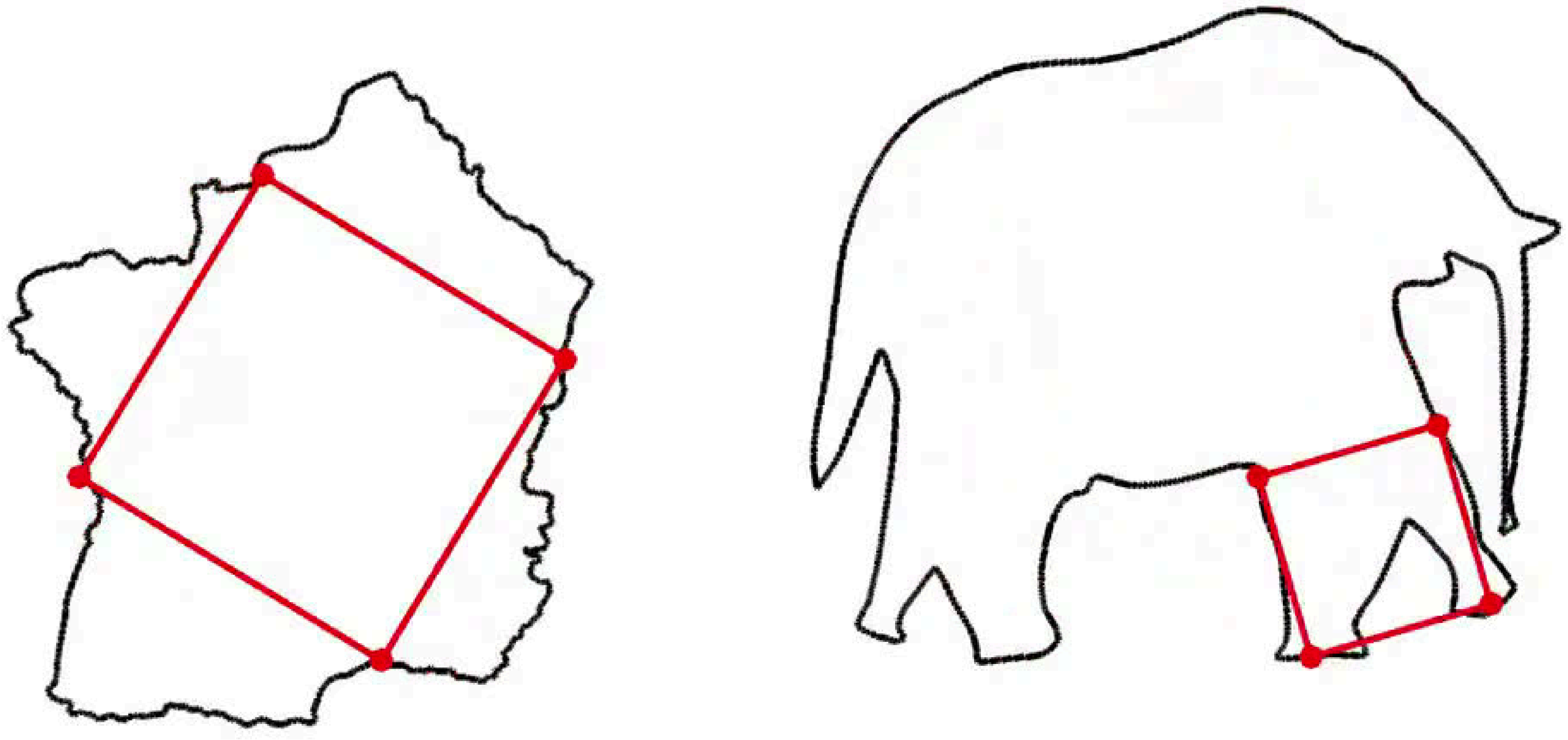
Inscribed Square Problem

(unsolved)



curves which are smooth are known to always contain inscribed squares

Inscribed Rectangle Problem

A slightly weaker question of Inscribed Square problem

“

It’s sort of weird; it was just the right idea for this problem.

”

Richard Schwartz, Brown University

“

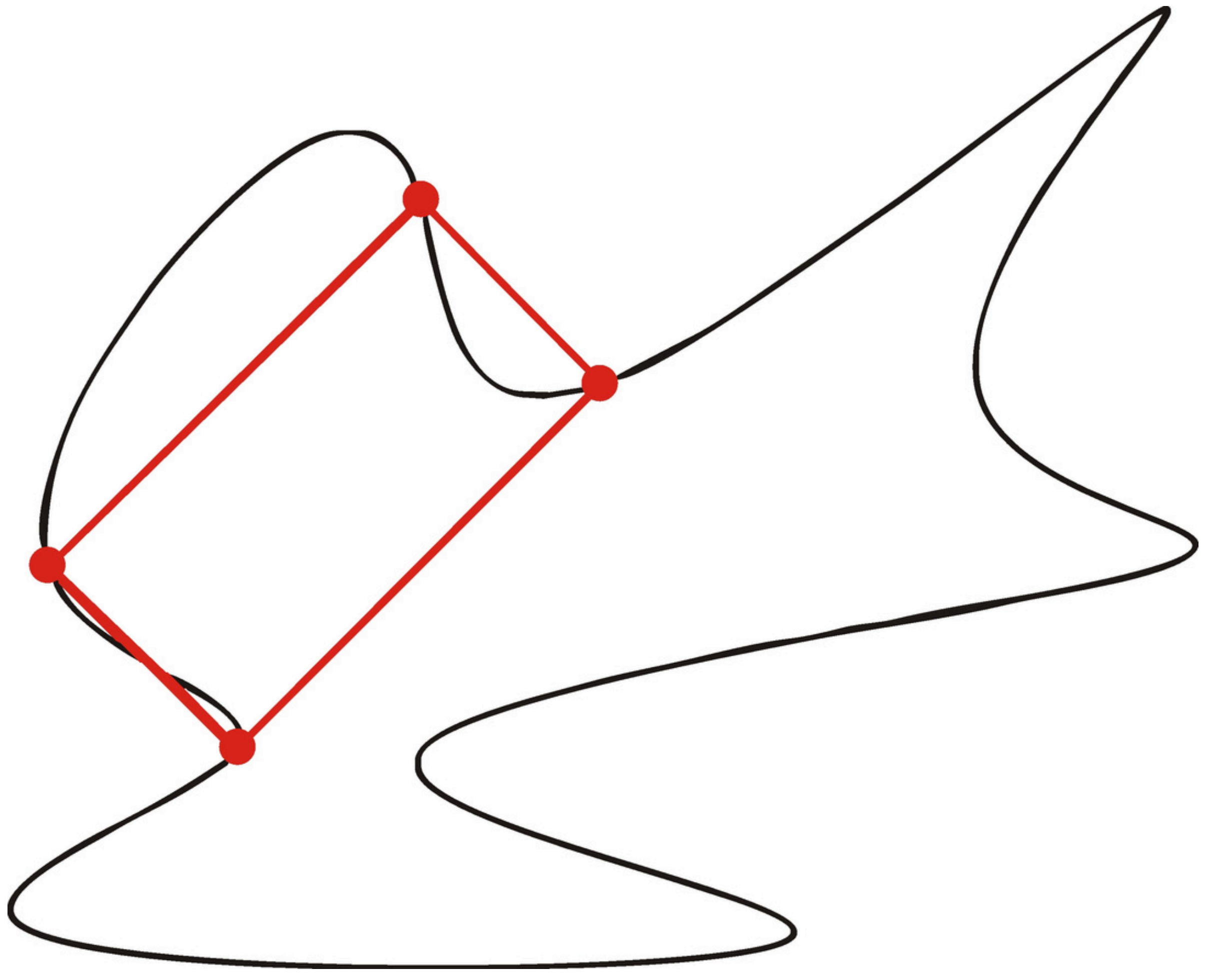
There was this pivotal insight to look at the problem from the perspective of symplectic geometry. That was just a game changer.

