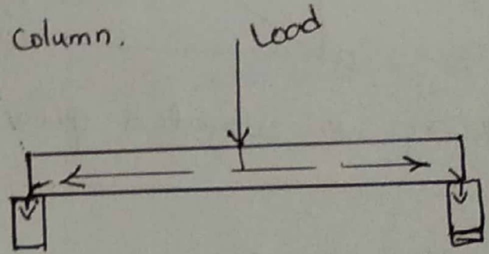
UNIT-III BENDING OF BEAMS

Syllabus: **Flexural Stresses**: Theory of Simple bending, Assumptions, Derivation of bending equation, Neutral axis, Determination of bending stresses, Section modulus of rectangular and Circular sections (solid and Hollow), I, T, Angle and Channel sections. Design of Simple beam sections.

Shear stresses: Derivation of Shear stress formula, Shear stress distribution across various beam sections like rectangular, circular, I, T, angle and channel sections.

Outcome: Analyze and design a structural member subjected to shear force and bending moment.

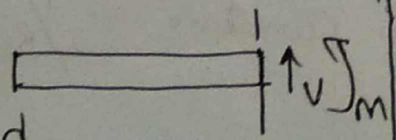
Introduction: Beam is a structural member which are used to support transverse load (or) vertical load. The function of the beam is to transfer the vertical load in to horizontal load and finally transfer to the supports (or) column.



When a beam is loaded what happens?

When a beam is loaded what happens? It is bent and subject to bending moments. Consequently longitudinal (or) bending stresses are induced in cross-section.

When a beam is loaded, the beam bents., If you cut the section at any point, there will be a shear force and bending moment.



Because of bending Moment bending stresses are developed
Because of shear force, shear stresses developed.

$$\sigma_b = \frac{M}{I} \cdot y$$

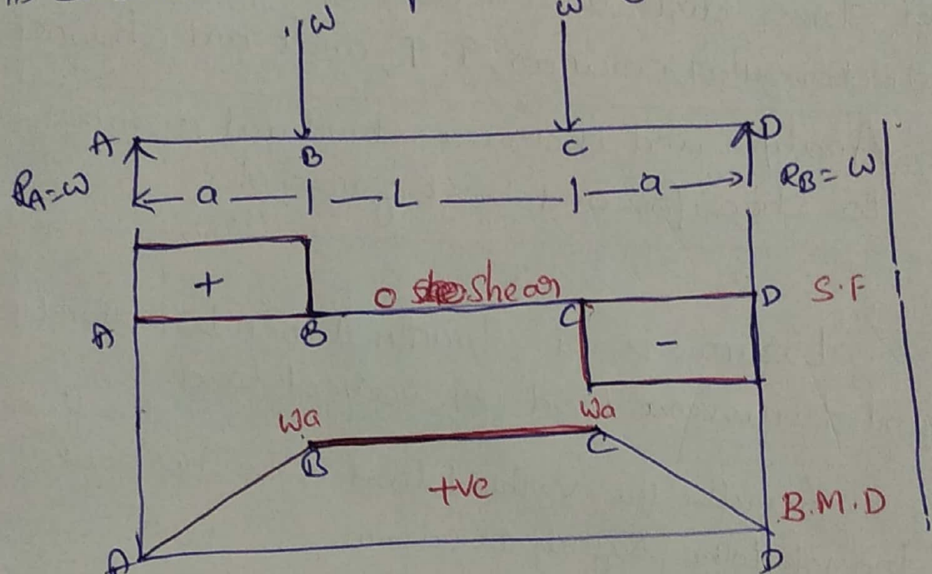
$$q = \frac{FQ}{Ib}$$

"Bending Stresses are the Internal resistance to external force which cause bending of Member". It is denoted by σ_b

Units will be N/mm^2

pure bending : when a beam is Subjected to such a system of bending ~~moments~~ loads so that the shear force in the beam is zero then the beam is said to be subjected to pure bending. In such cases the bending moment shall be constant in the beam.

"Beam Under Constant bending Moment with no shear force in it, is said to be Under pure bending / Simple bending."



B → C is Subjected pure bending (or) Simple bending.

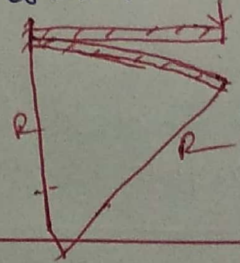
$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad \text{and} \quad F = \frac{dM}{dx}$$

no shear mean $F = 0$

$\frac{dM}{dx} = 0$ means M is Constant

If M is Constant then $\frac{d^2y}{dx^2}$ = Constant.

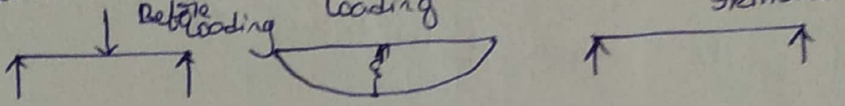
$\frac{d^2y}{dx^2}$ is Curvature = $1/R$ is Constant then R is Constant.



Assumptions in theory of Simple bending:

1. Material is homogeneous and Isotropic
   homogeneous → Same material throughout same Composition.
   Isotropic → Same properties in all directions. young modulus is same in all directions.
   Ex certain type of plastics, ceramics, glass, metals, alloys

2. Beam is Straight before loading and remains Straight even after load is removed.

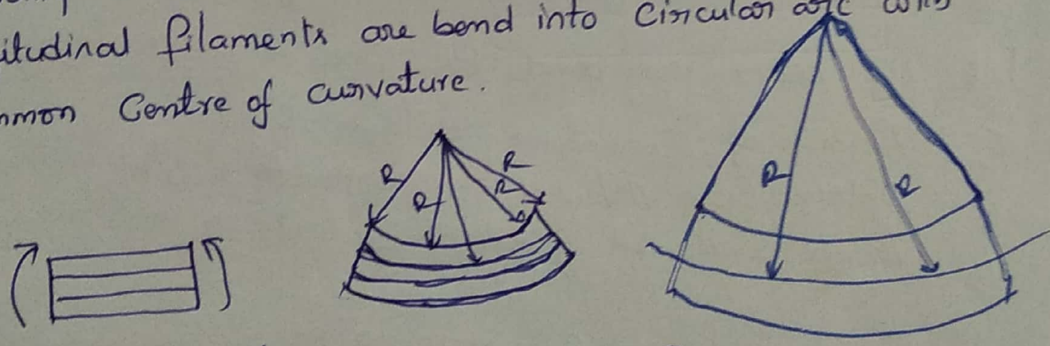


3. Beam is stressed with in the elastic limit, it means follows hook's law. $\sigma \propto \epsilon$

4. Beam is Subjected to pure bending (no shear)
   Shear stress are not to be Considered.
   stress is purely longitudinal tensile / compressive stress.

5. The layers at neutral axis does not take bending action.

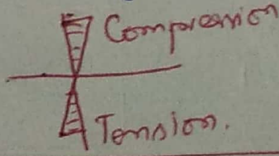
6. Initially the beam is straight and after bending all longitudinal filaments are bend into circular arc with Common Centre of curvature.



7. Radius of curvature $R >>> B$ (or) D.

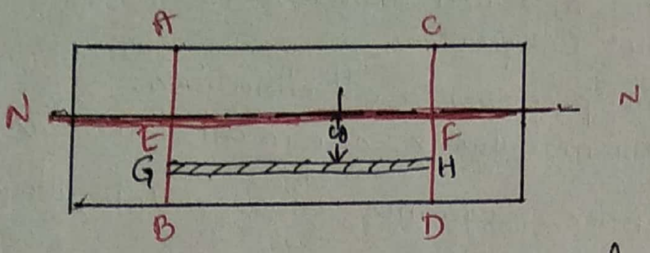
8. Beam must not twist while loading.

9. Bending stress is $\propto$ Distance to neutral axis $\sigma_b \propto y$



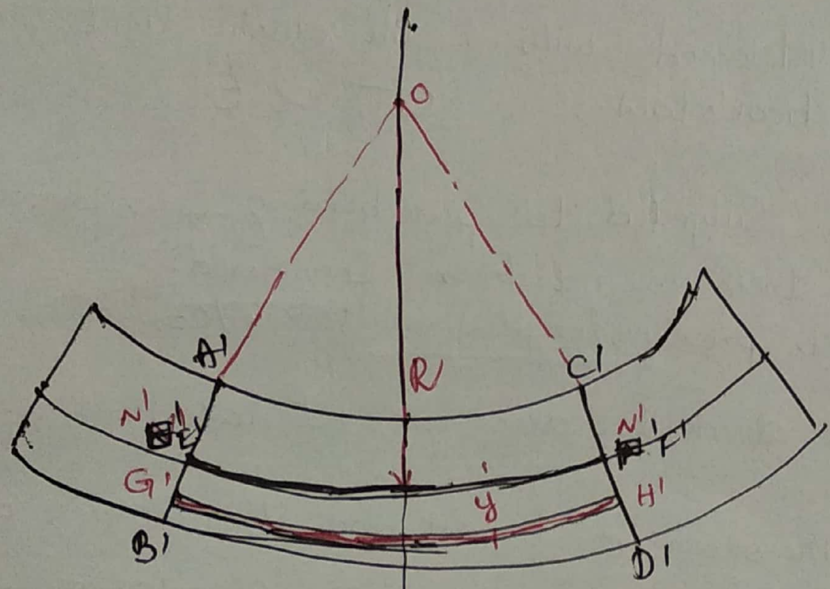
Derivation of Bending equation

Consider the beam section before bending as shown below.



Consider another layer at a distance 'y' from the neutral axis

Considering the Section of the beam after bending.



Let the beam may bend as show in above fig.

R : Radius of Curvature of layer $E'F'$

y : Distance from Neutral axis up to Layer $G'H'$

θ : Angle Subtended by arc at the Centre

$\therefore$ Strain in layer GH due to bending.

$$e = \frac{\text{Change in length}}{\text{Original length}} = \frac{G'H' - GH}{GH}$$

$$e = \frac{G'H' - EF}{EF} \quad \text{since } GH = EF \qquad (1)$$

Now, Length of arc $E'F'$ (or) EF is given by.

$$EF = R \times \theta = E'F'$$

Similarly length of arc $G'H'$ is given by.

$$G'H' = (R+y)\theta = R\theta + y\theta$$

Change in length $\delta l = G'H' - EF$

$$= \cancel{R\theta} + y\theta - \cancel{R\theta}$$

$$= y\theta$$

put all values in equation (1)

$$e = \frac{y\theta}{R\theta} = \frac{y}{R}$$

From hook's law $\sigma \propto e$

$$\sigma = Ee \qquad \Big| \qquad e = \frac{\sigma}{E}$$

$$\frac{\sigma}{E} = e = \frac{y}{R} \quad \therefore \quad \boxed{\frac{\sigma}{y} = \frac{E}{R}} \text{ — (ii)}$$

From equation no. 2 bending stress acting on the layer is given by

$$\sigma = \frac{Ey}{R} \text{ — (iii)} \qquad \sigma_b = \frac{E}{R} \cdot y$$

From equation No. 3. We see that

$\frac{E}{R}$ is Constant $\therefore$ $\sigma_b \propto y$

$\therefore$ Force acting the elemental strip because of bending.

$$dF = \sigma_b \times \text{Area} \qquad \left[\sigma = \frac{F}{A} \right]$$

$$dF = \frac{E}{R} \cdot y \cdot \times dA$$

Taking of Moment of Force about neutral axis.

$$dM = dF \times y$$

$$dM = \frac{E}{R} y \times y \cdot dA$$

$$dM = \frac{E}{R} y^2 dA$$

Integrating.

$$\int dM = \int \frac{E}{R} \cdot y^2 dA$$

$$M = \frac{E}{R} \cdot I$$

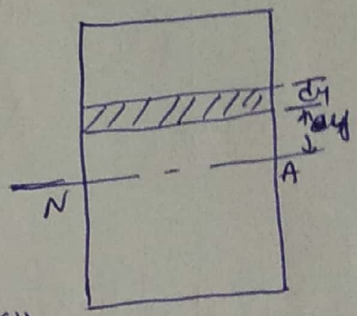
$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}}$$

* I = Second Moment of area

Neutral axis :

Neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section. It is written as N.A.

If a section of a beam is subjected to pure sagging moment, then the stresses will be compressive at any point above the neutral axis and tensile below the neutral axis. There is no stresses at the neutral axis.



$\therefore$ The stress at a distance y from neutral axis is given by $\sigma = \frac{E}{R} \times y$

Consider a small layer at a distance 'y' from the neutral axis

Let dA = Area of the layer

Force on the layer = Stress on layer × Area of layer

$= \sigma \times dA$

$dF = \frac{E}{R} \times y \times dA$

Total force on the beam section $= \int \frac{E}{R} \times y \times dA$

$= \frac{E}{R} \int y \cdot dA$

But in pure bending there is no force acting at neutral axis.

$\frac{E}{R} \int y \, dA = 0 \Rightarrow \boxed{\int y \cdot dA = 0}$

$\int y \times dA$ represent the moment of entire area of the section about neutral axis. But we know that moment of any area about an axis passing through the centroid, is also equal to zero. Hence neutral axis coinsides with the centroidal axis.

XX The centroidal axis of a section gives the position of neutral axis XX

Sectional Modulus:- It is ratio of Moment of Inertia of a Section about the neutral axis to the distance of the outermost layer from the neutral axis.

$$Z = \frac{I}{y_{max.}}$$

I = Moment of Inertia about neutral axis.

$y_{max.}$ = Distance of the outermost layer from the neutral axis.

$$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow \sigma = \frac{M}{I} \times y$$

$$= \frac{M}{\left(\frac{I}{y}\right)} = \frac{M}{Z}$$

$$\Rightarrow M = \sigma_b \times Z$$

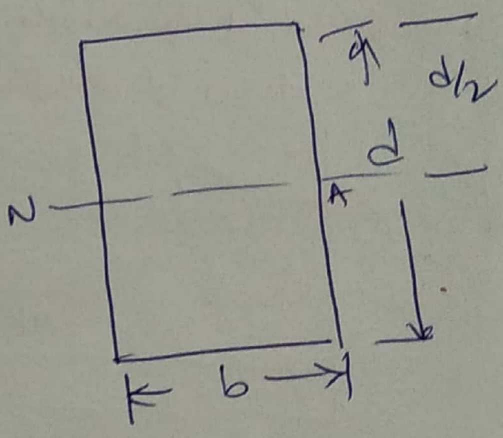
thus, Moment of resistance offered by the Section is maximum when Z is maximum and Hence Z represents the Strength of the Section.

1.. **Rectangular Section**:-

$$I = \frac{bd^3}{12}$$

$$y_{max} = d/2$$

$$Z = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^2}{6}$$

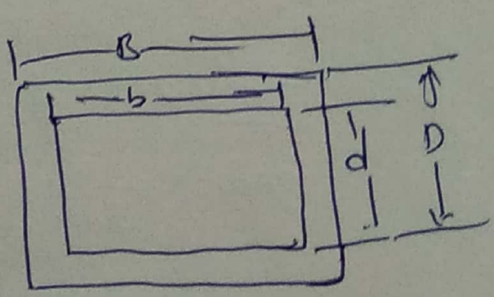


Hollow rectangular:-

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = D/2$$

$$Z = \frac{1}{6D} \cdot \left[BD^3 - bd^3\right]$$



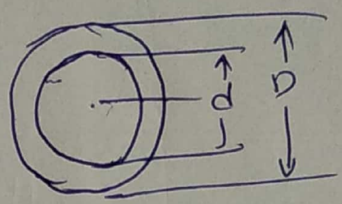
iii) Circular Section :

$$I = \frac{\pi}{64} d^4$$

$$y_{max} = d/2$$

$$Z = \frac{\pi}{32} d^3$$

iv) Hollow Circular Section



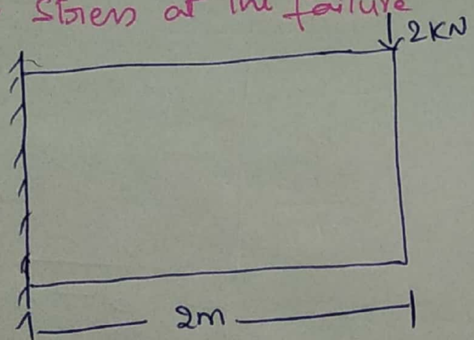
$$I = \frac{\pi}{64}(D^4 - d^4)$$

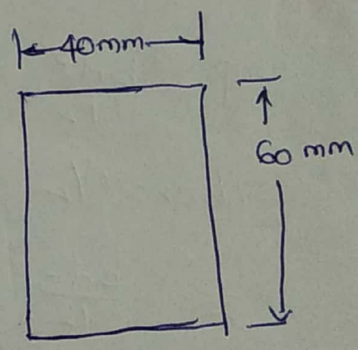
$$y_{max} = D/2$$

$$Z = \frac{\pi}{32D}(D^4 - d^4)$$

Q. A cantilever of length 2m fails when a load of 2 kN is applied at the free end. If the section of the beam is 40x60 mm², find the stress at the failure.

→





$$\sigma = \frac{E}{R} \cdot y$$

$$M = \sigma_{max} \times Z$$

$$= \sigma_{max} \times \frac{I}{y_{max}}$$

$$Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = \cancel{4 \times 10^{6}} \cancel{6 \times 10^{3}} = 24000 \text{ mm}^3$$

$$M = W \times L = 2000 \times 2 \times 10^3 = 4 \times 10^6 \text{ N-mm}$$

$$\sigma_{max} = \frac{4 \times 10^6}{24,000} = 166.67 \text{ N/mm}^2$$

Q. A 250 mm depth x 150 mm width rectangular beam is Subjected to Maximum Bending Moment of 750 kN-m. Determine

a) Maximum Stress in the beam

b) If the value of E for the beam material is 200 GN/m², find out the radius of curvature for that portion of the beam where the bending is maximum

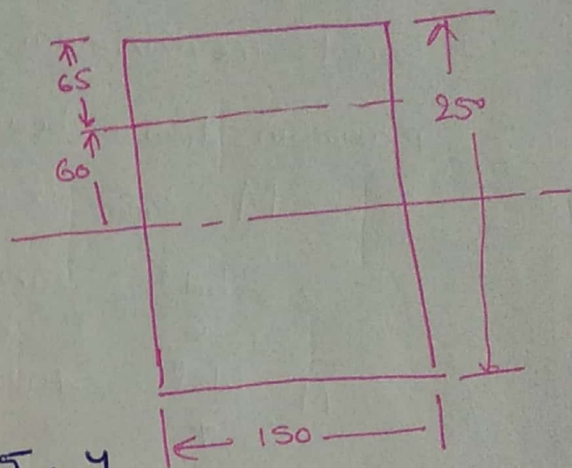
c) The value of the longitudinal Stress at a distance of 65 mm from the top Surface of the beam

→ $b = 150\,mm$

$d = 250\,mm$

$M = 750 \times 10^3 \times 10^3\ N\text{-}mm$

$E = 2 \times 10^5\ MPa\ (or)\ N/mm^2$



max. stress

$$\sigma = \frac{M}{I} \times y$$

$$I = \frac{bd^3}{12} = \frac{150 \times (250)^3}{12} = 1953 \times 10^5\ mm^4$$

$$y = d/2 = 125\ mm$$

$$\sigma = \frac{750 \times 10^6 \times 125}{1953 \times 10^5} = 480\ N/mm^2$$

b) Radius of curvature R.

$$R = \frac{EI}{M} \quad \left[\frac{M}{I} = \frac{E}{R}\right]$$

$$= \cancel{200 \times 10^9}\ \frac{2 \times 10^5 \times 1953 \times 10^5}{750 \times 10^6} = 52.08\ m$$

c) $$\sigma_1 = \frac{M y_1}{I} = \frac{750 \times 10^3 \times 60 \times 10^3 \times}{1953 \times 10^5}$$

$$= 230.4\ N/mm^2$$

Q. A Square beam 20×20 mm² in section and 2m long is Simply Supported at the ends. The beam fails when a point load of 400N is applied at the Centre of the beam. What Uniformly distributed load per meter length will break a Cantilever of the Same material 40mm × 60 mm deep and 3m long.

→ Square c/s = 20×20 mm , L = 2m W = 400N

Rectangular c/s = 40mm × 60mm L = 3m w = ?

Equate the maximum stress in the two Cases.

Maximum stress in beam of Square c/s

$$M = \sigma_{max} \times Z$$

$$Z = \frac{bd^2}{6} = \frac{20 \times 20^2}{6} = \frac{4000}{3} \text{ mm}^3$$

$$M = \frac{W \times l}{4} = \frac{400 \times 2000}{4} = 2 \times 10^5 \text{ N-mm}$$

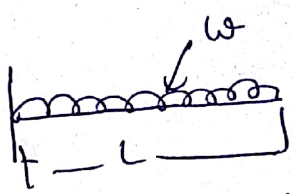
$$\sigma_{max} = \frac{2 \times 10^5 \times 3}{4000} = 150 \text{ N/mm}^2$$

maximum stress in beam of rectangular c/s

$$M = \sigma_{max} \times Z$$

$$Z = \frac{40 \times 60^2}{6} = 24,000 \text{ mm}^3$$

maximum bending moment for a Cantilever beam Loaded with UDL for the entire span is given by



$$M_{max} = \frac{wL^2}{2}$$

$$= \frac{w \times (3)^2}{2} = w(4.5) \text{ N-m}$$

$$\text{UDL} \cdot L/2 = \frac{wL^2}{2}$$

$$M_{max} = \cancel{\frac{w \times 3000 \times 3000}{2}} = \cancel{45 \times 10^5 \, w \text{ N-mm}}$$

$$= \cancel{45 \times 10^5 \, w \text{ N-mm}}$$

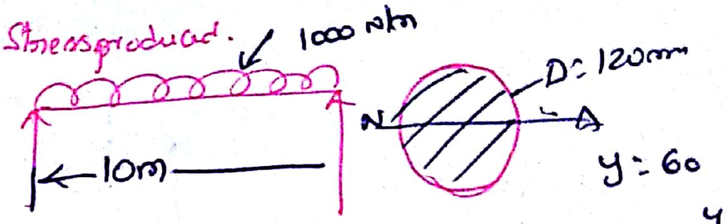
$$M = 4500 \times w \text{ N-mm}$$

$$M = \sigma \times Z$$

$$4500\,w = 150 \times 24,000$$

$$\boxed{w = 800 \text{ N/m}}$$

Q. A Circular beam of 120 mm diameter is Simply Supported over a Span of 10m and Carries a U.d.l. of 1000 N/m. find the maximum bending Stress produced.



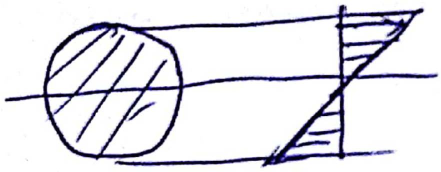
$$Z = \frac{\pi}{32} d^3 = \frac{\pi}{32} \times (120)^3 =$$

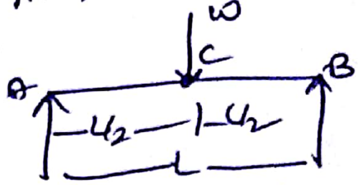
$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times (120)^4 = 10.18 \times 10^6 \ mm^4$$

$$M = \frac{1000 \times (10)^2}{8} = 12.5 \times 10^3 \ N\text{-}m \Rightarrow M = 12.5 \times 10^6 \ N\text{-}mm$$

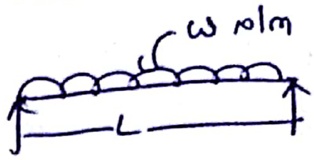
$$\sigma_b = \frac{12.5 \times 10^6}{10.18 \times 10^6} \times 60$$

$$\sigma_b = 73.68 \ N/mm^2$$

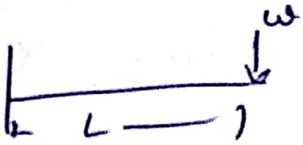


i) 

$M_{max} = \frac{WL}{4}$ at point

ii) 

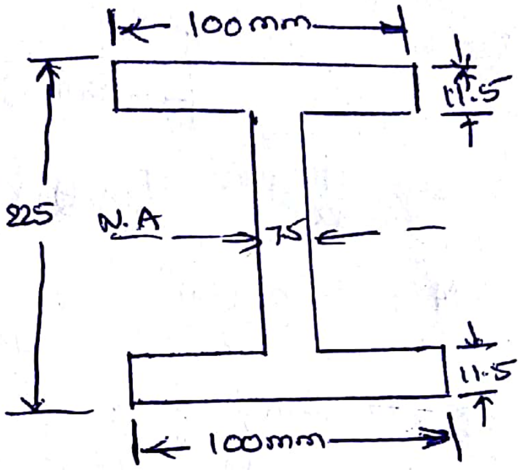
$M_{max} = \frac{wL^2}{8}$ at point

iii) 

$M_{max} = WL$ at point

iv)

Q. An I - Section as shown in fig. is Simply Supported over a span of 12m. If the maximum permissible bending stress is 80 N/mm² what Concentrated Load can be carried at a distance of 4m from one support ?



→ $L = 12m$

$\sigma = 80\ N/mm^2$

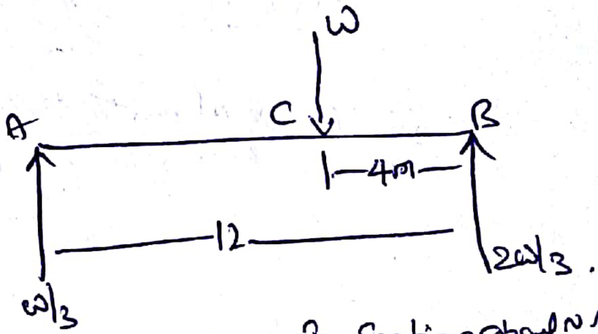
$W = ?$

$R_A + R_B = W$

$R_B \times 12 - W \times 8 = 0$

$R_B = \frac{8W}{12} = \frac{2W}{3}$

$R_A = W/3$



B.M at point C = $R_A \times W$

$= \frac{W}{3} \times 8 = \frac{8W}{3}$ N-m

$= \frac{8W}{3} \times 1000$ N-mm

Now find the moment of Inertia of the given I-Section about N.A

$$I = \frac{100 \times 225^3}{12} - \frac{(100 - 7.5) \times (225 - 2 \times 11.5)^3}{12}$$

$$= 94921875 - \frac{92.5 \times (202)^3}{12}$$

$$= 31386647.45\ mm^4$$

Now using relation $\frac{M}{I} = \frac{\sigma}{y}$

$$\frac{\frac{8000W}{3}}{31386647.45} = \frac{80}{112.5}$$

$$\boxed{W = 8369.77\ N}$$

Q. A beam is Simply Supported and carries a U.D.L of 40kN/m run over the whole span. The Section of the beam is rectangular having depth as 500 mm. If the max. stress in the material of the beam is 120 N/mm² and Moment of Inertia of the section is 7×10^{8} mm⁴, find the Span of the beam.

→ $w = 40\,kN/m = 40{,}000\,N/m.$

$d = 500\,mm$ | $y = \frac{d}{2} = 250\,mm$

$\sigma_b = 120\,N/mm^2$

$I = 7\times10^{8}$

$$Z = \frac{I}{y} = \frac{7\times10^{8}}{250} = 28\times10^{5}\,mm^3$$

The maximum B.M for a Simply Supported beam, carrying a U.D.L over the whole span is at Centre of the beam is equal to

$$BM = \frac{wl^2}{8} = \frac{40{,}000\,l^2}{8} = 5000\,l^2\,N\text{-}m$$
$$= 5\times10^{6}\,l^2\,N\text{-}mm$$

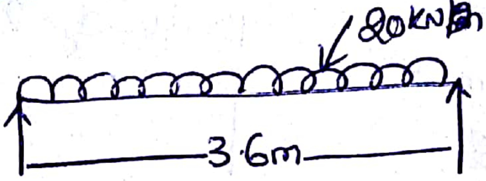
$$M = \sigma_{max}\cdot Z$$

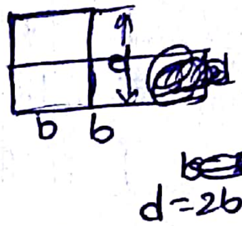
$$5\times10^{6}\,l^2 = 120\times28\times10^{5}$$

$$l^2 = 2.4\times28$$

$$\boxed{L = 8.19m \simeq 8.20m}$$

Q. A timber beam of rectangular section is to support a load of 20 kN uniformly distributed over a span of 3.6 m when beam is simply supported. If the depth of section is to be twice the breadth, and the stress in the timber is not to exceed 7 N/mm² find the dimensions of the cross section. How would you modify the c/s of the beam, if it carries a concentrated load of 20 kN placed at the centre with the same ratio of breadth to depth.





$W = 20\ kN = 20 \times 1000\ N$

$L = 3.6\ m$

$\sigma_{max} = 7\ N/mm^2$

$d = 2b$

Sectional modulus of rectangular beam $= \dfrac{bd^2}{6}$

$$Z = \frac{b \times (2b)^2}{6} = \frac{b \times 4b^2}{6} = \frac{2b^3}{3}$$

maximum B.M. $= \dfrac{wL^2}{8}$ (or) $\dfrac{w.L.L}{8} = \dfrac{W.L}{8}$

$$M = \frac{WL}{8} = \frac{20{,}000 \times 3.6}{8} = 9000\ N\text{-}m = 9 \times 10^6\ N\text{-}mm$$

$$M = \sigma_{max} \cdot Z$$

$$9 \times 10^6 = 7 \times \frac{2b^3}{3} \Rightarrow b^3 = \frac{3 \times 9 \times 10^6}{14} = 1.92857 \times 10^6$$

$\boxed{b = 124.47}$ mm. $d = 250$ mm

ii) point load at centre

$$\text{max. B.M} = \frac{WL}{4} = \frac{20{,}000 \times 3.6}{4} = 18{,}000\ N\text{-}m$$

$$= 18 \times 10^6\ N\text{-}mm$$

$\sigma_{max} = 7\ N/mm^2$

$Z = \dfrac{2b^3}{3}$

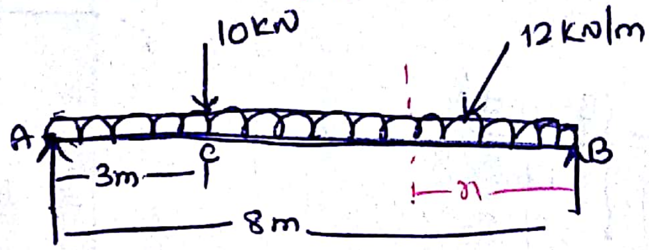
$\therefore$ $18 \times 10^6 = 7 \times \dfrac{2b^3}{3}$

$b^3 = 3.85714 \times 10^6$

$b = 156.82$ mm $\qquad d = 313.64$ mm.

Q. A timber beam of rectangular Section of Length 8m is Simply Supported. The beam carries a U.D.L of 12 kN/m run over the entire length and a point load of 10 kN at 3m from the left Support. If the depth is two times the width and the Stress in the timber is not to exceed 8 N/mm², find the Suitable dimensions of the Section

→



$w = 12$ kN/m
point load $= 10$ kN
$L = 8$m
UDL $= 12$ kN/m
$d = 2b$
$\sigma = 8$ N/mm²

$R_A + R_B = 12 \times 8 + 10 = 106$ kN

$R_B \times 8 - 12 \times 8 \times 4 - 10 \times 3 = 0$

$R_B \times 8 = 384 + 30 = 414 \Rightarrow R_B = 51.75$ kN

$R_A = 54.25$ kN

S.F at A: $R_A = 54.25$ kN

at LHS of C $= 54.25 - 12 \times 3 = 18.25$ kN

RHS of C $= 8.25$ kN

S.F at B $= -R_B = -51.75$ kN

It means shear force changes their sign between C & B

$\therefore$ shear force is zero between C&B.

From B

We Consider a Section at distance of x.

$12x - R_B = 0$

$x = R_B/12 = 4.3125$m

Max. B.M occurs at 4.3125 m from B.

Maximum B.M $= R_B \times 4.3125 - 12000 \times 4.3125 \times \frac{4.3125}{2}$

$= 111585.9375 \times 1000$ N-mm

$Z = \frac{bd^2}{6} = \frac{2b^3}{3}$ $\left(b = \frac{d}{2}\right)$

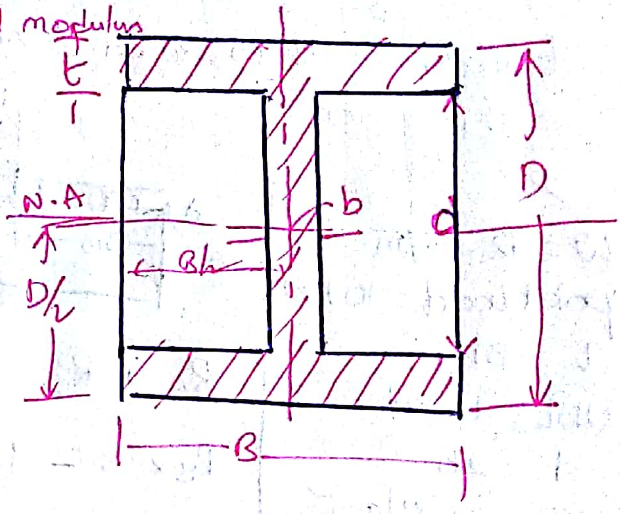
$M = \sigma_{max} \cdot Z$

$111585.93 \times 1000 = 8 \times \frac{2b^3}{3} \Rightarrow b^3 = 20.92 \times 10^6$

$b = 275.5$ mm

$d = 551$ mm.

Q. Fig. shows a Symmetrical I Section most Commonly Used as a Structural member. There are two flanges on the top and bottom of the dimensions B×t and one web in the Centre of dimensions b×d Since the Section is Symmetrical, it is C.G is located at the C.G of the web. calculate the Sectional modulus



→

$$I_{xx} = \frac{BD^3}{12} - \frac{(B-b).d^3}{12}$$

$y = D/2$

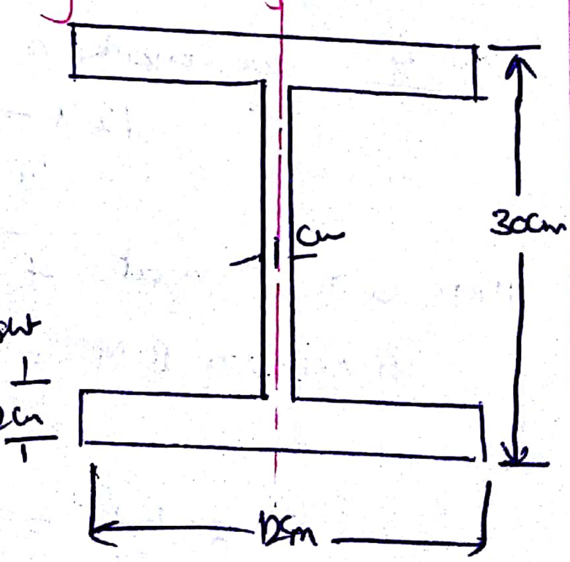
Sectional modulus

$$Z = \frac{I_{xx}}{D/2}$$

$$\boxed{Z = \frac{BD^3 - (B-b)d^3}{6D}}$$

Q. A beam of I Section 30cm×12cm, has flanges 2cm Thick and web 1cm Thick. Compare the its flexural strength asthmatically with that of a beam of rectangular section of the same weight the depth being twice the width. what will be the maximum stress developed in I section for a bending Moment of 30kN-m.

→



I Section

Area: $12\times2 + (30-4)\times1 + 12\times2$
$= 74\ cm^2$

Rectangular beam is of the Same Weight and Same material.

So Area of c/s of rectangle Section

$B\times D = 74\ cm^2$

$B\times 2B = 74$

$2B^2 = 74$ $\boxed{B = 6.08cm}$ $D = 12.16cm$

I Section

$B = 12cm$

$b = 1cm$

$D = 30cm$

$d = 26cm$

$$Z_i = \frac{12 \times 30^3 - (12-1)\,26^3}{6 \times 30}$$

$$= 725.9 \times 10^3 \; mm^3$$

Rectangular Section $\quad Z_R = \frac{BD^2}{6} = \frac{6.08 \times (12.16)^2}{6}$

$$= 149.84 \; cm^3$$

$$= 149.84 \times 10^3 \; mm^3$$

The flexural Strength of a beam is directly proportional to its Sectional modulus.

$$\frac{Z_i}{Z_R} = \frac{725.9}{149.84} = 4.84.$$

B.M on I Section $\quad M = 30 \, kN\text{-}m \quad = 30 \times 10^6 \; N\text{-}mm$

$$30 \times 10^6 = f_{max} \times 725.9 \times 10^3$$

$$f_{max} = \frac{30 \times 10^6}{725.9 \times 10^3} = 41.32 \; N/mm^2$$

H.W A beam is of I Section 20x15 cm with thickness of the flanges 2.5 cm and thickness of the web 1.5cm. Compare the flexural strength with that of a beam of Circular section of the same weight and same material. What will be the maximum Stress developed in I Section for a bending Moment of 2.5 tonne-metres

[7.14, 403.024 kg/cm²]

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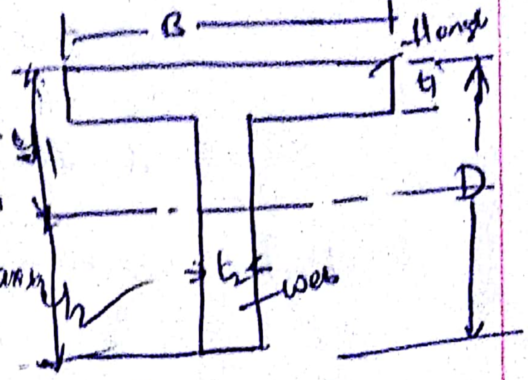
T- Section (Un Symmetrical about x-x axis)

Fig. shows a T- Section of breadth B and depth D. The thickness of the flange is t_1 and thickness of web is t_2



The Section is unsymmetrical about x-x axis passing through Centroid of the section

∴ To determine the distance of the C.G from lower edge.

area of flange $= a_1 = B \times t_1$

$y' =$ distance of C.G of the flange from lower edge

$= D - \frac{t_1}{2}$

Area of web $a_2 = t_2 \times (D - t_1)$

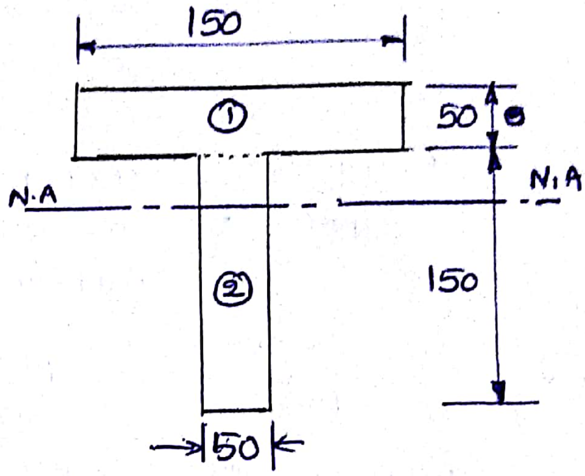
$y_2' = \left(\frac{D - t_1}{2}\right)$

C.G $\quad y_2 = \frac{a_1 y_1' + a_2 y_2'}{a_1 + a_2} \qquad y_1 = D - y_2$

$$I_{xx} = \frac{B t_1^3}{12} + B t_1 \left(y_1 - \frac{t_1}{2}\right)^2 + \frac{t_2 \times (D - t_1)^3}{12} + t_2 (D - t_1) \times (y_2 - y_2')^2$$

$$\boxed{Z_1 = \frac{I_{xx}}{y_1}, \quad Z_2 = \frac{I_{xx}}{y_2}}$$

Q. Two wooden planks 150 mm × 50 mm each are connected to form a T-Section of a beam as shown in fig. If a moment of 3.4 KN-m is applied around the horizontal neutral axis, inducing tension below the neutral axis. Find the stresses at the extreme fibres of cross-section. Also calculate the total tensile force on the cross section.



→

Moment = 3.4 KN-m
$= 3.4 \times 10^6$ N-mm

① step to find the centroidal axis (or) neutral axis.

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$A_1 = 150 \times 50 = 7500 mm^2$
$A_2 = 50 \times 150 = 7500 mm^2$
$y_1 = 175$ from bottom face
$y_2 = 75$

$$\bar{y} = \frac{7500 \times 175 + 7500 \times 75}{7500 + 7500} = 125 mm.$$

$$I_{xx} = \left[\frac{150 \times 50^3}{12} + 150 \times 50 \times (175 - 125)^2 + \frac{50 \times 150^3}{12} + 50 \times 150 \times (125 - 75)^2\right]$$

$= 53125 \times 10^4 mm^4$ $[2031.2 \times 10^4 + 3281.2 \times 10^4]$

Extreme fibre stress

Distance of C.G from the upper extreme fibre $y_c = 200 - 125 = 75$ mm
$y_t = 125$ mm

Now using Bending equation

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{(I/y)} = \frac{M}{I} \times y$$

$$\sigma_t = \frac{3.4 \times 10^6 \times 125}{5312.5 \times 10^4} = 8 \times 10^6 N/m^2 = 8 MPa$$

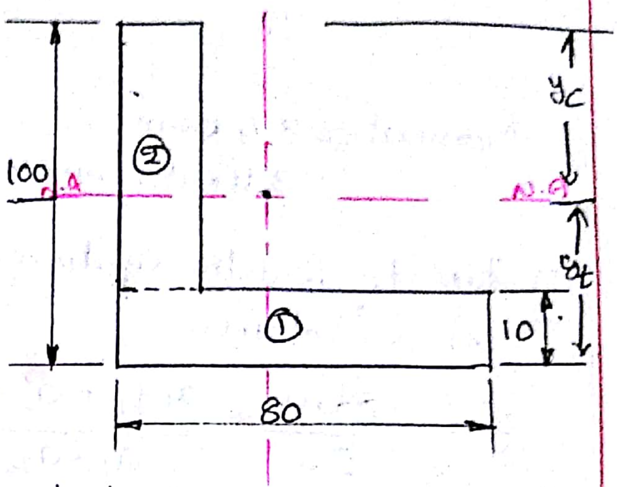
$$\sigma_c = \frac{3.4 \times 10^6 \times 75}{5312.5 \times 10^4} = 4.8 \times 10^6 N/m^2 = 4.8 MPa.$$

Tensile force = $\frac{8 \times 10^6}{2} \times 125 \times 50 \times 10^{-6}$
$= 25 kN$ ∵ Total tensile force = Avg tensile stress . Area.

Q. Find the position of C.G and calculate the moment of Inertia Ixx of an unequal angle of section 10cm×8cm×1cm. A beam of this angle section is used as a Cantilever of length 3 m Subjected to a turning Moment M at its free end. What is the maximum value of M of the stress in the section is not to exceed 70 MN/m².

→ $a_1 = 80 \times 10 = 800\ mm^2$

$a_2 = 90 \times 10 = 900\ mm^2$



$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = \frac{800\times40 + 900\times5}{800+900}$$

$= 21.47\ mm.$

$$\bar{y} = \frac{800\times5 + 900\times55}{1700} = 31.47$$

from bottom

$y_t = 31.47$ from bottom (subjected to tensile stress)

$y_c = 100 - 31.47 : 68.53\ mm.$ from the neutral axis.

Moment of Inertia $I_{xx} = \dfrac{80\times10^3}{12} + 80\times10\times(31.47-5)^2$

$+ \dfrac{10\times90^3}{12} + 900\times(55-31.47)^2$

$= 6666.66 + 560528.72$

$+ 607500 + 498294.8.$

$= \cancel{1339450}$

$= 167.\overset{299}{}\times 10^4\ mm^4$

stress $\sigma = 70\ MN/m^2$

$= 70\times10^6\ N/m^2 = 70\ N/mm^2$

maximum stress will occur at the top edge : $y_2 > y_1$

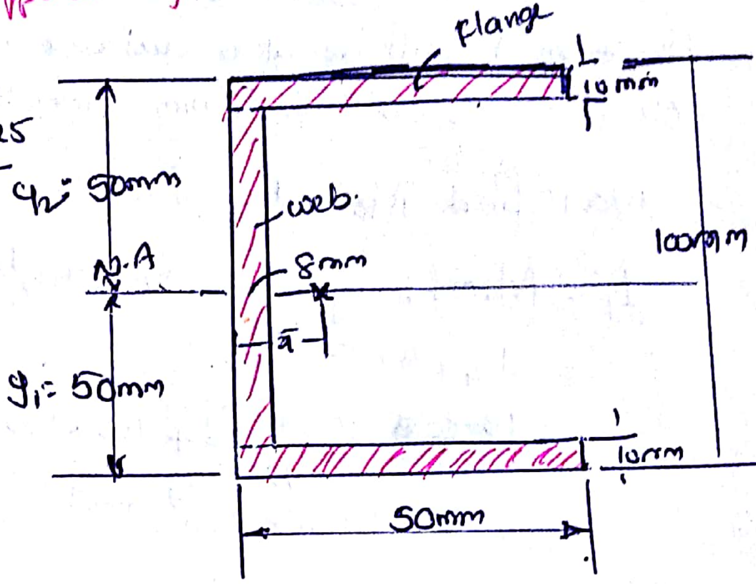
$$M = f_{max} \times \frac{I_{xx}}{y_2}$$

$$= \frac{70\times167.299\times10^4}{68.53} = 1708.88\times10^4\ N\text{-}mm$$

$= 17.08\ kN\text{-}m.$

Q. The thickness of flange and web of a channel section are 10mm and 8mm respectively, while its breadth and depth are 50mm and 100mm. Find the position of the C.G of the section and its I_{xx}, if a beam of this channel section is used, what maximum bending moment can be applied if the stress is not exceed 0.5 ~~tonne~~ tonne/cm².

→



$$\bar{x} = \frac{50\times10\times25 + 80\times8\times4 + 50\times10\times25}{50\times10 + 50\times10 + 80\times8}$$

$= 1680mm = 16.80mm$

$$I_{xx} = \frac{50\times100^3}{12} - \frac{42\times80^3}{12}$$

$= 237.467\times10^4\ mm^4$

$\sigma = 0.5\ tonn/cm^2$

$= 0.5\times1000\ \frac{kg}{cm^2}$

$= 0.5\times1000\times10^{-2}\ kg/mm^2$

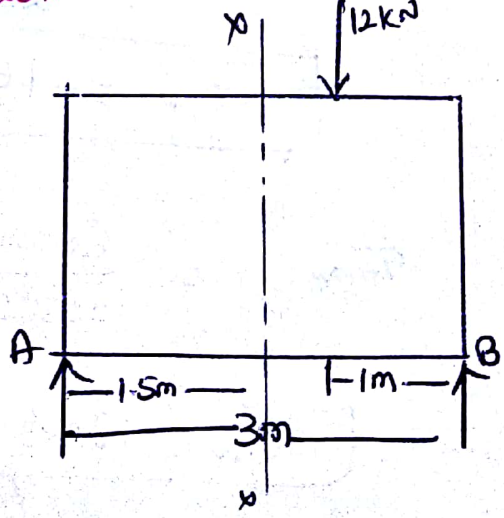
$= 0.5\times1000\times9.81\times10^{-2}\ N/mm^2$

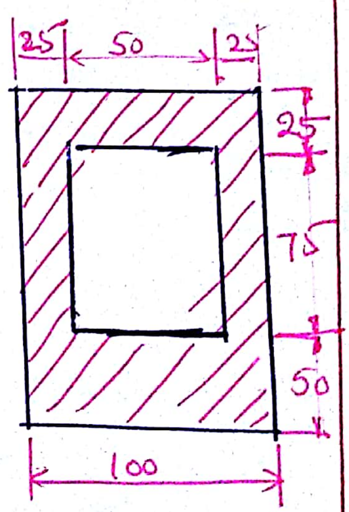
$$\therefore \frac{M}{I} = \frac{\sigma}{y} \Rightarrow M = \frac{\sigma}{y}\times I = \frac{0.5\times1000\times9.81\times10^{-2}\times237.467\times10^4}{50}$$

$$\boxed{M = 0.2374\ T\text{-}m}$$

Q. A simply supported beam of length 3 m carries a point load of 12 kN at a distance of 2 m from left support. The cross-section of the beam is shown in fig. Determine the maximum tensile and compressive stress at X–X.

→





$W = 12kN = 12000\ N$

$R_A + R_B = 12000$

$R_B\times3 - 12000\times2 = 0$

$R_B = 8000 \quad R_A = 4000$

B.M at X–X axis

$R_A\times1.5 = 4000\times1.5$

$= 6000\ N\text{-}m$

$= 6000\times10^3\ N\text{-}mm$

Find the N.A

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{150 \times 100 \times 75 - 50 \times 75 \times (50 + \frac{75}{2})}{15000 - 3750} = \frac{796875}{11250} = 70.83 mm$$

Hence N.A will lie at a distance of 70.83 from the bottom edge (or) $150 - 70.83 = 79.17$ mm from the top edge

Now find Moment of Inertia: $I = I_1 - I_2$

I_1 = M.O.I of outer rectangle about N.A

$$= I_G + Ah^2$$

$$= \frac{100 \times 150^3}{12} + 100 \times 150 \times (75 - 70.83)^2$$

$$= 2838.58 \times 10^4 \ mm^4$$

I_2 = M.O.I of cutout rectangular part about N.A

$$= \frac{50 \times 75^3}{12} + 50 \times 75 \times (50 + \frac{75}{2} - 70.83)^2 = 2799.8 \times 10^4 \ mm^4$$

$I = I_1 - I_2 = 2558.59 \times 10^4 \ mm^4$

Maximum tensile Stress $\sigma = \frac{M}{I} \times y_t$

$$= \frac{6 \times 10^6}{2558.5 \times 10^4} \times 70.83$$

$$\boxed{\sigma_{max\,t} = 16.60 \ N/mm^2}$$

$$\sigma_{max\,c} = \frac{6000000}{2558.5 \times 10^4} \times 79.17$$

$$\boxed{\sigma_{c\,max} = 18.56 \ N/mm^2}$$

Q Three beams have same length, same allowable bending stress and are subjected to the same bending moment. The c/s of the beam are a circle, a square, and rectangle with depth twice the width. Find the ratio of the weights of the circular and rectangular beams with respect to the square beam.

→ D = diameter of the circular section.

S : Length of each side of the square section

b : Width of the rectangular section

d : depth of the rectangular section = 2b

$\frac{M}{I} = \frac{\sigma}{y}$; I for circular section $= \frac{\pi}{64} D^4$ and $y = D/2$

$Z = \frac{\pi}{32} D^3$ | $Z_R = \frac{bd^2}{6}$

$Z_{squ} : \frac{bd^2}{6} = \frac{S \cdot S^2}{6} = \frac{S^3}{6}$ | $= \frac{b \times 4b^2}{6}$

$= \frac{2}{3} b^3 = 0.667 b^3$

∴ Same strength

$$\frac{\pi}{32} D^3 = \frac{S^3}{6}$$

$$\frac{D^3}{S^3} = \frac{32}{6\pi} \Rightarrow \frac{D}{S} = \left(\frac{32}{6\pi}\right)^{1/3} = 1.193$$

$$\boxed{D = 1.193 \times S}$$

l = length of each beam.

w : Weight per unit volume of each beam.

[Since allowable bending stress is same in each beam, they must be made of the same material having same weight per unit volume]

Weight of circular beam is given by

$$W_c = \frac{\pi}{4} D^2 \times l \times w = \frac{\frac{\pi}{4} D^2 \times l}{V} \times w$$

$$= \frac{\pi}{4} \cdot (1.193S)^2 \times l \times w$$

Weight of square beam is given by

$$W_s = s \times s \times l \times w = s^2 \times l \times w$$

$$\frac{W_c}{W_s} = \frac{\frac{\pi}{4}(1.1935)^2 \times \cancel{l \times w}}{s^2 \times \cancel{l \times w}} = 1.118$$

$$0.6667 b^3 = \frac{1}{6} s^3$$

$$\frac{s^3}{b^3} = 6 \times 0.6667 = 4 \quad \frac{s}{b} = (4)^{1/3} = 1.588$$

$$\boxed{s = 1.588b}$$

$$W_R = b \times d \times l \times w$$
$$= b \times 2b \times l \times w = 2b^2 l w$$

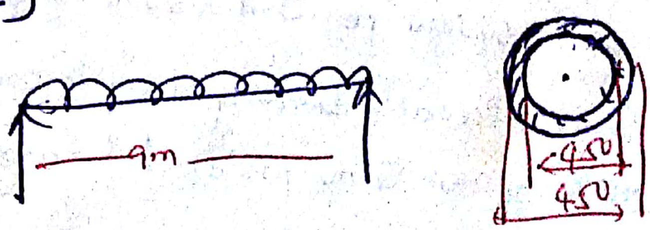
$$\frac{W_R}{W_s} = \frac{2b^2 \cancel{lw}}{s^2 \cancel{lw}} = \frac{2b^2}{s^2} = \frac{2b^2}{(1.588b)^2} = 0.793$$

$$\boxed{\frac{W_R}{W_s} = 0.793}$$

Q. A Cast Iron water pipe 450mm bore and 500mm outside diameter is Supported at two points 9m apart. Find the maximum Stress in the material when the pipe is running full. Density of CI is 7.2 gm/c.c and that of water is 1000 kg/m^3.

→ [density of pipe material is given, weight of the pipe is to Considered. Thy weight and the weight of water behave like total value of U.D.L]

density of CI = 7.2 gm/cc
= 7.2 × 1000 kg/m^3
= 7200 kg/m^3



density of water = 1000 kg/m^3

mass of water = density × volume

$$= 1000 \times \frac{\pi}{4} \times (.450)^2 \times 9. =$$

Weight of water = $1000 \times \pi/4 \times (.45)^2 \times 9 \times 9.81 = 14041.95N$

weight of CI = $7200 \times \frac{\pi}{4} \times [(.5)^2 - (.45)^2] \times 9 \times 9.81 = 23715.29N$

Total value of UDL = $23715.29 + 14041.9 = 37757.24 N$

Now, for a beam loaded with U.D.L over the entire length, maximum B.M is given by

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$I = \frac{\pi}{64}(D^2 - d^2) = \frac{\pi}{64}(500^4 - 450^4) = 1055072000 \text{ mm}^4$$

$$y = \frac{500}{2} = 250 \text{ mm}$$

$$M = \frac{Wl}{8} = \frac{37757.24 \times 9000}{8} = 42476895 \text{ N-mm}$$

$$\therefore \frac{42476895}{1055072000} = \frac{\sigma}{250} \Rightarrow \sigma = 10.06 \text{ N/mm}^2$$

~~H.W An I - Section is welded on the top~~