1 vectors

1. Find the angle between the line $\overrightarrow{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\overrightarrow{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

2. Using vectors, prove that the points $(2, -1, 3)$, $(3, -5, 1)$ and $(-1, 11, 9)$ are collinear.

3. For any two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, prove that $\left(\overrightarrow{a} \times \overrightarrow{b}\right)^2 = \overrightarrow{a}^2\overrightarrow{b}^2 - \left(\overrightarrow{a}.\overrightarrow{b}\right)^2$

4. Using vectors, find the value of X such that the four points A$(X, 5, -1)$, B$(3, 2, 1)$, C$(4, 5, 5)$ and D$(4, 2, -2)$ are coplanar.

2 Linear Forms

1. Find the co-ordinates of the point, where the line $\frac{X+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.

2. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

3 probability

1. Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards.Find the probability that at least three cards are of diamonds.

2. The probbility of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively.Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time.

3. If A and B are independent events with P$(A) = \frac{3}{7}$ and P$(B) = \frac{2}{5}$, then find P$(A' \cap B')$.

4 optimization

1. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 mintues each for assembling. souvenirs of type B require 8 minutes each for cutting and assemblig. The profit for type A souvenirs is ₹100 each and for type B souvenirs, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximise the profit? Formulate the problem as a LPP and then slove it graphically.

5 differentiation

1. Find the differential equation representing the family of curves $y = -A\cos 3x + B\sin 3x$.
2. Find the differential of the function $\cos^{-1}(\sin 2x)$ w.r.t.x.
3. Slove the following differential equation: $\left(y + 3x^2\right)\frac{dx}{dy} = x$.
4. Differentiate $\tan^{-1}\frac{3x-x^3}{1-3x^2}$, $\left|x\right| < \frac{1}{\sqrt{3}}$ w.r.t $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$.
5. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, $\left|x\right| < 1, \left|y\right| < 1$, show that $\frac{dx}{dy} = \sqrt{\frac{1-y^2}{1-x^2}}$.
6. Find the particular solution of the differential equation: $\left(1 + e^{2x}\right)dy + \left(1 + y^2\right)e^x dx = 0$, given that y(0) = 1.
7. Find the particular solution of the differential equation: $x\dfrac{dy}{dx}\sin\left(\dfrac{y}{x}\right) + x - y\sin\left(\dfrac{y}{x}\right) = 0$, given that $y(1) = \dfrac{\pi}{2}$.
8. If $y = (\sin x)^x + \sin^{-1}\left(\sqrt{1-x^2}\right)$, then find $\dfrac{dy}{dx}$.
9. Find the interval inwhich the funnction f given by $f(x) = \sin 2x + \cos 2x, 0 \leq x \leq \pi$ is strictly decreasing.

6 Integration

1. Find:
   $\int \frac{x-1}{(x-2)(x-3)}dx$
2. integrate:
   $\frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$ with respect to x.
3. Find: $\int e^x\left(\dfrac{2+\sin 2x}{2\cos^2 x}\right)dx$
4. Evaluate: $\int_1^5 \left(\left|x-1\right| + \left|x-2\right| + \left|x-4\right|\right)dx$
5. Find:
   $$\int \cos 2x \cos 4x \cos 6x dx$$
6. Using integration, find the area of the following region: $\left\{(x, y) : x^2 + y^2 \leq 16a^2 \text{and} y^2 \leq 6ax\right\}$
7. Using integration, find the area of area of triangle ABC bounded by the lines $4x - y + 5 = 0, x + y - 5 = 0 \text{and} x - 4y + 5 = 0$.

7 Function

1. If an operation $*$ on the set of integers Z is defined $a * b = 2a^2 + b$, then find (i) whether it is a binary or not, and (ii) if a binary, then is it commutative or not.

2. prove that:
   $\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}$

3. Prove that the relation R in the set A=$\{1, 2, 3, 4, 5, 6, 7\}$ given by R=$\left\{(a, b) : \left|a - b\right| iseven\right\}$ is an equivalence relation.

4. Show that the function f in A=R- $\left\{\frac{2}{3}\right\}$ defined as f(x)=$\frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.

8 Matrices

1. If A is a square matrix of order 2 and $\left|A\right|$=4, then find the value of $\left|2.AA'\right|$, where A' is the transpose of matrix A.

2. Find the value of $(x - y)$ from the matrix equation $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$.

3. prove that:
   $\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}$

4. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

5. Using matrices, solve the following system of linear equations:

$$\begin{aligned} 2x + 3y + 10z &= 4 \\ 4x - 6y + 5z &= 1 \\ 6x + 9y - 20z &= 2 \end{aligned}$$