ENGINEERING MECHANICS

-: Unit - I :-

-: Introduction to Mechanics:-

The branch of physical Science that deals with the State of rest or the state of motion is termed as "Mechanics". → which deals with the study of physical state of bodies at rest & in motion under the action of forces.

Classification:-

- Engg. Mechanics
  - Mechanics of Solids
    - Mechanics of Rigid bodies
      - Statics (Building)
      - Dynamics. (cars, vehicles)
        - Kinematics
        - Kinetics
    - Mechanics of Deformable bodies.
      - Theory of Elasticity
      - Theory of plasticity
      - S.O.M
  - Mechanics of fluids

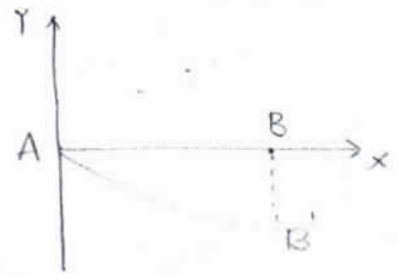
* The body which will not deform or the body in which deformation can be neglected, → Rigid bodies.
* The body is in rest position → Statics.
* The body is in motion position → Dynamics.
* Without (Referring) force the body (causing) is motion → Kinematics.
* with force " " " → kinetics

Basic Terminologies:-

* Mass:- The quantity of the matter possessed by a body is called "Mass".

* Space:- The geometric region in which study of body is involved is called "Space".

* Displacement:- It is defined as the distance moved by a body/particle in specified direction., Units → Metre



→ If a body moves from position A to B in the x-y plane, its displacement in
x-direction is AB
y-direction is BB'

* Velocity:- The rate of change of displacement w.r.t. time.

$$V = \frac{d}{dt}(s) \qquad s \rightarrow \text{displacement}$$

Units of velocity m/s.

* Acceleration:- The rate of change of velocity w.r.t. time.

$$a = \frac{d}{dt}(v) \qquad v \rightarrow \text{velocity}$$

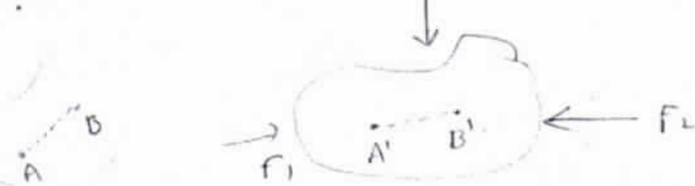
Units of Acceleration m/s^2

* Momentum:- product of mass & velocity.

Momentum = Mass × velocity

* Rigid body:- A body is said to be rigid, if the relative positions of any two particles do not change under the action of the forces.

$AB = A'B'$



Laws of Mechanics:-

→ **Newton's first Law:-** (Law of Inertia)

It states that everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it.

→ **Newton's Second law:-**

It States that the rate of change of momentum of a body is directly propotional to the impressed force & it takes place in direction of the force acting on it.

force $\propto$ rate of change of momentum

force $\propto$ mass × rate of change of velocity

force $\propto$ mass × Acceleration

force $\propto$ $m \times a$. $\boxed{F = ma}$

Newton's third law:-

It States that for every action there is an Equal and opposite reaction.

~~Force~~:- → If a force acting at a point on a rigid body is shifted to another point which is in the line of action of force, the unit effect of force on the body remain unchanged.

→ **Scalar Quantity:-** It is specified by the magnitude only and has no direction.

Ex:- Distance, speed, mass, temp, time, work, energy, power, density, volume etc.

Vector quantity:- It has magnitude & direction.

Ex:- Displacement, velocity, Acceleration, force, etc.

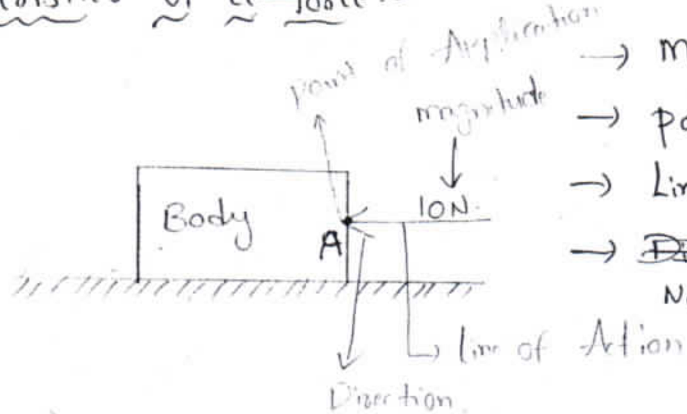
→ external agent which changes or tends to change the position of rest or uniform motion of a body upon which it act i.e parallel

Force:- force is the agency which tries to change state of rest or state of uniform motion of the body.

→ force is a vector quantity.

→ Units are Newton (or) $\frac{kg\text{-}m}{s^2}$ $\boxed{F = ma}$

Characteristics of a force:-



→ Magnitude (10N)

→ point of Application (A)

→ Line of Action (Horizontal)

→ ~~Direction~~ Nature (push (or) pull)

Systems of forces:-

When several forces act simultaneously on a body is called Systems of forces.

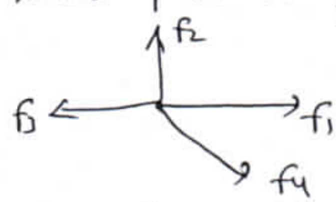
Systems of forces.

- Coplanar
  - Collinear
  - Concurrent
  - parallel
    - Like
    - Unlike
  - Non Concurrent Non parallel
- Non-Coplanar.
  - Concurrent
  - parallel
  - Non-concurrent Non-parallel

Collinear * The line of Action of all the forces act along the same line.

concurrent * Line of Action of All forces pass through a single point.

Collinear



Con-current

Parallel → **Like parallel**:- All forces are parallel to each other, acting in the same direction. $\rightrightarrows$ $\leftleftarrows$

→ **Unlike parallel**:- All forces are parallel to each other but acting in opposite direction. $\rightleftarrows$ $\leftrightarrows$

prefixes & symbols:-

$10^9 \rightarrow$ Giga (G)　　　$10^{-9} \rightarrow$ Nano (n)

$10^6 \rightarrow$ mega (M)　　　$10^{-6} \rightarrow$ micro (μ)

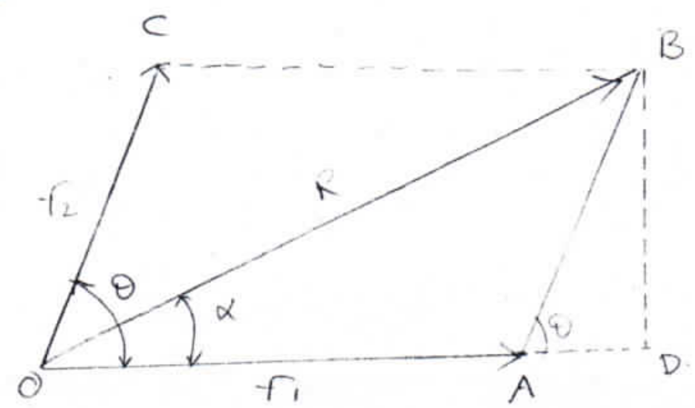
$10^3 \rightarrow$ kilo (k)　　　$10^{-3} \rightarrow$ milli (m)

Resultant of force:- If no. of forces act simultaneously on a particle, then it is possible to find out a single force to replace all those forces producing the same effect. The single force is called "resultant force".

Methods for Resultant of force:-

1) Graphical method
2) Analytical method.
    a) Parallelogram Law of forces　b) method of resoluti

Parallelogram Law of forces:-



It states that if two forces acting simultaneously on a body at a point are represented in magnitude & direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude & direction by the diagonal of the parallelogram which passes through the point of Intersection of the two sides of the forces.

Two forces f_1 & f_2 acting at point 'O' be represented magnitude f_1, f_2 & direction OA, OC. The resultant 'R' be represented magnitude and direction 'OB', adjacent sides of the parallelogram OACB is completed. The two forces makes an angle 'θ', One of the force & resultant makes angle 'α'.

from fig. $OA = F_1$, $OC = F_2$, $OB = R$.

$$OA = CB, \quad OC = AB,$$

from Δ^{le} OBD.

$$OB = \sqrt{OD^2 + BD^2}$$

$$OB = \sqrt{(OA + AD)^2 + BD^2}$$

from Δ^{le} BAD.

$$\cos\theta = \frac{AD}{AB} \Rightarrow AD = AB.\cos\theta$$
$$= f_2 \cos\theta$$

$$\sin\theta = \frac{BD}{AB} \Rightarrow BD = AB.\sin\theta$$
$$= f_2 \sin\theta$$

$$OB = \sqrt{(OA + AD)^2 + BD^2}$$

$$R = \sqrt{(F_1 + f_2\cos\theta)^2 + (f_2\sin\theta)^2}$$

$$R = \sqrt{f_1^2 + f_2^2\cos^2\theta + 2f_1 f_2\cos\theta + f_2^2\sin^2\theta}$$

$$R = \sqrt{f_1^2 + f_2^2(\sin^2\theta + \cos^2\theta) + 2f_1 f_2 \cos\theta}$$

$$= \sqrt{f_1^2 + f_2^2(1) + 2f_1 f_2 \cos\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$\boxed{R = \sqrt{f_1^2 + f_2^2 + 2f_1 f_2 \cos\theta}} \rightarrow \text{Magnitude of the Resultant}$$

Direction of Resultant:-

Δ^{le} BOD,
$$\tan\alpha = \frac{BD}{OD}$$

$$\tan\alpha = \frac{f_2 \sin\theta}{OA + AD}$$

$$\tan\alpha = \frac{f_2 \sin\theta}{f_1 + f_2\cos\theta} \Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{f_2 \sin\theta}{f_1 + f_2\cos\theta}\right)}$$

i) When $\theta = 90°$, $R = \sqrt{f_1^2 + f_2^2}$, $\alpha = \tan^{-1}\left(\frac{f_2}{f_1}\right)$

ii) When $\theta = 0°$, $R = \sqrt{f_1^2 + f_2^2 + 2f_1 f_2} \Rightarrow f_1 + f_2$, $\alpha = 0°$

iii) when $\theta = 180°$, $R = \sqrt{f_1^2 + f_2^2 - 2f_1 f_2} \Rightarrow f_1 - f_2$, $\alpha = 0°$

iv) Two forces are Equal

i.e $f_1 = f_2 = f$

$$R = \sqrt{f_1^2 + f_2^2 + 2f_1 f_2 \cos\theta}$$

$$= \sqrt{F^2 + F^2 + 2FF\cos\theta} = \sqrt{2F^2 + 2F^2\cos\theta}$$

$$= \sqrt{2F^2(1+\cos\theta)} = \sqrt{2F^2\left(2\cos^2\left(\tfrac{\theta}{2}\right)\right)} \quad \left(\because (1+\cos\theta) = 2\cos^2\tfrac{\theta}{2}\right)$$

$$= \sqrt{4F^2\cos^2\frac{\theta}{2}}$$

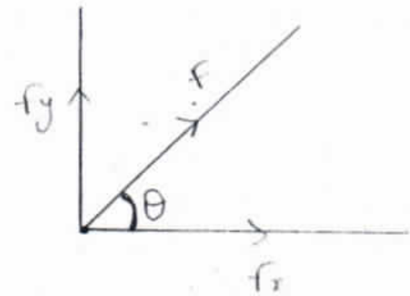
$$\boxed{R = 2F\cos\frac{\theta}{2}}, \quad \boxed{\alpha = \frac{\theta}{2}}$$

Method of Resolution:-

A single force acting on a particle may be replaced by two or more forces which together have the same effect as that of the single force. The process of doing this is called resolution of force.

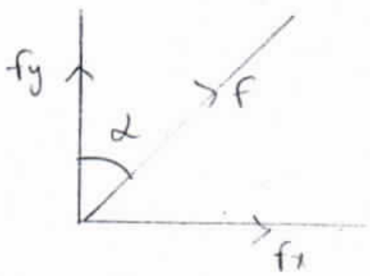
Examples:-

→



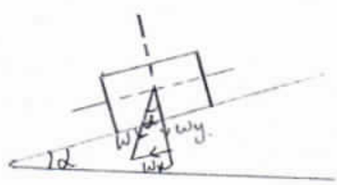
Along x-axis → $f_x = f\cos\theta$
Along y-axis → $f_y = f\sin\theta$.

→



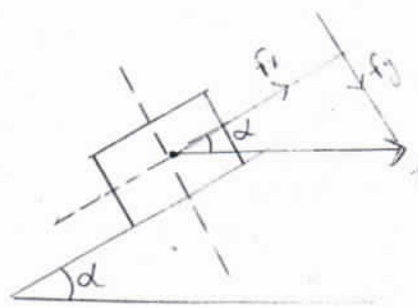
Along x-axis → $f_x = f\sin\alpha$
Along y-axis → $f_y = f\cos\alpha$.

→



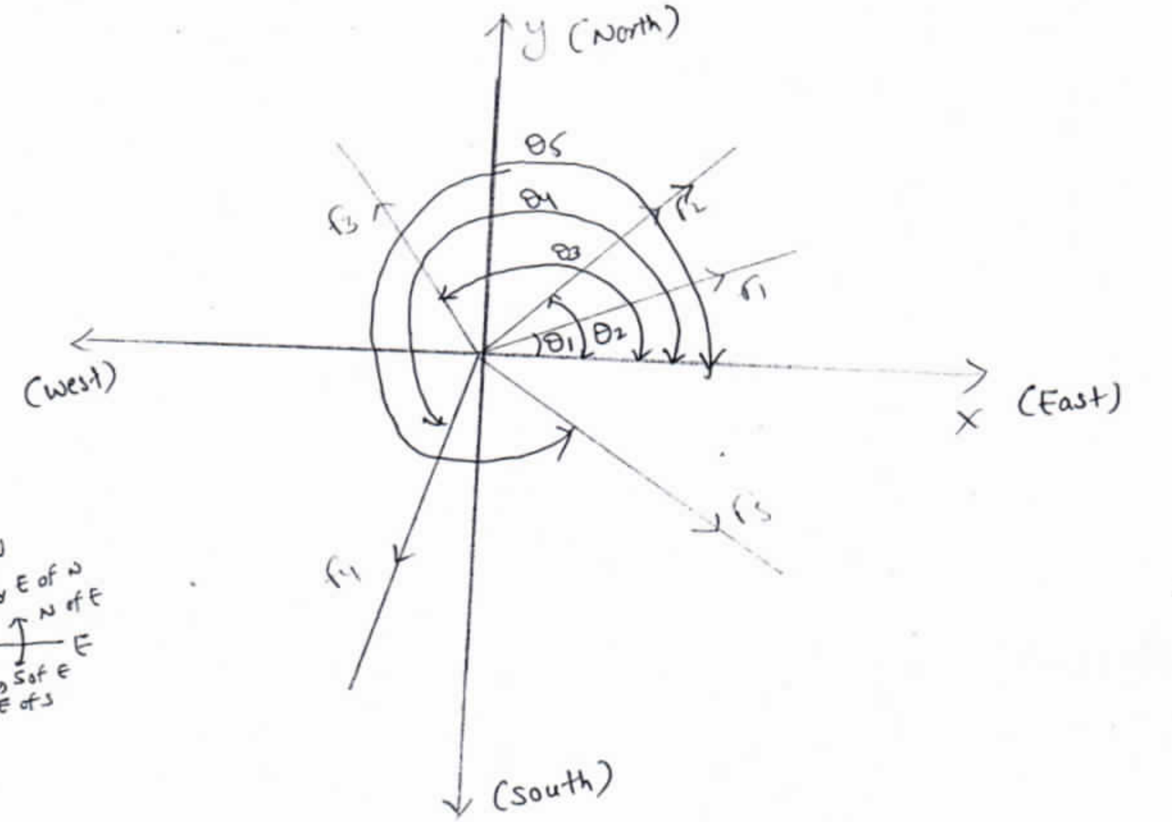
$W_x = W\sin\alpha$
$W_y = W\cos\alpha$.

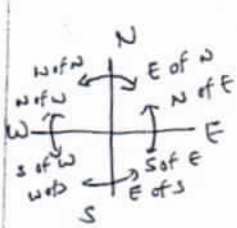
→



$f_x = f\cos\alpha$
$f_y = f\sin\alpha$.

Resultant of Concurrent force system using method of Resolution:-





If the no. of forces are more than two, then its resultant Can be found conveneienty by using method of resolution.

Resolve the forces Horizontally

$$\Sigma H = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4 + \cdots$$

Resolve the forces vertically

$$\Sigma V = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4 + \cdots$$

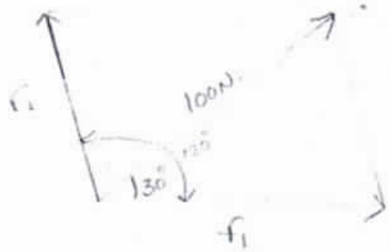
Resultant $\Rightarrow R = \sqrt{\Sigma H^2 + \Sigma V^2}$

Direction of resultant $\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

Problems:-

→ Resolve the 100N force acting a 30° horizontal into two components one along horizontal & ~~vertical~~ other along 120° to horizontal.

