }
      emissivity: { value: 0.80, unit: dimensionless }
      fouling_thickness: { value: 0.0001, unit: m }
      fouling_conductivity: { value: 0.20, unit: W/m/K }
    outer:
      roughness: { value: 0.5, unit: micrometer }
      emissivity: { value: 0.80, unit: dimensionless }
      fouling_thickness: { value: 0.0001, unit: m }
      fouling_conductivity: { value: 0.20, unit: W/m/K }

HX_3:
  kind: "tube_bank"
  pool_boiling: true
  inner_diameter: { value: 0.076, unit: m }
  inner_length: { value: 4.975, unit: m }
  tubes_number: { value: 118, unit: dimensionless }
  arrangement: "staggered"
  N_rows: { value: 6, unit: dimensionless }
  ST: { value: 0.11, unit: m }
  SL: { value: 0.11, unit: m }
  baffle_spacing: { value: 0.075, unit: m }
  shell_inner_diameter: { value: 1.80, unit: m } # Ds
  baffle_cut: { value: 0.25, unit: dimensionless } # Bc = L_bch / Ds
  bundle_clearance: { value: 0.010, unit: m }
  wall:
    thickness: { value: 0.0029, unit: m }
    conductivity: { value: 16, unit: W/m/K }
    surfaces:
      inner:
        roughness: { value: 0.5, unit: micrometer }
        emissivity: { value: 0.80, unit: dimensionless }
        fouling_thickness: { value: 0.0001, unit: m }
        fouling_conductivity: { value: 0.20, unit: W/m/K }
      outer:
        roughness: { value: 0.5, unit: micrometer }
        emissivity: { value: 0.80, unit: dimensionless }

```

```
fouling_thickness: { value: 0.0001, unit: m }  
fouling_conductivity: { value: 0.20, unit: W/m/K }
```

HX_4:

```
kind: "reversal_chamber"  
pool_boiling: true  
inner_diameter: { value: 1.6, unit: m }  
inner_length: { value: 0.8, unit: m }  
curvature_radius: { value: 0.8, unit: m }  
nozzles:  
  inlet:  
    k: { value: 1, unit: dimensionless }  
  outlet:  
    k: { value: 1, unit: dimensionless }  
wall:  
  thickness: { value: 0.0029, unit: m }  
  conductivity: { value: 16, unit: W/m/K }  
surfaces:  
  inner:  
    roughness: { value: 0.5, unit: micrometer }  
    emissivity: { value: 0.80, unit: dimensionless }  
    fouling_thickness: { value: 0.0001, unit: m }  
    fouling_conductivity: { value: 0.20, unit: W/m/K }  
  outer:  
    roughness: { value: 0.5, unit: micrometer }  
    emissivity: { value: 0.80, unit: dimensionless }  
    fouling_thickness: { value: 0.0001, unit: m }  
    fouling_conductivity: { value: 0.20, unit: W/m/K }
```

HX_5:

```
kind: "tube_bank"  
pool_boiling: true  
inner_diameter: { value: 0.076, unit: m }  
inner_length: { value: 5.620, unit: m }  
tubes_number: { value: 100, unit: dimensionless }  
arrangement: "staggered"  
N_rows: { value: 6, unit: dimensionless }  
ST: { value: 0.11, unit: m }  
SL: { value: 0.11, unit: m }  
baffle_spacing: { value: 0.075, unit: m }  
shell_inner_diameter: { value: 1.80, unit: m } #  $D_s$   
baffle_cut: { value: 0.25, unit: dimensionless } #  $B_c = L_{bch} / D_s$   
bundle_clearance: { value: 0.010, unit: m }  
wall:  
  thickness: { value: 0.0029, unit: m }
```

```

conductivity: { value: 16, unit: W/m/K }
surfaces:
  inner:
    roughness: { value: 0.5, unit: micrometer }
    emissivity: { value: 0.80, unit: dimensionless }
    fouling_thickness: { value: 0.0001, unit: m }
    fouling_conductivity: { value: 0.20, unit: W/m/K }
  outer:
    roughness: { value: 0.5, unit: micrometer }
    emissivity: { value: 0.80, unit: dimensionless }
    fouling_thickness: { value: 0.0001, unit: m }
    fouling_conductivity: { value: 0.20, unit: W/m/K }

HX_6:
  kind: "economiser"
  pool_boiling: false
  inner_diameter: { value: 0.076, unit: m } # unchanged
  inner_length: { value: 7.5, unit: m } # was 5.620
  tubes_number: { value: 160, unit: dimensionless } # was 100
  layout: "triangular"
  arrangement: "inline"
  N_rows: { value: 4, unit: dimensionless }
  ST: { value: 0.123, unit: m } # keep pitch
  SL: { value: 0.123, unit: m }
  baffle_spacing: { value: 0.085, unit: m } # was 0.105 → higher h
  shell_inner_diameter: { value: 1.80, unit: m }
  baffle_cut: { value: 0.25, unit: dimensionless }
  bundle_clearance: { value: 0.010, unit: m }
  wall:
    thickness: { value: 0.0025, unit: m } # thinner wall
    conductivity: { value: 30, unit: W/m/K }
    surfaces:
      inner:
        roughness: { value: 0.5, unit: micrometer }
        emissivity: { value: 0.80, unit: dimensionless }
        fouling_thickness: { value: 0.0, unit: m } # best case to raise UA
        fouling_conductivity: { value: 0.20, unit: W/m/K }
      outer:
        roughness: { value: 0.5, unit: micrometer }
        emissivity: { value: 0.80, unit: dimensionless }
        fouling_thickness: { value: 0.0, unit: m }
        fouling_conductivity: { value: 0.20, unit: W/m/K }

```

A.6 Water side properties (config/water.yaml)

```
enthalpy: { value: 300000, unit: J/kg }
pressure: { value: 1000000, unit: Pa }
composition:
  H2O: { value: 1.0, unit: dimensionless }
drum:
  flow_area: { value: 5, unit: m^2 }
```


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