Constraint eqn of optical flow :

- First we consider the 2 images and then we take that time t, and $t+\delta t$
- By considering 2 images and one small window at the same point in Both

i.e we get

x, y along with the $(x+\delta x, y+\delta y)$

the optical flow $u, v = \left(\frac{\delta x}{\delta t}, \frac{\delta y}{\delta t}\right)$ and the displacement is given as $(\delta x, \delta y)$

We are assuming that the Brightness

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x, y, t) \longrightarrow \text{eqn } ①$$

we also see that the displacements are very small

we apply the taylor's approximation,

as δx is small

$$f(x+\delta x) = f(x) + \frac{\partial f}{\partial x}\delta x + O \longrightarrow \text{small order}$$

then

$$f(x+\delta x, y+\delta y, t+\delta t) \simeq f(x, y, t) + \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y + \frac{\partial f}{\partial t}\delta t$$

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x, y, t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

We get $= I(x, y, t) + I_x \delta x + I_y \delta y + I_t \delta t$

$\longrightarrow$ eqn ②

Subtracting ① - ②

$$I_x \delta x + I_y \delta y + I_t \delta t = 0$$

divide by δt

$$\Rightarrow I_x \frac{\partial x}{\partial t} + I_y \frac{\partial y}{\partial t} + I_t = 0$$

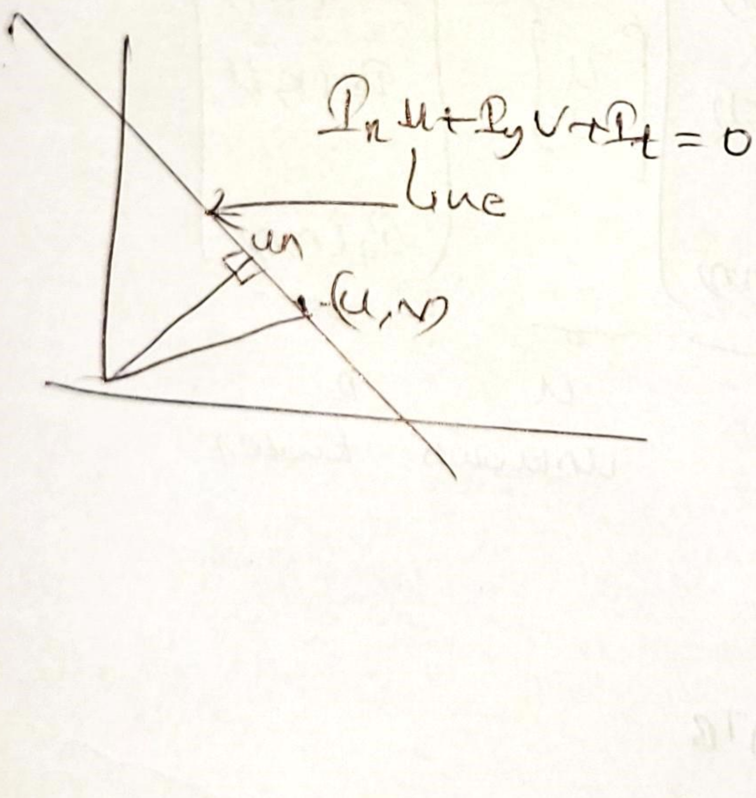
$$\boxed{I_x u + I_y v + I_t = 0}$$

Constraint eqn

u, v is optical flow

$I_x\ I_y\ I_t$ can be calculated from 2 frames

u, v lie in the line



Lucas Kanade algorithm :

Here we assume motion field and optical flow (u, v) is constant within a small neighbourhood ω.

· So for all $(k, l) \in \omega$

derivatives in x, y, t direction $= 0$

$$I_x(k,l)u + I_y(k,l)v + I_t(k,l) = 0$$

Considering for $(k, l) \in \omega$

let window size be $n \times n$

In matrix form

$$\underbrace{\begin{bmatrix} I_x(1,1) & I_y(1,1) \\ I_x(k,l) & I_y(k,l) \\ I_x(n,n) & I_y(n,n) \end{bmatrix}}_{\substack{A \\ \text{known}}} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\substack{u \\ \text{unknown}}} = \underbrace{\begin{bmatrix} I_t(1,1) \\ I_t(k,l) \\ \vdots \\ I_t(n,n) \end{bmatrix}}_{\substack{B \\ \text{known}}}$$

considering $Au = B$

$$A^T A u = A^T B$$

matrix

$$\begin{bmatrix} \Sigma_\omega I_x I_x & \Sigma_\omega I_x I_y \\ \Sigma_\omega I_x I_y & \Sigma_\omega I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\Sigma_\omega I_x I_t \\ -\Sigma_\omega I_y I_t \end{bmatrix}$$

$$u = (A^T A)^{-1} A^T B$$

① $A^T A$ must be invertible $\rightarrow \det(A^T A) \neq 0$

(2) A^TA must be small - conditioned

λ_1 and λ_2 are the eigen values of A^TA

then $\lambda_1 > \epsilon$ and $\lambda_2 > \epsilon$

$\lambda_1 \geq \lambda_2$ but not $\lambda_1 >> \lambda_2$