Sol G.D: $R = 100N$, $\theta = 120°$, $\alpha = 30°$



$$R = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos\theta}$$

$$100 = \sqrt{f_1^2 + f_2^2 + 2ff_2\cos 120°}$$

$$(100)^2 = f_1^2 + f_2^2 - f_1f_2 \quad (\because \cos 120° = -1/2) \longrightarrow ①$$

$$\tan\alpha = \frac{f_2\sin\theta}{f_1 + f_2\cos\theta} \Rightarrow \tan 30° = \frac{f_2\sin 120°}{f_1 + f_2\cos 120°}$$

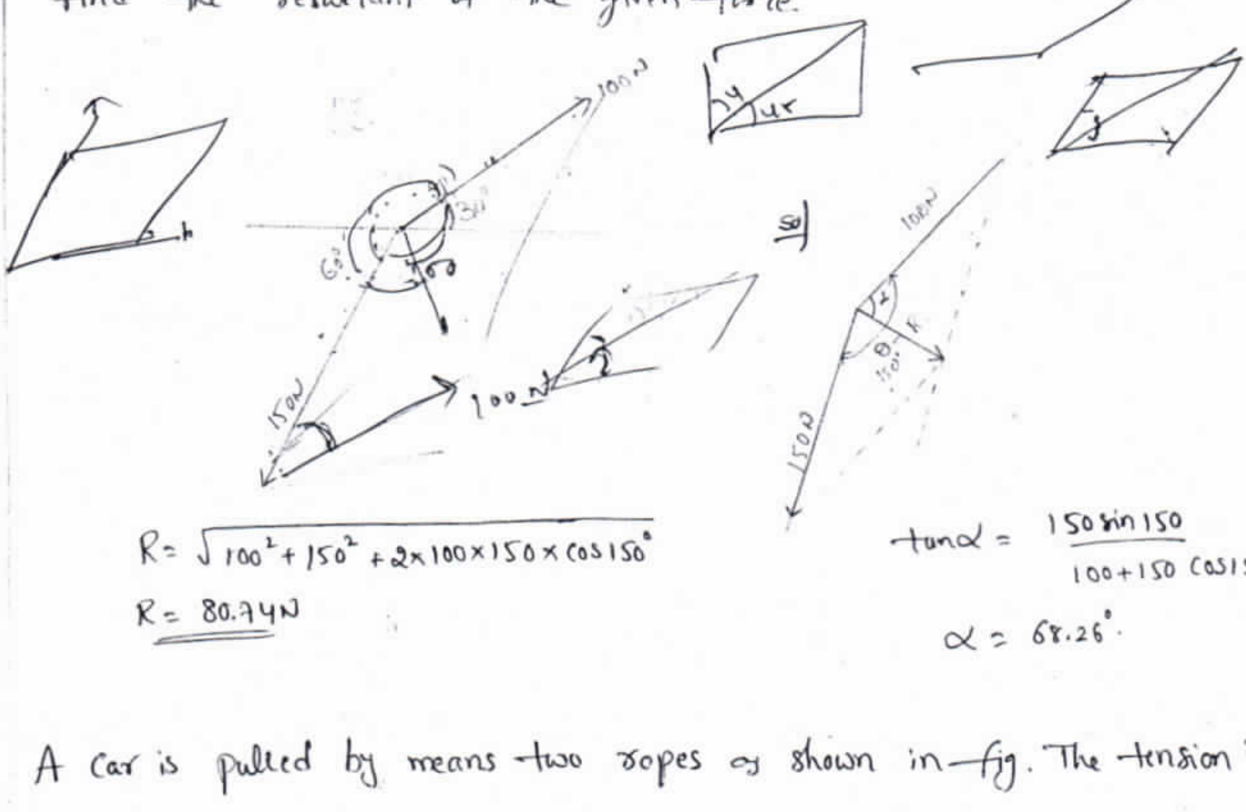
$$0.5774f_1 - 0.2887f_2 = 0.866f_2$$

Solving ① & ②

$$f_1 = 2f_2 \text{ --- } ②$$

$f_1 = 115.47N$, $f_2 = 57.76N$

→ find the resultant of the given force.



$$R = \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \times \cos 150°}$$

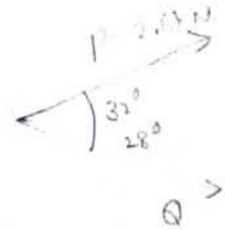
$R = 80.74N$

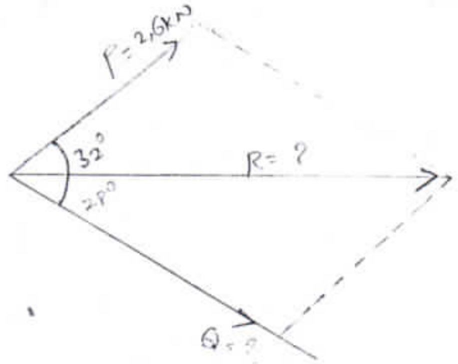
$$\tan\alpha = \frac{150\sin 150}{100 + 150\cos 150}$$

$\alpha = 68.26°$.

→ A car is pulled by means two ropes as shown in fig. The tension in one rope is $P = 2.6kN$. If the resultant of two forces applied at 'O' is directed along the x-axis of the car. find the tension in other rope & magnitude of the resultant.

Sol $P = 2.6\,kN$, $\theta = 60^\circ\ (28 + 32)$





$$\tan 28^\circ = \frac{2.6 \sin 60}{Q + 2.6 \cos 60^\circ} \Rightarrow Q = \underline{2.934\,kN}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{(2.6)^2 + (2.934)^2 + 2 \times 2.6 \times 2.934 \cos 60^\circ}$$

$$R = \underline{4.8\,kN}$$

→ A body is acted upon by forces as below. find the resultant of these forces.

i) 50 N acting due East ii) 100 N at 50° North of East

iii) 75 N at 20° West of North iv) 120 N at 30° South of West

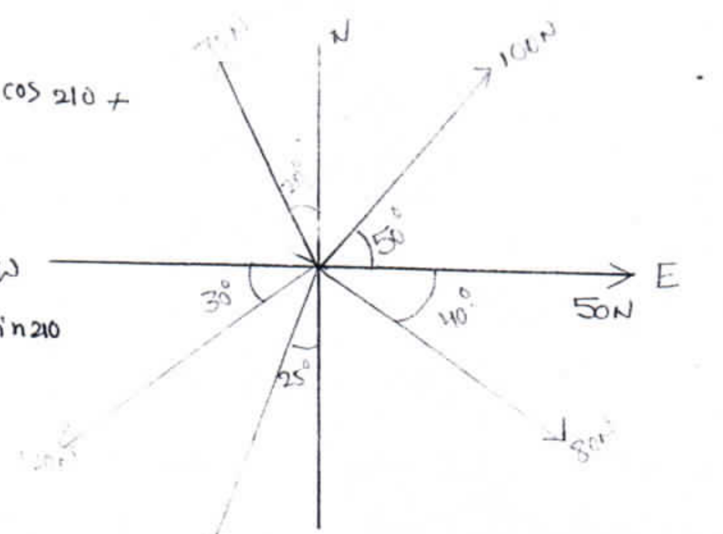
v) 90 N at 25° West of South vi) 80 N at 40° South of East.

Sol $\Sigma f_x = 50 + 100\cos 50^\circ + 75\cos 110^\circ + 120\cos 210 + 90\cos 245^\circ + 80\cos 320^\circ$

$\Sigma f_x = \underline{7.95\,N}\ (\rightarrow)$

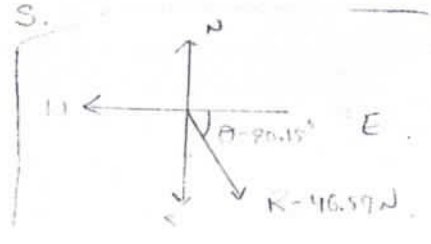
$\Sigma f_y = 50 + 100\sin 50^\circ + 75\sin 110^\circ + 120\sin 210 + 90\sin 245^\circ + 80\sin 320^\circ$

$\Sigma f_y = \underline{-45.91\,N}\ (\downarrow)$

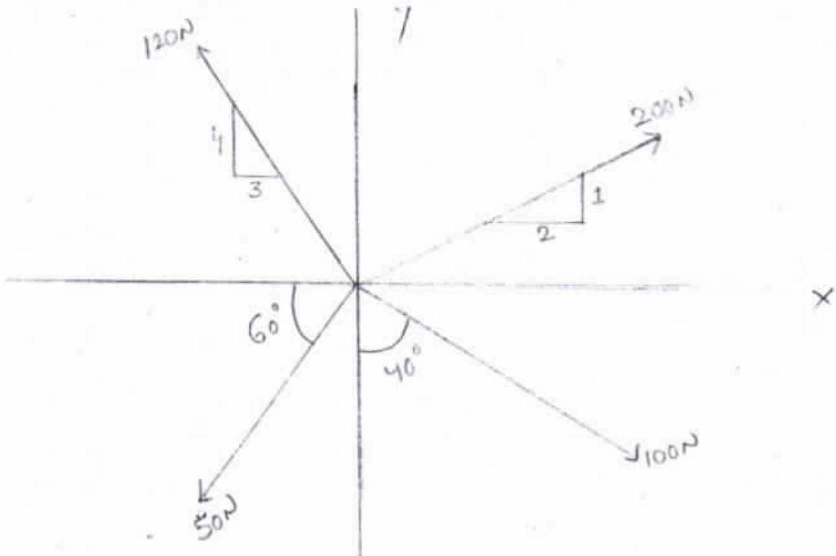


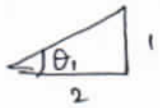
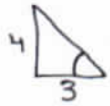
$$R = \sqrt{(7.95)^2 + (-45.91)^2} = 46.59\,N.$$

$$\theta = \tan^{-1}\left(\frac{45.91}{7.95}\right) = \underline{80.18^\circ}$$



→ A system of four forces acting at a point on a body as shown in fig
a) Determine the resultant.



So|  $\tan\theta_1 = \frac{1}{2} \Rightarrow \theta_1 = 26.56°$,  $\tan\theta_2 = \frac{4}{3} \Rightarrow \theta_2 = 53.13°$

$$\Sigma f_x = 200 \times \cos(26.56) + 120\cos(126.87) + 50\cos(240) + 100\cos(310)$$

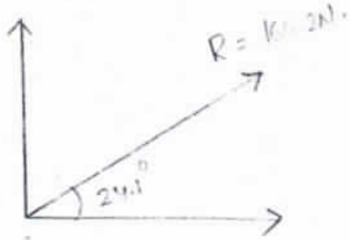
$$= 146.2\ N.$$

$$\Sigma f_y = 200\sin(26.56) + 120\sin(126.87) + 50\sin(240) + 100\sin(310)$$

$$\Sigma f_y = 65.5\ N$$

$$R = \sqrt{146.2^2 + 65.5^2} = 160.2N$$

$$\theta = \tan^{-1}\left(\frac{65.5}{146.2}\right) = 24.1°$$



Equilibrium & Equilibrant:-

when no. of forces acting on a particle, the resultant force will produce the same effect as produced by all the given forces.
That if the resultant of a no. of forces, acting on a particle is zero. The body is in Equilibrium condition.

Such a set forces, whose resultant is zero → Equilibrium forces

The force, which brings the set of forces in Equilibrium is Called an Equilibrant

~~Free~~ ~~be~~ Lami's Theorem:-

Condition for Equilibrium:

→ The Algebraic sum of horizontal components of all the forces must be zero, $\Sigma f_x = 0$

→ " " " Vertical " , $\Sigma f_y = 0$

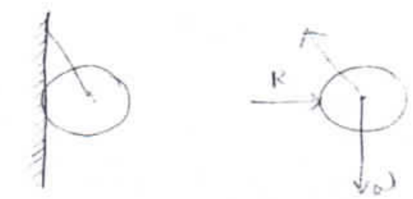
→ The Algebraic sum of moments of all the forces about any point in their plane is zero, $\Sigma m = 0$.

Free body diagram:

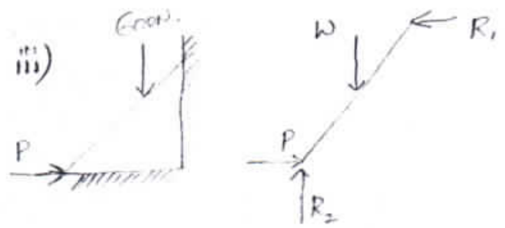
It is a sketch of the body showing all active & reactive forces that acts on it after removing all supports with consideration of Given.

Ex: i)

ii)

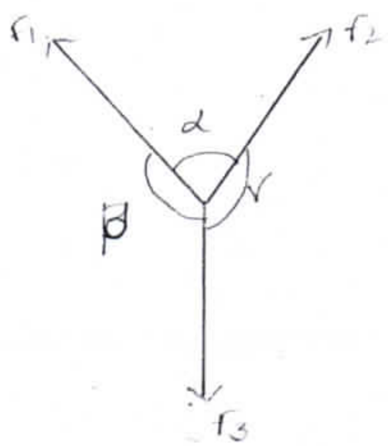


iii)



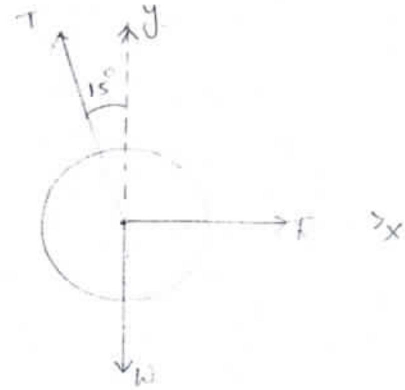
Lami's theorem:

It states that, if a body is in Equilibrium under the action of only three forces, each force is propotional to the sine of angle b/w the other two forces.



$$\boxed{\frac{f_1}{\sin\gamma} = \frac{f_2}{\sin\beta} = \frac{f_3}{\sin\alpha}}$$

→ A sphere weighing 100N is tied to a Smooth wall by a string as Shown in fig. find the tension T in the string & the reaction R from the wall



Left si (
Right side (
up (+ve),

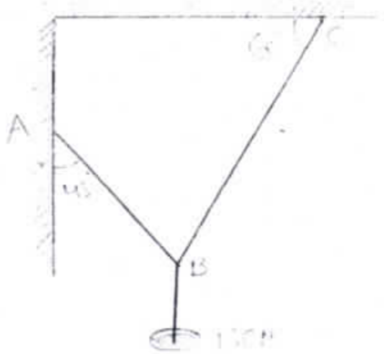
Sol| By Applying Lami's theorem,

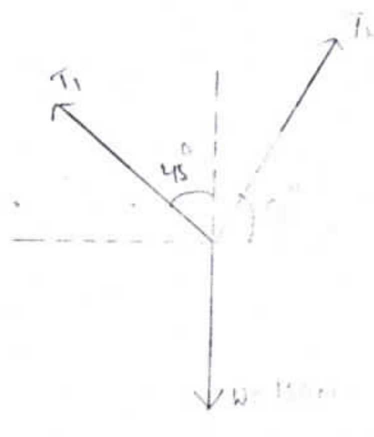
$$\frac{T}{\sin 90} = \frac{W}{\sin(90+15)} = \frac{R}{\sin(180-15)}$$

$$\frac{T}{\sin 90} = \frac{100}{\sin 105} \quad , \quad \frac{100}{\sin 105^\circ} = \frac{R}{\sin 165^\circ}$$

$$T = \underline{103.5\,N} \qquad R = \underline{26.8\,N}$$

→ find the forces developed in the wires, supporting an electric fixture as Shown in fig.



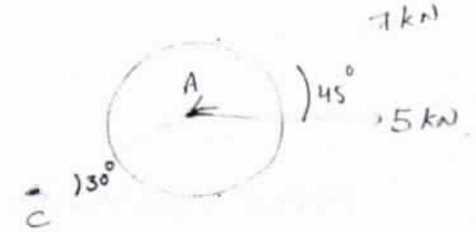


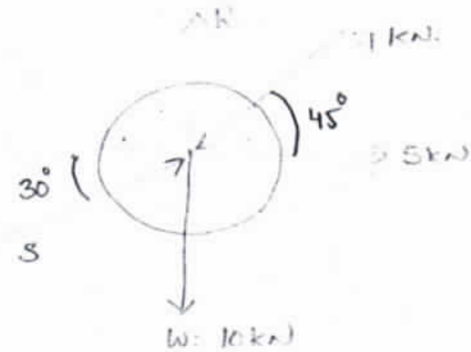
$$\frac{W}{\sin(45+30)} = \frac{T_1}{\sin(90+60)} = \frac{T_2}{\sin(90+45)}$$

$$\frac{150}{\sin 75^\circ} = \frac{T_1}{\sin 150^\circ} \quad , \quad \frac{150}{\sin 75^\circ} = \frac{T_2}{\sin 135^\circ}$$

$$T_1 = 77.6\,N \quad , \quad T_2 = \underline{109.8\,N}$$

→ A roller weighing 10kN rests on a smooth horizontal floor & is connected to the floor by the bar AC as shown in fig. Determine the force in the bar AC & reaction from the floor, if the roller is subject to a horizontal force of 5kN & an inclined force of 7kN as shown





Left side (-ve)
Right side (+ve)
up (+ve), down (-ve)

A bar AC can develop tensile force or compressive force. let the force developed be a compressive force 'S'.

$$\Sigma f_x = 0, \quad 5 + S\cos 30^\circ - 7\cos 45^\circ = 0$$

$$S = \frac{7\cos 45^\circ - 5}{\cos 30^\circ} = -0.058\ kN.$$

$$\underline{S = 0.058 kN\ (+ve)} \quad (\text{tensile force})$$

$$\Sigma f_y = 0, \quad R - 7\sin 45^\circ - W + S\sin 30^\circ = 0$$

$$R = 7\sin 45^\circ + 10 + 0.058 \sin 30$$

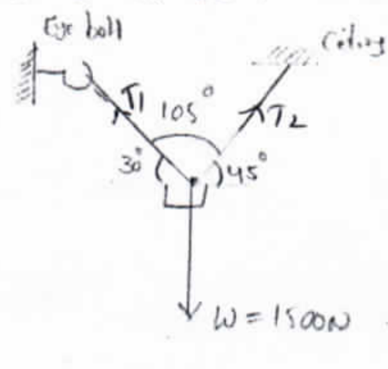
$$\underline{R = 14.98\ kN}$$

→ A rock weighing 1500N is supposted by two chains attached to some point on the rock. One of these ropes goes to the eye bolts in the wall & is inclined at 30° to the horizontal & other goes to the hook in ceiling & is inclined at 45° to horizontal. find the tensions in the two chains.

$$\frac{T_1}{\sin(90+45)} = \frac{T_2}{\sin(90+30)} = \frac{W}{\sin(105)}$$

$$T_1 = 1098.96\ N$$

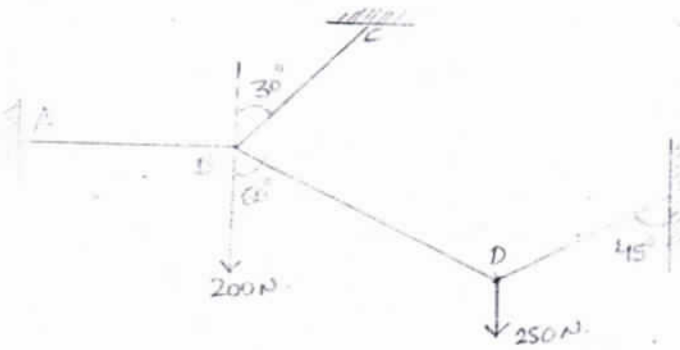
$$T_2 = 1346.11\ N.$$



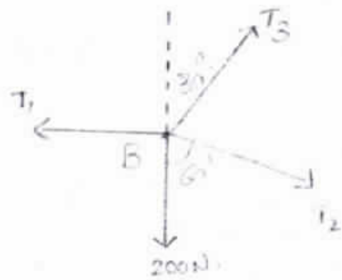
Equilibrium of Connected bodies:

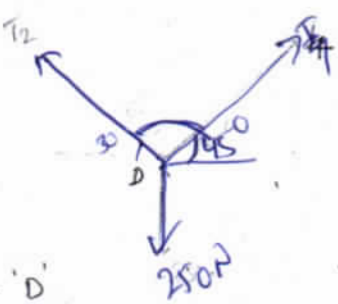
When two or more bodies are in contact with one another, the System of forces appear as though it is a non-Concurrent force system.

