Name (last, first):________________________

CS 201 Discrete Structures

Chapter 1: Logic and Proofs

1.5 Nested Quantifiers

Definition:

Rule for working with nested quantifiers:

$$\forall x\ \exists y\ \forall z\ [(x + y) \times z = 0]$$

What is the value of y?

Commutative Law for Addition:

Associative Law for Addition:

$\exists x\ \forall y\ (x \times y = 0)$

Read As:

What is the value of x?

Domain: all integers

Consider: $\forall x\ \exists y\ (x + y = 0)$

What is the value of y?

Domain: all integers

Transpose the Quantifiers: $\exists y\ \forall x\ (x + y = 0)$

What is the value of y?

P. 64 # 1 (a-c): Domain: all the real numbers, translate into English Statements:

a. $\forall$ **$x\ \exists y\ (x < y)$**
b.
c.

P. 66-67 #19(a-d) Express these statements using mathematical and logical operators, predicates and quantifiers:

a. " The sum of two negative integers is negative."
b.
c.
d.

P. 67 # 26 (a-h): What are the truth values?

<table>
<tr><td colspan="2">Domain for both x and y: all integers</td></tr>
<tr><td colspan="2">Let Q(x,y) be the statement "x+y = x-y"</td></tr>
<tr><td>a. Q(1,1)</td><td>f.</td></tr>
<tr><td>b.</td><td>g.</td></tr>
<tr><td>c. $\forall$ y Q(1,y)</td><td>h.</td></tr>
<tr><td>d.</td><td>i.</td></tr>
<tr><td>e.</td><td>j.</td></tr>
</table>

P. 67 # 29 (a-d): Write out these propositions using disjunctions and conjunctions.

Domain x: 1,2, or 3 Domain y: 1, 2, or 3
$P(x,y)$
a.
b. **$\exists x \exists y\ P(x,y)$**
c.
d.

P. 68 # 39 (a-c): Is there a counterexample? If yes, what is it?

Domain for x and y: all integers	
a) $\forall x \forall y (x^2 = y^2 \rightarrow x = y)$	
b) $\forall x \exists y (y^2 = x)$	
c) $\forall x \forall y (xy >= x)$	