A SIMPLE ALGEBRIC APPROACH OF CLASSICAL RANDOM SPHERE PACKING PROBLEM, CALCULATION OF PACKING FRACTION, HENCE THE CALCULATION OF MEAN DENSITY OF EARTH AS A FUNCTION OF CONSTITUIENT ELEMENTS' MEAN ATOMIC DENSITIES

ABSTRACT

This paper talks about utilizing this to calculate the *average density of the eart*h by looking at the problem as *classical random sphere packing*. All the atoms present inside earth is considered to have a same volume approximately and they are assumed to be perfectly spherical, in-compressible, in-elastic. These atoms are filled within the total volume of Earth. This method depends upon the calculation of the *total mass, mean atomic density and the number of atoms* of the most abundant individual elements present in the earth and by calculating the packing factor, we can determine the mean density of the earth as a function of atomic densities of the constituent elements. Also we can determine the total number of atoms and calculate the average mass of each atom. From there it can be concluded that the earth is indeed a rocky planet.

1. DETERMINATION METHODOLOGY

1.1. THEORY OF SPHERE PACKING WITH SPHERES

Let a large sphere of radius R is to be filled by much smaller spheres of radius r , where $R >> r$. But there are,

Let all the atoms have a same constant radius of r meter.

Let there be N_1 number of atoms have density d_1 .
Let there be N_2 number of atoms have density d_2 .
Let there be N_n number of atoms have density d_n .
Now mass of n-th atom is

$$m_n = vd_n = \frac{4}{3}\pi r^3 d_n$$

Also, the total number of atoms

$$N_{total} = \sum_1^N N_n = cons\tan t$$

The experimental mean density of the large object is $\rho_{Expt} = \frac{M}{V}$

Where, $v << V$ as $R >> r$

Hence, according to conservation of mass,

$$M = \sum_1^N m_n N_n$$

Or, $M = \sum_1^N N_n v_n d_n$

Or,

$$\rho = \sum_1^N \frac{N_n r^3 d_n}{R^3} = \frac{N_1 r^3}{R^3} d_1 + \frac{N_2 r^3}{R^3} d_2 + ... + \frac{N_n r^3}{R^3} d_n$$

Or,

$$\rho = \sum_1^N (PF)_n d_n = \frac{N_1 r^3}{R^3} d_1 + \frac{N_2 r^3}{R^3} d_2 + ... + \frac{N_n r^3}{R^3} d_n$$

Where, $(PF)_n = \frac{N_n r^3}{R^3}$

1.2. EXPERIMENTAL PROOF OF THE ABOVE THEORY

This is the following established data--

The mass of the Earth is approximately 5.98×10^{24} kg. In bulk, by mass, it is composed mostly of iron (32.1%), oxygen (30.1%), silicon (15.1%), magnesium (13.9%), sulfur (2.9%), nickel (1.8%), calcium (1.5%), and aluminium (1.4%); with the remaining 1.2% consisting of trace amounts of other elements.

Hence, we have the data as follows,

$$M = 5.98 \times 10^{24} kg$$

$$R = 6357000 m$$

$$V = 1.07608169 \times 10^{21} m^3$$

$$\rho_{Expt} = \frac{M}{V} = 5510 kg / m^3$$

ρ_{Expt} is the experimentally established value for the mean density of earth.

From that we can calculate the total mass of individual elements.

Also, mass of iron is

$M_{Fe} = 1.91958 \times 10^{24} kg$

mass of oxygen is

$M_O = 1.79998 \times 10^{24} kg$

mass of silicon is

$M_{Si} = 9.0298 \times 10^{23} kg$

mass of magnesium is

$M_{Mg} = 8.3122 \times 10^{23} kg$.

mass of sulphur is

$M_S = 1.734 \times 10^{23} kg$

mass of nickel is

$M_{Ni} = 1.076 \times 10^{23} kg$

mass of calcium is

$M_{Ca} = 8.97 \times 10^{23} kg$

mass of Aluminium is

$M_{Al} = 8.372 \times 10^{22} kg$

Others are neglected due to simplicity.

As we know the mass of those element's individual atoms.

