Assignment on Probability

Basic Probability

1. Two dies are rolled at once. Find out the probability for sum of numbers being even and one of the dice shows 6.

① $D_1 \rightarrow$ Event for dice showing 6

$D_2 \rightarrow$ Event for dice showing Even numbers

$$P(D_1 \cap D_2) = P(D_1) \cdot P(D_2) \quad \text{(Independent Events)}$$

$$= \frac{1}{6} \times \frac{3}{6}$$

$$= 1/12$$

2. Two dies are rolled at once. Find out the probability for sum of numbers being less than 7

② Possible Events = (1,1) (1,2) (1,3) (1,4) (1,5) (2,1) (2,2) (2,3)
(2,4) (3,1) (3,2) (3,3) (4,1) (4,2) (5,1)

Num of favourable events = 15

Total num of events = $6^2 = 36$

$$P(\text{Sum} < 7) = \frac{15}{36} = \frac{5}{12}$$

3. You toss a fair coin three times: Given that you have observed at least one heads, what is the probability t3hat you observe at least two heads?

3 Given

A → Observed atleast one head.

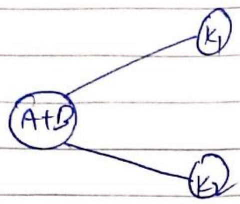
B → Event that we'll observe two heads atleast

$$P(B/A) = \frac{P(B \cap A)}{P(B)} = \frac{P(H \geq 2) \cap P(H \geq 1)}{P(H \geq 1)}$$

$$= \frac{4/8}{7/8} = \frac{4}{7}.$$

4. A and B are a married couple with two kids. One of them is a girl. What is the probability that their other kid is also a girl?

4



A → one kid is a girl

B → other kid is also a girl.

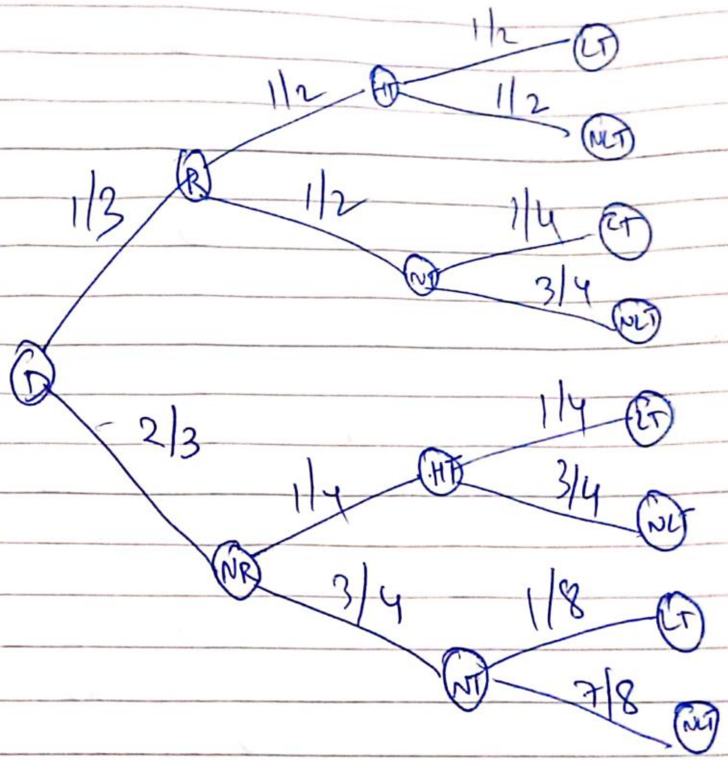
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{(1/4)}{3/4} = \frac{1}{3}$$

G B
B G x
G G
B Bx

Conditional, Joint and Marginal Probability

5. In my town, it's rainy for one third of the days. Given that it is rainy, there will be heavy traffic with probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2. On the other hand, the probability of being late is 1/8 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25, 0.25. You pick a random day.



Key :-

D → Day

R → Rainy

NR → No-rain

HT → Heavy traffic   LT → Late

i) What is the probability that it's not raining and there is heavy traffic and I am not late?

