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11/23/22
HW5

Problem 1

3

unusual

1/5 letters is u

1 2 3 4 5

cases: {3u, n, s, a, l}

(1) Take all 5 different letters

$\binom{5}{5} \cdot 5! = 120$

↓

* ways to arrange

(2) 1 pair of the one letter that shows up multiple times

2 u's & 3 diff

$\binom{1}{1} \cdot \binom{4}{3} \cdot \frac{5!}{2!} = 4 \cdot 60 = 240$

2! of the pairs ← ways to have 3 copies of 2 identical ways for of letters

(3) 3 u's & 2 diff

$\binom{1}{1} \cdot \binom{4}{2} \cdot \frac{5!}{3!} = 6 \cdot \frac{120}{6} = 120$

3! of the pair

$= 120 + 240 + 120 = \boxed{480}$

Problem 2

2 pairs of each value → 13 values → 26 pairs
+ diff from above bc above order matters
* 4 cards of each value
* 13 diff values

$$\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot \binom{44}{1} = 123552$$

↓ for this value

↓ for second value

Problem 3

1 couple at one song

$\binom{16}{1}$ → doesn't even matter bc always one song

stars & bars at least (15 songs & 6 carts)

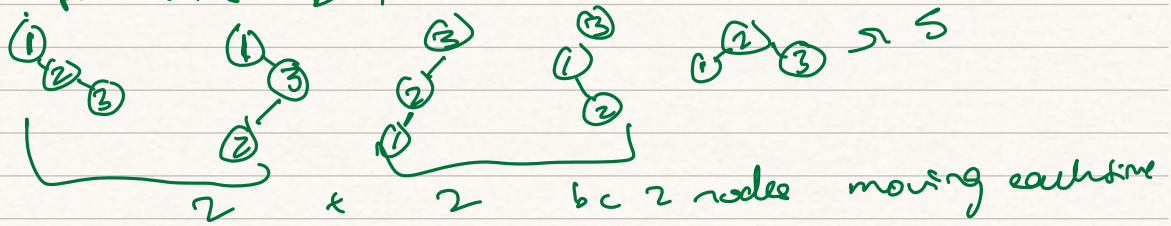
