In [ ]:
```
import pandas as pd, numpy as np, matplotlib.pyplot as plt
from statsmodels.tsa.api import VAR
```

In [ ]:
```
gdp_raw = pd.read_csv("GDP_Component.csv")


gdp_raw = gdp_raw[[
    "REF_DATE", "GEO", "Prices", "Seasonal adjustment",
    "Estimates", "VALUE"
]]
gdp_raw.head()
```

Out[ ]:

	REF_DATE	GEO	Prices	Seasonal adjustment	Estimates	VALUE
0	1990-01	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	892824
1	1990-04	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	882793
2	1990-07	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	888741
3	1990-10	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	889602
4	1991-01	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	875730

In [15]:
```
print(gdp_raw["GEO"].unique())
print(gdp_raw["Prices"].unique())
print(gdp_raw["Seasonal adjustment"].unique())
```

```
['Canada']
['Chained (2017) dollars']
['Seasonally adjusted at annual rates']
```

In [16]:
```
gdp = gdp_raw[
    (gdp_raw["GEO"] == "Canada") &
    (gdp_raw["Prices"] == "Chained (2017) dollars") &
    (gdp_raw["Seasonal adjustment"] == "Seasonally adjusted at annual rates")
].copy()

print("rows after filter:", len(gdp))
gdp.head()
```

```
rows after filter: 4402
```

Out[16]:

	REF_DATE	GEO	Prices	Seasonal adjustment	Estimates	VALUE
0	1990-01	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	892824
1	1990-04	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	882793
2	1990-07	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	888741
3	1990-10	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	889602
4	1991-01	Canada	Chained (2017) dollars	Seasonally adjusted at annual rates	Final consumption expenditure	875730

In [17]:
```
gdp_wide = (
    gdp.pivot(index="REF_DATE", columns="Estimates", values="VALUE")
      .reset_index()
)


gdp_wide["REF_DATE"] = pd.to_datetime(gdp_wide["REF_DATE"])
gdp_wide = gdp_wide.sort_values("REF_DATE").set_index("REF_DATE")

print("columns:", list(gdp_wide.columns)[:15])
gdp_wide.tail()
```

```
columns: ['Business gross fixed capital formation', 'Durable goods', 'Exports of good
s', 'Exports of goods and services', 'Exports of services', 'Farm', 'Final consumption
expenditure', 'Final domestic demand', 'General governments final consumption expendit
ure', 'General governments gross fixed capital formation', 'Goods', 'Gross domestic pr
oduct at market prices', 'Gross fixed capital formation', 'Household final consumption
expenditure', 'Imports of goods']
```

Out[17]:

Estimates	Business gross fixed capital formation	Durable goods	Exports of goods	Exports of goods and services	Exports of services	Farm	Final consumption expenditure	Final domestic demand
REF_DATE								
2024-04-01	412401	174453	563595	732534	173046	-2967	1938889	2443520
2024-07-01	409356	179925	564772	731408	170563	-2097	1961068	2462587
2024-10-01	418674	187224	578970	744109	168776	293	1981053	2494078
2025-01-01	413456	184389	587798	754705	170542	-1938	1982459	2488291
2025-04-01	410942	189009	533997	698086	168183	-2740	2004878	2509589

5 rows × 31 columns

In [ ]:
```
patterns = {
    "C": r"^(Final consumption expenditure|Household final consumption expenditure)$"
    "I": r"^Gross fixed capital formation$",
    "G": r"^(General governments final consumption expenditure|Government final consu
    "X": r"^Exports of goods and services$",
    "M": r"^Imports of goods and services$",
}

selected = {}
for short, pat in patterns.items():
    matched = gdp_wide.columns[gdp_wide.columns.str.fullmatch(pat)]
    if len(matched) == 0:
        matched = gdp_wide.columns[gdp_wide.columns.str.contains(pat.strip("^$").spli
    selected[short] = matched[0] if len(matched) else None

selected
```

Out[ ]:
```
{'C': 'Final consumption expenditure',
 'I': 'Gross fixed capital formation',
 'G': 'General governments final consumption expenditure',
 'X': 'Exports of goods and services',
 'M': 'Less: imports of goods and services'}
```

In [19]:
```
keep_cols = [v for v in selected.values() if v is not None]
gdp_block_level = gdp_wide[keep_cols].dropna()
```

```
print("Selected columns (in order C,I,G,X,M):", keep_cols)
gdp_block_level.tail()
```

```
Selected columns (in order C,I,G,X,M): ['Final consumption expenditure', 'Gross fixed capital formation', 'General governments final consumption expenditure', 'Exports of goods and services', 'Less: imports of goods and services']
```

Out[19]:

Estimates	Final consumption expenditure	Gross fixed capital formation	General governments final consumption expenditure	Exports of goods and services	Less: imports of goods and services
REF_DATE					
2024-04-01	1938889	509764	537645	732534	794175
2024-07-01	1961068	507456	544858	731408	792198
2024-10-01	1981053	518483	548051	744109	797086
2025-01-01	1982459	511943	547464	754705	804141
2025-04-01	2004878	511490	554367	698086	793594

In [20]:
```
gdp_block_growth = np.log(gdp_block_level).diff().dropna()
gdp_block_growth.tail()
```

Out[20]:

Estimates	Final consumption expenditure	Gross fixed capital formation	General governments final consumption expenditure	Exports of goods and services	Less: imports of goods and services
REF_DATE					
2024-04-01	0.006753	0.008166	0.013548	-0.012333	0.000020
2024-07-01	0.011374	-0.004538	0.013327	-0.001538	-0.002492
2024-10-01	0.010139	0.021497	0.005843	0.017216	0.006151
2025-01-01	0.000709	-0.012694	-0.001072	0.014139	0.008812
2025-04-01	0.011245	-0.000885	0.012530	-0.077985	-0.013203

In [21]:
```
from statsmodels.tsa.api import VAR
```

In [22]:
```
model = VAR(gdp_block_growth)
```

```
c:\Users\logc0\anaconda3\envs\hw6\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
473: ValueWarning: No frequency information was provided, so inferred frequency QS-OCT
will be used.
  self._init_dates(dates, freq)
```

In [23]:
```
lag_selection = model.select_order(maxlags=8)
print(lag_selection.summary())
```

```
 VAR Order Selection (* highlights the minimums)
=================================================
      AIC         BIC         FPE         HQIC
-------------------------------------------------
0     -42.27*     -42.16*  4.386e-19*     -42.23*
1      -42.27      -41.62   4.392e-19      -42.00
2      -42.08      -40.88   5.322e-19      -41.59
3      -42.02      -40.28   5.666e-19      -41.31
4      -42.03      -39.75   5.631e-19      -41.11
5      -42.07      -39.25   5.484e-19      -40.92
6      -41.90      -38.53   6.657e-19      -40.53
7      -41.70      -37.79   8.297e-19      -40.11
8      -41.68      -37.23   8.781e-19      -39.87
-------------------------------------------------
```

In [28]:
```
lag_opt = 2
var_fit = model.fit(lag_opt)
print(var_fit.summary())
```

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Mon, 13, Oct, 2025
Time:                      16:29:27
----------------------------------------------------------------------
No. of Equations:         5.00000    BIC:                   -40.9293
Nobs:                     139.000    HQIC:                  -41.6186
Log likelihood:           1994.13    FPE:                5.26094e-19
AIC:                     -42.0905    Det(Omega_mle):     3.59485e-19
----------------------------------------------------------------------
Results for equation Final consumption expenditure
=================================================================================================
                                                        coefficient       std. error
t-stat            prob
---------------------------------------------------------------------------------------------
---------------------------------
const                                                     0.008703         0.001521
5.723           0.000
L1.Final consumption expenditure                         -0.241576         0.164724
-1.467          0.142
L1.Gross fixed capital formation                          0.137295         0.076995
1.783           0.075
L1.General governments final consumption expenditure     -0.021730         0.172562
-0.126          0.900
L1.Exports of goods and services                         -0.007131         0.066322
-0.108          0.914
L1.Less: imports of goods and services                   -0.129785         0.083336
-1.557          0.119
L2.Final consumption expenditure                         -0.293120         0.160317
-1.828          0.067
L2.Gross fixed capital formation                          0.035428         0.076269
0.465           0.642
L2.General governments final consumption expenditure      0.053237         0.175842
0.303           0.762
L2.Exports of goods and services                          0.011954         0.064403
0.186           0.853
L2.Less: imports of goods and services                    0.035731         0.082667
0.432           0.666
=================================================================================================
=================================

