Assignment on Hypothesis Testing

1. The dean from UCLA is concerned that the student's grade point averages have changed dramatically in recent years. The graduating seniors' mean GPA over the last five years is 2.75. The dean randomly samples 256 seniors from the last graduating class and finds that their mean GPA is 2.85, with a sample standard deviation of 0.65.

a) What would be the null and alternative hypothesis for this scenario?

b) What would be the standard error for this particular scenario?

c) Describe in your own words how you would set the critical regions and what they would be at an alpha level of .05.

d) Test the null hypothesis and explain your decision.

① $\mu_0 = 2.75$, $\mu_1 = 2.85$, $\sigma = 0.65$, $n = 256$

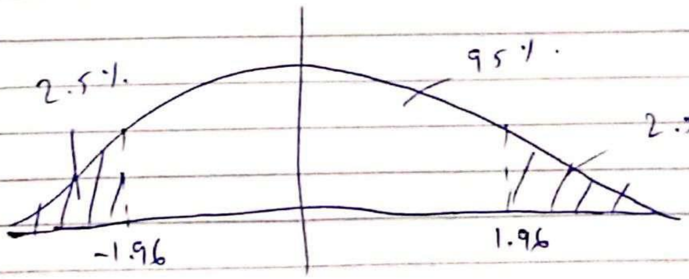
a) $H_0 : \mu_0 = 2.75$ $(\mu_0 = \mu_1)$ (Null hypothesis)

$H_1 : \mu_0 \neq 2.75$ $(\mu_0 \neq \mu_1)$ (Alternate hypothesis)

(b) standard-error (z-score) = $\dfrac{\mu_s - \mu_p}{\frac{\sigma}{\sqrt{n}}} = \dfrac{2.85 - 2.75}{\frac{0.65}{\sqrt{256}}} = 2.461$

(c) For $\alpha = 0.05$, $Z_{critical} = 1.96$

So, critical region would be set at $(-1.96, 1.96)$



(d) P-value = 1.3%

Hence, P-value < significance-level

We reject null hypothesis as p-value is significant And accept alternate hypothesis.

2. The College bookstore tells prospective students that the average cost of its textbooks is Rs. 52 with a standard deviation of Rs.4.50. A group of smart statistics students thinks that the average cost is higher. To test the bookstore's claim against their alternative, the students will select a random sample of size 100. Assume that the mean from their random sample is Rs. 52.80. Perform a hypothesis test at the 5% level of significance and state your decision.

(2) $\mu = 52$, $\sigma = 4.5$, $n = 100$, $\bar{x} = 52.80$, $\alpha = 0.05$

$H_0 : \mu_s = \mu_p$ $\quad H_1 : \mu_s \neq \mu_p$

$$Z = \frac{52.80 - 52}{\frac{4.5}{\sqrt{100}}} = 1.78$$

$Z_{critical} = 1.96$ $\quad$ P-value $= 7.5\%$

$Z_{calc} < Z_{critical}$.

Hence, we fail to reject null hypothesis.

3. A certain chemical pollutant in the Genesee River has been constant for several years with a mean of 34ppm (parts per million) and standard deviation of 8ppm. A group of factory representatives whose companies discharge liquids into the river is now claiming that they have lowered the average with improved filtration devices. A group of environmentalists will test to see if this is true at the 1% level of significance. Assume that their sample of size 50 gives a mean of 32.5ppm. Perform a hypothesis test at the 1% level of significance and state your decision.

3 $\mu = 34$, $\sigma = 8$, $\alpha = 0.01$, $n = 50$, $\bar{x} = 32.5$

$H_0 : \mu_s = \mu_p$

$H_1 : \mu_s \neq \mu_p$

$$Z_{Calc} = \frac{32.5 - 34}{\frac{8}{\sqrt{50}}} = -1.32 \qquad Z_{critical} = 2.326$$

P-value = 18.68 % $Z_{calc} < Z_{critical}$

Hence, we accept null hypothesis

4. Carry out a one-tailed test to determine whether the population proportion of traveller's check buyers who buy at least $2500 in checks when sweepstakes prizes are offered as at least 10% higher than the proportion of such buyers when no sweepstakes are on

Population 1: With sweepstakes

N1 = 300

X1 = 120

S1 = 0.53

Population 2: No sweepstakes

N2 = 700

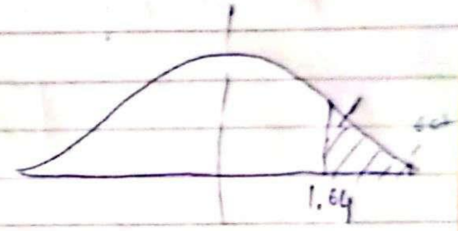
X2 = 140

S2 = 0.20

④ Pop 1 — Pop 2

Pop 1	Pop 2
$N_1 = 300$	$N_2 = 700$
$\bar{X}_1 = 120$	$\bar{X}_2 = 140$
$S_1 = 0.53$	$S_2 = 0.20$

$$\hat{P} = \frac{P_1 \times N_1 + P_2 \times N_2}{N_1 + N_2}$$



$$= 0.26$$

$$SE = \sqrt{P(1-P)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{0.26 \times 0.74 \left(\frac{1}{300}+\frac{1}{700}\right)}$$

$$= 0.03$$

$H_0: P_S - P_{NS} \leq 10$

$H_1: P_S - P_{NS} \geq 10$

$$Z = \frac{P_1 - P_2}{SE} = \frac{0.02}{0.03} = 6.67$$

$Z_{critical} = 1.64$ $\therefore$ P-value $< 5\%$.