Also, mass of iron atom is

$m_{Fe} = 9.2736 \times 10^{-26} kg$

mass of oxygen atom is

$m_O = 2.6567 \times 10^{-26} kg$

mass of silicon atom is

$m_{Si} = 4.69 \times 10^{-26} kg$

mass of magnesium atom is

$m_{Mg} = 4.0589 \times 10^{-26} kg$

mass of sulphur atom is

$m_S = 5.3548 \times 10^{-26} kg$

mass of nickel atom is

$m_{Ni} = 9.80179 \times 10^{-26} kg$

mass of calcium atom is

$m_{Ca} = 6.69302 \times 10^{-26} kg$

mass of Aluminium atom is

$m_{Al} = 4.50591 \times 10^{-26} kg$

The total number of atoms of each type of element can be determined.

So $N_1 = \frac{M_{Fe}}{m_{Fe}} = 2.06993 \times 10^{49}$

$$N_2 = \frac{M_O}{m_O} = 6.77509 \times 10^{49}$$

$$N_3 = \frac{M_{Si}}{m_{Si}} = 1.92521 \times 10^{49}$$

$$N_4 = \frac{M_{Mg}}{m_{Mg}} = 2.0478 \times 10^{49}$$

$$N_5 = \frac{M_S}{m_S} = 3.2385 \times 10^{48}$$

$$N_6 = \frac{M_{Ni}}{m_{Ni}} = 1.09816 \times 10^{48}$$

$$N_7 = \frac{M_{Ca}}{m_{Ca}} = 1.3402 \times 10^{48}$$

$$N_8 = \frac{M_{Al}}{m_{Al}} = 1.858 \times 10^{48}$$

The total number of atoms present in earth be,

$$N_{total} = \sum_1^N N_n = cons\tan t = 1.3776296 \times 10^{50}$$

let, m_{avg} be the average mass of the atoms present in earth.

the total mass $M = N_{total} m_{avg}$

So, $\frac{M}{N_{total}} = m_{avg} = 4.340789 \times 10^{-26} kg$

The over all Atomic mass number for earth is,

$A = \frac{m_{avg}}{m_{Hydrogen}} = 25.9927 \approx 26$ (Fe) for earth.

So it is clear that earth is made up of heavier elements hence is a rocky planet.

Next, the atomic radius is assumed to be a constant independent of the type of atom. Therefore, the volume and mean density of each atom can be found. Here the atoms are crudely assumed to be in-compressible and perfectly in-elastic spheres.

If the mean atomic radius is assumed to be $r \approx 1.2 \times 10^{-10} m$

The mean volume of each atom is

$v = \frac{4}{3}\pi r^3 = 7.23822947 \times 10^{-30} m^3$

Therefore,

$d_1 = \frac{m_{Fe}}{v} = 12811.972 kg/m^3$ mean atomic density of Fe

$d_2 = \frac{m_O}{v} = 3670.372 kg/m^3$ mean atomic density of O

$d_3 = \frac{m_{Si}}{v} = 6479.485 kg/m^3$ mean atomic density of Si

$d_4 = \frac{m_{Mg}}{v} = 5607.586 kg/m^3$ mean atomic density of Mg

$d_5 = \frac{m_S}{v} = 7397.941 kg/m^3$ mean atomic density of S

$d_6 = \frac{m_{Ni}}{v} = 13541.695 kg/m^3$ mean atomic density of Ni

$d_7 = \frac{m_{Ca}}{v} = 9246.764 kg/m^3$ mean atomic density of Ca

$d_8 = \frac{m_{Al}}{v} = 6225.154 kg/m^3$ mean atomic density of Al

In this theory the atoms of different elements have the same volume but different densities.

Now let there be N_1 number of Fe atoms, N_2 number of O atoms, …….., N_8 number of calcium atoms.

We know the individual element's mean atomic densities, from there our goal is to find the overall mean density of the earth, which is assumed to be near spherical.

From theory we have,

$$\rho = \sum_1^N (PF)_n d_n = \frac{N_1 r^3}{R^3} d_1 + \frac{N_2 r^3}{R^3} d_2 + ... + \frac{N_n r^3}{R^3} d_n$$

$$\rho = \frac{N_1 r^3}{R^3} d_1 + \frac{N_2 r^3}{R^3} d_2 + ... + \frac{N_8 r^3}{R^3} d_8$$

$\rho = 1783.8258 +$
$1672.638 +$
$839.071 +$
$772.406 +$
$161.152 +$
$100.022 +$
$83.357 +$
77.799

$\rho = 5329.1148 kg/m^3$

So the average density is found to be 5329.64kg/cubic meter. Which is in close approximation to the currently accepted value.

The percentage error is, $\mp 3.282\%$ the errors are due to some factors, such as, by neglecting the others elements' contribution, all the atomic radius is assumed to be same etc.

SaikatMohanta
Bongaon, WB, INDIA