(i) $P(\text{No-rain} \cap \text{Heavy-traffic} \cap \text{Not-Late})$

$= P(A) \cdot P(B/A) \cdot P\left(\frac{C}{A \cap B}\right) = \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{1}{8}$

ii) What is the probability that I am late?

$$P(LATE) = P(NR \cap NT \cap LT) + P(NR \cap HT \cap LT) + P(R \cap HT \cap LT) + P(R \cap NT \cap LT)$$

$$= \frac{2}{3} \times \frac{3}{4} \times \frac{1}{8} + \frac{2}{3} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{16} + \frac{\cancel{2}1}{24} + \frac{1}{24} + \frac{1}{12}$$

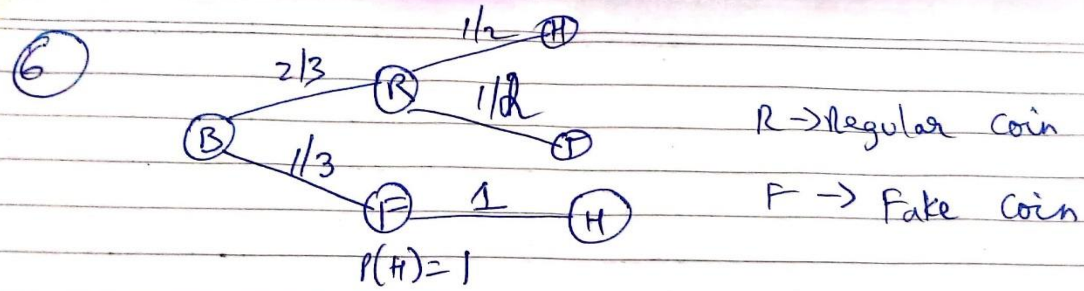
$$= 11/48$$

iii) Given that I arrived late at work, what is the probability that it rained that day?

$$P\left(\frac{RAINY}{I'm\ late\ to\ work}\right) = \frac{P(R \cap HT \cap LT) + P(R \cap NT \cap LT)}{P(R \cap HT \cap LT) + P(R \cap NT \cap LT) + P(NR \cap HT \cap LT) + P(NR \cap NT \cap LT)}$$

$$= \frac{3/24}{11/48} = \frac{6}{11}.$$

6. A box contains three coins: two regular coins and one fake two-headed coin (P (Heads) =1), you pick a coin at random and toss it.



i) What is the probability that it lands heads up?

(i) P(Coin lands up Head) = P(R gives H) + P(F gives H)

$$= \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1$$

$$= \frac{2}{3}$$

ii) You pick a coin at random and toss it and get heads. What is the probability that it is the two-headed coin?

(ii) $$P\left(\frac{\text{2 Headed Coin}}{\text{toss is head}}\right) = \frac{P(\text{2 Headed} \cap \text{Toss is head})}{P(\text{Toss is head})}$$

$$= \frac{1/3}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1}$$

$$= 1/3 \times \frac{3}{2} = \frac{1}{2}.$$

7. Suppose that, of all the customers at a coffee shop,

-> 70% purchase a cup of coffee;

-> 40% purchase a piece of cake;

-> 20% purchase both a cup of coffee and a piece of cake. Given that a randomly chosen customer has purchased a piece of cake, what is the probability that he/she also purchased a cup of coffee?

(7) $P(\text{Purchase coffee}) = 0.7$

$P(\text{Purchase Cake}) = 0.4$

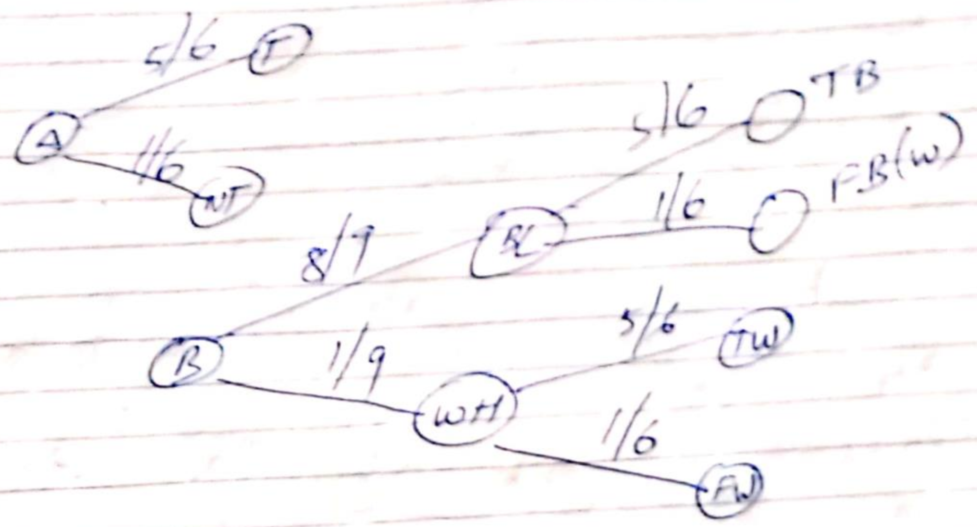
$P(\text{Coffee} \cap \text{Cake}) = 0.2$

