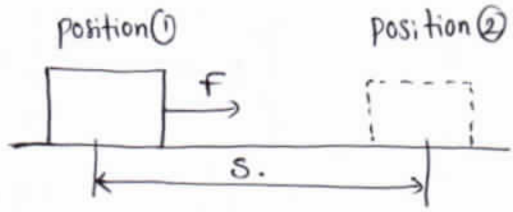
Unit-V

Work done by a force:-

If a particle is subjected to a force 'F' and particle is displaced by 's' position ① to position ② then work done 'U' is the product of force & displacement

Work done = force × displacement

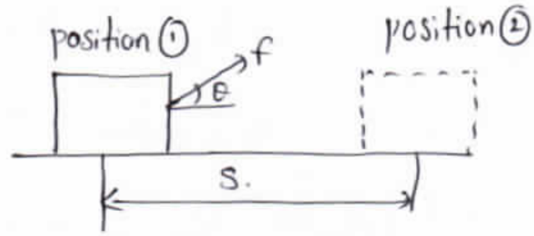
$U = F \times S$



$\text{workdone}\,(U) = F\cos\theta \times S$

Workdone is a scalar quantity.

Units N.m (or) Joule.



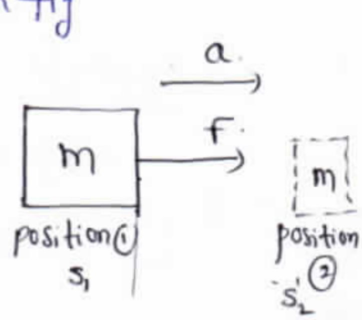
Work- Energy principle:-

Work done by the forces acting on a particle during some displa-cement is Equal to the change in kinetic Energy during that displacement.

proof:-

Consider the particle having mass 'm' is acted upon by a force 'F' and moving along a path as shown in fig

Let v_1 & v_2 be the velocities of the particle at position ① & ② and corresponding displacement s_1 & s_2 respectively.



By Newton's Second Law, we have

$$\Sigma f = ma$$

$$F = m\frac{dv}{dt}$$

$$f = m \cdot \frac{dv}{dt}$$

$$= m \cdot \frac{dv}{ds} \cdot \frac{ds}{dt}$$

$$= mv \cdot \frac{dv}{ds} \quad \left(\because v = \frac{ds}{dt}\right)$$

$$f\,ds = mv\,dv.$$

Integrating both sides, we have

$$\int_{s_1}^{s_2} f\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$f.S = \frac{1}{2} m (v_1^2 - v_2^2)$$

$$U_{1-2} = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_2^2 \quad (\because U = f \times s)$$

work done = Change in kinetic Energy

Kinetic Energy of a particle:-

It is the energy posssessed by a particle by virtue of its motion.

$$k.E = \frac{1}{2} m v^2$$

Potential Energy of a particle:-

It is the energy possessed by a particle by virtue of its position.

$$P.E = m.g.h$$

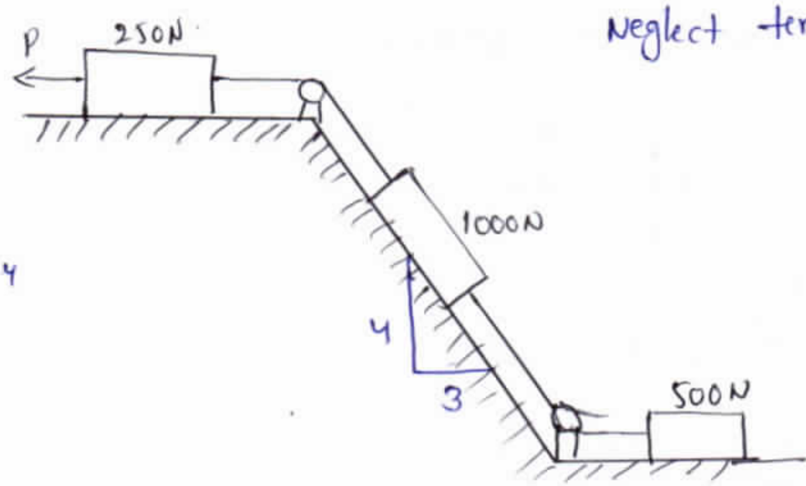
Principle of Conservation of Energy:-

It states that energy can ~~never~~ neither be created nor destroyed, but it can only be transformed from one form to other.

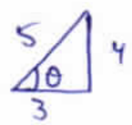
$$\text{Total energy} = k.E + P.E$$

$$= \frac{1}{2} m v^2 + mgh$$

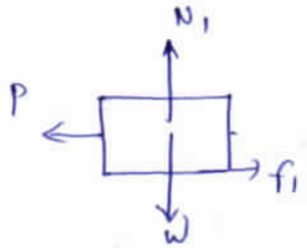
→ Determine the constant force 'P' that will give the system of bodies shown in fig. velocity of 3 m/s after moving 4.5 m from rest. Co-eff. of friction b/w 0.3.



Neglect tension force in rope.

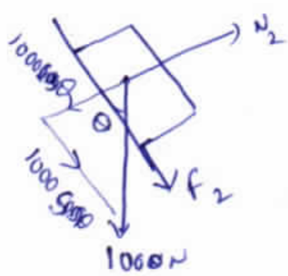


Sol



$N_1 = 250\,N$

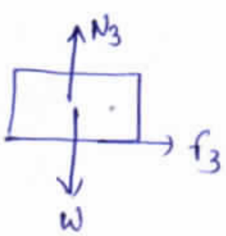
$f_1 = 0.3 \times 250 = 75\,N.$



$N_2 = 1000\cos\theta = 1000 \times \frac{3}{5} = 600N$

$f_2 = 0.3\,N_2$

$= 0.3\,(600) = 180N.$



$N_3 = 500N$

$f_3 = 0.3\,N_3 = 0.3\,(500) = 150\,N.$

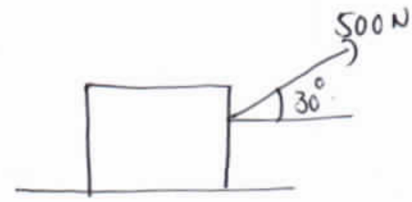
$$\left(-P + f_1 + f_2 + 1000\sin\theta + f_3\right) s = \frac{1}{2}(m_1 + m_2 + m_3)(v^2 - u^2)$$

$$\left(-P + 75 + 180 + 1000\left(\frac{4}{5}\right) + 150\right) 4.5 = \frac{1}{2 \times 9.8}(250 + 1000 + 800)(3^2 - 0^2)$$

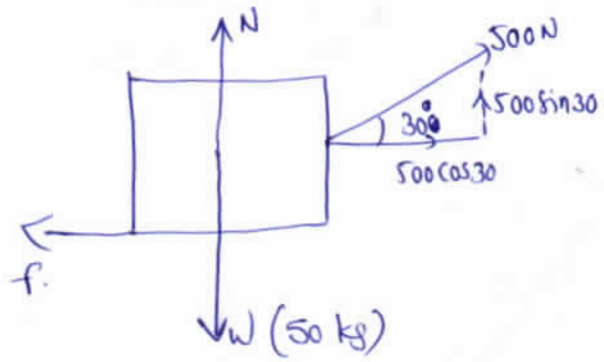
$$\underline{P = 1383.3\,N}$$

(2)

→ A force of 500 N is acting on block of mass 50 kg resting on a horizontal surface as shown in fig. Determine its velocity after the block has travelled at a distance of 10m. $\mu = 0.5$



Sol



By work energy principle:-

$\Sigma f_y = 0$; $N + 500 \sin 30 = W$

$N = (50 \times 9.81) - 500 \sin 30$

$= 240.5$ N

w.d = change in k.E

$(500 \times \cos 30 - f)\, s = \frac{1}{2} m (v_2^2 - v_1^2)$

$(500 \times \cos 30 - 0.5 (240.5)) \times 10 = \frac{1}{2} \times 50 (v_2^2 - 0^2)$

$v_2 = 11.18$ m/s.

By D' Alembert's principle:-

$\Sigma f_x = m a_x$

$500 \cos 30 - f = m a_x$

$500 \cos 30 - 0.5 (240.5) = 50 \times a_x$

$a_x = 6.25$ m/s^2

$v^2 - u^2 = 2as$

$v^2 - 0^2 = 2 \times 6.25 \times 10$

$\underline{v = 11.18 \text{ m/s}}$

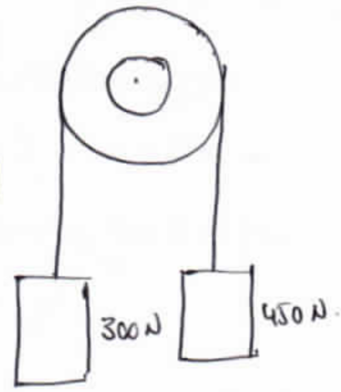
→ Two bodies weighing 300N & 450N are hung to the ends of a rope passing over an ideal pulley as shown in fig. How much distance the blocks will move an increasing the vel. of system from 2 m/s to 4m/s? How much is the tension in the string? use work energy method (3)

50

$$(450-300)S = \left[\frac{450}{2\times9.81}\right](v^2-u^2) + \left[\frac{300}{2\times9.81}\right](v^2-u^2)$$

$$150\ S = \left[\frac{450}{2\times9.81}\right](4^2-2^2) + \left[\frac{300}{2\times9.81}\right](4^2-2^2)$$

$$S = 3.058\ m$$



$$(450-T)\ S = \left[\frac{450}{2\times9.81}\right]\left[4^2-2^2\right]$$

$$(450-T)3.058 = \left[\frac{450}{2\times9.81}\right]\times12$$

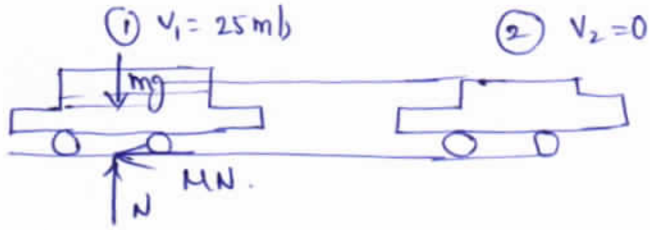
$$\underline{T = 360\ N}$$

→ i) Determine the distance in which a car moving at 90 kmph can comes to rest after the power is swithoff if $\mu = 0.8$.

ii) Determine the maxi. allowable speed of a car, if its is stop in the same distance as above, $\mu = 0.08$

50

i)



$$V_1 = 90\,kmph = \frac{90\times1000}{3600} = 25\ m/s$$

$$V_2 = 0$$

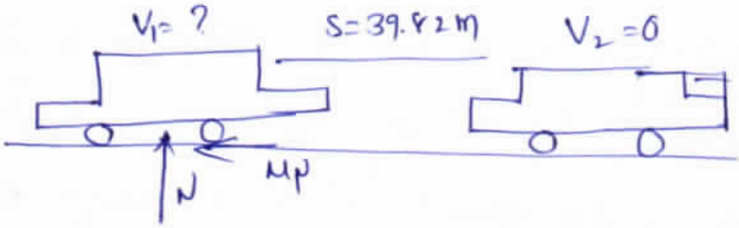
w.d = change in K.E

$$-\mu N\times S = 0 - \frac{1}{2}m(25)^2$$

$$-0.8\times mg\times S = -\frac{1}{2}m(625)$$

$$S = 39.82\ m.$$

ii)



$\mu = 0.08$, $V_1 = ?$, $V_2 = 0$, $S = 39.82\ m$

$$w.d = \text{change in } k.E$$

$$-\mu N \times S = 0 - \tfrac{1}{2} m \times V_1^2$$

$$-0.08 \times mg \times 39.8 = -\tfrac{1}{2} \times m \times V_1^2$$

$$V_1 = \underline{7.91\ m/s}$$

— A bullet of mass 50gms posses a kinemati Energy of 25000J. What is its vel.?

Sol **Given** $m = 50gm = 0.05\ kg$

$$K.E = 25{,}000$$

$$K.E = \tfrac{1}{2} m v^2$$

$$25{,}000 = \tfrac{1}{2}(0.05)(V)^2$$

$$V = 1000\ m/s$$

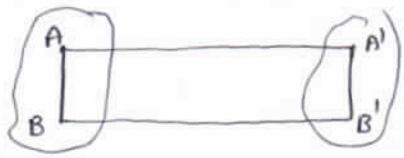
Unit-V

Kinetics

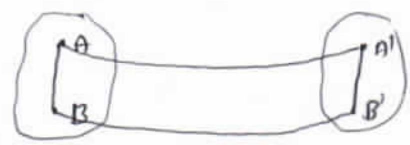
Types of Motion:-

1) Translation   2) Rotation   3) General plane motion

Translation:- A motion is said to be translation, if a straight line drawn on the moving body remains parallel to its original position at any time. During translation if the path traced by a point is a straight line, it is called Rectilinear ~~motion~~ translation & if the path is curve one it is called curvilinear translation.

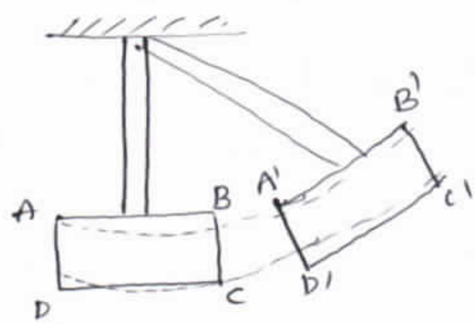


Rectilinear



Curvilinear

Rotation:- A motion is said to be rotation if all particles of a rigid body move in a concentric circle.



General plane motion:- It is a combination of both translation & rotation.

→ **Displacement:-** Linear distance b/w two points (or) positions of the body

Velocity:- Rate of change of displacement w.r.t. time. → $v = \frac{ds}{dt}$

Acceleration:- Rate of change of velocity w.r.t. time. → $a = \frac{dv}{dt}$

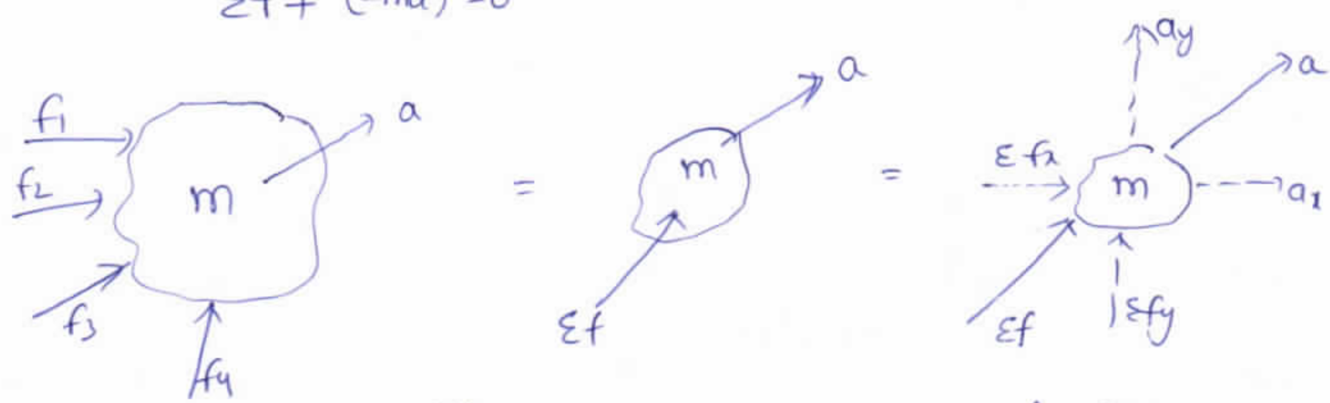
Negative acceleration is called Retardation.

Dynamic Equilibrium:- The force system consisting of external forces and Inertia force can be considered to keep the particle in Equilibrium Since the resultant force externally acting on the particle is not zero, the particle is said to be in Dynamic Equilibrium.

D'Alembert's principle:-

The algebraic sum of external force (Σf) and Inertia force ($-ma$) is equal to zero.

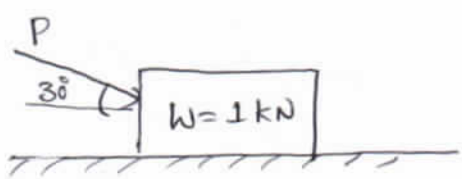
$$\Sigma f + (-ma) = 0$$

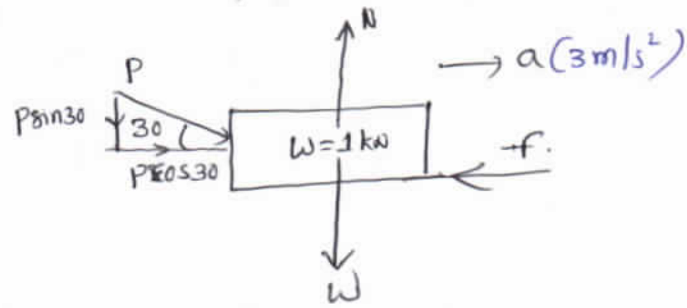


$$\Sigma f = ma$$

$$\Sigma f_x = ma_x$$
$$\Sigma f_y = ma_y.$$

→ A block weighing 1 kN rests on a horizontal plane as shown in fig. find the magnitude of the force 'p' required to give the block an acceleration of $3\,m/s^2$ to the right. $\mu = 0.25$.





$$\Sigma f_x = ma_x$$

$$P\cos 30 - f = \frac{1}{9.81} \times 3$$

$$P\cos 30 - 0.25\,(P\sin 30 - \cancel{1}) = \frac{3}{9.81}$$

$$P = 0.561\ kN$$

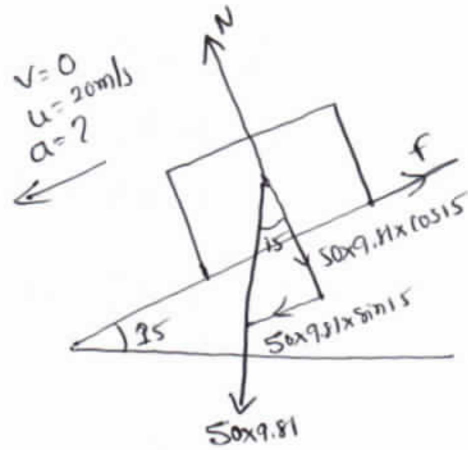
$$\Sigma f_y = 0$$

$$N = P\sin 30 - W \rightarrow (1)$$

$$N = P\sin 30 - 1$$

→ A 50 kg block kept on the top of a 15° sloping surface is push down the plane with an initial vel. of 20 m/s. If $\mu_k = 0.4$, determ the distance travelled by the block & the time it will take as it Comes to rest. ②

Sol



$\Sigma f_y = 0$

$N - 50 \times 9.81 \times \cos 15 = 0$

$N = 473.7\ N$

$\Sigma f_x = -ma_x$

$f - 50 \times 9.81 \times \sin 15 = -50 \times a$

$0.4(473.7) - 126.9 = -50 \times a$

$a = -1.25\ m/s^2$ (Retardation)

$V = u + at$

$0 = 20 + (-1.25)t$

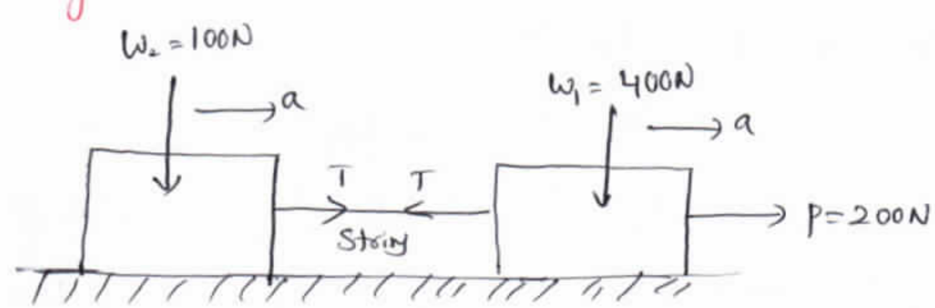
$t = 16$ sec.

$S = ut + \frac{1}{2}at^2$

$= (20 \times 16) + \frac{1}{2}(-1.25)(16)^2$

$S = 160m$

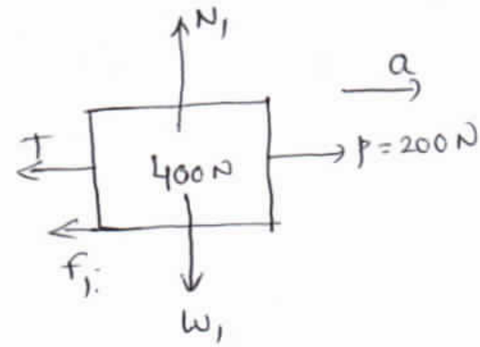
→ Two weights $W_1 = 400N$ & $W_2 = 100N$ are connected by a string & move along a horizontal plane under the action of force $P = 200N$ applied horizontally to the weight W_1. The co-eff. of friction b/w weights & plane is 0.25. Determine the acceleration of the weights & the tension in the string. Will the acceleration & tension in the string remain the same if the weights are interchanged.



Case i:-

$\Sigma f_y = 0$

$N_1 = 400\,N$

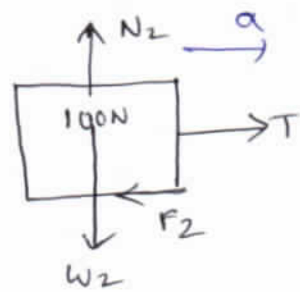


$\Sigma f_x = ma_x$

$200 - T - F_1 = ma_x$

$200 - T - \mu N_1 = \frac{400}{9.81} \times a$

$200 - T - 0.25 \times 400 = 40.78 \times a \Rightarrow 100 - T = 40.78a \longrightarrow (i)$



$\Sigma f_y = 0$

$N_2 = 100N$

$\Sigma f_x = ma_x$

$T - \mu N_2 = \frac{100}{9.81} \times a$

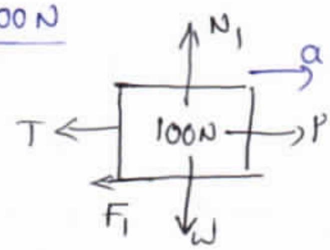
$T - 25 = 10.19a \longrightarrow (ii)$

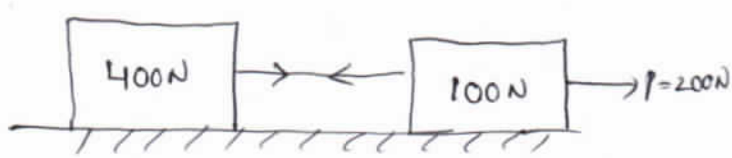
from (i) & (ii)

$T = 39.98N, \quad a = 1.47\ m/s^2$

Case ii: Weights Interchanged

F.B.D of 100N



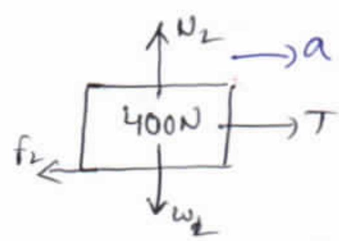


$\Sigma f_y = 0; \quad N_1 = 100N$

$\Sigma f_x = ma_x; \Rightarrow \quad P - T - F_1 = ma$

$200 - T - 0.25 \times 100 = \frac{100}{9.81} a$

$175 - T = 10.19a \longrightarrow (i)$



$\Sigma f_y = 0; \quad N_2 = 400\,N$

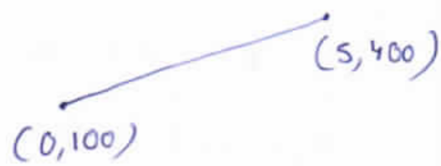
$\Sigma f_x = ma_x; \quad T - \mu N_2 = \frac{400}{9.8} \times a$

~~T = 400N~~ $\quad T - 100 = 40.78a \longrightarrow (ii)$

from (i) & (ii) $\quad T = 160N, \quad a = 1.47\ m/s^2$ ($\because$ During both conditions only tension changes, from the graph

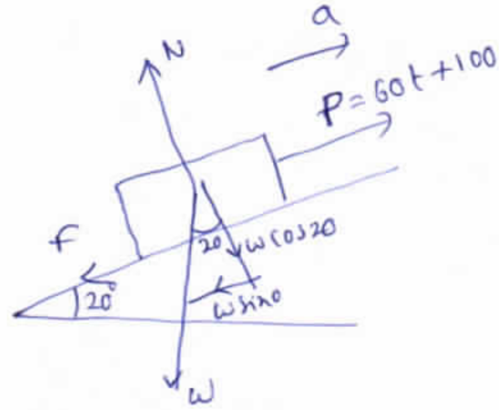
$$y = mx + C$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{400-100}{5-0} = 60,$$



$$\cancel{4}00 = 60 \times 5 + C$$

$$\underline{C = 100}$$



f.B.D

$$\Sigma f_x = m a_x$$

$$P - F - W \sin 20^\circ = 20 \times a$$

$$60t + 100 - 0.2 \times W \cos 20 - W \sin 20 = 20 \times a$$

$$60t - 4 = 20a \Rightarrow a = 3t - 0.2$$

$$a = 3(5) - 0.2 = 14.8 \text{ m/s}^2$$

$$a = \frac{dv}{dt}$$

$$dv = (3t - 0.2)\,dt$$

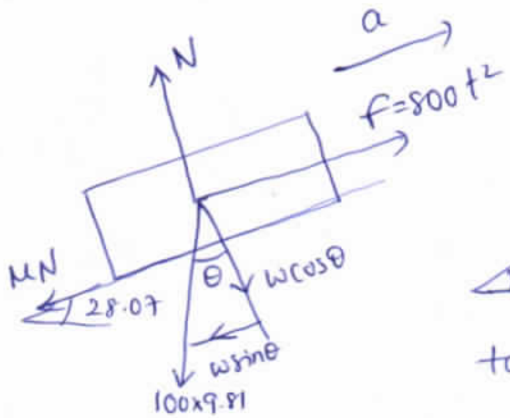
$$\int_{v_1 = 4}^{v_2 = ?} dv = \int_{t_1 = 0}^{t_2 = 5} (3t - 0.2)\,dt \Rightarrow V_2 - 4 = \left[\frac{3t^2}{2} - 0.2t\right]_0^5$$

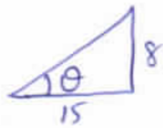
$$V_2 - 4 = \left[\frac{3 \times 5^2}{2} - 0.2 \times 5\right]$$

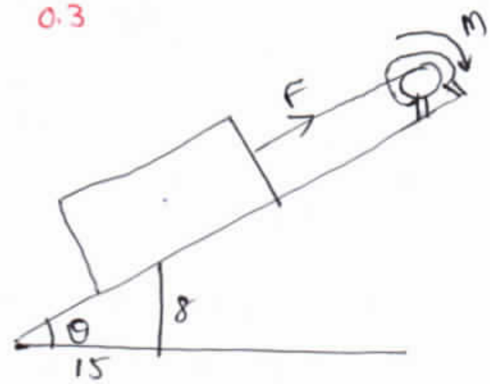
$$V_2 = 32.5 \text{ m/s}$$

→ The 100kg crate as shown in fig is pulled up by the incline (3) using the cable & motor M. for a short time, the force in the cable is $F = 800t^2$ N where t is in sec, if the crate has an initial vel. $v_1 = 2$ m/s, when $t=0$ sec, determine the vel when $t=2$ sec, The coeff of kinetic friction b/w the crate & incline is 0.3

Sol







$\tan\theta = 8/15, \ \theta = 28.07^\circ$

$\Sigma f_y = 0$; $N - 100\times9.81\cos\theta = 10$

$N = 865.53$

$\Sigma f_x = ma_x$; $f - 100\times9.81\sin\theta - \mu N = 100\times a$

$800t^2 - 100\times9.81\sin\theta - 0.3\,(865.53) = 100\times a$

$a = 8t^2 - 7.213$

$\frac{dv}{dt} = 8t^2 - 7.213$

$$\int_{v_1=2\,m/s}^{v_2=?} dv = \int_{t_1=0}^{t_2=2 sec} 8t^2 - 7.213$$

$$V_2 - V_1 = \left[\frac{8t^3}{3} - 7.213t\right]_0^2$$

$$V_2 - 2 = \left[\frac{8\times2^3}{3} - 7.213(2)\right] \Rightarrow V_2 = \underline{8.91\ m/s}$$

→ A crate of mass 20kg is pulled up the inclined at 20° by force P which vary as per graph shown in fig. find the acce. & vel. of the crate at $t=5$sec. knowing that its vel. was 4m/s at $t=0$, $\mu=0.2$