A System of connected flexible cables shown in fig. is supporting two vertical forces 200N & 250N at points B & D. Determine the forces in various Segments of the cable.



Sol





Apply Lami's theorem at point 'D'.

$$\frac{T_4}{\sin(180-60)} = \frac{T_2}{\sin(90+45)} = \frac{250}{\sin(60+45)}$$

$$T_4 = 224.1N, \quad T_2 = 183N.$$

Now consider the System of forces acting at B.

$\Sigma F_H = 0$, $\quad T_3 \sin 30 + T_2 \sin 60 - T_1 = 0$

$$\boxed{\cancel{T_3 \sin 30 + \overset{183}{(224.1)} \sin 60 - 224.1 = 0 \quad T_3 = 3}}$$

$$T_3 \sin 30 + 183 \sin 60 - T_1 = 0. \rightarrow (1)$$

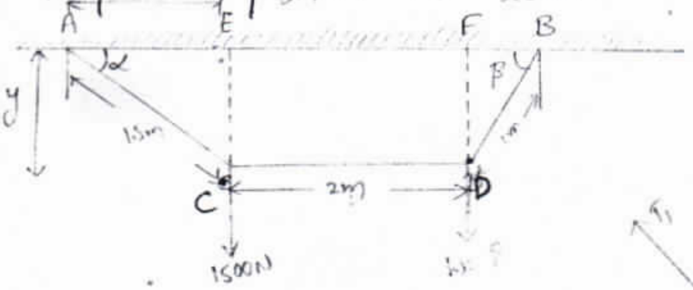
$\Sigma F_V = 0$; $\quad T_3 \cos 30 - T_2 \cos 60^\circ - 200 = 0$

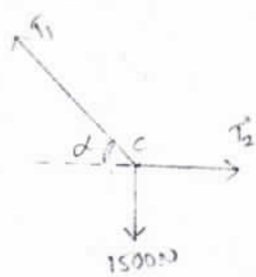
$$\underline{T_3 = 336.6N}$$

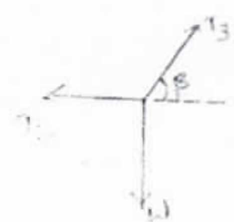
Sub in (1)

$$\underline{T_1 = 326.8N}$$

→ Rope AB shown in fig. is 4.5m long and is connected at two points A & B at the same level 4m apart. A load of 1500N is suspended from a point C on the rope at 1.5m from A. What load connected at point D on the rope, 1 m from B will be necessary to keep the position CD level?







Sol

from fig $CE = DF = y$.

$AE = x$, $AB = 4m$,

$AC + CD + DB = 4.5m$, $CD = 2m$, $AC = 1.5m$, $DB = 1m$.

$EF = 2m$.

$$BF = AB - (AE + EF) = 4 - (x+2) = 2-x.$$

from Δ^{le} BFD

$$BF^2 + FD^2 = BD^2$$

$$(2-x)^2 + y^2 = 1^2 \rightarrow (1) \Rightarrow y^2 = 1 - (2-x)^2 \rightarrow (1)$$

from Δ^{le} AEC, $AE^2 + EC^2 = AC^2$

$$x^2 + y^2 = 1.5^2 \rightarrow (2)$$

Sub (1) in (2)

$$x^2 + 1 - (2-x)^2 = 1.5^2$$

$$\cancel{x^2} + 1 - 2^2 - \cancel{x^2} + 4x = 2.25$$

$$1 - 4 + 4x = 2.25$$

$$4x = 2.25 + 3 = 5.25$$

$$\underline{x = 1.3125}$$

$$\cos\alpha = \frac{1.3125}{1.5} \Rightarrow \alpha = 28.95^\circ$$

$$\cos\beta = \frac{2 - 1.3125}{1.5} \Rightarrow \beta = 46.56^\circ$$

Applying Lami's theorem at point 'C' & D.

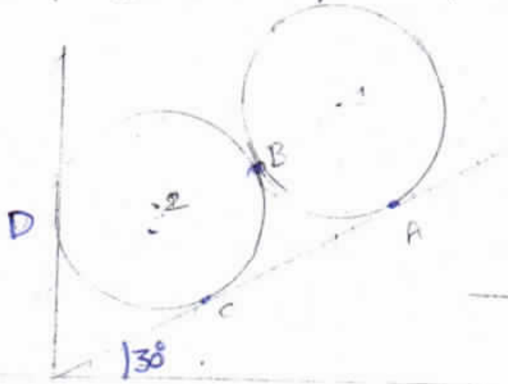
$$\frac{T_1}{\sin 90^\circ} = \frac{T_2}{\sin(90+28.95)} = \frac{1500}{\sin(180-28.95)}$$

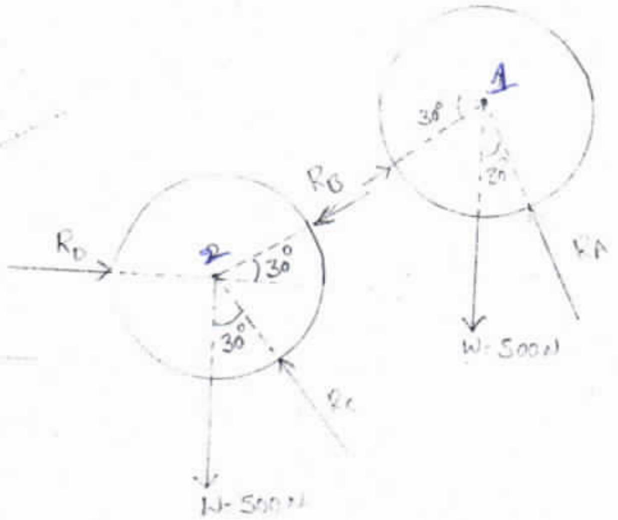
$T_1 = 3098.4\,N$, $T_2 = 2711.1\,N$.

$$\frac{T_3}{\sin 90} = \frac{W}{\sin(180-46.56^\circ)} = \frac{T_2}{\sin(90+46.56^\circ)}$$

$T_3 = 3943.4\,N$, $W = 2863.6\,N$

→ Two identical cylinders, each weighing 500N are placed in a trough as shown in fig. Determine the reactions developed at contact points A, B, C & D. Assume all points of contact are smooth





Consider body ①

$\Sigma f_x = 0$

$R_B \cos 30 - R_A \cos 60 - 500 \cos 90 = 0$

$R_B \cos 30 - R_A \cos 60 = 0$ —— ①

$\Sigma f_y = 0$

$R_B \sin 30 + R_A \sin 60 - 500 = 0 \Rightarrow R_B \sin 30 + R_A \sin 60 = 500$ —— ②

Solve ① & ②

$R_B = 250N$, $R_A = 433\,N$.

Consider body (2)

$\Sigma f_x = 0, \quad -R_B \cos 30 - R_C \cos 60 + R_D \cos 0 + W \cos 90 = 0$

$-R_B \cos 30 - R_C \cos 60 + R_D = 0 \longrightarrow (3)$

$\Sigma f_y = 0, \quad -R_B \sin 30 + R_D \sin 0 - W \sin 90 + R_C \sin 60 = 0$

$-R_B \sin 30 - 500 + R_C \sin 60 = 0 \quad (\because R_B = 250\,N)$

$-250 \sin 30 - 500 + R_C \sin 60 = 0$

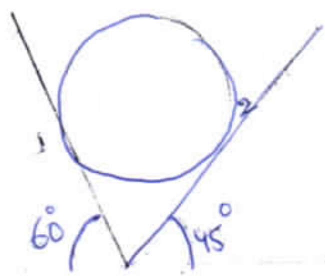
$R_C = \underline{721.68\ N}$

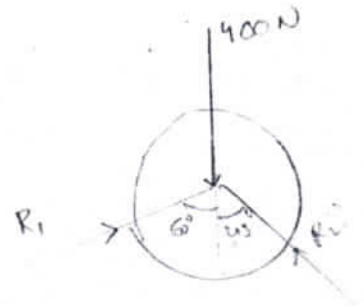
Sub R_c in (3)

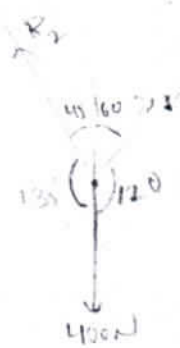
$-250 \cos 30 - 721.68 \cos 60 + R_D = 0$

$R_D = \underline{577.4\ N}$

→ A 400N sphere is resting in a trough as shown in fig. Determine the reactions developed at contact surfaces. Assume all contact surfaces are smooth





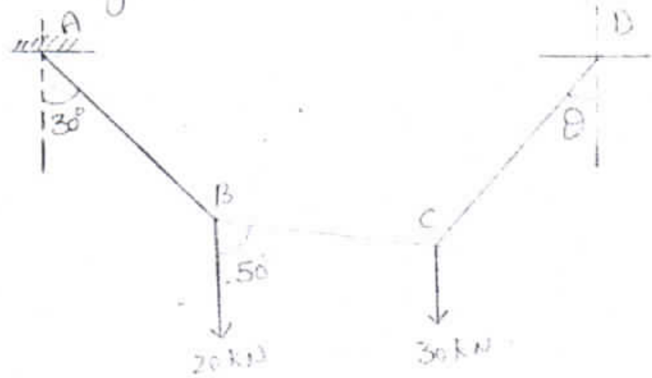


Applying Lami's theorem

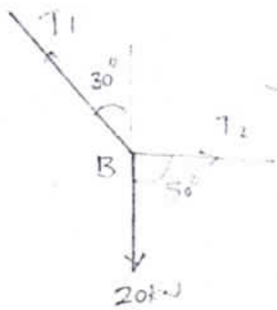
$$\frac{W}{\sin(105)} = \frac{R_1}{\sin 135} = \frac{R_2}{\sin 120}$$

$R_1 = 292.8\,N, \quad R_2 = 358.6\,N$

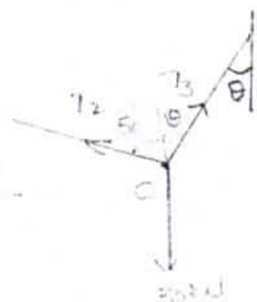
→ A wire rope is fixed at two points A & D shown in fig. weights 20 kN & 30 kN are attached to it at B & C respectively. The weights rest with portions AB & BC inclined at 30° & 50° resp. to the vertical. Find the tensions in AB, BC & CD of the wire. Determine the inclination of the segment CD to vertical.



At point B



At point C



Sol Lami's theorem for the system of forces at B

$$\frac{T_1}{\sin 50} = \frac{T_2}{\sin 150} = \frac{20}{\sin(160)}$$

$T_1 = 44.8\ kN \qquad T_2 = 29.2\ kN.$

Assume equilibrium at 'C'.

$\Sigma f_x = 0$

$T_3 \cos(90-\theta) - T_2 \cos 40 = 0 \Rightarrow T_3 \sin\theta = 22.36 \to (1)$

$\Sigma f_y = 0$

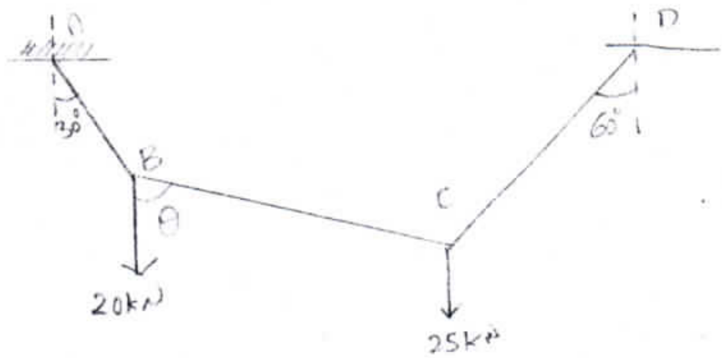
$T_3 \sin(90-\theta) + T_2 \sin 40 - 30 = 0 \Rightarrow T_3 \cos\theta = 11.2 \to (2)$

from (1) & (2)

$\tan\theta = 1.99 \Rightarrow \theta = 63.39°$

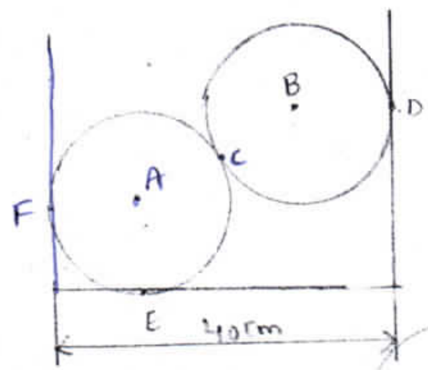
Sub 'θ' in (1) $\quad T_3 \sin\theta = 22.36 \Rightarrow T_3 = 25.04\ kN$

→ A wire is fixed at A & D as shown in fig. weight 20 kN & 25 kN are supported at B & C resp. when equilibrium is reached, it is found that inclination of AB is 30°, CD is 60° to vertical. Determine tensions & inclination to the vertical.



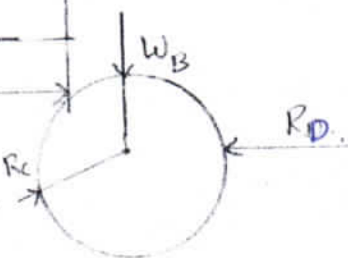
Ans:- $T_1 = 38.97$ kN
$T_2 = 23.84$ kN
$T_3 = 22.5$ kN.
$\theta = 54.78°$

→ find the reaction forces as shown in fig.

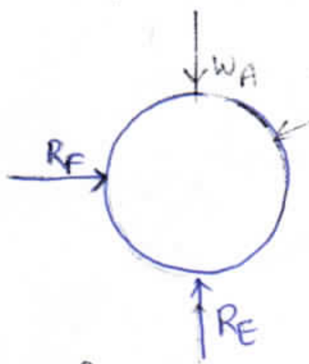


radius of ball A = 12 cm
" " " B = 10 cm
weight of ball A = 50N
" " " B = 40 N.