Results for equation Gross fixed capital formation
=================================================================================================
=================================
                                                        coefficient       std. error
t-stat            prob
---------------------------------------------------------------------------------------------
---------------------------------
const                                                     0.009985         0.002841
3.515           0.000
L1.Final consumption expenditure                         -0.372268         0.307730
-1.210          0.226
L1.Gross fixed capital formation                          0.344013         0.143837
```

```
2.392           0.017
L1.General governments final consumption expenditure      -0.510639        0.322373
-1.584          0.113
L1.Exports of goods and services                           0.076655        0.123899
0.619           0.536
L1.Less: imports of goods and services                    -0.142666        0.155685
-0.916          0.359
L2.Final consumption expenditure                          -0.199252        0.299496
-0.665          0.506
L2.Gross fixed capital formation                           0.085953        0.142483
0.603           0.546
L2.General governments final consumption expenditure      -0.026395        0.328499
-0.080          0.936
L2.Exports of goods and services                          -0.126242        0.120314
-1.049          0.294
L2.Less: imports of goods and services                     0.062433        0.154435
0.404           0.686
================================================================================
================================

Results for equation General governments final consumption expenditure
================================================================================
================================
                                                        coefficient       std. error
t-stat            prob
--------------------------------------------------------------------------------
--------------------------------
const                                                     0.005176        0.000976
5.305           0.000
L1.Final consumption expenditure                          -0.129020        0.105678
-1.221          0.222
L1.Gross fixed capital formation                           0.100120        0.049396
2.027           0.043
L1.General governments final consumption expenditure       0.010786        0.110707
0.097           0.922
L1.Exports of goods and services                          -0.035849        0.042548
-0.843          0.399
L1.Less: imports of goods and services                    -0.056762        0.053464
-1.062          0.288
L2.Final consumption expenditure                          -0.106439        0.102850
-1.035          0.301
L2.Gross fixed capital formation                          -0.023638        0.048930
-0.483          0.629
L2.General governments final consumption expenditure       0.173460        0.112811
1.538           0.124
L2.Exports of goods and services                          -0.003996        0.041317
-0.097          0.923
L2.Less: imports of goods and services                     0.010380        0.053035
0.196           0.845
================================================================================
================================

Results for equation Exports of goods and services
================================================================================
================================
                                                      coefficient       std. error
```

```
t-stat          prob
-------------------------------------------------------------------------------------------
--------------------------------
const                                                  0.011979         0.003309
3.620           0.000
L1.Final consumption expenditure                      -0.015105         0.358447
-0.042          0.966
L1.Gross fixed capital formation                       0.284337         0.167543
1.697           0.090
L1.General governments final consumption expenditure  -0.732030         0.375503
-1.949          0.051
L1.Exports of goods and services                       0.079456         0.144319
0.551           0.582
L1.Less: imports of goods and services                -0.192903         0.181343
-1.064          0.287
L2.Final consumption expenditure                      -0.646046         0.348856
-1.852          0.064
L2.Gross fixed capital formation                      -0.116343         0.165965
-0.701          0.483
L2.General governments final consumption expenditure  -0.018935         0.382639
-0.049          0.961
L2.Exports of goods and services                      -0.111874         0.140143
-0.798          0.425
L2.Less: imports of goods and services                 0.383100         0.179887
2.130           0.033
===========================================================================================
================================

Results for equation Less: imports of goods and services
===========================================================================================
================================
                                                      coefficient       std. error
t-stat          prob
-------------------------------------------------------------------------------------------
--------------------------------
const                                                  0.015142         0.003916
3.867           0.000
L1.Final consumption expenditure                      -0.105226         0.424186
-0.248          0.804
L1.Gross fixed capital formation                       0.287572         0.198271
1.450           0.147
L1.General governments final consumption expenditure  -0.551882         0.444370
-1.242          0.214
L1.Exports of goods and services                       0.108479         0.170787
0.635           0.525
L1.Less: imports of goods and services                -0.306846         0.214602
-1.430          0.153
L2.Final consumption expenditure                      -0.555662         0.412835
-1.346          0.178
L2.Gross fixed capital formation                       0.068746         0.196403
0.350           0.726
L2.General governments final consumption expenditure  -0.254482         0.452815
-0.562          0.574
L2.Exports of goods and services                      -0.087087         0.165845
-0.525          0.600
L2.Less: imports of goods and services                 0.210812         0.212879
```

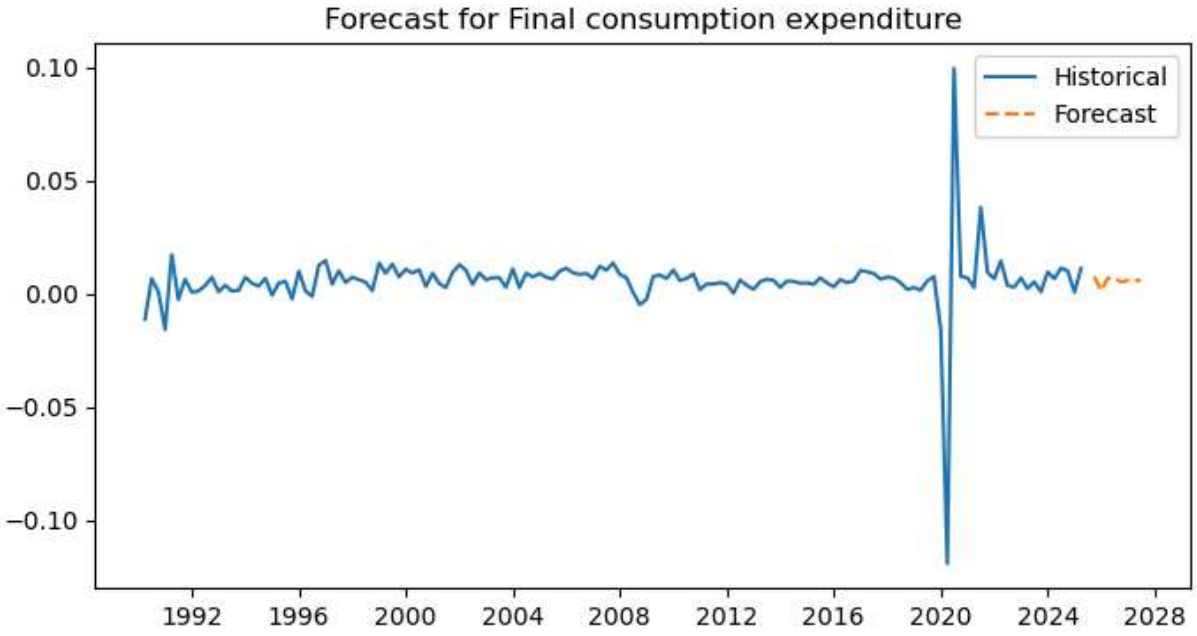
```
0.990                0.322
==========================================================================================
================================