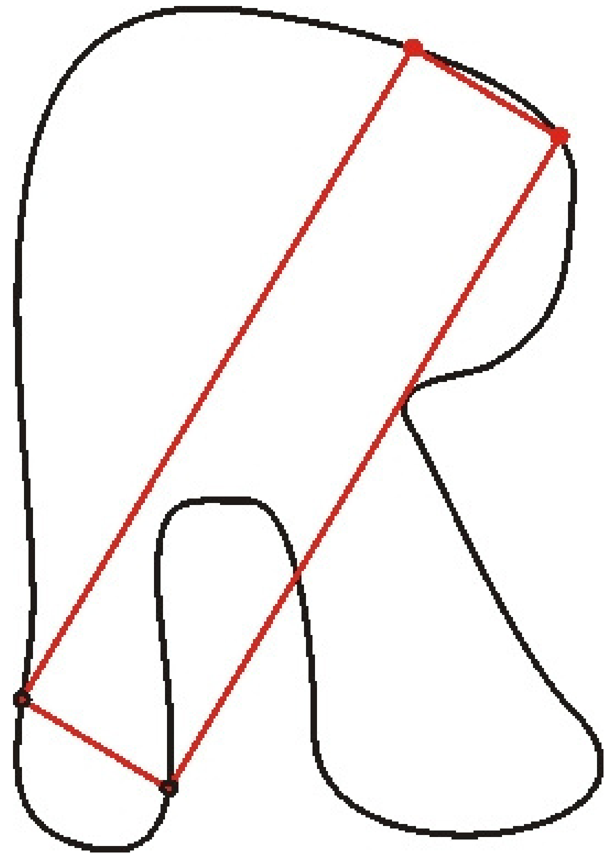
”

Joshua Greene, Boston College

Problem : For any *Jordan curve* C there exist a rectangle whose vertices lie on C

A Jordan curve is a simple closed curve in the plane, which means that it is a continuous, non-self-intersecting loop