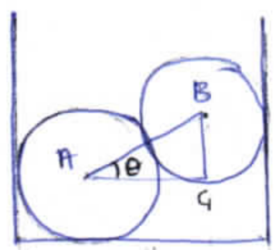
Sol



f.B.D of B



f.B.D of A



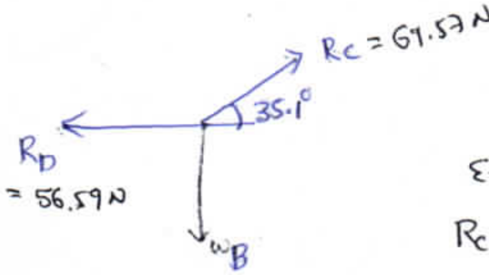
$AB = 10+12 = 22$ cm

$AG = 40-(12+10) = 1$[illegible]

$\cos\theta = \frac{AG}{AB} \Rightarrow \frac{18}{22}$

$\theta = 35.1°$

Consider the Equilibrium at 'B'



$\Sigma f_x = 0$

$R_C \cos(35.1) - R_D = 0 \longrightarrow (1)$

$\Sigma f_y = 0$

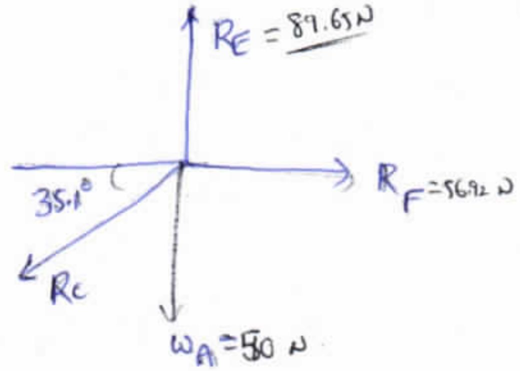
$R_C \sin(35.1) - W_B = 0 \Rightarrow R_C = 40/\sin 35.1 \Rightarrow$

$R_C = 69.57$ N

R_C Sub in (1)

$R_D = 69.57\cos(35.1) = 56.592$ N

Consider the Equilibrium of 'A'



$\Sigma f_x = 0$

$R_F - R_C \cos 35.1 = 0$

$R_F = 69.57 \cos 35.1 = \underline{56.92\,N}$

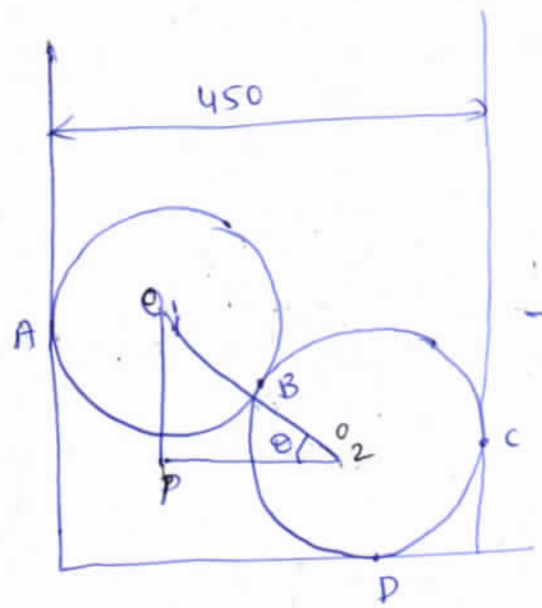
$\Sigma f_y = 0$

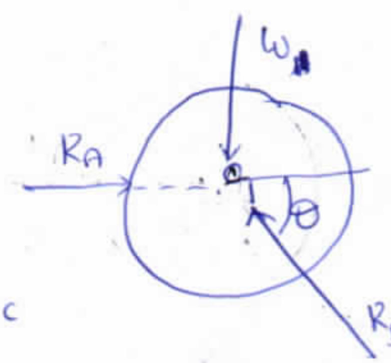
$R_E - R_C \sin 35.1 - W_A = 0$

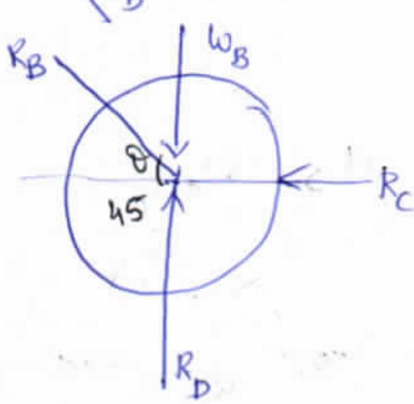
$R_E - 69.57 \sin 35.1 - 50 = 0$

$\underline{R_E = 89.65\,N}$

→ cylinder 1 of diameter 200mm & cylinder 2 of diameter 300mm are placed in a trough as shown in fig. If cylinder 1 weight 800N and cylinder 2 weights 1200N, determine the reactions developed at contact surfaces A, B, C & D. Assume all contact surfaces are smooth.







Sol/ from Δ^{le} O_1O_2P

$$\cos\theta = \frac{450 - 100 - 150}{100 + 150} = 0.8$$

$\theta = 36.87°$

Consider the Equilibrium of cylinder 1

$\Sigma f_H = 0$; $\quad R_A - R_B\cos(36.87) = 0 \rightarrow (1)$

$\Sigma f_V = 0$; $\quad R_B \sin(36.87) - 800 = 0$.

$R_B = 800/\sin(36.87) = 1333.3\,N$.

Sub R_B in (1)

$R_A - 1333.3\cos(36.87) = 0$

$R_A = 1066.7\,N$.

Consider the Equilibrium of cylinder (2)

$\Sigma f_H = 0$; $\quad R_D \cos45 - R_c + R_B\cos(36.857) = 0 \longrightarrow (3)$

$\Sigma f_V = 0$; $\quad R_D \sin45 - R_B \sin(36.87) - 1200 = 0$

$$R_D = \frac{1333.3\sin(36.87) + 1200}{\sin 45^\circ}$$

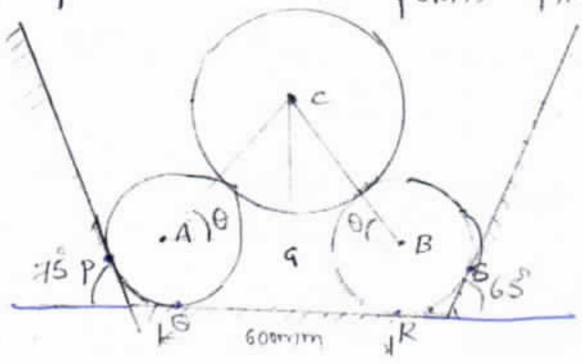
$R_D = 2828.4\,N$

Sub R_D & R_B in (3)

$2828.4\cos45 - R_c + 1333.3\cos(36.87) = 0$

$R_c = 3066.7\,N$

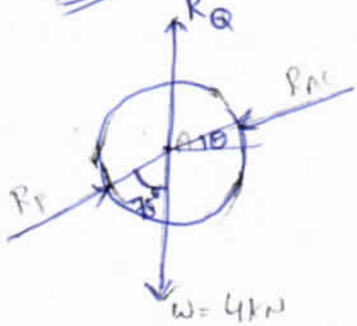
→ Three spheres A, B & C having their diameters 500mm, 500mm & 800mm resp are placed in trough with smooth side walls & floor as shown in fig. The centre to centre distance of spheres A & B is 600mm. The cylinders A, B, & C weights 4kN, 4kN & 8kN resp. Determine the reactions developed at contact points P, Q, R & S.



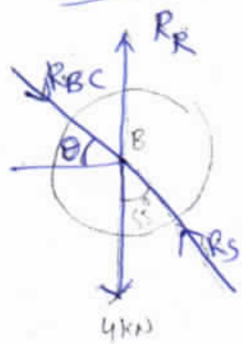
$$\cos\theta = \frac{AG}{AC} = \frac{300}{650}$$

$\theta = 62.51^\circ$

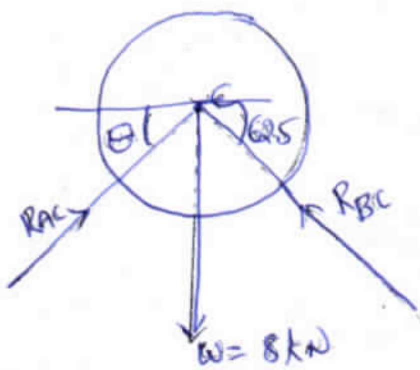
F.B.D of A



F.B.D of B



F.B.D. of C



Applying Lami's theorem on body ③

$$\frac{W}{\sin 55} = \frac{R_{AC}}{\sin(152.5)} = \frac{R_{BC}}{\sin(152.5)} \Rightarrow \begin{array}{l} R_{AC} = 4.5\ kN \\ R_{BC} = 4.5\ kN \end{array}$$

at F.B.D. ②

$\Sigma f_x = 0;\quad R_{BC}\cos 62.5 - R_S \cos 25 = 0$

$$R_S = \underline{2.29\ kN}$$

$\Sigma f_y = 0;\quad R_R - R_{BC}\sin 62.5 - 4 + R_S \sin 25 = 0$

$$R_R = \underline{7.02\ kN}$$

at F.B.D ①

$\Sigma f_x = 0;\quad -R_{AC}\cos 62.5 + R_P \cos 15 = 0$

$$R_P = \underline{2.15\ kN}$$

$\Sigma f_y = 0;\quad -R_{AC}\sin 62.5 + R_Q + R_P \sin 15 - 4 = 0 \Rightarrow R_Q = \underline{7.43\ kN}$

→ A roller of radius $r = 300mm$ & weighing 2000N is to be pulled over a curb of height 150mm, as shown in fig. A Horizontal force f' applied to the end of a string wound around the circumference of the roller. find the magnitude of force f' required to start the roller mo over the curb. what is the least pull f through the centre of the wheel to just turn the roller over the curb.

F.B.D

Case i:-

Sol from Δ^{le} AOC

$$OA^2 = AC^2 + OC^2$$

$$300^2 = AC^2 + 150^2 \Rightarrow AC = 259.8$$

From Δ^{le} ABC

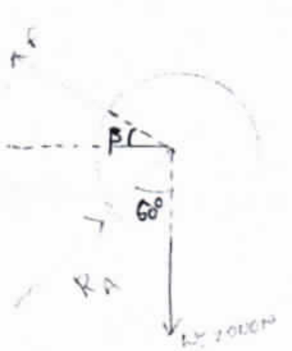
$$\tan\theta = \frac{AC}{BC} \Rightarrow \theta = \tan^{-1}\left(\frac{259.8}{450}\right) \Rightarrow \theta = 29.9^\circ$$

$$\frac{2000}{\sin(119.9)} = \frac{R_A}{\sin 90} = \frac{f}{\sin(150.1)}$$

$$R_A = 2307.07\ N.$$

$$f = \underline{1150.05\ N}$$

Case ii force is applied at centre of roller.



from Δ^{le} AOC

$$\cos\alpha = \frac{OC}{AO} \Rightarrow \alpha = \cos^{-1}\left(\frac{150}{300}\right) = 6($$

$\Sigma f_x = 0$

$$R\cos 30 - f\cos\beta = 0 \Rightarrow \quad R = \frac{f\cos\beta}{\cos 30}$$

$\Sigma f_y = 0$; $\quad -f\sin\beta + R\sin 30 - 2000 = 0$

$$f\sin\beta + \frac{f\cos\beta}{\cos 30}\sin 30 - 2000 = 0$$

$$f\sin\beta + f\cos\beta\tan 30 = 2000$$

$$f(\sin\beta + \cos\beta\tan 30) = 2000$$

$$\sin\beta + \cos\beta\tan 30 = \frac{2000}{f} = \frac{W}{f} \rightarrow (1)$$

$$\frac{d}{d\beta}\left(\frac{W}{f}\right) = 0$$

$$\cos\beta + \tan 30(-\sin\beta) = 0$$

$$\cos\beta = \sin\beta \cdot \tan 30$$

$$\cot\beta = \tan 30 \qquad\qquad \tan 30 = \cot 60$$

$$\beta = \cancel{\cot^{-1}(\tan 30)}$$

$$\cot\beta = \cot 60 \Rightarrow \underline{\beta = 60^\circ}$$

Sub $\beta = 60^\circ$ in (1)

$$\frac{W}{f} = \sin\beta + \cos\beta\tan 30$$

$$\frac{2000}{f} = \sin 60 + \cos 60\tan 30$$

$$\underline{f = 1732.05\ N}$$

Moment of a force:

Moment of a force about a point is defined as the product of the magnitude of the force & $\perp^{ar}$ distance of the point from the line of action of force.

In other words the rotational effect produced by a force is called moment of a force.

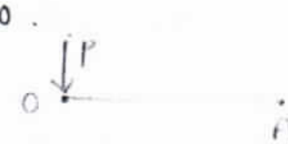
moment of force = force × $\perp^{ar}$ distance

$= P \times OA$

$= P \times l$

Units → N-m (or) kN-m

Note:- If a point lies on the line of action of a force, the moment of the force about that point is zero.

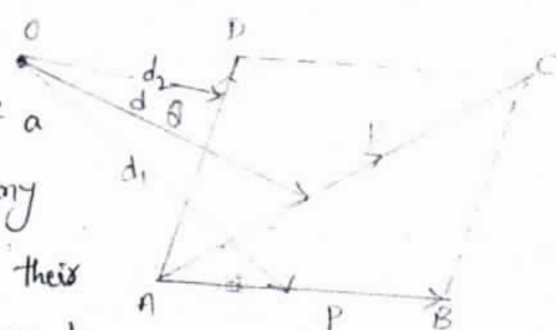


Moment about 'O' $= P \times 0$ (zero)

$= 0$

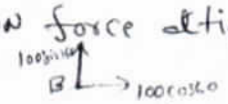
Varignon's theorem:-

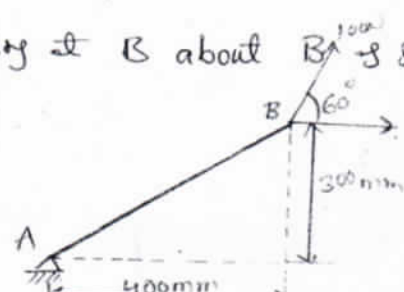
It states that " The algebraic sum of the moments of a system of coplanar forces about any point is equal to the moment of their resultant force about the same point.



$$\boxed{R.d = P \times d_1 + \theta.d_2}$$

→ Determine the moment of 100N force acting at B about B as shown in fig.





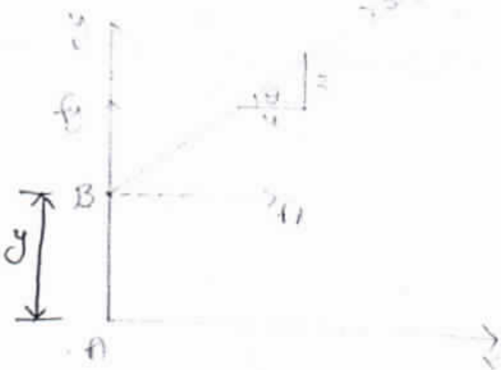
Sol

$m_A = 100\cos 60 \times 300 - 100 \sin 60 \times 400$

$= -28301$ N-mm

$m_A = 28301$ N-mm (Anticlockwise)

→ what will be the y-intercept of 5000 N force shown in fig., if its moment about 'A' is 8000 N-m?



Sol) $\cos\theta = 4/5$, $\sin\theta = 3/5$

$f_x = 5000\cos\theta = 4000\,N$

$f_y = 5000\sin\theta = 3000\,N.$

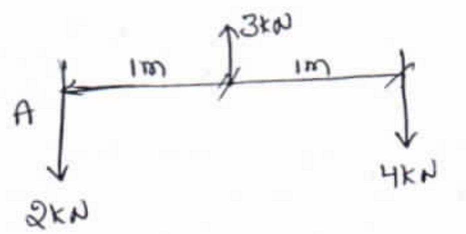
By Varignon's theorem

about A $\quad 8000 = 4000 \times y + 3000 \times 0$

$y = 2m$

Couple:-

→ find the resultant & moment about point 'A' as shown in fig



Sol) Resultant ($\uparrow$ +ve & $\downarrow$ -ve)

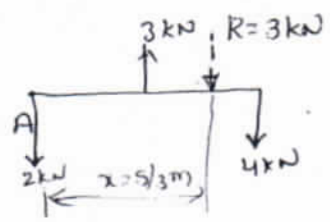
$R = -2 + 3 - 4 = -3\,kN\ (\downarrow)$

By Varignon's theorem of moments

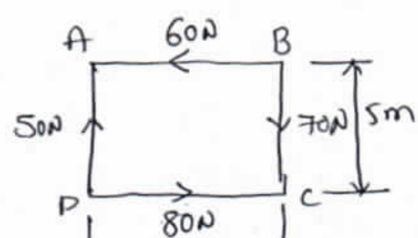
$\Sigma M_A = R \times x \qquad (\circlearrowright +ve, \circlearrowleft -ve)$

$-3\times1 + 4\times2 = 3\times x$

$x = 5/3$ w.r.t. 'A'.



→ Calculate the moment about point 'A' as shown in fig



$M_A = ?$

Taking moments about point 'A'

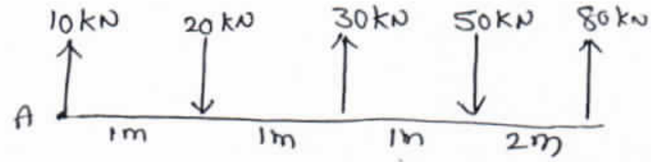
50N & 60N are meet at point 'A', so there is no ⊥ distance, so not consider 50N & 60N

$$m_A = (70\times5) - (80\times5) = (-50\ N\text{-}m)\ (\text{Anticlockwise})$$

→ Five parallel forces 10, 20, 30, 50 & 80 kN are acting on a beam. Distances of forces from 10 kN forces are 1m, 2m, 3m & 5m resp. Forces 20 kN & 50 kN are acting downwards & remaining forces acting upwards. Find the resultant in magnitude, direction & location w.r.t. 10 kN force

Sol)

$$R = 10 - 20 + 30 - 50 + 80$$
$$= 50\ kN\ (\uparrow)$$



By Varignon's theorem about point 10 kN force.

$$\Sigma M_A = R \times x$$

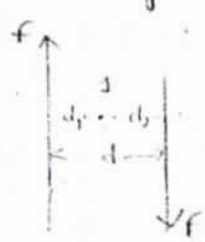
$$(-80\times5) + (50\times3) - (30\times2) + (20\times1) = -50\times x$$

$$x = 5.8m$$

Couple:-

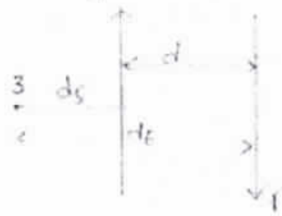
Two Equal Unlike parallel forces, whose line of Action are not the same (i.e Not collinear), form a couple. The effect of a couple is to cause the rotation of a body.

