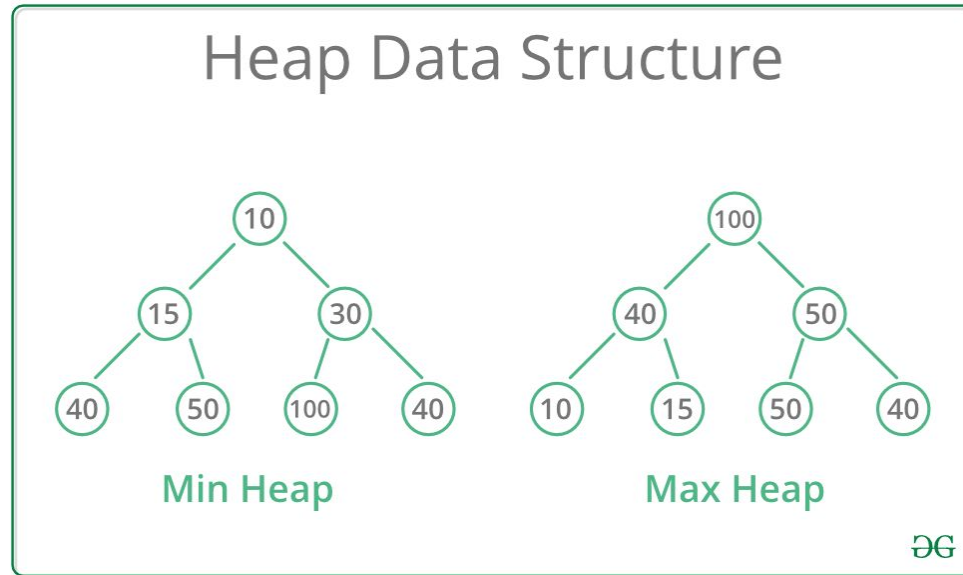


Heaps

Session 27

What is a heap

A Heap is a special Tree-based data structure in which the tree is a complete binary tree. This property of Binary Heap makes them suitable to be stored in an array. (Level Order)



Heaps in Array : Introduction

- $\text{Arr}[0]$: Root element.
- If i th index node
- $\text{Arr}[(i-1)/2]$: Returns the parent node
- $\text{Arr}[(2*i)+1]$:Returns the left child node
- $\text{Arr}[(2*i)+2]$: Returns the right child node

Types of heap

Max heap: In a Max-Heap the key present at the root node must be greatest among the keys present at all of its children. The same property must be recursively true for all sub-trees in that binary tree.

Min heap: In a Min-Heap the key present at the root node must be minimum among the keys present at all of its children. The same property must be recursively true for all sub-trees in that binary tree.

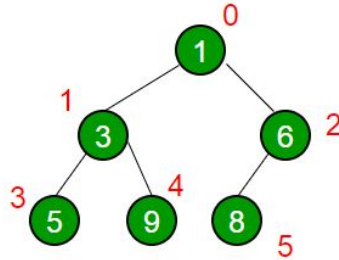
Important operations for heap data structure

- **Heapify** : Process of creating heap from an array
- **Insertion** : Process to insert an element in existing heap
- **Deletion** : Deleting the max / min element in heap and then reorganisation of heap so that next max / min element comes at top

Heapify

- Heaps can be implemented using custom defines class as well as array, since implementation using primary data structure (array) can be done we prefer to implement it that way.
- To represent the binary tree as array we do the following:

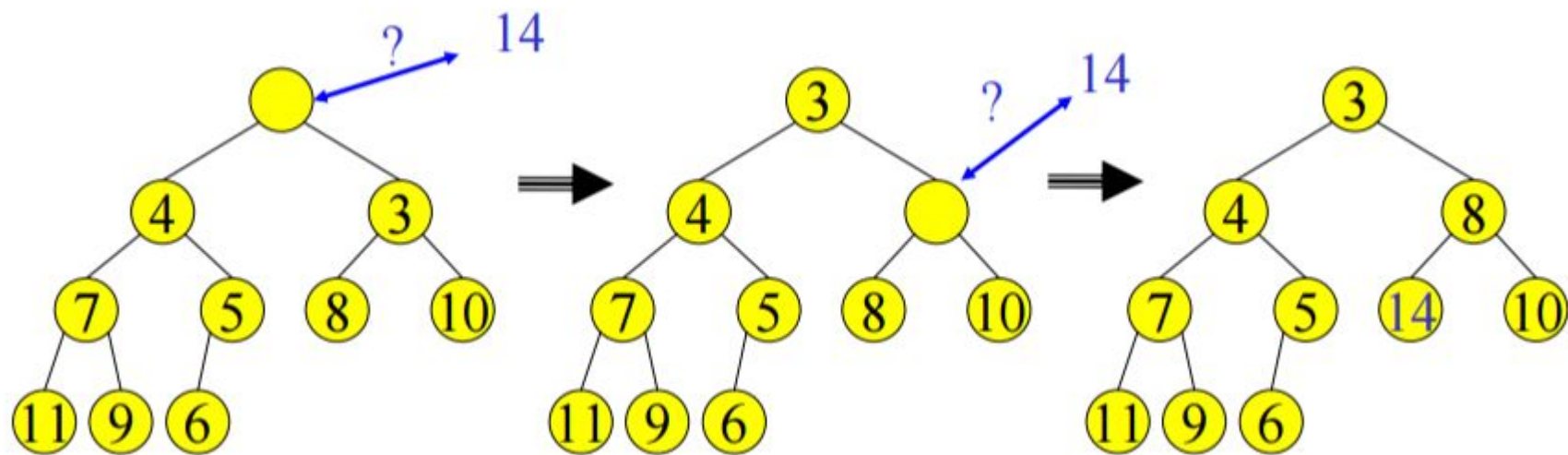
0 based indexing,
child of index i are $2i + 1$ and $2i + 2$,
parent of i is $(i - 1) / 2$



