

NUMBER SYSTEM

BINARY NUMBER SYSTEM

→ Has only two digits [0 & 1]

→ Base number is "2"

$$\underline{\text{Ex:}} (1111000)_2$$

CONVERSION

[1] BINARY TO DECIMAL

$$\begin{array}{l} \text{Ex: } 1001 \\ \quad | \quad | \quad | \\ \quad 1 \times 2^0 = 1 \times 1 = 1 \\ 0 \times 2^1 = 0 \times 2 = 0 \\ 0 \times 2^2 = 0 \times 4 = 0 \\ 1 \times 2^3 = 1 \times 8 = 8 \end{array}$$

$$\Rightarrow 1 + 0 + 0 + 8 = 9.$$

[2] BINARY TO OCTAL

Binary -	011	100	101
Code -	421	421	421
Octal -	3	4	5

[3] BINARY TO HEXADECIMAL

Binary -	1110	0101	0111	1111
Code -	8421	8421	8421	8421
Hexadecimal -	E	5	7	F

1's Complement

$$\begin{array}{ccccccc} 0 & 1 & 0 & 1 & 0 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 1 & 0 & 1 & 1 \end{array}$$

2's Complement

Adding 1 to 1's complement is 2's complement.

OCTAL NUMBER SYSTEM

→ Has only eight [8] digits [0 to 7]

→ Base number is "8"

$$\underline{\text{Ex:}} [360]_8$$

CONVERSION

[1] OCTAL TO BINARY

OCTAL →	3	4	5
CODE →	421	421	421
BINARY →	011	100	101

$$\Rightarrow [011100101]_2$$

[2] OCTAL TO HEXADECIMAL

OCTAL →	1110	0101	1111
CODE →	8421	8421	8421
HEXA DECIMAL →	E	5	F

$$\Rightarrow [ESF]_{16}$$

[3] OCTAL TO DECIMAL

$$\underline{\text{Ex:}} 345$$

$$\Rightarrow (3 \times 8^2) + (4 \times 8^1) + (5 \times 8^0)$$

$$\Rightarrow (3 \times 64) + 32 + 40$$

$$\Rightarrow 229$$

$$\Rightarrow (229)_{10}$$

DECIMAL NUMBER SYSTEM

→ Has only ten [10] digits [0 to 9]

→ Base number is "10"

$$\underline{\text{Ex:}} [240]_{10}$$

CONVERSION

[1] DECIMAL TO BINARY

$$\underline{\text{Ex:}} (24)_{10}$$

$$\begin{array}{r} 2 | 24 \quad -0 \\ 2 | 12 \quad -0 \\ 2 | 6 \quad -0 \\ 2 | 3 \quad -1 \\ \quad \quad 1 \end{array} \Rightarrow [11000]_2$$

[2] DECIMAL TO OCTAL

$$\underline{\text{Ex:}} [12345]_{10}$$

$$\begin{array}{r} 8 | 12345 \quad -1 \\ 8 | 1543 \quad -7 \\ 8 | 192 \quad -0 \\ 8 | 24 \quad -0 \\ \quad \quad 3 \end{array} \Rightarrow (30071)_8$$

[3] DECIMAL TO HEXADECIMAL

$$\underline{\text{Ex:}} (725)_{10}$$

$$\begin{array}{r} 16 | 725 \quad -5 \rightarrow S \\ 16 | 45 \quad -13 \rightarrow D \\ \quad \quad 2 \end{array} \rightarrow 2$$

$$\Rightarrow [2DS]_{16}$$

Activity:

$$1. (423)_5 = (\quad)_7$$

$$2. (323)_4 = (\quad)_9$$

Activity:

HEXADECIMAL NUMBER SYSTEM

→ Has sixteen [16] Alphanumeric values from (0 to 9)₈ (A to F)

→ Base number is "16"

$$\underline{\text{Ex:}} (FO)_{16}$$

CONVERSION

[1] HEXADECIMAL TO BINARY

Hexa -	A	2	D	E
Code -	8421	8421	8421	8421
Binary -	1010	0010	1101	1110

$$\Rightarrow [11000]_2$$

[2] DECIMAL TO OCTAL

$$\underline{\text{Ex:}} [12345]_{10}$$

$$\Rightarrow (1010001011011110)_2$$

$$\Rightarrow (001010001011011110)_2$$

∴ put 0's in front of 1.

$$\Rightarrow [121336]_8$$

TO DECIMAL

$$\underline{\text{Ex:}} [FO]_{16} = 0[F \times 16^3] + (0 \times 16^0)$$

$$= (5 \times 16) + 0 = (240)_{10}$$

2's Complement

$\underline{\text{Ex:}}$	0 1 0 1 0 0
	↓ ↓ ↓ ↓ ↓ ↓
	1 0 1 0 1 1
	1 +
	1 0 1 1 0 0

→ 1's complement

BOOLEAN FUNCTIONS SIMPLIFICATION AND IMPLEMENTATION

Boolean algebra :-

Deals with binary Variable and Logic operation

Boolean function :- Consists of binary variables, Constants (0/1) and Logic Operators

$$\text{Ex :- } Y = ab' + b'c$$

Simplifications

→ Using boolean axioms and laws

→ K-map

→ Tabulation method

Different form of Boolean functions :-

1. Canonical form

↳ Sum of minterms

↳ Product of maxterms

2. Standard form

↳ SOP

↳ POS

Postulates and theorems of Boole

$$1. x + 0 = x \quad x \cdot 1 = x$$

$$2. x + x' = 1 \quad x \cdot x' = 0$$

$$3. x + x = x \quad x \cdot x = x$$

$$4. x + 1 = 1 \quad x \cdot 0 = 0$$

$$5. Idempotent (x')' = x$$

$$6. Commutative x + y = y + x$$

$$7. Associative x + (y + z) = (x + y) + z$$

$$8. Distributive x(y + z) = xy + xz$$

$$x + (yz) = (x + y)(x + z)$$

$$9. De Morgan's (x + y)' = x'y'$$

$$(x'y)' = x + y'$$

$$10. Absorption x + xy = x$$

$$x(x + y) = x$$

Conversion of standard form into Canonical form.

$$\text{Ex :- } 1 \quad Y = a'b + bc \text{ (Standard form)}$$

$$\begin{aligned} \text{Ans :- } Y &= a'b(b(c+c')) + (a+a')bc \\ &= a'b(c + a'b + ab + a'b) \\ &= a'bc + a'b c' + abc \text{ (Canonical)} \\ &\quad \begin{matrix} 0 & 1 & 0 & 1 & 1 & 1 \\ m_3 & m_2 & m_1 \end{matrix} \end{aligned}$$

$$Y = \sum_m(2, 3, 7) \rightarrow (CF)$$

Truth table for Boolean function

$$\text{Ex :- } Y = a'b + bc$$

A :- Convert it into CF

$$Y = \sum_m(2, 3, 7)$$

→ place 1 against this minterm in DIP

a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

POS form of given Boolean Exp

$$F = \sum_m(0, 3, 5, 7, 9, 11)$$

A :- Identify no. of Variables
Place other no. other than min term given

$$\text{Ans :- } f = \sum_m(1, 2, 4, 6, 8, 10, 12, 13, 14, 15)$$

SOP and POS form of BF

$$f = \sum_m(1, 3)$$

$$= a'b + ab - SOP$$

$$b(a+a') = b \quad (\text{or})$$

$$f = \sum_m(0, 2)$$

$$= (a+b), (a'+b) - POS$$

$$= aa' + ab + a'b + b'b$$

$$= a'b + ab + b = b(1 + a + a')$$

$$= b$$

Boolean for Simplification using BF

1. Given in SOP form

$$f_1 = a'bc + a'b'c + c'$$

$$f_1' = a'c(b + b') + c'$$

$$= a'c + c' = c' + ca'$$

$$= c' + a'$$

2. Given in POS form

$$f_2 = (a' + b + c')(a' + b')(c + a)$$

$$f_2' = (a'A' + a'B' + a'B + BB' + A'A'c' +$$

$$+ B'c')(c + a)$$

$$= (a'(a'B' + a'B + B'c') + B'c)(c + a)$$

$$= a'B'c + a'BC + A'cc' + B'cc' +$$

$$AA'B' + AA'B + AA'C' + AB'C'$$

$$A'C + AA'$$

$$= A'BC + A'BC + ABC' + A'c$$

$$= A'C(B' + B + 1) + AB'C'$$

$$= A'C + ABC'$$

Simplification Using K-map

Step 1 :- Identify no. of input variables

Variable in given function

Step 2 :- Bring given function in Canonical form

Step 3 :- Draw Suitable n-variable K-map for Simplification

Ex :- find minimal SOP and POS form of given function

$$1. \quad Y = a'b'c + bc' + ac$$

A :- Bring it to Canonical form

$$Y = a'b'c + (a + a')bc' + a(b + b')c$$

$$= a'b'c + abc' + a'b'c' + a'b'c' + a'b'c$$

$$+ ab'c$$

$$= \sum_m(2, 3, 5, 6, 7)$$

→ Given func is 3 variable func hence draw 3 variable K-map.

abc	000	011	111	100
0
1	1	1	1	1

$$Y = b + ac \rightarrow SOP$$

abc	000	011	111	100
0	0	0	0	0
1	0	1	1	1

$$Y = (a + b) \cdot (b + c) \rightarrow POS$$

Ex :- Q :- find the minimal SOP and POS form of given fun

$$Y_2 = \sum_m(1, 3, 5, 7, 8, 9, 12, 13)$$

A :- Given fn. is in Canonical form and 4 variable DP fn. hence use 4 Variable K-map

abcd	0000	0111	1110	0011
00	0	0	0	1
01	0	0	1	1
11	0	1	1	1
10	0	0	1	1

$$Y_2 = (a + d')(a' + c)$$

$$Y_2 = a'd' + ac$$

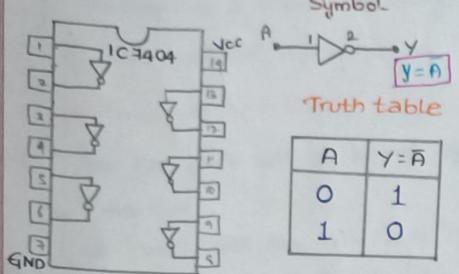
min POS

min SOP

Logic Gates

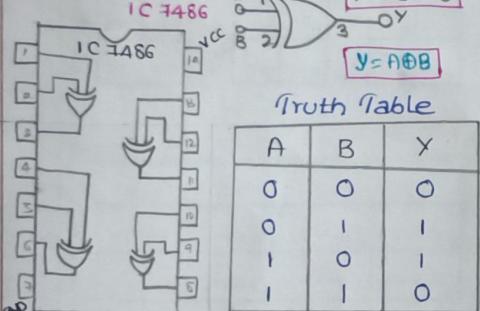
NOT Gate

PIN DIAGRAM - IC 7404



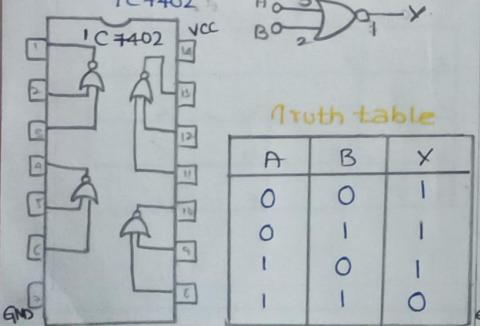
X-OR Gate

PIN DIAGRAM - IC 7486



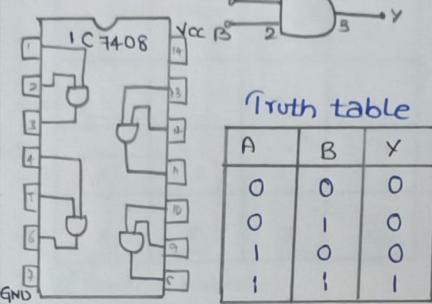
NOR Gate

PIN DIAGRAM - IC 7402

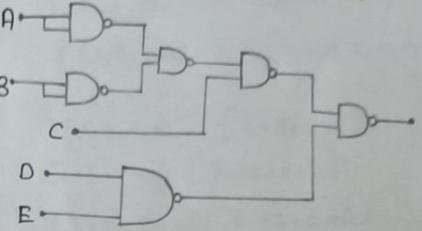


AND Gate

PIN DIAGRAM - IC 7408



* Implement boolean function $f = (A+B)C + DB$ using NAND gates.