Consider the moment of the two forces constituting a couple about point 1 as shown in fig.



$$M_1 = Fd_1 + Fd_2$$
$$= F(d_1 + d_2) = Fd$$

$$M_2 = F(d_3) - Fd_4 =$$
$$= F(d_3 - d_4) = Fd.$$



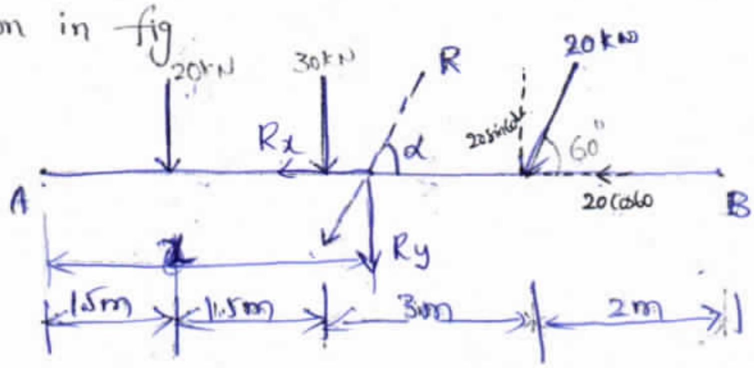
$$m_3 = Fd_6 - Fd_5$$
$$= F(d_6 - d_5) = Fd$$

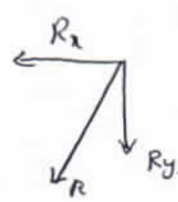
Thus it can be observed that moment of a couple about any point is same.

Properties of a couple:-

- A couple consists a pair of equal & opposit parallel forces which are seperated by a definite distance.
- Translatory effect of a couple on the body is zero.
- Rotational effect (moment) of a couple about any point is a const & it is equal to the product of magnitude of forces & ⊥ distance
- A couple is not balanced by a single force.

→ Determine the resultant of the system of forces acting on a beam as shown in fig





Sol

$$R_x = \Sigma f_x = -20\cos 60 = -10\,kN = \overleftarrow{10\,kN}$$

$$R_y = \Sigma f_y = -20 - 30 - 20\sin 60 = -67.32\,kN = 67.32\,kN \downarrow$$

$$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2} = \sqrt{10^2 + 67.32^2} = 68.06\,kN.$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right) \Rightarrow \alpha = 81.55^\circ$$

Taking moments about point 'A'

$$\Sigma M_A = (20 \times 1.5) + (30 \times 3) + (20 \sin 60 \times 6)$$

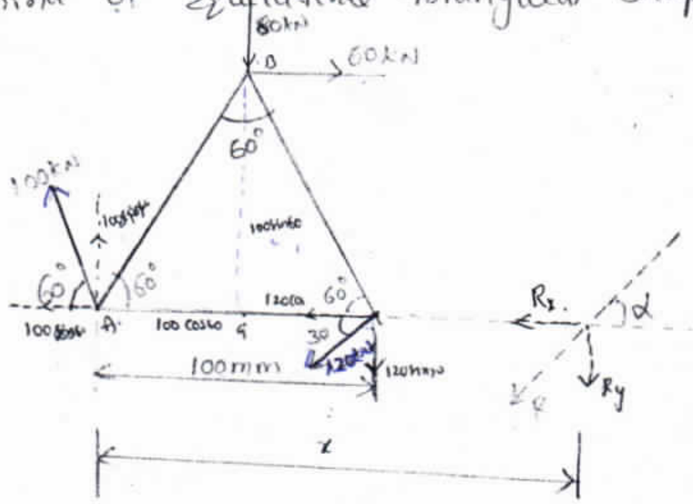
$$= 223.92 \text{ kN-m}$$

By varignon's theorem.

$$\Sigma M_A = R_y \times x.$$

$$223.92 = 67.32 \times x \Rightarrow x = 3.326\text{m}.$$

→ find the resultant of the force system as shown in fig. acting on a Lamina of equilateral triangular shape.



$\cos 60 = \frac{AG}{100}$

$AG = 100 \cos 60$

$\sin 60 = \frac{BG}{100}$

$BG = 100 \sin 60$

Sol

$$R_x = \Sigma f_x = 60 - 100 \cos 60 - 120 \cos 30 = -93.9\text{kN} = 93.9\text{kN} \leftarrow$$

$$R_y = \Sigma f_y = -80 + 100 \sin 60 - 120 \sin 30 = -53.4\text{kN} = 53.4\text{kN} \downarrow$$

$$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2} = \sqrt{93.9^2 + 53.4^2} = 108 \text{ kN}$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right) \Rightarrow \alpha = 29.62°$$

Taking moments about point 'A'

$$\Sigma M_A = (80 \times 50) + (60 \times 100 \sin 60) + (120 \sin 30 \times 100)$$

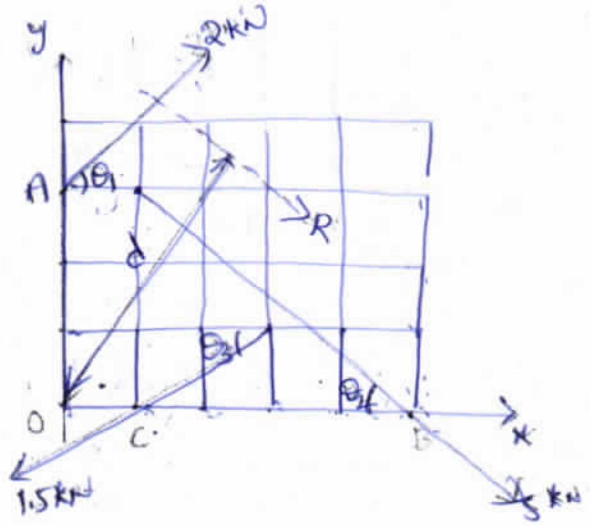
$$= 15196.15 \text{ N-mm}$$

By varignon's theorem

$$\Sigma M_A = R_y \times x \Rightarrow 15196.15 = 53.4 \times x$$

$$x = 284.6\text{mm}$$

→ find the resultant of the system of coplanar forces acting on a Lamina as shown in fig. Each square has a side of 10mm.



$\tan\theta_1 = \frac{10}{10} \Rightarrow \theta_1 = 45^0$

$\tan\theta_2 = \frac{30}{40} \Rightarrow \theta_2 = 36.87^0$

$\tan\theta_3 = \frac{10}{20}$

$\theta_3 = 26.56^\circ$

$R_x = \Sigma f_x = 2\cos 45 + 5\cos 36.87 - 1.5\cos 26.56$

$= 4.072 \text{ kN}$

$R_y = \Sigma f_y = 2\sin 45 - 5\sin 36.87 - 1.5\sin 26.56$

$= -2.257 \text{ kN} = 2.257 \text{ kN} \downarrow$

$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2} = \sqrt{4.07^2 + 2.25^2} = 4.655 \text{ kN}$

$\alpha = \tan^{-1}\left(\frac{2.257}{4.07}\right) = 29^\circ$

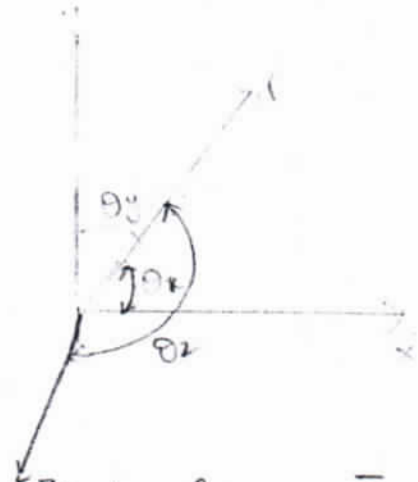
$\Sigma m_O = (2\cos 45 \times 30) + (5\sin 36.87 \times 50) + (1.5\sin 26.56 \times 10)$

$= 199.13 \text{ kN-mm}$

$\Sigma m_O = R \times d$

$d = \frac{199.13}{4.655} = 42.8\text{mm}$

Analysis of Spatial force System:-



$F_x = F\cos\theta_x$

$F_y = F\cos\theta_y$

$F_z = F\cos\theta_z$.

'F' is in vector form $\Rightarrow \bar{F} = F_x\bar{i} + F_y\bar{j} + F_z\bar{k}$

$|\bar{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$ → magnitude

$F_x = F\cos\theta_x \Rightarrow \theta_x = \cos^{-1}\left(\frac{F_x}{F}\right)$

$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$

$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right)$

lie b/w 0 to 180

$F^2 = F_x^2 + F_y^2 + F_z^2$

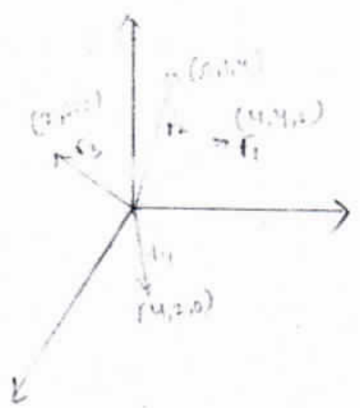
$F^2 = (F\cos\theta_x)^2 + (F\cos\theta_y)^2 + (F\cos\theta_z)^2$

$\cancel{F^2} = \cancel{F^2}(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)$

$$\boxed{1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z}$$

$$\bar{F} = \frac{F(x\bar{i} + y\bar{j} + z\bar{k})}{\sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}}$$

- find the resultant of magnitude & direction of the forces shown in fig. $F_1, F_2, F_3, \& F_4$ are 10, 12, 14 & 16 kN.



$$\bar{F}_1 = \frac{F_1(x\bar{i} + y\bar{j} + z\bar{k})}{\sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}}$$

$$\bar{F}_1 = \frac{10(4\bar{i} + 4\bar{j} + 2\bar{k})}{\sqrt{4^2 + 4^2 + 2^2}}$$

$$\bar{F}_1 = \frac{10(4\bar{i} + 4\bar{j} + 2\bar{k})}{6}$$

Similarly $\bar{F}_2 = \frac{12(1\bar{j}+4\bar{k})}{\sqrt{0^2+1^2+4^2}} = \frac{12(1\bar{j}+4\bar{k})}{4.123}$

$\bar{F}_3 = \frac{14(2\bar{i}+2\bar{k})}{\sqrt{2^2+2^2}} = \frac{14(2\bar{i}+2\bar{k})}{2.828}$

$\bar{F}_4 = \frac{16(4\bar{i}+2\bar{j})}{\sqrt{4^2+2^2}} = \frac{16(4\bar{i}+2\bar{j})}{4.472}$

$\bar{F}_1 = 6.67\bar{i} + 6.67\bar{j} + 3.3\bar{k}$

$\bar{F}_2 = 2.91\bar{j} + 11.64\bar{k}$

$\bar{F}_3 = 9.9\bar{i} + 9.9\bar{k}$

$\bar{F}_4 = 14.31\bar{i} + 7.156\bar{j}$

$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$

$= 30.88\bar{i} + 16.736\bar{j} + 24.87\bar{k}$

$|\bar{R}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$= \sqrt{30.88^2 + 16.736^2 + 24.87^2}$

$|\bar{R}| = 43.047\,kN.$

$R_x \cancel{F_x} = \cancel{R}\cos\theta_x$ $\quad$ $R_y \cancel{F_y} = R\cos\theta_y$ $\quad$ $R_z \cancel{F_z} = R\cos\theta_z$

$\frac{30.88}{43.04} = \cos\theta_x$ $\quad$ $\frac{16.736}{43.04} = \cos\theta_y$ $\quad$ $\frac{24.87}{43.04} = \cos\theta_z$

$\theta_x = 44.15^\circ$ $\quad$ $\theta_y = 67.11^\circ$ $\quad$ $\theta_z = 54.69^\circ$

→ Determine the magnitude & direction of the force

$f = 345i + 150j - 290k$.

Sol

$$\bar{F} = f_x i + f_y j + f_z k.$$

$\therefore f_x = 345, \quad f_y = 150, \quad f_z = 290.$

$$|\bar{F}| = \sqrt{f_x^2 + f_y^2 + f_z^2}.$$

$$= \sqrt{345^2 + 150^2 + 290^2} = 475.$$

$$f_x = \bar{F}\cos\theta_x \Rightarrow 345 = 475.\cos\theta_x$$

$$\theta_x = 43.42^\circ$$

$$f_y = \bar{F}\cos\theta_y \Rightarrow 150 = 475.\cos\theta_y$$

$$\theta_y = 71.59^\circ$$

$$f_z = \bar{F}\cos\theta_z \Rightarrow -290 = 475.\cos\theta_z$$

$$\theta_z = 127.63^\circ$$

— A force acts at the origin in a direction defined by the angles $\theta_x = 70.9^\circ$, $\theta_y = 144.9^\circ$. Knowing that z component of the force is -52 N. determine a) the angle θ_z b) the other components & magnitude of the force.

Sol G.D: $\theta_x = 70.9^\circ$, $\theta_y = 144.9^\circ$, $f_z = -52N$

$\theta_z = ?$, $f_x, f_y, f = ?$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2\theta_z = 1 - \cos^2\theta_x - \cos^2\theta_y$$

$$\cos^2\theta_z = 0.22 \Rightarrow$$

$$\cos\theta_z = \pm 0.47.$$

F → should be +ve so $\cos\theta$ value should be in −ve

$\cos\theta_z = -0.47 \Rightarrow \theta_z = 118.03^0$

$f_z = f\cos\theta_z \Rightarrow f = \frac{f_z}{\cos\theta_z} = \frac{-52}{-0.47}$

$f = 110.64\ N$

$f_x = f\cos\theta_x \Rightarrow f_x = 110.64 \times \cos(70.9)$

$f_x = 36.2\ N$

$f_y = f\cos\theta_y \Rightarrow 110.64 \times \cos(144.9)$

$f_y = -90.52\ N.$

→ force of 800N acts along AB, A(3,2,-4) and B(8,-5,+6) write the force vector

Sol] $\bar{F} = F \times \bar{e}_{AB}$

$$\bar{e}_{AB} = \frac{(x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k}{\sqrt{(x_2-x_1)^2 + (y_2-y_1) + (z_2-z_1)}}$$

$$= \frac{(8-3)i + (-5-2)j + (6-(-4))k}{\sqrt{(8-3)^2 + (-5-2)^2 + (6+4)^2}}$$

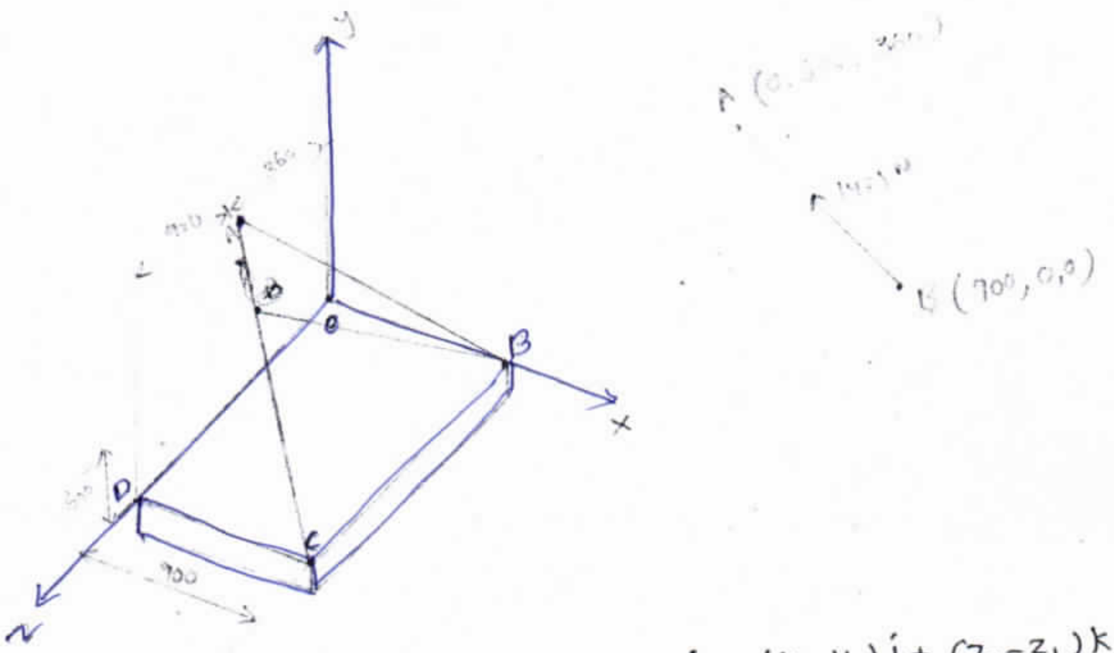
$$= \frac{5i - 7j + 10k}{\sqrt{174}}$$

$\bar{F} = F.\bar{e}_{AB}$

$$= 800\left(\frac{5i - 7j + 10k}{\sqrt{174}}\right)$$

$\bar{F} = 303.24i - 424.54j + 606.48k$

→ knowing that the tension in cable AB is 1425 N, determine the components of the force exerted on the plate at B



unit vector $\bar{e}_{BA} = \dfrac{(x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$

$$= \frac{-900i + 600j + 360k}{\sqrt{(-900)^2 + (600)^2 + (360)^2}}$$

$$= -\frac{900i + 600j + 360k}{1140}$$

force vector $= \bar{T}_{BA} = T \cdot \bar{e}_{BA}$

$$= 1425 \cdot \left(-\frac{900i + 600j + 360k}{1140}\right)$$

$$\bar{T}_{BA} = -1125i + 750j + 450k$$

$f_x = -1125\ N$, $f_y = 750\ N$, $f_z = 450\ N$

Unit Vector $\bar{e}_{CA} = \dfrac{-900i + 600j - 920k}{\sqrt{(-900)^2 + 600^2 + (-920)^2}}$ $\quad T_{AC}$