Correlation matrix of residuals
                                                 Final consumption expenditure  Gr
oss fixed capital formation  General governments final consumption expenditure  Export
s of goods and services  Less: imports of goods and services
Final consumption expenditure                                         1.000000
0.638020                                      0.625426                      0.58
1832                                  0.769686
Gross fixed capital formation                                         0.638020
1.000000                                      0.339162                      0.51
3539                                  0.786366
General governments final consumption expenditure                     0.625426
0.339162                                      1.000000                      0.25
2789                                  0.395796
Exports of goods and services                                         0.581832
0.513539                                      0.252789                      1.00
0000                                  0.758241
Less: imports of goods and services                                   0.769686
0.786366                                      0.395796                      0.75
8241                                  1.000000
```

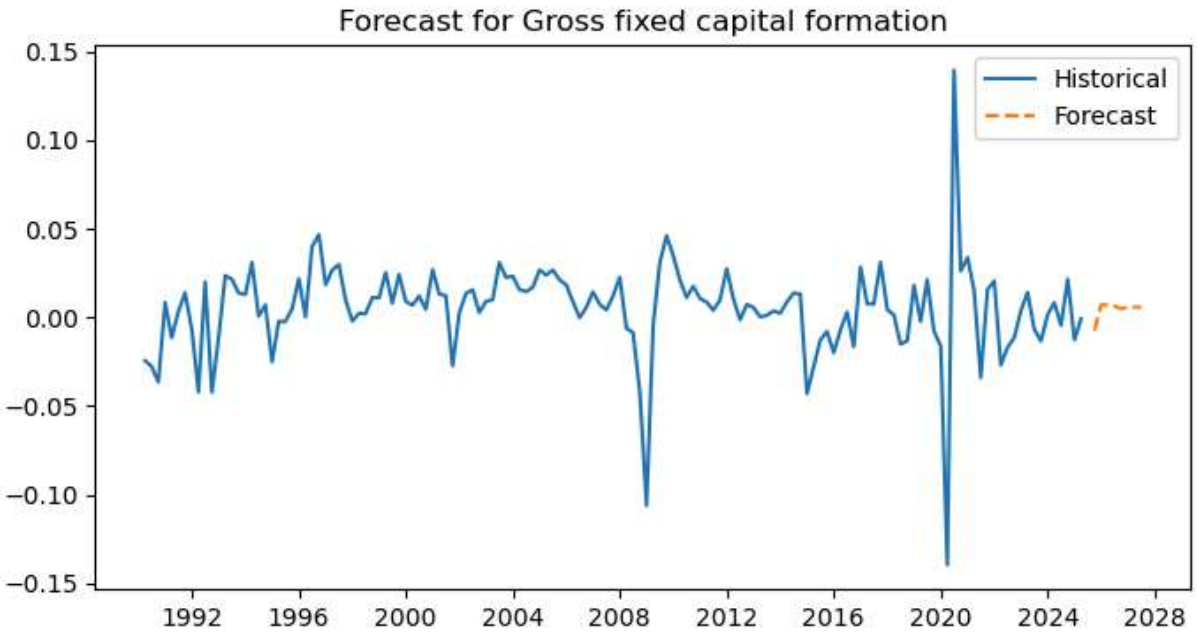
In [ ]:
```
lag_used = var_fit.k_ar
forecast = var_fit.forecast(gdp_block_growth.values[-lag_used:], steps=8)
forecast_df = pd.DataFrame(forecast, columns=gdp_block_growth.columns)


last_per = gdp_block_growth.index.to_period('Q')[-1]
forecast_df.index = pd.period_range(last_per + 1, periods=8, freq='Q')
forecast_df
```

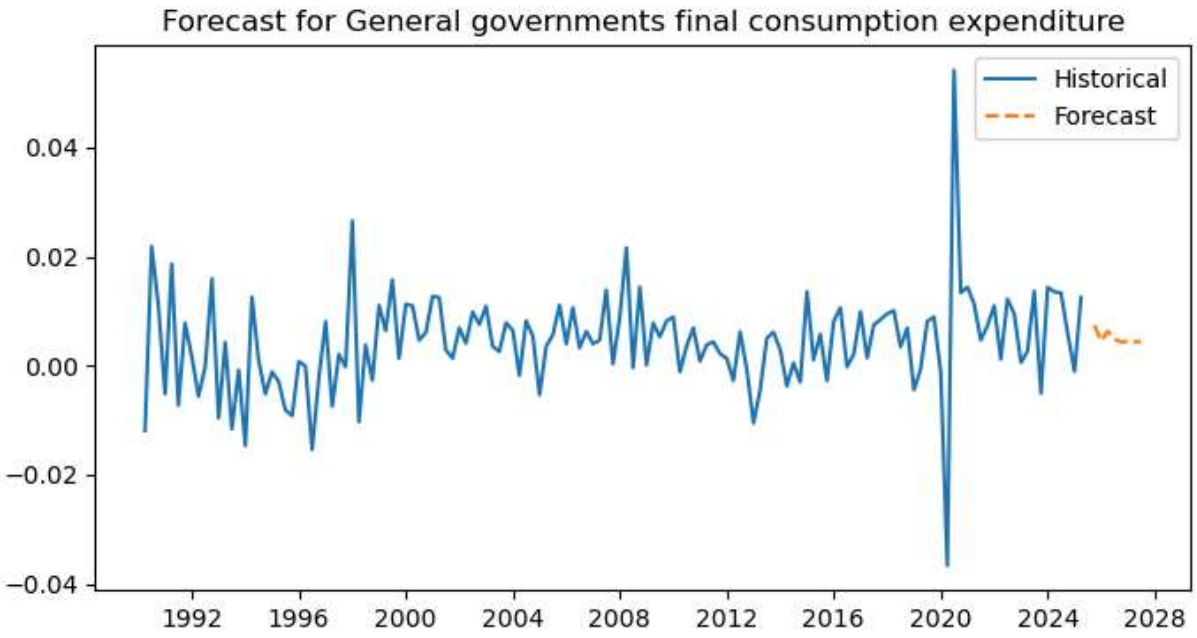
Out[ ]:

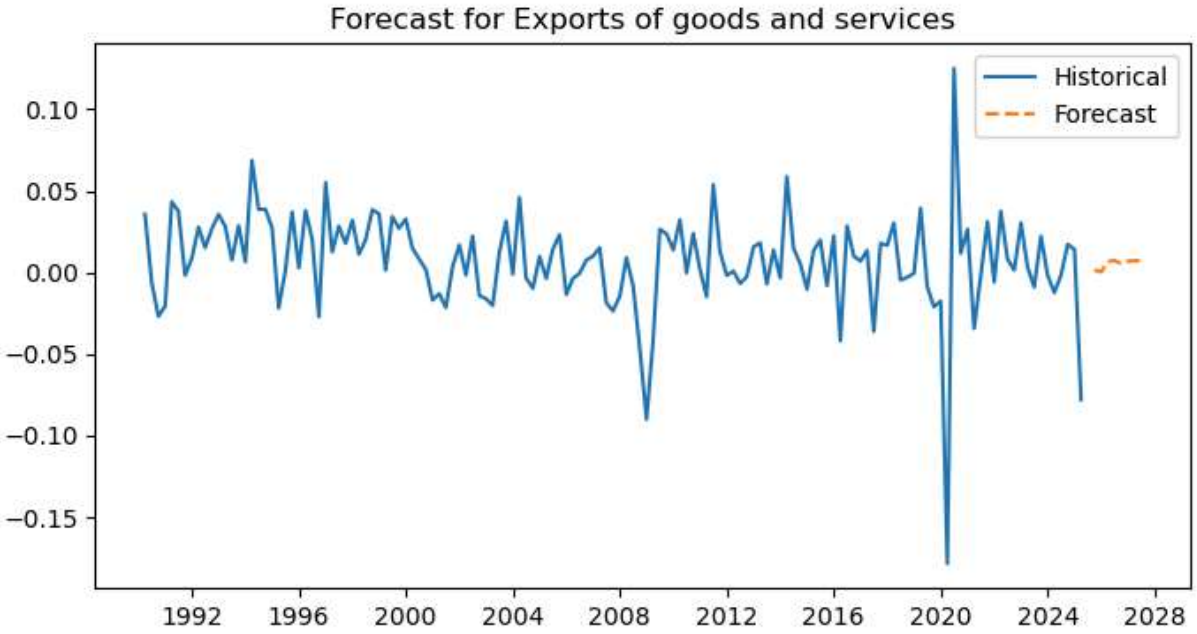
Estimates	Final consumption expenditure	Gross fixed capital formation	General governments final consumption expenditure	Exports of goods and services	Less: imports of goods and services
2025Q3	0.007631	-0.007438	0.007390	0.001568	0.002013
2025Q4	0.001341	0.007018	0.004528	0.000342	0.002185
2026Q1	0.006942	0.006875	0.006278	0.006636	0.007543
2026Q2	0.006986	0.006302	0.004868	0.007336	0.010346
2026Q3	0.005271	0.004828	0.004347	0.005432	0.007184
2026Q4	0.005919	0.005596	0.004453	0.006940	0.008817
2027Q1	0.005930	0.005719	0.004403	0.007166	0.009057
2027Q2	0.005832	0.005612	0.004334	0.007157	0.008949

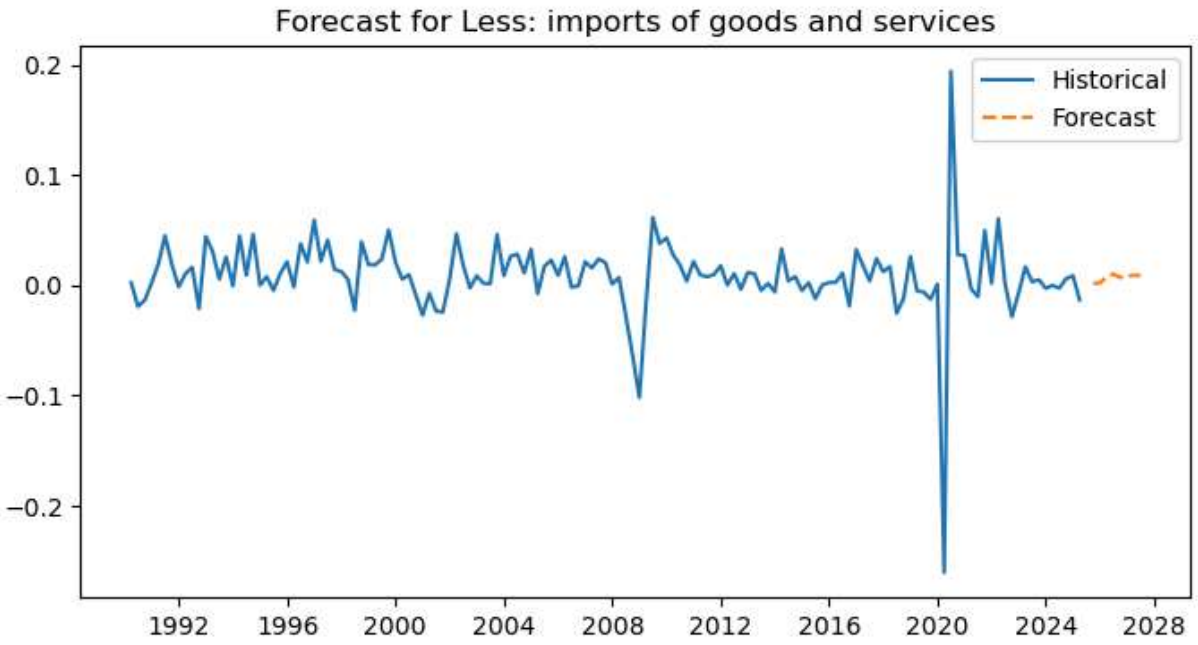
In [31]:
```
import matplotlib.pyplot as plt

