# **Assignment 1: PCA**

# What you will learn

- · Working with images using scikit-image
- PCA using scikit-learn
- · Practical applications of PCA

# **Setup**

- Download Anaconda Python 3.6 (https://www.anaconda.com/download/) for consistent environment.
- If you use pip environment then make sure your code is compatible with versions of libraries provided within Anaconda's Python 3.6 distribution.

## **Submission**

- · Do not change any variable/function names.
- · Just add your own code and don't change existing code
- Save this file and rename it to be **studentid\_lastname.ipynb** (student id (underscore) last name.ipynb) where your student id is all numbers.
- Export your .ipynb file to PDF (File > Download as > PDF via Latex). Please don't leave this step for final minutes.
- Submit both the notebook and PDF files.
- If you happen to use any external library not included in Anaconda (mention in Submission Notes section below)

# **Submission Notes**

(Please write any notes here that you think I should know during marking)

# [NO MARKS] PCA Warming Up (MUST READ)

I'm adding some code to illustrate examples of PCA using sklearn library.

Let's create some random 5d data

```
In [1]: import numpy as np
    from sklearn.decomposition import PCA

# 100 points of 5d data
    data = np.random.rand(100, 5)
```

### Lets convert this 5d data to 2d using PCA'

In code above when we call fit, it populates two things in pca:

```
1. mean_
```

2. components

Now we are ready to transform 5d from data into 2d using following code

```
In [5]: data_to_reduce = data[:10]
    reduced_data = np.dot(data_to_reduce - pca.mean_, pca.components_.T)

# reduced_data from 5d to 2d
    reduced_data.shape
Out[5]: (10, 2)
```

You can accomplish the same using transform function provided in pca

```
In [6]: pca.transform(data_to_reduce).shape
Out[6]: (10, 2)
```

Time for inverse transform or changing 2d data back to 5d

Compression --> Decompression

```
In [7]: decompressed data = np.dot(reduced data, pca.components )+pca.mean
        decompressed data
Out[7]: array([[0.11268137, 0.5292387, 0.68228227, 0.466384, 0.87516734],
               [0.7603341, 0.48098921, 0.4484759, 0.39819963, 0.30084041],
               [0.55407687, 0.67735918, 0.64023356, 0.73288934, 0.38366914],
               [0.82195761, 0.454136, 0.41180261, 0.35321804, 0.25850254],
               [0.47939664, 0.63278575, 0.63470282, 0.65405938, 0.47761484],
               [0.38519453, 0.42707774, 0.53085565, 0.29615115, 0.67876708],
               [0.55501762, 0.39380899, 0.45618786, 0.24262316, 0.5395701 ],
               [0.93990411, 0.33789436, 0.29958842, 0.15500011, 0.21332105],
               [0.45572109, 0.51196045, 0.56380734, 0.44458237, 0.566389
               [0.68642325, 0.81049071, 0.68511959, 0.96620297, 0.18724737]])
In [8]: # same can we accomplished using inverse transform
        pca.inverse_transform(pca.transform(data_to_reduce))
Out[8]: array([[0.11268137, 0.5292387, 0.68228227, 0.466384, 0.87516734],
               [0.7603341 , 0.48098921, 0.4484759 , 0.39819963, 0.30084041],
               [0.55407687, 0.67735918, 0.64023356, 0.73288934, 0.38366914],
               [0.82195761, 0.454136, 0.41180261, 0.35321804, 0.25850254],
               [0.47939664, 0.63278575, 0.63470282, 0.65405938, 0.47761484],
               [0.38519453, 0.42707774, 0.53085565, 0.29615115, 0.67876708],
               [0.55501762, 0.39380899, 0.45618786, 0.24262316, 0.5395701 ],
               [0.93990411, 0.33789436, 0.29958842, 0.15500011, 0.21332105],
               [0.45572109, 0.51196045, 0.56380734, 0.44458237, 0.566389
               [0.68642325, 0.81049071, 0.68511959, 0.96620297, 0.18724737]])
In [9]: # Lets find compression decompression error (absolute mean error)
        np.sum(np.abs(data to reduce - decompressed data))/data to reduce.size
Out[9]: 0.13884886747264605
```

# Questions [8 marks]

Answer the questions below as follows:

```
1) What is 2+2
```

- 4
- 5
- 6

```
In [10]: ans1 = 4 ans1
```

```
Out[10]: 4
```

2) (2 marks) For a n-D data, you can ALWAYS reconstruct the data with 0\% error if all n PCAs are used.