$= \dfrac{-900i + 600j - 920k}{1420}$

$\bar{T}_{CA} = T_{AC} \times \bar{e}_{CA}$

$= T_{AC} \left(\dfrac{-900i + 600j - 920k}{1420} \right)$

$= T_{AC} \left(-0.633i + 0.422j - 0.647k \right)$

Unit Vector $\bar{e}_{DA} = \dfrac{+600j - 920k}{\sqrt{600^2 + (-920)^2}}$

$= \dfrac{600j - 920k}{1098.36}$

$\bar{T}_{DA} = T_{AD} \left(\dfrac{600j - 920k}{1098.36} \right)$

$= T_{AD} \left(0.54j - 0.83k \right)$

$\Sigma f_x = 0 \Rightarrow -1125 - 0.633\, T_{AC} = 0$

$T_{AC} = 1777.25$

$\Sigma f_y = 0 \Rightarrow 750 + 0.422\, T_{AC} + 0.54\, T_{AD} = 0$

$T_{AD} = 2777.7$

Unit vector $\overline{e}_{AD} = \dfrac{-63i - 16j - 72k}{\sqrt{63^2 + 16^2 + 72^2}}$

$= -0.64i - 0.164j - 0.74k.$

force vector $(T_{AD}) = T_{AD} \times \overline{e}_{AD}$

$= T_{AD} \left(-0.64i - 0.16j - 0.74k\right)$

$\Sigma f_x = 0$:-

$\cancel{1340.2j}$ $T_{AB}(0.57)i - T_{AD}(0.64)i = 0$

$T_{AB} = \dfrac{0.64\,T_{AD}}{0.57} \Rightarrow 1.12\,T_{AD}$

$\Sigma f_y = 0$;

$1340.2 - 0.34\,T_{AB} - 0.164\,T_{AD} = 0$

$1340.2 - 0.34\,(1.12\,T_{AD}) - 0.164\,T_{AD} = 0$

$1340.2 - 0.38\,T_{AD} - 0.164\,T_{AD} = 0$

$T_{AD} = 2463.6N$

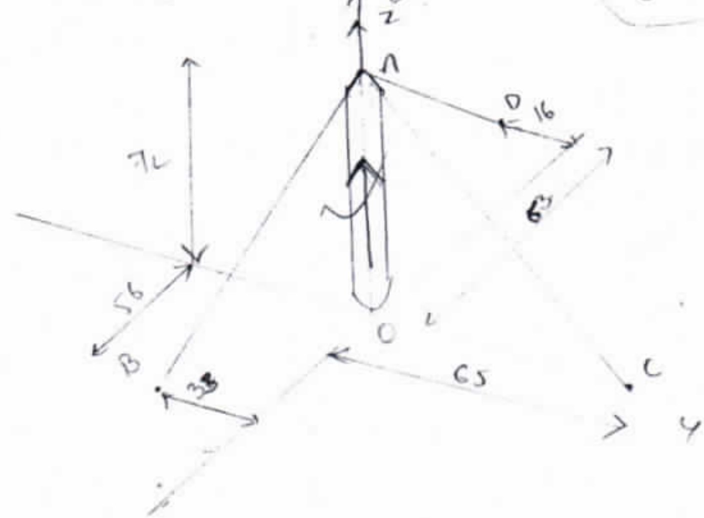
$T_{AB} = 1.12\,(T_{AD}) = 2759.23\ N$.

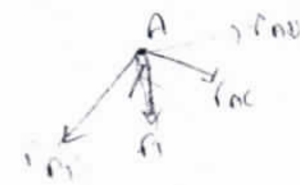
$\Sigma f_z = 0$;-

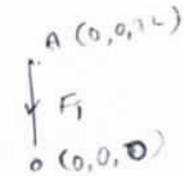
$-1484.5 - 0.74\,T_{AB} - 0.74\,T_{AD} - f_{AO} = 0$

$f_{AO} = -2380.39\ N$

7) The tension in wire $T_{AC} = 2000N$, determine the components of force exerted B & D and magnitude of force

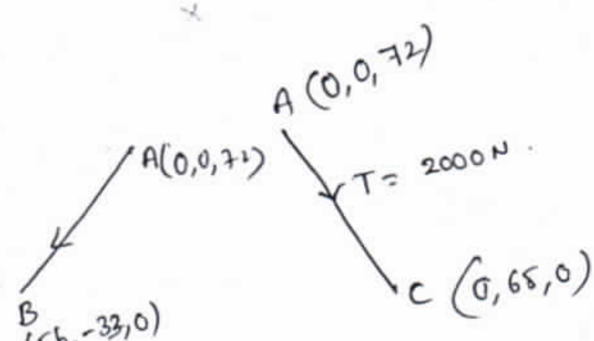






Unit vector $\bar{e}_{AO} = \frac{0i+0j-72k}{\sqrt{72^2}}$

$0i+0j-72k$



force $\bar{F}_{AO} = F_{AO} \times (-k)$

$= -F_{AO}k$

Unit vector $\bar{e}_{AC} = \frac{0i+65j+72k}{\sqrt{65^2+72^2}}$

$= \frac{+65j+72k}{97}$

force vector $\bar{T}_{AC} = T \times \bar{e}_{AC}$

$= 2000 \times \left(\frac{+65j+72k}{97}\right)$

$= +1340.2j + 1484.5k$

Unit vector $\bar{e}_{AB} = \frac{+56i+33j+72k}{\sqrt{56^2+33^2+72^2}} = \frac{+56i+33j+72k}{97}$

force vector $\bar{T}_{AB} = T_{AB} \times \bar{e}_{AB}$

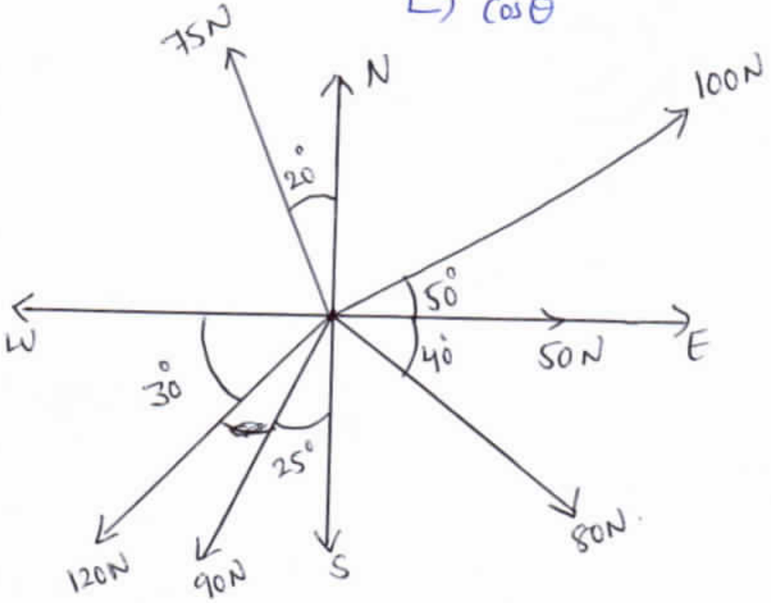
$= T_{AB}(+0.57i + 0.34j + 0.74k)$

Problems:-

Resolution of force:- Σf_x (Right (+ve), left (−ve)) , Σf_y (up (+), Down (−))
↳ $\cos\theta$ ↳ $\sin\theta$

$\theta \rightarrow$ makes angle with Horizontal

1)



$$\Sigma f_x = 50\cos 0^\circ + 100\cos(50) - 75\cos(70) - 120\cos(30) - 90\cos(65^\circ) + 80\cos 40$$

$$= 7.95\,N\ (\rightarrow)$$

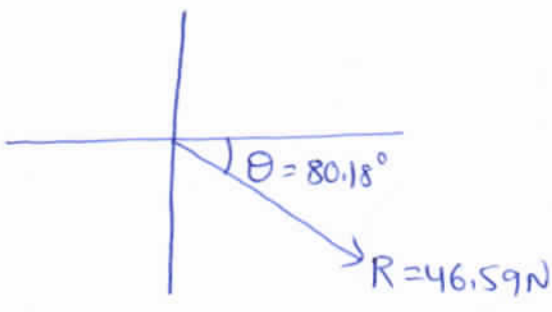
$$\Sigma f_y = 50\sin 0^\circ + 100\sin(50) + 75\sin(70) - 120\sin(30) - 90\sin(65) - 80\sin(40)$$

$$= -45.91\,N\ (\downarrow)$$

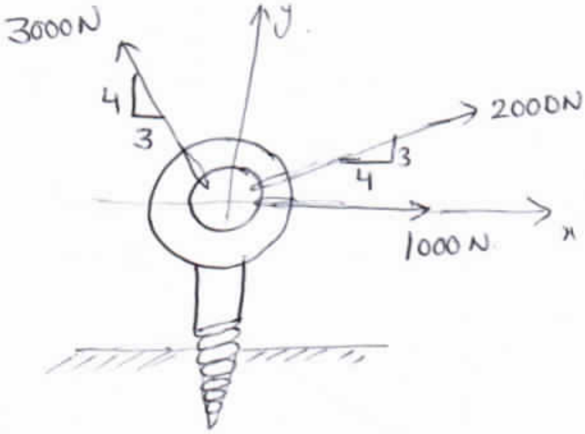
$$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2} = 46.59\,N.$$

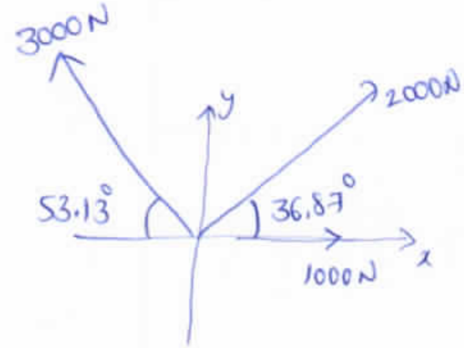
$$\theta = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right) = 80.18^\circ$$

Σf_x is +ve, Σf_y is −ve , So Resultant will be in IVth Quadrant

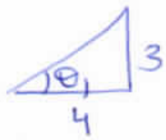


2) An eye bolt is being pulled from ground by three forces as shown in fig. Determine the ~~equilibrium~~ Resultant

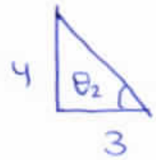




Sol)



$\Rightarrow \tan\theta_1 = 3/4 \Rightarrow \theta_1 = 36.87^\circ$

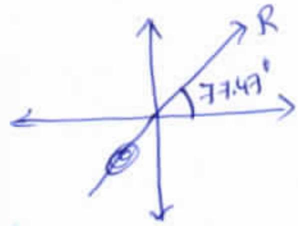


$\Rightarrow \tan\theta_2 = 4/3 \Rightarrow \theta_2 = 53.13^\circ$

$\Sigma f_x = 1000\cos 0^\circ + 2000\cos(36.87^\circ) - 3000\cos(53.13^\circ) = 800\,N\ (\rightarrow)$

$\Sigma f_y = 1000\sin 0^\circ + 2000\sin(36.87^\circ) + 3000\sin(53.13^\circ) = 3600\,N\ (\uparrow)$

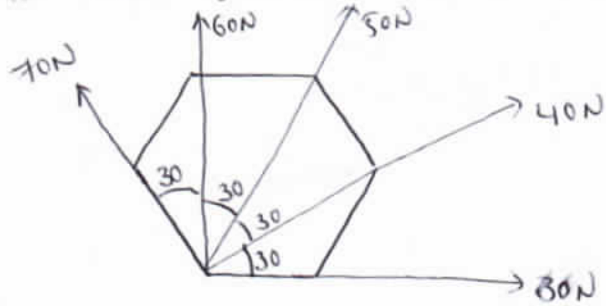
$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2} = 3687.82\,N$



$\theta = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right) = 77.47^\circ$

Σf_x (+ve), Σf_y (+ve), 'R' is in I Quadrant

3) forces 30N, 40N, 50N, 60N & 70N are acting at one of the angular points of a rectangular Hexagon, towards the other five angular points, taken in order. find the magnitude & direction of Resultant



Sol

$\Sigma f_x = 30\cos 0^\circ + 40\cos 30^\circ + 50\cos(60^\circ) + 60\cos(90^\circ) - 70\cos(60^\circ)$ ⊖

$= 54.64\ N \cdot (\rightarrow$

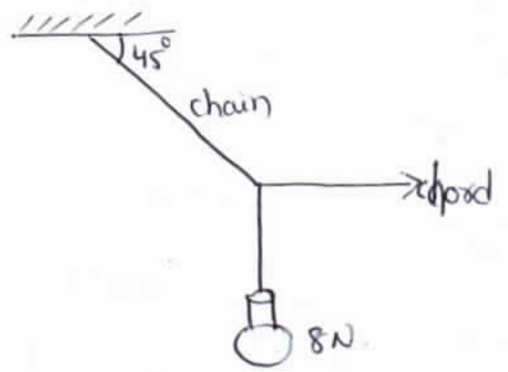
$\Sigma f_y = 30\sin(0^\circ) + 40\sin(30^\circ) + 50\sin(60^\circ) + 60\sin(90^\circ) + 70\sin(60^\circ)$

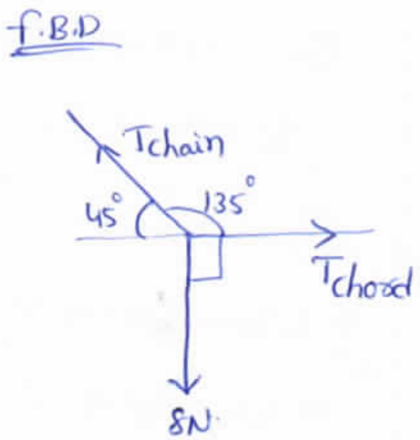
$= 183.92\ N\ (\uparrow)$

$R = 191.86 N, \quad \theta = 73.4^\circ$

Problems on Lami's theorem:-

→ find the tensions in the chord & chain





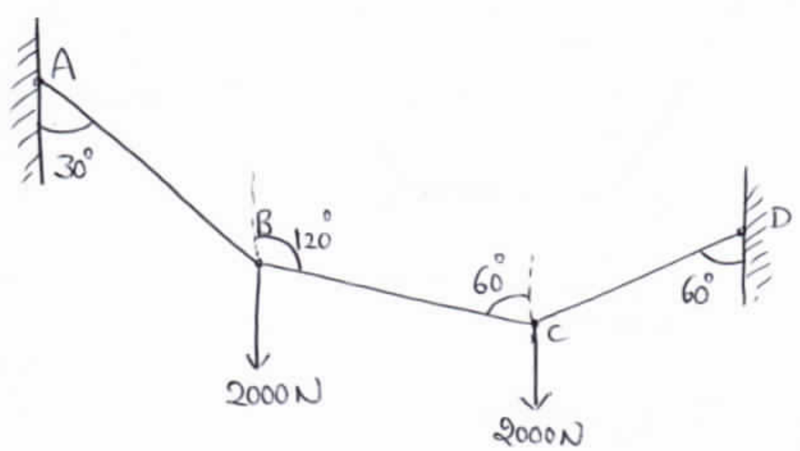
Sol

$$\frac{8}{\sin(135^\circ)} = \frac{T_{chain}}{\sin(90^\circ)} = \frac{T_{chord}}{\sin(135^\circ)}$$

$T_{chain} = 11.31\ N, \quad T_{chord} = 8N.$

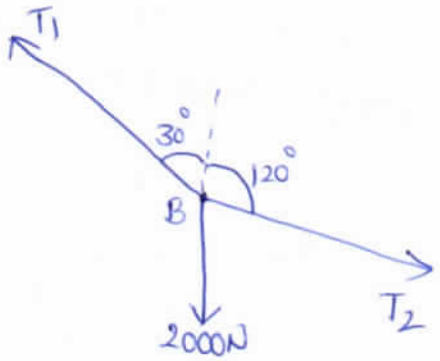
Equilibrium of connected bodies:-

→ find the tensions in the position AB, BC & CD of the string

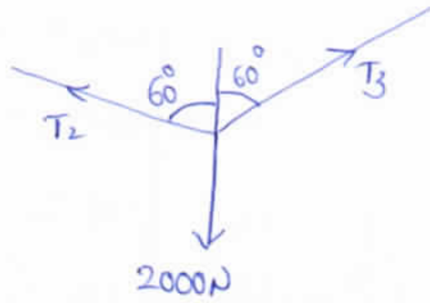


Sol

f.B.D of 'B'



f.B.D of C



$\Sigma f_x = 0;$

$T_2 \cos(30) - T_1 \cos(60) + 2000 \cos(90) = 0$

$T_2 \cos(30) - T_1 \cos(60) = 0 \rightarrow (1)$

$\Sigma f_y = 0;$

$-T_2 \sin(30) + T_1 \sin(60) - 2000 \sin(90) = 0$

$-T_2 \sin(30) + T_1 \sin(60) = 2000 \sin(90) \rightarrow (2)$

Solving (1) & (2)

