## HYDROSTATIC EQUILIBRIUM

Let there be a gas planet whose mass is $M(R)$

The acceleration due to gravity for a non rotating planet be, $g(r) = \frac{Gm(r)}{r^2} = \frac{g(R)}{R} r$

G is the Universal gravitational constant.
The surface gravity is defined as,

$$g(R) = \frac{GM(R)}{R^2} = g_s \text{ ---------------------------------------------3}$$

M(R) is the total mass of the body. R is the mean radius of the large celestial body.

Now if we consider the rotation of the planet with time period $T = \frac{2\pi}{\omega}$

The angular velocity of rotation be $\omega$

The expression for $g(r)$ becomes

$$g(r,\phi) = g(r) - r\omega^2 \cos^2\phi$$

$$g(r,\phi) = [\frac{g_s}{R} - \omega^2 \cos^2\phi] r = A_\phi r$$

Where

$$A_\phi = [\frac{g_s}{R} - \omega^2 \cos^2\phi]$$

The equation of state for an ideal gas be,

$$p(r) = \frac{kT(r)}{Am_H} \rho(r) \text{ -------------------------------------------------5}$$

P(r) is the pressure at r
ρ ( r ) is the density at r
T(r ) is the temperature at r
k is the Boltzmann constant
A is the atomic number of the elements involved in the process.

$m_H$ is the rest mass of an Hydrogen atom

$$\rho(r) = \frac{Am_H}{kT(r)} p(r) \text{ -------------------------------------------------6}$$

Again we have,

$$\frac{\rho_c}{p_c} = \frac{Am_H}{kT_c} \text{ -----------------------------------------7}$$

$\rho_c$ is the mass density at center

$p_c$ is the central pressure

$$p_c = \frac{2}{3}\pi G\bar{\rho}^2 R^2 \approx \frac{3GM^2(R)}{4\pi R^4} \text{ -----------------------------------------8}$$

Equation 8 represents the expression for central pressure.
R is the radius of the body

$$\rho_c = \frac{Am_H}{kT_c} \cdot \frac{3GM^2(R)}{4\pi R^4} \text{ ---------------------------------------------9}$$

The temperature is very badly assumed to be in the form,

$T(r) = \frac{a}{r}$ ------------------------------------------------------------10

The temperature profile is evaluated by the formula,

$v_{orbit} = v_{rms}$

$R\sqrt{\frac{g_s}{r}} = \sqrt{\frac{3kT(r)}{Am_H}}$ ----------------------------------------------10

$T(r) = \frac{a}{r}$

Where a is a constant, $a = \frac{Am_H g_s R^2}{3k}$

$T(0) = T_c$

The central temperature be,

The surface temperature be,

$$T(R) = T_S = \frac{a}{R} = \frac{Am_H g_S R}{3k}$$

The modified First Stellar equation may be written as,

$$dp = -g(r,\phi)\rho(r)dr = -[\frac{g_s}{R} - \omega^2 \cos^2\phi]r.\frac{Am_H}{k.\frac{Am_H g_S R^2}{3kr}}.p(r)dr$$

$$\frac{dp}{p} = -\frac{3}{g_S R^2}[\frac{g_s}{R} - \omega^2\cos^2\phi].r^2dr = -3[\frac{1}{R^3} - \frac{\omega^2\cos^2\phi}{g_S R^2}]r^2dr$$

$$\int_{p_c}^{p}\frac{dx}{x} = -3[\frac{1}{R^3} - \frac{\omega^2\cos^2\phi}{g_S R^2}]\int_0^r y^2dy$$

$$\ln\frac{p}{p_c} = -[\frac{1}{R^3} - \frac{\omega^2\cos^2\phi}{g_S R^2}].r^3 = -\alpha_\phi r^3$$

$$p(r) = p_c\exp[-\alpha_\phi r^3]$$

Where, $\alpha_\phi = \frac{1}{R^3} - \frac{\omega^2\cos^2\phi}{g_S R^2}$

As the pressure and density profile is same, So the density profile may be written as,

$\rho(r) = p_c\exp[-\alpha_\phi r^3]$

The second stellar equation may be written as,

$\frac{d}{dr}M(r) = 4\pi.r^2\rho(r)$ ---------------------------------------18

The mass included with in the radius r is,

$$M(r) = 4\pi\rho_c\int_0^r r^2\exp[-\alpha_\phi r^3]dr$$

$$M(r) = \frac{4\pi\rho_c}{3\alpha_\phi}(1-\exp[-\alpha_\phi r^3])$$

The expression for the total mass may be written as r=R,

$$M(R) = \frac{4\pi\rho_c}{\frac{3}{R^2}[\frac{1}{R} - \frac{R\omega^2\cos^2\phi}{g_S}]}(1 - \exp[-\alpha_\phi R^3]) = \frac{4\pi\rho_c}{3\alpha_\phi}\{1 - \exp[-(1 - \frac{R\omega^2\cos^2\phi}{g_S R^2})]\}$$

$$M(R) = \frac{4\pi R^3}{3}.\frac{\rho_c}{1 - \frac{R\omega^2\cos^2\phi}{g_S}}\{1 - \exp[-(1 - \frac{R\omega^2\cos^2\phi}{g_S})]\}$$

$$\overline{\rho}(R) = \frac{M(R)}{V(R)} = \frac{\rho_c}{1 - \frac{R\omega^2\cos^2\phi}{g_S}}\{1 - \exp[-(1 - \frac{R\omega^2\cos^2\phi}{g_S})]\}$$

$$\frac{\overline{\rho}}{\rho_c} = \frac{1 - \exp[-(1 - \frac{R\omega^2\cos^2\phi}{g_S})]}{1 - \frac{R\omega^2\cos^2\phi}{g_S}}$$

The expression for the total mass may be rearranged

$$\overline{\rho} = \frac{1 - \exp[-(1 - \frac{R\omega^2\cos^2\phi}{g_S})]}{1 - \frac{R\omega^2\cos^2\phi}{g_S}}\rho_c$$

$$\frac{M(r)}{V(R)} = \frac{1 - \exp[-(1 - \frac{R\omega^2\cos^2\phi}{g_S})]}{1 - \frac{R\omega^2\cos^2\phi}{g_S}}\frac{Am_H}{kT_c}\frac{3GM^2(R)}{4\pi R^4}$$

$$M(R) = \frac{kT_c}{GAm_H}\frac{(1 - \frac{R\omega^2\cos^2\phi}{g_S})R}{1 - \exp[-(1 - \frac{R\omega^2\cos^2\phi}{g_S})]}$$

The expression for central temperature may be written as ,

$$M(R) = \frac{kRT_c}{AGm_H} \frac{(1 - \frac{R\omega^2 \cos^2 \phi}{g_S})}{1 - \exp[-(1 - \frac{R\omega^2 \cos^2 \phi}{g_S})]}$$

$$T_c \frac{(1 - \frac{R\omega^2 \cos^2 \phi}{g_S})}{1 - \exp[-(1 - \frac{R\omega^2 \cos^2 \phi}{g_S})]} = \frac{AGm_H M(R)}{kR} = \lambda = cons\tan t$$

$$T_c(\omega) = T_c(\frac{1}{T_{day}}) = \frac{AGm_H M(R)}{kR} \frac{1 - \exp[-(1 - \frac{R\omega^2 \cos^2 \phi}{g_S})]}{(1 - \frac{R\omega^2 \cos^2 \phi}{g_S})}$$