for col in forecast_df.columns:
    plt.figure(figsize=(8,4))
    plt.plot(gdp_block_growth[col], label='Historical')
    plt.plot(forecast_df[col], label='Forecast', linestyle='--')
    plt.title(f"Forecast for {col}")
    plt.legend()
    plt.show()
```






Forecast for General governments final consumption expenditure
Historical
Forecast
0.04
0.02
0.00
−0.02
−0.04
1992
1996
2000
2004
2008
2012
2016
2020
2024
2028


Forecast for Exports of goods and services
Historical
Forecast
0.10
0.05
0.00
−0.05
−0.10
−0.15
1992
1996
2000
2004
2008
2012
2016
2020
2024
2028



In [32]:
```
last_level = gdp_block_level.iloc[-1].copy()
last_level
```

Out[32]:
```
Estimates
Final consumption expenditure                          2004878
Gross fixed capital formation                           511490
General governments final consumption expenditure       554367
Exports of goods and services                           698086
Less: imports of goods and services                     793594
Name: 2025-04-01 00:00:00, dtype: int64
```

In [33]:
```
level_forecast = pd.DataFrame(index=forecast_df.index, columns=forecast_df.columns, c


current = last_level.copy()
for i, idx in enumerate(forecast_df.index):
    current = current * (1.0 + forecast_df.iloc[i])
    level_forecast.loc[idx] = current

level_forecast.head()
```

Out[33]:

Estimates	Final consumption expenditure	Gross fixed capital formation	General governments final consumption expenditure	Exports of goods and services	Less: imports of goods and services
2025Q3	2.020177e+06	507685.700399	558463.775725	699180.526312	795191.425369
2025Q4	2.022886e+06	511248.591523	560992.359205	699419.948813	796928.563254
2026Q1	2.036928e+06	514763.631260	564514.227085	704061.491116	802939.797245
2026Q2	2.051158e+06	518007.864902	567262.355408	709226.192903	811246.790814
2026Q3	2.061970e+06	520508.805425	569728.214699	713079.021600	817074.922402

In [34]:
```
import_col = [c for c in level_forecast.columns if "imports of goods and services" in


total_gdp_level = (
    level_forecast["Final consumption expenditure"]
    + level_forecast["Gross fixed capital formation"]
    + level_forecast.filter(like="final consumption", axis=1).filter(like="government
    + level_forecast["Exports of goods and services"]
    - level_forecast[import_col]
)

total_gdp_level.name = "GDP (level, chained 2017$)"
total_gdp_level.head()
```

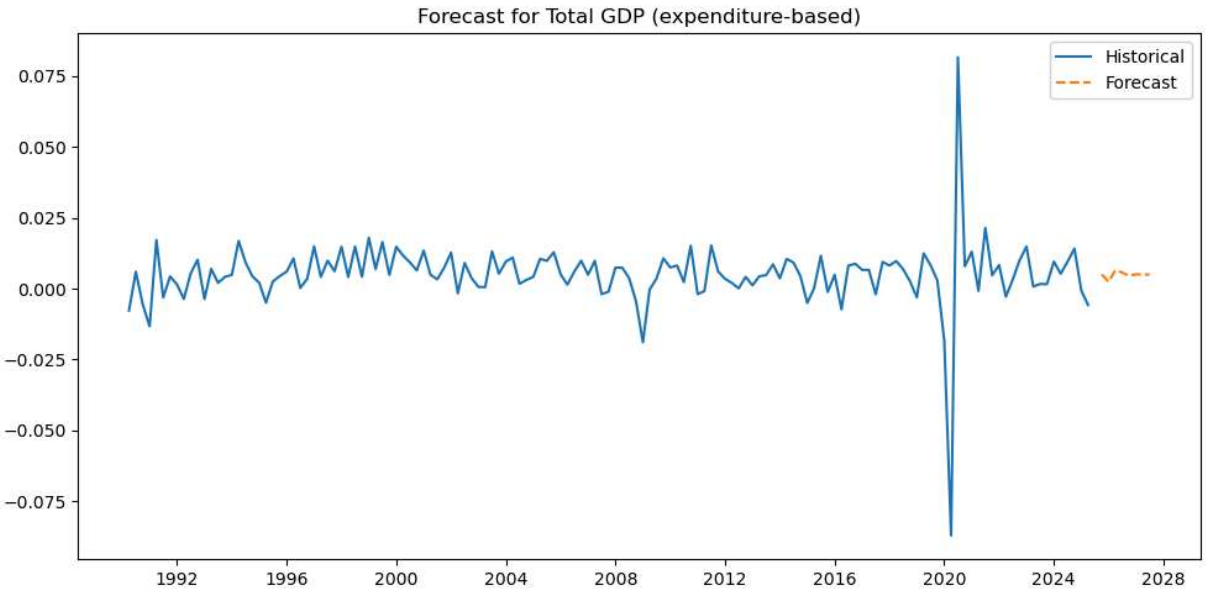
Out[34]:
```
2025Q3    2.990316e+06
2025Q4    2.997618e+06
2026Q1    3.017328e+06
2026Q2    3.034408e+06
2026Q3    3.048212e+06
Freq: Q-DEC, Name: GDP (level, chained 2017$), dtype: float64
```

In [ ]:
```
total_gdp_growth = np.log(total_gdp_level).diff()
total_gdp_growth.name = "GDP growth (Δlog)"
total_gdp_growth.dropna().head()
```

Out[ ]:
```
2025Q4    0.002439
2026Q1    0.006554
2026Q2    0.005645
2026Q3    0.004539
2026Q4    0.005039
Freq: Q-DEC, Name: GDP growth (Δlog), dtype: float64
```

In [39]:
```
hist_total_level = (
    gdp_block_level["Final consumption expenditure"]
  + gdp_block_level["Gross fixed capital formation"]
  + gdp_block_level["General governments final consumption expenditure"]
  + gdp_block_level["Exports of goods and services"]
  - gdp_block_level["Less: imports of goods and services"]
)
```

