

Naïve Bayes Algorithm



NAIVE BAYES CLASSIFIER
WORKS ON THE PRINCIPLES OF
CONDITIONAL PROBABILITY AS
GIVEN BY THE BAYES' THEOREM

BEFORE WE MOVE AHEAD, LET
US GO THROUGH SOME OF THE
SIMPLE CONCEPTS IN
PROBABILITY THAT WE WILL BE
USING

LET US CONSIDER THE
FOLLOWING EXAMPLE OF
TOSsing TWO COINS



Here, the sample space is:

{HH, HT, TH, TT}

1. $P(\text{Getting two heads}) = 1/4$
2. $P(\text{At least one tail}) = 3/4$
3. $P(\text{Second coin being head given first coin is tail}) = 1/2$
4. $P(\text{Getting two heads given first coin is a head}) = 1/2$

Bayes' Theorem gives the conditional probability of an event A given another event B has occurred

Bayes Theorem

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

where:

$P(A|B)$ = Conditional Probability of A given B

$P(B|A)$ = Conditional Probability of B given A

$P(A)$ = Probability of event A

$P(B)$ = Probability of event A

LET US APPLY BAYES THEOREM TO OUR EXAMPLE



Here, the sample space is:

{HH, HT, TH, TT}

1. $P(\text{Getting two heads}) = 1/4$

2. $P(\text{Atleast one tail}) = 3/4$

3. $P(\text{Second coin being head given first coin is tail}) = 1/2$

4. $P(\text{Getting two heads given first coin is a head}) = 1/2$

THESE TWO USE SIMPLE
PROBABILITIES CALCULATED DIRECTLY
FROM THE SAMPLE SPACE

LET US APPLY BAYES THEOREM TO OUR EXAMPLE



Here, the sample space is:

$\{HH, HT, TH, TT\}$

1. $P(\text{Getting two heads}) = 1/4$

2. $P(\text{Atleast one tail}) = 3/4$

3. $P(\text{Second coin being head given first coin is tail}) = 1/2$

4. $P(\text{Getting two heads given first coin is a head}) = 1/2$

THIS USES CONDITIONAL
PROBABILITY. LET US
UNDERSTAND THIS IN DETAIL



IN THIS SAMPLE SPACE, LET **A** BE THE
EVENT THAT SECOND COIN IS HEAD
AND **B** BE THE EVENT THAT FIRST COIN
IS TAIL



In the sample space:

{HH, HT, TH, TT}

P(Second coin being head given first coin is tail)

