Saihaj Gulati
11/23/22
HW5

Problem 1

Probability is $\frac{|P|}{|E|}$

probability of choosing 8 distinct of 15

$= \frac{\text{ways to choose 8 distinct}}{\text{ways to choose 8 w repl}} = \frac{{}^{15}P_8}{15^8}$

$= \boxed{0.1012}$

Problem 2

- even
- start w/ 2 odd digits
- all digits unique
- 5 digit #

*generate 8
*get 5 that match criteria

possible #s that meet reqs: even means odd options for first digit

$5 \cdot 5 \cdot 4 \cdot 7 \cdot 6$

even options for ones; odd but not first; not ones or first two; unique too

All possible #s: 0-99999 → 100000 #s

Prob of # that meet criteria: $\frac{5 \cdot 5 \cdot 4 \cdot 7 \cdot 6}{100000} = 0.042$

Bernoulli Trial w/ $p = 0.042$

$$P = (0.042)^5 \cdot (1-0.042)^3 \binom{8}{5} = \boxed{6.485 \times 10^{-6}}$$

Problem 3

$\{1, 2, 3, \underbrace{4, 5, 6}\}$

P(A) = At least 2/3 dice show 4+

$= P(\text{exactly } 2 \text{ dice } 4+) \cup P(\text{exactly } 3 \text{ dice } 4+)$

$$= \binom{3}{2}\left(\frac{3}{6}\right)^2 \cdot \left(\frac{3}{6}\right)^1 + \binom{3}{3}\left(\frac{3}{6}\right)^3\left(\frac{3}{6}\right)^0$$

$= 0.5$ ↗

P(B) = all 3 same value

$$= \frac{\text{ways to get all 3 same value}}{\text{ways to rolls of 3 die}}$$

$$= \frac{6 \cdot 1 \cdot 1}{6 \cdot 6 \cdot 6} = \frac{6}{216} = \frac{1}{36}$$

P(A|B) = At least 2/3 dice show 4+ given all are same

$P(A|B) = \frac{3}{6} = 0.5 = P(A)$ → independent

$\{1, 2, 3, \underbrace{4, 5, 6}\}$

6 cases of all same → 3 of them show 4+

$\boxed{\text{Independent}}$

Problem 4 (1?) ← it's a typo on codio

X is # of hands to get a flush

$P(X=1)=$ prob of 5/5 of same suit

$= \dfrac{\binom{4}{1} \cdot \binom{13}{5}}{\binom{32}{5}}$ → all ways choosing same hand & 5 of 13 from that hand

→ all ways to select 5

≈ 0.00198 is probability of flush

$P(X=2)= 0.00198 \cdot (1-0.00198) \cdot \binom{1}{1}$ ← not $\binom{2}{1}$ bc is prob that get in that one specifically

$P(X=3) = 0.00198 \cdot (1-0.00198)^2 \cdot \binom{1}{1}$

$E(X) = \sum_{i=1} i \cdot (1-p)^{i-1} \cdot p = \dfrac{1}{(\cancel{1}-(\cancel{1}-p))^2} \cdot p = \dfrac{1}{p}$

so, $\dfrac{1}{\sim 0.00198} =$ $\boxed{504.85}$ is expected # of hands

used actual number, not this rounded #

Problem 5 (2?) → it's a typo on codio

Team won 4 out of 5 → 80%.

$\boxed{\text{Chance to play games is } 75\%.}$

when play → 70%. when not → 50%.

If plays:

$$(0.7)^4 \cdot (0.3)^1 \binom{5}{4} = \underset{\text{prob win}}{0.36015}$$

If doesn't:

$$(0.5)^4 \cdot 0.5 \binom{5}{4} = 0.15625$$

$$P(\text{played} \mid \text{won 4 out 5}) = \frac{P(\text{played \& won 4 of 5})}{P(\text{won 4 out of 5})}$$

$$= \frac{P(\text{won 4 of 5 given (played)}) \cdot P(\text{played})}{P(\text{won 4 of 5})}$$

$$= \frac{P(\text{won 4 of 5 given (played)}) \cdot P(\text{played})}{\underbrace{P(\text{won 4 of 5} \mid \text{played}) \cdot P(\text{played}) + P(\text{won 4 of 5} \mid \text{not played}) \cdot P(\text{not played})}_{= \text{total prob}}}$$

$$= \frac{0.36015 \cdot 0.75}{0.36015 \cdot 0.75 + 0.15625 \cdot 0.25} = \boxed{0.874}$$