Hence, we can safely reject null hypothesis and say that higher traveler's cheque buyers have higher prizes offered.

5. A sample of 100 voters are asked which of four candidates they would vote for in an election. The number supporting each candidate is given below:-

Higgins	Reardon	White	Charlton
41	19	24	16

Do the data suggest that all candidates are equally popular? Chi-Square = 14.96, with 3 df < 0.05.

(5)

	Observed	Expected
Higgins	41	25
Reardon	19	25
White	24	25
Charlton	16	25

$\boxed{n = 100}$

$\alpha = 0.05$

$df = 4 - 1 = 3$

$H_0: P_1 = P_2 = P_3 = P_4 = 0.25$

$H_1: H_0$ is false

$$\chi^2 = \frac{(41-25)^2}{25} + \frac{(19-25)^2}{25} + \frac{(24-25)^2}{25} + \frac{(16-25)^2}{25}$$

$\chi^2 = 14.96$ $\quad\chi^2_{critical} = 7.815$

P-value = 0.001851

Hence, null hypothesis is rejected

So, Data doesn't suggest that candidates are equally popular.

6. Fifteen trainees in a technical program are randomly assigned to three different types of instructional approaches, all of which are concerned with developing a specified level of skill in computer-assisted design. The achievement test scores at the conclusion of the instructional unit are reported in Table along with the mean performance score associated with each instructional approach. Use the analysis of variance procedure to test the null hypothesis that the three sample means were obtained from the sample population, using the 5 percent level of significance for the test.

Instructional method	Test scores					Total scores	Mean test scores
A_1	86	79	81	70	84	400	80
A_2	90	76	88	82	89	425	85
A_3	82	68	73	71	81	375	75

⑥ $\alpha = 0.05$, $df = (5-1)(3-1) = 8$

$\chi^2_{critical} = \cancel{12.592}\ 15.507$

$H_0 : S_1, S_2, S_3 \in S_p$

$H_1 : S_1, S_2, S_3 \notin S_p$

						Expected
A_1	86	79	81	~~84~~ 70	84	80
A_2	90	76	88	82	89	85
A_3	82	68	73	71	81	75

$$\chi^2 = \sum_{i=1}^{n} \frac{(O-E_i)^2}{E_i} = \frac{(86-80)^2}{80} + \frac{(79-80)^2}{80} + \_\_ + \frac{(90-85)^2}{85} + \_\_$$

$$+ \frac{(82-75)^2}{75} + \_\_\_ + \frac{(81-75)^2}{75}$$

$$= 1.925 + 1.647 + 2.05 = 5.625$$

p-value $= 0.69$

Hence, we fail to reject null hypothesis.

7. The school nurse thinks the average height of 7th graders has increased. The average height of a 7th grader five years ago was 145 cm with a standard deviation of 20 cm. She takes a random sample of 200 students and finds that the average height of her sample is 147 cm. Are 7th graders now taller than they were before? Conduct a single-tailed hypothesis test using a .05 significance level to evaluate the null and alternate hypothesis.

(4) (7) $\mu = 145$, $\sigma = 20$, $n = 200$, $\bar{x} = 147$, $\alpha = 0.05$

$H_0: \mu_s = \mu_p$

$H_1: \mu_s \neq \mu_p$

$$Z_{calc} = \frac{147 - 145}{\frac{20}{\sqrt{200}}} = 1.414$$

$Z_{critical} = 1.96$

P-value $= 0.157/2 = 7.85\%$

Hence, null hypothesis can be accepted.

8. A farmer is trying out a planting technique that he hopes will increase the yield on his pea plants. The average number of pods on one of his pea plants is 145 pods with a standard deviation of 100 pods. This year, after trying his new planting technique, he takes a random sample of his plans and finds the average number of pods to be 147. He wonders whether or not this is a statistically significant increase. What are his hypothesis and test statistic?

(8) $\mu = 145$, $\sigma = 100$, $\bar{x} = 147$, $\alpha = 0.05$

$H_0 : \mu_s = \mu_p$

$H_1 : \mu_s \neq \mu_p$

$$Z_{calc} = \frac{147-145}{\frac{100}{\sqrt{1}}} = 0.02$$

$Z_{critical} = 1.96$

P-value $= 98.4\%$

Hence, we fail to reject null hypothesis.
(Not a statistically significant increase)

9. You have just taken ownership of a pizza shop. The previous owner told you that you would save money if you bought the mozzarella cheese in a 4.5 pound slab. Each time you purchase a slab of cheese, you weigh it to ensure that you are receiving 72 ounces of cheese. The results of 7 random measurements are 70, 69, 73, 68, 71, 69 and 71 ounces. Are these differences due to chance or is the distributor giving you less cheese than you deserve?

a) State the hypothesis

b) Calculate the test statistic

c) Would the null hypothesis be rejected at the 10% level? The 5% level? The 1% level.

9 $\mu = 72$, $n = 7$, $\bar{x} = 70.14$, $\sigma = 1.676$

(a) $H_0 : \mu_s = \mu_p$

$H_1 : \mu_s \neq \mu_p$

(b) $$Z_{score} = \frac{70.14 - 72}{\frac{1.676}{\sqrt{7}}} = -2.9236$$

(c)

For Significance levels		P-value (for - one tail)
10 %	—	0.17 %
5 %	—	0.17 %
1 %	—	0.17 %

For all the Significance levels, the null hypothesis Can be rejected and all alternate hypothesis Can be accepted.

So, the distributor is giving less cheese than you deserve.