$\underline{T_2 = 2000N}, \; \underline{T_1 = 3464.1\,N}$

$\Sigma f_x = 0;$

$T_3 \cos(30) - T_2 \cos(30) + 2000 \cos 90 = 0$

$T_3 \cos(30) - T_2 \cos(30) = 0 \rightarrow (3)$

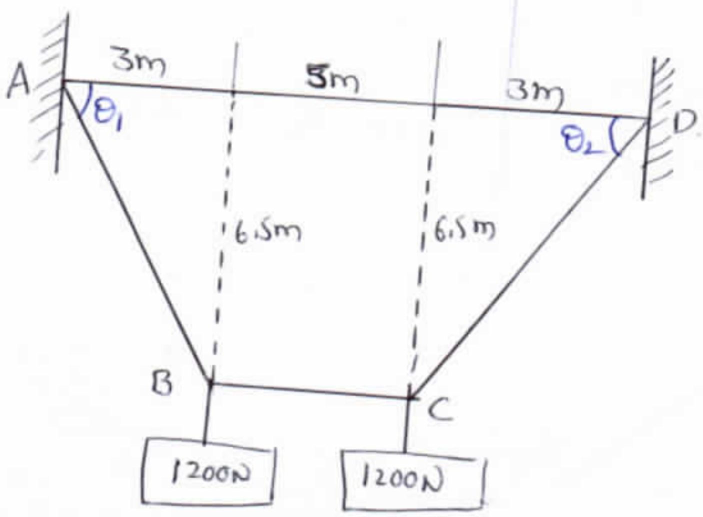
$\Sigma f_y = 0;$

$T_3 \sin(30) + T_2 \sin(30) - 2000 \sin 90 = 0$

$T_3 \sin(30) + T_2 \sin(30) = 2000 \rightarrow (4)$

$\underline{T_3 = 2000\,N}$

→ find the tensions in the postions AB, BC & CD

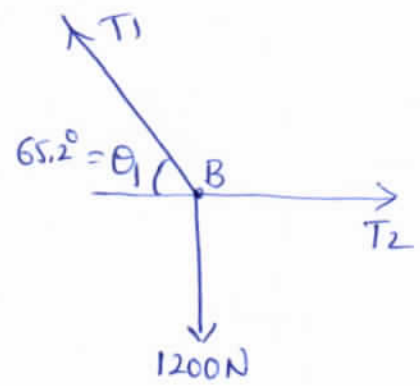


Sol

$\tan\theta_1 = 6.5/3 \Rightarrow \theta_1 = 65.22^\circ$

$\tan\theta_2 = 6.5/3 \Rightarrow \theta_2 = 65.22^\circ$

F.B.D of B



$\Sigma f_x = 0;$

$T_2 \cos(0) - T_1 \cos(65.2) + 1200 \cos(90) = 0$

$T_2 \cos(0) - T_1 \cos(65.2) = 0 \rightarrow (1)$

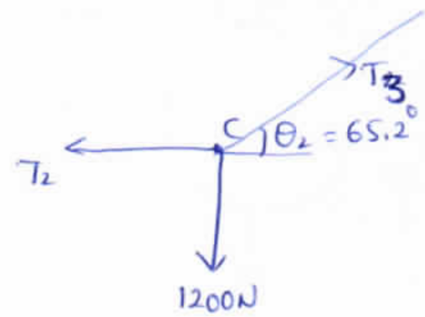
$\Sigma f_y = 0;$

$T_2 \sin(0) + T_1 \sin(65.2^\circ) - 1200 \sin(90) = 0$

$T_2 \sin(0) + T_1 \sin(65.2^\circ) = 1200 \sin(90) \rightarrow (2)$

$T_2 = 1321.69N, \; T_1 = 553.96N$

F.B.D of C



$\Sigma f_x = 0;$

$-T_2 \cos(0) + T_3 \cos(62.52) + 1200 \cos(90) =$

$-T_2 \cos(0) + T_3 \cos(62.52) = 0 \rightarrow (3)$

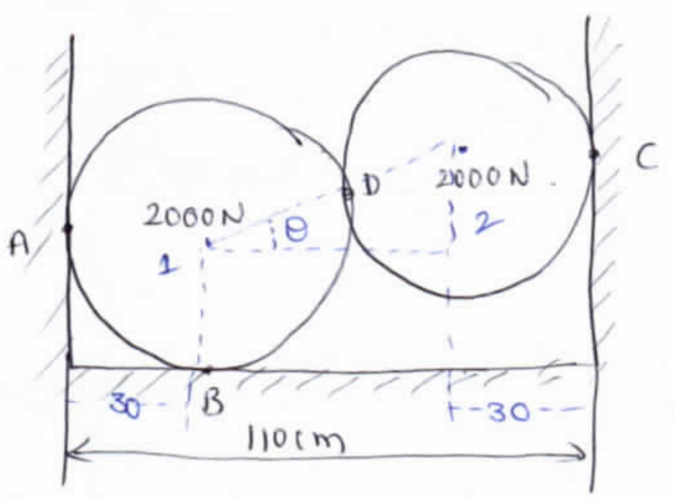
$\Sigma f_y = 0;$

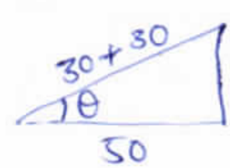
$T_3 \sin(62.52) + T_2 \sin(0) - 1200 \sin(90) = 0$

$T_3 \sin(62.52) + T_2 \sin(0) = 12000 \rightarrow (4)$

$T_3 = 1321.69N, \; T_2 = 553.96N$

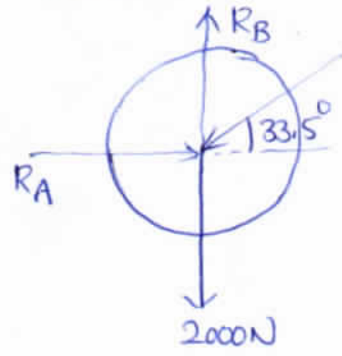
→ find the reactions at surface contact A, B & C, Radius → 30cm (Two spheres)





$\cos\theta = 50/60 \Rightarrow \theta = 33.5^\circ$

f.B.D of ①



$\Sigma f_x = 0;$

$-R_D \cos(33.5) + R_B \cos(90) + R_A \cos(0) + 2000\cos(90) = 0.$

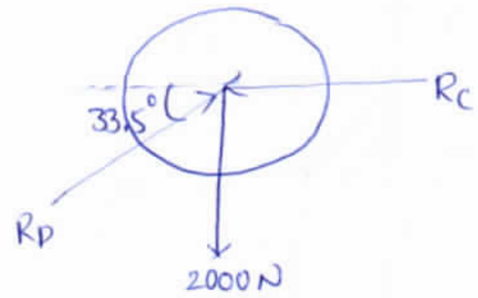
$R_B \cos(90) + R_A \cos(0) = R_D \cos(33.5) \to ①$

$\Sigma f_y = 0;$

$-R_D \sin(33.5) + R_B \sin(90) - 2000 \sin(90) + R_A \sin(0) = 0 \to ②$

$R_B = 3999.9\,N, \quad R_A = 3021.66\,N$

f.B.D of ②



$\Sigma f_x = 0;$

$-R_C \cos(0) + R_D \cos(33.5) + 2000 \cos(90) = 0$

$-R_C \cos(0) + R_D \cos(33.5) = 0 \to ①$

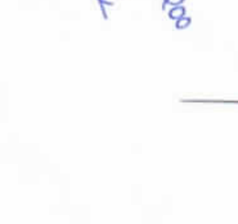
$\Sigma f_y = 0;$

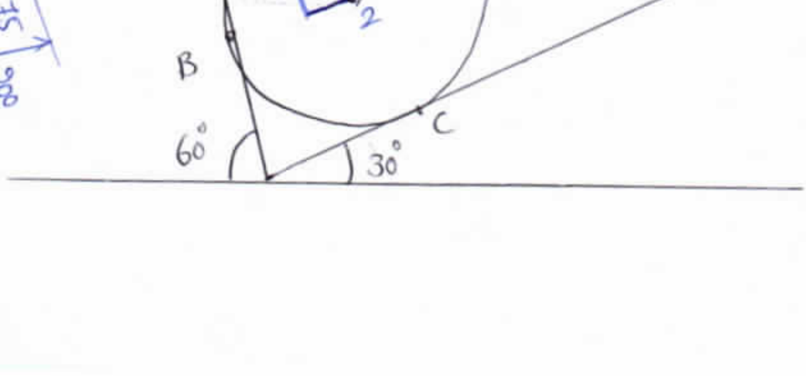
$R_C \sin(0) + R_D \sin(33.5) - 2000 \sin(90) = 0$

$R_C \sin(0) + R_D \sin(33.5) = 2000 \to ②$

$R_C = 3021.67\,N, \quad R_D = 3623.6\,N$

→ Two smooth cylinders with diameters 250mm & 400mm resp. are kept in a groove with slanting surfaces making angles 60° & 30° as shown in fig. Determine the reactions at contact points A, B, C & D.



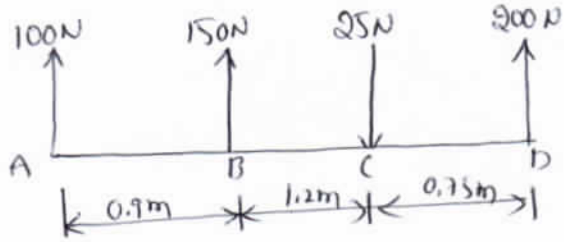


$\sin\theta = \frac{75}{325}$

$\theta = 13.34^\circ$

Moment of a force & couple:-

→ four parallel forces are acting as shown in fig. find the magnitude & position of resultant.

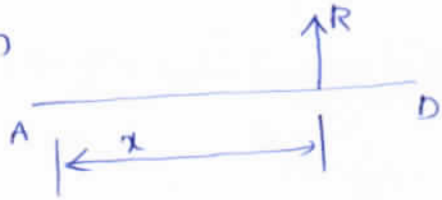


Sol

$\Sigma f_x = 0$; → No Horizontal forces

$\Sigma f_y = R = 100 + 150 - 25 + 200 = 425N$ (↑)

$M_A = R \times x$ → Acc. to varignon's therem

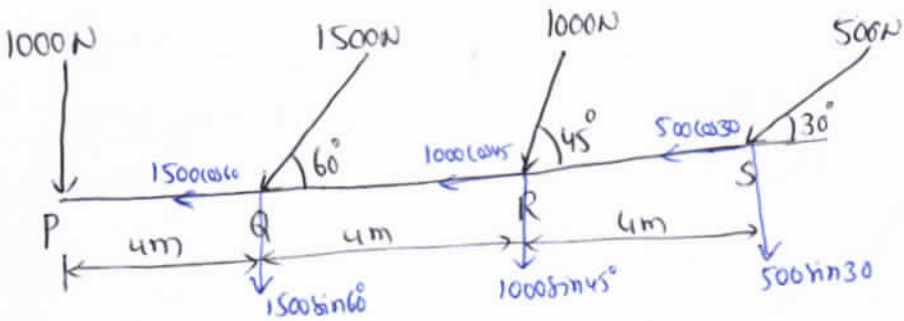


$(-150 \times 0.9) + (25 \times 2.1) - (200 \times 2.85) = -652.5 N\text{-}m$

$-652.5 = -425 \times x$

$x = 1.53m$

→ find the magnitude, direction & position of Resultant from point P



Sol

$\Sigma f_x = -1500\cos 60 - 1000\cos 45 - 500\cos 30 = -1890.1N$ (←)

$\Sigma f_y = -1000 - 1500\sin 60 - 1000\sin 45 - 500\sin 30 = -3256N$ (↓)

$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2} = 3764.79 N$

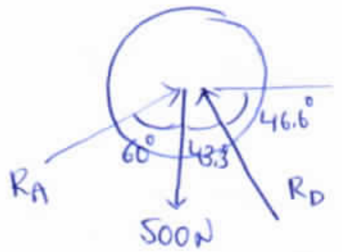
$\theta = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right) = 59.86^\circ$

$\Sigma M_P = R_y \times x$

$(1500\sin 60 \times 4) + (1000\sin 45 \times 8) + (500\sin 30 \times 12) = 3256$

$x = 4.25 m$

f.B.D of ①



$\Sigma f_x = 0;$

$R_A\cos(30) - R_D\cos(46.6) + 500\cos(90) = 0$

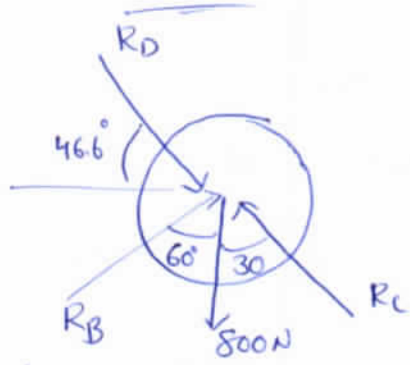
$\Sigma f_y = 0;$

$R_A\sin(30) + R_D\sin(46.6) - 500\sin(90) = 0$

$R_A = 353.15\,N, \quad R_D = 445.13\,N$

f.B.D of ②

④



$\Sigma f_x = 0;$

$R_D\cos(46.6) + R_B\cos(30) - R_C\cos(60) = 0$

$305.84 + R_B\cos(30) - R_C\cos(60) = 0 \quad \to ③$

$\Sigma f_y = 0;$

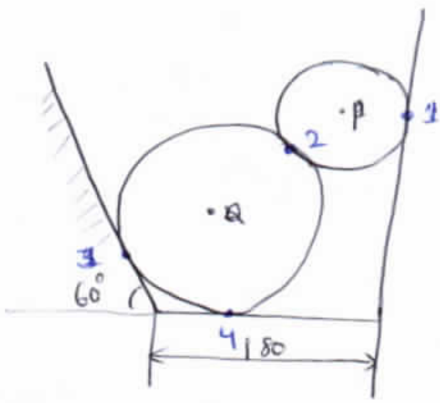
$R_D\sin(46.6) + R_B\sin(30) + R_C\sin(60) = 800$

$R_B\sin(30) + R_C\sin(60) = 1123.42 \quad \to ③$

$R_B = 297.84\,N$

$R_C = 1125.83\,N$

→



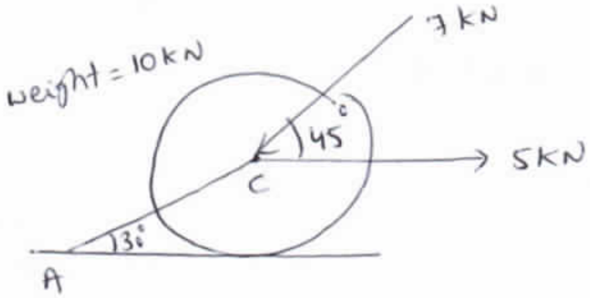
P → Dia = 100mm, weight = 200N

Q → Dia = 180mm, weight = 500N

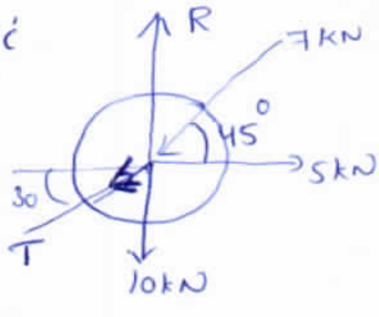
Sol

Answers :- $R_1 = 134.2\,N$, $R_2 = 240.8\,N$, $R_3 = 154.9\,N$, $R_4 = 622.5\,N$

→



f.B.D 'C'



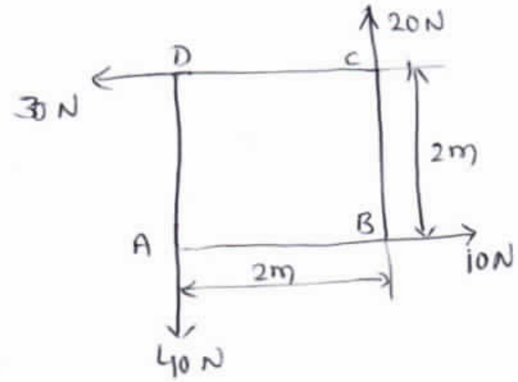
$\Sigma f_x = 0;$ $5 - 7\cos(45) - T\cos(30) = 0$

$T = 0.058\,kN$

$\Sigma f_y = 0;$ $-7\sin(45) + R - T\sin(30) - 10 = 0$

$R = 14.97\,kN$

→ Calculate magnitude, direction & position of resultant $\hat{}$'s shown in fig (5)



Sol

$\Sigma f_x = -30 + 10 = -20N$ (←) $R = 2.82N$

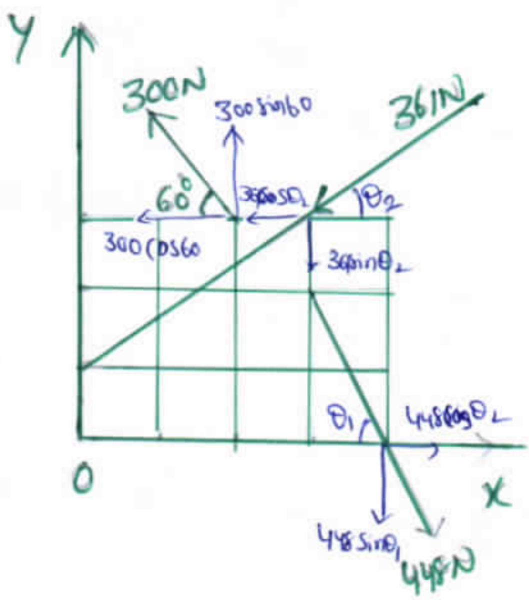
$\Sigma f_y = -40 + 20 = -20N$ (↓) $\theta = 45^0$

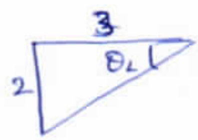
$\Sigma m_A = R \times x$

$(40 \times 0) - (30 \times 2) + (10 \times 0) - (20 \times 2) = 2.82 \times x$

$x = 35.46\, m$

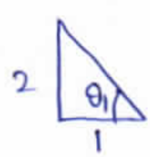
→ A flat plate is sub to the coplanar System of forces shown in fig. The inscribed grid with each Square having a length of 1m. Determine the Resultant & its x & y intercepts.