* Implement the following Boolean function with NOR-NOR logic.

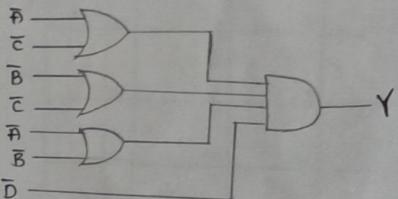
$$Y = AC + BC + AB + D$$

Sol:-

Step 1:- Express Boolean function in POS form

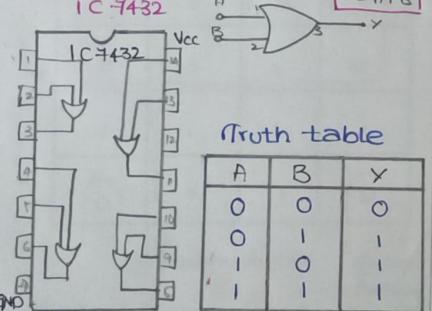
$$\bar{Y} = (\bar{A} + \bar{C})(\bar{B} + \bar{C})(\bar{A} + \bar{B})\bar{D}$$

Step 2:- Implement Boolean function with OR-NAND logic.



OR Gate

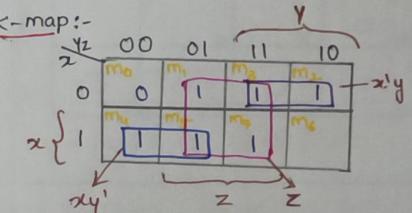
PIN DIAGRAM - IC 7432



* Implement the following Boolean function with NAND gates:

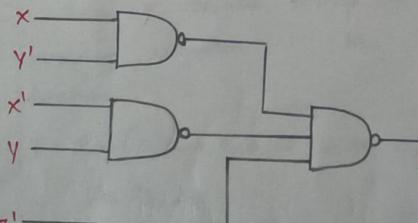
$$F(x_1, y_1, z) = (1, 2, 3, 4, 5, 7)$$

K-map:-



$$F = xy' + x'y + z$$

Logic Diagram:-



Activity:-

① Implement the following Boolean function using NAND-NAND

$$F(x_1, y_1, z) = (0, 2, 3, 4, 5, 7)$$

② NOR-NOR implementation

$$F(A, B, C) = T1 M(0, 2, 4, 5, 6)$$

* Implement the following Boolean function with NOR-NOR logic.

K-Map

SIMPLIFICATION

4

PRODUCT OF SUMS (POS)

REDUCTION USING K-MAP

K-map is a table consisting of cells which represents a minterm or maxterm.

For POS each cell in k-map represents a maxterm.

We will fill the cells with '0' whose maxterm output is '0'.

TWO VARIABLE K-MAP IN POS

$x'y$	y'	$y'y$	$y'y'$	$x'y$	$x'y'$	$y'y$	$y'y'$
$x \times x+y$	$x+y$	$x+y$	$x+y'$	$x \times x+y$	$x+y$	$x+y$	$x+y'$
$x' \times x+y$	$x+y$	$x+y$	$x+y'$	$x' \times x+y$	$x+y$	$x+y$	$x+y'$

THREE VARIABLE K-MAP IN POS

$x'y'z$	$y'z$	$y'z'$	$y+z'$	$x+y+z$	$x+y+z'$	$x+y'+z$	$x+y'+z'$
$x \times x+y+z$	$x+y+z$	$x+y+z'$	$x+y'+z$	$x \times x+y+z$	$x+y+z$	$x+y+z'$	$x+y'+z$
$x' \times y+y+z$	$y+y+z$	$y+y+z'$	$y+y'+z$	$x' \times y+y+z$	$y+y+z$	$y+y+z'$	$y+y'+z$

FOUR VARIABLE K-MAP IN POS

$w'y'z'$	$w'y'z$	$w'yz'$	wyz	$wy'z'$	wyz'	$w'y'z'$	$w'y'z$
$[00] \times y+z'$	$[00] y+z'$	$[11] y'+z'$	$[10] y'+z$	$[00] y+z'$	$[00] y+z$	$[11] y'+z'$	$[10] y'+z$
$[00]$	0	1	3	2			
$[01]$	4	5	7	6			
$[10]$	12	13	15	14			
$[11]$	8	9	11	10			

REDUCING RULES FOR POS

USING K-MAP		① Pair reduction	
$w+x$	$y+z$	$y+z'$	$y'+z'$
$w+x'$	$y+z'$	$y+z$	
$w+x$	$y+z$	$y+z'$	
$w+x'$	$y+z'$	$y+z$	

② Quad reduction	
$w+x$	$y+z$
$w+x'$	$y+z'$
$w+x$	$y+z'$
$w+x'$	$y+z$

③ Octet reduction		④ Map rolling reduction	
$w+x$	$y+z$	$y+z'$	$y'+z'$
$w+x'$	$y+z'$	$y+z$	
$w+x$	$y+z$	$y+z'$	
$w+x'$	$y+z'$	$y+z$	

Overlapping groups.	
$w+x$	$y+z$
$w+x'$	$y+z'$
$w+x$	$y+z'$
$w+x'$	$y+z$

PROBLEMS

Reduce the POS function using K-map.

$$f(A, B, C, D) = \prod M(2, 6, 8, 9, 10, 11, 14)$$

$$f(A, B, C, D) = (A+B+\bar{C}+D)$$

$$(A+\bar{B}+\bar{C}+D) (\bar{A}+B+C+\bar{D})$$

$$(\bar{A}+B+\bar{C}+D) (\bar{A}+B+C+\bar{D})$$

$$(A+\bar{B}+\bar{C}+D) (\bar{A}+B+C+D)$$

ST	00	01	11	10
00	1	0	1	1
01	1	4	15	1
11	1	12	13	15
10	1	8	9	11

P=0

ST	00	01	11	10
00	1	0	1	1
01	1	5	1	6
11	1	12	13	15
10	1	8	9	11

P=1

5 VARIABLE K-MAP OF SOP

$2^5 = 32$ cells to fill each minterm

PROBLEM:

Solve 5-variable K-map for maximum reduction, the function

$$F = \sum m(0, 2, 4, 7, 8, 10, 12, 14, 18, 20, 22, 24)$$

SUBCUBE 1: 0, 4, 8, 12, 16, 20, 24

SUBCUBE 2: 0, 2, 8, 10

SUBCUBE 3: 18, 24

SUBCUBE 4: 7, 23

$$\bar{S}\bar{T} + QRS\bar{T} + Q\bar{R}\bar{S}\bar{T} + \bar{Q}\bar{R}S\bar{T}$$

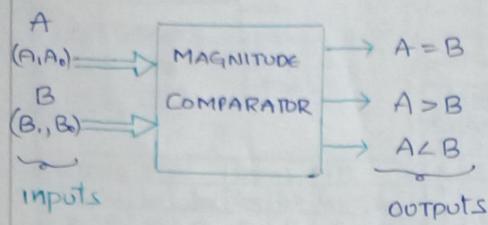
PROBLEMS

$$f(A, B, C, D, E) = \sum m(0, 2, 4, 5, 9$$

$$, 11, 13, 15, 17, 21, 25, 27, 29, 31)$$

MAGNITUDE COMPARATOR

2-BIT COMPARATOR



TRUTH TABLE

INPUT			OUTPUT		
			A = B	A > B	A < B
A ₁	A ₀	B ₁	B ₀		
0	0	0	0	1	0 0
0	0	0	1	0	0 1
0	0	1	0	0	0 1
0	0	1	1	0	0 1
0	1	0	0	0	1 0
0	1	0	1	1	0 0
0	1	1	0	0	1 0
0	1	1	1	0	0 1
1	0	0	0	1	0
1	0	0	1	0	1
1	0	1	0	1	0
1	0	1	1	0	1
1	1	0	0	1	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

K-MAP

A = B

	B ₁ B ₀			
A ₁ A ₀	1	0	0	0
A ₁ A ₀	0	1	0	0
A ₁ A ₀	0	0	1	0
A ₁ A ₀	0	0	0	1

A > B

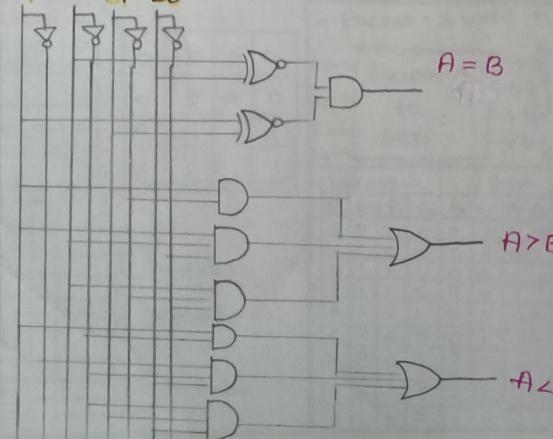
	B ₁ B ₀			
A ₁ A ₀	0	0	0	0
A ₁ A ₀	1	0	0	0
A ₁ A ₀	1	1	0	1
A ₁ A ₀	1	1	0	0

A < B

	B ₁ B ₀			
A ₁ A ₀	0	1	1	1
A ₁ A ₀	0	0	1	1
A ₁ A ₀	0	0	0	0
A ₁ A ₀	0	0	1	0

Logic Diagram:-

A₁ A₀ B₁ B₀



PRIORITY ENCODER

→ In priority Encoder, if two or more inputs are equal to 1 at the same time, the input having the highest priority will take precedence.

TRUTH TABLE

INPUTS				OUTPUTS		
D ₀	D ₁	D ₂	D ₃	X	Y	V
0	0	0	0	0	0	0
0	0	0	1	0	0	1
1	0	0	0	0	0	0
1	0	0	1	0	1	0
0	1	0	0	0	1	0
0	1	0	1	1	0	1
0	1	1	0	0	1	0
1	1	0	0	1	0	1
1	1	0	1	0	1	0
1	1	1	0	1	0	0
1	1	1	1	0	0	0

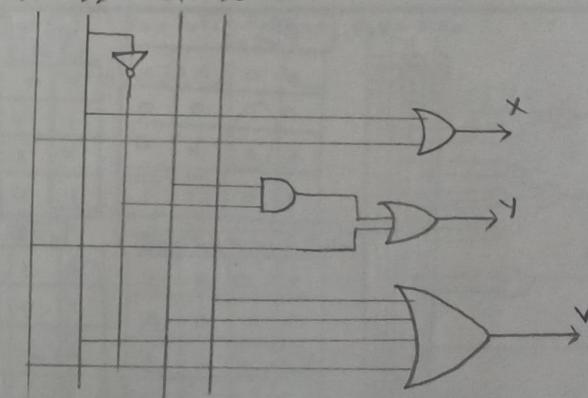
K-MAP SIMPLIFICATION EQUATIONS

$$\text{For } X \Rightarrow X = D_2 + D_3$$

$$\text{For } Y \Rightarrow Y = D_3 + D_1 D_2$$

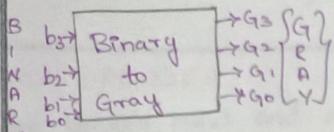
$$\text{For } V \Rightarrow V = D_0 + D_1 D_2 + D_3$$