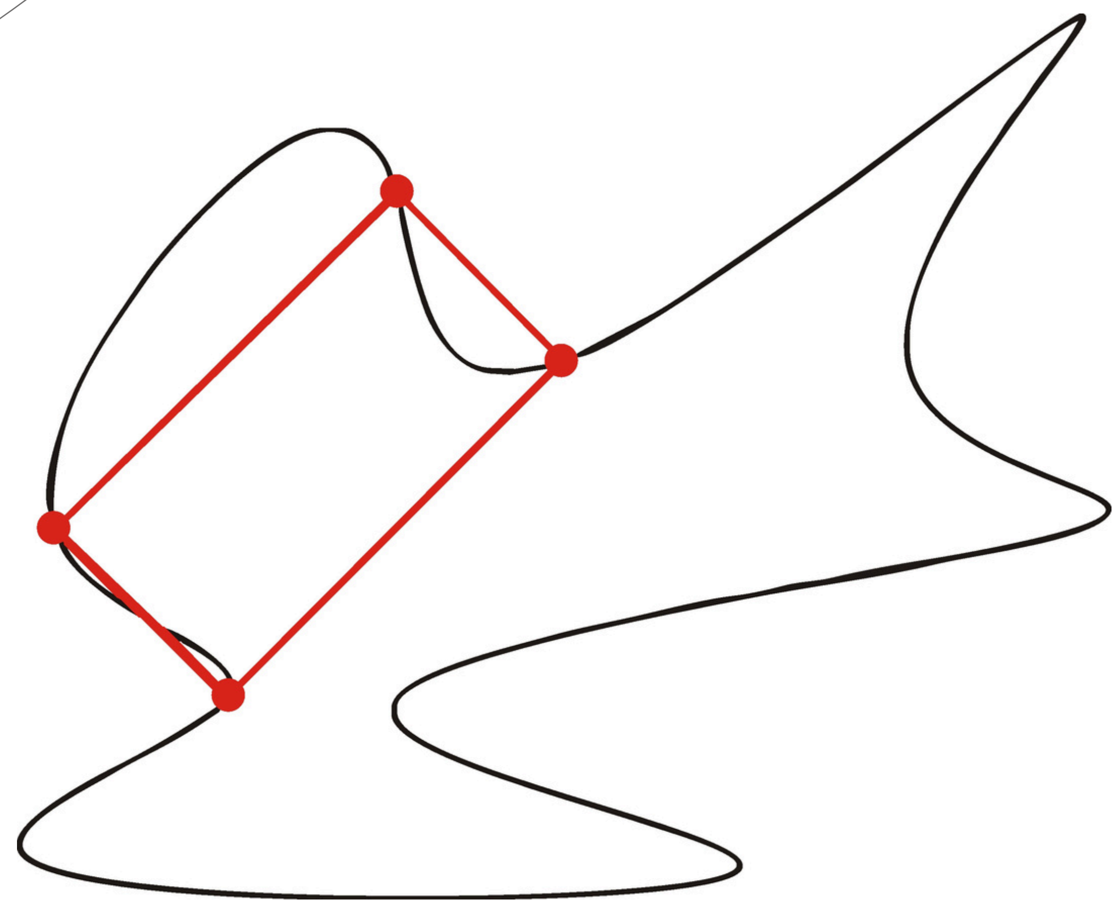


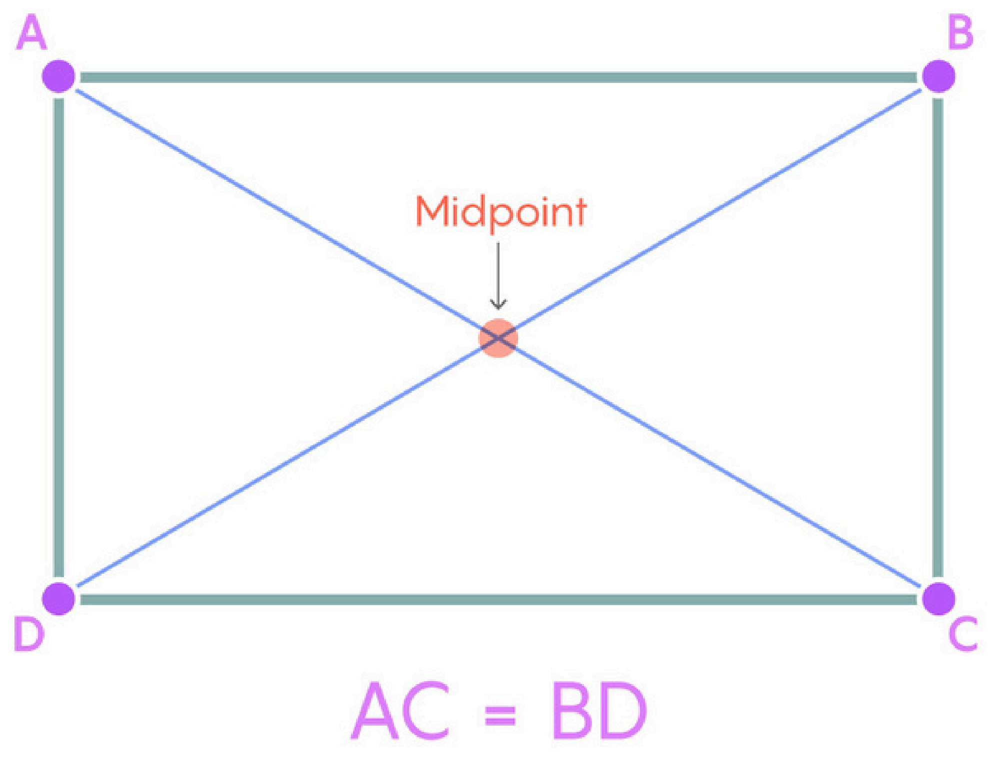


To Prove : there exist a rectangle, whose vertices lie on closed continuous curve C

Proof :

Consider a jordan curve C lies on XY plane,



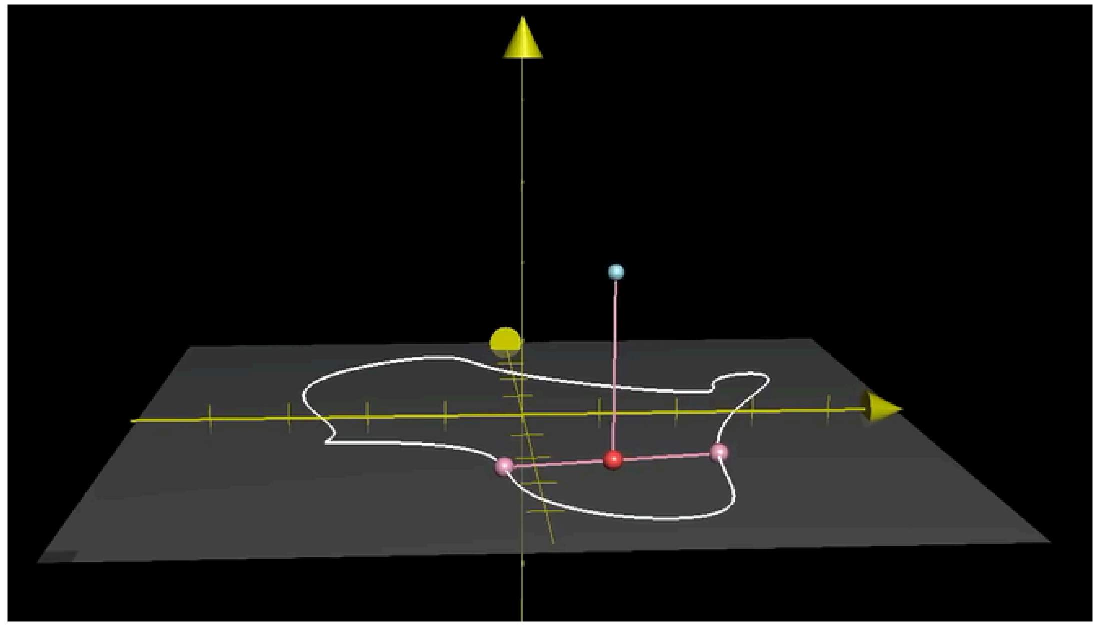


If rectangle R exist on curve C whose vertices lies on C, then the diagonals of R can be represented as two equal length line segment whose end points lies on curve C.

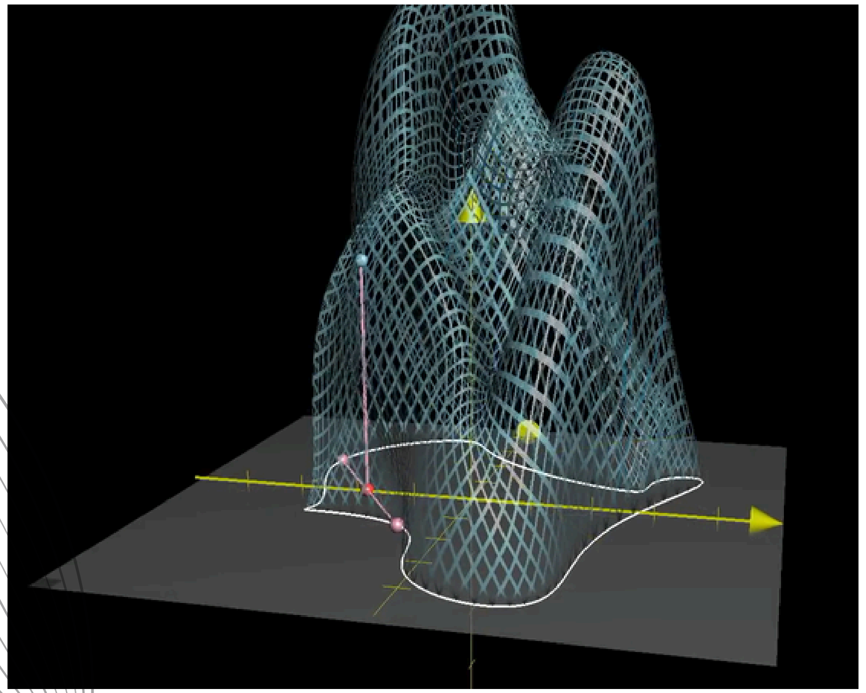
We only have to prove that there exist two distinct (unordered) pair of points on curve C form two line segments having equal length and intersect at their mid-point.