- True
- False

```
In [11]: ans2 = True
ans2
Out[11]: True
```

3) (2 marks) From the 2nd tutorial, we ran PCA alorithm on faces. We called the extracted PCs--Eigenfaces. What is the value of a dot product between arbitary two eigen faces?

```
In [12]: ans3 = 0
ans3
Out[12]: 0
```

4) (4 marks) Using the probablity, find the expected value for the function below:

**Note:** Lets say for a coin toss, head = 0 and tail = 1. Then the expected value for a coin-toss will be p(x=tail)\*1 + p(x=head)\*0 = 1/2\*1 = 0.5.

```
In [13]: import operator as op
         from functools import reduce
         def func():
             arr = [49, 8, 48, 15, 47, 4, 16, 23, 43, 44, 42, 45, 46]
             np.random.shuffle(arr)
             print(arr[0:6])
             return min(arr[0:6])
         def ncr(n, r):
             r = min(r, n-r)
             numer = reduce(op.mul, range(n, n-r, -1), 1)
             denom = reduce(op.mul, range(1, r+1), 1)
             return numer / denom
         # Sorted: [4, 8, 15, 16, 23, 42, 43, 44, 45, 46, 47, 48, 49]
         # ie. 4 is picked as min in the total set of 13
         # only 12 more numbers higher than 4 exist and only need to choose 5 mor
         e numbers
         # because the array is length 6 and one num (the min) was already chosen
         # additionally, only need account up to 44 because if 44 is the min the
          5 numbers above it
         # would create the rest of the subset of six numbers
         # expected value is calculated by taking the value of the discrete varia
         ble chosen
         # and multiplying it by the probability of it being chosen
         top = 4*ncr(12,5)+8*ncr(11,5)+15*ncr(10,5)+16*ncr(9,5)+23*ncr(8,5)+42*nc
         r(7,5)+43*ncr(6,5)+44*ncr(5,5)
         bot = ncr(13, 6)
         res = top / bot
         res
```

Out[13]: 8.818181818181818

# **Programming Tasks [92 marks]**

## Task 1: Building an Image Compression Algorithm

In this section you will build you own compression algorithm for images using PCA.

#### STEP 1: Read the image

- 1. Use imread function from skimage.io to read leena.jpg.
- 2. Show the image using show image function provided to you

```
In [14]: import matplotlib.pyplot as plt
# make matplotlib to show plots inline
%matplotlib inline

from skimage.io import imread

def show_image(img):
    if len(img.shape) == 2:
        plt.imshow(img, cmap='Greys_r')
    else:
        plt.imshow(img)
    plt.axis('off')
    plt.title('Image Dimension: {0},{1}'.format(img.shape[0], img.shape[1]), fontsize=20)

# !!ADD CODE HERE!!
image = imread('leena.jpg')
# show_image(image)
```

Image Dimension: 350,780



#### STEP 2: Compressing an image using PCA

Image is generally made of 3 (or 4) channels (RGB), we will build a compression algorithm that applies one channel at a time to compress multi-channel image.

In order to compress entire image you will compress all the channels within the image one by one and then serialize compressed channels and any auxiliary data required for decompression (for ex. principle components (components\_), means (means\_), original image size).

In order to decompress, you will deserialize the data, then uncompress the compressed channel one by one and stack them up to rebuild the uncompressed version of the original image.

#### Compression Strategy using PCA

- The above image you have read is of size  $350 \times 780$ , i.e. width is 780 and height is 350
- Patch the image into 10x10 patches yielding 35x78=2730 patches in total.
- Flatten each patch (10x10) into 100 dimensional vector.
- Now (for each channel) you will have 2730 number of 100-d vectors.
- · Apply PCA on these vectors.
- Reduce the dimensionality of 100-d vector to 5-d.

I've given you two functions patchify and depatchify.

patchify creates 100-d vectors from all the patches from a given image and depatchify combines these 100-d patches back to the image of the given size.

Please read through code below and figure out how these two functions work.