$$P\left(\frac{\text{Purchase coffee}}{\text{Purchase cake}}\right) = \frac{P(\text{coff} \cap \text{cake})}{P(\text{cake})}$$

$$= \frac{0.2}{0.4} = 0.5.$$

8. A is known to tell the truth in 5 cases out of 6 and he states that a white ball was drawn from a bag containing 8 blacks and 1 white ball. Find the probability that the white ball was drawn.

(8)



Key:-

B → BALL   TW - TRUE WHITE

BL → BLACK   FW - FALSE WHITE

WH → WHITE

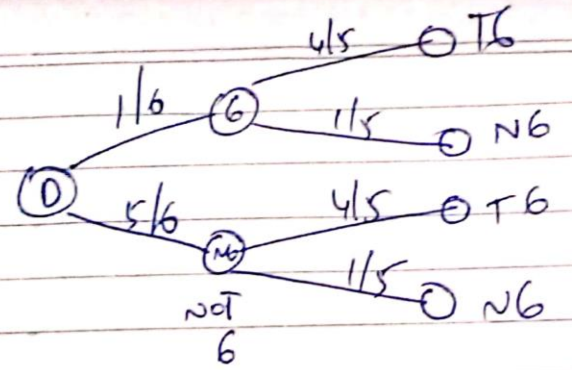
(i) P(WHITE BALL)

$$= \frac{1}{9} \times \frac{5}{6} + \frac{8}{9} \times \frac{1}{6} = \frac{13}{54}$$

~~WHITE BALL IS DRAWN~~ ⇒

9. A speaks the truth 4 out of 5 times. A die is tossed. A report that it is a 6. What are the chances that there actually was a 6?

9



$$P\left(\frac{\text{Dice roll is 6}}{\text{A tells truth}}\right) = \frac{P(D6 \cap AT)}{P(AT)}$$

$$= \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}} = \frac{4}{30} \times \frac{30}{9} = \frac{4}{9}$$

10. in a class, 40% of the student's study math and science. 60% of the student's study math. What is the probability of a student studying science given he/she is already studying math?

10

A — Math Study $\quad P(A) = 0.6$

B — science Study $\quad P(B) = ?$

$P(A \cap B) = 0.4$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.4}{0.6} = \frac{2}{3}$$

11. Below is a table of graduates and post graduates

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

i) What is the probability that a randomly selected individual is a male and a graduate? What kind of probability is it (Marginal/Joint/Conditional)

(i) $P(\text{male} \cap \text{Grad}) = \frac{19}{31}$ (Joint)

ii) What is the probability that a randomly selected individual is a male?

(ii) $P(\text{Male}) = \frac{60}{100}$ (Marginal)

iii) What is the probability of a randomly selected individual being a graduate? What kind of probability is this?

(iii) $P(\text{Grad}) = \frac{31}{100}$ (Marginal)

iv) What is the probability that a randomly selected person is a female given that the selected person is a post graduate? What kind of probability is this?