Let denote arbitrary points on C as $P_1 = (x_1, y_1, 0)$ and $P_2 = (x_2, y_2, 0)$

Define a point P,



$$P = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right)$$



Then, surface S = collection of all points in 3d space formed by taking every unordered pair of points from curve C

The boundary of the surface S consist of points on curve C

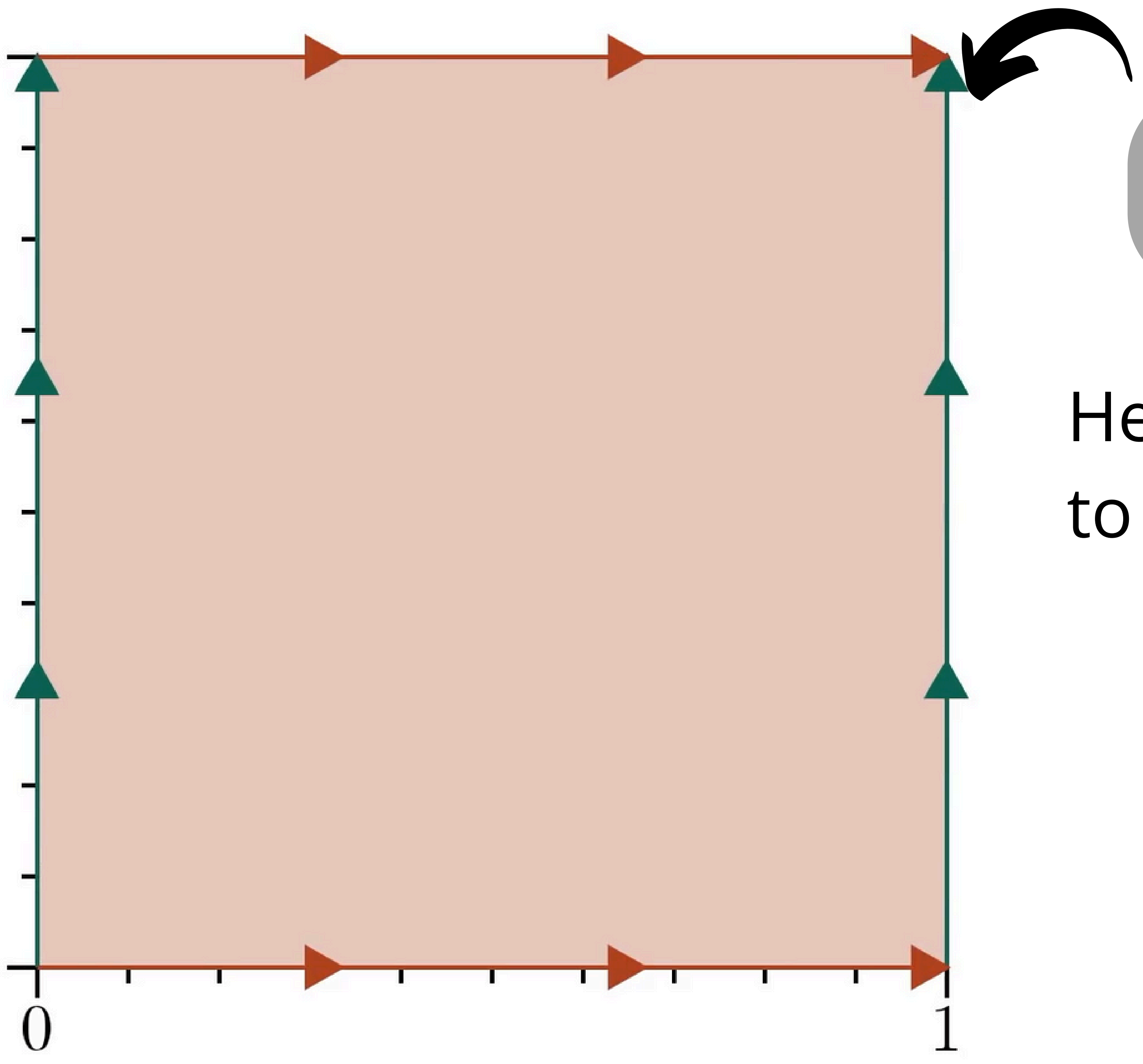
Now, we only have to prove that two distinct pairs (unordered) of points have same point on surface S

Let define a continuous function $f:[0,1]\to R^2$ such that $f(0)=f(1)$

Now define, $\lambda : C\times C\to[0,1]\times[0,1]$ as,
for any two points p and q on the loop L, let m and n be their corresponding parameters on the loop
i.e. $f(m)=p$ and $f(n)=q$

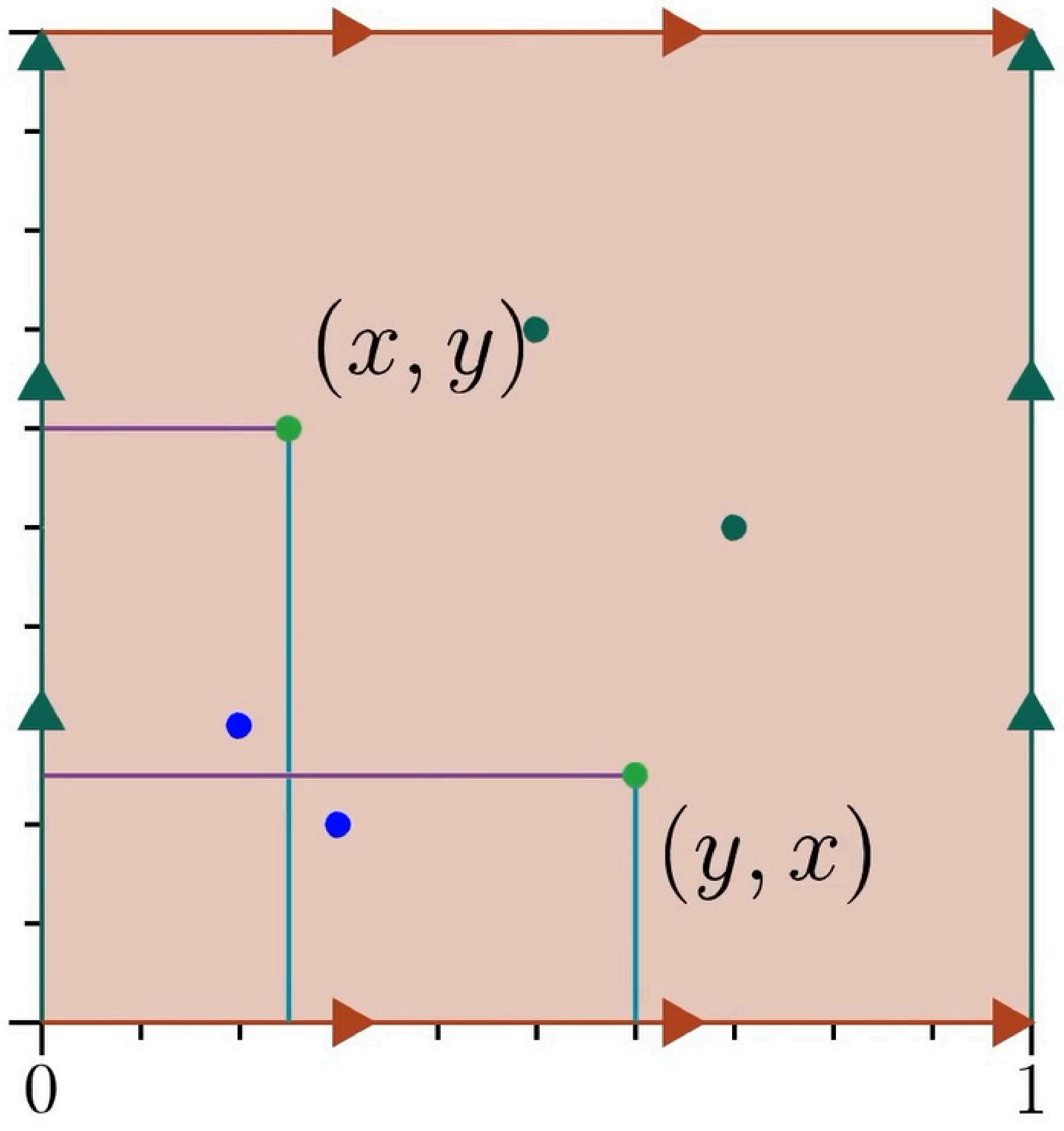
then the mapping $\lambda : (p,q)=(m,n)$ establishes a bijection between the set of points on the loop and the unit square.

