BUSN 33946 & ECON 35101

International Macroeconomics and Trade

Jonathan Dingel

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The University of Chicago Booth School of Business

Probabilistic Ricardian models

Today we study a class of Ricardian models that make more structural assumptions to facilitate quantitative exercises

- Eaton and Kortum (2012) call this "putting Ricardo to work"
- Probabilistic Ricardian models do not attempt to predict which countries make which goods
- Akin to Wilson (1980), we focus on what account of trade patterns is needed to conduct certain counterfactuals
- Borrow tools from discrete-choice models to deliver closed-form solutions that depend on few key parameters
- Over time, we've learned that some of this is not so different – an Armington or CGE model in Fréchet clothing

Armington model with CES prefs (Anderson 1979)

- Each country produces its own "signature" good (others have zero productivity in this good; maximal absolute advantage)
- Consumers in each country have identical CES preferences over the N goods with elasticity of substitution σ
- Bilateral trade costs of the iceberg form τ_{ij}
- Consumer demand (see Dingel 2009 for refresher)

$$X_{ij} = \frac{(p_i \tau_{ij})^{1-\sigma}}{\sum_\ell (p_\ell \tau_{\ell j})^{1-\sigma}} X_j = \frac{(p_i \tau_{ij})^{1-\sigma}}{P_j^{1-\sigma}} X_j$$

- Economy i endowed with Q_i units so GDP Y_i and $p_i = Y_i / Q_i$

$$X_{ij} = \frac{Y_i^{1-\sigma}}{Q_i^{1-\sigma}} \frac{X_j}{P_j^{1-\sigma}} \tau_{ij}^{1-\sigma}$$

- Balanced-trade equilibrium is $\{Y_i\}_i$ such that $X_i = Y_i = \sum_j X_{ij}$
- A Ricardian "corner" case gives us a "gravity" equation for multiple countries with trade costs that we can take to data

The logit model of discrete choice

Individual i considers choice j (see Train 2009)

- Utility $U_{ij} = V_{ij} + \epsilon_{ij}$
- Assume error is iid T1EV: $F(\epsilon_{ij}) = \exp(-\exp(-\epsilon_{ij}))$
- Choice probabilities are

$$\Pr(U_{ij} > U_{ij'} \ \forall j' \neq j) = \frac{\exp(V_{ij})}{\sum_{j'} \exp(V_{ij'})}$$

Now try a cost-minimization problem with multiplicative error term

- Cost $\ln c_{ji} = \ln c_j + \ln \tau_{ji} - \epsilon_{ji}$
- Least-cost probability

$$\begin{aligned}
&\Pr(-\ln c_{ji} > -\ln c_{j'i} \ \forall j' \neq j) \\
&= \Pr(\ln c_{ji} < \ln c_{j'i} \ \forall j' \neq j) = \frac{1/(c_j \tau_{ji})}{\sum_{j'} 1/(c_{j'} \tau_{j'i})}
\end{aligned}$$

Eaton & Kortum (Ecma 2002): Environment

Before we start: Questions about the big picture?

- N countries indexed by $i = 1, \dots, N$
- Continuum of goods indexed by $u \in [0, 1]$ [EK use "j"]
- CES preferences

$$U = \left(\int_0^1 Q(u)^{\frac{\sigma-1}{\sigma}} du \right)^{\frac{\sigma}{\sigma-1}}$$

- Trade costs "d_{ni}" (as in Train book) from i to n (contra "τ_{ij}")
- One factor of production labor with wage w_i (perhaps intermediate goods too)
- c_i is the unit cost of sole (or composite) input in i
- Perfect competition with good-specific idiosyncratic productivity $z_i(u)$ so that cost of delivering u to n from i is

$$c_{ni}(u) = c_i d_{ni} / z_i(u)$$

Eaton & Kortum (2002): Probabilistic technology

- $Z_i(u)$ is drawn independently from a Fréchet distribution

$$F_i(z) = \exp\left(-T_i z^{-\theta}\right), \qquad T_i > 0,\ \theta > \sigma - 1$$

- T_i is distribution's location parameter (shifts absolute advantage for all goods)
- θ is distribution's shape parameter (scope of comparative advantage)
- The Fréchet distribution is a "max stable" distribution, and this is the key property that delivers the results that follow

Price distribution

The distribution of prices offered by i to n is

$$G_{ni}(p) = \Pr(p_{ni}(u) \leq p) = \Pr\left(\frac{d_{ni}c_i}{z_i(u)} \leq p\right) = 1 - \Pr\left(z_i(u) \leq \frac{d_{ni}c_i}{p}\right)$$
$$= 1 - \exp\left(-T_i (d_{ni}c_i)^{-\theta} p^{\theta}\right)$$

The distribution of minimum prices in n is

$$G_n(p) = 1 - \prod_{i=1}^{N}(1 - G_{ni}(p)) = 1 - \prod_{i=1}^{N}\exp\left(-T_i (d_{ni}c_i)^{-\theta} p^{\theta}\right)$$
$$= 1 - \exp\left(-\sum_{i=1}^{N} T_i (d_{ni}c_i)^{-\theta} p^{\theta}\right)$$
$$= 1 - \exp\left(-\Phi_n p^{\theta}\right)$$

where Φ_n summarizes n's "market access", which depends on all partners' technologies, input costs, and bilateral trade costs

Allocation of purchases

Probability that i provides good u at the lowest price in n is

$$\pi_{ni} = \frac{T_i \left(d_{ni} c_i\right)^{-\theta}}{\Phi_n}$$

Proof:

$$\begin{aligned}
\pi_{ni} &= \Pr\left(p_{ni} \leq \min_{i' \neq i} p_{ni'}\right) = \prod_{i' \neq i} \Pr\left(p_{ni'} \geq p_{ni}\right) \\
&= \prod_{i' \neq i} \left[1 - G_{ni'}(p)\right] = \exp\left(-\Phi_{n,\neg i} p_{ni}^{\theta}\right)
\end{aligned}$$

where $\Phi_{n,\neg i} \equiv \sum_{i' \neq i} T_{i'} \left(d_{ni'} c_{i'}\right)^{-\theta}$. Integrate over all p.

$$\begin{aligned}
&\int_0^\infty \exp\left(-\Phi_{n,\neg i} p^{\theta}\right) dG_{ni}(p) \\
&= \int_0^\infty \exp\left(-\Phi_{n,\neg i} p^{\theta}\right) \theta p^{\theta-1} T_i \left(d_{ni} c_i\right)^{-\theta} \exp\left(-T_i \left(d_{ni} c_i\right)^{-\theta} p^{\theta}\right) dp \\
&= T_i \left(d_{ni} c_i\right)^{-\theta} \int_0^\infty \exp\left(-\Phi_n p^{\theta}\right) \theta p^{\theta-1} dp = \pi_{ni} \int_0^\infty dG_n(p) = \pi_{ni}
\end{aligned}$$

Bilateral import price distributions

Prices of goods imported by n from i also have distribution $G_n(p)$.
What is probability i is least-cost supplier at price p?

- Goods imported by n from i also have price distribution $G_n(p)$.
- What is probability i is least-cost supplier given its price is q?
  $\Pr\left(q \leq \min_{i' \neq i} p_{ni'}\right) = \exp\left(-\Phi_{n,\neg i} q^{\theta}\right)$
- Joint probability that i is least-cost supplier at price q is
  $\exp\left(-\Phi_{n,\neg i} q^{\theta}\right) dG_{ni}(q)$
- Integrate this probability from 0 to p to find CDF of bilateral import prices

$$\int_0^p \exp\left(-\Phi_{n,\neg i} q^{\theta}\right) dG_{ni}(q) = \pi_{ni} G_n(p)$$

- Since π_{ni} is the probability that any good is imported from i, the conditional distribution of the price paid by n for goods actually imported from i is $G_n(p)$

All action is on the extensive margin of varieties purchased. Price distributions are independent of exporter. Expenditure shares are π_{ni}.

The CES price index

- The CES price index in country n is

$$P_n^{1-\sigma} = \int_0^1 p_n^{1-\sigma}(u)du = \int_0^\infty p^{1-\sigma} dG_n(p)$$
$$= \int_0^\infty p^{1-\sigma} e^{-\Phi_n p^\theta} \Phi_n \theta p^{\theta-1} dp$$

- Change of variable $x = \Phi_n p^\theta \Rightarrow dx = \Phi_n \theta p^{\theta-1} dp$

$$P_n^{1-\sigma} = \int_0^\infty \left(\frac{x}{\Phi_n}\right)^{\frac{1-\sigma}{\theta}} e^{-x} dx = \Phi_n^{-\frac{1-\sigma}{\theta}} \int_0^\infty x^{\frac{1-\sigma}{\theta}} e^{-x} dx$$

- Integral is finite if $\theta > \sigma - 1$

$$P_n = \Phi_n^{-\frac{1}{\theta}} \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)$$

where $\Gamma()$ is the gamma function, $\Gamma(a) \equiv \int_0^\infty x^{a-1} \exp(-x) dx$

Equilibrium

- Let X_{ni} be n's expenditure on imports from i
- $X_n \equiv \sum_i X_{ni}$ is n's total expenditure
- Since $X_{ni}/X_n = \pi_{ni}$, we obtain a gravity equation

$$X_{ni} = \frac{T_i \left(d_{ni} c_i\right)^{-\theta}}{\Phi_n} X_n = T_i c_i^{-\theta} \frac{X_n}{\Phi_n} d_{ni}^{-\theta}$$

- Suppose no intermediate goods so that $c_i = w_i$.
- The value of sales by i is $w_i L_i = \sum_n X_{ni}$
- By budget balance, $w_i L_i = X_i$.
- We get a system of equations in the wage vector

$$w_i L_i = \sum_n \frac{T_i \left(d_{ni} w_i\right)^{-\theta}}{\sum_j T_i \left(d_{nj} w_{ij}\right)^{-\theta}} w_n L_n$$

We will discuss gravity more in week 4. Note the similarity to Armington system of equations.

Estimating the trade elasticity θ

Write bilateral trade flows as "normalized import shares"

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{P_i d_{ni}}{P_n}\right)^{-\theta}$$

$$\ln\left(X_{ni}X_i/X_nX_{ii}\right) = -\theta \ln P_i + \theta \ln P_n - \theta \ln d_{ni}$$

The trade elasticity $-\theta$ governs how bilateral trade flows respond to bilateral trade costs. Note the absence of preference parameter σ.

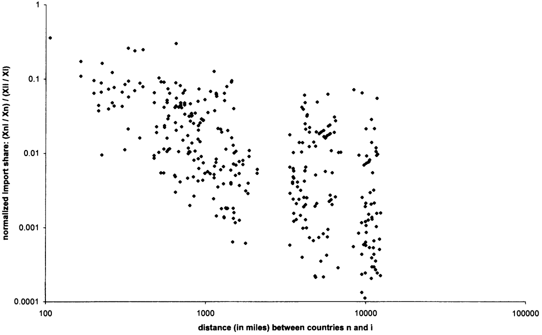

FIGURE 1.—Trade and geography.

A proxy for d_{ni} (e.g., distance) won't deliver an estimate of θ

Estimating the trade elasticity θ

Eaton and Kortum (2002) infer d_{ni} from price data for 50 goods and a price-differential inequality: $p_n(u)/p_i(u) \leq d_{ni}$

$$\widehat{\ln d_{ni}} = \max 2_u \left\{ \ln p_n(u) - \ln p_i(u) \right\}$$

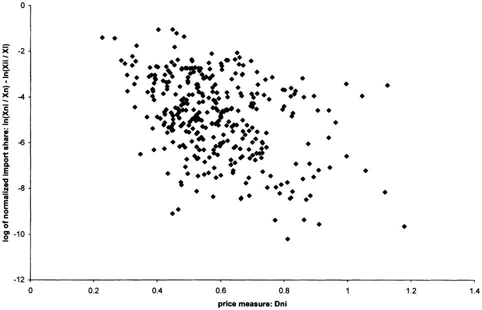

FIGURE 2.—Trade and prices.

- Eaton and Kortum (2002) get $\hat{\theta} = 8.28$.
- Simonovska and Waugh (2014) refine max estimator of the inequality: $\hat{\theta} \in [3, 5]$
- See Atkin and Donaldson (2015) on inferring trade costs from price gaps

Intermediates à la Ethier (1982)

- Imagine that goods are produced using labor and a composite intermediate that is a CES aggregate coinciding with the consumption good (as in Ethier AER 1982)
- Thus, the composite may be consumed or used as an intermediate
- Assume Cobb-Douglas: $c_i = w_i^{\beta} p_i^{1-\beta}$

Eaton and Kortum (2002) counterfactuals

Three counterfactuals in Eaton and Kortum (2002):

- Autarky: $d_{ni} \to \infty \ \forall i \neq n$. Single-digit percentage-point welfare loss for most countries.
- Free trade: $d_{ni} = 1 \ \forall n, i$. Welfare gains on order of 20%.
- Technological advances: Increase T_{US} and T_{Germany} 20%. Favors trading partners.

We have since learned from Dekle, Eaton, and Kortum (2008) that some of this model's counterfactual predictions can be obtained without knowing all parameter values by a procedure that we now call "exact hat algebra"

Exact hat algebra (in traditional "ij" notation)

Start from the market-clearing condition and the gravity equation:

$$w_i L_i = \sum_{j=1}^{N} \lambda_{ij} w_j L_j \qquad \lambda_{ij} = \frac{T_i \left(\tau_{ij} w_i\right)^{-\epsilon}}{\sum_{l=1}^{N} T_l \left(\tau_{lj} w_l\right)^{-\epsilon}}$$

We consider a shock to $\hat{T}_i \equiv \frac{T_i'}{T_i}$. By assumption, $\hat{\tau} = 1$ and $\hat{L} = 1$. We want to solve for the endogenous variables $\hat{\lambda}_{ij}, \hat{X}_{ij}$ and $\hat{w}_i$. In the following derivation, define "sales shares" by $\gamma_{ij} \equiv \frac{X_{ij}}{Y_i}$.

$$w_i L_i = \sum_{j=1}^{N} \lambda_{ij} w_j L_j, \quad w_i' L_i' = \sum_{j=1}^{N} \lambda_{ij}' w_j' L_j = \sum_{j=1}^{N} X_{ij}'$$

$$\hat{w}_i \hat{L}_i = \sum_{j=1}^{N} \frac{X_{ij}'}{w_i L_i} = \sum_{j=1}^{N} \frac{X_{ij}}{w_i L_i} \hat{X}_{ij} \equiv \sum_{j=1}^{N} \gamma_{ij} \hat{X}_{ij} \tag{1}$$

Exact hat algebra (in traditional "ij" notation)

$$\lambda_{ij} = \frac{T_i \left(\tau_{ij} w_i\right)^{-\epsilon}}{\sum_{l=1}^{N} T_l \left(\tau_{lj} w_l\right)^{-\epsilon}}, \quad \lambda'_{ij} = \frac{T'_i \left(\tau_{ij} w'_i\right)^{-\epsilon}}{\sum_{l=1}^{N} T'_l \left(\tau_{lj} w'_l\right)^{-\epsilon}}$$

$$\hat{\lambda}_{ij} \equiv \frac{\lambda'_{ij}}{\lambda_{ij}} = \hat{T}_i \hat{w}_i^{-\epsilon} \hat{\tau}_{ij}^{-\epsilon} \frac{\sum_{l=1}^{N} T_l \left(\tau_{lj} w_l\right)^{-\epsilon}}{\sum_{l=1}^{N} T'_l \left(\tau_{lj} w'_l\right)^{-\epsilon}} = \frac{\hat{T}_i \hat{w}_i^{-\epsilon} \hat{\tau}_{ij}^{-\epsilon}}{\sum_{l=1}^{N} \lambda_{lj} \hat{T}_l \hat{w}_l^{-\epsilon} \hat{\tau}_{lj}^{-\epsilon}} \tag{2}$$

Combining equations (1) and (2) under the assumptions that $\hat{Y}_i = \hat{X}_i$ and $\hat{\tau} = \hat{L} = 1$, we obtain a system of equation characterizing an equilibrium $\hat{w}_i$ as a function of shocks $\hat{T}_i$, initial equilibrium shares λ_{ij} and γ_{ij}, and the trade elasticity ϵ:

$$\hat{w}_i \hat{L}_i = \sum_{j=1}^{N} \gamma_{ij} \hat{X}_{ij} = \sum_{j=1}^{N} \gamma_{ij} \hat{\lambda}_{ij} \hat{w}_j \Rightarrow \hat{w}_i = \sum_{j=1}^{N} \frac{\gamma_{ij} \hat{T}_i \hat{w}_i^{-\epsilon} \hat{w}_j}{\sum_{l=1}^{N} \lambda_{lj} \hat{T}_l \hat{w}_l^{-\epsilon}}$$

If we use data to pin down ϵ, λ_{ij}, and γ_{ij}, then we can feed in productivity shocks $\hat{T}$ and solve for $\hat{w}$.

Can you comptue the free-trade counterfactual outcome? No.

Empirical estimation of Ricardian predictions

Alan Deardorff (*Handbook*, 1984) on "Testing Trade Theories and Predicting Trade Flows":

> *The intuitive content of most trade theories is quite simple and straightforward... seldom stated in forms that are compatible with the real world complexities that empirical research cannot escape.*

- Given difficulty of testing $p^a \cdot T \leq 0$, we typically model p^a as function of primitives
- In the Ricardian model, this seems simple: relative prices equal relative labor costs (in both trade and autarky)
- Model predicts which goods countries trade (not with whom or how much)

Empirical challenges for Ricardian models

- Specialization is selection: If countries don't produce some goods in the trade equilibrium, we cannot infer relative labor costs.[1]
  - In fact, data suggests that countries aren't specializing at commodity-code level (intraindustry trade)
- Suspicions that relative labor costs in trade equilibrium do not reveal relative labor costs in autarky
  - Multiple factors of production
  - Relative costs endogenous to trade flows
- Dimensionality mismatch: How to take two-country predictions to many-country data?

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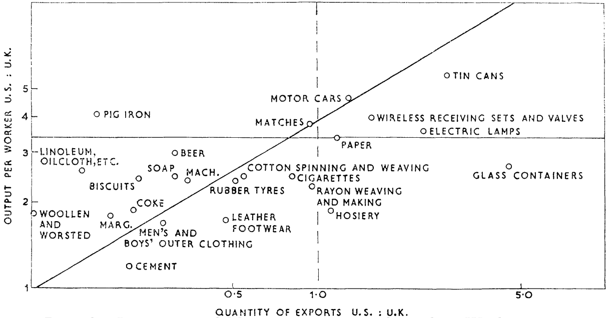
[1] Sattinger (1993, p.832): "Empirical modeling of the distribution of earnings requires the econometric specification of worker alternatives, even though only the chosen sector or job is observed. This generates a set of econometric problems that have been addressed in applications of Roy's and Tinbergen's models."

Ad hoc regressions

Challenges evident in early empirical work (Deardorff 1984)

- MacDougall (1951, 1952), Stern (1962), and Balassa (1963) regress relative exports volumes on relative productivities
- 95% of US and UK trade are with third markets, so regress relative exports to third markets on relative productivities
- Absent trade costs, the Ricardian model predicts no overlap in exports to third markets



Regressions of exports on sector-country interactions

A wave of papers in the 2000s examined sector-country interactions to test whether observable sources of comparative advantage govern trade patterns

- Rajan and Zingales (1998) is early, non-trade exemplar
- Romalis (2004) interacts sectoral factor intensity with country factor abundance (we discuss factor-proportions theory in week 5)
- Costinot (2009) provides the general logic: log-supermodularity
- This has been described as "the typical way trade economists would explore" comparative advantage
- Ciccone and Papaioannou raise concern that sectoral characteristics aren't commonly ordered across countries
- As one example of these trade papers, Nunn (2007) looks at countries' contract enforcement and the relationship specificity of goods' intermediate inputs

Nunn (2007)

"I find that countries with better contract enforcement export relatively more in industries for which relationship-specific investments are most important"

$$\ln x_{ic} = \alpha_i + \alpha_c + \beta_1 z_i Q_c + \beta_2 h_i H_c + \beta_3 k_i K_c + \epsilon_{ic},$$

where z_i is inputs' relationship specificity, h_i is skill intensity, and k_i is capital intesity

- z_i is average of inputs' Rauch (1999) indicators weighted by input cost shares from US input-output table
- Rauch (1999) indicator say commodity is neither 'sold on an organized exchange' nor 'reference priced in industry journals'
- Q_c is investors' perceptions from World Bank survey; uses legal origin as IV for Q_c
- No explicit model linking lower relative unit costs to export volume; appeals to Romalis (2004) two-country model

Nunn (2007): Sectoral characteristic

TABLE II
THE TWENTY LEAST AND TWENTY MOST CONTRACT INTENSIVE INDUSTRIES

Least contract intensive: lowest z_i^{rs1}		Most contract intensive: highest z_i^{rs1}	
z_i^{rs1}	Industry description	z_i^{rs1}	Industry description
.024	Poultry processing	.810	Photographic & photocopying equip. manuf.
.024	Flour milling	.819	Air & gas compressor manuf.
.036	Petroleum refineries	.822	Analytical laboratory instr. manuf.
.036	Wet corn milling	.824	Other engine equipment manuf.
.053	Aluminum sheet, plate & foil manuf.	.826	Other electronic component manuf.
.058	Primary aluminum production	.831	Packaging machinery manuf.
.087	Nitrogenous fertilizer manufacturing	.840	Book publishers
.099	Rice milling	.851	Breweries
.111	Prim. nonferrous metal, excl. copper & alum.	.854	Musical instrument manufacturing
.132	Tobacco stemming & redrying	.872	Aircraft engine & engine parts manuf.
.144	Other oilseed processing	.873	Electricity & signal testing instr. manuf.
.171	Oil gas extraction	.880	Telephone apparatus manufacturing
.173	Coffee & tea manufacturing	.888	Search, detection, & navig. instr. manuf.
.180	Fiber, yarn, & thread mills	.891	Broadcast & wireless comm. equip. manuf.
.184	Synthetic dye & pigment manufacturing	.893	Aircraft manufacturing
.190	Synthetic rubber manufacturing	.901	Other computer peripheral equip. manuf.
.195	Plastics material & resin manuf.	.904	Audio & video equipment manuf.
.196	Phosphatic fertilizer manufacturing	.956	Electronic computer manufacturing
.200	Ferroalloy & related products manuf.	.977	Heavy duty truck manufacturing
.200	Frozen food manufacturing	.980	Automobile & light truck manuf.

The contract intensity measures reported are rounded from seven digits to three digits.

Nunn (2007): “Determinants of Comparative Advantage”

TABLE IV
THE DETERMINANTS OF COMPARATIVE ADVANTAGE

	(1)	(2)	(3)	(4)	(5)
Judicial quality interaction: $z_i Q_c$	.289**	.318**	.326**	.235**	.296**
	(.013)	(.020)	(.023)	(.017)	(.024)
Skill interaction: $h_i H_c$			.085**		.063**
			(.017)		(.017)
Capital interaction: $k_i K_c$			.105**		.074
			(.031)		(.041)
Log income × value added: $va_i \ln y_c$				−.117*	−.137*
				(.047)	(.067)
Log income × intra-industry trade: $iit_i \ln y_c$				.576**	.546**
				(.041)	(.056)
Log income × TFP growth: $\Delta tfp_i \ln y_c$				.024	−.010
				(.033)	(.049)
Log credit/GDP × capital: $k_i CR_c$				.020	.021
				(.012)	(.018)
Log income × input variety: $(1 - hi_i) \ln y_c$				.446**	.522**
				(.075)	(.103)
Country fixed effects	Yes	Yes	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes	Yes	Yes
R^2	.72	.76	.76	.77	.76
Number of observations	22,598	10,976	10,976	15,737	10,816

Dependent variable is $\ln x_{ic}$. The regressions are estimates of (1). The dependent variable is the natural log of exports in industry i by country c to all other countries. In all regressions the measure of contract intensity used is z_i^{rs1}. Standardized beta coefficients are reported, with robust standard errors in brackets. * and ** indicate significance at the 5 and 1 percent levels.

CDK: “A Quantitative Exploration of Ricardo’s Ideas”

Costinot, Donaldson, and Komunjer (2012) introduce a multi-sector model in which each sector behaves like Eaton and Kortum (2002)

- EK’s Ricardian model says nothing about a key Ricardian question: what’s the pattern of specialization and trade?
- In CDK, while specialization within industries is indeterminate, now model predicts (aggregate) sectoral trade flows

The structural model guides the empirical estimation

- Derive a valid estimating equation from theory (and contrast with ad hoc regressions)
- Think about the error term’s contents and plausibility of orthogonality requirements
- Explicitly tackle the selection problem associated with unobserved productivities
- Quantify welfare importance of Ricardian comparative advantage under model assumptions

CDK model: Technology

- Index countries by i, industries by k, and varieties by ω
- Labor is sole factor of production, endowed in quantity L_i and paid wage w_i
- Unit cost is $w_i/z_i^k(\omega)$ with productivity $z_i^k(\omega)$ randomly drawn
- CDK's notation for the Fréchet distribution is

$$F_i^k(z) = \exp\left[-\left(z/z_i^k\right)^{-\theta}\right]$$

- z_i^k is the "fundamental productivity", an industry-country location parameter, that generates Ricardian comparative advantage by sector
- θ governs idiosyncratic comparative advantage across varieties within sector, as in EK (2002). Note that it does not vary.

CDK model: Rest of the setup

- Iceberg trade costs d_{ij}^k from i to j with $d_{ii}^k = 1$ and triangle inequality
- Perfect competition: $p_j^k(\omega) = \min_i \left[c_{ij}^k(\omega)\right] = \min_i \left[w_i d_{ij}^k / z_i^k(\omega)\right]$
- See paper for Bertrand competition case (à la BEJK 2003)
- Preferences: Cobb-Douglas upper tier and CES lower tier

$$x_j^k(\omega) = \left[\frac{p_j^k(\omega)}{p_j^k}\right]^{1-\sigma_j^k} \alpha_j^k w_j L_j$$

- Trade is balanced (*not* sector by sector!)

CDK Lemma 1: Trade and fundamental productivities

Start from gravity equation:

$$x_{ij}^k = \frac{\left(w_i d_{ij}^k / z_i^k\right)^{-\theta}}{\sum_{i'} \left(w_i d_{ij}^k / z_i^k\right)^{-\theta}} \alpha_j^k w_j L_j$$

This implies a difference-in-differences version:

$$\ln\left(\frac{x_{ij}^k x_{i'j}^{k'}}{x_{ij}^{k'} x_{i'j}^k}\right) = \theta \ln\left(\frac{z_i^k z_{i'}^{k'}}{z_i^{k'} z_{i'}^k}\right) - \theta \ln\left(\frac{d_{ij}^k d_{i'j}^{k'}}{d_{ij}^{k'} d_{i'j}^k}\right)$$

But we don't observe z_i^k and we need a measure of d_{ij}^k

- We cannot observe fundamental productivity $z_i^k = \mathbb{E}\left[z_i^k(\omega)\right]$
- We observe the endogenous object $\tilde{z}_i^k = \mathbb{E}\left[z_i^k(\omega) | \Omega_i^k\right]$ where Ω_i^k is the set of varieties produced in equilibrium

CDK Theorem 1: Trade and observed productivities

CDK show that

$$\frac{\tilde{z}_i^k}{\tilde{z}_{i'}^k} = \left(\frac{z_i^k}{z_{i'}^k}\right)\left(\frac{\pi_{ii}^k}{\pi_{i'i'}^k}\right)^{-1/\theta}$$

Plug that in

$$\ln\left(\frac{\tilde{x}_{ij}^k \tilde{x}_{i'j}^{k'}}{\tilde{x}_{ij}^{k'} \tilde{x}_{i'j}^k}\right) = \theta \ln\left(\frac{\tilde{z}_i^k \tilde{z}_{i'}^{k'}}{\tilde{z}_i^{k'} \tilde{z}_{i'}^k}\right) - \theta \ln\left(\frac{d_{ij}^k d_{i'j}^{k'}}{d_{ij}^{k'} d_{i'j}^k}\right)$$

where $\tilde{x}_{ij}^k = x_{ij}^k / \pi_{ii}^k$

- Special case of $d_{ij}^k = d_{ij} d_j^k$ is illuminating
- We get pairwise predictions that feel like 2-by-2 Ricardian story but are for quantities, destination by destination

Can also state as gravity regression

$$\ln \tilde{x}_{ij}^k = \gamma_{ij} + \gamma_j^k + \theta \ln \tilde{z}_i^k - \theta \ln d_{ij}^k$$

CDK's structural answers to specification questions

Gravity regression, so this isn't a test of Ricardian story vs other stories that deliver a similar gravity equation.

$$\ln\left(x_{ij}^k/\pi_{ii}^k\right) = \gamma_{ij} + \gamma_j^k + \theta \ln \tilde{z}_i^k - \theta \ln d_{ij}^k$$

But it gives structural answers to many questions that lurk in prior empirical work.

- What's the appropriate dependent variable?
- How do we aggregate across multiple countries/destinations?
- What is the meaning of the slope coefficient?
- Levels vs logs vs semi-log
- What fixed effect are required?
- What is in the error term?

Data on productivity $\tilde{z}_i^k$

- Tough to compare productivity across industries and countries (need producer price deflators to get physical quantities)
- CDK use International Comparisions of Output and Productivity (ICOP) Industry Database from GGDC (Groningen)
- 1997 cross section has 21 OECD countries for 13 manufacturing industries
- Since wages are common across industries in a one-factor model, relative producivity $\tilde{z}_i^k$ shows up straightforwardly in relative (inverse) producer prices

Estimating equation

Empirical specification is

$$\ln\left(x_{ij}^k/\pi_{ii}^k\right) = \gamma_{ij} + \gamma_j^k + \theta \ln \tilde{z}_i^k + \epsilon_{ij}^k$$

- Given fixed effects, log producer price $\ln p_i^k$ is a measure of $-\ln \tilde{z}_i^k$
- Industry-pair specific trade costs $\ln d_{ij}^k$ are sitting in the error term, threating OLS assumption that $\mathbb{E}\left[\ln p_i^k \epsilon_{ij}^k | \gamma_{ij}, \gamma_j^k\right] = 0$
- (Classical) measurement error in $\ln p_i^k$ will attenuate $\hat{\theta}$
- Exporting and productivity may be simultaneously determined
- CDK employ R&D expenditure as IV for productivity (why isn't R&D endogenous to trade?)

Estimates of θ

TABLE 3
Cross-sectional results—baseline

Dependent variable	log (corrected exports)	log (exports)	log (corrected exports)	log (exports)
	(1)	(2)	(3)	(4)
log (productivity based on producer prices)	1·123***	1·361***	6·534***	11·10***
	(0·0994)	(0·103)	(0·708)	(0·981)
Estimation method	OLS	OLS	IV	IV
Exporter × importer fixed effects	YES	YES	YES	YES
Industry × importer fixed effects	YES	YES	YES	YES
Observations	5652	5652	5576	5576
R^2	0·856	0·844	0·747	0·460

Notes: Regressions estimating equation (18) using data from 21 countries and 13 manufacturing sectors (listed in Table 1) in 1997. "Exports" is the value of bilateral exports from the exporting country to the importing country in a given industry. "Corrected exports" is "exports" divided by the share of the exporting country's total expenditure in the given industry that is sourced domestically (equal to one minus the country and industry's IPR). "Productivity based on producer prices" is the inverse of the average producer price in an exporter–industry. Columns (3) and (4) use the log of 1997 R&D expenditure as an instrument for productivity. Data sources and construction are described in full in Section 4.1. Heteroskedasticity-robust standard errors are reported in parentheses. ***Statistically significantly different from zero at the 1% level.

CDK's welfare counterfactuals

- "According to our estimates, the removal of Ricardian comparative advantage at the industry level would only lead, on average, to a 5.3% decrease in the total gains from trade."
- Some countries actually gain from eliminating comparative advantage
- See section 5.3 discussion of heterogeneous trade costs and heterogeneous tastes as potential explanations
- The key is that CDK's double-differenced gravity regression did not restrict Cobb-Douglas shares nor $\ln d_{ij}^k$

Caliendo & Parro (2015) model

Multi-sector extension of Eaton & Kortum (2002) with intersectoral linkages used to assess NAFTA (Antras & Chor 2021 notation)

- EK assumptions sector-by-sector: CES aggregators over varieties, Fréchet distributions of productivities (T_i^s, θ^s)
- Cobb-Douglas preferences (α_j^s) and production functions (γ_j^{sr})

$$c_j^s = \Gamma_j^s w_j^{1-\sum_{r=1}^S \gamma_j^{rs}} \prod_{r=1}^{S} \left(P_j^r\right)^{\gamma_j^{rs}}$$

- Gravity equation for sectoral expenditure share:

$$\pi_{ij}^s = \frac{T_i^s \left(c_i^s \tau_{ij}^s\right)^{-\theta^s}}{\sum_{k=1}^J T_k^s \left(c_k^s \tau_{kj}^s\right)^{-\theta^s}}$$

- Clear markets for each industry in each country

$$X_j^s = \sum_{r=1}^{S} \gamma_{j^{sr}} \underbrace{\sum_{i=1}^{J} X_i^r \pi_{ji}^r}_{\text{gross output } Y_j^r} + \alpha_j^s \left(w_j L_j + D_j\right)$$

CP 2015: Estimation, calibration, counterfactuals

- Estimate sectoral trade elasticities θ^s using a gravity equation assuming non-tariff trade costs are symmetric (see next week)
- Compute counterfactual by writing everything in relative changes and initial shares (extends DEK's "exact hat algebra" to multi-sector model)
- Use 1993 baseline for 30 countries and 40 sectors
- Compute counterfactual for all tariff changes 1993-2005 and for NAFTA tariff changes 1993-2005
- Some trade diversion, but effects on rest of world are small
- Larger welfare and trade effects than in one-sector and no-input-output-linkages models

Caliendo & Parro (2015) caveats

- Final-use and intermediate sourcing shares are identical (see Antras and Chor 2021 on extensions that relax)
- Roundabout production structure: goods are produced via an endless sequence of steps, with each stage using inputs from prior stages in an infinite loop
- Compute counterfactuals "without needing to estimate parameters which are difficult to identify in the data, as productivities and iceberg trade costs" (p.11)
- Antras and Chor (2021): "the lack of 'external' evidence supporting the out-of-sample performance of these models remains problematic, and a clear area with room for improvement in future research"

Next week

- Gravity and gains from trade in more detail
- Submit your Armington model comprehension check
- Submit your DFS 1977 computational assignment