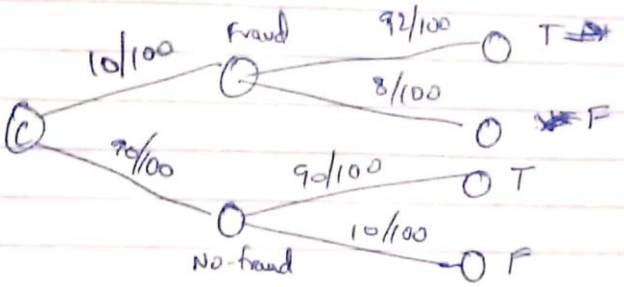
(iv) $P\left(\frac{\text{Female}}{PG}\right) = \frac{P(F \cap PG)}{P(PG)} = \frac{28}{69}$

(Conditional)

Bayes Theorem

12. You need to figure out whether a company is fraud based on the legal charges they filed. We have the knowledge that, the chances a company submitting fraudulent filings is 0.1. There exists an algorithm that can predict fraud. This algorithm returns a correct positive result in 92% of the cases in which the fraud is present, and correct negative results in 90% of the cases where the fraud is not present. Suppose we observe a company for whom the algorithm test returns a fraud result. Calculate the posterior probability that this company truly did frau in their filings.

(12)



Posterior Probability

H → Company did fraud

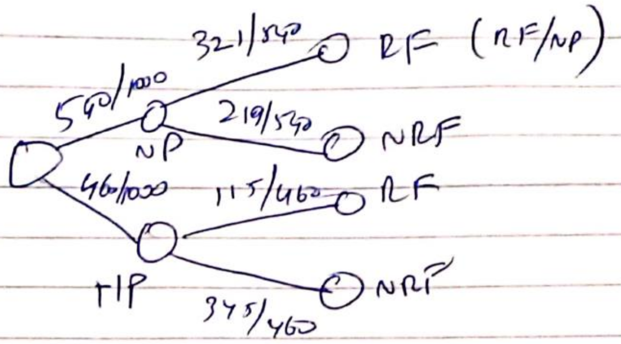
D → Actual Fraud happened

$$P\left(\frac{H}{D}\right) = \frac{P(D/H) \cdot P(H)}{P(D)}$$

$$= \frac{0.92 \times 0.1}{(0.92 \times 0.1) + (0.9 \times 0.9)} = 10.12\%$$

13. In a particular region during a 1 year period, there were 1000 deaths. It was observed that 321 people died of a renal failure and 460 men had at least one parent with renal failure. Of these 460 men, 115 died of renal failure. Calculate that he dies of renal failure if neither of his parents had a renal failure

13



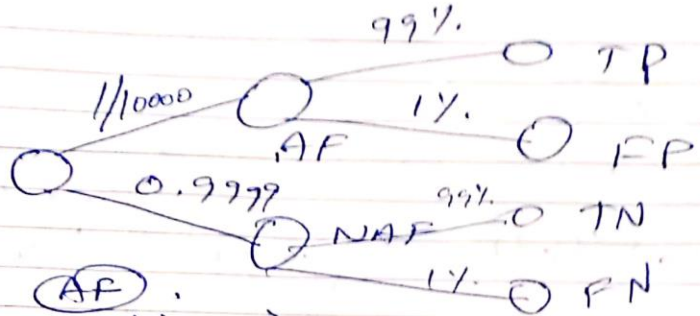
$$P\left(\frac{NP}{RF}\right) = \frac{P\left(\frac{RF}{NP}\right) \cdot P(NP)}{P(RF)}$$

$$= \frac{\frac{540}{1000} \times \frac{321}{540}}{\frac{540}{1000} \times \frac{321}{540} + \frac{115}{460} \times \frac{460}{1000}}$$

$$= \frac{0.321}{0.321 + 0.115} = 73.62\%$$

14. You go to see the doctor about an ingrowing toe nail. The doctor selects you at random to have a blood test for swine flu, which for the purposes of this exercise we will say is currently suspected to affect 1 in 10,000 people in Australia. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is zero. You test positive. What is the new probability that you have swine flu?

(14)



$$P\left(\frac{\text{Have disease (AF)}}{TP}\right) = \frac{P\left(\frac{TP}{AF}\right) \times P(AF)}{P(TP)}$$

$$= \frac{\frac{99}{100} \times \frac{1}{10000}}{P\left(\frac{TP}{AP}\right) \times P(AF) + P\left(\frac{FN}{NAT}\right) \times P(NAP)}$$

$$= \frac{\frac{0.99}{10000}}{0.99 \times \frac{1}{10000} + \frac{0.01}{0.9999}}$$

$$\frac{1}{0.010100}$$

$$= 98.0198\%.$$

$$P(\text{Have disease}) = 100\%. \quad \text{If } FP = 0$$