```
hist_total_growth = np.log(hist_total_level).diff().dropna()


fc_total_level = (
    level_forecast["Final consumption expenditure"]
  + level_forecast["Gross fixed capital formation"]
  + level_forecast["General governments final consumption expenditure"]
  + level_forecast["Exports of goods and services"]
  - level_forecast["Less: imports of goods and services"]
)


combo_level = pd.concat([hist_total_level.iloc[-1:], fc_total_level])
fc_total_growth = np.log(combo_level).diff().iloc[1:]


hist_idx = hist_total_growth.index.to_timestamp(how="end") if isinstance(hist_total_g
fc_idx   = fc_total_growth.index.to_timestamp(how="end")   if isinstance(fc_total_gro

plt.figure(figsize=(10,5))
plt.plot(hist_idx, hist_total_growth, label="Historical")
plt.plot(fc_idx,   fc_total_growth,   linestyle="--", label="Forecast")
plt.title("Forecast for Total GDP (expenditure-based)")
plt.legend()
plt.tight_layout()
plt.show()
```

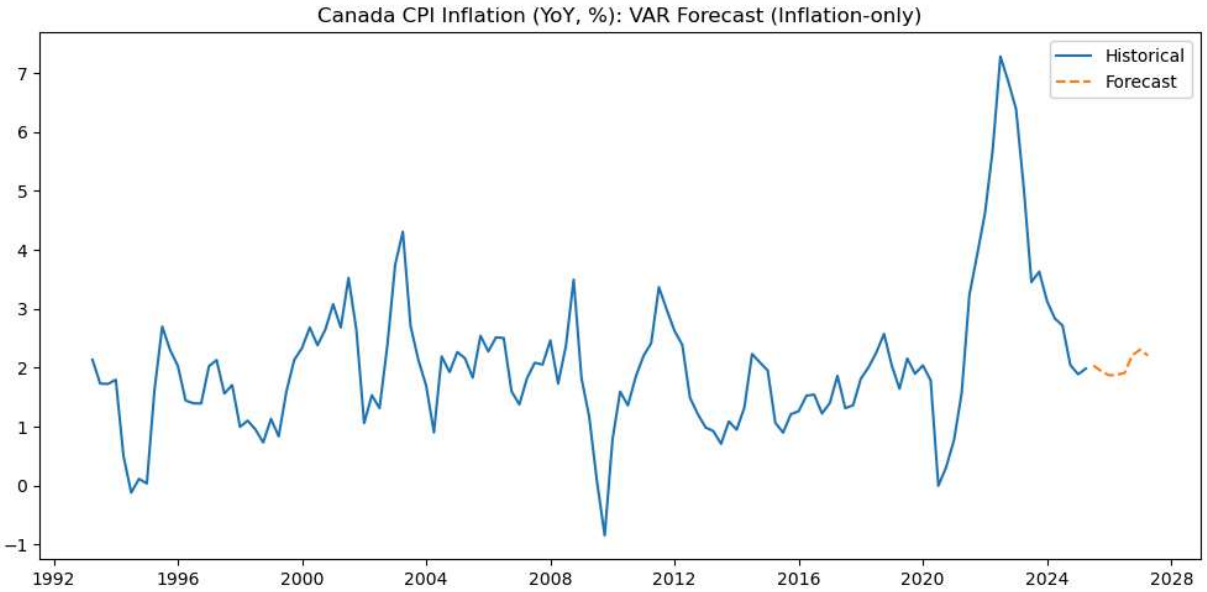


In [ ]:

```
# CPI → Canada, All-items
cpi = pd.read_csv("Inflation.csv")[["REF_DATE","GEO","Products and product groups","V
cpi = cpi[(cpi["GEO"]=="Canada") & (cpi["Products and product groups"]=="All-items")]
cpi["REF_DATE"] = pd.to_datetime(cpi["REF_DATE"])
cpi = cpi.sort_values("REF_DATE").set_index("REF_DATE").rename(columns={"VALUE":"CPI"

# Monthly MoM% and YoY%
infl_mom = (np.log(cpi["CPI"]) - np.log(cpi["CPI"].shift(1))) * 100
infl_yoy = (np.log(cpi["CPI"]) - np.log(cpi["CPI"].shift(12))) * 100
```

```
# Quarterly (average)
iq_mom = infl_mom.dropna().resample("Q").mean()
iq_yoy = infl_yoy.dropna().resample("Q").mean()
iq_mom.index = iq_mom.index.to_period("Q")
iq_yoy.index = iq_yoy.index.to_period("Q")

# VAR dataset: inflation-only (YoY & MoM)
var_data = pd.concat([iq_yoy.rename("Infl_YoY_%"), iq_mom.rename("Infl_MoM_%")], axis

# Fit VAR
sel = VAR(var_data).select_order(8)
lag = max(1, sel.selected_orders.get("aic", 2))
fit = VAR(var_data).fit(lag)

# Forecast
h = 8
yhat = fit.forecast(var_data.values[-lag:], steps=h)
fc_idx = pd.period_range(var_data.index[-1]+1, periods=h, freq="Q")
fc = pd.DataFrame(yhat, columns=var_data.columns, index=fc_idx)

# Plot: YoY only
hist = var_data.to_timestamp("Q")
fut  = fc.to_timestamp("Q")
plt.figure(figsize=(10,5))
plt.plot(hist.index, hist["Infl_YoY_%"], label="Historical")
plt.plot(fut.index,  fut["Infl_YoY_%"], linestyle="--", label="Forecast")
plt.title("Canada CPI Inflation (YoY, %): VAR Forecast (Inflation-only)")
plt.legend(); plt.tight_layout(); plt.show()
```

```
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\2353328134.py:15: FutureWarning: 'Q'
is deprecated and will be removed in a future version, please use 'QE' instead.
  iq_mom = infl_mom.dropna().resample("Q").mean()
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\2353328134.py:16: FutureWarning: 'Q'
is deprecated and will be removed in a future version, please use 'QE' instead.
  iq_yoy = infl_yoy.dropna().resample("Q").mean()
```



In [50]:
```
print("lag (selected):", lag)
print("lag (fit.k_ar):", fit.k_ar)
print(sel.summary())
```

```
lag (selected): 8
lag (fit.k_ar): 8
 VAR Order Selection (* highlights the minimums)
=================================================
      AIC         BIC         FPE         HQIC
-------------------------------------------------
0      -3.266      -3.220     0.03816      -3.247
1      -5.095      -4.956    0.006129      -5.038
2      -5.402      -5.171    0.004509      -5.308
3      -6.286      -5.963    0.001862      -6.155
4      -6.396      -5.980    0.001669      -6.227
5      -6.512      -6.004    0.001487      -6.305
6      -6.465      -5.865    0.001559      -6.221
7      -6.717     -6.023*    0.001214     -6.435*
8     -6.726*      -5.940   0.001204*      -6.406
-------------------------------------------------
```

In [ ]:
```
print(fit.is_stable())
print(fit.roots)
```