$$= P(A|B)$$

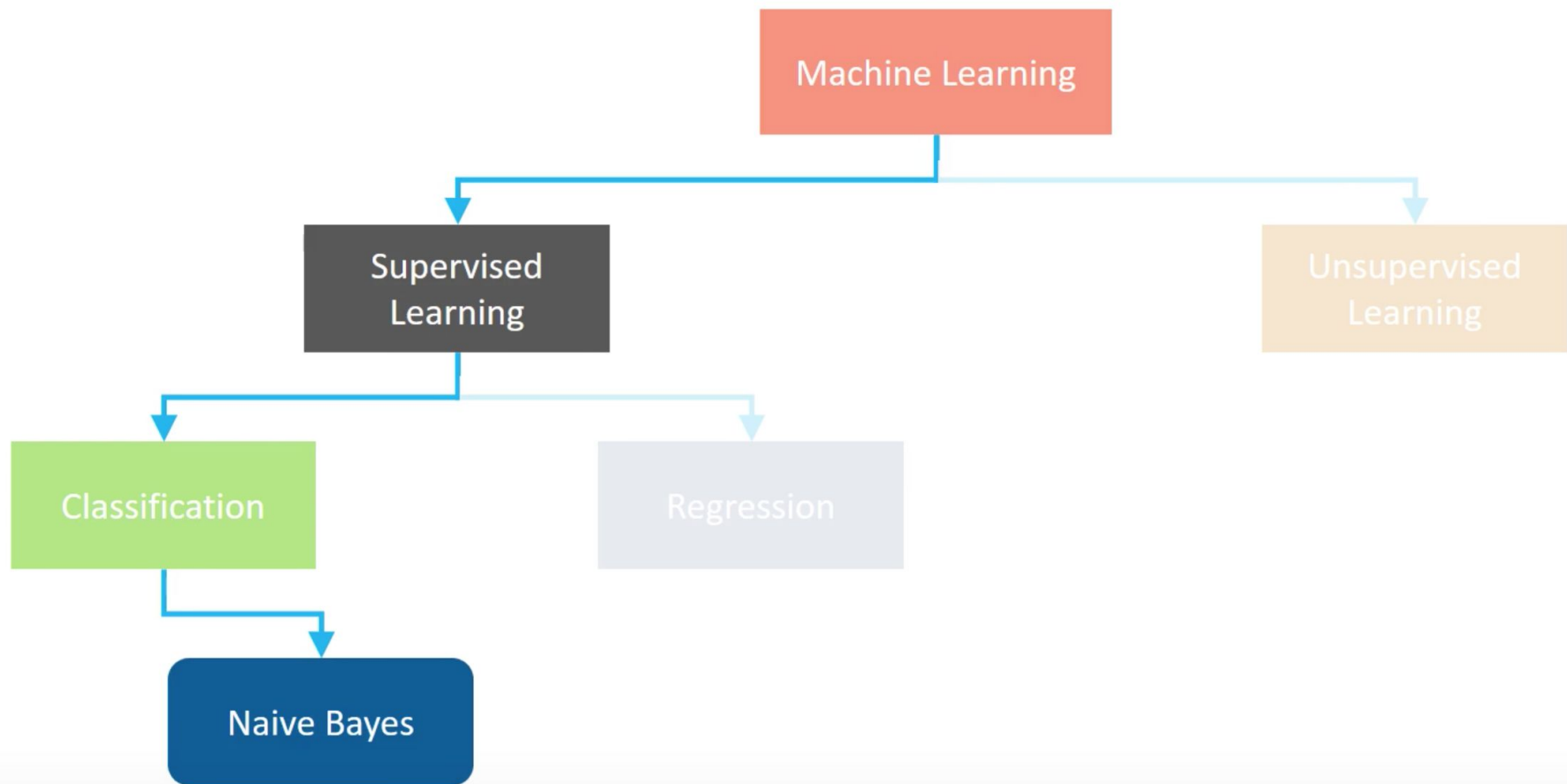
$$= [P(B|A) * P(A)] / P(B)$$

$$= [P(\text{First coin being tail given second coin is head}) * P(\text{Second coin being head})] / P(\text{First coin being tail})$$

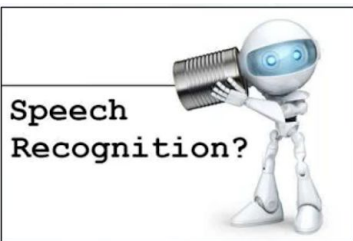
$$= [(1/2) * (1/2)] / (1/2)$$

$$= 1/2 = 0.5$$

Understanding Naïve Bayes Classifier



Where is it used?



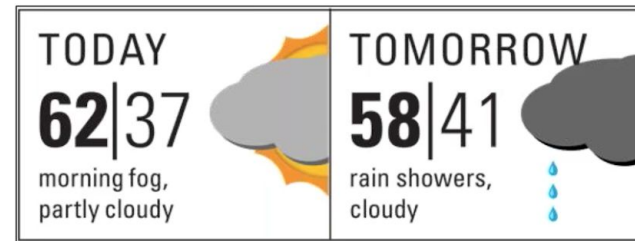
Speech Recognition



Face Recognition



Anti Virus



Weather Prediction

Let us understand how **Bayes' Theorem** can be used in **Naive Bayes classifier**:

The diagram illustrates the components of Bayes' Theorem for a Naive Bayes classifier. The equation is presented as follows:

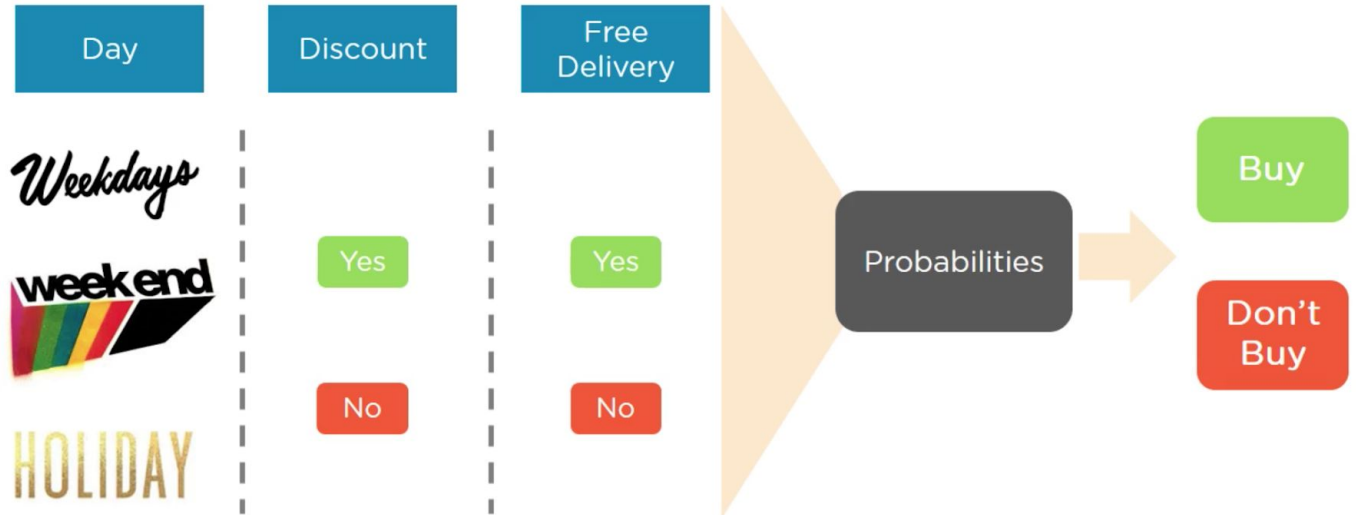
$$P(c|x) = \frac{P(x|c) P(c)}{P(x)}$$

Annotations with arrows pointing to the equation components:

- Likelihood** points to $P(x|c)$.
- Class Prior Probability** points to $P(c)$.
- Posterior Probability** points to $P(c|x)$.
- Predictor Prior Probability** points to $P(x)$.

**Let us learn with an
Example - 1**

To predict whether a person will purchase a product on a specific combination of Day, Discount and Free Delivery using Naive Bayes Classifier



We have a small sample dataset of 30 rows for our demo

	A	B	C	D
1	Day	Discount	Free Delivery	Purchase
2	Weekday	Yes	Yes	Yes
3	Weekday	Yes	Yes	Yes
4	Weekday	No	No	No
5	Holiday	Yes	Yes	Yes
6	Weekend	Yes	Yes	Yes
7	Holiday	No	No	No
8	Weekend	Yes	No	Yes
9	Weekday	Yes	Yes	Yes
10	Weekend	Yes	Yes	Yes
11	Holiday	Yes	Yes	Yes
12	Holiday	No	Yes	Yes
13	Holiday	No	No	No
14	Weekend	Yes	Yes	Yes
15	Holiday	Yes	Yes	Yes
Naive_Bayes_Dataset				

**Converting it to frequency
table on each category**

Based on this dataset containing three input types of *Day*, *Discount* and *Free Delivery*, we will populate frequency tables for each attribute

Frequency Table		Buy	
		Yes	No
Discount	Yes	19	1
	No	5	5

Frequency Table		Buy	
		Yes	No
Free Delivery	Yes	21	2
	No	3	4

Frequency Table		Buy	
		Yes	No
Day	Weekday	9	2
	Weekend	7	1
	Holiday	8	3

Based on this dataset containing three input types of *Day*, *Discount* and *Free Delivery*, we will populate frequency tables for each attribute

Frequency Table		Buy	
		Yes	No
Discount	Yes	19	1
	No	5	5

Frequency Table		Buy	
		Yes	No
Free Delivery	Yes	21	2
	No	3	4

Frequency Table		Buy	
		Yes	No
Day	Weekday	9	2
	Weekend	7	1
	Holiday	8	3

Diagram illustrating the construction of frequency tables for the attributes *Discount*, *Free Delivery*, and *Day*, based on the *Buy* event (A). The tables show the count of occurrences for each combination of attribute value and *Buy* status. Dashed blue lines indicate the flow of data from the attribute labels to the corresponding frequency tables and then to the event *Buy* (A).