this bijection preserves the **topological properties** *(continuity and homeomorphism)* loop and the unit square

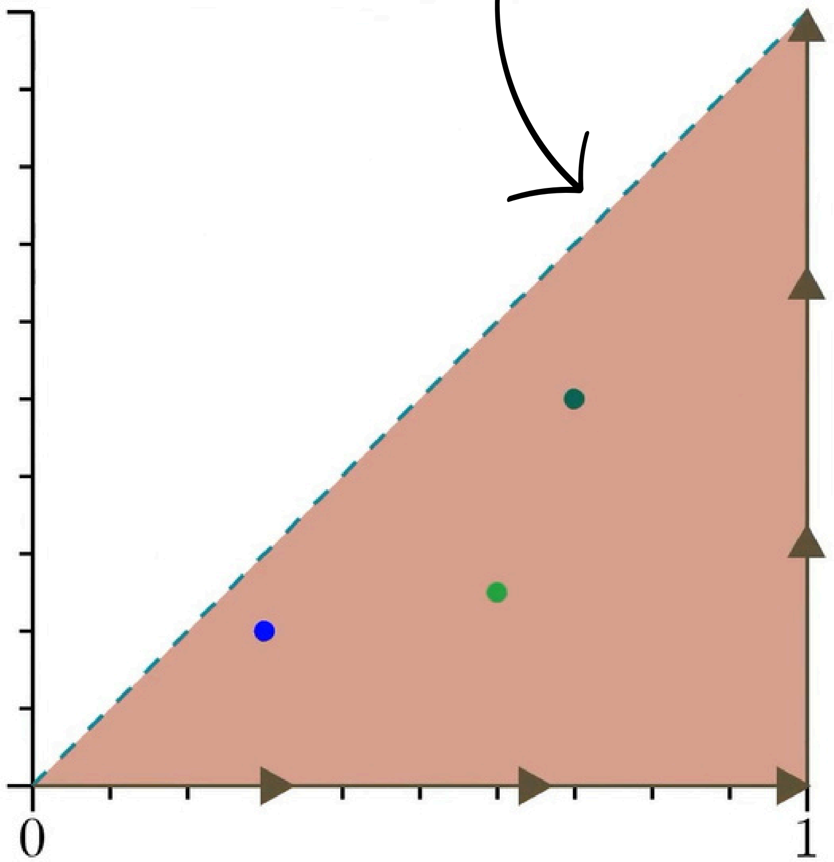


We also consider the orientation while mapping the pairs of loop onto unit square

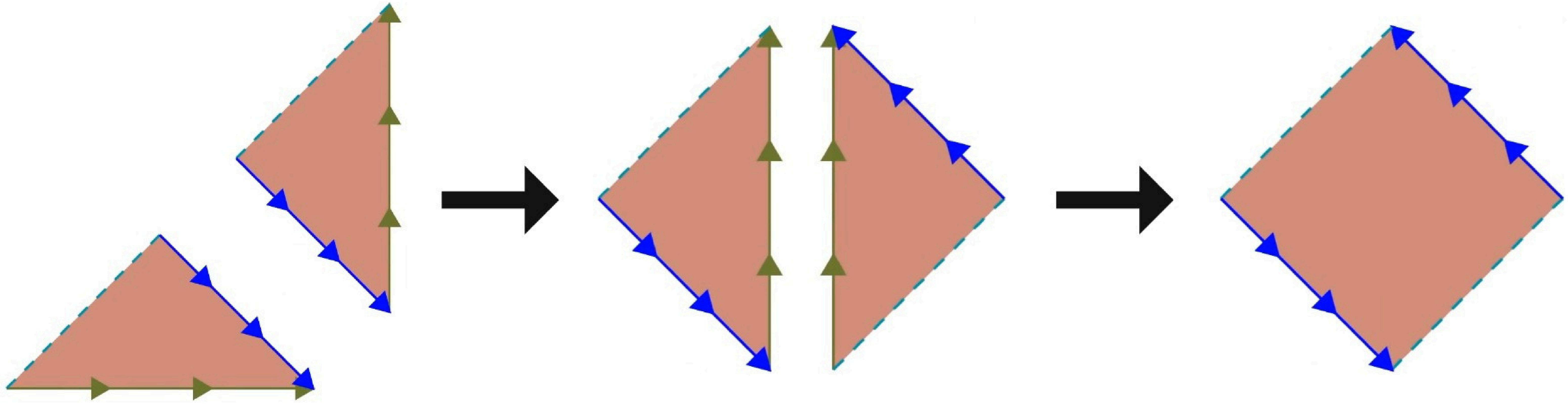
Here, Opposite edges of unit square corresponds to same points on loop C



Also, here (x,y) and (y,x) represents same unordered pair of points on the loop C *this dotted line represents (x,x) points i.e. Single points of loop*

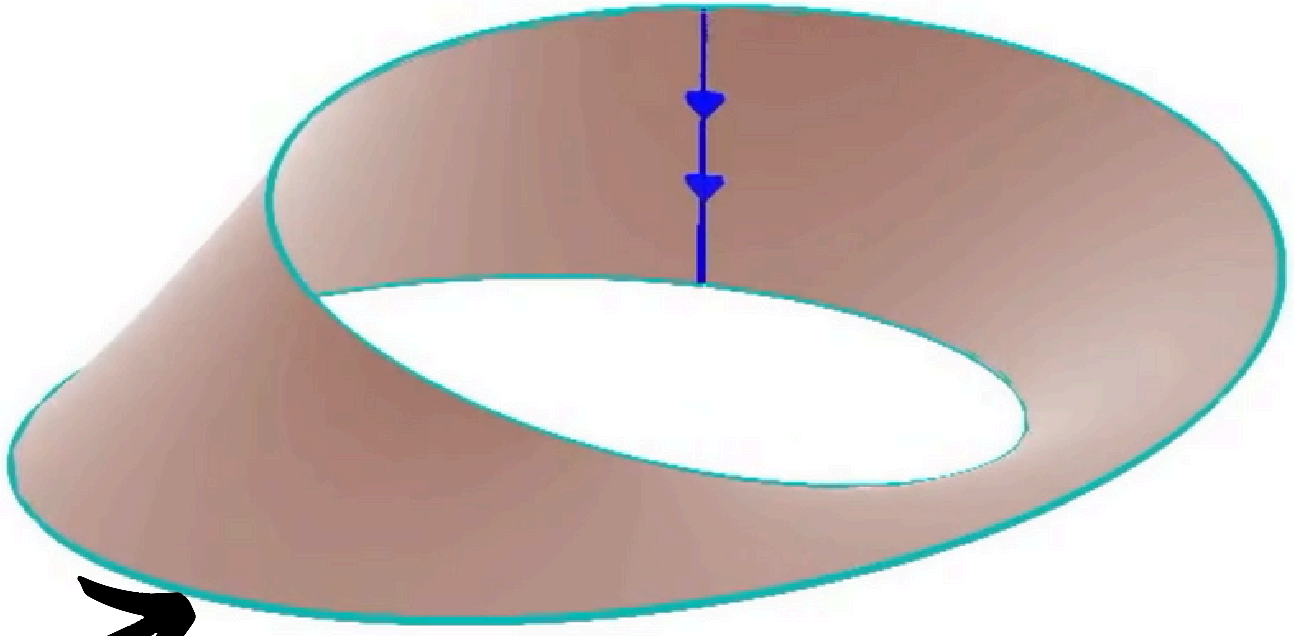


- Now we are considering unordered pair of points
- make a new diagonal cut
- To glue back what we just cut, we don’t simply connect the edges. We have to make a twist! Doing this in 3D space, the shape we get is a ____!!!!!