```
True
[ 4.2964697 -0.j         -1.26047078+0.8193306j  -1.26047078-0.8193306j
 -0.91085167+0.93526031j -0.91085167-0.93526031j  1.24752342-0.j
 -1.00991733+0.63379071j -1.00991733-0.63379071j  1.06879054+0.49713113j
  1.06879054-0.49713113j -0.52423984+1.001158j   -0.52423984-1.001158j
  0.74411672+0.84486558j  0.74411672-0.84486558j  0.44118693+1.00355731j
  0.44118693-1.00355731j]
```

In [52]:
```
print(fit.summary())
```

Summary of Regression Results

Model:	VAR		
Method:	OLS		
Date:	Mon, 13, Oct, 2025		
Time:	17:22:44		
No. of Equations:	2.00000	BIC:	-5.93996
Nobs:	121.000	HQIC:	-6.40649
Log likelihood:	97.5126	FPE:	0.00120436
AIC:	-6.72555	Det(Omega_mle):	0.000925909

Results for equation Infl_YoY_%

	coefficient	std. error	t-stat	prob
const	0.237881	0.097870	2.431	0.015
L1.Infl_YoY_%	-0.250265	0.211297	-1.184	0.236
L1.Infl_MoM_%	3.700723	0.495058	7.475	0.000
L2.Infl_YoY_%	-0.026225	0.185510	-0.141	0.888
L2.Infl_MoM_%	4.262309	0.759946	5.609	0.000
L3.Infl_YoY_%	0.078205	0.185236	0.422	0.673
L3.Infl_MoM_%	3.996342	0.938199	4.260	0.000
L4.Infl_YoY_%	-0.777706	0.172501	-4.508	0.000
L4.Infl_MoM_%	2.637617	1.088465	2.423	0.015
L5.Infl_YoY_%	-0.138504	0.196411	-0.705	0.481
L5.Infl_MoM_%	2.856044	1.083580	2.636	0.008
L6.Infl_YoY_%	0.056538	0.126749	0.446	0.656
L6.Infl_MoM_%	2.766208	0.985072	2.808	0.005
L7.Infl_YoY_%	0.162689	0.127235	1.279	0.201
L7.Infl_MoM_%	2.221850	0.831217	2.673	0.008
L8.Infl_YoY_%	-0.109323	0.080746	-1.354	0.176
L8.Infl_MoM_%	0.242537	0.586313	0.414	0.679

Results for equation Infl_MoM_%

	coefficient	std. error	t-stat	prob
const	0.110875	0.042000	2.640	0.008
L1.Infl_YoY_%	-0.087860	0.090675	-0.969	0.333
L1.Infl_MoM_%	0.305390	0.212448	1.437	0.151
L2.Infl_YoY_%	-0.018411	0.079609	-0.231	0.817
L2.Infl_MoM_%	0.447258	0.326121	1.371	0.170
L3.Infl_YoY_%	0.052090	0.079492	0.655	0.512
L3.Infl_MoM_%	0.404299	0.402616	1.004	0.315
L4.Infl_YoY_%	-0.089869	0.074027	-1.214	0.225
L4.Infl_MoM_%	0.131315	0.467101	0.281	0.779
L5.Infl_YoY_%	-0.073230	0.084287	-0.869	0.385
L5.Infl_MoM_%	0.542152	0.465005	1.166	0.244
L6.Infl_YoY_%	-0.005517	0.054393	-0.101	0.919
L6.Infl_MoM_%	0.469569	0.422731	1.111	0.267
L7.Infl_YoY_%	0.055267	0.054601	1.012	0.311
L7.Infl_MoM_%	0.314828	0.356706	0.883	0.377
L8.Infl_YoY_%	-0.012889	0.034651	-0.372	0.710
L8.Infl_MoM_%	-0.086698	0.251609	-0.345	0.730

```
================================================================================

Correlation matrix of residuals
              Infl_YoY_%  Infl_MoM_%
Infl_YoY_%      1.000000    0.894893
Infl_MoM_%      0.894893    1.000000
```

In [54]:
```
employment = pd.read_csv("employment.csv")
employment.columns
```

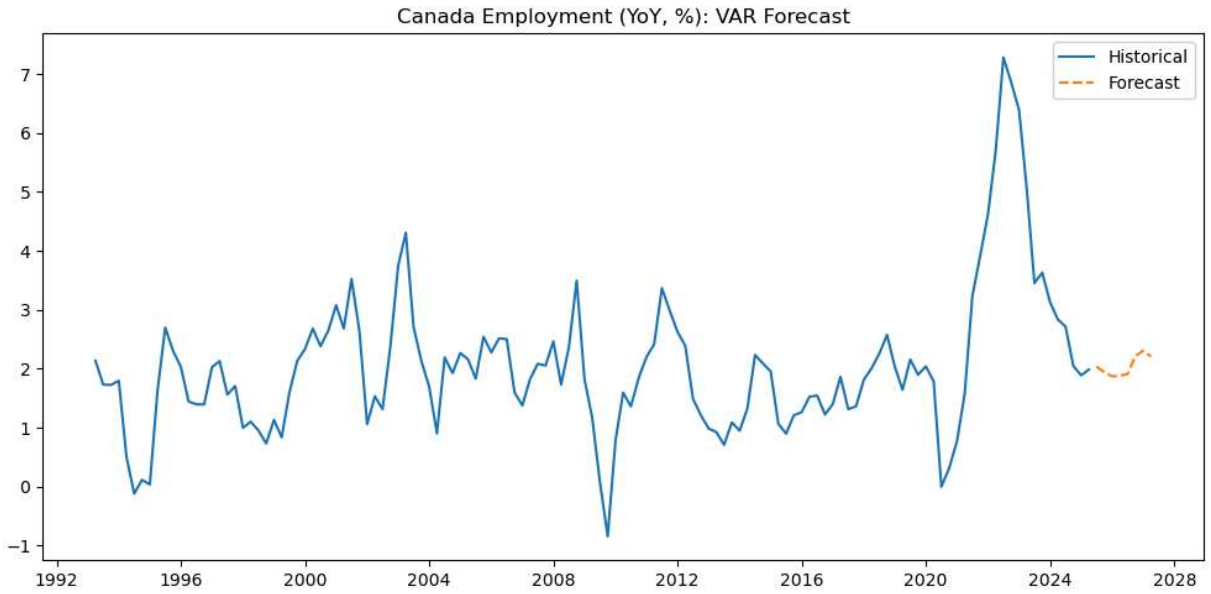
Out[54]:
```
Index(['REF_DATE', 'GEO', 'DGUID', 'Products and product groups', 'UOM',
       'UOM_ID', 'SCALAR_FACTOR', 'SCALAR_ID', 'VECTOR', 'COORDINATE', 'VALUE',
       'STATUS', 'SYMBOL', 'TERMINATED', 'DECIMALS'],
      dtype='object')
```

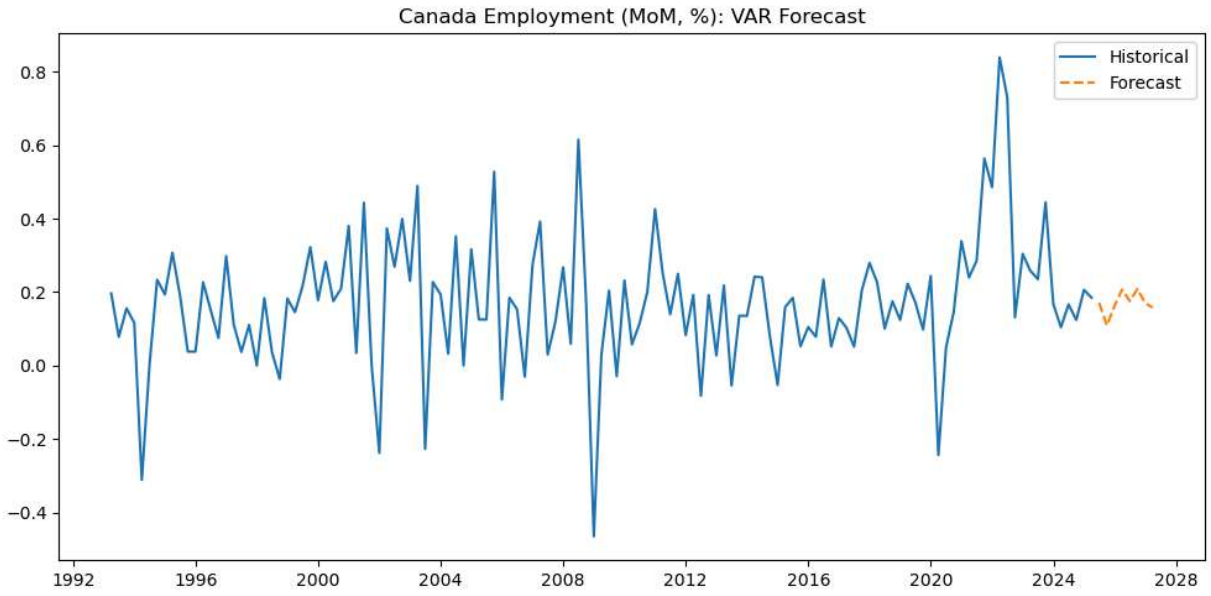
In [56]:
```
emp = pd.read_csv("employment.csv")
emp = emp[emp["GEO"]=="Canada"].copy()
emp["REF_DATE"] = pd.to_datetime(emp["REF_DATE"])
emp = emp.sort_values("REF_DATE")