15 stars & 5 bars

$1 \cdot \binom{20}{15} = \boxed{15504}$

Problem 4

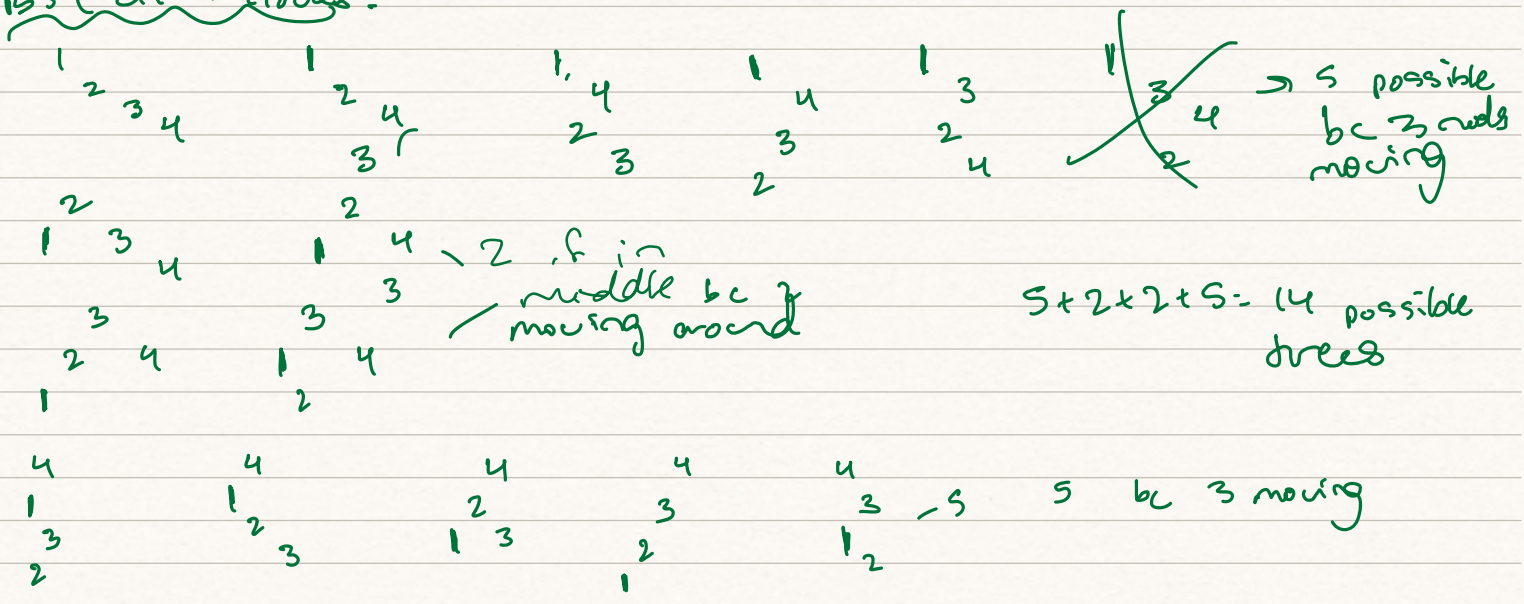
BST of two nodes: 2 possibilities

BST of 3 nodes:



2 × 2 bc 2 nodes moving around

BST w/ 4 nodes:



5 possible bc 3 nodes moving

2 if in middle bc 3 moving around

5+2+2+5 = 14 possible trees

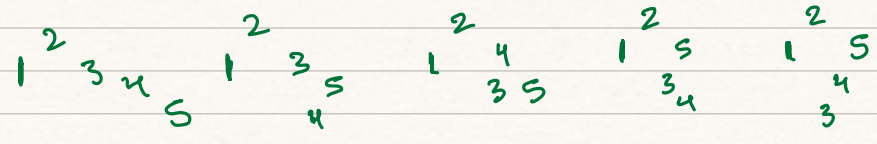
5 bc 3 moving

BST w/ 5 nodes

w/ 1 on top: 14 options (bc 14 nodes & one on top like head as always one side of mountain)

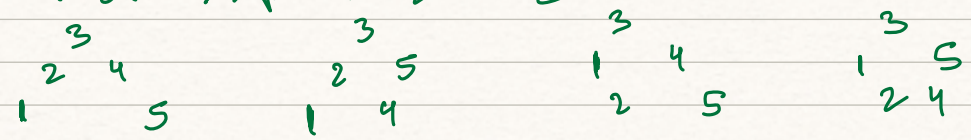
w/ 5 on top: 14 options

w/ 2 on top: 5 options bc end up w/ 3 moving around

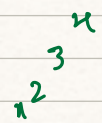


5·1

w/ 3 on top → 2 · 2

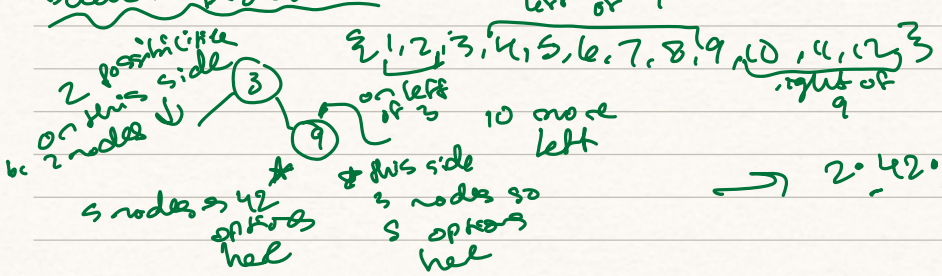


w/ 4 on top: 5 options bc 3 moving around



14+14+5+4+5 = 42 possible trees

Back to problem:



→ 2 · 42 · 5 = 420 possible trees

Problem 5

C1: 3 nurses & 10 ppl

$10 - 3 = 7$ ppl

7 stars 2 bars

$$10 \binom{9}{2} = 360$$

C2: 4 nurses & 10 ppl

$10 - 4 = 6$ ppl

6 stars 3 bars

$$10 \binom{9}{6} = 840$$

$840 + 360 = \boxed{1200}$