Laser Welding Theory: Optical and Thermal Calculations

Saikat Mohanta

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Abstract

This document presents the theoretical basis of laser welding, covering both optical and thermal aspects. Analytical formulas for beam focusing, spot size, energy requirements, and heat distribution are derived. Numerical methods and an example C code are also provided for estimating temperature rise during the welding process.

1 Introduction

Laser welding involves high-precision energy focusing on metallic surfaces, with applications in manufacturing, micro-fabrication, and electronics. This study outlines the fundamental optical calculations governing beam focusing, followed by the thermal modeling necessary to evaluate heat transfer and temperature evolution during the process.

2 Optical Calculations

Let:

- λ be the wavelength of the incident laser beam.
- d be the radius of the incident beam.
- b be the radius of the focused spot.
- f be the focal length of the lens.
- u and v be object and image distances respectively.

The Rayleigh length is approximated as:

$$l_R \approx \frac{\pi d^2}{2M^2\lambda}$$

For a lens with surfaces of radii r_1 and r_2 and refractive index n, the focal length is given by:

$$f = \frac{1}{p_1 + p_2}, \quad \text{where } p_1 = \frac{n-1}{r_1}, \quad p_2 = \frac{n-1}{r_2}$$

Cross-sectional areas:

$$A_{\text{in}} \approx \pi d^2, \quad A_{\text{out}} \approx \pi b^2$$

Using the lens geometry and Gaussian approximation:

$$b = \frac{fd}{\sqrt{(u-f)^2 + l_R^2}}, \quad v = f\left(1 + \frac{(u-f)^2 + l_R^2}{(u-f)^2 - l_R^2}\right)$$

3 Thermal Calculations

Let:

- x be the thickness of the welding sheet.
- ρ be the material density.
- c_p be the specific heat.
- L be the latent heat of fusion.
- ΔT be the temperature rise.

Volume:

$$V = \pi b^2 x$$

Mass:

$$m = \rho \pi b^2 x$$

Total heat required:

$$Q \approx m(c_p \Delta T + L) = \rho \pi b^2 x (c_p \Delta T + L)$$

Laser power required over time t:

$$P = \frac{Q}{t} = \frac{\rho \pi b^2 x (c_p \Delta T + L)}{t}$$

4 Temperature Distribution

Using Carslaw and Jaeger's solution:

$$\Delta T(y,t) = \frac{2H}{k\sqrt{\pi \alpha t}} \left[\operatorname{erfc}\left(\frac{y+b}{2\sqrt{\alpha t}}\right) - \operatorname{erfc}\left(\frac{y-b}{2\sqrt{\alpha t}}\right) \right]$$

Where:

$$\alpha = \frac{k}{\rho c_p}, \quad H = (1-R) I_{\text{out}}, \quad I_{\text{out}} = \frac{P}{\pi b^2}$$

5 Numerical Estimation in C

Below is the provided C program to estimate the center temperature rise:

Listing 1: C Code for Temperature Estimation

```
#include <math.h>
#include <stdio.h>
#include <conio.h>

#define PI 3.141
#define ROOTPI sqrt(3.141)
#define ROOTA sqrt(27.41625)

void main(void)
{
    double a0=0.3480242, a1=0.0958798, a2=0.7478556, p=0.47047;
```

```
    double erf_pi, ierfc_pi, j, b=0.0005, k=80000.0, a_out, optical_pow=50.0;
    double h, t=1.0, g_pi, g_j, erf_j, ierfc_j, del_t;

    clrscr();
    a_out=PI*b*b;
    h=0.35*(optical_pow/a_out);

    g_pi=1/(1+(p/ROOTPI));
    erf_pi=1-(((a0*g_pi)+(a1*g_pi*g_pi)+(a2*g_pi*g_pi*g_pi))*exp(-(1/ROOTPI)*(1/ROOTPI)));
    ierfc_pi=(1/ROOTPI)*exp(-(1/ROOTPI)*(1/ROOTPI)) - (1/ROOTPI)*(1-erf_pi);

    j=b/(2.0*ROOTA*sqrt(t));
    g_j=1/(1+p*j);
    erf_j=1-(((a0*g_j)+(a1*g_j*g_j)+(a2*g_j*g_j*g_j))*exp(-j*j));
    ierfc_j=(1/ROOTPI)*exp(-j*j) - (j*(1-erf_j));

    del_t=(2.0*h/k)*ROOTA*sqrt(t)*(ierfc_pi-ierfc_j);
    del_t=fabs(del_t);
    printf("\nTemperature increase: %lf K", del_t);
    getch();
}
```

6 Conclusion

This document outlines the theoretical background necessary to analyze and design laser welding systems. By combining beam optics with thermal modeling, engineers can predict temperature distributions and energy requirements effectively.