

T6H

$$T_n = T_{n-1} + 1 + T_{n-1}$$

$$= 2T_{n-1} + 1 \rightarrow \text{Open Form}$$

$$T_4 = 2T_3 + 1$$

$$= 2(2T_2 + 1) + 1$$

$$= 4T_2 + 3$$

$$= 4(2T_1 + 1) + 3$$

$$= 8T_1 + 7$$

$$= 8(2^{T_0} + 1) + 7$$

$$= 15$$

$$\# T_0 = 0, T_1 = 1, T_2 = 3, T_3 = 7, T_4 = 15$$

$$T_n = 2^n - 1 \rightarrow \text{Close form}$$

Prove by mathematical induction

Basis \rightarrow Hypothesis \rightarrow Induction [Steps]

Basis: for the lowest value of n.

$$T_0 = 2^0 - 1 = 0$$

Hypothesis: Assuming for all values of n ranging from 1 to n-1, $2^n - 1$ is true.

$$\left(\frac{2^1 + 2^2 + \dots + 2^n}{2^n} \right) S =$$

$$(2^1 + 2^2 + \dots + 2^n) S =$$

Induction:

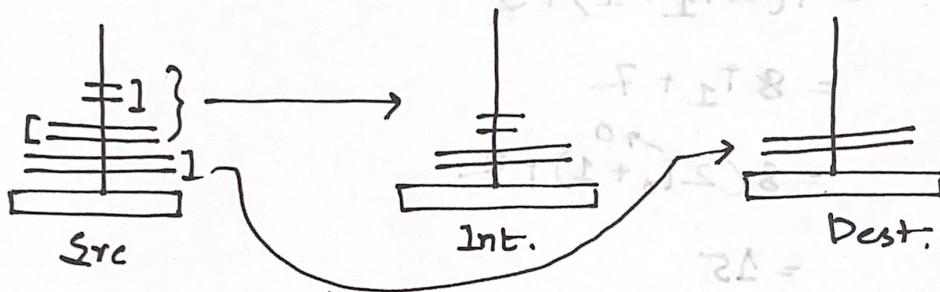
from the recurrence relation,

$$T_n = 2T_{n-1} + 1$$

$$= 2(2^{n-1} - 1) + 1$$

$$T_n = 2^n - 1$$

DTOH



$$T_n = T_{n-1} + 2 + T_{n-1} \quad \downarrow \quad T_n = \varepsilon^T, \varepsilon = \varepsilon^T, \varepsilon = \varepsilon^T, \varepsilon = \varepsilon^T, \varepsilon = \varepsilon^T$$

$$= 2 T_{n-1} + 2$$

$$= 2(2T_{n-2} + 2) + 2 = 2^2 T_{n-2} + 2^2 + 2$$

$$= 2^3 T_{n-3} + 2^3 + 2^2 + 2.$$

... it is to follow + record SNT-YOF: circosil

$$= 2^n T_{n-n} + 2^n + 2^{n-1} + \dots + 2^1$$

$$= 2^n \cdot 0 + 2^n + 2^{n-1} + \dots + 2^2 + 2$$

$$= 2 \left(z^{n-1} + \dots + \right)$$

$$= 2^{n+1} - 2 \longrightarrow \text{Closed form / Eq } n$$

Prove by mathematical induction for TTOH that

$$T_n = 2(2^n - 1) \text{ holds.}$$

Basis: $n=0 \dots$

Hypothesis: Assuming for $n-1$ true, $T_{n-1} = 2 \cdot (2^{n-1} - 1)$

Induction:

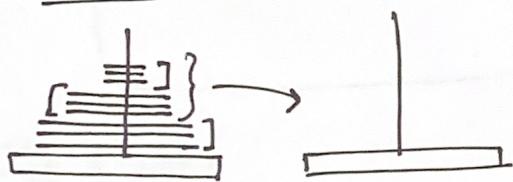
from recurrence $T_n = 2T_{n-1} + 2$

$$= 2(2(2^{n-1} - 1)) + 2$$

$$= 2(2^n - 2) + 2 =$$

$$= 2(2^n - 1)$$

TTOH



$$T_n = T_{n-1} + 3 + T_{n-1}$$

$$= 2T_{n-1} + 3(1-n)$$

$$= 2(2T_{n-2} + 3) + 3$$

$$= 2^2 T_{n-2} + 2 \cdot 3 + 3$$

$$= 2^3 T_{n-3} + 2^2 \cdot 3 + 2 \cdot 3 + 3$$

$$= 2^n T_{n-n} + 2^{n-1} \cdot 3 + 2^{n-2} \cdot 3 + \dots + 2^1 \cdot 3 + 2^0 \cdot 3$$

$$= 3(2^0 + 2^1 + \dots + 2^{n-1})$$

$$= 3\left(\frac{2^{n-1+1}-1}{2-1}\right) = 3(2^n - 1) \rightarrow \text{closed form}$$

Prove by math. induction

Lines in the plane

$$\begin{aligned}
 L_n &= L_{n-1} + n && \text{Recurrence} \\
 (L - L^{(0)}s) &= L_{n-2} + (n-1) + n \\
 &= L_{n-3} + (n-2) + (n-1) + n \\
 &\quad \downarrow \\
 &= L_0 + 1 + 2 + \dots + (n-2) + (n-1) + n \\
 &= 1 + \frac{n(n+1)}{2}
 \end{aligned}$$

