unit - 3

① state parallel axis thoerem?

A. The parallel axis theorem states that the moment of inertia of a plane area with respect to any reference axis in its plain is equal to the sum of moment of inertia at a parallel centroidal axis and product of the total area and the square of the distance between two axis.

$$I_{AB} = I_G + Ar^2$$

(b) Determine the area moment of inertia of a rectangle with respect to its centroidal x-axis parallel to the base.

(A) moment of inertia of rectangle about parallel centroidal axis to the base by parallel axis theorem.

$$I_{AB} = I_G + Ar^2$$

$$I_G = I_{AB} - Ar^2$$

$$I_G = \frac{bh^3}{3} - bh \times \left(\frac{h}{2}\right)^2$$

$$I_G = \frac{bh^3}{3} - \frac{bh^3}{4}$$

$$I_G = \frac{bh^3}{12}$$

(c) Find the area moment of inertia of a I-section about x-x and y-y axis passing through the centroid of th section.

(A)

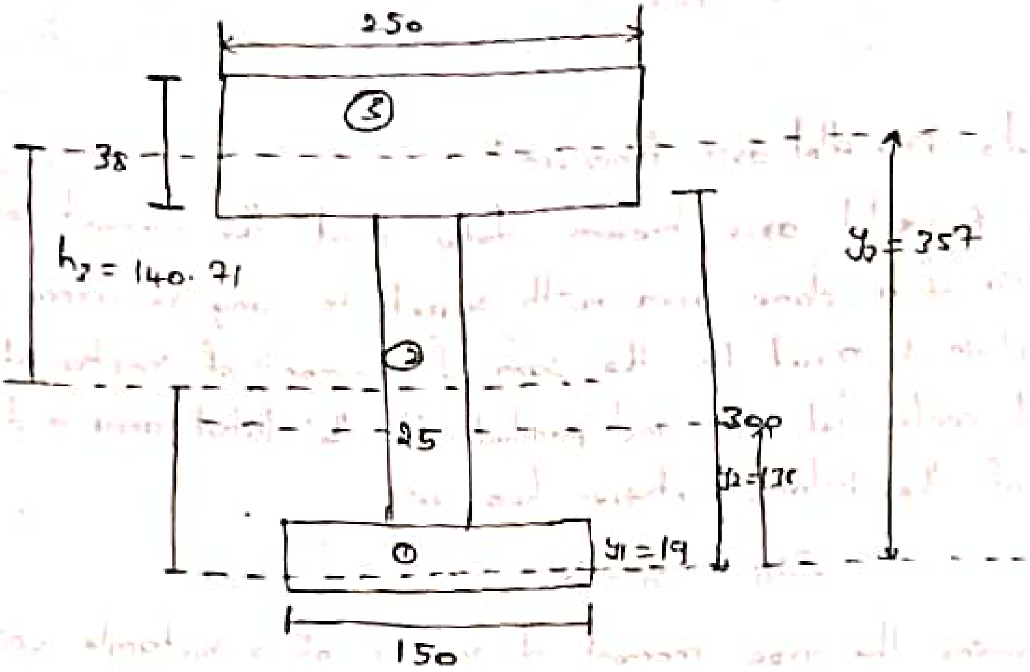
$$A_1 = l \times b$$

$$A_1 = 150 \times 38$$

$$A_1 = 5700\,mm^2$$

$$y_1 = \frac{h}{2}$$

$$y_1 = \frac{38}{2} = 19$$



$A_2 = l \times b$

$A_2 = 25 \times 300$

$A_2 = 7500 mm^2$

$y_2 = \frac{h}{2}$

$y_2 = \frac{300}{2} + 38$

$y_2 = 188$

$A_3 = l \times b$

$= 250 \times 38$

$= 9500 mm^2$

$y_3 = 38 + 300 + \frac{38}{2}$

$y_3 = 357$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(5700 \times 19) + (7500 \times 188) + (9500 \times 357)}{5700 + 7500 + 9500}$$

$\bar{y} = 216.29 mm$

$I_{xx} = I_G + A r^2$

$$I_{xx} = \left[\frac{150 \times 38^3}{12} + 150 \times 38 \times (197.29)^2\right] + \left[\frac{25 \times 300^3}{12} + 25 \times 300 \times (28.19)^2\right] +$$

$$\left[\frac{250 \times 38^3}{12} + 250 \times 38 \times (140.71)^2\right]$$

$$I_{xx} = 222.548961.4 + 6221 0070.75$$

$$I_{xx} = 47399 5587.8mm$$

$$I_{yy} = \frac{hb^3}{12} + \frac{hb^3}{12} + \frac{hb^3}{12}$$

$$= \frac{38 \times 150^3}{12} + \frac{300 \times 25^3}{12} + \frac{38 \times 250^3}{12}$$

$$I_{yy} = 60557291.67 mm$$

(2) state perpendicular axis theorem?

