UNIT-V THIN CYLINDERS & THICK CYLINDERS

Syllabus:- Thin Seamless cylindrical shells, Derivation of formula for longitudinal and Circumferential stresses. longitudinal and volumetric strains, Change in diameter and volume of thin cylinders. Riveted boiler shells, thin Spherical shells. A thick cylinder lame's equation, cylinders Subjected to inside and outside pressures, Compound cylinders.

Outcome:- Design and Analysis of thin and thick cylinders under fluid pressure.

Introduction:- In order to meet with Several requirements the fluids are stored under pressure in pressure vessels (or) shells and transmitted from one place to the other through pipes.

Vessels are cylindrical and Spherical form are used for storing fluids. under Pressure.

Eg. steam boilers, air Compressors, tanks and watertanks.

Liquids and gases Causing internal pressure in a closed Vessels. when it is a gas, the Pressure is Constant in all parts of vessels. In Case of liquid, the Pressure is lowest at the top and increases with the depth.

Pressure vessels are two types.

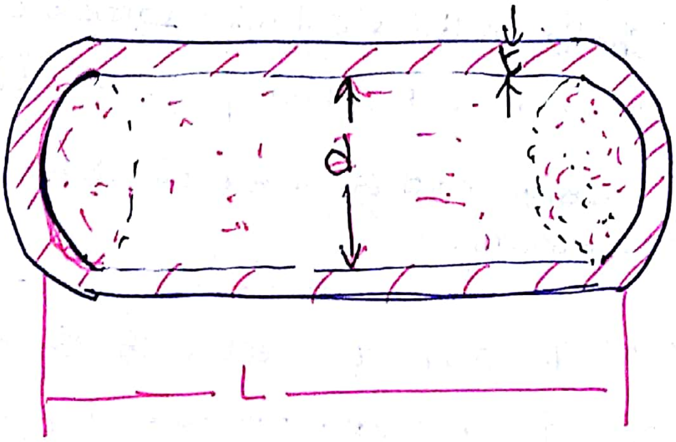
1. Cylindrical — Thin cylindrical / Thick cylindrical.
2. Spherical.

Thin cylindrical Sheels

Thin cylindrical Sheels are cylinders in which the thickness is 20^{th} part of diameter of cylinder. (or) even less.

$\boxed{t < \frac{d}{20}} \Rightarrow \frac{t}{d} < \frac{1}{20}$

t = thickness of cylinder in mm.
d = Internal diameter in mm



Some important points :-

1) In thin cylinder, the thickness is very Less compared to the diameter

2) Thin cylinders are used in L.P.G cylinders, boiler, I.C engines

3) Thin cylindricals are Subjected to Internal pressure

4) Internal pressure gives rise to internal stresses.

5) The Thickness should be strong enough to resist internal pressure

6. They should be non-corrosive, non-toxic & Should not react Chemically with the fluids.

Differentiate between thin cylinder & thick cylinder.

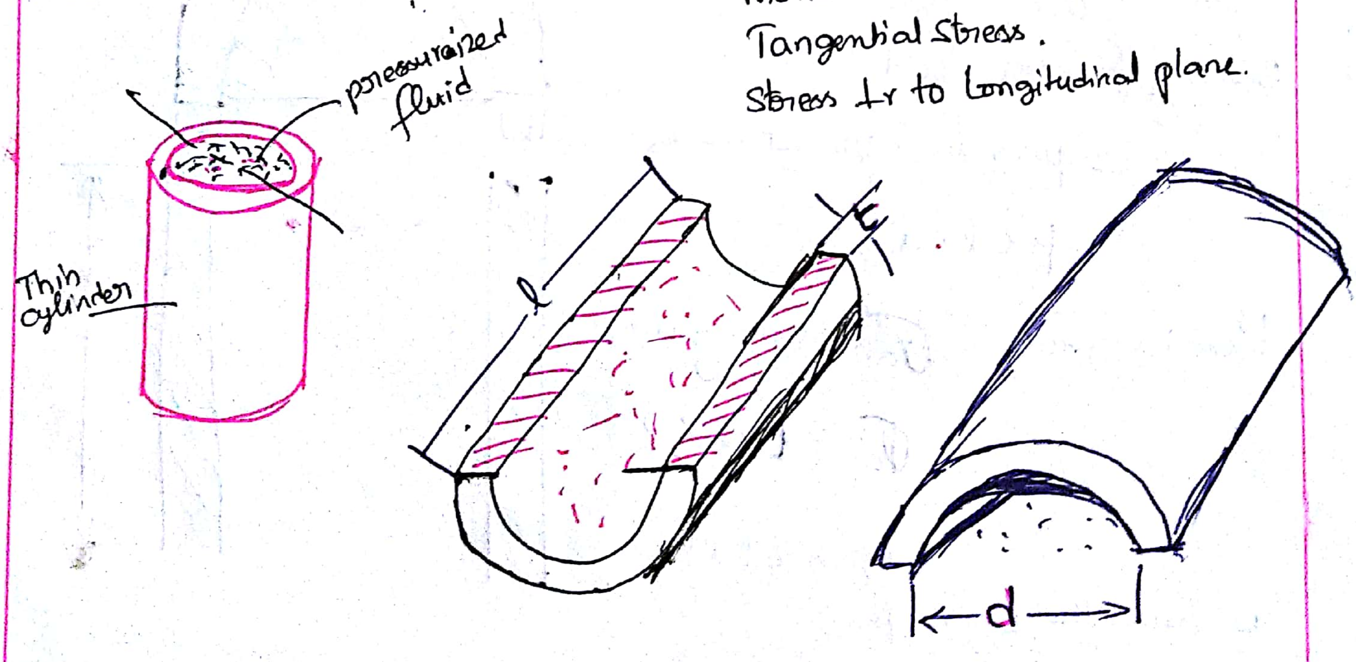
Thin cylinder	Thick cylinder.
1) $\frac{t}{d} < \frac{1}{20}$	1) $t/d > \frac{1}{20}$
2. Stress distribution is uniform throughout thickness	2) Stress distribution is non uniform. max. stress at inner surface min. stress at outer surface
3. Can resist only internal pressures	3) can resist internal as well as external pressures.

Stresses developed in thin cylinders :-

3 Type of stress.

1. Hoop stresses (σ_h)
2. longitudinal stresses (σ_l)
3. Radial stress (σ_r)

Hoop stresses :- It is also called Circumferential stress.
meridional stress.
Tangential stress.
Stress ⊥r to longitudinal plane.

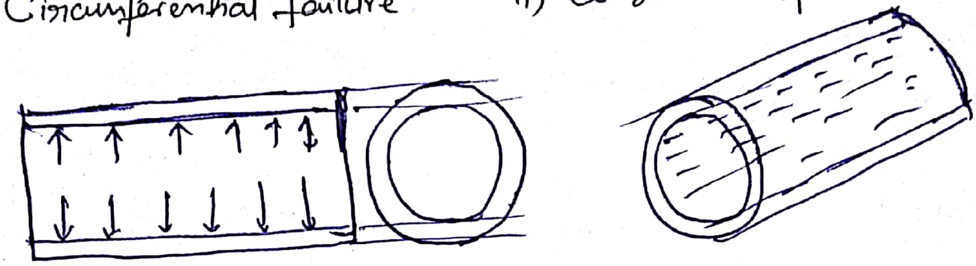


Cylinder is breaking into two parts. due to stress.

Various kinds of failures :

There are two types of failures in pressure vessels. due to Internal pressure.