1	3	6	5	9	8
0	1	2	3	4	5

Percolate Down

- We do the following step under percolate down algo (if max heap):
 - Pick the max of value of both the child if one child is there pick it's value
 - Compare this with the value of current node where you are
 - If the value of child is more swap parent with child
 - Continue doing the same
- We do the following step under percolate down algo (if min heap):
 - Pick the min of value of both the child if one child is there pick it's value
 - Compare this with the value of current node where you are
 - If the value of child is less swap parent with child
 - Continue doing the same.



Max heapify an array

Let's say we want to make a max heap

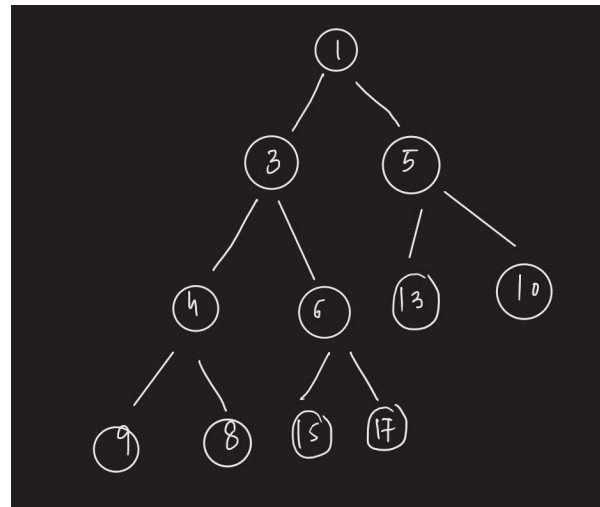
- Start at the last index make sure subtree at that index is max heap
- Move to the previous element
- Recursively apply this step

Example:

11

1 3 5 4 6 13 10 9 8 15 17

https://www.codingninjas.com/studio/problems/build-heap_975375

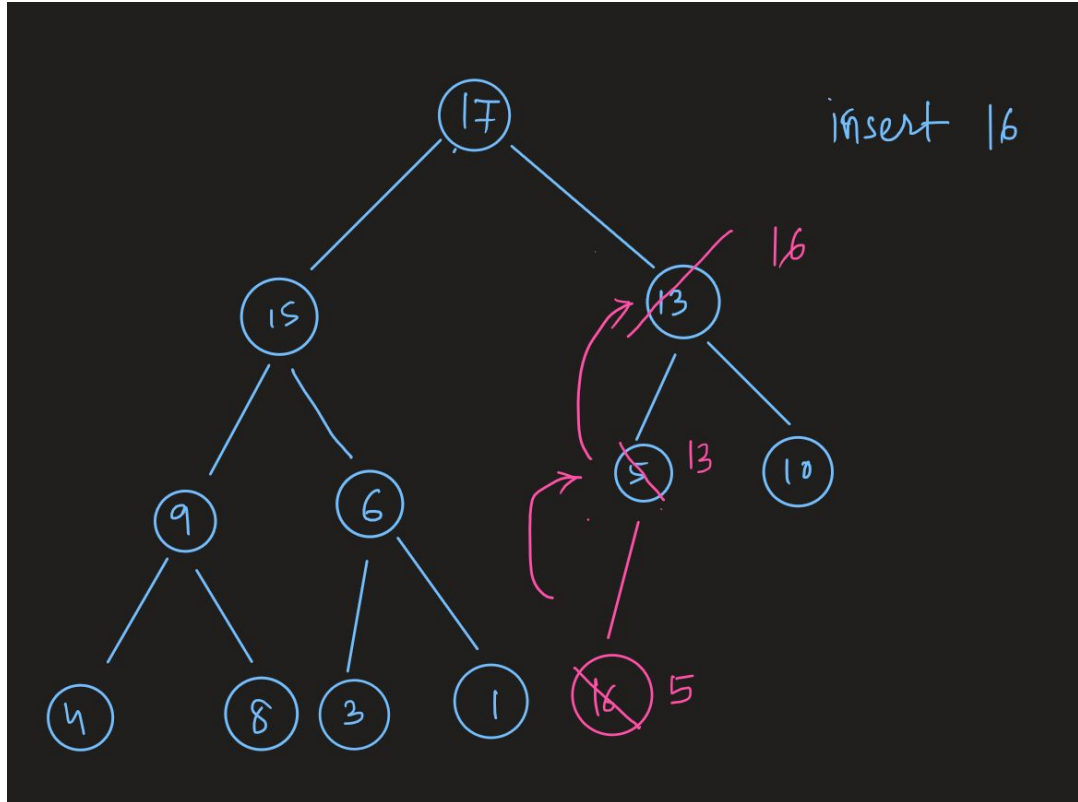


```
1  #include <bits/stdc++.h>
2  void help(int i, vector<int> &arr, int n){
3      int large = i;
4      int left = 2*i+1;
5      int right = 2*i+2;
6      if(left<n&&arr[left]>arr[large]){
7          large = left;
8      }
9      if(right<n&&arr[right]>arr[large]){
10         large = right;
11     }
12     if(large!=i){
13         swap(arr[i],arr[large]);
14         help(large,arr,n);
15     }
16 }
17 vector<int> buildHeap(vector<int> arr, int n)
18 {
19     // Write your code here
20     for(int i=n-1;i>=0;i--){
21         help(i,arr,n);
22     }
23     return arr;
24 }
```

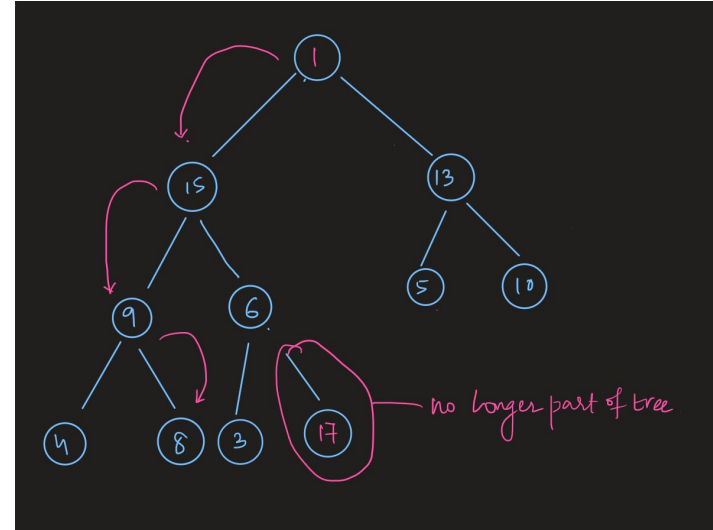
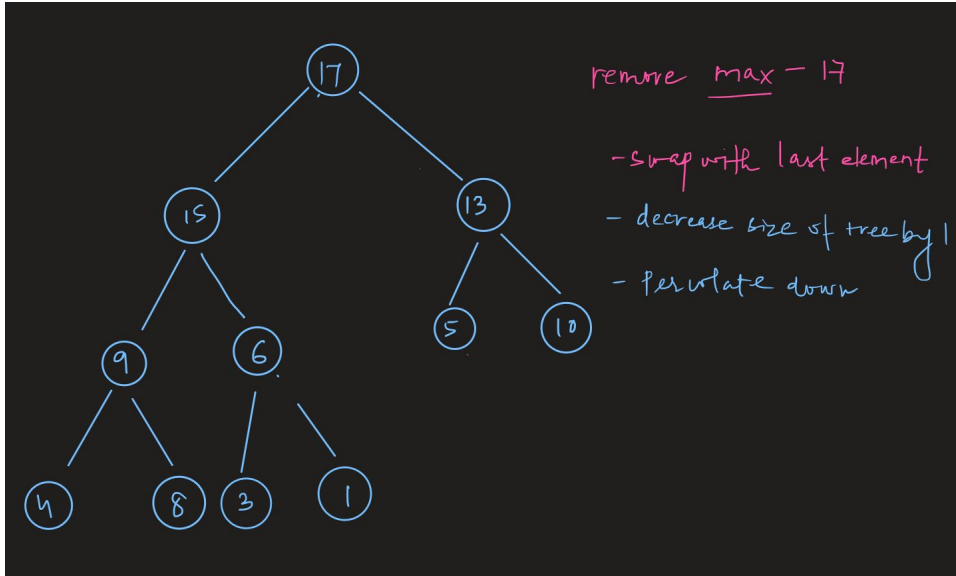
Percolate Up