The perpendicular axis theorem states that the moment of inertia of an area of with respect to an axis perpendicular to the x-y plane and passing through origin will be equal to the sum of moment of inertia of the same area about x-x and y-y axis.

$$I_{zz} = I_{xx} + I_{yy}.$$

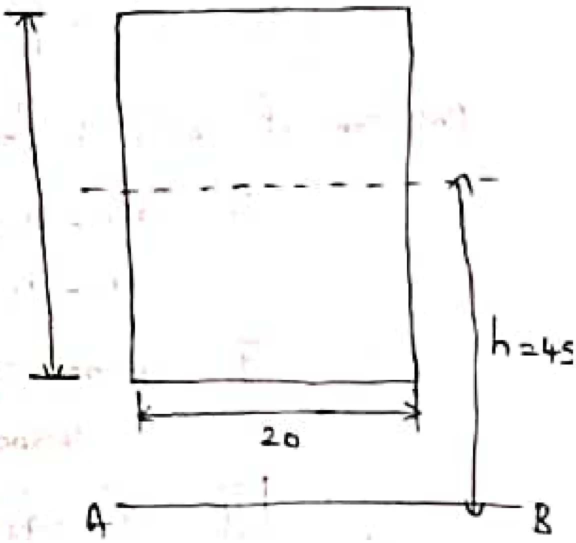
it is also called as polar moment of inertia.

(6) calculate the area moment of inertia of the following rectangle about a given axis which is at a distance of 45mm from its centroid.?

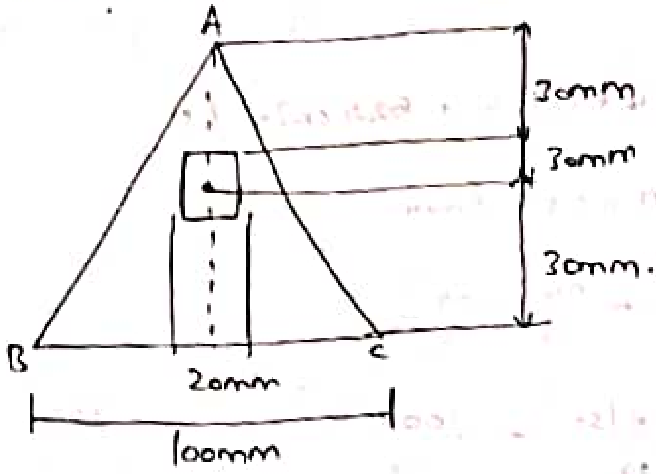
$$I_{AB} = I_G + Ah^2$$

$$I_{AB} = \frac{20 \times 30^3}{12} + 20 \times 30 \times (45)^2$$

$$I_{AB} = 1260000mm$$



(c) Determine the moment of inertia and radius of gyration of the section about its horizontal centroidal axis.



$$a_1 = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 100 \times 90$$

$$a_1 = 4500 mm^2$$

$$y_1 = \frac{h}{3}$$

$$y_1 = \frac{90}{3}$$

$$\boxed{y_1 = 30mm}$$

$$a_2 = l \times b$$

$$a_2 = 20 \times 30$$

$$\boxed{a_2 = 600 mm^2}$$

$$y_2 = \left(30 + \frac{30}{2}\right)$$

$$\boxed{y_2 = 45mm}$$

Position of centroid from base.

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$\bar{y} = \frac{4500 \times 30 - 600 \times 45}{4500 - 600.}$$

$$\boxed{\bar{y} = 27.6923 mm}$$

$$I_{xx} = \left[I_G + Ax^2\right] - \left[I_G + Ax^2\right]$$

$$= \left[\frac{100\times 90^3}{36} + \frac{1}{2}\times 100\times 90\times (2-31)^2\right] - \left[\frac{20\times 30^3}{12} + 20\times 30\times (17-31)^2\right]$$

$I_{xx} = 184\ 230.79\,mm$.

Radius of gyration, $k_{xx} = \sqrt{\frac{I_{xx}}{2A}}$

$$k_{xx} = \sqrt{\frac{1824230.79}{(3900)}}$$

$$\boxed{k_{xx} = 21.62\,mm}$$

③

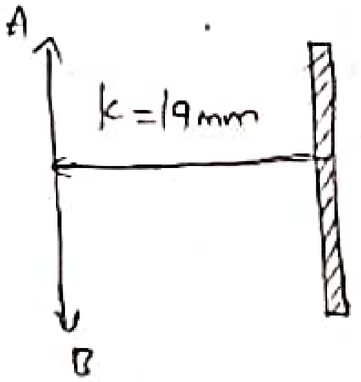
ⓐ Define the radius of gyration.