**NOTE**: convert\_to\_cf is important function to note (in cell below). It converts channel last format image to channel first format. Images that you read through imread function returns array of shape X x Y x 3 channel is last axis, it is easier if channel were first axis then to extract any channel you can do use first indexer.

```
In [15]: from sklearn.decomposition import PCA
         import numpy as np
         def patchify(img, ps=(10, 10)):
             patches = []
             h, w = img.shape
             for i in range(0, h, ps[0]):
                 for j in range(0, w, ps[1]):
                      patches.append(img[i:i+ps[0], j:j+ps[1]].ravel())
             return np.array(patches)
         def depatchify(patches, patch size=(10, 10), img size=(350, 780)):
             # normalize
             patches[patches > 255.] = 255.
             patches[patches < 0.] = 0.</pre>
             # convert to unint8
             patches = patches.astype('uint8')
             rec_img = np.zeros(img_size, dtype='uint8')
             ph, pw = patch_size
             h, w = img size
             x = 0
             for i in range(0, h, ph):
                  for j in range(0, w, pw):
                     rec_img[i:i+ph, j:j+pw] = patches[x].reshape((ph, pw))
                     x += 1
             return rec img
         def convert to cf(img cl):
             # convert image to channel first
             img_cf = np.swapaxes(img_cl.T, 1, 2)
             return imq cf
         image=imread('leena.jpg')
         img cf = convert to cf(image)
         # I'll patchify each channel and depatchify them
         # Then I'll stack them together to create original image back
         ch1 = depatchify(patchify(img cf[0])) # first channel
         ch2 = depatchify(patchify(img cf[1])) # second channel
         ch3 = depatchify(patchify(img cf[2])) # third channel
         # combine them now
         rec img = np.dstack((ch1, ch2, ch3))
         plt.figure(figsize=(12, 12))
         show image(rec img)
```

Image Dimension: 350,780



### STEP 3: Compressing single channel of an image (FILL TWO FUNCTIONS BELOW)

Now you are familiar with how patchify and depatchify work. Do the following:

- 1. Write a function compress that will take one of the channel of the image as input and outputs compressed channel and auxiliary data required for decompressing. Return type of this function should be dictionary.
- 2. Write another function decompress that will take whatever dictionary data you returned from previous compress function and decompresses it into image channel (that was compressed).
- 3. PCA compression is lossy compression algorithm -- means you will loose the information during decompression.
- 4. I should be able to call two of your functions like so decompress (compress (img\_ch[0])) to compress and decompress the given image's channel.

#### Compress: Pseudo code

- Patchify the given image's channel (input).
- Run PCA on the patches (you may use sklearn's implementation) to reduce them to 5d vectors.
- You will need basis vectors or principal components and mean in order to reconstruct the data back to 100d
- Return dictionary: {'compressed\_patches': 5d vectors, 'aux\_data': principal components/basis vectors/means/final size of image}
- By converting 100d to 5d, you reduce size by 20 times.

#### **Decompress: Pseudo code**

- Input is dictionary as returned by your compress function.
- Use 5d vectors and do inverse PCA (check tutorial 2) and convert them back to 100d vectors.
- Use depatchify function to convert these reconstructed 100d vectors into an image channel

**Tip**: Good code is always modular and easy to read.

```
In [16]: # COMPLETE FOLLOWING FUNCTIONS
         from sklearn.decomposition import PCA
         def compress(img_ch, n_components=5):
             Inputs
                  img ch: one of the channel of given image
                 n components: number of components returned by PCA
             Returns
                 comp data: Some data structure (may be dict.) that represents co
         mpressed form of given input
                 along with auxiliary data required for decompression (components
         _, mean_ and shape of input image)
             # patchify img ch
             patches = patchify(img_ch)
             # ADD CODE HERE!!
             # 1) Get PCA components and means
             pca = PCA(n components)
             pca.fit(patches)
             # 2) Compress the patches
             y = pca.transform(patches)
             # 3) Return data
             aux data = [
                 pca.components_, # basis vecs
                 pca.mean , # mean
                 img ch.shape # shape of image
              ]
             return {'y': y, 'aux data': aux data }
             pass
         def decompress(comp data):
             Inputs
                 comp data: data structure that is returned by `compress` functio
             Returns
                 img ch: decompressed form of channel compressed and contained in
         side `comp_data` data structure
              11 11 11
             # 1) Get compressed data, y, and the aux data
             y = comp data['y']
             components, means, img size = comp data['aux data']
             # 2) recontruct the patches to 100d using aux data and inverse PCA
             rec patches = np.dot(y, components) + means
             # 3) depatchify and return img ch
             img ch = depatchify(rec patches)
             return img ch
```

```
# Red channel
# visualize your compression and decompression
img_ch = img_cf[0]
plt.figure(figsize=(10, 10))
show_image(img_ch)

plt.figure(figsize=(10, 10))
show_image(decompress(compress(img_ch, n_components=5)))
```





Image Dimension: 350,780



#### STEP 3: Compress and decompress entire image (FILL TWO MORE FUNCTIONS)

Write a compress\_image function that:

- takes channel last (regular image read from imread) representation of an image
- convert channel last to channel first representation using convert\_to\_cf function defined previously
- compresses each of the channel using compress function
- outputs a one dictionary that contains all auxiliary data required to reconstruct/decompress entire image back.