FOR OUR BAYES THEOREM, LET THE EVENT *BUY* BE **A** AND THE INDEPENDENT VARIABLES, *DISCOUNT*, *FREE DELIVERY* AND *DAY* BE **B**



Creating a Likelihood table

Now let us calculate the Likelihood table for one of the variable,
Day which includes *Weekday*, *Weekend* and *Holiday*

Frequency Table		Buy		
		Yes	No	
Day	Weekday	9	2	11
	Weekend	7	1	8
	Holiday	8	3	11
		24	6	30

Likelihood Table		Buy		
		Yes	No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

$$P(B) = P(\text{Weekday}) \\ = 11/30 = 0.37$$

$$P(A) = P(\text{No Buy}) \\ = 6/30 = 0.2$$

$$P(B|A) \\ = P(\text{Weekday} \mid \text{No Buy}) \\ = 2/6 = 0.33$$

Based on this likelihood table, we will calculate conditional probabilities as below

Frequency Table		Buy		
		Yes	No	
Day	Weekday	9	2	11
	Weekend	7	1	8
	Holiday	8	3	11
		24	6	30

Likelihood Table		Buy		
		Yes	No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

$$P(B) = P(\text{Weekday}) = 11/30 = 0.367$$

$$P(A) = P(\text{No Buy}) = 6/30 = 0.2$$

$$P(B|A) = P(\text{Weekday} | \text{No Buy}) = 2/6 = 0.33$$

$$\begin{aligned}
 P(A|B) &= P(\text{No Buy} | \text{Weekday}) \\
 &= P(\text{Weekday} | \text{No Buy}) * P(\text{No Buy}) / P(\text{Weekday}) \\
 &= (0.33 * 0.2) / 0.367 = 0.179
 \end{aligned}$$



Based on this likelihood table, we will calculate conditional probabilities as below

Frequency Table		Buy		
		Yes	No	
Day	Weekday	9	2	11
	Weekend	7	1	8
	Holiday	8	3	11
		24	6	30

Likelihood Table		Buy		
		Yes	No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

$$P(B) = P(\text{Weekday}) = 11/30 = 0.367$$

$$P(A) = P(\text{Buy}) = 24/30 = 0.8$$

$$P(B|A) = P(\text{Weekday} | \text{Buy}) = 2/6 = 0.375$$

If A equals Buy, then

$$\begin{aligned}
 P(A|B) &= P(\text{Buy} | \text{Weekday}) \\
 &= P(\text{Weekday} | \text{Buy}) * P(\text{Buy}) / P(\text{Weekday}) \\
 &= (0.375 * 0.8) / 0.367 = 0.817
 \end{aligned}$$

As the Probability(Buy | Weekday) is more than Probability(No Buy | Weekday), we can conclude that a customer will most likely buy the product on a Weekday

Similarly, we can find the likelihood of occurrence of an event involving all three variables

WE HAVE THE FREQUENCY TABLES OF ALL THE THREE INDEPENDENT VARIABLES. WE WILL NOW CONSTRUCT LIKELIHOOD TABLES FOR ALL THE THREE

Frequency Table		Buy	
		Yes	No
Day	Weekday	3	7
	Weekend	8	2
	Holiday	9	1

Likelihood Table		Buy		
		Yes	No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	



Likelihood Tables

Likelihood Table		Buy		
		Yes	No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

Frequency Table		Buy		
		Yes	No	
Discount	Yes	19/24	1/6	20/30
	No	5/24	5/6	10/30
		24/30	6/30	

Frequency Table		Buy		
		Yes	No	
Free Delivery	Yes	21/24	2/6	23/30
	No	3/24	4/6	7/30
		24/30	6/30	

Calculating Conditional Probability of purchase on the following combination of day, discount and free delivery:

Where B equals:

- Day = **Holiday**
- Discount = **Yes**
- Free Delivery = **Yes**

Let A = **No Buy**

$P(A|B) = P(\text{No Buy} \mid \text{Discount} = \text{Yes}, \text{Free Delivery} = \text{Yes}, \text{Day} = \text{Holiday})$

$$\begin{aligned}
 &= \frac{P(\text{Discount} = \text{Yes} \mid \text{No}) * P(\text{Free Delivery} = \text{Yes} \mid \text{No}) * P(\text{Day} = \text{Holiday} \mid \text{No}) * P(\text{No Buy})}{P(\text{Discount} = \text{Yes}) * P(\text{Free Delivery} = \text{Yes}) * P(\text{Day} = \text{Holiday})} \\
 &= \frac{(1/6) * (2/6) * (3/6) * (6/30)}{(20/30) * (23/30) * (11/30)} \\
 &= 0.178
 \end{aligned}$$

Likelihood Tables

Likelihood Table		Buy		
		Yes	No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

Frequency Table		Buy		
		Yes	No	
Discount	Yes	19/24	1/6	20/30
	No	5/24	5/6	10/30
		24/30	6/30	

Frequency Table		Buy		
		Yes	No	
Free Delivery	Yes	21/24	2/6	23/30
	No	3/24	4/6	7/30
		24/30	6/30	

Calculating Conditional Probability of purchase on the following combination of day, discount and free delivery:

Where B equals:

- Day = **Holiday**
- Discount = **Yes**
- Free Delivery = **Yes**

Let A = **Buy**

$P(A|B) = P(\text{Yes Buy} \mid \text{Discount} = \text{Yes}, \text{Free Delivery} = \text{Yes}, \text{Day} = \text{Holiday})$

$$= \frac{P(\text{Discount} = \text{Yes} \mid \text{Yes}) * P(\text{Free Delivery} = \text{Yes} \mid \text{Yes}) * P(\text{Day} = \text{Holiday} \mid \text{Yes}) * P(\text{Yes Buy})}{P(\text{Discount} = \text{Yes}) * P(\text{Free Delivery} = \text{Yes}) * P(\text{Day} = \text{Holiday})}$$

$$= \frac{(19/24) * (21/24) * (8/24) * (24/30)}{(20/30) * (23/30) * (11/30)}$$

$$= 0.986$$

PROBABILITY OF PURCHASE = 0.986
PROBABILITY OF NO PURCHASE = 0.178

FINALLY, WE HAVE CONDITIONAL
PROBABILITIES OF PURCHASE
ON THIS DAY!

LET US NOW NORMALIZE THESE
PROBABILITIES TO GET THE
LIKELIHOOD OF THE EVENTS

SUM OF PROBABILITIES

$$= 0.986 + 0.178 = 1.164$$

LIKELIHOOD OF PURCHASE

$$= 0.986 / 1.164 = 84.71 \%$$

LIKELIHOOD OF NO PURCHASE

$$= 0.178 / 1.164 = 15.29 \%$$

PROBABILITY OF PURCHASE = 0.986

PROBABILITY OF NO PURCHASE = 0.178

AS 84.71% IS GREATER THAN 15.29%,
WE CAN CONCLUDE THAT AN AVERAGE
CUSTOMER WILL BUY ON A HOLIDAY
WITH DISCOUNT AND FREE
DELIVERY

Example 2

From the dataset we have obtained, we will populate frequency tables for each of the attribute

Frequency Table		Play	
		Yes	No
Sunny	Yes	3	4
	No	6	1

Frequency Table		Play	
		Yes	No
Windy	Yes	6	2
	No	3	3

Frequency Table		Play	
		Yes	No
Season	Summer	3	2
	Monsoon	4	0
	Winter	2	3

For each of the frequency tables, we will find the likelihoods for each of the cases

Here, $c = \text{Play}$ and $x = \text{Variables like Season, Sunny \& Windy}$.