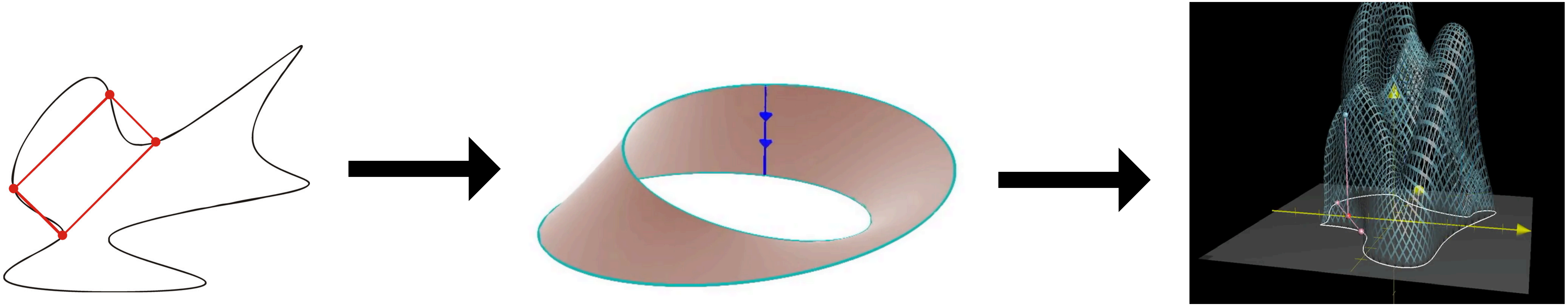
Möbius strip

the boundary line of mobius strip corresponds to the points of loop C



there is continuous 1-to-1 association between points on the Möbius strip and pairs of points on the loop

i.e. the surface which represents all possible unordered pairs of points on a loop is the Möbius strip.



Now, The Möbius strip can be continuously mapped onto a surface, so that every point on the Möbius strip aligns with its corresponding point on the 3D surface. where the boundary of mobius strip maps perfectly on boundary of S

Here, the strip intersects itself (mapping is not one-one) *(why?)*

it means at least two distinct pairs of points correspond to the same output on this surface.

Hence, there exist two pairs of points corresponds to same midpoint and have equal length

Therefore, there exist a rectangle, whose vertices lie on closed continuous curve C

(How did we know that the strip intersects itself?)

(Is our intuition is correct ?)

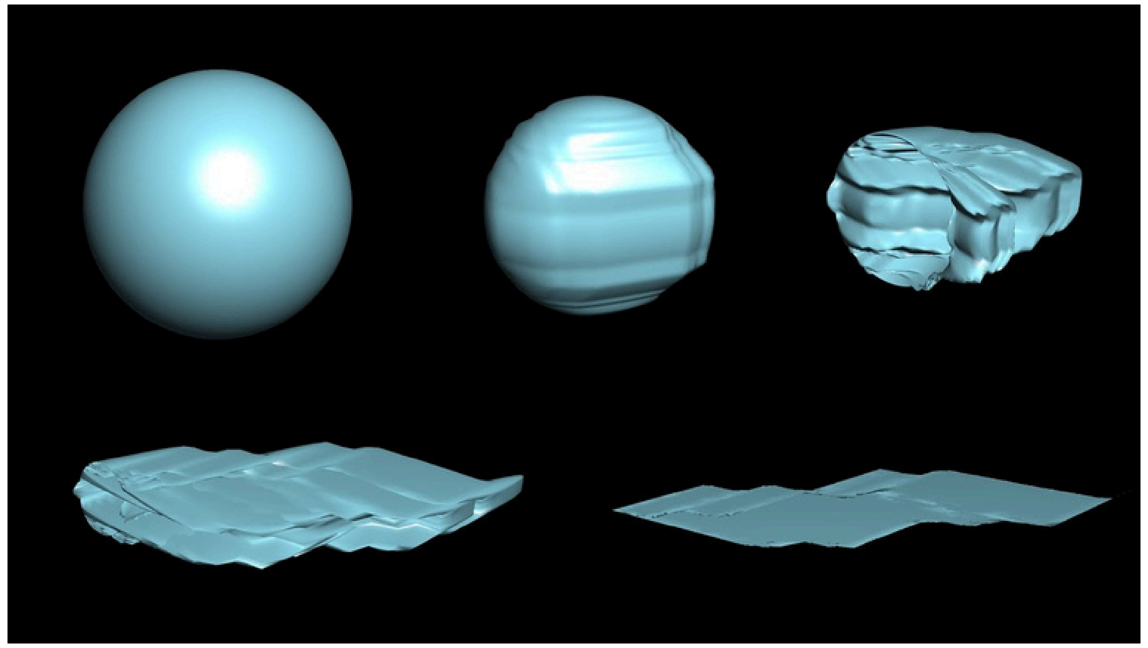
(for that we have to understand some concepts)