main_name = None
cands = emp["Products and product groups"].dropna().unique().tolist()
if "Employment" in cands:
    main_name = "Employment"
else:
    cand_list = [c for c in cands if "employment" in c.lower()]
    main_name = cand_list[0] if cand_list else cands[0]

emp = emp[emp["Products and product groups"]==main_name][["REF_DATE","VALUE"]]
emp = emp.set_index("REF_DATE").rename(columns={"VALUE":"EMP"}).astype(float)


emp_yoy_m = (np.log(emp["EMP"]) - np.log(emp["EMP"].shift(12))) * 100
emp_mom_m = (np.log(emp["EMP"]) - np.log(emp["EMP"].shift(1 ))) * 100


emp_yoy_q = emp_yoy_m.dropna().resample("Q").mean()
emp_mom_q = emp_mom_m.dropna().resample("Q").mean()
emp_yoy_q.index = emp_yoy_q.index.to_period("Q")
emp_mom_q.index = emp_mom_q.index.to_period("Q")

var_data = pd.concat([emp_yoy_q.rename("Emp_YoY_%"),
                      emp_mom_q.rename("Emp_MoM_%")], axis=1).dropna()


T, k = var_data.shape
maxlags_safe = max(1, min(8, T // (k + 1) - 1))
sel = VAR(var_data).select_order(maxlags_safe)
lag = max(1, sel.selected_orders.get("aic", 2))
fit = VAR(var_data).fit(lag)

h = 8
yhat = fit.forecast(var_data.values[-lag:], steps=h)
```

```
fc_idx = pd.period_range(var_data.index[-1]+1, periods=h, freq="Q")
emp_fc = pd.DataFrame(yhat, columns=var_data.columns, index=fc_idx)

print("selected_lag:", lag, "| stable:", fit.is_stable())


hist = var_data.to_timestamp("Q")
fut  = emp_fc.to_timestamp("Q")

plt.figure(figsize=(10,5))
plt.plot(hist.index, hist["Emp_YoY_%"], label="Historical")
plt.plot(fut.index,  fut["Emp_YoY_%"], linestyle="--", label="Forecast")
plt.title("Canada Employment (YoY, %): VAR Forecast")
plt.legend(); plt.tight_layout(); plt.show()

plt.figure(figsize=(10,5))
plt.plot(hist.index, hist["Emp_MoM_%"], label="Historical")
plt.plot(fut.index,  fut["Emp_MoM_%"], linestyle="--", label="Forecast")
plt.title("Canada Employment (MoM, %): VAR Forecast")
plt.legend(); plt.tight_layout(); plt.show()
```

```
selected_lag: 8 | stable: True
```

```
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\1252648259.py:22: FutureWarning: 'Q'
is deprecated and will be removed in a future version, please use 'QE' instead.
  emp_yoy_q = emp_yoy_m.dropna().resample("Q").mean()
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\1252648259.py:23: FutureWarning: 'Q'
is deprecated and will be removed in a future version, please use 'QE' instead.
  emp_mom_q = emp_mom_m.dropna().resample("Q").mean()
```





In [58]:
```
pol = pd.read_csv("population.csv")
pol.columns
```

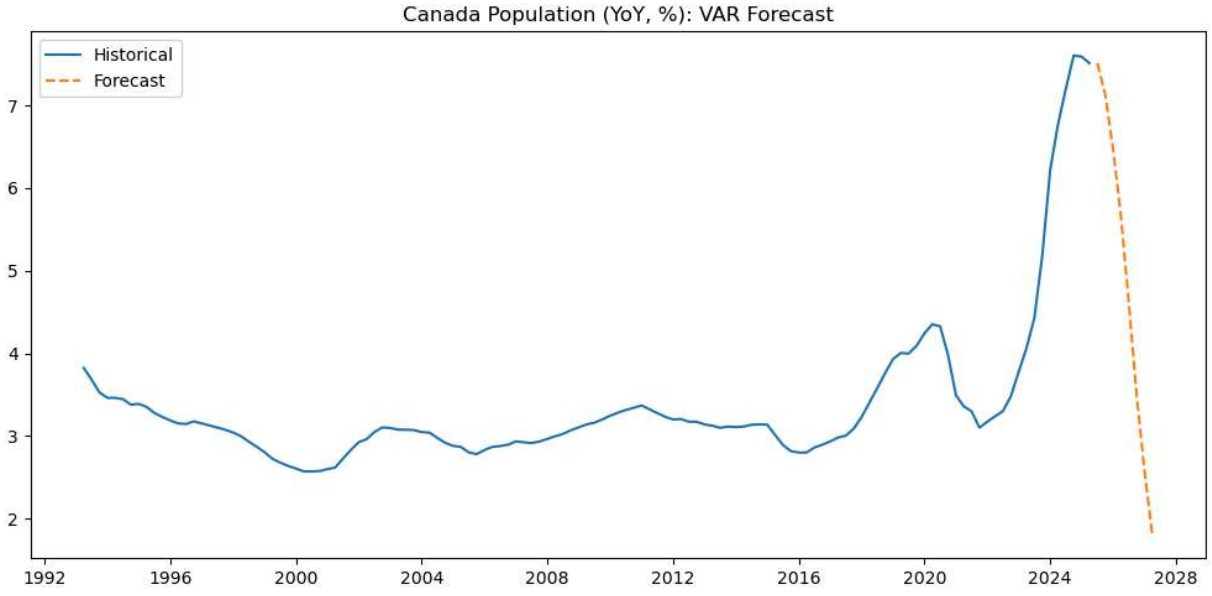
Out[58]:
```
Index(['REF_DATE', 'GEO', 'DGUID', 'UOM', 'UOM_ID', 'SCALAR_FACTOR',
       'SCALAR_ID', 'VECTOR', 'COORDINATE', 'VALUE', 'STATUS', 'SYMBOL',
       'TERMINATED', 'DECIMALS'],
      dtype='object')
```

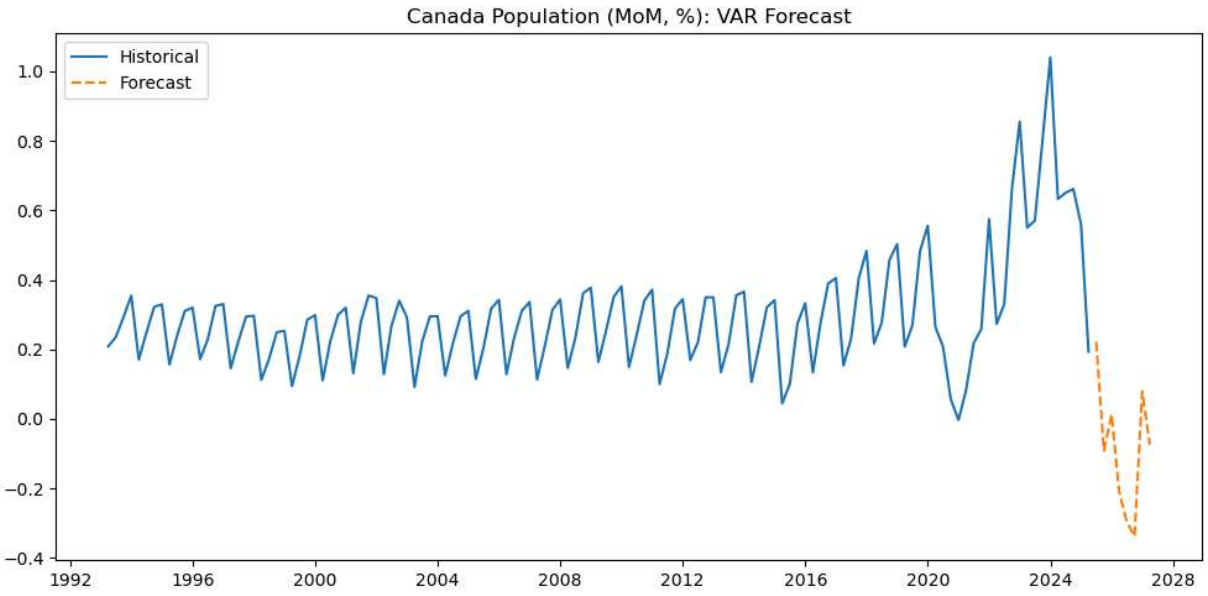
In [59]:
```
pop = pd.read_csv("population.csv")
pop = pop[pop["GEO"]=="Canada"][["REF_DATE","VALUE"]].copy()
pop["REF_DATE"] = pd.to_datetime(pop["REF_DATE"])
pop = pop.sort_values("REF_DATE").set_index("REF_DATE").rename(columns={"VALUE":"POP"