1. Circumferential failure   ii) Longitudinal failure.

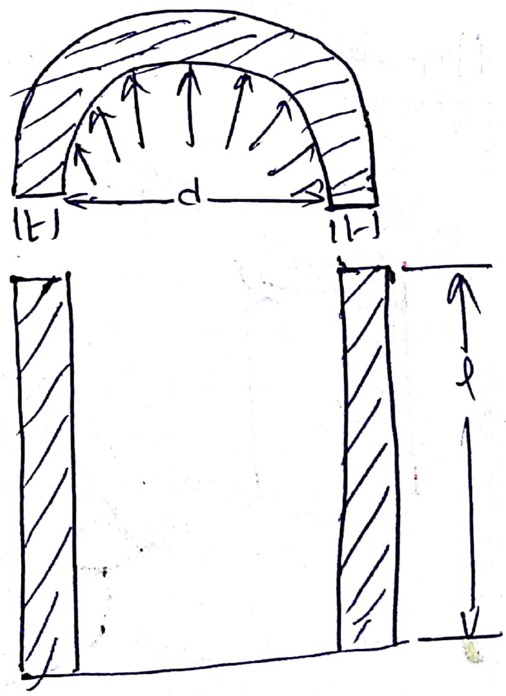


Because of Circumferential failure stresses are induced is called Circumferential stress (or) hoop stress. failure occur in Circumferential direction.

Circumferential stress is responsible for change of diameter.

Longitudinal fail occur. due to longitudinal stresses induced

Circumferential Stress



Load acting on a thin cylinder.

Bursting force (or) load

= pressure × projected area

= $p \times l \times d$

Resistance force = $\sigma_c \times$ Resisting area

= $\sigma_c \times t \times l \times 2$

= $\sigma_c \times 2tl$

To avoid the circumferential failure

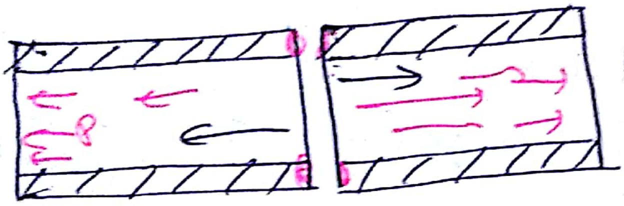
Bursting force = Resistance force.

$$P \times \cancel{l} \times d = \sigma_h \times 2 \times t \times \cancel{l}$$

$$\boxed{\sigma_h = \frac{Pd}{2t}}$$

Unit : N/mm^2

Longitudinal stress :



Bursting force = $P \times \frac{\pi}{4} d^2$

Resisting force : $\sigma_l \times$ Resisting area

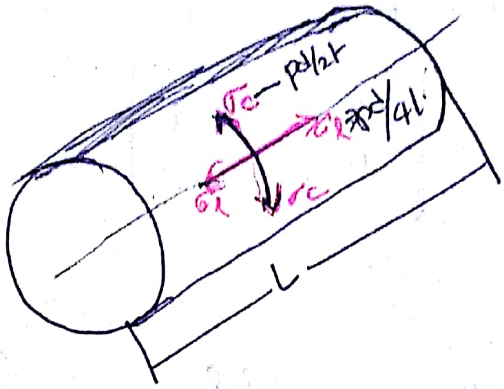
$= \sigma_l \times \pi d \times t$

$$\therefore P \times \frac{\pi}{4} d^{\cancel{2}} = \sigma_l \times \cancel{\pi} \cancel{d} t$$

$$\boxed{\sigma_l = \frac{Pd}{4t}}$$

Strains

Circumferential Strain :



σ_c is also called maximum principal stress.

σ_l is also called minor principal stress. $= Pd/4t$

$$\text{Circumferential Strain} = \frac{\text{Change in Circumference}}{\text{Original Circumference}} = \frac{F.C - O.C}{O.C}$$

$= \underline{\pi d}$ is a circumference.

Initial (or) Original Circumference $= \pi d_i$

Final Circumference $: \pi(d_f) = \pi(d_i + \delta d)$

$$\epsilon_c = \frac{\pi(d_i + \delta d) - \pi d_i}{\pi d_i}$$

$$\epsilon_c = \epsilon_h = \frac{\cancel{\pi}\,\delta d}{\cancel{\pi} d_i} = \frac{\delta d}{d_i} = \frac{\delta d}{d}$$

Longitudinal Strain : (ϵ_L)

L_o : Original length

L_f : final length

ΔL : Increase in length

$$\epsilon_L = \frac{L_f - L_i}{L_i} = \frac{\delta L}{L}$$

Volumetric strain : $\epsilon_V = \frac{\delta V}{V}$

Volume of cylinder $V_i = \frac{\pi}{4} d^2 \times L$

final volume : $V_f = \frac{\pi}{4} (d+\delta d)^2 \times (L+\delta L)$

$$\frac{\delta V}{V} = \frac{\pi/4\,(d+\delta d)^2 \times (L+\delta L) - \pi/4\, d^2 L}{\pi/4\, d^2 L}$$

$$= \frac{\frac{\pi}{4}\left[d^2 + \delta d^2 + 2d\,\delta d\right](L+\delta L) - \pi/4\, d^2 L}{\pi/4\, d^2 L}$$

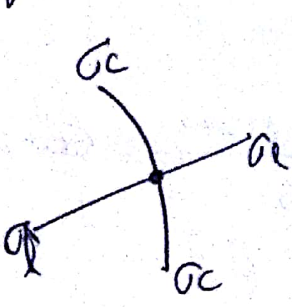
$$\frac{\delta V}{V} = \frac{\frac{\pi}{4}\left[d^2 L + \delta d^2 L + 2d\,\delta d L + d^2 \delta L + \cancel{\delta d^2 \delta L + 2d\,\delta d\,\delta L}\right] - \pi/4\, d^2 L}{\pi/4\, d^2 L}$$

$$\frac{\delta V}{V} = \frac{\cancel{\frac{\pi}{4}}\left[\cancel{d^2 L} + \cancel{\delta d^2 L} + 2d\,\delta d L + d^2 \delta L - \cancel{d^2 L}\right]}{\cancel{\pi/4}\, d^2 L}$$

$$\frac{\delta V}{V} = \frac{2d\,\delta d L + d^2 \delta L}{d^2 L}$$

$$\boxed{\frac{\delta V}{V} = \frac{\delta L}{L} + 2 \cdot \frac{\delta d}{d}} \Rightarrow \boxed{\epsilon_V = \epsilon_L + 2\epsilon_c}$$

Volumetric strain
hoop strain
longitudinal strain
} in terms of P, d, t, E



Strain in the circumferential direction

$$\epsilon_c = \frac{\sigma_c}{E} - \mu \cdot \frac{\sigma_l}{E} = \frac{Pd}{2tE} - \mu \frac{Pd}{4tE} = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

Strain in the longitudinal ~~stress~~ direction

$$\epsilon_l = \frac{\sigma_l}{E} - \mu \cdot \frac{\sigma_c}{E} = \frac{Pd}{4tE} - \mu \cdot \frac{Pd}{2tE} = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$\epsilon_c = \frac{pd}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$\epsilon_l = \frac{pd}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$\boxed{\delta d = \frac{pd^2}{2tE}\cdot\left[1-\frac{\mu}{2}\right]}$$

$$\frac{\delta l}{l} = \frac{pd}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$\boxed{\delta l = \frac{pdl}{2tE}\left[\frac{1}{2}-\mu\right]}$$

$$\epsilon_v = 2\epsilon_c + \epsilon_l$$

$$= 2\times\frac{pd}{2tE}\left[1-\frac{\mu}{2}\right] + \left[\frac{1}{2}-\mu\right]\frac{pd}{2tE}$$

$$= \frac{pd}{2tE}\left[2-\mu+\frac{1}{2}-\mu\right]$$

$$\boxed{\epsilon_v = \frac{pd}{2tE}\left[\frac{5}{2}-2\mu\right]}$$

$$\frac{\delta V}{V} = \frac{pd}{2tE}\cdot\left[\frac{5}{2}-2\mu\right]$$

$$\boxed{\delta V = \frac{pd}{2tE}\cdot V\left[\frac{5}{2}-2\mu\right]}$$

Q. A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to internal fluid pressure of 1.2 N/mm². Determine

1) Longitudinal stress developed in the pipe
11) Circumferential stress developed in the pipe

→ $d = 1.5\,m$

$t = 1.5\,cm = 1.5\times10^{-2}\,m$

$p = 1.2\,N/mm^2$

$$\frac{t}{d} = \frac{1.5\times10^{-2}}{1.5} = \frac{1}{100} < \frac{1}{20}$$

then it is a thin cylinder.