- We do the following step under percolate up algo (max heap):
 - Pick the value of parent
 - Compare this with the value of current node where you are
 - If the value of parent is less swap parent with child
 - Continue doing the same
- We do the following step under percolate up algo (min heap):
 - Pick the value of parent
 - Compare this with the value of current node where you are
 - If the value of parent is more swap parent with child
 - Continue doing the same

Addition of a value to heap



Removal of a value from heap



C++ stl equivalent of heap

```
std::priority_queue<T, vector<T>, Compare>
```

```
class Compare {  
    public:  
        bool operator()(T a, T b){  
            if(cond){  
                return true;  
            }  
            return false;  
        }  
}
```

Time complexity

- Heapify : $O(N \log N)$
- Insertion of element : $O(\log N)$
- Deletion of element (min / max) : $O(\log N)$

Basic Operation On Binary Heap

- Heap sort :

[https://practice.geeksforgeeks.org/problems/heap-sort/1/?page=1&difficulty\[\]=1&status\[\]=solved&category\[\]=Heap&sortBy=submissions](https://practice.geeksforgeeks.org/problems/heap-sort/1/?page=1&difficulty[]=1&status[]=solved&category[]=Heap&sortBy=submissions)

- Insertion and Deletion. <https://practice.geeksforgeeks.org/problems/operations-on-binary-min-heap/1>


```

14 //Heapify function to maintain heap property.
15 void heapify(int arr[], int n, int i)
16 {
17     // Your Code Here
18     int large = i;
19     int left = 2*i+1;
20     int right = 2*i+2;
21     if(left<n&&arr[left]>arr[large]){
22         large = left;
23     }
24     if(right<n&&arr[right]>arr[large]){
25         large = right;
26     }
27     if(large!=i){
28         swap(arr[i],arr[large]);
29         heapify(arr,n,large);
30     }
31 }
32
33 public:
34 //Function to build a Heap from array.
35 void buildHeap(int arr[], int n)
36 {
37     // Your Code Here
38     for(int i=n-1;i>=0;i--){
39         heapify(arr,n,i);
40     }
41 }
42
43
44 public:
45 //Function to sort an array using Heap Sort.
46 void heapSort(int arr[], int n)
47 {
48     //code here
49     buildHeap(arr,n);
50     for(int i=n-1;i>=0;i--){
51         swap(arr[i],arr[0]);
52         heapify(arr,i,0);
53     }
54 }
55 };
56

```

```

91 //next minimum value at first index.
92 int MinHeap::extractMin()
93 {
94     // Your code here
95     if(heap_size<=0)
96         return -1;
97     if(heap_size==1){
98         heap_size--;
99         return harr[0];
100     }
101     int ans = harr[0];
102     harr[0]=harr[heap_size-1];
103     harr[heap_size-1]=0;
104     heap_size--;
105     MinHeapify(0);
106     return ans;
107 }
108
109 //Function to delete a key at ith index.
110 void MinHeap::deleteKey(int i)
111 {
112     // Your code here
113     if(i<heap_size){
114         decreaseKey(i,INT_MIN);
115         extractMin();
116     }
117 }
118
119 //Function to insert a value in Heap.
120 void MinHeap::insertKey(int k)
121 {
122     // Your code here
123     // if(heap_size==capacity){
124     //     return;
125     // }
126     heap_size++;
127     harr[heap_size-1]=k;
128     int idx = heap_size-1;
129     while(idx>0&&harr[idx]<harr[parent(idx)]){
130         swap(harr[idx],harr[parent(idx)]);
131         idx = parent(idx);
132     }
133 }
134 }
135

```

Question

<https://leetcode.com/problems/kth-largest-element-in-a-stream/description/>

```
1  class KthLargest {
2  public:
3      priority_queue<int,vector<int>, greater<int>> pq;
4      int maxSize;
5      KthLargest(int k, vector<int>& nums) {
6          for(auto i:nums)
7              pq.push(i);
8          maxSize = k;
9          while(pq.size()>k){
10             pq.pop();
11         }
12     }
13
14     int add(int val) {
15         pq.push(val);
16         if(pq.size()<maxSize)
17             return -1;
18         while(pq.size()>maxSize){
19             pq.pop();
20         }
21         return pq.top();
22     }
23 };
24
25 /**
26  * Your KthLargest object will be instantiated and called as such:
27  * KthLargest* obj = new KthLargest(k, nums);
```