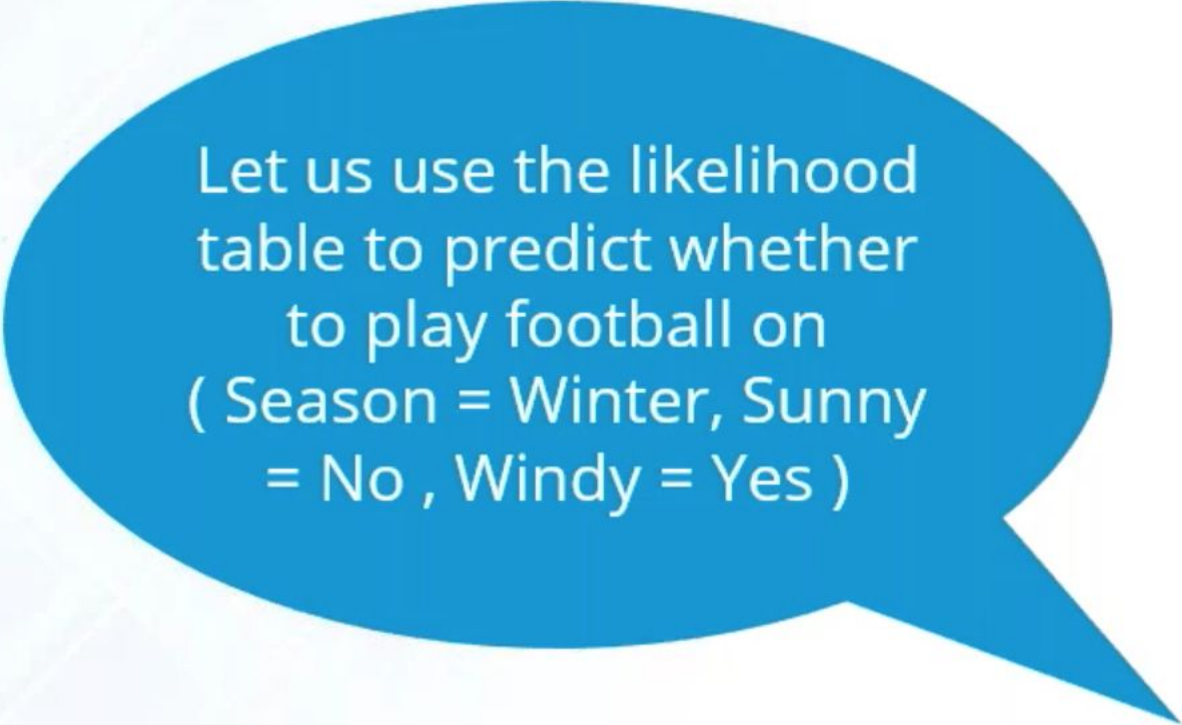
Frequency Table		Play	
		Yes	No
Season	Summer	3	2
	Monsoon	4	0
	Winter	2	3

Likelihood Table		Play		
		Yes	No	
Season	Summer	3/9	2/5	5/14
	Monsoon	4/9	0/5	4/14
	Winter	2/9	3/5	5/14
		9/14	5/14	

$P(x \mid c) = P(\text{Summer} \mid \text{Yes}) = 3/9 = 0.33$
 $P(x) = P(\text{Summer}) = 5/14 = 0.36$
 $P(c) = P(\text{Yes}) = 9/14 = 0.64$

Likelihood of 'Yes' given Summer is:

$$P(c \mid x) = P(\text{Yes} \mid \text{Summer}) = P(\text{Summer} \mid \text{Yes}) * P(\text{Yes}) / P(\text{Summer}) = (0.33 \times 0.64) / 0.36 = 0.60$$



Let us use the likelihood
table to predict whether
to play football on
(Season = Winter, Sunny
= No , Windy = Yes)

$$P(c \mid x) = P(\text{Play} = \text{Yes} \mid \text{Winter}, \text{Sunny} = \text{No}, \text{Windy} = \text{Yes})$$

$$= \frac{P(\text{Winter} \mid \text{Yes}) * P(\text{Sunny} = \text{No} \mid \text{Yes}) * P(\text{Windy} = \text{Yes} \mid \text{Yes}) * P(\text{Yes})}{P(\text{Winter}) * P(\text{Sunny} = \text{No}) * P(\text{Windy} = \text{Yes})}$$

$$= (2/9) * (6/9) * (6/9) * (9/14) / (5/14) * (7/14) * (8/14) = 0.6223$$

Since the probability is greater than 0.5, we should play football on that day.

Advantages of Naive Bayes Classifier