Longitudinal Stress $\sigma_2 = \frac{pd}{4t} = \frac{1.2\times1.5}{4\times1.5\times10^{-2}} = 30\,N/mm^2$

Circumferential Stress $= \frac{pd}{2t} = \frac{1.2\times1.5\times1000}{2\times15} = 60\,N/mm^2$

Q. A thin cylinder of internal diameter 1.25 m contains a fluid at an internal pressure of 2 N/mm². Determine the maximum thickness of the cylinder. If
a) The longitudinal stress is not to exceed 30 N/mm²
b) The circumferential stress is not to exceed 45 N/mm²

→

$d = 1.25\,m = 1250\,mm$

$p = 2\,N/mm^2$

$\sigma_2 = 30\,N/mm^2$

$\sigma_1 = 45\,N/mm^2$

$$\sigma_1 = \frac{pd}{2t} \Rightarrow t = \frac{pd}{2\sigma_1} = \frac{1250 \times 2}{2 \times 45} = 0.0277\,m = 2.77\,cm$$

$$\sigma_2 = \frac{pd}{4t} = t = \frac{pd}{4\sigma_2} = \frac{2 \times 1250}{4 \times 30} = 2.08\,cm$$

∴ maximum thickness = 2.77 cm.

Q. A water main 80 cm diameter contains water at a pressure head of 100 m. If the weight density of water is 9810 N/m³, find the thickness of the metal required for the water main, given the permissible stress as 20 N/mm²

→ $d = 80\,cm$, $h = 100\,m$ } w = weight density $= \rho g = 1000 \times g = 9810\,N/m^3$

$\sigma = 20\,N/mm^2$

pressure of water inside water main

$$P = \rho \times g \times h = 9810 \times 100 = 981000\,N/m^2$$
$$= 0.981\,N/mm^2$$

Thickness of the metal required t.

$$\sigma_1 = \frac{Pd}{2t}$$

$$t = \frac{Pd}{2\sigma_1} = \frac{0.981 \times 80}{2 \times 20} = 2\,cm = 20\,mm$$

$= 1.96 \simeq 2\,cm$..

(H.W)

Q! Find the thickness of metal necessary for a cylindrical shell of internal diameter 160 mm to withstand an internal fluid pressure of 8 N/mm². The maximum allowable (or) permissible (or) hoop stress in the section is not exceeding 35 N/mm². [Ans: 20.95 mm]

→

Q. A thin cylindrical shell, 2 m long has 200 mm diameter and thickness of metal 10 mm. It is subjected to an internal pressure of 4.18 MPa ?. Find the hoop stress developed and changes in diameter and length.

Q. Calculate i) the change in diameter ii) change in length iii) change in volume of thin cylindrical shell of 100 cm diameter 1 cm thick and 5 m long when subjected to internal pressure of 3 N/mm². Take the value of $E = 2\times10^5$ N/mm² and poisson's ratio is 0.3

→ $d = 100$ cm $= 1000$ mm

$t = 1$ cm $= 10$ mm

$L = 5$ m $= 5000$ mm

$p = 3$ N/mm²

$E = 2\times10^5$ N/mm² $\mu = 0.3$

1) change in diameter $\delta d = \dfrac{pd^2}{2tE}\left(1-\dfrac{\mu}{2}\right)$

$$= \frac{3\times(1000)^2}{2\times10\times2\times10^5}\left[1-\frac{0.3}{2}\right] = 0.6375 \text{ mm}$$

2. Change in length $\delta l = \dfrac{pdl}{2tE}\left(\dfrac{1}{2}-\mu\right)$

$$= \frac{3\times1000\times5000}{2\times10\times2\times10^5}\left(\frac{1}{2}-0.3\right) = 0.75 \text{ mm}$$

3. Change in volume $\delta v = V(2e_c + e_l)$

$$= V\left(2\frac{\delta d}{d} + \frac{\delta l}{l}\right)$$

$$= V(1.275\times10^{-3} + 0.15\times10^{-3})$$

$$= 5.594\times10^{-3}\times10^{9}$$

$$: 5.594\times10^{6} \text{ mm}^3$$

$V = \dfrac{\pi}{4}\times(1000)^2\times5000$

$= 3.926\times10^{9}$ mm³

Q. A cylindrical shell 90 cm long & 20 cm internal diameter having thickness of metal as 8 mm is filled with fluid at atmospheric pr. If an additional 20 cm^3 of fluid is pumped into cylinder,
1) the pressure exerted by the fluid on the cylinder
2) the hoop stress induced. Take $E = 2\times10^5$ N/mm^2 and $\mu = 0.3$

→ $l = 90$ cm $= 900$ mm

$d = 20$ cm $= 200$ mm

$t = 8$ mm

Volume of additional fluid $= 20\ cm^3 = 20\times10^3\ mm^3$

Volume of cylinder : $V = \frac{\pi}{4} d^2 \times L = \frac{\pi}{4}\times(200)^2\times 900$

$= 28.27\times10^6\ mm^3$

$\delta V = 20\times10^3\ mm^3$

Volumetric strain $= \frac{\delta V}{V} = 2e_1 + e_2$

$$\frac{20\times10^3}{28.27\times10^6} = 2e_1 + e_2$$

$$\frac{20\times10^3}{28.27\times10^6} = \frac{2pd}{2tE}\left[1-\frac{1}{2}\times\mu\right] + \frac{pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$= \frac{2\times p\times 20\times10^{2}}{2\times8\times2\times10^5}\left[1-\frac{1}{2}\times0.3\right] + \frac{p\times200}{2\times8\times2\times10^5}\left[\frac{1}{2}-0.3\right]$$

$$7.07\times\cancel{10^{-4}} = \cancel{10^{-4}}p + 0.125p\times\cancel{10^{-4}}$$

$$= 1.125p = 7.07$$

$$p = 5.656\ N/mm^2$$

II) Hoop stress $\sigma_c = \frac{pd}{2t} = \frac{5.656\times200}{2\times8} = 70.7\ N/mm^2$

Q A cylindrical vessel whose ends are closed by means of rigid flange plates. is made of steel, plate 3 mm thick. The length and internal diameter of the vessel are 50 cm and 25 cm respectively. Determine the longitudinal and hoop stress in the cylindrical shell due to an internal fluid pressure of 3 N/mm². Also calculate the increase in length, diameter and volume of the vessel, Take $E = 2\times10^5$ N/mm² and $\mu = 0.3$

→ $t = 3$ mm

$l = 500$ mm

$d = 250$ mm

$p = 3$ N/mm²

hoop stress $\sigma_c = \dfrac{3\times\cancel{250}^{125}}{2\times\cancel{3}} = 125$ N/mm²

Longitudinal stress $\sigma_l = \dfrac{125}{2} = 62.5$ N/mm²

$\dfrac{\delta V}{V} = 2e_c + e_l \Rightarrow \quad \epsilon_V = 2\epsilon_c + \epsilon_l$

$e_c = \dfrac{\sigma_c}{E} - \mu\cdot\dfrac{\sigma_l}{E} = \dfrac{125}{2\times10^5} - 0.3\times\dfrac{62.5}{2\times10^5} = \dfrac{106.25}{2\times10^5}$

$= 5.31\times10^{-4}$

$\epsilon_c = \dfrac{\delta d}{d} = 5.31\times10^{-4} \Rightarrow \delta d = 5.31\times10^{-4}\times250 = 0.132$ mm

$\epsilon_l = \dfrac{\sigma_l}{E} - \mu\cdot\dfrac{\sigma_c}{E} = \dfrac{62.5}{2\times10^5} - 0.3\times\dfrac{125}{2\times10^5} = \dfrac{25}{2\times10^5} = 12.5\times10^{-5}$

$\epsilon_l = \dfrac{\delta l}{l} = 12.5\times10^{-5} \Rightarrow \delta l = 12.5\times10^{-5}\times500 = 0.0625$ mm

$\dfrac{\delta V}{V} = 2\times\dfrac{\delta d}{d} + \dfrac{\delta l}{l} = 2\times5.31\times10^{-4} + 1.25\times10^{-4}$

$= 11.87\times10^{-4}$

$\delta V = 11.87\times10^{-4}\times V$

$V = \dfrac{\pi}{4}d^2\times l$

$= \dfrac{\pi}{4}\times(250)^2\times500$

$= 2.45\times10^7$ mm³

$\therefore \delta V = 11.87\times10^{-4}\times2.45\times10^7$

$= 29081.5$ mm³

Q. A hollow cylindrical drum 600 mm in diameter and 3 m long, has a shell thickness of 10 mm. If the drum is subjected to an internal air pressure of 3 N/mm^2, determine the increase in its volume. Take $E = 2\times10^5 N/mm^2$ and poisson's ratio = 0.3 for the material

→ $D = 600mm$

$L = 3m = 3000mm$

$t = 10mm$

$p = 3\ N/mm^2$

$E = 2\times10^5\ N/mm^2$

$\mu = 0.3$

$d = D - 2t = 600 - 2\times10 = 580\ mm.$

$$\frac{\delta v}{V} = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$= \frac{3\times580}{2\times10\times2\times10^5}\cdot\left[\frac{5}{2} - 2\times0.3\right]$$

$$= 0.00043\times1.9 = 0.0008265$$

$\delta v = 0.0008265\times V$

$= 0.0008265\times\frac{\pi}{4}d^2L = 0.0008265\times\left(\frac{\pi}{4}580^2\times3000\right)$

$= 792623000\ mm^3.$

$= 6.55\times10^5\ cm^3$

H.W

1.Q. A cylindrical shell 100 cm long 20 cm internal diameter having thickness of metal as 10 mm is filled with fluid at atmospheric pressure of an additional 20 cm^3 of fluid is pumped into cylinder find;

I) The pressure exerted by the fluid on the cylinder

II) hoop stress Take $E = 2\times10^5 N/mm^2$ and $\mu = 0.3$

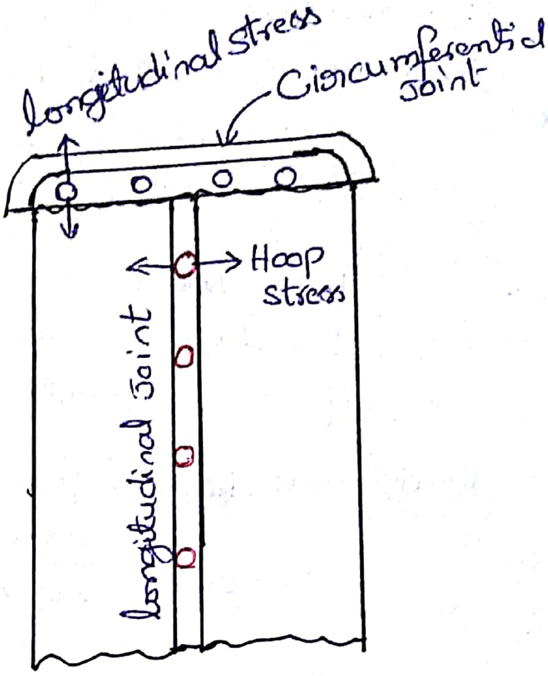
[Ans 10.05 N/mm^2
100.52 N/mm^2]

2. A thin cylindrical shell of 120 cm diameter, 1.5 mm thick and 6 m long is subjected to internal fluid pressure of 2.5 N/mm^2. If the value $E = 2\times10^5 N/mm^2$ and poisson ratio = 0.3 calculate

I) Change in diameter II) Change in length III) Change in volume.

[Ans: 0.051 m II) 0.06 cm
III) 6449.7 cm^3]

Riveted Boiler shells :



If Joints in the plates of a cylinder to be considered, the strength of the plates is reduced owing to decrease in area due to holes in the plates.
For example, in boilers, there are two type of Joints
1. longitudinal Joint
2. Circumferential Joint.

If the efficiency of a Joint is defined as strength of Jointed plate to that of the solid plate

$$\sigma_c = \frac{Pd}{2t\eta_l} \text{ (or) } \eta_l = \frac{\frac{Pd}{2t}}{\sigma_c} \Rightarrow \boxed{\sigma_c = \frac{Pd}{2t\eta_l}}$$

$$\sigma_l = \frac{Pd}{4t\eta_c} \text{ (or) } \eta_c = \frac{\frac{Pd}{4t}}{\sigma_l} \Rightarrow \boxed{\sigma_l = \frac{Pd}{4t\eta_c}}$$

Note
1) In longitudinal Joint, the Circumferential stress is developed where as in Circumferential Joint, the longitudinal stress is developed.
2. efficiency of a Joint means the efficiency of longitudinal Joint
3. If efficiencies of a Joint are given, the thickness of the thin shell is determined from equation. $\sigma_c = \frac{Pd}{2t\eta_l}$.

Q. A boiler is subjected to an internal steam pressure of $2\ N/mm^2$ the thickness of boiler plate is 2 cm and permissible tensile stress is $120\ N/mm^2$. Find out the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.

→ $P = 2\ N/mm^2$ $\quad \eta_l = 0.90$

$t = 2\ cm = 20\ mm$ $\quad \eta_c = 0.40$

$\sigma_t = 120\ N/mm^2$

Find max. diameter for circumferential stress.

$$\sigma_c = \frac{p \times d}{2 \times t \times \eta_l} \Rightarrow 120 = \frac{2 \times d}{2 \times 20 \times 0.9}$$

$$d = \frac{120 \times \cancel{2} \times 20 \times 0.9}{\cancel{2}}$$

$$= 2160\ mm$$

Max. diameter for longitudinal stress

$$\sigma_l = \frac{Pd}{4t\,\eta_c} \Rightarrow 120 = \frac{2 \times d}{4 \times 20 \times 0.4}$$

$$d = \frac{120 \times \cancel{4}^{2} \times 20 \times 0.4}{\cancel{2}}$$

$$= 1920\ mm$$

The longitudinal (or) circumferential stresses induced in the material are directly proportional to diameter(d) σ_c & $\sigma_l \propto d$. Hence the stress induced will be less if the value of 'd' is less. Hence take the minimum value of 'd'

Note

If d is taken as equal to 2160mm

$$\sigma_l = \frac{Pd}{4t \times \eta_c} = \frac{2 \times 2160}{4 \times 20 \times 0.4} = 135\ N/mm^2$$

$\sigma_l = 135\ N/mm^2 > 120\ N/mm^2$ then the component not safe. That's why $\boxed{d = 1920\ mm}$

Q. A boiler shell is to be made of 15 mm thick plate having tensile stress of 120 MN/m². If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively determine.
1) Max. Permissible diameter of the shell for an internal Pr of 2 MN/m²
2) Permissible intensity of internal Pr when shell dia is 1.5 m.

→ 1) Max. Permissible diameter of $P = 2\times10^6\ N/m^2 = 2\ N/mm^2$

Consider hoop stress

$t = 15\ mm$
$\sigma_t = 120\ N/mm^2$
$\eta_c = 30\% = 0.3$
$\eta_l = 0.7$

$$\sigma_c = \frac{Pd}{2t\eta_l}$$

$$d = \frac{2\times\sigma_c\times t\times\eta_l}{P}$$

$$= \frac{2\times120\times15\times0.7}{2} = 1260\ mm = 1.26\ m$$

Consider longitudinal stress

$$\sigma_l = \frac{Pd}{4t\eta_c} \Rightarrow d = \frac{\sigma_l\times4\times t\times\eta_c}{P}$$

$$= \frac{120\times\overset{2}{\cancel{4}}\times15\times0.3}{\cancel{2}} = 1080\ mm = 1.08\ m$$

In order to satisfy both the conditions ~~Max~~ min. diameter d = 1.08 m

Note [If you provide bigger diameter 1.26 m $\sigma_l = \frac{Pd}{4t\eta_c} = 140\ MN/m^2 > 120\ MN/m^2$ not safe]

ii) Max. Permissible pressure

Consider hoop stress $\sigma_c = \frac{Pd}{2t\eta_l} \Rightarrow P = \frac{2\times\sigma_c\times t\times\eta_l}{d}$

$$= \frac{2\times120\times15\times0.7}{1500}$$

$$P = 1.68\ N/mm^2$$

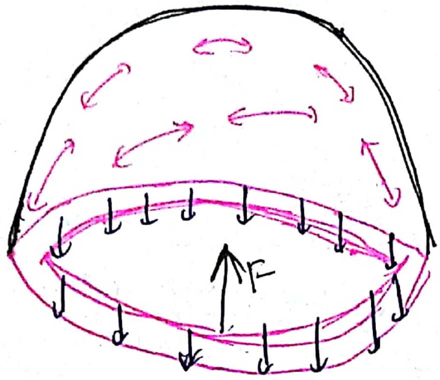
Consider longitudinal stress $\sigma_l = \frac{Pd}{4t\eta_c} \Rightarrow P = \frac{4\times\sigma_l\times t\times\eta_c}{d}$

$$= \frac{4\times120\times15\times0.3}{1500}$$

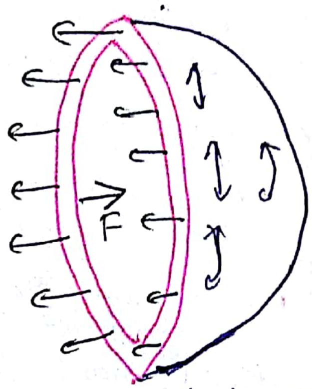
$$P = 1.44\ N/mm^2$$

The permissible intensity of Pr = 1.44 N/mm².

Thin Spherical shells :



Circumferential direction.



longitudinal direction.

σ_c = hoop stress
p = Pressure of Intensity
t = thickness
d = internal diameter of spherical shell.

Note

$\sigma_c = \sigma_e = \frac{pd}{4t\eta}$

Bursting force = Resisting force

$$p \times \frac{\pi}{4} d^2 = \pi d \cdot t \cdot \sigma_c$$

$$\boxed{\sigma_c = \frac{pd}{4t}} \quad (\text{or}) \quad \sigma_c = \frac{pd}{4t\eta}$$

(If the shell is seamless)
- η is Joint η efficiency)

Change in dimensions :

Strain in any diametrical plane axis $e = \frac{\delta d}{d} = \frac{\sigma_c}{E} - \mu \cdot \frac{\sigma_e}{E}$

$$= \frac{pd}{4tE} - \mu \cdot \frac{pd}{4tE}$$

$$= \frac{pd}{4tE}\left[1-\mu\right]$$

$$\delta d = \frac{pd^2}{4tE}(1-\mu) \quad (\text{or}) \quad \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$$

e_v = Algebraic sum of strains in three axis
$= e + e + e = 3e$

$$e_v = \frac{\delta V}{V} \Rightarrow \delta V = e_v \cdot V$$

$$\boxed{\delta V = 3 \frac{pd}{4tE}\left[1 - \frac{1}{m}\right] \cdot V}$$

& $V = \frac{4}{3}\pi r^3$
$= \pi d^3/6$

$$\delta V = \frac{\cancel{3} pd}{4tE} \cdot \frac{\pi d^3}{\cancel{6}\,2}\left[1 - \frac{1}{m}\right] = \frac{\pi p d^4}{8tE}\left[1 - \frac{1}{m}\right]$$

Q. A thin spherical shell of internal diameter 1.5m and thickness 8 mm is subjected to an internal pressure of 1.5 N/mm². Determine the increase in diameter and increase in volume. Take $E = 2\times10^5$ N/mm² and $\mu = 0.3$ [0.369mm, 1304×10^3 mm³]

$\rightarrow$

Q. A thin spherical shell 1 m in diameter with its wall of 1.2 cm thickness is filled with a fluid at atmospheric pressure. What intensity of pressure will be developed in it if 175 cm³ more of fluid is pumped into it. Also calculate the circumferential stress at that pressure and the increase in diameter. E = 200GPa and 1/m = 0.3

$\rightarrow$

$d = 1000$ mm $\quad \frac{1}{m} = 0.3 \quad E = 2\times10^5$ MPa

$t = 12$ mm

$\delta V = 175\times10^3$ mm³

$$\text{Volumetric strain } e_v = \frac{\delta V}{V} = \frac{175\times10^3}{\frac{4}{3}\times\pi\times(500)^3} = 3.34\times10^{-4}$$

$$e_v = 3\times e$$

$$e = \frac{e_v}{3} = 1.114\times10^{-4}$$

$$\frac{\delta d}{d} = e \Rightarrow \delta d = 1.114\times10^{-4}\times1000 = 0.111\text{mm}$$

Circumferential Stress (σ_c)

$$e = \frac{\sigma_c}{E}(1-1/m) \Rightarrow \sigma_c = 31.714 \text{ N/mm}^2$$

Pressure p :

$$\sigma_c = \frac{pd}{4t}$$

$$p = \frac{4t\sigma_c}{d} = \frac{31.714\times4\times0.012}{1000}$$

$$\boxed{p = 1.522 \text{ N/mm}^2}$$

	V UNIT QUESTIONS
1	The internal diameter of a cylindrical shell is 1 m and its length is 3 m, the plates being 1.5 cm thick. Determine the circumferential and longitudinal stresses set up and the changes in Dimensions of the shell when a fluid is introduced in it at a pressure of 1.5 N/mm^2. Take E = 200 kN/mm^2 and 1/m = 0.3.
2	A cylindrical vessel is 1.6m diameter and 5m long is closed at ends by rivets. It is subjected to an internal pressure of 4N/mm^2.If the maximum principal stress is not to exceed 120 N/mm^2, find the thickness of the shell. Assume E = 2 x 105 N/mm^2 and Poisson's ratio = 0.25. Find the change in diameter, length and volume of the shell.
3	Vertical cylindrical gasoline storage tank made of 20mm thick mild steel plate has to withstand maximum pressure of 1.5 MN/m^2. Calculate the diameter of the tank if stress 240 MN/m^2, factor of Safety 2 and joint efficiency 70%.

PROBLEM 1:- A cylindrical pipe of diameter 1.5m and thickness 1.5cm is subjecte to an internal fluid pressure of 1.2N/mm^2. Determine:

(i) Longitudinal stress developed in the pipe, and

(ii) Circumferential stress developed in the pipe.

(Answer: -$\sigma_1 = 60N/mm^2, \sigma_2 = 30N/mm^2$)

PROBLEM 2:- A cylinder of internal diameter 2.5m and of thickness 5cm contain a gas. If the tensile stress in the material is not to exceed 80N/mm^2. Determine the internal pressure of the gas.

(Answer: -$p = 3.2N/mm^2$)

PROBLEM 3:- A cylinder of internal diameter 0.5m contains air at a pressure of 7N/mm^2 (gauge). If the maximum permissible stress induced in the material is 80N/mm^2. Find the thickness of the cylinder.

(Answer: -t=2.2 cm)

1) A cylindrical boiler is 2.5 m in diameter and 20 mm in thickness and it carries steam at a pressure of 1.0 N/mm^2. Find the stresses in the shell.
Ans: Longitudinal stress=31.25 N/mm2 ; Hoop stress= 62.5 N/mm2

2) A thin cylindrical vessel of 2 m diameter and 4 m length contains a particular gas at a pressure of 1.65 N/mm2. If the permissible tensile stress of the material of the shell is 150 N/mm2, find the maximum thickness required.
Ans: thickness=11mm

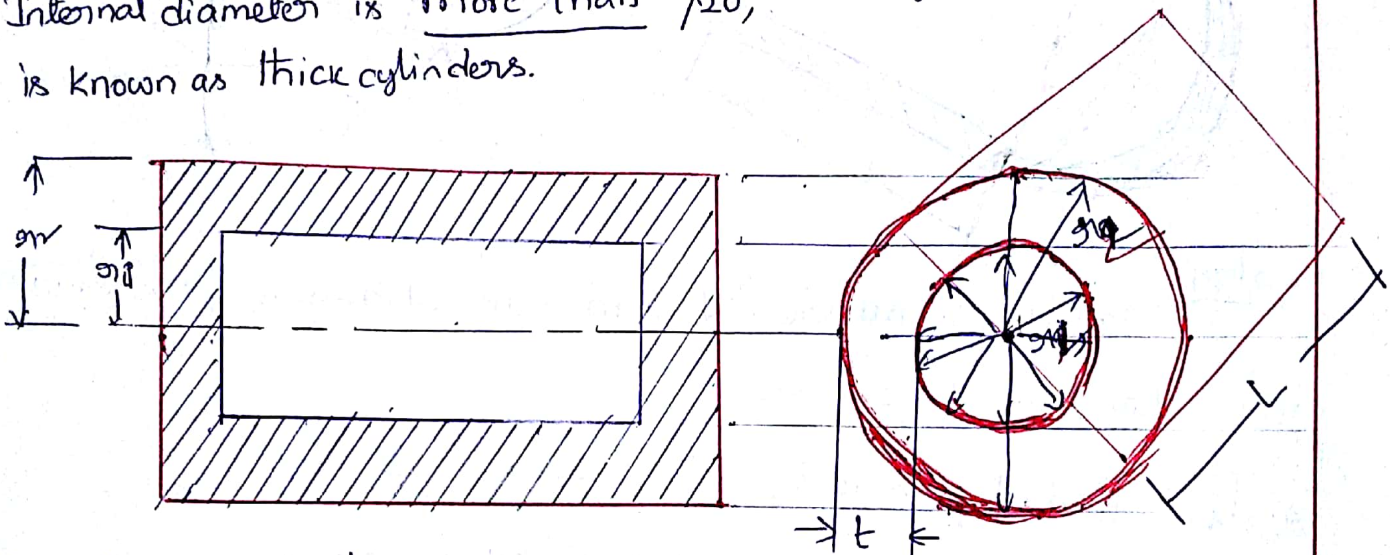
3) A cylindrical compressed air drum is 2 m in diameter with plates 12.5 mm thick. The efficiencies of the longitudinal (ηl) and circumferential (ηc) joints are 85% and 45% respectively. If the tensile stress in the plating is to be limited to 100 MN/m^2, find the maximum safe air pressure.
Ans: maximum safe air pressure = 1.063 N/mm^2

4) A cylindrical shell, 0.8 m in a diameter and 3 m long is having 10 mm wall thickness. If the shell is subjected to an internal pressure of 2.5 N/mm^2, determine
(a) Change in diameter,
(b) Change in length, and
(c) Change in volume.
Take E = 200 GPa and Poisson's ratio = 0.25.

Ans: Change in diameter=0.35mm, Change in length=0.375mm, Change involume=1507e3 mm3

Thick cylindrical shell :-

Thick cylindrical shell, [the ratio of thickness to internal diameter of cylindrical shell is Less than about 1/20, the cylindrical shell is known as thin cylinders, If the ratio of thickness to Internal diameter is more than 1/20, then cylindrical shell is known as thick cylinders.



In thick cylinder, we are going to analyze the pressure and hoop stress induced in the inside of the cylinder.

We Use the Lame's Theorem to analyze.

Assumption :- 1) fluid is in Compressible

2. fluid is Homogeneous.

3. pressure is Constant

4. longitudinal strain is Constant.

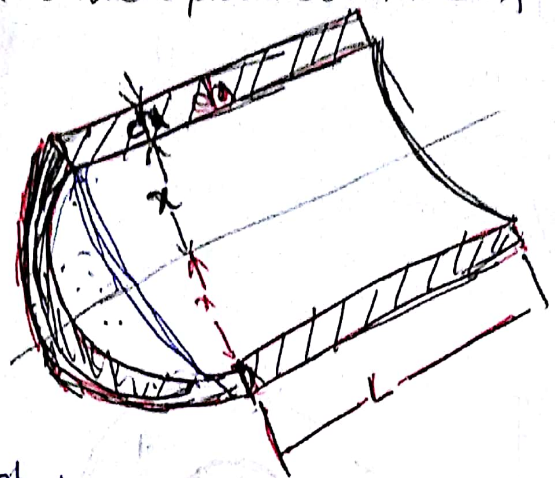
Consider an elementary ring of the cylinder of radius x and thickness dx as shown in fig.

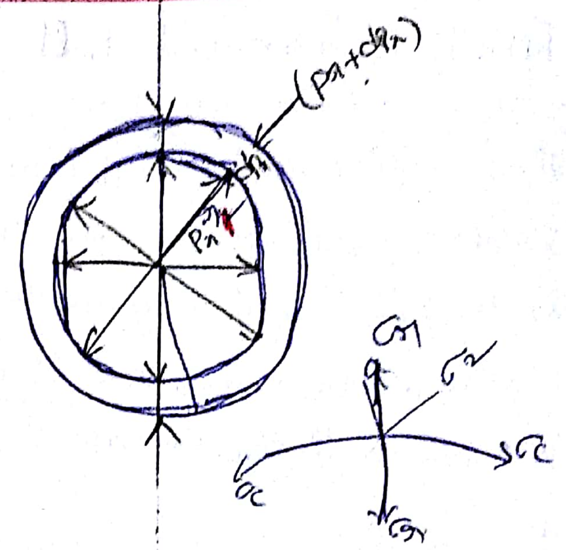
Let P_x : Radial pressure on the inner Surface of the ring

$P_x + \delta P_x$: Radial pressure on the outer Surface of the ring.

σ_x : hoop stress induced in the ring

P_x : Radial pressure on inside Surface.





Step1: r_2 & r_1 are ~~ext~~ external and internal radius of the cylinder

2. L : Length of the cylinder

3. P_x = Radial pressure of fluid at inner Surface

4. $P_x + dP_x$: Radial pressure ~~of fluid~~ at outer Surface

5. σ_x : hoop Stress induced in the ring

6. Bursting force = $P_x \cdot 2x \cdot L - [(p+dp_x)\cdot 2(x+dx)\cdot L]$

7. Resisting force : $\sigma_x \cdot 2dx \cdot L$

8. For Safe design.

(9) $\sigma_x \cdot \cancel{2} \cdot dx \cdot \cancel{L} = P_x \cdot \cancel{2}x \cdot \cancel{L} - (p+dp_x)\cdot \cancel{2}(x+dx)\,\cancel{L}$

10 $\sigma_x \cdot dx = P_x \cdot x - [(P_x+dP_x)(x+dx)]$

$= \cancel{P_x x} - [\cancel{P_x x} - dP_x \cdot x - P_x \cdot dx - dP_x \cdot dx$

$= -P_x \cdot dx - dP_x \cdot x$

(11) $\sigma_x = -P_x - \frac{dP_x}{dx} \cdot x$ —— (1)

(12) Consider longitudinal Stress σ_l

longitudinal forces due to longitudinal strain & fluid pressure

(13) force due to longitudinal Stress = force of fluid (Resultant)

(14) $\sigma_L \cdot \cancel{\pi}(r_2^2 - r_1^2) = P \times \cancel{\pi} r_1^2$

(15) $$\sigma_L = \frac{P r_1^2}{r_2^2 - r_1^2} \quad \text{---} (2)$$

(16) longitudinal Strain

$$\epsilon_L = \frac{\sigma_L}{E} - \mu \cdot \frac{\sigma_r}{E} - \mu \cdot \frac{-P_r}{E}$$

$$= \frac{\sigma_L}{E} - \mu \cdot \frac{\sigma_r}{E} + \mu \cdot \frac{P_r}{E}$$

(17) But σ_L, μ & E are constant

$\therefore \quad \sigma_r - P_r = \text{Constant}$

(18) $\sigma_r - P_r = 2A$

$$\sigma_r = P_r + 2A \quad \text{---} (3)$$

(19) Comparing equation (1) & (3)

$$-P_r - \frac{dP_r}{dr} \cdot r = P_r + 2A$$

$$r \cdot \frac{dP_r}{dr} = -P_r - P_r - 2A.$$

$$r \cdot \frac{dP_r}{dr} = -2P_r - 2A$$

$$\frac{dP_r}{dr} = \frac{-2P_r}{r} - \frac{2A}{r} = \frac{-2(P_r + a)}{r}$$

$$\frac{dP_r}{P_r + a} = -\frac{2\,dr}{r}$$

$$\frac{dP_x}{P_x + a} = -\frac{2\,dx}{x}$$

Integrating the above equation, we get

$$\log(P_x + a) = -2\log_e x + \log_e b$$

$$= \log_e\left(\frac{b}{x^2}\right)$$

$$\therefore P_x + a = \frac{b}{x^2}$$

$$\boxed{P_x = \frac{b}{x^2} - a}$$

$$\boxed{\sigma_x = \frac{b}{x^2} - a + 2a = \frac{b}{x^2} + a}$$

above equations are called Lame's equations of given radial pressure & hoopstress at any radius x.

a, b are constants that are obtained by applying boundary conditions

1) at $x = r_1$, $P_x = P_0$ (or) Pressure of fluid inside cylinder

2 at $x = r_2$, $P_x = 0$ (or) atmosphere pressure

Q. Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm². Also sketch the radial pressure distribution and hoop stress distribution across the section.

→ Internal diameter = 400 mm

radius r_1 = 200 mm

t_1 = 100 mm

External radius = $r_1 + t$ = 200 + 100 = 300 mm

Fluid pressure = 8 N/mm²

at $x = r_1$, $P_x = P_0$ = 8 N/mm²

Radial pressure $P_x = \frac{b}{x^2} - a$

apply the boundary conditions to the above equation.

At $x = r_1 = 200$ mm, $P_x = 8$ N/mm²

$x = r_2 = 300$ mm, $P_x = 0$

$$8 = \frac{b}{(200)^2} - a \quad = \frac{b}{40000} - a = 8$$

$$0 = \frac{b}{(300)^2} - a \quad = \frac{b}{90000} - a = 0$$

$$\frac{b}{40{,}000} - \frac{b}{90{,}000} = 8$$

$$\frac{9b - 4b}{360{,}000} = 8 \Rightarrow \frac{5b}{3{,}60{,}000} = 8$$

$$b = \frac{8 \times 360000}{5} = 5{,}76{,}000$$

Substituting this value in equation

$$0 = \frac{5{,}76{,}000}{90{,}000} - a \Rightarrow a = \frac{576000}{90{,}000} = 6.4$$

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hoop stress

$$\sigma_x = \frac{b}{x^2} + a$$

~~x = 200~~ $\sigma_x = \dfrac{5,76,000}{x^2} + 6.4$

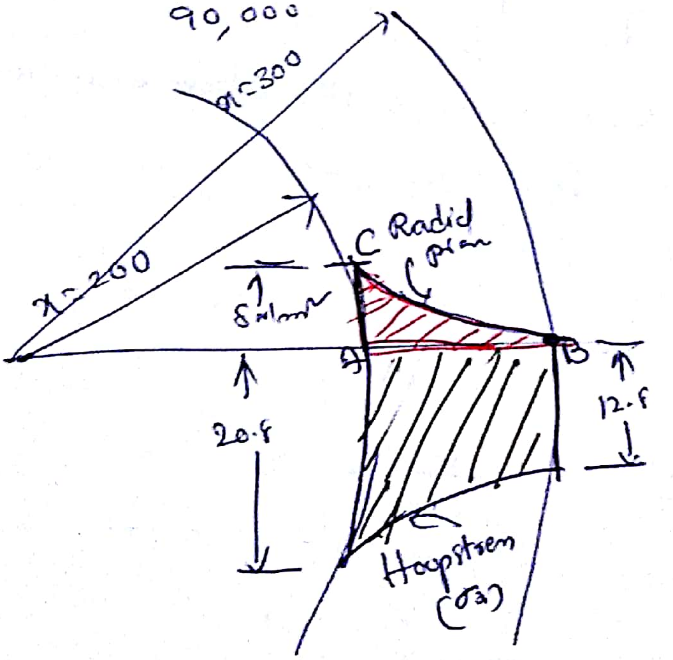
at $x = 200$ $\sigma_x = \dfrac{5,76,000}{(200)^2} + 6.4 = 20.8 \text{ N/mm}^2$

at $x = 300$ $\sigma_x = \dfrac{5,76,000}{(300)^2} + 6.4 = 12.8 \text{ N/mm}^2$

$$P_x = \frac{5,76,000}{x^2} - 6.4$$

at $x = 200$: $\dfrac{5,76,000}{40,000} - 6.4 = 8 \text{ N/mm}^2$

$x = 300$: $\dfrac{5,76,000}{90,000} - 6.4 = 0 \text{ N/mm}^2$



AB is taken a horizontal line

$AC = 8 \text{ N/mm}^2$

The variation between B and C is parabolic, the curve BC shows a variation of radial pressure across AB.

The curve DE which is also parabolic shows the variation of hoop stress across AB. Values $BD = 12.8 \text{ N/mm}^2$ $AE = 20.8 \text{ N/mm}^2$. The radial pressure is compressive where as the hoop stress is tensile.

Q. Find the thickness of metal necessary for a cylindrical shell of Internal diameter 160 mm to withstand an internal fluid pressure of 8 N/mm². The maximum hoop stress in section is not exceed 35 N/mm²

→ Given

$r_1 = 80\ mm$

$P_x = P_0 = 8\ N/mm^2$

Maximum hoop stress $\sigma_{x\,max} = 35\ N/mm^2$

$$P_x = \frac{b}{x^2} - a$$

$$\sigma_x : \frac{b}{x^2} + a$$

$x = 80\ mm \quad P_x = 8 \quad \& \quad \sigma_x = 35\ N/mm^2$

$$8 = \frac{b}{(80)^2} - a \qquad (8) \qquad \cancel{+27} = -2a \qquad \boxed{a = 13.5}$$

$$35 = \frac{b}{(80)^2} + a$$

$$43 = 2 \cdot \frac{b}{(80)^2} \Rightarrow b = \frac{43 \times (80)^2}{2} = 1,37,600$$

$$P_x = \frac{1,37,600}{x^2} - 13.5 \qquad \sigma_x = \frac{1,37,600}{x^2} + 13.5$$

$x = r_2 \quad P_x = 0$

$$0 = \frac{1,37,600}{r_2^2} - 13.5 \Rightarrow r_2^2 = \frac{1,37,600}{13.5}$$

$$r_2 = 100.96\ mm$$

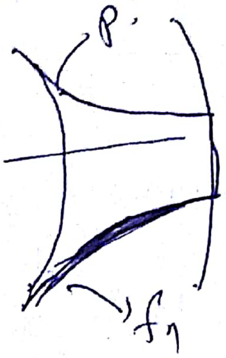
Thickness of the shell $t = r_2 - r_1$

$= 100.96 - 80 = 20.96\ mm$

H.W. Find the thickness of metal necessary for a cylindrical shell of Internal diameter 150 mm to withstand an internal pressure of 50 N/mm². The maximum hoop stress in the section is not to exceed 150 N/mm². [Ans : 31 mm]

Thick Compound cylinders :

In the thick cylinders when the cylindrical shell subjected to internal fluid pressure, the hoop stress is maximum at inner hoop circumference and it decreases towards the outer circumference.

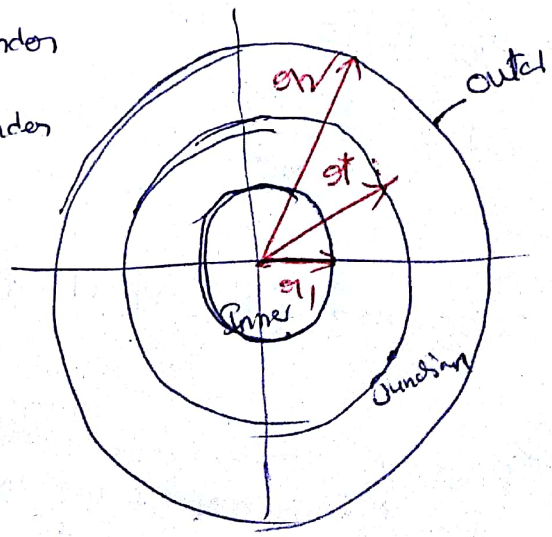


Hence the max. pressure inside the shell is limited corresponding to condition that the hoop stress at inner circumference reaches the permissible value.

But suppose the shell is made of shrinking one tube over the other. This will initially introduce hoop compressive stress in the inner tube and hoop tensile stresses in outer tube. If now the compound tube is subjected to internal pressure, both the inner and outer tubes will be subjected to hoop tensile stress due to internal pressure alone.

Adding the internal stresses caused while shrinking and the stresses due to internal pressure alone, the final hoop stresses can be determined.

A Fig. shows a compound thick cylinder made up of two cylinders.



Let r_2 = outer radius of compound cylinder

r_1 : inner radius of compound cylinder

r^* : radius of the junction of the two cylinders.

p^* = radial pressure at the junction of the two cylinders.

Let us apply Lame's equation, only shrinking.

1) $\boxed{\text{For outer cylinder}}$ $\quad P_r = \frac{b_1}{r^2} - a_1$ (i) $\quad \sigma_r = \frac{b_1}{r^2} + a_1$ (ii)

Lame's equation at a radius r for outer cylinders.

a_1, b_1 are Constants for outer cylinder.

At $r = r_2$ $P_x = 0$ and $r = r^*$ $P_x = p^*$

$$0 = \frac{b_1}{r_2^2} - a_1 \quad \text{(iii)} \qquad p^* = \frac{b_1}{r^{*2}} - a_1 \quad \text{(iv)}$$

From the equations (iii) & (iv) the Constant a_1, b_1 Can be determined. These values are Substituted in equation (ii) And then hoop stresses in the outer cylinder due to Shrinking Can be obtained.

(ii) **For inner cylinder**

$$P_x = \frac{b_2}{r^2} - a_2, \qquad \sigma_x = \frac{b_2}{r^2} + a_2$$

$r = r_1$, $P_x = 0$ as fluid Under pressure is not admitted into the cylinder.

$r = r^*$ $P_x = p^*$

$$0 = \frac{b_2}{r_2^2} - a_2 \quad \text{(v)} \qquad p^* = \frac{b_2}{r^{*2}} - a_2 \quad \text{(vi)}$$

From the equation (v) & (vi) the Constants a_2, b_2 Can be determined. From the equation $\sigma_x = \frac{b_2}{r^2} + a_2$, hoop Stress. These values are Substituted in inner cylinder are obtained. developed in inner cylinder due to internal pressure alone

Hoop Stresses in Compound cylinder due to internal pressure alone

When the fluid under pressure is admitted into the Compound cylinder. The hoop Stresses are set in the Compound cylinder. To find these stresses, the inner cylinder and outer cylinder will be Considered as one thick shell.

$$P_x = \frac{B}{r^2} - A \quad \text{(vii)} \qquad \sigma_x = \frac{B}{r^2} + A \quad \text{(viii)}$$

$r = r_2$ $P_x = 0$ $\qquad$ $r = r_1$ $P_x = P$

$$0 = \frac{B}{r_2} - A \quad \text{(ix)} \qquad \boxed{P = \frac{B}{r_1^2} - A} \quad \text{(x)}$$

From the equation (ix) and (x) the Constants A & B Can be determined. These values are Substituted in the equation (viii) and then hoop Stress across the Section Can be obtained.

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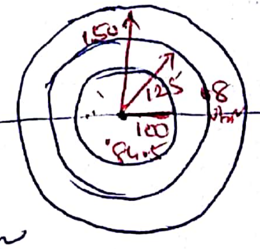
The resultant hoop stresses will be the algebraic sum of the hoop stresses caused due to shrinking and those due to internal fluid pressure.

1) A Compound cylinder is made by shrinking a cylinder of external diameter 300 mm and internal diameter of 250 mm. The radial pr at the junction after shrinking is 8 N/mm². The outer cylinder shrinking a another cylinder having outer diameter 250mm and internal diameter 200mm. Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm²

→



For outer cylinder

$r_2 = 150\text{mm}$ $\quad r^* = 125\text{mm}$

For internal cylinder

$r_1 = 100\text{ mm}$ $\quad r^* = 125\text{ mm}$ $\quad p^* = 8\ \text{N/mm}^2$

Fluid pressure in the compound cylinder $p = 84.5\ \text{N/mm}^2$

1) Stresses due to shrinking in the outer and inner cylinders before fluid admitted.

a) Lame's equation for outer cylinder

$P_x = \frac{b_1}{x^2} - a_1$ — (i) $\qquad \sigma_x = \frac{b_1}{x^2} + a_1$ — (ii)

$x = 150 \quad P_x = 0$

$0 = \frac{b_1}{(150)^2} - a_1$ — (iii) $\qquad 8 = \frac{b_1}{(125)^2} - a_1$ — (iv)

$b_1 = 409090.9$ $\qquad a_1 = 18.18$

$$\boxed{\sigma_x = \frac{409090.9}{x^2} + 18.18}$$

$\sigma_{150} \Rightarrow 36.36\ \text{N/mm}^2$

$\sigma_{125} \Rightarrow 44.36\ \text{N/mm}^2$

(ii) Lame's equation for inner cylinder

$P_x = \frac{b_2}{x^2} - a_2 \qquad \sigma_x = \frac{b_2}{x^2} + a_2$

$x = r_1 = 100\text{mm} \quad P_x = 0$

$0 = \frac{b_2}{100^2} - a_2 = \frac{b_2}{10000} - a_2$

$x = r^* \quad P_x = P = 8\ \text{N/mm}^2$

$8 = \frac{b_2}{125^2} - a_2$

$b_2 = -222222.2 \qquad a_2 = -22.22$

$$\sigma_{125} = \frac{-222222.2}{125^2} - 22.2$$
$$= -36.44 \text{ N/mm}^2 \text{ (Compressive)}$$

$$\sigma_{100} = \frac{-222222.2}{100^2} - 22.22 = -44.44 \text{ N/mm}^2 \text{ (Compressive)}$$

Stresses due to fluid pressure alone

$$P_x = \frac{B}{x^2} - A \qquad \sigma_x = \frac{B}{x^2} + A$$

$x = 150 \quad P_x = 0$

$x = 100 \quad P_x = P = 84.5$

$$0 = \frac{B}{150^2} - A = \frac{B}{22500} - A$$

$$84.5 = \frac{B}{10000} - A$$

$$B = 1521000$$
$$A = 67.6$$

$$\sigma_x = \frac{1521000}{x^2} + 67.6$$

$x = 100$

$$\sigma_{100} = \frac{1521000}{(100)^2} + 67.6 = 219.7 \text{ N/mm}^2$$

$$\sigma_{125} = \frac{1521000}{125^2} + 67.6 = 164.94 \text{ N/mm}^2$$

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6 = 135.2 \text{ N/mm}^2$$

The resultant Stresses

Inner cylinder: $\sigma_{100} = -44.44 + 219.7 = 175.26$ N/mm²

$\sigma_{125} = -36.44 + 164.94 = 128.5$ N/mm²

Outer cylinder:

$\sigma_{125} = 44.36 + 164.94 = 209.3$ N/mm²

$\sigma_{150} = 36.36 + 135.2 = 171.56$ N/mm²

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(H.W). A Compound tube is Composed of a tube 25 cm internal diameter and 2.5 cm thick shrunk on a tube of 25 cm external diameter and 2.5 cm thick. The radial pressure at the junction is 80 kg/cm². The compound tube is subjected to an internal fluid pressure of 845 kg/cm². Find the variation of the hoop stress over the wall of the compound tube.

Outer: σ_{15} : 363.6 + 1352.2 = 1715.8 kg/cm²

$\sigma_{12.5}$: 443.6 + 1650.1 = 2093.7 kg/cm²

Inner: $\sigma_{12.5}$ = −364.2 + 1650.1 = 1285.9 kg/cm²

σ_{10} : −444.2 + 2198.1 = 1753.9 kg/cm²