D₃ D₂ D₁ D₀



CODE CONVERTERS

1. Binary to Gray :-



Binary I/P				Gray O/P			
b3	b2	b1	b0	G3	G2	G1	G0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	1	0
0	1	1	1	0	1	0	0
1	0	0	1	0	0	0	0
1	0	0	1	1	0	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	0	0
1	1	0	0	1	0	1	0
1	1	0	1	0	1	1	1
1	1	1	0	0	0	1	0
1	1	1	1	1	0	0	0

$$G_0 = b_1 \oplus b_0$$

$$G_1 = b_1 \oplus b_2$$

0	1	0	1	0	0	1	1
0	1	0	1	1	0	0	0
0	1	0	1	1	1	0	0
0	1	0	1	0	0	1	1

$$G_2 = b_3 \oplus b_2$$

$$G_3 = b_3$$

0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1

$$b_3, b_2, b_1, b_0$$

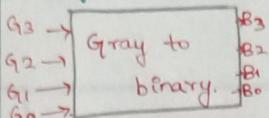
$$G_2 = b_3 \oplus b_2$$

$$G_3 = b_3$$

$$G_2 = b_1 \oplus b_2$$

$$G_0 = b_1 \oplus b_0$$

2. Gray to Binary :-



Gray I/P				Binary O/P			
G3	G2	G1	G0	B3	B2	B1	B0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	1
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	1
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	1
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	1	0
1	1	0	0	0	1	1	1
1	1	0	1	0	1	1	0
1	1	1	0	0	1	0	0
1	1	1	1	1	1	1	1

$$b_2 = G_3 G_2 + G_3 G_2 + G_3 \oplus G_2$$

$$b_2 = G_3$$

$$G_3 = b_3$$

$$G_2 = b_3 \oplus b_2$$

$$G_1 = b_1 \oplus b_2$$

$$G_0 = b_1 \oplus b_0$$

$$b_3$$

$$b_2$$

$$b_1$$

$$b_0$$

$$G_3 = b_3$$

$$G_2 = b_1 \oplus b_2$$

$$G_1 = b_1 \oplus b_0$$

$$G_0 = b_1 \oplus b_0$$

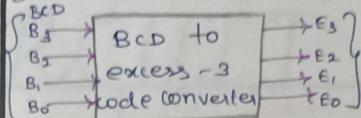
$$b_3$$

$$b_2$$

$$b_1$$

$$b_0$$

3. BCD To Excess 3 code converter :-



BCD				Excess 3 code			
B3	B2	B1	B0	E3	E2	E1	E0
0	0	0	0	1	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0
0	0	1	1	0	1	1	1
0	1	0	0	1	0	1	0
0	1	0	1	1	0	0	1
0	1	1	0	0	1	1	0
0	1	1	1	1	1	0	0
1	0	0	0	0	1	1	1
1	0	0	1	1	0	0	0
1	0	1	0	0	0	1	1
1	0	1	1	1	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	1	0	0	1
1	1	1	0	0	1	1	0
1	1	1	1	1	1	0	0

B3	B2	B1	B0	E3	E2	E1	E0
0	0	0	0	1	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0
0	0	1	1	0	1	1	1
0	1	0	0	1	0	1	0
0	1	0	1	1	0	0	1
0	1	1	0	0	1	1	0
0	1	1	1	1	1	0	0
1	0	0	0	0	1	1	1
1	0	0	1	1	0	0	0
1	0	1	0	0	0	1	1
1	0	1	1	1	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	1	0	0	1
1	1	1	0	0	1	1	0
1	1	1	1	1	1	0	0

$$B_3$$

$$B_2$$

$$B_1$$

$$B_0$$

$$E_3$$

$$E_2$$

$$E_1$$

$$E_0$$

$$F_3$$

$$F_2$$

$$F_1$$

$$F_0$$

$$Y_3$$

$$Y_2$$

$$Y_1$$

$$Y_0$$

$$B_3$$

$$B_2$$

$$B_1$$

$$B_0$$

$$E_3$$

$$E_2$$

$$E_1$$

$$E_0$$

$$F_3$$

$$F_2$$

$$F_1$$

$$F_0$$

$$Y_3$$

$$Y_2$$

$$Y_1$$

$$Y_0$$

$$B_3$$

$$B_2$$

$$B_1$$

$$B_0$$

$$E_3$$

$$E_2$$

$$E_1$$

$$E_0$$

$$F_3$$

$$F_2$$

$$F_1$$

$$F_0$$

$$Y_3$$

$$Y_2$$

$$Y_1$$

$$Y_0$$

$$B_3$$

$$B_2$$

$$B_1$$

$$B_0$$

$$E_3$$

$$E_2$$

$$E_1$$

$$E_0$$

$$F_3$$

$$F_2$$

$$F_1$$

$$F_0$$

$$Y_3$$

$$Y_2$$

$$Y_1$$

$$Y_0$$

$$B_3$$

$$B_2$$

$$B_1$$

$$B_0$$

$$E_3$$

$$E_2$$

$$E_1$$

$$E_0$$

$$F_3$$

$$F_2$$

$$F_1$$

$$F_0$$

$$Y_3$$

$$Y_2$$

$$Y_1$$

$$Y_0$$

$$B_3$$

$$B_2$$

$$B_1$$

$$B_0$$

$$E_3$$

$$E_2$$

$$E_1$$

$$E_0$$

$$F_3$$

$$F_2$$

$$F_1$$

$$F_0$$

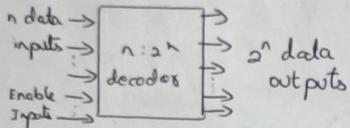
$$Y_3$$

$$Y_2$$

COMBINATIONAL LOGIC CIRCUIT

BINARY DECODER

n bit binary input and a one activated output out of 2^n outputs.

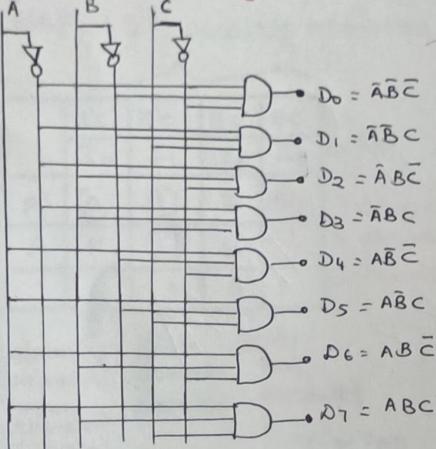


3 to 8 Decoder - Truth Table

A	B	C	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

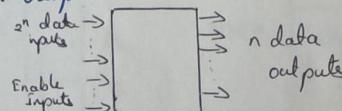
A 3 to 8 Decoder has
→ 3 Inputs
→ 8 Outputs.

3 to 8 Decoder Logic



BINARY ENCODER

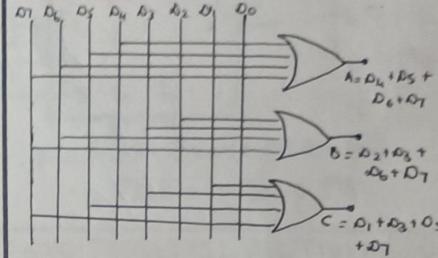
A binary encoder has 2^n input lines and n output lines.



TRUTH TABLE

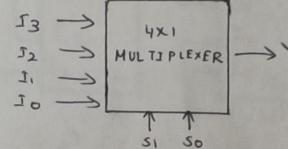
INPUTS						OUTPUTS		
D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	A	B	C
1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1
0	0	1	0	0	0	0	1	0
0	0	0	1	0	0	0	1	1
0	0	0	0	1	0	1	0	0
0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	1

LOGIC CIRCUIT



MULTIPLEXER

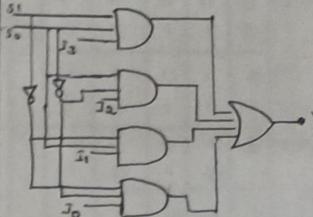
A combinational circuit that has maximum of 2^n data inputs, ' n ' selection lines and single output line.



Selection lines		Output
S ₁	S ₀	Y
0	0	I ₀
0	1	I ₁
1	0	I ₂
1	1	I ₃

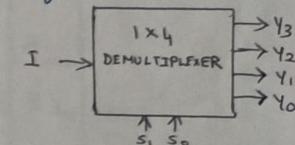
$$Y = S_1 S_0 I_0 + S_1 S_0 I_1 + S_1 S_0 I_2 + S_1 S_0 I_3$$

LOGIC CIRCUIT



DEMULTIPLEXER

1 input, 2^n output
'n' selection lines, 2^n combination of zeros and ones.



TRUTH TABLE

INPUTS		OUTPUTS			
S ₁	S ₀	Y ₃	Y ₂	Y ₁	Y ₀
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

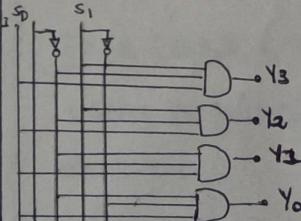
$$Y_3 = S_1 S_0 I$$

$$Y_2 = S_1 S_0 I'$$

$$Y_1 = S_1' S_0 I$$

$$Y_0 = S_1' S_0 I'$$

LOGIC CIRCUIT



IMPLEMENTATION OF BOOLEAN FUNCTIONS USING DECODER : MUX

IMPLEMENTATION USING MUX

Step 1: Find the no. of select lines & i/p lines of the Mux.

Eg: In 8:1 Mux No. of select lines: 3 if no. of i/p = 4 (ie n)
No. of select lines = $2^{(n-1)}$

Step 2: Formation of implementation table

1 Variable written in the first column of the table
All the combinations of $n-1$ variables are written column wise.

Step 3: Write all the mors in the cells of the Imp table Encircle the given minterm

Step 4: Draw the circuit

PROBLEM: Implement the function
 $f(A, B, C) = \Sigma (0, 1, 3, 5, 7)$

Step 1:

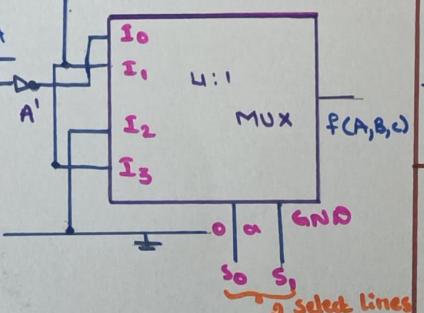
No. of i/p: 5;

No. of Select Lines 2
(ie 3-1)

Step 2; 3 possible Minterm

	B'C	B'C	B'C	BC
	I ₀	I ₁	I ₂	I ₃
A'	①	②	2	③
A	4	⑤	6	⑦
A'	1	0	1	

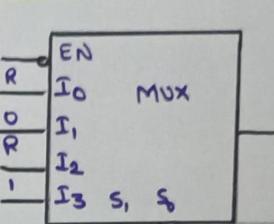
Element in upper row circled
Both elements both not encircled
Step 4:
↑
Both elements not encircled
↑ or VDD



GATE PROBLEM

A Mux is shown where S₀ → Select lines, I₀ to I₅ - data lines, F(P, Q, R) is the o/p

Find F



Sol:

$$I_0 = \bar{P} \bar{Q}$$

$$I_1 = \bar{P} Q$$

$$I_2 = P \bar{Q}$$

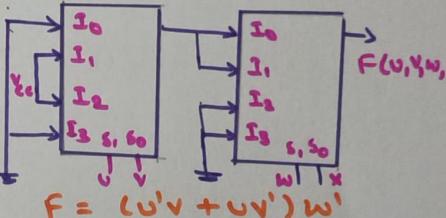
$$I_3 = P Q$$

$$\therefore F = \bar{P} \bar{Q} R + \bar{P} Q \bar{R} + P \bar{Q} R + P Q$$

$$= \bar{P} \bar{Q} R + P \bar{Q} R + P Q$$

GATE PROBLEM: ACTIVITY

A 4-Variables Boolean fn is realized using 4:1 Muxes. find F



**IMPLEMENTATION
USING DECODER**

Steps

No. of i/p - n

No. of o/p from decoders - 2^n

Act as minterm

Necessary minterm combined using OR gate.

PROBLEM

Implement full adder using decoder.

I Draw the truth table

A	B	C	S	Cy
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \Sigma (1, 2, 4, 7)$$

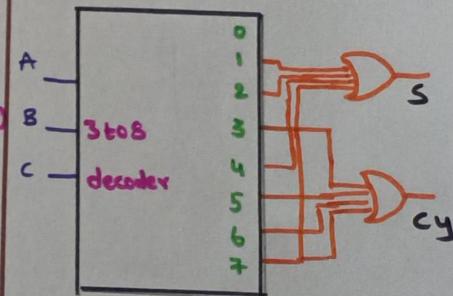
$$Cy = \Sigma (3, 5, 6, 7)$$

S → Sum

Cy → carry

1, 2, 3 etc - Minterm nos

II 3 i/p s, so a 3-to-8 decoder needed



PROBLEM:

Implement the functions

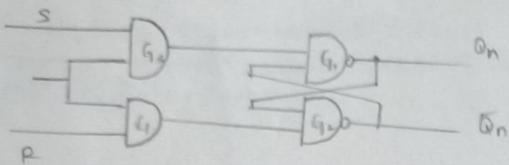
$$F = \Sigma (0, 3, 5, 7, 9, 11, 13)$$

using a suitable decoder.

