Single-Stage Electric Coilgun: Theoretical Modelling

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Abstract

This paper presents a theoretical study of a single-stage coilgun. Both the electrical discharge characteristics and magnetic field-based acceleration of a ferromagnetic projectile are examined. Key cases such as overdamped, critically damped, and underdamped (oscillatory) discharges are analyzed with derived formulas for current, force, acceleration, and velocity.

1 Introduction

A coilgun is an electromagnetic launcher that uses magnetic force generated by a current-carrying solenoid to accelerate a ferromagnetic projectile. In this study, we investigate the dynamics of a single-stage coilgun with a DC capacitor-based pulse discharge.

Let:

- R = resistance,
- L' = inductance,
- C = capacitance,
- E = supply voltage,
- $q(t)$ = instantaneous charge on capacitor.

2 Electrical Discharge Model

2.1 Capacitor Discharge Equation

When the switch is set to discharge mode, the capacitor discharges into the RL circuit. Using Kirchhoff's laws:

$$L'\frac{di}{dt} + Ri + \frac{q}{C} = 0 \tag{1}$$

2.2 Generalized Charge Equation

$$\frac{d^2q}{dt^2} + b\frac{dq}{dt} + \omega_0^2 q = 0 \tag{2}$$

where:

$$b = \frac{R}{L'}, \quad \omega_0^2 = \frac{1}{L'C}$$

General solution:

$$q(t) = C_0 e^{(-b+n)t/2} + D_0 e^{(-b-n)t/2}$$

2.3 Discharge Cases

Overdamped: $b^2 > 4\omega_0^2$; real exponential decay.

$$i(t) = \frac{d}{dt}q(t) = e^{-\frac{bt}{2}}\left(Cn_0 e^{nt} + Dn_0 e^{-nt}\right)$$

Critically damped: $b^2 = 4\omega_0^2$;

$$q(t) = q_0(1 + bt)e^{-bt}, \quad i(t) = q_0 bte^{-bt}$$

Underdamped (Oscillatory): $b^2 < 4\omega_0^2$;

$$q(t) = q_0 e^{-bt}\sin(\omega t + \alpha), \quad i(t) = -q_0\omega e^{-bt}\sin(\omega t)$$

3 Magnetic Force and Acceleration

3.1 Lorentz Force

$$\vec{F} = \vec{i} \times \vec{L} \cdot \vec{B}, \quad \text{with } \theta = \frac{\pi}{2} \Rightarrow F = iLB$$

3.2 Solenoid Field at Center

$$B = \frac{\mu_0 Ni}{l} \cdot \frac{l}{\sqrt{l^2 + 4r^2}} = \frac{\mu_0 Ni}{\sqrt{l^2 + 4r^2}}$$

3.3 Force at Center

$$F = iLB = i \cdot L \cdot \frac{\mu_0 Ni}{\sqrt{l^2 + 4r^2}} \Rightarrow F = ki^2$$

Where:

$$k = \frac{\mu_0 NL}{\sqrt{l^2 + 4r^2}}$$

3.4 Acceleration

$$\frac{d^2x}{dt^2} = \frac{F}{m} = \frac{ki^2}{m}$$

4 Casewise Acceleration and Velocity

4.1 Critically Damped Case

$$i(t) = q_0 bte^{-bt}, \quad a(t) = \frac{kq_0^2 b^2 t^2 e^{-2bt}}{m}$$

$$v(t) = \int a(t)\, dt = \frac{kq_0^2 b^2}{m} \int t^2 e^{-2bt}\, dt$$

4.2 Underdamped Case

$$i(t) = -q_0 \omega e^{-bt} \sin(\omega t)$$

$$a(t) = \frac{kq_0^2 \omega^2 \sin^2(\omega t) e^{-2bt}}{m}$$

4.3 Overdamped Case

Complex combination of exponential terms; acceleration derived from:

$$i(t) = (\text{combination of exponentials}), \quad a(t) = \frac{ki(t)^2}{m}$$

5 DC Supply Mode

From:

$$L' \frac{di}{dt} + Ri = E \Rightarrow i(t) = \frac{E}{R}(1 - e^{-Rt/L'})$$

5.1 Velocity Estimation

$$v(t) = \int \frac{ki(t)^2}{m}\, dt = \frac{kE^2}{mR^2} \int (1 - e^{-Rt/L'})^2 dt$$

Assuming $t \gg L'/R$, steady-state approximation gives:

$$v(t) \approx \frac{kE^2 t}{mR^2}$$

6 Conclusion

The dynamics of a single-stage coilgun depend heavily on the damping conditions of the capacitor discharge circuit. For optimal acceleration and control, the critically damped case offers the best balance between speed and simplicity. Analytical expressions derived here provide a foundation for designing practical electromagnetic launch systems.