It states that the distance from the reference axis at which the given area is assumed to be compressed and kept as a thin strip, such that there is no change in its moment of inertia from the definition

$$I_{AB} = Ak^2$$

where k is radius of gyration.

ⓑ The radius of gyration of a rectangular channel 19mm and area is $35\,mm^2$ Determine the MI of channel.



area = $35\,mm^2$

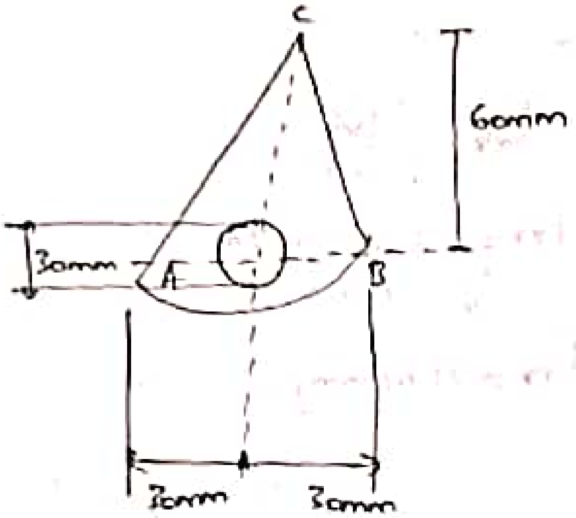
radius of gyration $(k) = 19\,mm$

$I_{AB} = k^2 A$

$= 35\times 19^2$

$= 12635\,mm^4$

(c) calculate the moment of inertia of the lamina with a circular hole of the 30mm diameter about the axis AB as shown in figure.



divide the given section into triangle, semicircle, circle.

$a_1 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 60 \times 60$

$\boxed{a_1 = 1800 mm^2}$

$y_1 = \frac{60}{3}$

$\boxed{y_1 = 20}$

$a_2 = \frac{\pi r^2}{2}$

$a_2 = \frac{\pi (30)^2}{2}$

$\boxed{a_2 = 1413.71 mm^2}$

$y_2 = -\frac{4r}{3\pi}$

$y_2 = -\frac{4(30)}{3\pi}$

$\boxed{y_2 = -12.73 mm}$

$a_3 = \pi r^2$

$a_3 = \pi (15)^2$

$\boxed{a_3 = 706.55 mm^2}$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{A_1 + A_2 - A_3}$$

$$\bar{y} = \frac{(1800 \times 20) - (1413.71 \times 12.73) - (706.55 \times 0)}{1800 + 1413.71 - 206.55}$$

$$\boxed{\bar{y} = 7.18 mm}$$

$$I_{xx} = \frac{bh^3}{12} - \frac{\pi r^4}{8} - \frac{\pi r^4}{4}$$

$$I_{xx} = \frac{60 \times 60^3}{12} - \frac{\pi(30)^4}{8} - \frac{\pi(15)^4}{4}$$

$$\boxed{I_{xx} = 722152.9618 mm}$$

(4)

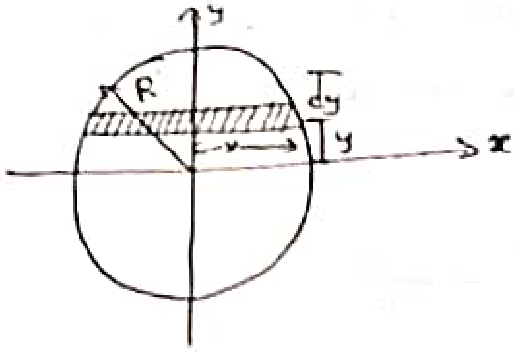
(a) Explain the significance of moment of inertia?

The significance of moment of inertia is similar to the mass in translational motion. In translational motion, the mass of a body is used for measuring inertia. In rotational motion, the greater the moment of inertia, the angular accerlation are require more for torque.

(b) Explain transfer formula for mass moment of inertia.

It states that moment of inertia of a body about an axes at a distance (d) and parallel to the centroidal axis is equal to the of moment of inertia about centroidal axis and product of mass and square of distance of parallel axis $\boxed{I_A = I_G + md^2}$

© estimate mass moment of inertia of a solid sphere having a radius R and mass m about its diametral axis.