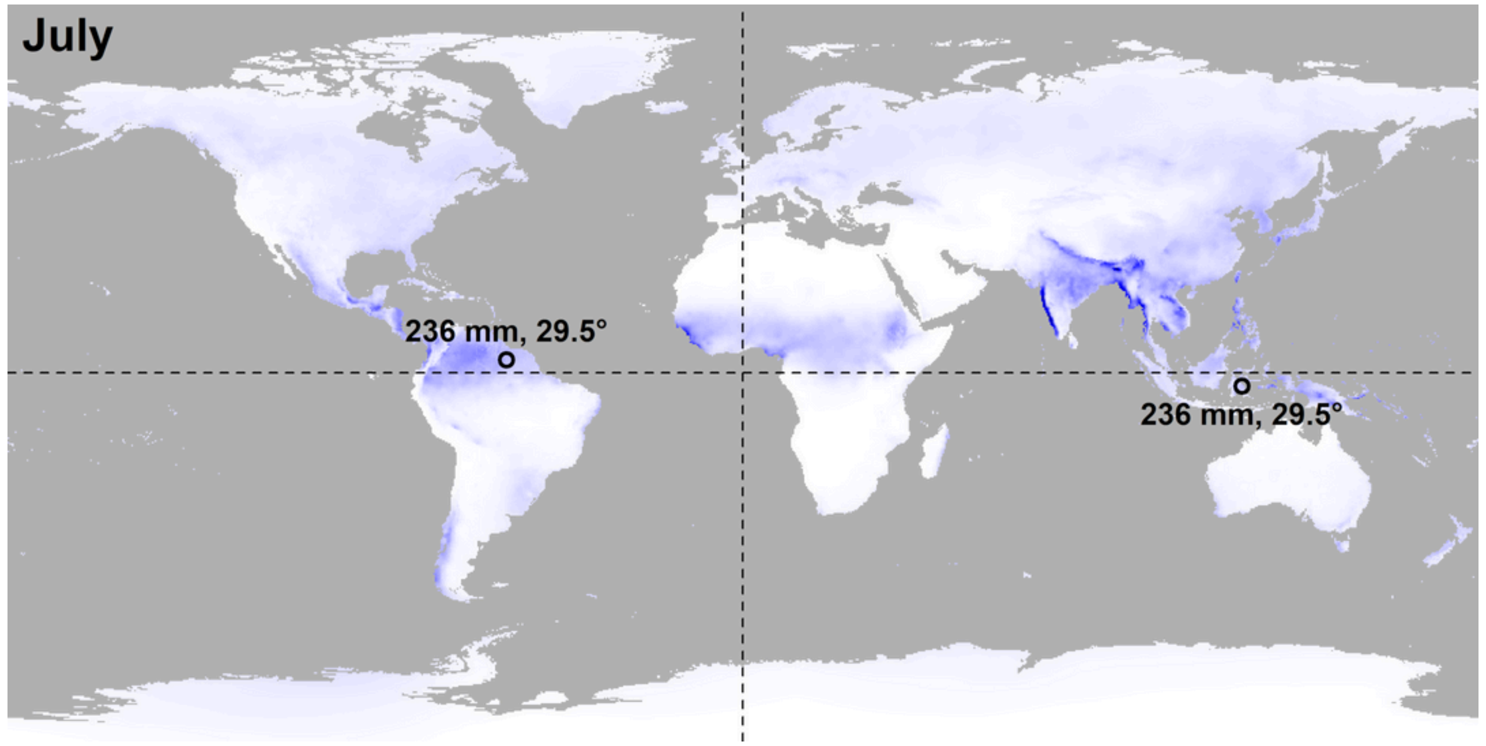
One interesting theorem

- **the Borsuk–Ulam theorem** states that every continuous function from an n-sphere into Euclidean n-space maps some pair of **antipodal** points to the same point.

Two points on a sphere are called antipodal if they are in exactly opposite directions from the sphere's center.

An Example :





At any moment, there is always a pair of points on exactly opposite sides of the Earth's surface which have equal temperatures and equal barometric pressures

BORSUK-ULAM THEOREM ON A MÖBIUS STRIP

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The Borsuk-Ulam theorem stands for a general principle able to describe a large amount of brain functions. However, when managing the neurodata extracted from EEG and fMRI, the BUT method, based on projections and mappings between different functional brain dimensions, in impractical and computationally expensive. Here we show how the BUT's antipodal features with matching description (say, two brain areas with the same values of entropy) can be described in terms of closed paths on aMöbius strip. This allows to assess the dynamics of the nervous system in terms of trajectories taking place in a well-established, easily manageable phase space.

The Borsuk-Ulam theorem (BUT) has been proven useful in the description of several brain activities and functions. The BUT suggests that the neural properties endowed in the physical and biological spaces of the brain can be translated to abstract mathematical ones, and*vice versa*. To make a few examples, Tozzi and Peters (2016b) studied the logistic maps of neural chaotic activities and showed how some nonlinear dynamics can be described in purely linear terms. It has also been showed how the BUT is able to unveil the mystery of (spatial) fractals and (temporal) power laws (Tozzi and Peters, 2016b), that are ubiquitous during brain oscillations (Friston and Ao, 2012; Beggs and Timme, 2012). Tozzi and Peters (2016b) proposed that a symmetry stands for two features with matching description lying in higher dimensions, while a symmetry break for a single feature lying one dimension lower. These symmetries, described in terms of BUT, have been correlated with neural thermodynamic activity and energy requirements/constraints during spontaneous and evoked brain activity (Tozzi and Peters, 2017b). A BUT framework allows also to understand how the brain perceives "sharp" objects and solves the Kullback-Leibler perceptual divergence (Tozzi and Peters, 2016b). Further, it has been shown how a symmetric, topological approach is able to elucidate the puzzling phenomenon of multisensory information integration

RESULTS

If we embed the trajectories of two BUT matching functions x and –x (Figure A) on a Möbius strip, we achieve a closed, continuous loop where the opposite functions are allowed to travel along constrained trajectories. We can easily see that a piece of strip of a given length, standing for a time interval, displays both x and –x at the same time (Figure B). The BUT dictates are preserved because, even in the two matching features are simultaneous, they do not have points in common: indeed, they lie on the opposite surfaces of the strip.