Prove by induction:

$$L_n = \frac{n(n+1)}{2} + 1$$



Basis: $n=0$

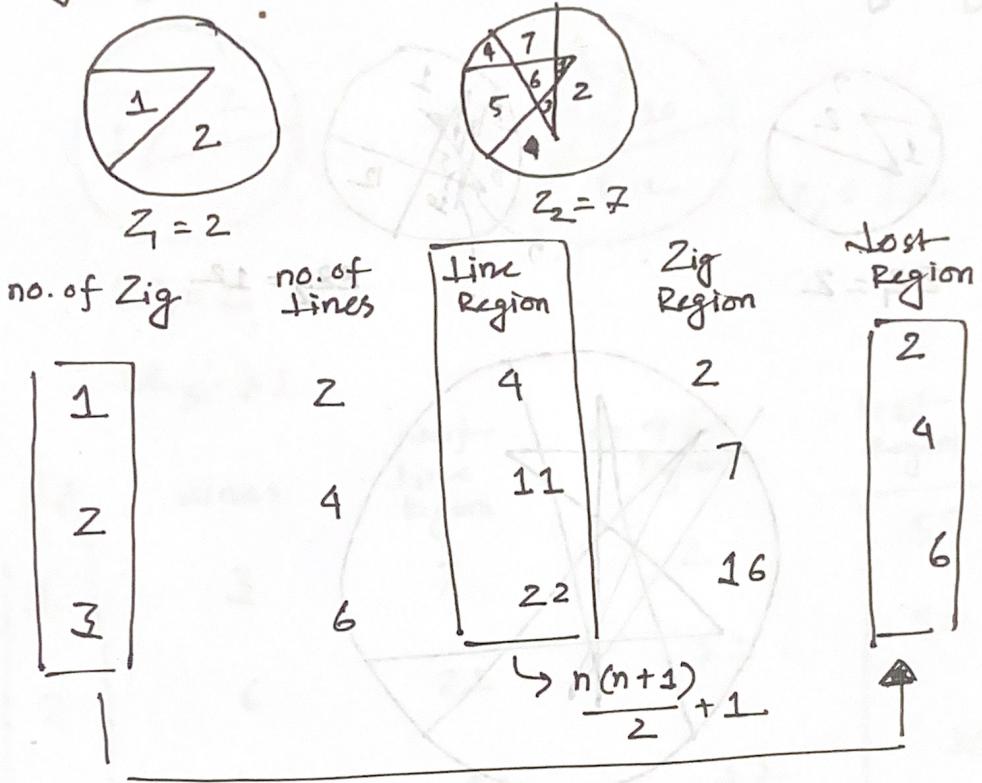
$$L_0 = 1$$

Hypothesis: Assuming until $(n-1)$, the eqn holds

Induction:

$$\begin{aligned}
 L_n &= L_{n-1} + n \\
 &= \frac{(n-1)(n-1+1)}{2} + 1 + n \\
 &= \frac{n^2 - n + 2 + 2n}{2} = \frac{n(n+1)}{2} + 1
 \end{aligned}$$

Zig :



Zig Regions

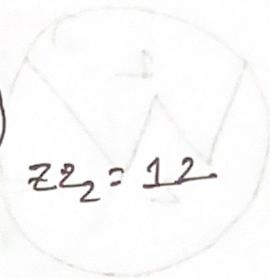
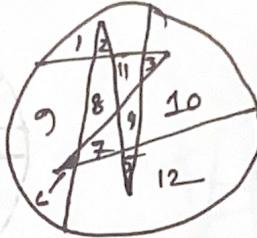
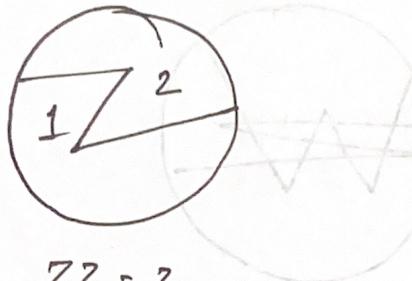
$$Z_n = L_n - 2n$$

$$= \frac{2n(2n+1)}{2} + 1 - 2n$$

$$= 2n^2 + n + 1 - 2n$$

$$= 2n^2 - n + 1$$

Zig Zag:



$$ZZ_3 = 31$$

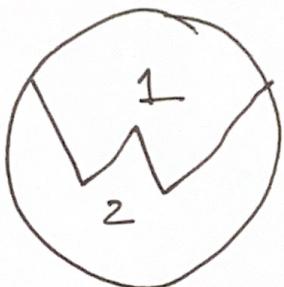
ZZ	lines	no. of line Region	no. of ZZ Region	lost Region
1	3	7	2	
2	6	22	12	
3	9	46	31	

$$ZZ_n = l_{3n} - 5n = \frac{3n(3n+1)}{2} + 1 - 5n$$

$$= \frac{9n^2 + 3n + 2 - 10n}{2}$$

$$= \frac{9n^2 - 7n + 2}{2}$$

ω -object:



$$\omega_1 = 2$$



$$\omega_2 = 19$$

ω	no. of Line	no. of Line Region	ω Region	lost Region
	4	11	2	
	8	37	19	

$$\omega_n = l_{4n} - g_n$$

$$= 1 + \frac{4n(4n+1)}{2} - g_n$$

$$= 1 + 2n(4n+1) - g_n$$

$$= 8n^2 + 2n + 1 - g_n$$

$$= 8n^2 - 7n + 1$$