pop_yoy = (np.log(pop["POP"]) - np.log(pop["POP"].shift(12))) * 100
pop_mom = (np.log(pop["POP"]) - np.log(pop["POP"].shift(1 ))) * 100

pop_yoy_q = pop_yoy.dropna().resample("Q").mean()
pop_mom_q = pop_mom.dropna().resample("Q").mean()
pop_yoy_q.index = pop_yoy_q.index.to_period("Q")
pop_mom_q.index = pop_mom_q.index.to_period("Q")

var_data = pd.concat([pop_yoy_q.rename("Pop_YoY_%"),
                      pop_mom_q.rename("Pop_MoM_%")], axis=1).dropna()


T, k = var_data.shape
maxlags_safe = max(1, min(8, T // (k + 1) - 1))
sel = VAR(var_data).select_order(maxlags_safe)
lag = max(1, sel.selected_orders.get("aic", 2))
fit = VAR(var_data).fit(lag)

h = 8
yhat = fit.forecast(var_data.values[-lag:], steps=h)
fc_idx = pd.period_range(var_data.index[-1]+1, periods=h, freq="Q")
pop_fc = pd.DataFrame(yhat, columns=var_data.columns, index=fc_idx)
```

```
print("selected_lag:", lag, "| stable:", fit.is_stable())


hist = var_data.to_timestamp("Q")
fut  = pop_fc.to_timestamp("Q")

plt.figure(figsize=(10,5))
plt.plot(hist.index, hist["Pop_YoY_%"], label="Historical")
plt.plot(fut.index,  fut["Pop_YoY_%"], linestyle="--", label="Forecast")
plt.title("Canada Population (YoY, %): VAR Forecast")
plt.legend(); plt.tight_layout(); plt.show()

plt.figure(figsize=(10,5))
plt.plot(hist.index, hist["Pop_MoM_%"], label="Historical")
plt.plot(fut.index,  fut["Pop_MoM_%"], linestyle="--", label="Forecast")
plt.title("Canada Population (MoM, %): VAR Forecast")
plt.legend(); plt.tight_layout(); plt.show()
```

```
selected_lag: 8 | stable: False
```

```
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\3406423.py:10: FutureWarning: 'Q' is deprecated and will be removed in a future version, please use 'QE' instead.
  pop_yoy_q = pop_yoy.dropna().resample("Q").mean()
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\3406423.py:11: FutureWarning: 'Q' is deprecated and will be removed in a future version, please use 'QE' instead.
  pop_mom_q = pop_mom.dropna().resample("Q").mean()
```





In [61]:
```
pop = pd.read_csv("population.csv")
pop = pop[pop["GEO"]=="Canada"][["REF_DATE","VALUE"]].copy()
pop["REF_DATE"] = pd.to_datetime(pop["REF_DATE"])
pop = pop.sort_values("REF_DATE").set_index("REF_DATE").rename(columns={"VALUE":"POP"

pop_yoy = (np.log(pop["POP"]) - np.log(pop["POP"].shift(12))) * 100
pop_mom = (np.log(pop["POP"]) - np.log(pop["POP"].shift(1 ))) * 100


pop_yoy_q = pop_yoy.dropna().resample("Q-DEC").mean()
pop_mom_q = pop_mom.dropna().resample("Q-DEC").mean()
pop_yoy_q.index = pop_yoy_q.index.to_period("Q-DEC")
pop_mom_q.index = pop_mom_q.index.to_period("Q-DEC")

var_data = pd.concat([pop_yoy_q.rename("Pop_YoY_%"),
                      pop_mom_q.rename("Pop_MoM_%")], axis=1).dropna()

T, k = var_data.shape
maxlags_safe = max(1, min(8, T // (k + 1) - 1))  # estimation limit

def choose_stable_var(data, maxlags=maxlags_safe, ic="bic"):
    for p in range(maxlags, 0, -1):              # try from high to low
        fit = VAR(data).fit(p)
        if fit.is_stable():
            return fit
    # if none stable, fall back to p=1
    return VAR(data).fit(1)


fit_try = VAR(var_data).select_order(maxlags_safe)
p0_bic  = max(1, fit_try.selected_orders.get("bic", 1))
p0_bic  = min(p0_bic, maxlags_safe)
fit = choose_stable_var(var_data, max(p0_bic, 2))   # start near BIC, allow step-dowr

print("chosen lag:", fit.k_ar, "| stable:", fit.is_stable())

h = 8
```

```
yhat = fit.forecast(var_data.values[-fit.k_ar:], steps=h)
fc_idx = pd.period_range(var_data.index[-1] + 1, periods=h, freq=var_data.index.freq)
pop_fc = pd.DataFrame(yhat, columns=var_data.columns, index=fc_idx)


hist = var_data.to_timestamp()
fut  = pop_fc.to_timestamp()

plt.figure(figsize=(10,5))
plt.plot(hist.index, hist["Pop_YoY_%"], label="Historical")
plt.plot(fut.index,  fut["Pop_YoY_%"], linestyle="--", label="Forecast")
plt.title("Canada Population (YoY, %): VAR Forecast (stability-checked)")
plt.legend(); plt.tight_layout(); plt.show()
```

```
chosen lag: 6 | stable: True
```

```
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\3678002316.py:10: FutureWarning: 'Q-DEC' is deprecated and will be removed in a future version, please use 'QE-DEC' instead.
  pop_yoy_q = pop_yoy.dropna().resample("Q-DEC").mean()
C:\Users\logc0\AppData\Local\Temp\ipykernel_32060\3678002316.py:11: FutureWarning: 'Q-DEC' is deprecated and will be removed in a future version, please use 'QE-DEC' instead.
  pop_mom_q = pop_mom.dropna().resample("Q-DEC").mean()
```