way to quantify the uncertainity
says something likely to happen

0 → it will not happen
1 → it will happen
Anything in between → Indicates the likelihood of event occuring

Mathematical Definition :

$$P(A) = \frac{\text{Number of favourable Outcomes}}{\text{Total number of Outcomes}}$$

Probabity of Indian winnig a Test match tommorow

outcoms ⇒ Win, loss, Draw, tie

$$\frac{1}{4}$$

Types of Events:

Independent events

if occurrence of one event does not affect other

Flopping a coin

$$\frac{1}{2} \,\Big|\, \frac{1}{2} \,\Big|\, \cdot \frac{1}{2}$$

Dependent Events

occurrence of one events affects the other

Drawing two Cards without replacement

$$\frac{1}{52} \,\Big|\, \frac{1}{51}$$

	Passed	Failed	Total
Studied	45	15	60
Didn't Studied	5	35	40
	50	50	100

1. Marginal Problablity

Single event happening

$$P(\text{Passing}) = \frac{\text{Total Passing}}{\text{Total student}} = \frac{50}{100}$$

$$P(\text{studied}) = \frac{\text{Total studied}}{\text{Total student}} = \frac{60}{100}$$

One event by itself

2. Joint Probability

Two Events happening together

$$P(\text{studied and Passed}) = \frac{\text{Students who studied and Passed}}{\text{Total students}}$$

$$\frac{45}{100} = 0.45$$

3. Conditional Probability

Probability of one event happening given another event has already happed

$$P(\text{Passed} \mid \text{Studied}) = \frac{P(\text{studied and Passed})}{P(\text{studied})}$$

$$= \frac{45}{60} = 0.75$$

$$P(\text{Failed} \mid \text{Didnt Studied}) = \frac{P(\text{Failed and Didt Studies})}{P(\text{Didt Studing})}$$

$$= \frac{35}{40}$$

$$= 0.87$$

$$P(\text{Passed} \mid \text{Studied}) = \frac{P(\text{studied and Passed})}{P(\text{studied})}$$

$$\text{Conditional Probablity} = \frac{\text{Joint Probabity}}{\text{Marginal Probablity}}$$

Baye's Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$P(A|B) \Rightarrow$ probablity of A happening, given that B happened

$P(B|A) \Rightarrow$ probablity of B happening, given that A happened

$P(A) \Rightarrow$ prior probability of A happening

$P(B) \Rightarrow$ Overall probabity of B happeng

$$P(\text{Studied} \mid \text{Passed})$$

$$= \frac{P(\text{Passed} \mid \text{Studied}) \cdot P(\text{Studied})}{P(\text{passed})}$$

$$= \frac{\frac{45}{60} \times \frac{60}{100}}{\frac{50}{100}}$$

$$= \frac{0.75 \times 0.6}{0.5} = 0.9$$

If a student pass the exam, there's 90% chance they have studied

Detective problem

$$\boxed{P(\text{Guilty} \mid \text{Blood on clothes})}$$

$\frac{20}{200}$

Known information

$P(\text{Guilty}) = \underline{1\%} = 0.01$ ✓

$P(\text{Blood on clothes} \mid \text{Guilty}) = 90\% = 0.90$ ✓

$P(\text{Blood on clothes}) = \underline{5\%} = 0.05$ ✓

200 Cases

20 → Guilty $\quad \frac{20}{200}$

18 — Blood on clothes $\quad \frac{18}{20}$

$$P(\text{Guilty} / \text{Blood on clothes})$$

$$= \frac{P(\text{Guilty}) \cdot P(\text{Blood on cloths} / \text{Guilty})}{P(\text{Blood on clothes})}$$

$$= \frac{0.01 \times 0.9}{0.05} = \frac{0.009}{0.05}$$

$$= 0.18$$

18% chance they are guilty

Spam Detection

$P(\text{spam} \mid \text{given a word 'Free'})$

$$= \frac{P(\text{spam}) \cdot P(\text{given 'Free'} \mid \text{spam})}{P(\text{'Free'})}$$

$$\Rightarrow \frac{0.2 * 0.7}{0.18}$$

$= 0.77$

probability of email spam give 'Free' 77%

$P(\text{spam}) = 0.2$

$P(\text{Not spam}) = 0.8$

$P(\text{'Free'} \mid \text{spam}) = 0.7$

$P(\text{'Free'} \mid \text{Not spam}) = 0.05$

$$P(\text{'Free'}) = P(\text{'Free'} \mid \text{spam}) \times P(\text{spam}) + P(\text{'Free'} \mid \text{Not spam}) * P(\text{Not spam})$$

$$= 0.7 * 0.2 + 0.05 \times 0.8$$

$$= 0.4 + 0.04$$

$$= 0.18$$