Similarly, write decompress\_image function that:

- decompresses the image compressed by compress image function
- return channel last image

**NOTE:** You would patchify each channel and implement PCA on each channel thus for decompression you need components , mean for each channel and you need shape of input image as well.

```
In [17]: def compress_image(img, n_components=5):
             Inputs:
                  img cf: `Channel last` image data
             img_cf = convert_to_cf(img)
             return {
                  '1': compress(img_cf[0], n_components),
                  '2': compress(img_cf[1], n_components),
                  '3': compress(img_cf[2], n_components)
             }
         def decompress_image(comp_img):
             Returns:
                  img_rec: Decompressed `channel last` image
             ch1 = decompress(comp_img['1'])
             ch2 = decompress(comp_img['2'])
             ch3 = decompress(comp img['3'])
             return np.dstack((ch1,ch2,ch3))
         plt.figure(figsize=(10, 10))
         show_image(
             decompress_image(
                 compress_image(image)
         )
```





#### STEP 4: Serialization and de-serialization

You can easily (de)serialize dictionary using np.save and np.load (see example below)

compress\_and\_serialize should:

- Read image given by inp path
- Use compress\_image to compress it
- Serialize the compressed data using np.save to a file specified by out\_path

NOTE: All the data required for deserialization must be saved to a one single file only.

deserialize\_and\_decompress should:

- Read the file specified by inp\_path using np.load
- Use decompress\_image function to decompress it
- Return image (channel last as returned by decompress\_image) function

```
In [18]: # EXAMPLE OF SAVING AND LOADING DICTIONARY OBJECT TO/FROM FILESYSTEM
         d = \{ 'a' : [1, 2, 3], 'b' : [4, 5, 6, 7, 8] \}
         np.save('example.npy', d)
         ds = np.load('example.npy').item()
         ds
Out[18]: {'a': [1, 2, 3], 'b': [4, 5, 6, 7, 8]}
```

```
In [19]: import os
# COMPLETE THESE TWO FUNCTIONS
def compress_and_serialize(inp_path='leena.jpg', out_path='output.bin'):
    image = imread(inp_path)
    comp_img = compress_image(image)
    np.save(out_path, comp_img)
    # rename the np
    os.rename(out_path+".npy", out_path)

def deserialize_and_decompress(inp_path='output.bin'):
    compressed = np.load(inp_path).item()
    return decompress_image(compressed)

compress_and_serialize()
    show_image(deserialize_and_decompress())
```

Image Dimension: 350,780



#### STEP 5: What is size of output.bin file?

Did you end of in compressing anything? Conclude your experiments!

#### Conclusion

Write your conclusion here!!

- the compressed bytes is higher than the original image, but this is because output bin has all the metadata
- the compressed image size should be compared instead of the .bin file

```
In [20]: import os
    print('Original', os.path.getsize('leena.jpg'), 'bytes')
    print('Compressed', os.path.getsize('output.bin'), 'bytes')

Original 35740 bytes
    Compressed 498280 bytes
```

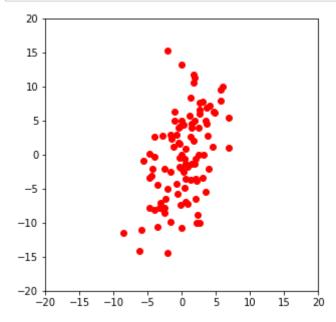
# Task 2: Rotation and Translation Invariance in PCA

STEP 1 (done already): Create normally distributed data

```
In [21]: mean = [0, 0]
    cov = [[10, 10], [10, 40]] # diagonal covariance
    x, y = np.random.multivariate_normal(mean, cov, 100).T
    X = np.array(list(zip(x, y)))

def plot_data(X):
    plt.figure(figsize=(5, 5))
    plt.plot(X[:, 0], X[:, 1], 'ro')
    plt.xlim([-20, 20])
    plt.ylim([-20, 20])
    plt.gca().set_aspect('equal', adjustable='box')
    plt.show()

plot_data(X)
```