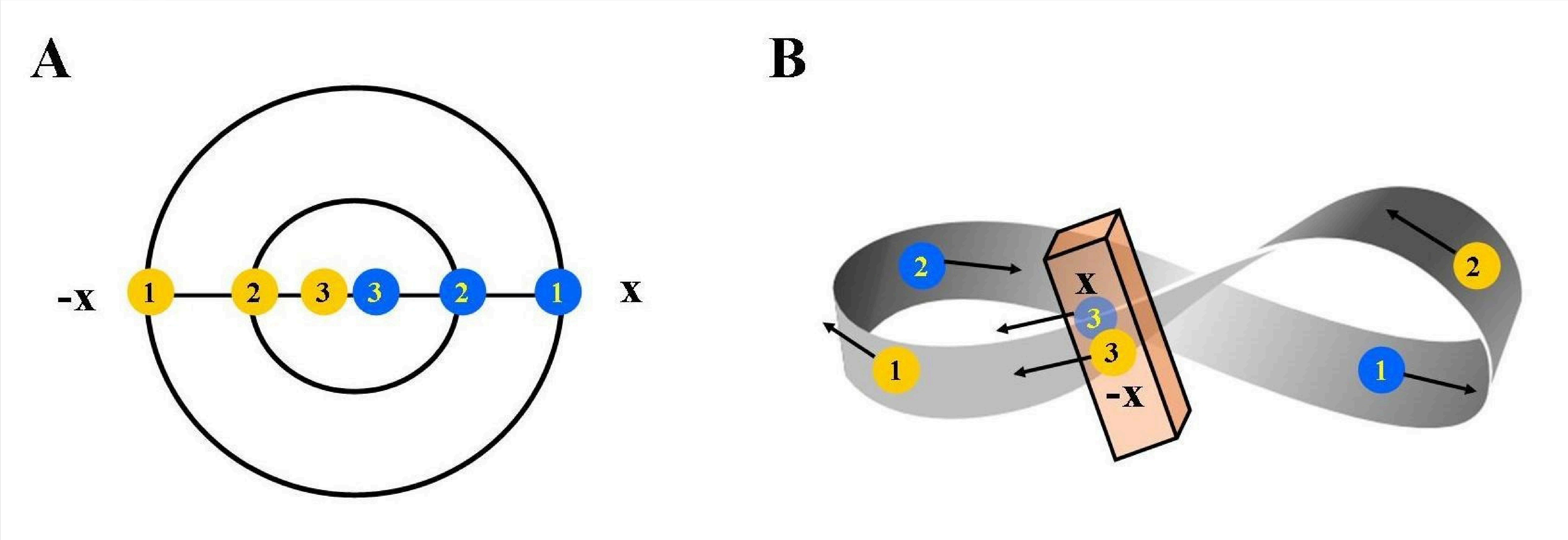


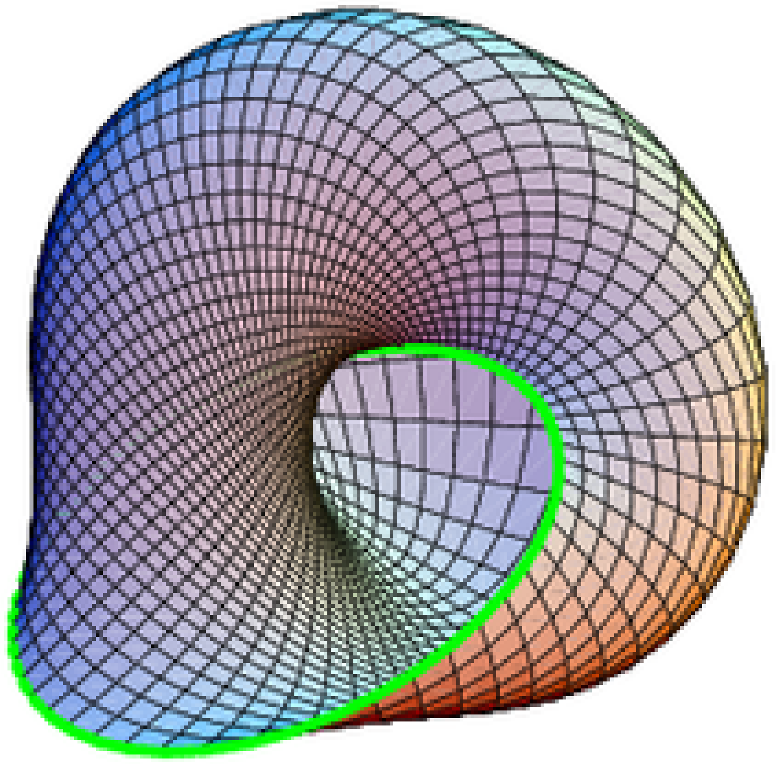
Figure. Transport of the BUT theorem on aMöbius strip.Figure A. Changing the radius of the hypersphere makes the antipodal points more or less close. Close to the center, the two points (marked with the number 3) are almost superimposed.Figure B. The movements of the antipodal points are described in terms of

From that we can comprehend,

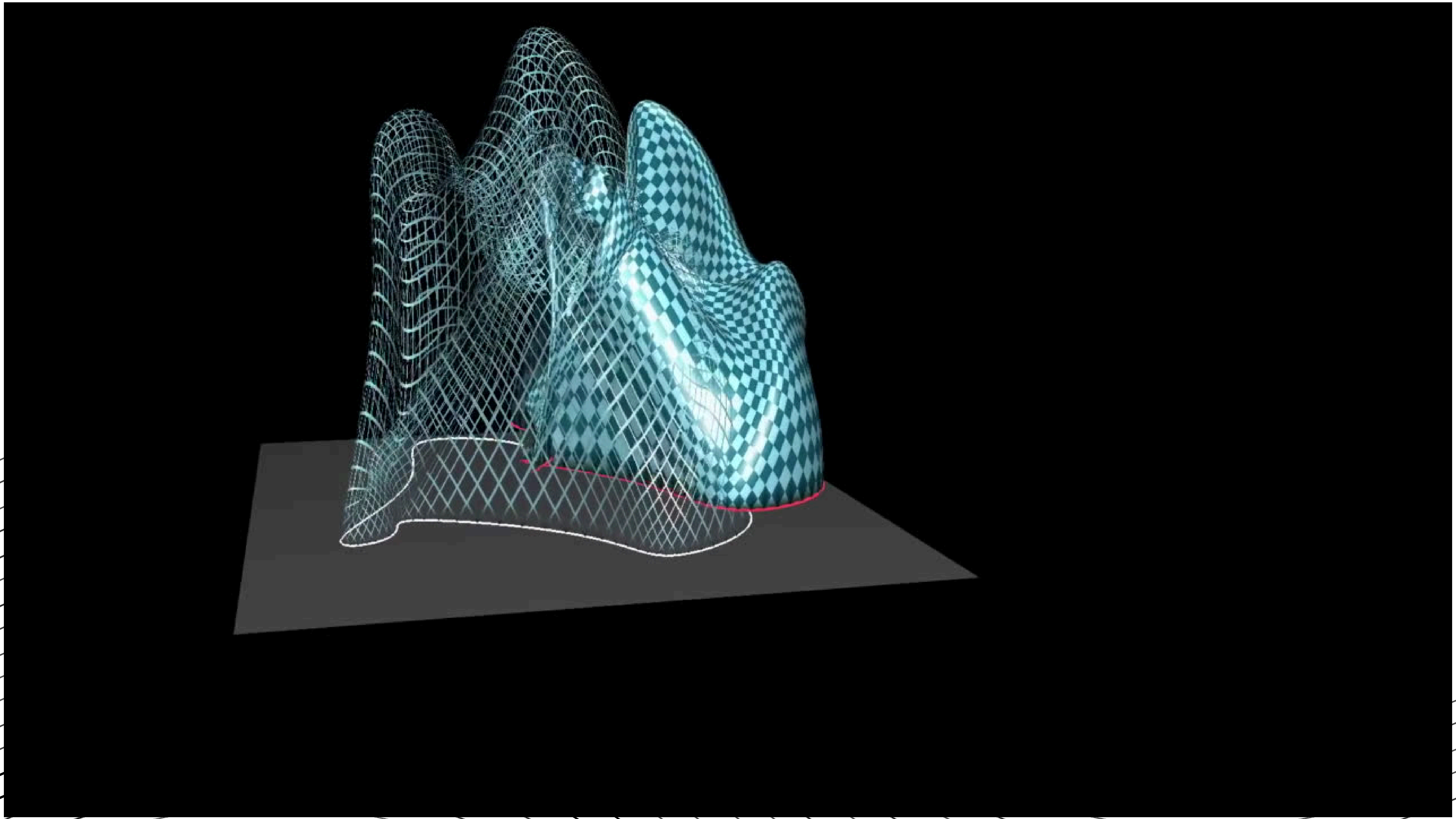
Any continuous mapping from the Möbius strip into the *half 3D space* where the $z \geq 0$ which maps the boundary of the Möbius strip to the xy-plane must have two inputs which map to the same output

Hence, the mapping cannot be injective

(why half space?)
because there exist The Sudanese Möbius band (one example of a Möbius strip) with its edge embedded perfectly in a 2D plane



A visual of continuous mapping (for better understanding)



THANK YOU

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