FLIP FLOPS

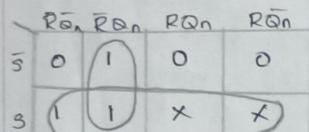
SR FLIPFLOP

LOGIC DIAGRAM.



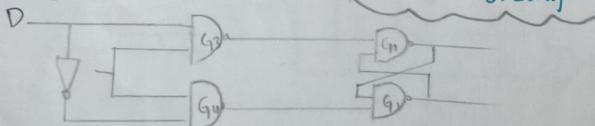
S	R	Q _{n+1}	state
0	0	Q _n	Nocharge
0	1	0	reset
1	0	1	set
1	1	X	undefined

Q _n	Q _{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0



$$Q_{n+1} = S + \bar{R}Q_n.$$

D FLIPFLOP

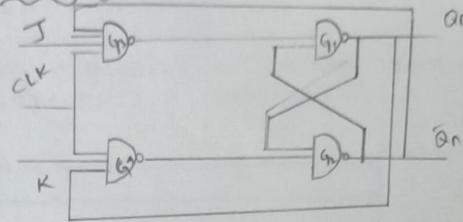


D	Q _{n+1}	state
0	0	reset
0	1	set
1	0	set

D	Q _{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

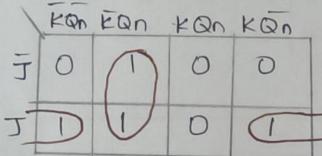
$$Q_{n+1} = D$$

JK FLIPFLOP.



J	K	Q _{n+1}	STATE
0	0	Q _n	Nocharge
0	1	0	Reset
1	0	1	Set
1	1	Q _n	Toggles

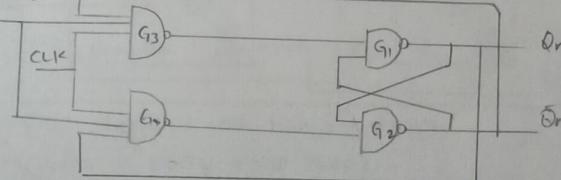
Q _n	Q _{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0



$$Q_{n+1} = J\bar{Q}_n + KQ_n$$

Characteristic Equation

T FLIP FLOP

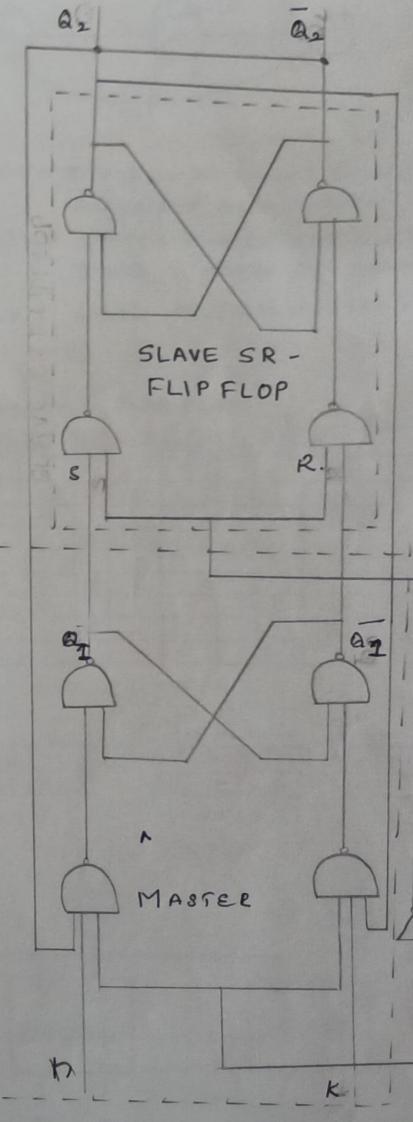


T	Q _{n+1}	state
0	Q _n	Nocharge
1	Qn-bar	Toggles

Characteristic Equation \Rightarrow

$$Q_{n+1} = T\bar{Q}_n + \bar{T}Q_n$$

MASTER SLAVE JK FLIPFLOP



SLAVE SR - FLIPFLOP

MASTER

MOORE & MEALY MODEL

MOORE CIRCUIT

- * CLOCKED SYNCHRONOUS CIRCUIT
- * OUTPUT DEPENDS ONLY ON PRESENT STATE OF THE FLIP-FLOPS.

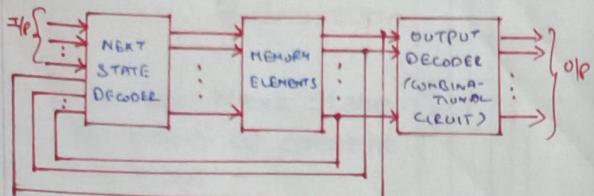


FIG: MOORE CIRCUIT WITH AN OUTPUT DECODER

$$\begin{aligned} \text{NEXT STATE} &= f\{\text{PRESENT STATE, INPUTS}\} \\ \text{OUTPUT} &= f\{\text{PRESENT STATE}\} \end{aligned}$$

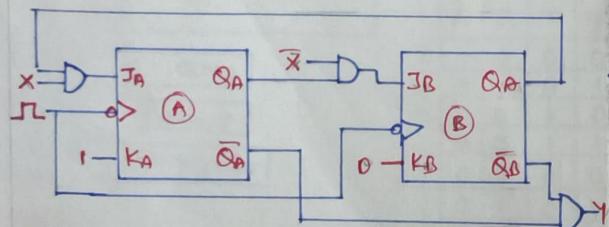


FIG: EXAMPLE OF MOORE CIRCUIT.

- * BOTH FLIP FLOPS GETS CLOCK.
- * CLOCKED CIRCUIT
- * OUTPUT $Y = Q_B \cdot Q_A$ - PRESENT STATE ONLY
- * INPUT CHANGES DOES NOT AFFECT THE OUTPUT
- * REQUIRES MORE NUMBER OF STATES FOR IMPLEMENTING SAME FUNCTION.
- * IT VARIES IN SYNCHRONISM WITH THE CLOCK INPUT.

MEALY CIRCUIT

- * CLOCKED SEQUENTIAL CIRCUIT
- * OUTPUT DEPENDS ON PRESENT STATE OF FLIP-FLOP(S) & INPUT(S).

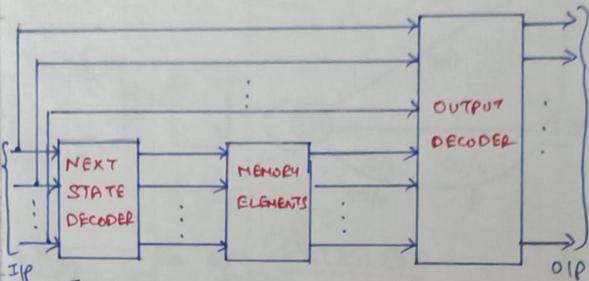


FIG: MEALY CIRCUIT MODEL

$$\begin{aligned} \text{NEXT STATE} &= f\{\text{PRESENT STATE, INPUTS}\} \\ \text{OUTPUT} &= f\{\text{PRESENT STATE, INPUTS}\} \end{aligned}$$

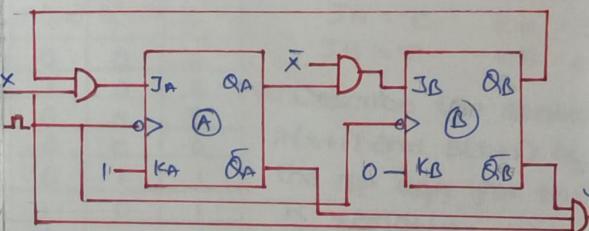


FIG: EXAMPLE OF MEALY CIRCUIT.

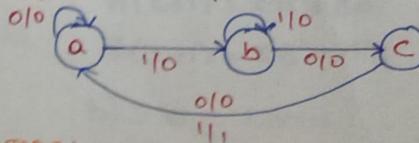
- * CLOCK - BOTH FLIP FLOPS
- * CLOCKED CIRCUIT
- * OUTPUT $Y = Q_B \cdot \bar{Q}_A \rightarrow$ PRESENT STATE & INPUTS
- * INPUT CHANGES AFFECT THE OUTPUT OF THE CIRCUIT
- * IT REQUIRES LESS NUMBER OF STATE FOR IMPLEMENTING SAME FUNCTION.

DESIGN 101 SEQUENCE DETECTORS

MEALY MACHINE

INPUT: 011010101100

STEP 1:- DEVELOP THE STATE DIAGRAM.



STEP 2:- CODE ASSIGNMENT

RULE1:- 2 STATES - SAME NEXT STATES - ADJACENT ASSIGNMENTS.

RULE2:- 3 STATES - NEXT STATE - SINGLE STATE - ADJACENT ASSIGNMENT.

RULE 1 GIVEN PREFERENCE OVER RULE 2

STEP 3:- MAKE PRESENT STATE / NEXT STATE TABLE.

USE D - FLIP FLOP

PRESNT STATE	IP	NEXT STATE	FLIP FLOP	OIP
X Y	X' Y'	Dx Dy		
0 0	0 0	0 0	0 0	0
0 0	1 1	0 0	1 0	0
0 1	0 0	0 0	0 0	0
0 1	1 0	0 0	0 0	0
1 0	0 0	1 0	0 0	1
1 0	0 1	0 1	0 1	0
1 0	1 1	0 1	1 0	0
1 1	0 X	X X	X X	X
1 1	1 X	X X	X X	X

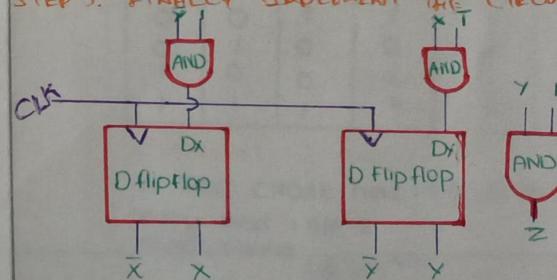
STEP 4:- DRAW K MAPS FOR Dx, Dy & Z

$$Dx = \bar{Y} \cdot 1$$

$$Dy = x \cdot \bar{I}$$

$$Z = 4 \cdot 1$$

STEP 5:- FINALLY IMPLEMENT THE CIRCUIT



ANALYSIS OF SEQUENTIAL CIRCUITS

Analysis - What a given circuit will do under certain operating conditions

State equation or Transition equation

→ Specifies the Next State as a function of present State as input

Steps to analyse a clocked Sequential Circuit

Sequential Circuit if NOT given obtain it from state QNS

↓
State \in QNS

↓
State Table

↓
State Diagram

PROBLEM

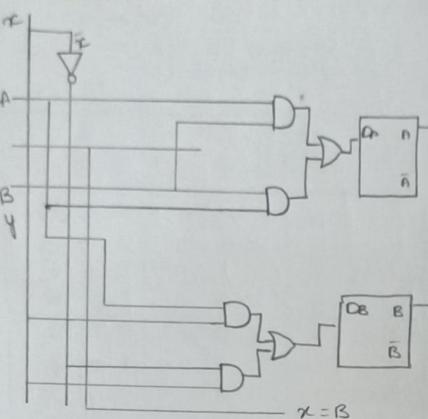
A Sequential Circuit with 2D F/F A and B 2 inputs $x, y, 10/1P$ Z is specified by its for Next State o/p equations.

$$A(t+1) = \bar{x}y + zA$$

$$B(t+1) = \bar{x}A + \bar{x}B; Z = B$$

- Draw the logic diagram
- List the State table
- Draw the state diagram.

a) Logic Diagram

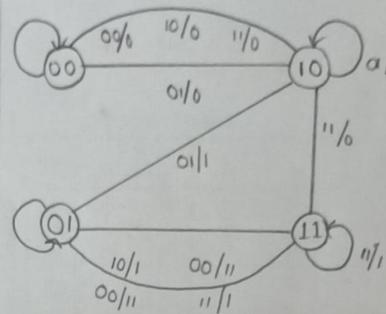


b) State table :-

A	B	x	y	A(t+1)	B(t+1)	Z
0	0	0	0	0	0	0
0	0	0	1	1	0	0
0	0	1	0	0	0	0
0	0	1	1	0	0	0
0	1	0	0	0	1	1
0	1	0	1	1	0	1
0	1	1	0	0	1	1
0	1	1	1	1	0	1
1	0	0	0	0	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	1	1

A,B - States

$x \& y$ - 3/ps



PROBLEM

A Sequential Circuit has 2 JK F/F, A,B are input x JK Circuit is described by the for f/f input eqn

$$JA = x, \quad KA = B$$

$$JB = \bar{x}, \quad KB = A$$

- Describe the state eqn $A(t+1)$ and $B(t+1)$ by substituting the 1/P eqn for the J and K Variables
- Draw the state table State diagram.