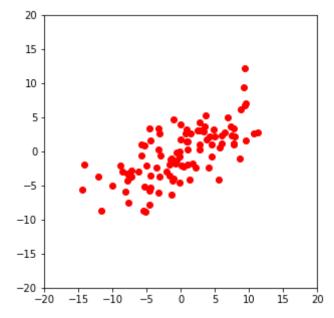


STEP 2: Rotate the data by 45 degrees and create new array X\_rot

Write code to rotate X by 45

- Use rotation matrix
- Or individually rotate each point in X by 45 degrees
- Use plot data function defined in cell above to visualize the rotated data

```
In [22]: theta = np.radians(45)
# WRITE CODE HERE
# Code to rotate X
c, s = np.cos(theta), np.sin(theta)
R = np.array(((c,-s), (s, c)))
# populate new matrix X_rot
X_rot = X.dot(R)
plot_data(X_rot)
```



### STEP 3 (done but need explanation): Perform pca with n\_components=2

Do you see anything interesting?

Explain the code below and write down your observations (2-3 lines only).

### **Observations**

- pca fit\_transform of X is the same as X\_rot
- pca is a rotational invariant

```
In [23]: import pandas as pd

def visualize_components(X, X_rot):
    pca = PCA(n_components=2)

    pca.fit(X)
    df1 = pd.DataFrame(pca.fit_transform(X))
    df2 = pd.DataFrame(pca.fit_transform(X_rot))

    return pd.concat((df1, df2), axis=1)[:20]

visualize_components(X, X_rot)
```

### Out[23]:

	0	1	0	1
0	5.504224	-3.355173	5.504224	-3.355173
1	-3.005763	-6.036108	-3.005763	-6.036108
2	-2.203019	3.725383	-2.203019	3.725383
3	-5.549313	-2.025345	-5.549313	-2.025345
4	-3.981244	1.758984	-3.981244	1.758984
5	7.939445	0.652354	7.939445	0.652354
6	-9.341564	-3.055980	-9.341564	-3.055980
7	0.929962	-0.591284	0.929962	-0.591284
8	0.677020	-3.966748	0.677020	-3.966748
9	-5.069111	0.140076	-5.069111	0.140076
10	-3.876856	-2.599697	-3.876856	-2.599697
11	-7.850321	-1.374726	-7.850321	-1.374726
12	0.706440	-1.476864	0.706440	-1.476864
13	2.391178	2.181455	2.391178	2.181455
14	-1.792900	1.169575	-1.792900	1.169575
15	4.287491	-1.385176	4.287491	-1.385176
16	2.883898	-2.829643	2.883898	-2.829643
17	-10.914111	1.405779	-10.914111	1.405779
18	-8.255857	-1.699187	-8.255857	-1.699187
19	-0.833272	-2.108724	-0.833272	-2.108724

### STEP 4: Perform PCA again and find angle between principal components

- Perform PCA on X and X\_rot with n\_components=1
- It will give one basis vector for each of data (X and X\_rot)
- · Find the angle between these two basis vectors
- Explain your observations?

#### **Observations**

- the two basis vectors are 45 degrees to each other
- this is because the X\_rot is 45 degrees rotation of the X
  - the original 45 degrees rotation is reflected in the basis vectors

```
In [24]:
         import math
         from numpy.linalg import norm
         def angle_between(a,b):
             # COMPLETE THIS FUNCTION
             # CALCULATE ANGLE IN RAD BETWEEN TWO VECTORS
             dotprod = np.dot(a,b)
             lengthA = norm(a)
             lengthB = norm(b)
             return math.acos(dotprod / (lengthA*lengthB))
         pca = PCA(n components=1)
         pca.fit(X)
         c1 = pca.components_
         pca.fit(X rot)
         c2 = pca.components
         np.rad2deg(angle between(c1[0], c2[0]))
```

Out[24]: 45.0

### (NO MARKS) STEP 5: Repeat these experiments for translation as well

There is not marks for this part. You can do this for your own learning.

Now translate every point in x by fixed x and y amount.

```
X = X + [1, 2]
```

like so and repeat all the above experiments in cell below and write down your observations:

```
In [25]: # PERFORM EXPERIMENTS WITH TRANSLATION HERE
# NO MARKS FOR DOING THIS
```

# TASK 3: Recovery of corrupted images using PCA

Check the code below.

It corrupts the leena.jpg image that you worked on before.

Check very carefully what below code is doing on the image and answer:

Is it same as rotation of data points (like done before), if yes explain (just one liner)?