$\tan\theta_2 = 2/3$

$\theta_2 = 33.7^0$



$\tan\theta_1 = 2/1$

$\theta_1 = 63.4^0$

$\Sigma f_x = -300\cos 60 - 361\cos(33.7) + 448\cos(63.4) = -249.7\,N$ (←)

$\Sigma f_y = 300\sin 60 + 361\sin(33.7) - 448\sin(63.4) = -341.1N$ (↓)

$R = 422.73\,N, \quad \theta = 53.8^0$

$$\Sigma m_0 = (-300\sin(60) \times 2) + (361 \sin 33.7 \times 3) + (448 \sin 63.4 \times 4)$$
$$- (300\cos 60 \times 3) - (361 \cos(33.7) \times 3)$$
$$= 332.5 \text{ N-m}$$

x - Intercept:-

$$\Sigma m_0 = R_y \times x$$

$$332.5 = 341.1 \times x \Rightarrow x = 0.975 \text{ m}$$

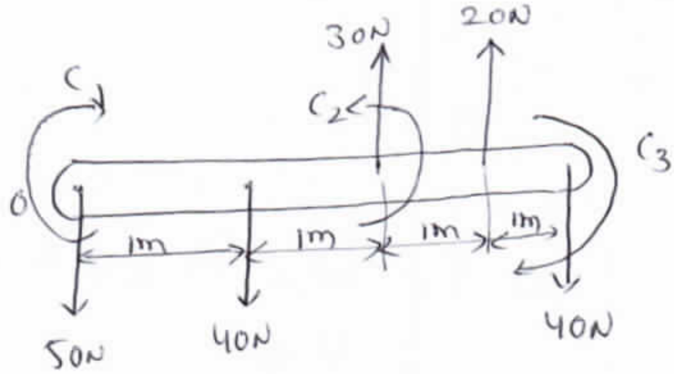
y - Intercept:-

$$\Sigma m_0 = R_x \times y$$

$$332.5 = 249.7 \times y \Rightarrow y = 1.33 \text{ m}$$

→ Replace the force system acting on a bar as shown in fig. by a Single force

$C_1 = 85$ N-m
$C_2 = 65$ N-m
$C_3 = 90$ N-m



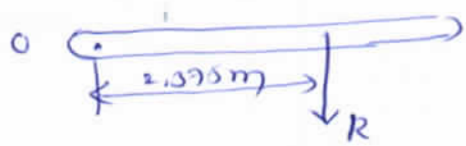
Sol

$$R = -50 - 40 + 30 + 20 - 40 = -80\text{N} (\downarrow)$$

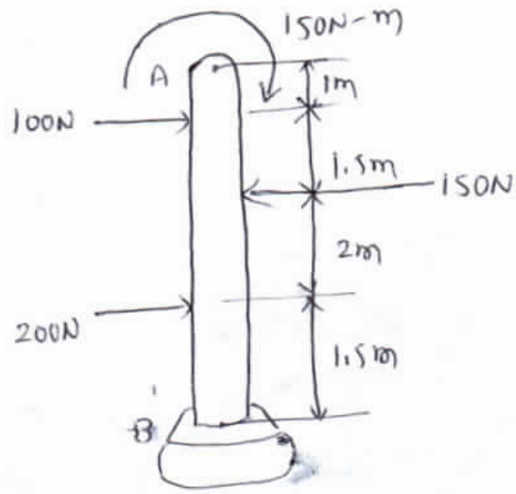
$$\Sigma m_0 = +(40 \times 1) - (30 \times 2) - (20 \times 3) + (40 \times 4) + 85 - 65 + 90$$
$$= +190 \text{ N-m} (\circlearrowright)$$

$$\Sigma m_0 = R \times d$$

$$190 = 80 \times d \Rightarrow d = 2.375 \text{ m}$$



→ find the resultant of given active forces w.r.t point B



So) $R = 100 + 200 - 150 = 150\,N\ (\rightarrow)$

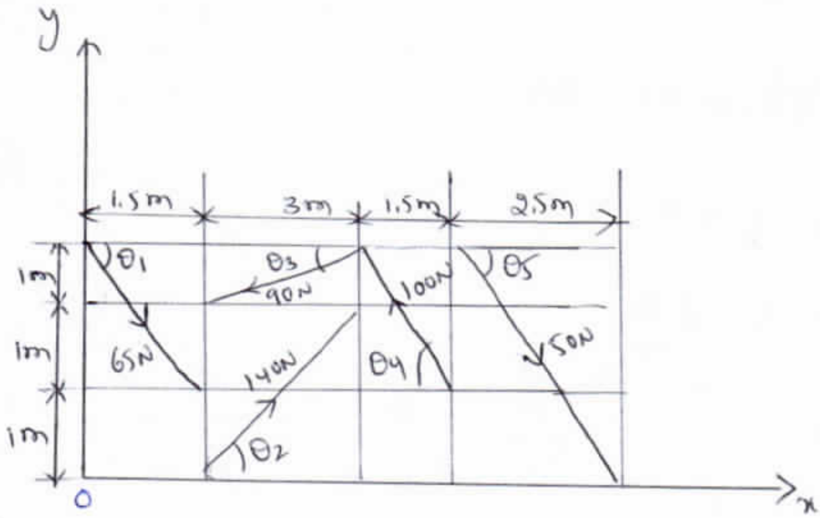
$\Sigma m_B = (+150) + (100 \times 5) - (150 \times 3.5) + (200 \times 1.5) = 425\,N\text{-}m\ (\circlearrowleft)$

$\Sigma m_B = R \times h$

$425 = 150 \times h$

$h = \underline{\underline{2.83m}}$

→ Replace the force system as shown in fig by a single force



$\tan\theta_1 = 2/1.5 = 53.13^\circ$, $\tan\theta_2 = 2/3 \Rightarrow 33.69^\circ$, $\tan\theta_3 = 1/3 \Rightarrow 18.44^\circ$

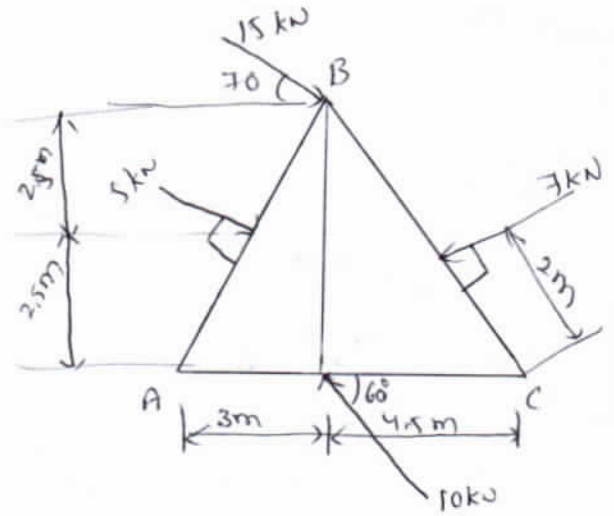
$\tan\theta_4 = 2/1.5 \Rightarrow 53.13^\circ$, $\tan\theta_5 = 3/2.5 \Rightarrow 50.19^\circ$

$\Sigma f_x = 42.12N$, $\Sigma f_y = 38.78N$, $R = 57.25N$, $\theta = 42.64^\circ$

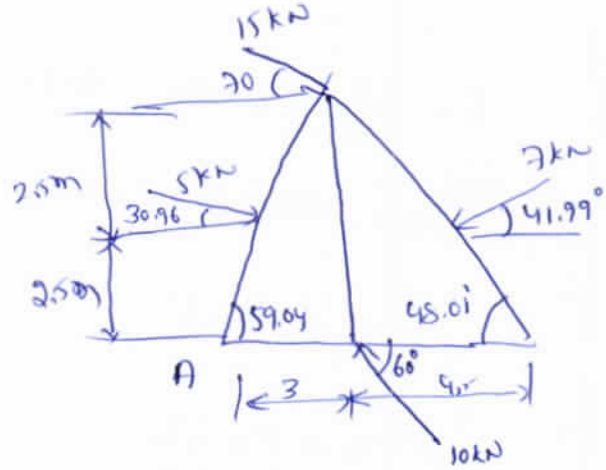
$\Sigma m_o = -341.03\ (\circlearrowright)$, $\Sigma m_o = R_y \times x$

$x = 8.79m$

→ find the Resultant & position w.r.t 'A'



Sol



$\Sigma f_x = 0.78 \text{ kN } (\leftrightarrow)$

$\Sigma f_y = -12.69 \text{ kN } (\downarrow)$

$R = 12.71 \text{ kN}$

$\theta = 86.48^\circ$

$$\Sigma M_A = (5\cos 30.96 \times 2.5) + (5\sin 30.96 \times 1.5) + (15\cos 70 \times 5) + (15\sin 70 \times 3) - (10\sin 60 \times 3) - (7\cos 41.99 \times 2\sin 48.01') + (7\sin 41.99) \times (7.5 - 2\cos 48.01')$$

$= 77.66 \text{ kN-m}$

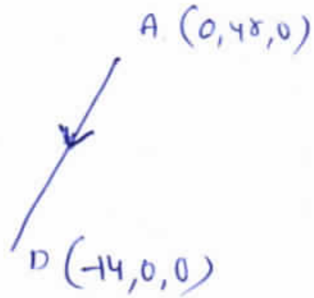
$\Sigma M_A = f_y \times x$

$x = \underline{6.12 \text{ m}}$

$\overline{T_{AD}} = (T_{AB})\ (\overline{e_{AD}})$

$= T_{AD}\left(\dfrac{-14i - 48j + 0k}{\sqrt{14^2+48^2}}\right)$

$= T_{AD}(-0.28i - 0.96j + 0k)$



Since the resultant is vertical (ie) acting along y-axis.

$\Sigma f_z = 0\ ;\ \Sigma f_x = 0$

$\Sigma f_z = 0$; k terms

$-8.56k + 0.231\,T_{AB} + 0 = 0 \Rightarrow T_{AB} = 37.09$ kN

$\Sigma f_x = 0$; i terms

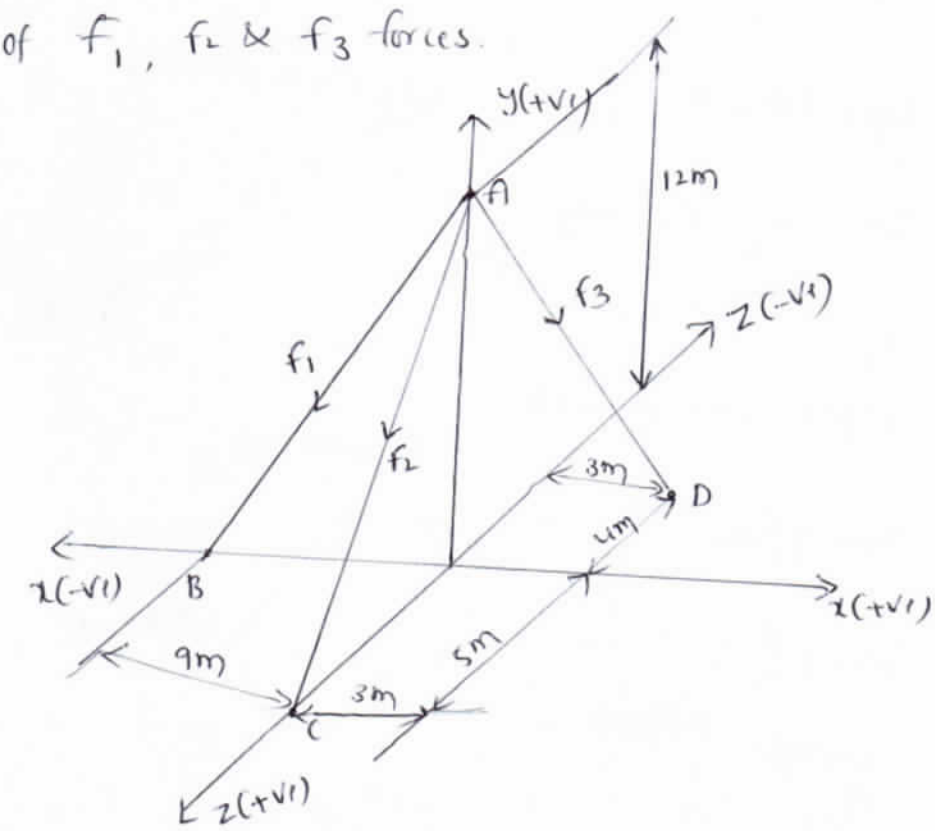
$5.712 + 0.308\,T_{AB} - 0.28\,T_{AD} = 0 \Rightarrow T_{AD} = 61.12$ kN.

$R = \Sigma f_y$; j terms

$R = -17.136 - 0.923(T_{AB}) - 0.96\,T_{AD}$

$= -110.05$ kN

→ The resultant of three forces at A $\overline{R} = (-758j)$N find the magnitude of f_1, f_2 & f_3 forces.

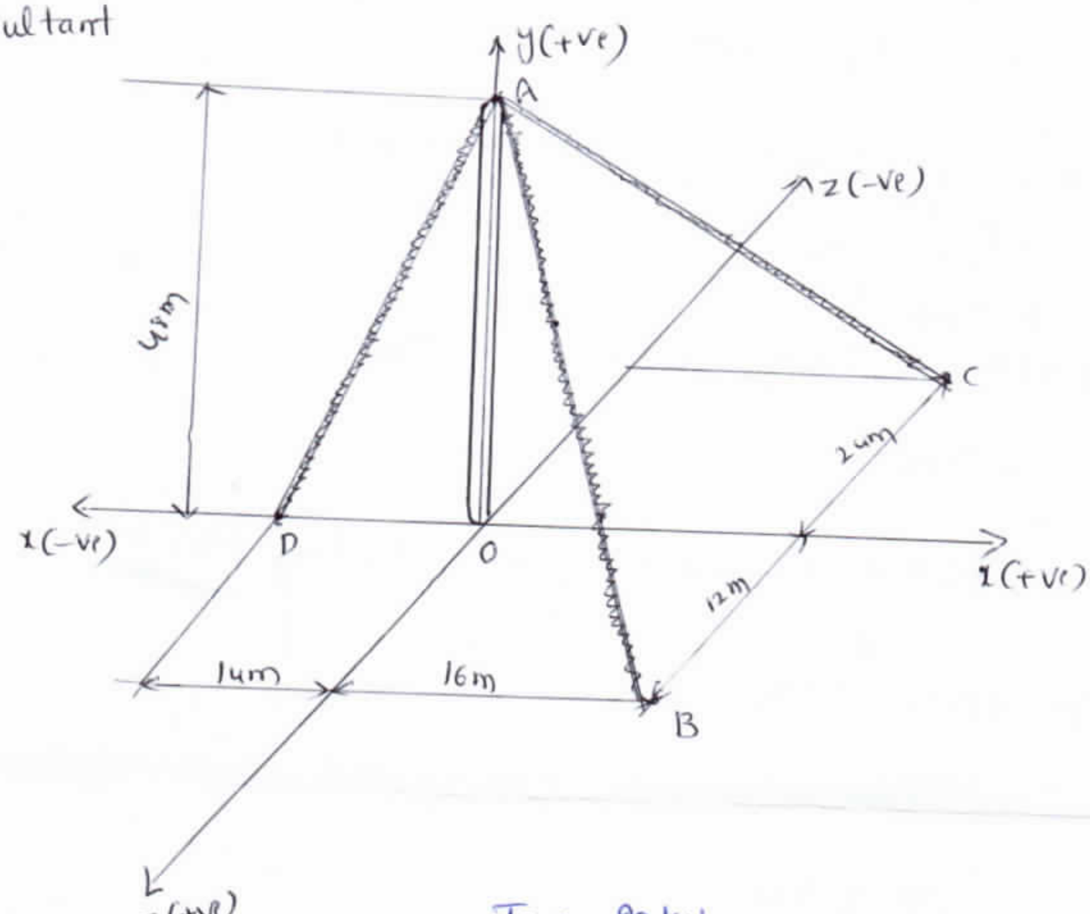


O(0,0,0), A(0,12,0)
B(-9,0,0), C(0,0,5), D(3,0,-4)

$f_1 = 154$ N
$f_2 = 320$ N
$f_3 = 400$ N

Forces in Space:-

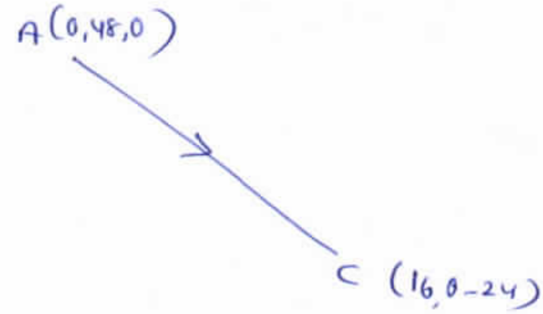
→ The tension in AC is $T_{AC} = 20\,kN$, determine T_{AB} & T_{AD} so that the resultant of three forces applied at A is vertical & calculate resultant



$T_{AC} = 20\,kN$.

Co-ordinates O(0,0,0), A(0,48,0), B(16,0,12), C(16,0,-24), D(-14,0,0)

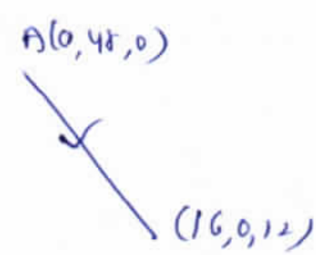
$\overline{T_{AC}} = (T_{AC})(\bar{e}_{AC})$



$= 20\left(\frac{16i - 48j - 24k}{\sqrt{16^2 + 48^2 + 24^2}}\right)$

$= 5.712i - 17.13j - 8.56k$

$\overline{T_{AB}} = T_{AB}(\bar{e}_{AB})$



$= T_{AB}\left(\frac{16i - 48j + 12k}{\sqrt{16^2 + 48^2 + 12^2}}\right)$

$= T_{AB}(0.308i - 0.923j + 0.231k)$