Consider a sphere of radius R

$$\text{mass of sphere} = \rho \times \frac{4}{3}\pi R^3$$

let us consider a thin plate at a distance 'y' from x-axis. thickness of plate 'dy' and radius of thin plate 'x'.

mass moment of inertia about y-axis $= \int_{-R}^{R} \frac{1}{2} \times dm \times x^2$

$$dm = \rho \times \pi x^2 \times dy.$$

$$= \int_{-R}^{R} \frac{1}{2} \times \rho \times \pi\, x^2 \times dy \times x^2$$

$$= \int_{-R}^{R} \frac{1}{2} \times \rho\, \pi x^4 \cdot dy$$

from figure $R^2 = x^2 + y^2$

$$x^2 = R^2 - y^2$$

$$I_{yy} = \int_{0}^{R} \rho\pi\, (R^2 - y^2) \cdot dy$$

$$I_{yy} = \int_{0}^{R} \rho\pi \left(R^4 + y^4 - 2R^2y^2\right) dy$$

$$I_{yy} = \rho\pi \left[R^4 y + \frac{y^5}{5} - \frac{2R^2y^3}{3}\right]_0^R$$

$$I_{yy} = 8\pi\left[\frac{15R^5 + 3R^5 - 10R^5}{15}\right]$$

$$I_{yy} = \frac{8\pi\, 8R^5}{15}$$

$$I_{yy} = \frac{4}{3}\pi R^3 \times \frac{8\,2R^2}{5}$$

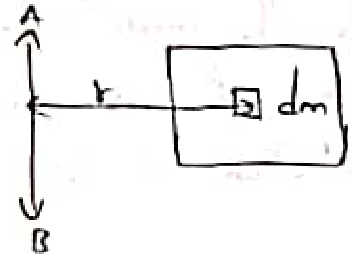
$$\boxed{I_{yy} = \frac{2MR^2}{5}}$$

(5)

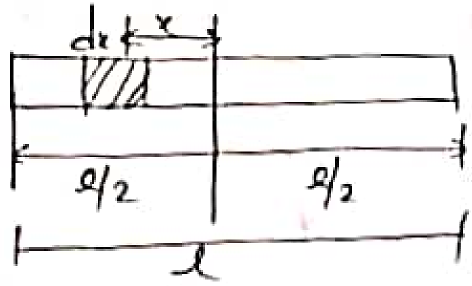
(a) Define mass moment of inertia?

mass moment of inertia of a body about an axis is defined as the sum total of product of its elemental masses and square of their distances.

$$I_{AB} = \int r^2 \cdot dm$$



(b) calculate the mass moment of inertia of a uniform radius of length (l) about axis normal to its centroid



Consider a uniform radius of AB of length (l) with its midpoint 'o'.

let M = total mass of the rod = ml.

m = mass of rod per unit length.

$$I = \int_{-l/2}^{l/2} x^2 \cdot dm$$

$$I = \int_{-l/2}^{l/2} x^2 \cdot m \cdot dx$$

$$I = m\left[\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$I = \frac{m}{3}\left[\left(\frac{l}{2}\right)^3 + \left(\frac{l}{2}\right)^3\right]$$

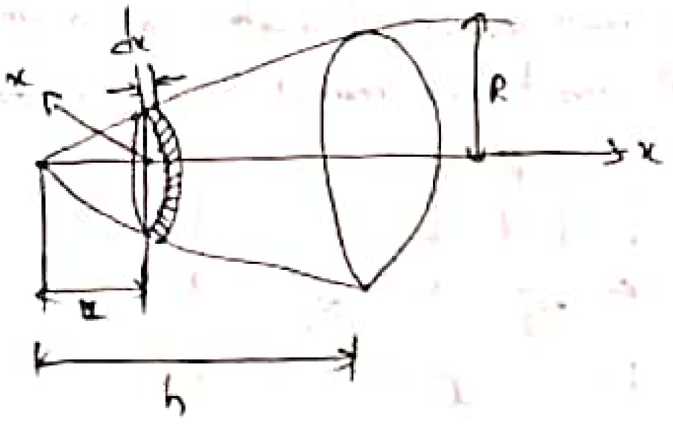
$$I = \frac{m}{3}\left[2\left(\frac{l^3}{8}\right)\right]$$

$$I = \frac{m}{3}\left[\frac{l^3}{4}\right]$$

$$I = \frac{m \cdot l \cdot l^2}{4}$$

$$\boxed{I = \frac{ml^2}{12}}$$

(c) calculate the mass moment of inertia of a right circular cone of base radius R and mass M about the axis of rotation of the cone.