- State eq'n for JK F/F

$$A(t+1) = JA' + KA$$

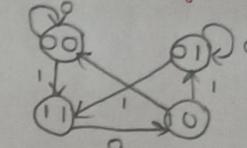
Sub JA, KA F/F

$$A(t+1) = \bar{x}A' + BA$$

$$B(t+1) = \bar{x}B' + A'B$$

b) State table

c) State diagram



JK F/F is used

- Either state eqn can be used
- Excitation tables can be used to find Next state

PROBLEM

Complete the state table

→ Use the excitation table

A	Z	J	JA	KA
0	0	1	1	x
0	1	0	0	x
1	0	0	x	1
1	1	1	x	0

Problem:

A Seq Circuit has 1 F/F 2 1/P x, y and 1 0/P Z State table



SYNCHRONOUS COUNTER

COUNTER

- Counts clock pulses.
- Group of flip flops.

TYPES

- Synchronous
- Asynchronous

DESIGN OF COUNTERS

- Identify the number of variables for the given problem
- Construct state table using flip flop excitation table
- Get state equation for flip flop i/p variable using k-map
- Construct sequential diagram using flip flops.

EXCITATION TABLE

OF ALL FLIP FLOPS

$Q(1)$	$Q(0)$	J	K	SR	T	D
0	0	0	X	0	X	0
0	1	1	X	1	0	1
1	0	X	1	0	1	1
1	1	X	0	X	0	0

EXAMPLE: 1

→ Design a 3 bit Synchronous counter using T-Flip Flop

Ans: - No. of variable is 3
Hence $2^3 = 8$

2) State Table

Seq.	PS			NS			T _A	T _B	T _C
	A	B	C	A	B	C			
0	0	0	0	0	0	1	0	1	1
1	0	0	1	0	1	0	0	1	0
2	0	1	0	0	1	1	1	1	1
3	0	1	1	0	0	0	1	0	0
4	1	0	0	1	0	0	1	1	0
5	1	0	1	1	0	0	0	1	1
6	1	1	0	0	0	0	0	0	0
7	1	1	1	1	1	0	1	1	0

3) State Rqmn.

for T_A

A	B	C	BC
0	0	0	00
1	0	0	11

for T_B

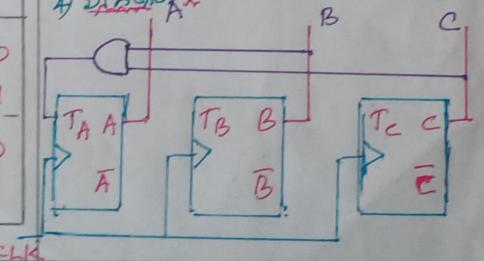
A	B	C	BC
0	0	0	00
1	0	0	11

for T_C

A	B	C	BC
0	0	0	00
1	1	1	11

$$\therefore T_A = BC \\ T_B = C \\ T_C = 1$$

4) Diagram



DESIGN OF COUNTER FOR RANDOM SEQUENCE

→ Design a syn. counter for the following sequence 0, 3, 2, 7, 6, 5 using D FF

State Table

Seq.	PS			NS			DA	DB	DC
	A	B	C	A	B	C			
0	0	0	0	0	1	1	0	1	1
1	0	1	1	0	1	0	0	1	0
2	0	1	0	1	1	1	1	1	1
3	1	1	1	1	1	0	1	1	0
4	1	1	0	0	0	0	0	0	0
5	1	0	0	1	0	1	1	0	1
6	0	0	1	0	0	1	0	1	0
7	0	1	0	0	1	0	0	0	0

→ 1, 4, 5: → don't cares

for D_A

A	B	C	BC
0	0	0	00
1	X	X	11

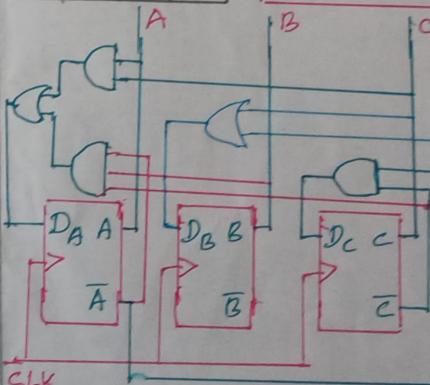
for D_B

A	B	C	BC
---	---	---	----

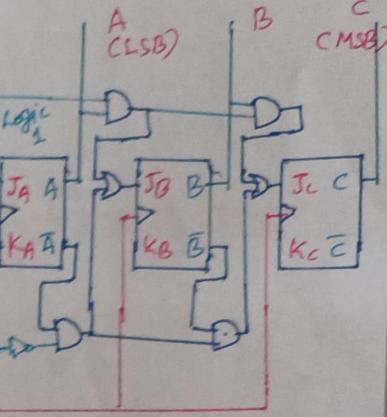
A	B	C	BC
---	---	---	----

for D_C

$$\therefore DA = AB + AC \\ DB = A' + C \\ DC = A'C'$$

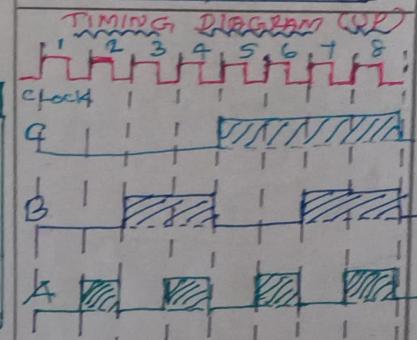


BINARY UP-DOWN COUNTER

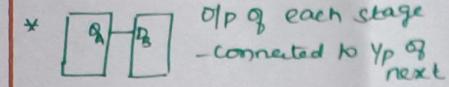


STATE TABLE

UP COUNTER			DOWN COUNTER		
C	B	A	C	B	A
0	0	0	1	1	1
0	0	1	1	1	0
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	0	1
1	1	0	0	1	0
1	1	1	0	0	0



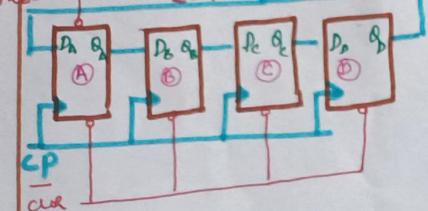
RING COUNTER



A circuit diagram showing a T flip-flop. It has four output terminals labeled Q_A , Q_B , Q_C , and Q_D . The inputs are labeled \overline{PRE} and \overline{CLR} . There is also a small circle with a dot inside, likely indicating a ground connection or common reference point.

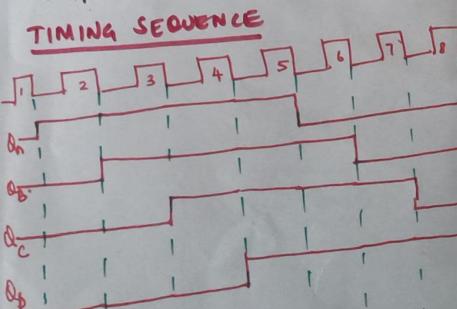
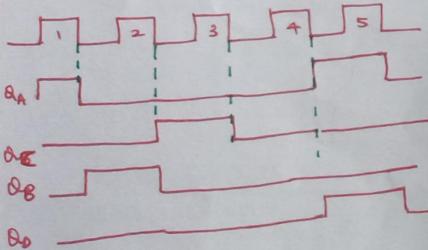
* FIRST CLOCK PULSE $Q_3 = 1$ OTHERS 0

→ Next CP - '1' shifted around
LOGIC DIAGRAM



CP	Q _A	Q _B	Q _C	Q _D
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1

TIMING SEQUENCE

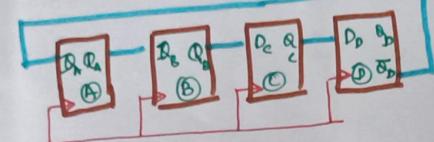


JOHNSON COUNTER

2 K Bit Ring | Shift / Turned ring |
switch tail counters

- \Rightarrow  $O/p(B_n)$ is connected to $D\,y\,p\,q$ next stage

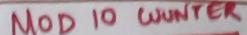
D_{in} D_{out} $\oplus P(\bar{Q})$ of last FF - connected to D input of first FF.



C_P	B_+	B_0	B_c	B_d
0	0	0	0	0
-1	-1	-1	0	0
2	-1	-1	1	0
3	-1	-1	-1	1
4	0	-1	-1	-1
5	0	0	-1	-1
6	0	0	0	1
7	0	0	0	1

- Initially all FF = 0 $Q_A = Q_B = Q_C = Q_D = 0$
- Next CPs $Q_A = \bar{Q}_D =$
- CLK1 - $Q_A = 1 \quad Q_B = Q_C = Q_D = 0$
- CLK2 - $Q_A = 1 \quad Q_B = 1 \quad Q_C = Q_D = 0$
- After 8 States - sequence repeats

TIMING SEQUENCE



- DIVIDE BY 10
- Decade Counter - 10 states
- DECIMAL WINTER / BCD SYN Counter

Step 1: $2^n > N$ $N = 10 \Rightarrow n = 4$

$$2^n > N \quad N = 10 \Rightarrow \underline{n \geq 4}$$

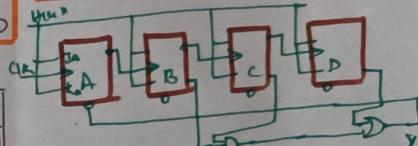
Step 2: Type of FF

Step 3: excitation table 109 FF

<u>PRESIDENT STATE</u>	<u>NEXT STATE</u>
D ₁	D ₂
D ₂	D ₃
D ₃	D ₁
D ₄	D ₅
D ₅	D ₄

Q ₃	Q ₂	Q ₁	Q ₀	Q ₃	Q ₂	Q ₁	Q ₀
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	0	0	0	0	0

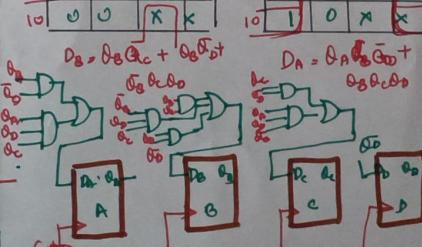
$$Y = \bar{Q}_D + Q_C \bar{Q}_L$$



BINARY RIPPLE COUNTER

- Binary - 2bit \rightarrow 4 states 2^2

* for $gk \Rightarrow J=1, k=1, l_{n+1} = \bar{a}_n$
 needed to prevent going cp. so a_0 together



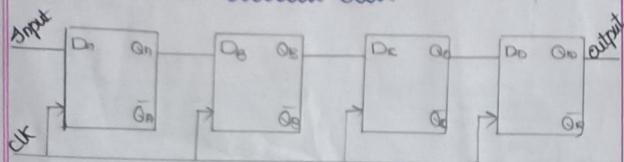
Current error
00,01,40,11 2
restated 400

SHIFT REGISTERS

Shift Registers

- * Flip Flop can be used to store a single bit
- * N Flip flops are to be connected
- * It is a group of flip-flops used to store multiple bits of data.

Serial In Serial Out



Truth table

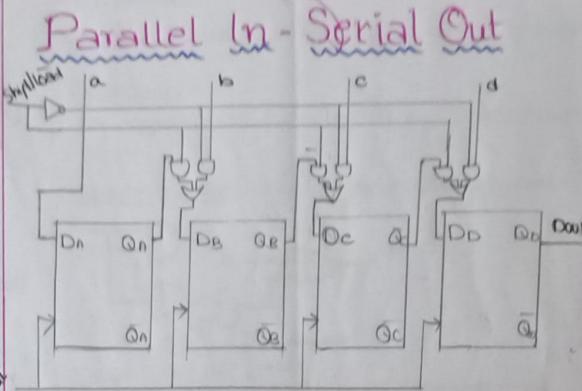
4P	Din	CLK	QA	QB	QC	QD	O/P Dout
1	1	1	0	0	0	0	0
1	2	1	1	0	0	0	0
1	3	1	1	1	0	0	0
1	4	1	1	1	1	1	1

Serial-In Parallel Out



Truth table

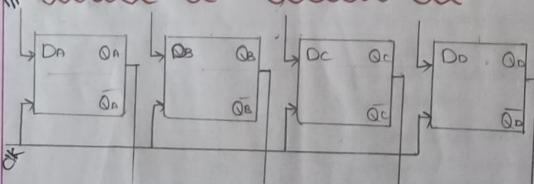
Input Din	CLK	Output			
		QA	QB	QC	QD
1	1	1	0	0	0
1	2	1	1	0	0
1	3	1	1	1	0
1	4	1	1	1	1



Truth table

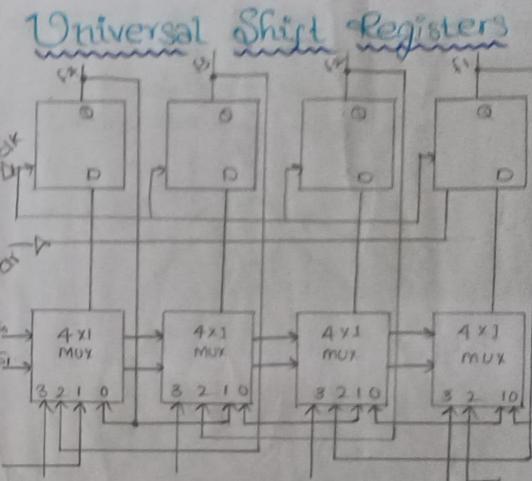
CLK Pulse	QA	QB	QC	QD
0	1	0	0	1
1	1	1	0	0
2	0	1	1	0
3	0	0	1	1

Parallel In - Parallel Out

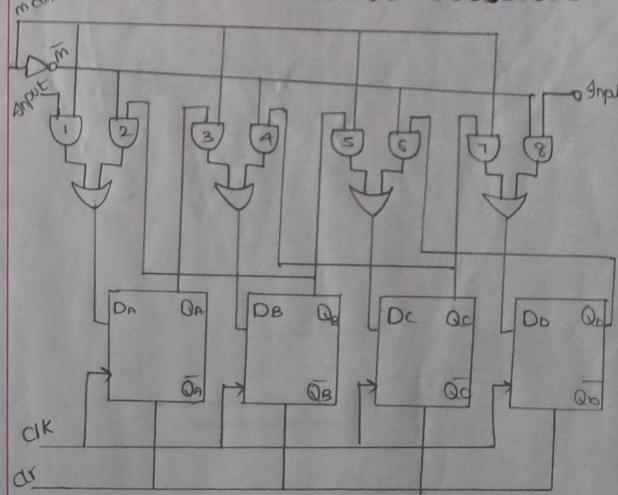


Truth table

CLK Pulse	QA	QB	QC	QD
0	0	0	0	0
1	1	1	0	1



Bidirectional Shift Register



Asynchronous Sequential Circuit

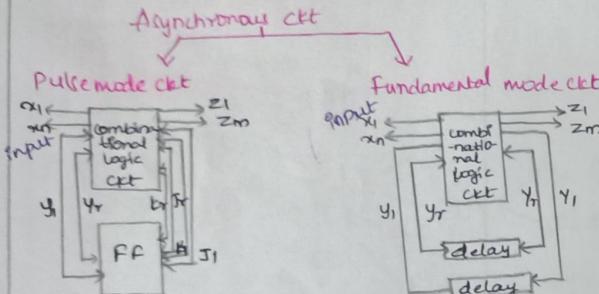
Asynchronous Sequential Circuit

Asynchronous Seq. Circuit :-

1. Memory element - unclocked flipflop or delay elements.
2. Input changes - memory element changes
3. State change - delay time

Types of Synchronous Circuits

- combinational circuit and delay element.



Excitation variables: $J_1-J_n, E_1-E_n, Y_1-Y_n$

Secondary variables: Y_1-Y_n

Input variables x_1-x_n ; output variable z_1-z_m

* Input variable changes, Secondary variable does not change - delay.

* Excitation variable - next state of the ckt.

* at Steady state (stable) $Y_i=Y'_i$

* unstable $Y_i \neq Y'_i$ (continuous transition)

for proper operation:-

1. only one I/P variable change at a time.
2. Input - wait to react stable state
3. time between two input - longer than to react stable state.

Analysis :-

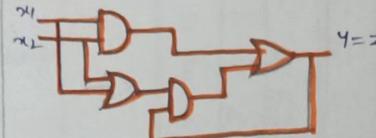
1. Determine next secondary state and output equations.

2. State table 3. transition table 4. output map

Problems :-

An asynchronous ckt given by $Y = x_1 x_2 + (x_1 + x_2) Y, Z = Y$.

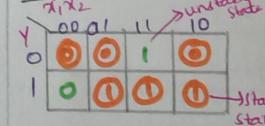
Solution:- Step 1: Draw the logic diagram



Step 2: State table

Present State	Next State	Stable State	Output Z
$y\ x_1\ x_2$	Y	Yes/No	Z
0 0 0	0	Yes	0
0 0 1	0	Yes	0
0 1 1	1	No	1
0 1 0	0	Yes	0
1 0 0	0	No	0
1 0 1	1	Yes	1
1 1 1	1	Yes	1
1 1 0	1	Yes	1

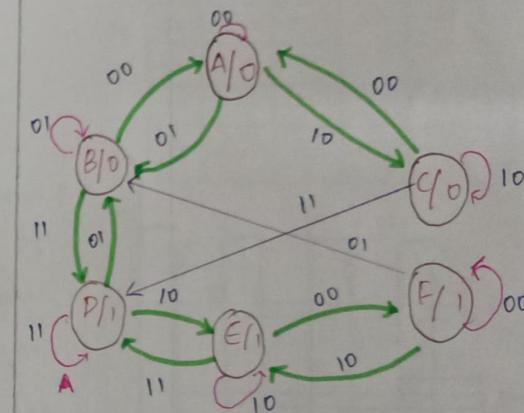
Step 3: Transition state



Step 4: Behaviour of the ckt.

* The circuit gives carry output of the full circuit.

' Draw the State diagram for the logic assume a asynchronous Sequential ckt with two inputs x and y and with one output z. whenever y is 1, input x is transferred to z. When y is 0, the output does not change for any change in x.'

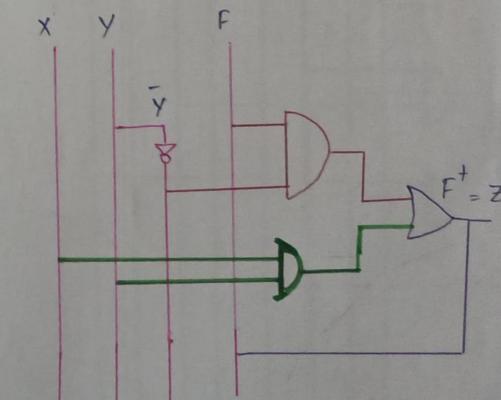


LOGIC DIAGRAM :-

Step 4: Output Map
* output mapped for all stable states

* For unstable states output is mapped unspecified.

$x_1\ x_2$	00	01	11	10
0 0	0	0	-	0
1 0	-	1	1	1



DESIGN OF ASYNCHRONOUS SEQUENTIAL CIRCUIT

18

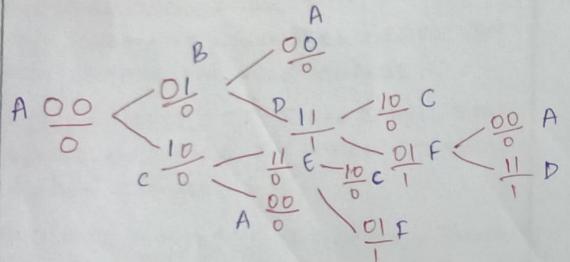
Design of asynchronous sequential circuit

1. State diagram
2. Primitive table
3. State assignment
4. Analyse primitive table

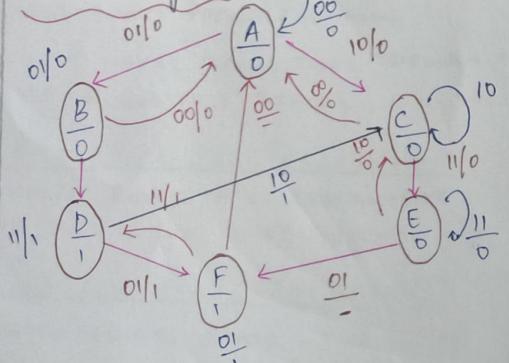
Problems :- Design an asynchronous sequence circuit has two inputs X_2 and X_1 and one output Z . When $X_1 = 0$ the output $Z = 0$. The first change in X_2 occurs while X_1 is 1 will cause output Z to be 1. The output Z will remain 1 until X_1 returns to 0.

Step-1 State diagram Input $X_2 X_1$

Flow Table



State Diagram



Step-2 :- Primitive flow table

(Circle) Present state
Stable state

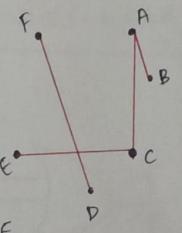
Present state	00	01	11	10
A	(A, 0)	B, 0	--	C, 0
B	A, 0	(B, 0)	D, -	--
C	A, 10	--	F, 0	(C, 0)

two bit doing
change care

Implicit table Compare the two rows A, B of Primitive flow table and put ✓ if same ✗ if different.

B ✓ (A, B)	
C	✓ ✗
D	✗ ✗
E	✗ ✗
F	✗ ✗ ✗ ✗

Merge graph



$$A = B$$

$$C = E$$

$$D = F$$

Reduce Primitive Flow Table :-

	00	01	11	10
$A=B$	A, 0	B, 0	D, -	C, 0
$C=E$	A, 0	F, -	E, 0	C, 0
$D=F$	A, -	F, 1	D, 1	C, -

Step 3: State assignment

Assume

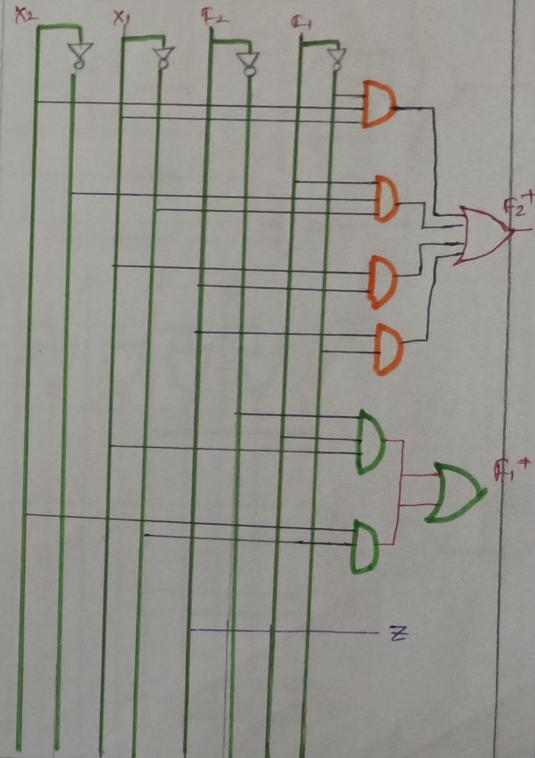
$$A = B \Rightarrow S_0$$

$$C = E \Rightarrow S_1$$

$$D = F \Rightarrow S_2$$

	00	01	11	10
S ₀	S ₀ , 0	S ₀ , 0	S ₂ , -	S ₁ , 0
S ₁	S ₁ , 0	S ₂ , -	S ₁ , 0	S ₁ , 0
S ₂	S ₀ , -	S ₂ , 1	S ₂ , 1	S ₁ , -

$F_2 F_1$	00	01	11	10
S ₀	00, 0	00, 0	10, -	01, 0
S ₁	00, 0	11, -	01, 0	01, 0
S ₂	-1-	10, -	-1-	01, -
	10	00, -	10, 1	10, 1



RACES

- * TWO OR MORE STATE VARIABLES CHANGE
- * CHANGE IN INPUT VARIABLES.
- * STATE VARIABLE CHANGES - UNPREDICABLE MANNER

Eg: IF STATE VARIABLE CHANGES FROM
 $00 \rightarrow 11$ FOR A GIVEN INPUT - DELAY.
 1st / 2nd VARIABLE CHANGE FASTER
 $00 \rightarrow 01 \rightarrow 11$
 $00 \rightarrow 10 \rightarrow 11$

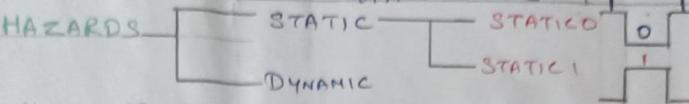
- * NON CRITICAL RACE - IF FINAL STABLE STATE DOES NOT DEPEND ON ORDER IN WHICH STATE VARIABLE CHANGE.
- * CRITICAL RACE: IF FINAL STABLE STATE DEPENDS ON ORDER IN WHICH STATE VARIABLE CHANGE.
- * AVOID RACE - CIRCUIT DIRECTED - THROUGH INTERMEDIATE UNSTABLE STATES WITH UNIQUE STATE VARIABLE CHANGE.
- * CIRCUIT MAKES TRANSITIONS THROUGH A SERIES OF UNSTABLE STATES - CYCLE.

CRITICAL RACE FREE ASSIGNMENT

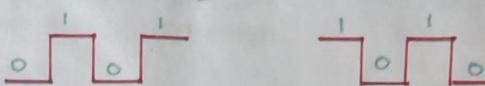
- SHARED ROW STATE ASSIGNMENT
- ONE HOT STATE ASSIGNMENT

SHARED ROW STATE ASSIGNMENT

INTERMEDIATE STATE - ONE STATE VARIABLE CHANGE - ONE TIME WHEN STATE TRANSITIONS.

RACES AND HAZARDS

- UNWANTED SWITCHING TRANSIENT / SPIKE / GLITCH
- APPEAR AT THE OUTPUT OF THE CIRCUIT.
- CAUSE - DIFFERENCE IN PROP DELAY AT DIFFERENT PATH.

DYNAMIC HAZARD

CIRCUIT GOES THROUGH THREE / MORE TRANSIENTS

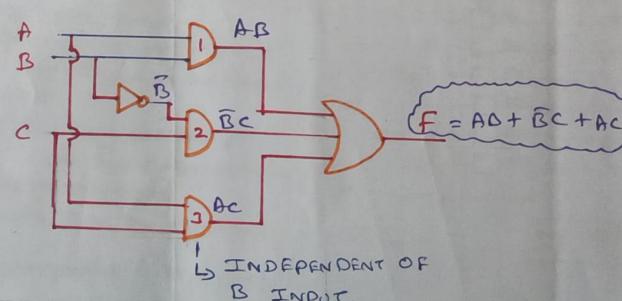
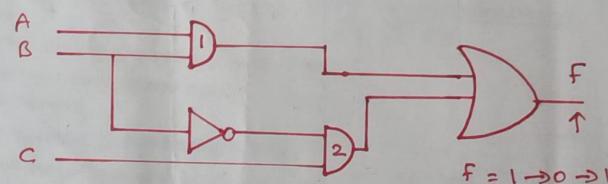
ELIMINATING HAZARDS

CONSIDER $F(A, B, C) = \sum m(1, 5, 6, 7)$

	BC	00	01	11	10
A	0	0	1	0	0
B	0	1	1	1	1
C	1	1	1	1	1

→ HAZARD FREE

ACTIVITY
 FIND A CIRCUIT THAT HAS NO STATIC HAZARDS & IMPLEMENTS THE BOOLEAN FUNCTION
 $F(A, B, C, D) = \sum m(0, 2, 6, 7, 8, 10, 12)$



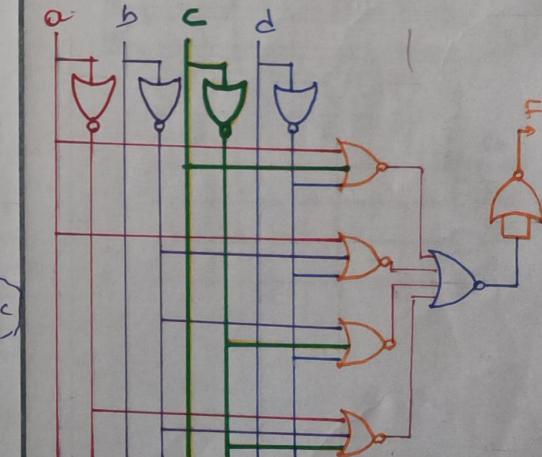
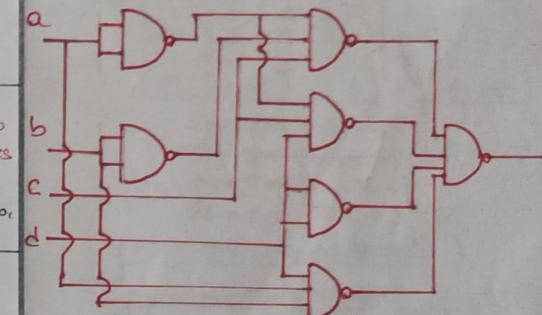
FIND A STATIC AND DYNAMIC HAZARD FREE REALIZATION FOR THE FOLLOWING FUNCTION USING (I) NAND (II) NOR

$$F(a, b, c, d) = \sum m(1, 5, 7, 14, 15)$$

$$F = \bar{a}\bar{c}d + \bar{a}\bar{b}d + bcd + abc$$

ab	cd	00	01	11	10
00	00	1			
01	01		1		
11	11			1	
10	10				1

1. COMPLEMENT INPUT
2. COMPLEMENT OUTPUT
3. REPLACE NAND GATE BY NOR GATE.

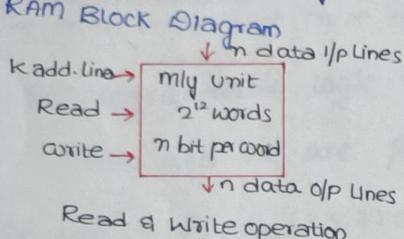


MEMORY DEVICES

Memory Devices

- ROM (Read only memory)
- RAM (Random access memory)

Memory: 1 byte = 8 bits
1 word = 2 bytes.



* Store word into mly:

Apply Binary add (Desired word) → Add. lines
Data bits stored mly to the data I/O lines
Activate the write I/O

* Store word out of mly:

Apply Binary Add (DW) → Add. lines
Activate the read I/O

RAM MEMORY:-

- Each word location is separated with space
- Each word occupying one particular location.

Types of RAM

- Dynamic RAM
- Static RAM

• DRAM loses its data in a very short time (milli sec)

ROM MEMORY:-

- Read Only Memory
- Non Volatile memory
- Used for permanent storage
- Random Access Property

ROM

Binary information must be specified by the designer

↓
Embedded in the unit off from the required interconnection pattern

↓
Data stays within the unit even when power is turned off and on again

Block diagram

K input (address) → 2^{kn} ROM → n output (data)

Types of ROM

NRAM
PROM
EPROM
EEPROM

* ROM (Hashed ROM)

- Hardwired device
- Pre programmed set of data as instruction.

PROM (Programmable ROM)

- Data can be modified only once by a user.
- Inside the PROM chip there are small fuses which are burnt open during programming.

EPROM (Erasable programmable ROM)

- Data can be erased by exposing it to ultra violet light for a duration of upto 40 min
- During programming an electric charge is trapped in an insulated gate region

EEPROM (Electrically Erasable PROM)

- EEPROM can be programmed & erased electrically
- 10,000 times can be reprogrammed
- Erasing required 4 to 10 ms

Applications of Static ROM

- Uses transistors, NO Capacitors.
- Transistors do not require power to prevent leakage
- SRAM not required refresh on regular basis.

Q1 ROM Problem:

Using ROM realize the following expression $f_1(a,b,c) = \sum m(0,1,3,5,7)$

$$f_2(a,b,c) = \sum m(1,2,5,6)$$

Sol:- $2^3 - 8$ minterms

Block diagram Truth table

A ₂	A ₁	A ₀	F ₁	F ₂
0	0	0	1	0
0	0	1	1	1
0	1	0	0	1
0	1	1	0	0
1	0	0	0	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

Q2 ROM Problem:

Design a Combinational using a PROM. The circuit accepts 3 bit binary number and generates its equivalent Excess-3 code.

Sol:- $2^3 - 8$ (0-7)

Input			Output				
B ₂	B ₁	B ₀	E ₃	E ₂	E ₁	E ₀	
0	0	0	0	0	1	1	
0	0	1	0	1	0	0	
0	1	0	0	1	0	1	
0	1	1	0	1	1	0	
1	0	0	0	1	0	1	
1	0	1	1	0	0	0	
1	1	0	1	0	0	0	
1	1	1	0	1	0	1	

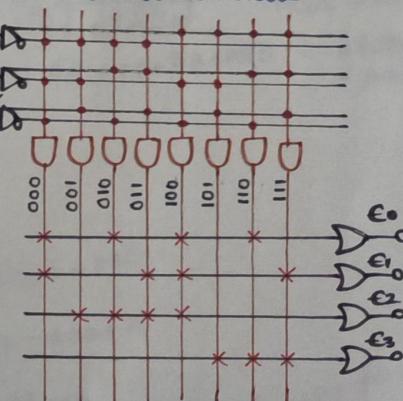
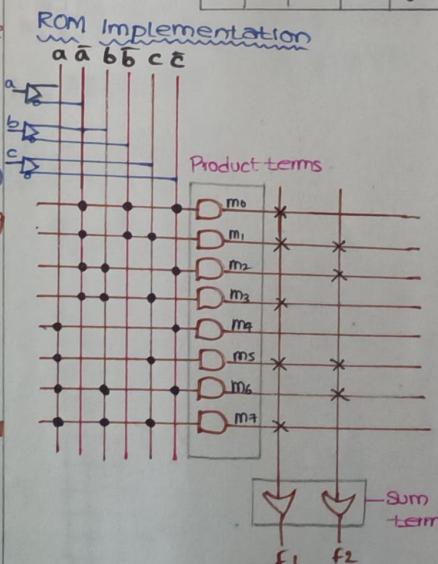
$$E_0 = \sum m(0,2,4,6)$$

$$E_1 = \sum m(0,1,3,4,7)$$

$$E_2 = \sum m(1,2,3,4)$$

$$E_3 = \sum m(5,6,7)$$

PROM Implementation



Verilog HDL

VERILOG HDL :

- * verification logic HDL
- * allows different levels of abstraction to be mixed in the same model.

Built in data types :

NETS : represents structural connectivity

Registers : abstract storage elements

wire : connectivity with no logical behaviour.

Variable type	input	output	inout
Net	Yes	Yes	Yes
Register	No	Yes	No

Memory declaration : (2D array)

- * provides an extension to the register variable declaration

Eg : - reg [31:0]

strings : sized by procedural assignment statement.

constant : declared by the keyword parameter.

Eg : parameter byte_size = 8

operators :

- ⇒ Arithmetic operators :

+,-,* , / (divide), % (modulo)

- ⇒ Logical operators :

&& (AND), || (OR), ! (negation)

- ⇒ Bitwise operators :

& (EX-OR), ~& (EX-NOR), & (AND)

- ⇒ Relational operators :

>, <, >=, <=

Equality operators :

== (logic equality), === (case equality)

Shift operators :

<< (left shift), >> (right shift)

Module name

port list, port declaration (if port present parameters (optional))

Declaration of wire registers and variables

Data flow statements (assign)

Instantiation of lower level models

always & initial blocks. At Block → behavioural statements

Tasks & Functions.

end module statements.

Modelling styles :

- 1) Behavioural model level - describe how the hardware should behave without any interface to digital hardware.

- 2) Data flow: provides the means of describing combinational circuits by their functional feature rather than by their gate structures.

- 3) Gate level: each gate and its interconnections are explicitly specified.

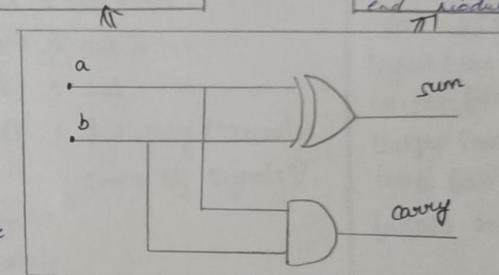
★ Registers normally represent storage elements, they need clock signals to update their output value.

Data flow modelling :

```
module half-adder (sum, carry, a,b);
input a,b;
output sum, carry;
assign sum = a^b;
assign carry = a&b;
end module
```

Gate level modelling :

```
module half-adder (sum, carry, a,b);
input a,b;
output sum, carry;
xor sum, (a^n~b);
and carry, ((a&b));
end module
```



* Initial statements are executed once and always statements are executed repetitively.

* Procedural and continuous assignments can co-exist with in a module.

Behavioural level modelling :

```
module Half-adder (sum, carry, a,b);
input a,b;
output sum, carry;
always @ (a or b)
assign {carry,sum} = a+b;
end module
```

VERILOG PROGRAM FOR LOGIC GATES.

AND GATE :

```
module and_dataflow (y, a,b);
input a,b;
output y;
assign y = a&b;
end module
```

NAND GATE :

```
module nand_gate (y,a,b);
input a;
input b;
output y;
assign y = ~ (a & b);
end module
```

OR GATE :

```
module or_dataflow (y, a,b);
input a,b;
output y;
assign y = a|b;
end module
```

NOR GATE :

```
module nor_gate (y, a,b);
input a;
input b;
output y;
assign y = ~ (a | b);
end module
```

NOT GATE :

```
module not_dataflow (y, a);
input a;
output y;
assign y = ~ a;
end module
```

XOR GATE :

```
module xorgate (y,a,b);
input a;
input b;
output y;
assign y = a ^ b;
end module
```

XNOR GATE :

```
module xnorgate (y,a,b);
input a;
input b;
output y;
assign y = ~ (a ^ b);
end module
```

VERILOG PROGRAM FOR COMBINATIONAL CIRCUITS

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Half Adder

```
module halfadder(a,b,s,c);
    input a,b;
    output s,c;
    assign s=a&b;
    assign c=a&b;
endmodule
```

Full Adder

```
module fulladder(a,b,c,sum,carry);
    input a,b,c;
    output sum,carry;
    assign sum = a&b&c;
    assign carry = (a&b)|(b&c)|(c&a);
endmodule
```

Half subtractor

```
module halfsub(a,b,d,br);
    input a,b;
    output d,br;
    wire d,br;
    assign d=a&b;
    assign br=(~a)&b;
endmodule
```

Full subtractor

```
module fullsub(a,b,bin,diff,bout);
    input a,b,bin;
    output diff,bout;
    wire diff,bout;
    assign diff = bin ^ a ^ b;
    assign bout = (na&bin)|(na&b)|(b&bin);
endmodule
```

2 to 4 Decoder

```
module decoder24_assign(en,a,b,y);
    input en,a,b;
    output [3:0]y;
    wire enb, na, nb;
    assign enb = ~en;
    assign na = ~a;
    assign nb = ~b;
    assign y[0] = ~enb & na & nb;
    assign y[1] = ~enb & na & b;
    assign y[2] = ~enb & a & nb;
    assign y[3] = ~enb & a & b;
endmodule
```

Priority Encoder

```
module priorityencoder(en,i,y);
    input en;
    input [7:0]i;
    output [2:0]y;
    assign y[2] = i[4]|i[5]|i[6]|i[7]&en;
    assign y[1] = i[2]|i[3]|i[6]|i[7]&en;
    assign y[0] = i[1]|i[3]|i[5]|i[7]&en;
endmodule
```

4:1 Multiplexer

```
module mux4to1(out,i0,i1,i2,i3,
                s1,s0);
    output out;
    input i0,i1,i2,i3;
    input s1,s0;
    reg out;
    begin
        if(s0==0)
            out = i0;
        else
            out = i1;
    end
endmodule
```

always @ (s1 or s0 or i0 or i1 or i2 or i3)

```
case ({s1,s0});
    2'b00 : out = i0;
    2'b01 : out = i1;
    2'b10 : out = i2;
    2'b11 : out = i3;
default : $display ("Invalid
    Combi of Signals");
endcase
endmodule
```

8 Bit Ripple Carry adder

```
module ripple8bitadder(a,b,
                        cin,sum,cout);
    input [7:0]a;
    input [7:0]b;
    input cin;
    output [7:0]sum;
    output cout;
    wire [6:0]c;
```

```
fulladd a1(a[0],b[0],cin,
            sum[0],c[0]);
fulladd a2(a[1],b[1],c[0],
            sum[1],c[1]);
```

```
fulladd a3(a[2],b[2],c[1],
            sum[2],c[2]);
fulladd a4(a[3],b[3],c[2],
            sum[3],c[3]);
```

```
fulladd a5(a[4],b[4],c[3],
            sum[4],c[4]);
fulladd a6(a[5],b[5],c[4],
            sum[5],c[5]);
```

```
fulladd a7(a[6],b[6],c[5],
            sum[6],c[6]);
fulladd a8(a[7],b[7],c[6],
            sum[7],c[7]);
```

```
fulladd a9(a[8],b[8],c[7],
            sum[8],c[8]);
fulladd a10(a[9],b[9],c[8],
            sum[9],c[9]);
```

```
fulladd a11(a[10],b[10],c[9],
            sum[10],c[10]);
fulladd a12(a[11],b[11],c[10],
            sum[11],c[11]);
```

```
fulladd a13(a[12],b[12],c[11],
            sum[12],c[12]);
fulladd a14(a[13],b[13],c[12],
            sum[13],c[13]);
```

```
fulladd a15(a[14],b[14],c[13],
            sum[14],c[14]);
fulladd a16(a[15],b[15],c[14],
            sum[15],c[15]);
```

```
fulladd a17(a[16],b[16],c[15],
            sum[16],c[16]);
fulladd a18(a[17],b[17],c[16],
            sum[17],c[17]);
```

```
fulladd a19(a[18],b[18],c[17],
            sum[18],c[17]);
fulladd a20(a[19],b[19],c[18],
            sum[19],c[18]);
```

```
module fulladd(a,b,cin,sum,cout);
    input a;
    input b;
    input cin;
    output sum,cout;
    assign sum = (a&b^cin) | (a&~b) | (~a&b);
    assign cout = ((a&b)&(b&cin)) | (a&cin);
```

VERILOG PROGRAM FOR SEQUENTIAL CIRCUITS

Verilog HDL Design of Sequential Circuits:

Up Counter:

```
module counter (clk, clka, q);
    input clk, clka;
    output [3:0] q;
    reg [3:0] tmp;
    always @ (posedge clk or posedge clka)
        begin
            if (clka)
                temp <= 4'b0000;
            else
                temp <= temp + 1'b1;
        end
    assign q = temp;
endmodule
```

Down Counter:

```
module counter (clk, s, q);
    input clk, s;
    output [3:0] q;
    reg [3:0] tmp;
    always @ (posedge clk)
        begin
            if (s)
                temp <= 4'b1111;
            else
                temp <= temp - 1'b1;
        end
    assign q = temp;
endmodule
```

serial in Serial out shift

Register:

```
module SISO (clk, rst, qin, qout, qoutba);
    input clk;
    input rst;
    input qin;
    output qout;
    output qoutba;
    wire qout1, qout2, qout3, qoutb1, qoutb2,
          qoutb3;
    dflipflop1 a_dflipflop_1 (clk, qin, rst, qout1, qoutb1);
    dflipflop2 a_dflipflop_2 (clk, qout1, rst, qout2,
                           qoutb2);
    dflipflop3 a_dflipflop_3 (clk, qout2, rst, qout3,
                           qoutb3);
    dflipflop4 a_dflipflop_4 (clk, qout3, rst,
                           qout, qoutba);
endmodule
```

```
module dflipflop1 (clk, Din, rst, Dout, Doutba);
    input clk;
    input Din;
    input rst;
    output Dout;
    output Doutba;
    reg Dout;
    reg Doutba;
    always @ (posedge clk)
        if (rst == 0)
            begin
                Dout <= Din;
                Doutba <= ~Din;
            end
        else
            begin
                Dout <= Dout;
                Doutba <= Doutba;
            end
    end
endmodule
```

Parallel in serial out shift

Register

```
module PISO (clk, rst, key, qin1, qin2,
             qin3, qin4, qout);
```

```
input clk;
input rst;
input key;
input qin1;
input qin2;
input qin3;
input qin4;
input qout;
reg qout1, qout2, qout3, qout;
```

always @ (posedge clk)

if (rst == 0)

begin

qout1 <= qin1;

qout2 <= qin2;

qout3 <= qin3;

qout <= qin4;

end

else

begin

qout1 <= qout;

qout2 <= qout1;

qout3 <= qout2;

qout <= qout3;

end

end

else

qout <= 1'b0;

endmodule

//testbench

module PISO_tst();

reg clk, key, rst, qin1, qin2, qin3,
 qin4, qouts;

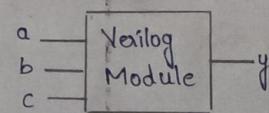
PISO PI(clk, rst, key, qin1, qin2, qin3,
 qin4, qouts);

initial

```
begin
    clk = 1'b0;
    rst = 1'b1;
    key = 1'b1;
end
initial
begin
    # 100 rst = 1'b0;
    qin1 = 1'b0;
    qin2 = 1'b1;
    qin3 = 1'b0;
    qin4 = 1'b1;
    # 100 key = 1'b0;
    # 100
    key = 1'b1;
    qin1 = 1'b1;
    qin2 = 1'b0;
    qin3 = 1'b1;
    qin4 = 1'b1;
```

```
# 100 key = 1'b0;
# 100 key = 1'b1;
qin1 = 1'b1;
qin2 = 1'b0;
qin3 = 1'b1;
qin4 = 1'b1;
# 100 Key = 1'b0;
# 10000 $stop;
end
always
# 5 clk = ~clk;
endmodule
```

⇒ Module is main building
Block in verilog

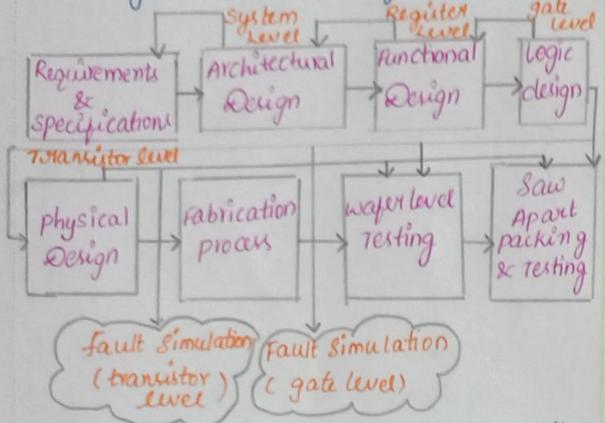


inputs - a,b,c output - y

Application And Simulation Tools

Example of digital System design

The integrated circuit design process



* Note all the simulation (design verification)

- helps to ensure the design works and assists in debugging design errors.

* In order to simulate a circuit, we must describe it in a manner that can be interpreted and understood by the simulator.

Applications and advantages of digital systems

→ A digital system is a combination of devices designed to manipulate logical information or physical quantities.

→ These devices are most often electronic, but they can also be mechanical, magnetic or pneumatic.

The chief reasons for the shift to digital technology are:

→ Digital systems are generally easier to design. The circuits used in digital systems are switching circuits, where exact values of voltage or current are not important, only the range (High or Low) in which they fall.

→ Information storage is easy. This is accomplished by special devices and circuits that can latch onto digital information and hold it for as long as necessary, and mass storage techniques that can store billions of bits of information in a relatively small physical space.

→ Accuracy and precision are easier to maintain throughout the system.

→ Operations can be programmed.

→ Digital circuits are less affected by noise.

→ More digital circuitry can be fabricated on IC chips.

Design capture with Hardware Description Language

- netlist
 - connections via signal name
- Schematic
 - connections either explicit or via signal name

- produces a netlist for simulation
- Higher level language (VHDL or Verilog)
 - synthesis to gate level netlist
- All of these design descriptions can go to simulation for design verifications

Types of Simulators

compiled simulator

No changing logic values within circuit

All gates in circuit have a finite unit delay

Short simulation times

Circuit may not be stable when input changes

Gates have real delays base on intrinsic & extrinsic factors

Ongoing Research in digital systems

- Quantum computing
- Quantum technology
- composite semiconductors
- Devices such as FINFET, High power MOSFET's
- modelling of devices
- Nano technology