CS 201-4 Thursday 13th September 2018

Discrete Structures

Rules of Inference – A special set of tautologies used to prove the validity of logic arguments.

$$\begin{array}{l} p \\ \underline{p \to q} \\ \therefore q \end{array}$$

Modus Ponens

P_1 p	C q	P_2 $p \to q$	A $P_1 \wedge P_2$	$A \to C$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Tautology

Cummutative Law – Allows us to switch the order of the "AND" premises (like multiplication)

Modus Ponens –

$$\begin{array}{l} h \to m \\ \underline{h} \\ \therefore m \end{array} \quad \text{same as} \Rightarrow \quad \begin{array}{l} h \\ \underline{h \to m} \\ \therefore m \end{array}$$

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Commutative Law – $[(h) \wedge (h \rightarrow m)] \rightarrow (m)$

antecedent consequent

Modus Ponens – Latin, literally 'mode that affirms'

Whenever a conditional statement AND its antecedent are given to be true may be validly inferred."

We cannot infer anything from a true conditional statement.

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If George does not have 8 legs, then he is not a spider.
George is a spider.
Therefore George has 8 legs

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Page 78: 2

If George does not have 8 legs, then he is not a spider.
George is a spider.
Therefore George has 8 legs

p = George does not have 8 legs
q = he is not a spider.

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \neg p \end{array}$$

p = George has 8 legs
q = George is a spider

$$\begin{array}{l} \neg p \rightarrow \neg q \\ q \\ \hline \therefore p \end{array}$$

Modus tollens
"mode that denies"

When opposite I can conclude the opposite of the antecedent.

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Whenever a conditional statement and the negation of its consequent are given to be true, the negation of its antecedent may be validly inferred.

Example -

If it rains today, then we will not have a barbecue today.

If we do not have a barbecue today, then we will not have a barbecue tommorrow.

Therefore if it rains today, then we will not have a barbecue tommorrow.

p = "It is raining today."
q = "We will not have a barbecue today."
r = "We will not have a barbecue tommorrow".

Hypothetical Syllogism "Chain rule"

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

"Cut out the middleman".

Discrete Structures

The antecedent of one premise must watch the consequent of the other, thus it is like cancelling out the "link" between the two.

Example -

"It is not sunny this afternoon and it is colder than yesterday."
"We will go swimming only if it is sunny this afternoon."
"If we do not go swimming, then we will take a canoe trip."
and "If we take a canoe trip, then we will be at home by sunset"
lead to the conclusion
"We will be home by sunset."

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p = "It is sunny this afternoon"
q = "It is colder than yesterday.
r = "We will go swimming
s = "We will take a canoe trip"
t = "We will be at home by sunset".

$$\begin{array}{l} \neg p \land q \\ r \to p \\ \neg r \to s \\ s \to t \\ \hline \therefore\ t \end{array}$$

Simplification: $\neg p$
Modus tollens: $\neg p \land r \to p \to \neg r$
Modus ponens: $\neg r \land \neg r \to s \to s$
Modus ponens: $s \land s \to t \to t$

Rules of Inference to prove conclusion

$$[(\neg p \land q) \land (r \to p) \land (\neg r \to s) \land (s \to t)] \to t$$

Simplification $\neg p$

Modus tollens $\neg r$

Hypothetical Syllogism $\neg r \to t$

Modus ponens t

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Direct Proof :

- $p \Rightarrow q$, and the first step is to assume p is true.
- Example Prove "If n is an odd int, then n^2 is odd"
- $p = n$ is an odd int
- $q = n^2$ is odd.
- Not even is defined as $n = 2k$, thus odd is defined as $n = 2k + 1$

- Square both sides : $n^2 = (2k+1)^2$
- Use "foil" method $(2k+1)(2k+1) = n^2 = 4k^2 + 4k + 1$
- Factor out 2 : $n^2 = 2(2k^2 + 2k) + 1$

Thus n^2 is odd as it is one more than twice an integer.