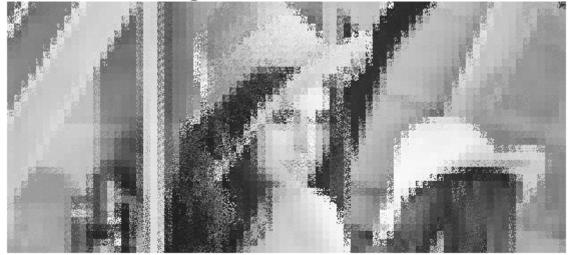
No, this corruption is random

```
In [26]: # Code for corrupting the leena image
         image = convert_to_cf(imread('leena.jpg'))[0]
         def corrupt_image(img):
             # lets patchify R channel with patches of 35x78 patches
             patches = patchify(img)
             # Noise 1:
             # lets jumble pixels of patches now
             jumble_idx = list(range(len(patches[0])))
             np.random.shuffle(jumble_idx)
             jumbled patches = np.array([patch[jumble_idx] for patch in patches])
             rec_jumbled_image = depatchify(jumbled_patches, img_size=img.shape)
             return rec_jumbled_image
         cimage = corrupt_image(image)
         plt.figure(figsize=(10,10))
         show_image(image)
         plt.figure(figsize=(10,10))
         show_image(cimage)
```

Image Dimension: 350,780



Image Dimension: 350,780



### Recovering from the corruption

Use the compress function you coded earlier to get compressed form of original leena image and corrupted leena image.

We will try to recover the corrupted leena given the compressed version of original leena.

Notice in code below, I replace aux\_data of corrupted version with aux\_data of original version.

Does the code below code work in recovering the original leena image back?

### Explain how below code works and show it actually works?

Yes the code works in getting the original leena image back. It compresses the corrupted image and then replaces the corrupted image's aux\_data with the original image's aux\_data.

Image Dimension: 350,780







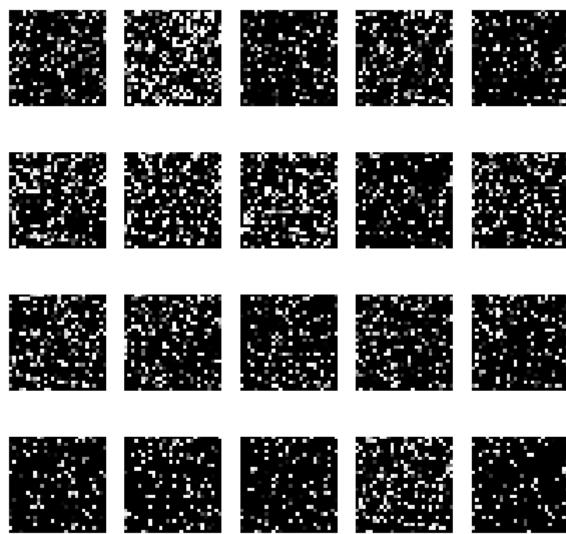
# Task 4: Decrypting manuscript of the lost civilization

You belong to one of the advanced civilization, while exploring the universe you land on the planet Earth. However, there is no inhabitants on the planet anymore. While exploring Earth, you come across some damaged hard drive that contains various images. You suspect that these images form a manuscript of how "humans" use to write different digits in maths. You recovered all the data from the hard drive safely. You opened up your jupyter notebook (python being universal language and popular among alien species), you started plotting the images you just recovered.

```
In [28]: rimages = np.load('recovered_images.npy')
    rimages.shape
Out[28]: (20, 28, 28)
```

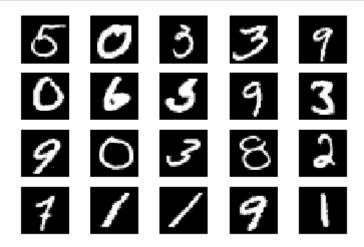
```
In [31]: def plot_manuscript(images):
    for i, img in enumerate(images):
        plt.subplot(4, 5, i+1)
        plt.imshow(img, cmap='Greys_r')
        plt.axis('off')

plt.figure(figsize=(10, 10))
    plot_manuscript(rimages)
```



You being a smart alient, figured out that these images are encrypted and not in their original form. You also figured out that encryption is just fixed jumbling of the pixels of the images. Since you're unaware of the manuscript, you cannot decrypt the images just by themselves even if they follow the fixed jumbling pattern. However, fortunately you found the principal components and mean of the the original manuscript somewhere in the same harddisk. Now, your task is to recover all the 20 images and plot them nicely in cell below.

Use plot manuscript function from above to plot the recovered manuscript.



In [ ]: