Assignment 3, part 1 of 2

1. Using the notation seen in class, let $\mathcal{L}_3 = \{\{1,2,3\}, \{1,2,5\}, \{1,3,4\}, \{2,3,4\}, \{2,4,5\}, \{2,5,6\}\}$. Compute the set of candidates $\mathcal{C}_4$.

The set of candidates $\mathcal{C}_4$ obtained by joining every joinable pair of itemsets from $\mathcal{L}_3$:

$\mathcal{C}_4 = \{\{1,2,3,5\}, \{2,3,4,5\}, \{1,2,3,4\}, \{2,4,5,6\}, \{1,2,4,5\}, \{1,2,5,6\}\}$

However, the 3-itemsets {2,3,5}, {3,4,5}, {1,2,4}, {4,5,6}, {1,2,4}, {1,2,6} are not frequent from each set in $\mathcal{C}_4$ respectively, since they do not belong to $\mathcal{L}_3$. The Apriori property guarantees that the 4-itemsets obtained in $\mathcal{C}_4$ are also not frequent.

Therefore, C4 = ∅.

2. In the transaction dataset used in the tutorial presented above, for a support threshold of 40%, is the itemset {{Kidney Beans}, {Eggs}, {Yogurt}} considered frequent?

CODE:

```
dataset = [['Milk', 'Onion', 'Nutmeg', 'Kidney Beans', 'Eggs', 'Yogurt'],
           ['Dill', 'Onion', 'Nutmeg', 'Kidney Beans', 'Eggs', 'Yogurt'],
           ['Milk', 'Apple', 'Kidney Beans', 'Eggs'],
           ['Milk', 'Unicorn', 'Corn', 'Kidney Beans', 'Yogurt'],
           ['Corn', 'Onion', 'Onion', 'Kidney Beans', 'Ice cream', 'Eggs']]

from mlxtend.preprocessing import TransactionEncoder

te = TransactionEncoder()
te_ary = te.fit_transform(dataset)
print(te_ary)
```

OUTPUT:

```
[[False False False  True False  True  True  True  True False  True]
 [False False  True  True False  True False  True  True False  True]
 [ True False False  True False  True  True False False False False]
 [False  True False False False  True  True False False  True  True]
 [False  True False  True  True  True False False  True False False]]
```

CODE:

```
import pandas as pd
```

```
df = pd.DataFrame(te_ary, columns=te.columns_)
display(df)
```

OUTPUT:

```
   Apple   Corn   Dill   Eggs  Ice cream  Kidney Beans   Milk  Nutmeg  Onion  \
0  False  False  False   True      False          True   True    True   True
1  False  False   True   True      False          True  False    True   True
2   True  False  False   True      False          True   True   False  False
3  False   True  False  False      False          True   True   False  False
4  False   True  False   True       True          True  False   False   True

   Unicorn  Yogurt
0    False    True
1    False    True
2    False   False
3     True    True
4    False   False
```

CODE:

```
from mlxtend.frequent_patterns import apriori

frequent_itemsets = apriori(df, min_support=0.4)
display(frequent_itemsets)

itemset = frequent_itemsets.loc[5]
print('Itemset: {0}. Support: {1}.'.format(itemset['itemsets'],
itemset['support']))
```

OUTPUT:

```
    support         itemsets
0       0.4              (1)
1       0.8              (3)
2       1.0              (5)
3       0.6              (6)
4       0.4              (7)
5       0.6              (8)
6       0.6             (10)
7       0.4           (1, 5)
8       0.8           (3, 5)
9       0.4           (3, 6)
10      0.4           (3, 7)
11      0.6           (8, 3)
12      0.4          (10, 3)
13      0.6           (5, 6)
14      0.4           (5, 7)
```

```
15       0.6            (8, 5)
16       0.6           (10, 5)
17       0.4           (10, 6)
18       0.4            (8, 7)
19       0.4           (10, 7)
20       0.4           (8, 10)
21       0.4         (3, 5, 6)
22       0.4         (3, 5, 7)
23       0.6         (8, 3, 5)
24       0.4        (10, 3, 5)
25       0.4         (8, 3, 7)
26       0.4        (10, 3, 7)
27       0.4        (8, 10, 3)
28       0.4        (10, 5, 6)
29       0.4         (8, 5, 7)
30       0.4        (10, 5, 7)
31       0.4        (8, 10, 5)
32       0.4        (8, 10, 7)
33       0.4      (8, 3, 5, 7)
34       0.4     (10, 3, 5, 7)
35       0.4     (8, 10, 3, 5)
36       0.4     (8, 10, 3, 7)
37       0.4     (8, 10, 5, 7)
38       0.4  (3, 5, 7, 8, 10)

Itemset: frozenset({8}). Support: 0.6.
```

CODE:

```
frequent_itemsets = apriori(df, min_support=0.4, use_colnames=True)
display(frequent_itemsets)
```

OUTPUT:

```
    support                                          itemsets
0       0.4                                            (Corn)
1       0.8                                            (Eggs)
2       1.0                                    (Kidney Beans)
3       0.6                                            (Milk)
4       0.4                                          (Nutmeg)
5       0.6                                           (Onion)
6       0.6                                          (Yogurt)
7       0.4                              (Kidney Beans, Corn)
8       0.8                              (Kidney Beans, Eggs)
9       0.4                                      (Milk, Eggs)
10      0.4                                    (Nutmeg, Eggs)
11      0.6                                     (Onion, Eggs)
12      0.4                                    (Eggs, Yogurt)
```

```
13      0.6                    (Milk, Kidney Beans)
14      0.4                  (Nutmeg, Kidney Beans)
15      0.6                   (Onion, Kidney Beans)
16      0.6                  (Kidney Beans, Yogurt)
17      0.4                          (Milk, Yogurt)
18      0.4                         (Onion, Nutmeg)
19      0.4                        (Nutmeg, Yogurt)
20      0.4                         (Onion, Yogurt)
21      0.4              (Milk, Kidney Beans, Eggs)
22      0.4            (Nutmeg, Kidney Beans, Eggs)
23      0.6             (Onion, Kidney Beans, Eggs)
24      0.4            (Eggs, Kidney Beans, Yogurt)
25      0.4                   (Onion, Eggs, Nutmeg)
26      0.4                  (Nutmeg, Eggs, Yogurt)
27      0.4                   (Onion, Eggs, Yogurt)
28      0.4            (Milk, Kidney Beans, Yogurt)
29      0.4           (Onion, Kidney Beans, Nutmeg)
30      0.4          (Nutmeg, Kidney Beans, Yogurt)
31      0.4           (Onion, Kidney Beans, Yogurt)
32      0.4                 (Onion, Yogurt, Nutmeg)
33      0.4     (Onion, Kidney Beans, Eggs, Nutmeg)
34      0.4    (Nutmeg, Eggs, Kidney Beans, Yogurt)
35      0.4     (Onion, Eggs, Kidney Beans, Yogurt)
36      0.4           (Onion, Eggs, Yogurt, Nutmeg)
37      0.4   (Onion, Kidney Beans, Yogurt, Nutmeg)
38      0.4  (Onion, Nutmeg, Yogurt, Kidney Beans, Eggs)
```

CODE:

```
frequent_itemsets['length'] = frequent_itemsets['itemsets'].apply(lambda x:
len(x)) # length of each frozenset
print('Frequent 3-itemsets:')
display(frequent_itemsets[frequent_itemsets['length'] == 3])
```

OUTPUT:

```
Frequent 3-itemsets:

    support                        itemsets  length
21      0.4      (Milk, Kidney Beans, Eggs)       3
22      0.4    (Nutmeg, Kidney Beans, Eggs)       3
23      0.6     (Onion, Kidney Beans, Eggs)       3
24      0.4    (Eggs, Kidney Beans, Yogurt)       3
25      0.4           (Onion, Eggs, Nutmeg)       3
26      0.4          (Nutmeg, Eggs, Yogurt)       3
27      0.4           (Onion, Eggs, Yogurt)       3
28      0.4    (Milk, Kidney Beans, Yogurt)       3
29      0.4   (Onion, Kidney Beans, Nutmeg)       3
```

```
30       0.4  (Nutmeg, Kidney Beans, Yogurt)       3
31       0.4   (Onion, Kidney Beans, Yogurt)       3
32       0.4         (Onion, Yogurt, Nutmeg)       3
```

Therefore, for a support threshold of 40%, is the itemset {{Kidney Beans}, {Eggs}, {Yogurt}} will be considered frequent.

3. Implement a function that receives a binary `DataFrame` of transactions and an association rule (represented by a `frozenset` of antecedents and a `frozenset` of consequents). This function should return the corresponding Kulczynski measure.

CODE:

```
frequent_itemsets = apriori(df, min_support=0.6)
display(frequent_itemsets)

itemset = frequent_itemsets.loc[5]
print('Itemset: {0}. Support: {1}.'.format(itemset['itemsets'],
itemset['support']))
```

OUTPUT:

```
    support   itemsets
0       0.8        (3)
1       1.0        (5)
2       0.6        (6)
3       0.6        (8)
4       0.6       (10)
5       0.8     (3, 5)
6       0.6     (8, 3)
7       0.6     (5, 6)
8       0.6     (8, 5)
9       0.6    (10, 5)
10      0.6  (8, 3, 5)

Itemset: frozenset({3, 5}). Support: 0.8.
```

CODE:

```
frequent_itemsets = apriori(df, min_support=0.6, use_colnames=True)
display(frequent_itemsets)
```

OUTPUT:

```
    support                      itemsets
0       0.8                        (Eggs)
```

```
1       1.0                (Kidney Beans)
2       0.6                        (Milk)
3       0.6                       (Onion)
4       0.6                      (Yogurt)
5       0.8          (Kidney Beans, Eggs)
6       0.6                 (Onion, Eggs)
7       0.6          (Milk, Kidney Beans)
8       0.6         (Onion, Kidney Beans)
9       0.6        (Kidney Beans, Yogurt)
10      0.6  (Onion, Kidney Beans, Eggs)
```

CODE:

```
frequent_itemsets['length'] = frequent_itemsets['itemsets'].apply(lambda x: len(x)) # length of each frozenset
print('Frequent 3-itemsets:')
display(frequent_itemsets[frequent_itemsets['length'] == 3])
```

OUTPUT:

```
Frequent 3-itemsets:

    support                     itemsets  length
10      0.6  (Onion, Kidney Beans, Eggs)       3
```

CODE:

```
support = {}
for _, row in frequent_itemsets.iterrows():
    support[row['itemsets']] = row['support']

itemset = frozenset(['Onion', 'Eggs'])
print('Itemset: {0}. Support: {1}.'.format(itemset, support[itemset]))
```

OUTPUT:

```
Itemset: frozenset({'Onion', 'Eggs'}). Support: 0.6.
```

CODE:

```
from mlxtend.frequent_patterns import association_rules

strong_rules = association_rules(frequent_itemsets, metric="confidence", min_threshold=0.7)
display(strong_rules)
```

OUTPUT:

```
               antecedents             consequents  antecedent support  \
0           (Kidney Beans)                  (Eggs)                 1.0
1                   (Eggs)          (Kidney Beans)                 0.8
2                  (Onion)                  (Eggs)                 0.6
3                   (Eggs)                 (Onion)                 0.8
4                   (Milk)          (Kidney Beans)                 0.6
5                  (Onion)          (Kidney Beans)                 0.6
6                 (Yogurt)          (Kidney Beans)                 0.6
7    (Onion, Kidney Beans)                  (Eggs)                 0.6
8            (Onion, Eggs)          (Kidney Beans)                 0.6
9     (Kidney Beans, Eggs)                 (Onion)                 0.8
10                 (Onion)    (Kidney Beans, Eggs)                 0.6
11                  (Eggs)   (Onion, Kidney Beans)                 0.8

    consequent support  support  confidence  lift  leverage  conviction
0                  0.8      0.8        0.80  1.00      0.00         1.0
1                  1.0      0.8        1.00  1.00      0.00         inf
2                  0.8      0.6        1.00  1.25      0.12         inf
3                  0.6      0.6        0.75  1.25      0.12         1.6
4                  1.0      0.6        1.00  1.00      0.00         inf
5                  1.0      0.6        1.00  1.00      0.00         inf
6                  1.0      0.6        1.00  1.00      0.00         inf
7                  0.8      0.6        1.00  1.25      0.12         inf
8                  1.0      0.6        1.00  1.00      0.00         inf
9                  0.6      0.6        0.75  1.25      0.12         1.6
10                 0.8      0.6        1.00  1.25      0.12         inf
11                 0.6      0.6        0.75  1.25      0.12         1.6
```

CODE:

```python
def cal_kulczynski(itemset, antecendents, consequents):

    rules = association_rules(itemset, metric="confidence", min_threshold=0)

    # Forward & Backward Rule: - Assumed always to return a strong rule
    fwd_rule = rules[(rules['antecedents'] == antecendents) &
(rules['consequents'] == consequents)]
    bck_rule = rules[(rules['antecedents'] == consequents) &
(rules['consequents'] == antecendents)]

    # Confidences:
    conf_fwd = fwd_rule['confidence'].values[0]

    conf_bck = bck_rule['confidence'].values[0]
```

```
    kulc = (conf_bck + conf_fwd) / 2

    return kulc

ants = frozenset(['Onion'])
cons = frozenset(['Kidney Beans','Eggs'])
print('Kulczynski measure: ' + str(cal_kulczynski(frequent_itemsets, ants, cons)))
```

OUTPUT:

```
Kulczynski measure: 0.875
```

4. Implement a function that receives a binary `DataFrame` of transactions and an association rule (represented by a `frozenset` of antecedents and a `frozenset` of consequents). This function should return the corresponding imbalance ratio.

CODE:

```
def cal_imbal_ratio(itemset, antecendents, consequents):

    ab_set = frozenset.union(antecendents,consequents)

    a = itemset[itemset['itemsets'] == antecendents]
    b = itemset[itemset['itemsets'] == consequents]
    ab = itemset[itemset['itemsets'] == ab_set]

    supporta = a['support'].values[0]
    supportb = b['support'].values[0]
    supportab = ab['support'].values[0]

    return abs(supporta-supportb) / (supporta+supportb-supportab)

ants = frozenset(['Onion'])
cons = frozenset(['Kidney Beans','Eggs'])
print('Imbalance ratio: ' + str(cal_imbal_ratio(frequent_itemsets, ants, cons)))
```

OUTPUT:

```
Imbalance ratio: 0.2500000000000001
```

5. The Apriori property allows us to take advantage of our knowledge of small frequent itemsets to reduce the search space for larger candidate itemsets. Specifically, if we know that an itemset A is not frequent, then we know that none of its supersets can be frequent, so we can discard them. Can we do the converse? That is, if we know an itemset A is not frequent, can we use the Apriori property to reduce the search space of smaller itemsets?

According to the Apriori property, all subsets of a frequent itemset must be frequent. If an itemset is infrequent, all its supersets will be infrequent. Therefore, if we know an itemset A is not frequent, we cannot use the Apriori property to reduce the search space of smaller itemsets beacuse we cannot really determine all its subsets to be infrequent.

6. Consider a data set consisting of N transactions and M items. Suppose we want to enumerate all frequent itemsets for a given support threshold τ.
    a. Consider a naive algorithm which simply checks the support of all imaginable non-empty itemsets. How many itemsets would it inspect?
    b. The Apriori algorithm attempts to avoid checking some of these itemsets, in order to enumerate the frequent ones faster. How many itemsets does Apriori check in the worst case?

A naïve algorithm which simply checks the support of all imaginable non-empty itemsets will inspect N * (2^M – 1) itemsets.

The Apriori algorithm in the worst case checks (${}^MC_1$ + ${}^MC_2$ + ... + ${}^MC_{M-1}$) itemsets.

7. Consider the items from the example in section 1 of this notebook. Construct the smallest possible data set (in the number of transactions) on which Apriori exhibits the worst-case behaviour you described in the previous exercise. You can choose the support threshold freely, so as to achieve this behaviour. Try running the algorithm on this data set and check whether the result matches your expectations.

CODE:

```
dataset = [['Milk', 'Onion', 'Nutmeg', 'Kidney Beans', 'Eggs',
           'Yogurt','Dill','Apple','Ice cream','Corn','Unicorn']]

transaction = TransactionEncoder()
trans_array = transaction.fit_transform(dataset)
dataframe = pd.DataFrame(trans_array, columns=transaction.columns_)
display(dataframe)
```

```
frequent = apriori(dataframe, min_support=1, use_colnames=True)
frequent
```

OUTPUT:

```
   Apple  Corn  Dill  Eggs  Ice cream  Kidney Beans  Milk  Nutmeg  Onion  \
0   True  True  True  True       True          True  True    True   True   

   Unicorn  Yogurt  
0     True    True  

      support                                          itemsets
0         1.0                                           (Apple)
1         1.0                                            (Corn)
2         1.0                                            (Dill)
3         1.0                                            (Eggs)
4         1.0                                       (Ice cream)
...       ...                                               ...
2042      1.0  (Ice cream, Dill, Unicorn, Nutmeg, Onion, Yogu...
2043      1.0  (Ice cream, Unicorn, Nutmeg, Onion, Yogurt, Ap...
2044      1.0  (Ice cream, Dill, Unicorn, Nutmeg, Onion, Yogu...
2045      1.0  (Ice cream, Dill, Unicorn, Nutmeg, Onion, Yogu...
2046      1.0  (Ice cream, Dill, Unicorn, Nutmeg, Onion, Yogu...

[2047 rows x 2 columns]
```

Assignment 3, part 2 of 2

1. The monthly rainfall in the London borough of Tower Hamlets in 2019 had the following amount of precipitation (measured in mm, values from January-December 2018): {22.93, 20.69, 25.75, 23.84, 25.34, 3.25, 23.55, 28.28, 23.72, 22.42, 26.83, 23.82}. Assuming that the data is based on a normal distribution, identify outlier values in the above dataset using the maximum likelihood method.

CODE:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats

rainfall_data = np.array([22.93, 20.69, 25.75, 23.84, 25.34, 3.25, 23.55,
28.28, 23.72, 22.42, 26.83, 23.82])

mu = np.mean(rainfall_data)
sigma = np.std(rainfall_data)
```

```
Z_data = (rainfall_data - mu)/sigma
print('All values: ')
print(Z_data)# 5th index is an outlier
```

OUTPUT:

```
All values:
[ 0.06443672 -0.30097656  0.52446592  0.21288586  0.45758224 -3.14597989
  0.16557789  0.93718716  0.19331015 -0.01876006  0.70064732  0.20962324]
```

2. Using the house prices dataset from Section 3 of this lab notebook, use PCA to obtain the first 2 principal components (remember that PCA should only be applied on the input attributes, and not the target; remember also to normalise using z-scores for better results). Then, perform outlier detection on the pre-processed dataset using the k-nearest neighbours approach using k=2. Display a scatterplot of the two principal components, where each object is colour-coded according to the computed outlier score.

CODE:

```
from pandas import read_csv

url =
'https://raw.githubusercontent.com/jbrownlee/Datasets/master/housing.csv'
df = read_csv(url, header=None)

data = df.values

X, y = data[:, :-1], data[:, -1]

print(X.shape, y.shape)

mu = np.mean(X, axis=0)
sigma = np.std(X, axis=0)

Z = (X-mu)/sigma

from sklearn.decomposition import PCA

numComponents = 2
pca = PCA(n_components=numComponents)
pca.fit(Z)

projected = pca.transform(Z)
project_h =
pd.DataFrame(projected,columns=['pc_1','pc_2'],index=range(1,len(Z)+1))
```

```
display(project_h)
```

OUTPUT:

```
(506, 13) (506,)

         pc_1      pc_2
1   -2.098297  0.773113
2   -1.457252  0.591985
3   -2.074598  0.599639
4   -2.611504 -0.006871
5   -2.458185  0.097712
..        ...       ...
502 -0.314968  0.724285
503 -0.110513  0.759308
504 -0.312360  1.155246
505 -0.270519  1.041362
506 -0.125803  0.761978

[506 rows x 2 columns]
```

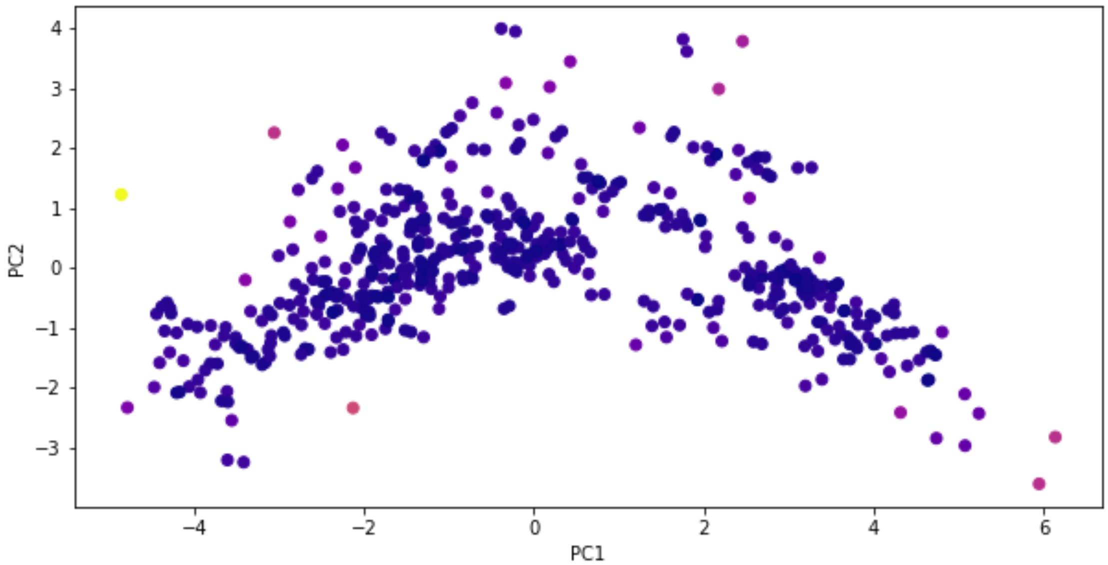
CODE:

```
from sklearn.neighbors import NearestNeighbors
import numpy as np
from scipy.spatial import distance
import matplotlib.pyplot as plt

knn = 2
nbrs = NearestNeighbors(n_neighbors=knn,
metric=distance.euclidean).fit(projected)
distances, indices = nbrs.kneighbors(projected)
outlier_score = distances[:,knn-1]

fig = plt.figure(figsize=(10,5))
p = plt.scatter(project_h['pc_1'], project_h['pc_2'], c=outlier_score,
cmap='plasma')
plt.xlabel('PC1')
plt.ylabel('PC2')
plt.show()
```

OUTPUT:



3. Consider the *absenteeism.csv* data set.

CODE:

```
import numpy as np
import pandas as pd
df = pd.read_csv("absenteeism.csv", header="infer", delimiter=";")
df
```

OUTPUT:

```
     ID  Reason for absence  Month of absence  Day of the week  Seasons  \
0    11                  26                 7                3        1
1    36                   0                 7                3        1
2     3                  23                 7                4        1
3     7                   7                 7                5        1
4    11                  23                 7                5        1
..   ..                 ...               ...              ...      ...
735  11                  14                 7                3        1
736   1                  11                 7                3        1
737   4                   0                 0                3        1
738   8                   0                 0                4        2
739  35                   0                 0                6        3

     Transportation expense  Distance from Residence to Work  Service time  \
0                       289                               36            13
1                       118                               13            18
2                       179                               51            18
3                       279                                5            14
4                       289                               36            13
```

```
..                      ...                              ...            ...
735                     289                               36             13
736                     235                               11             14
737                     118                               14             13
738                     231                               35             14
739                     179                               45             14

     Age  Work load Average/day   ...  Disciplinary failure  Education  Son
\
0     33                239.554  ...                     0          1    2
1     50                239.554  ...                     1          1    1
2     38                239.554  ...                     0          1    0
3     39                239.554  ...                     0          1    2
4     33                239.554  ...                     0          1    2
..   ...                    ...  ...                   ...        ...  ...
735   33                264.604  ...                     0          1    2
736   37                264.604  ...                     0          3    1
737   40                271.219  ...                     0          1    1
738   39                271.219  ...                     0          1    2
739   53                271.219  ...                     0          1    1

     Social drinker  Social smoker  Pet  Weight  Height  Body mass index  \
0                 1              0    1      90     172               30
1                 1              0    0      98     178               31
2                 1              0    0      89     170               31
3                 1              1    0      68     168               24
4                 1              0    1      90     172               30
..              ...            ...  ...     ...     ...              ...
735               1              0    1      90     172               30
736               0              0    1      88     172               29
737               1              0    8      98     170               34
738               1              0    2     100     170               35
739               0              0    1      77     175               25

     Absenteeism time in hours
0                            4
1                            0
2                            2
3                            4
4                            2
..                         ...
735                          8
736                          4
737                          0
738                          0
739                          0

[740 rows x 21 columns]
```

a. The "ID" column is the employee identifier. Consider the number of absences for each employee and find outliers among those values. Use Grubb's test with $\alpha = 0.1$ to find said outliers.

CODE:

```
from outliers import smirnov_grubbs as grubbs
absentees_count = pd.DataFrame(df.groupby('ID')['ID'].count())
test = grubbs.min_test(absentees_count['ID'], alpha=0.1)
absentees_count['grubs_test'] = test
print(absentees_count)
print("113 is an outlier")
```

OUTPUT:

```
     ID  grubs_test
ID
1    23        23.0
2     6         6.0
3   113         NaN
4     1         1.0
5    19        19.0
6     8         8.0
7     6         6.0
8     2         2.0
9     8         8.0
10   24        24.0
11   40        40.0
12    7         7.0
13   15        15.0
14   29        29.0
15   37        37.0
16    2         2.0
17   20        20.0
18   16        16.0
19    3         3.0
20   42        42.0
21    3         3.0
22   46        46.0
23    8         8.0
24   30        30.0
25   10        10.0
26    5         5.0
27    7         7.0
28   76        76.0
29    5         5.0
30    7         7.0
31    3         3.0
```

```
32    5          5.0
33   24         24.0
34   55         55.0
35    1          1.0
36   34         34.0
113 is an outlier
```

b. The attribute "Reason for absence" is categorical. The label "26" corresponds to "Unjustified absence". Again, consider the number of absences for each employee. However, now you must find contextual outliers among cases of unjustified absences, again using Grubb's test with $\alpha = 0.1$.

CODE:

```
absentees = df.loc[df['Reason for absence'] == 26]
absentees_count = pd.DataFrame(absentees.groupby('ID')['ID'].count())
test = grubbs.min_test(absentees_count['ID'], alpha=0.1)
absentees_count['grubs_test'] = test
print(absentees_count)
print("no outliers found")
```

OUTPUT:

```
    ID  grubs_test
ID
1    2           2
3    1           1
5    9           9
11   6           6
13   2           2
18   2           2
20   4           4
24   2           2
33   1           1
34   3           3
36   1           1
no outliers found
```