$$dm = m\pi r^2 . dx$$

$$I_{xx} = \frac{1}{2} dm \times r^2$$

$$I_{xx} = \frac{1}{2} \times m\pi r^2 . dx \times r^2$$

$$I_{xx} = \frac{1}{2} m\pi r^4 . dx$$

$$I_{xx} = \int_0^h \frac{1}{2} m\pi r^4 . dx$$

$$I_{xx} = \left[\frac{m\pi R^4}{2h^4} \left[\frac{x^5}{5} \right]_0^h \right]$$

$$I_{xx} = \frac{m\pi R^4 \times h^5}{2h^4 \times 5}$$

$$I_{xx} = \frac{m\pi R^4 h}{10} \times \frac{3}{3}$$

$$I_{xx} = \frac{3m\pi R^4 h}{3 \times 10}$$

$$I_{xx} = \frac{3m\pi R^2 . hR^2}{3 \times 10}$$

$$\boxed{I_{xx} = \frac{3mR^2}{10}}$$

Total mass $(m) = \int_0^h m . \pi r^2 . dx$

$$m = \int_0^h m\pi R^4 . \frac{x^2}{R^2} . dx$$

$$m = \frac{m\pi R^2}{h^2} \left[\frac{x^3}{3} \right]_0^h$$

$$m = \frac{m\pi R^2}{h^2} \times \frac{h^3}{3}$$

$$\boxed{m = \frac{m\pi R^2 h}{3}}$$

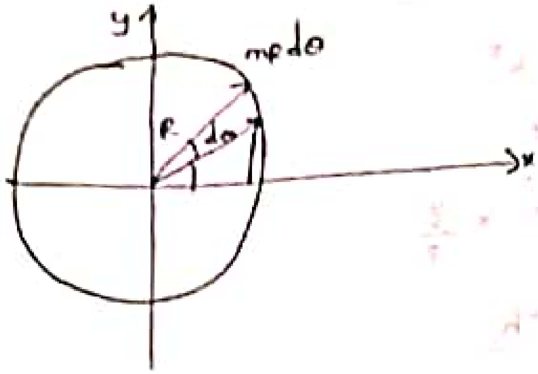
6

a) Define product of inertia.

A quantity that characterizes the mass distribution in a body for mechanical system. Product of inertia is the sum of the product formed by multiplying the mass mk of each poin of the body (or) system by the product of two of the coordinates x_k, y_k, z_k of the point.

$$I_{yy} = \Sigma m_k x_k y_k$$

(b) find the mass moment of inertia of a circular ring.



$$M = 2\pi R \cdot m$$

$$dm = mR\,d\theta$$

$$I_{xx} = \int_0^{2\pi} (R\sin\theta)^2 \cdot mR\,d\theta$$

$$I_{xx} = \int_0^{2\pi} R^3 \sin^2\theta \cdot m \cdot d\theta$$

$$= mR^3 \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{mR^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{mR^3}{\cancel{2}} \cdot \cancel{2}\pi$$

$$I_{xx} = \frac{2}{2} mR^3 \pi$$

$$= 2mR\pi \cdot \frac{R^2}{2}$$

$$\boxed{I_{xx} = \frac{m \cdot R^2}{2}}$$

(c) calculate the mass moment of inertia of a rectangular plate of a size (a and b) and thickness 't' about its centroidal axis.

sol : elemental mass → uniform rod.

differential mass $dm = m \times (bt\, dy)$

Total mass $(m) = mbta$

$$I_{xx} = \int_{-a/2}^{a/2} y^2 \cdot dm$$

$$I_{xx} = \int_{-a/2}^{a/2} y^2 \cdot mbt \cdot dy$$

$$I_{xx} = mbt \left[\frac{y^3}{3}\right]_{-a/2}^{a/2}$$

$$I_{xx} = \frac{mbt}{3}\left[\frac{a^3}{8} + \frac{a^3}{8}\right]$$

$$I_{xx} = \frac{mbt}{3}\left[\frac{a^3}{4}\right]$$

$$I_{xx} = \frac{mbt\, a^3}{12}$$

$$I_{xx} = \frac{mbt \cdot a \cdot a^2}{12}$$

$$\boxed{I_{xx} = \frac{m \cdot a^2}{12}}$$

$$\boxed{I_{yy} = \frac{m \cdot b^2}{12}}$$

$$I_{zz} = I_{xx} + I_{yy}$$

$$= \frac{m \cdot a^2}{12} + \frac{m \cdot b^2}{12}$$

$$\boxed{I_{zz} = \frac{m}{12}\left[a^2 + b^2\right]}$$