Question Q#1

(a) Sol

we have $A(4,-4,1)$
$B(-4,3,-4)$
$C(4,-1,-2)$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$$

$$= -8i + 7j - 5K$$

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$= \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$$

$$= 3j - 3K$$

$$n = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & K \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix}$$

$$= i(-4+15) - (24-0)j + (-24)K$$

$$= -6i - 24j - 24K$$

$$= i + 4j + 4K$$

$$d = a.n = (4i - 4j + K) \cdot (i + 4j + 4K)$$

$$= 4 - 16 + 4 = -8$$

Equation of Plane

$$r.n = d$$

$$r(i + 4j + 4K) = -8$$

$$(x\hat{i} + y\hat{j} + zK) \cdot (i + 4j + 4K) = -8$$

$$x + 4y + 4z = -8$$

$$x + 4y + 4z + 8 = 0$$

(B) Perp distance $= \frac{d}{|n|} = \frac{8}{\sqrt{1^2+4^2+4^2}}$

$= \frac{8}{3.3} = 1.39$

(c). line OD :

$r = a \times \lambda b$

$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$

as same value of λ

$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$

$= 2\lambda + 12\lambda - 12\lambda = -8$

$= 2\lambda = -8$

$\lambda = -4$

$r = \begin{bmatrix} 2(-4) \\ 2(-4) \\ -3(-4) \end{bmatrix} = (-8, -12, 12)$

Question no 3:

(a) $l_1 = t\hat{i} + \hat{j} \qquad -2\hat{i} - \hat{j}$

$l_2 = \hat{j} + t\hat{k} \qquad -2\hat{j} + \hat{k}$

The shortest distance

l_2 is $\sqrt{21}$

$r_1 = OA + \lambda AB$

$r_2 = OA + \lambda AB$

$r_1 = t\hat{i} + \hat{j} + \lambda 2\hat{i} - \hat{j}$

$l_1 = r_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -10 \\ 0 \end{bmatrix}$

$l_2 \; r_2 = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + u \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{(b_1 \times b_2)}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & K \\ -2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} \; j \begin{vmatrix} -2 & 0 \\ 0 & 1 \end{vmatrix} + K \begin{vmatrix} -2 & -1 \\ 0 & -2 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(-2) + K(4)$$

$$= -\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(-1)^2 + (2)^2 + (4)^2}$$

$$= \sqrt{1+4+16}$$

$$= \sqrt{21}$$

$$(a_2 - a_1) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} t \\ 0 \end{bmatrix}$$

$$= -t\hat{i} + t\hat{k}$$

$$D = \frac{(-\hat{i} + 2\hat{j} + 4\hat{k}) \cdot (-t\hat{i} + t\hat{k})}{\sqrt{21}}$$

$$\sqrt{21} = t + 4j / \sqrt{21}$$

$$21 = t + 4j$$

$$21 = 5t$$

$$t = 21/5$$

(B) $\gamma_1 = \frac{21}{5}\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j})$

$$\gamma_2 = j - \frac{21}{5}\hat{k} + u(-2\hat{j} + \hat{k})$$

$$\pi_1 = \gamma = a\dot{i} + \lambda\overrightarrow{AB} + \mu AC$$

$$\vec{r} = \frac{-21}{5}\hat{i} + \hat{j} + \lambda(-2\hat{i} - \hat{j}) + \mu(-2\hat{j} + k)$$

(C) $\lambda_2 = 5x - 6y + 5z = 0$

$$= \lambda_2 = \frac{x-0}{0}, \quad \lambda_2 = \frac{4-1}{-2}$$

$$\lambda_2 = \frac{2-4.2}{1}$$

from l_2 direction vector is

$= \{0, -2, 1\}$

from $\pi_2 = (5, -6, 7)$

$$\cos\theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{|a|\ |b|}$$

$$(a \cdot b) = \begin{bmatrix} 0 \\ 1 \\ -21/5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$= -6 + 29.4 = 23.4$

$|a| = \sqrt{1^2 + 21/5^2} = \sqrt{1 + 17.64}$

$\boxed{|a| = 4.3}$

$|b| = \sqrt{5^2 + 6^2 + 7^2}$

$|b| = 0.49$

$$\theta = \cos^{-1}\left(\frac{23.4}{4.3 \times 10.49}\right)$$

$$\theta = \cos^{-1} \frac{23.4}{45.11}$$

$\boxed{\theta = 59.34°}$

Pard (D)

$$\pi_1 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix}, \quad \pi_2 \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$\pi_1 \cdot \pi_2 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$\boxed{= -21 - 6 \Rightarrow -27}$

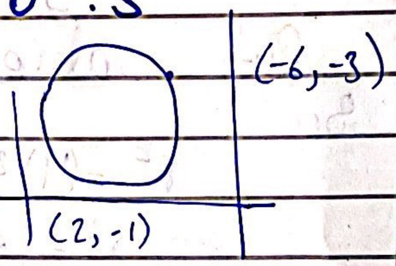
$|a| = \sqrt{(21/5)^2 + (1)^2} = \sqrt{17.64 + 1} = 4.3$

$|b| = \sqrt{5^2 + (-6)^2 + 7^2} = 10.49$

$\theta = \cos^{-1} \dfrac{-27}{4.3 \times 10.49}, \quad \theta = 126.78$

Question No :5

(a) Mid Point



$\left(\dfrac{-2-6}{2}\right) \left(\dfrac{-1-3}{2}\right)$

$-\dfrac{8}{2}, \quad \dfrac{-4}{2}$

$(-4, -2)$

eq of circle $= (x+h)^2 + (y-k)^2 = r^2 \rightarrow (1)$

$(x+4)^2 + (y+2)^2 = r^2$

$x(x, y) = (-2, -1)$

$(-2+4)^2 + (-1+2)^2 = r^2$

$4 + 1 = r^2$

$5 = r^2$

$r = \sqrt{5}$

Put the values

$(x+4)^2 + (y+2)^2 = 5$

(B) eq of circle

$(x-h)^2 + (y-k)^2 = r^2$

let $x = 0, \quad y = 6$

at Point $(4, 0)$

$4^2 + (-b)^2 = r^2$

$16 + b^2 = r^2$

at Point $(0, 2)$

$(0)^2 + (2-b)^2 = r^2$

$(2-b)^2 = r^2$

compare

$16 + b^2 = (2-b)^2$

$16 + \cancel{b^2} - \cancel{b^2} - 4 + 4b = 0$

$12 + 4b = 0$

$4b = -12$

$b = -3$

Put values in eq

So

$r^2 = (4)^2 + (-3)^2$

$r^2 = 16 + 9$

$r = \pm 5$

(c)

$y^2 = 100x$

$y^2 = 4ax$

$4a = 100$

$\boxed{a = 25}$

eq of directx $= x = -a$

$x = -25$

(d) $x^2 = 24y$

$x^2 = 4ay$

$4a = 24$

$a = 8$

$x = -a$

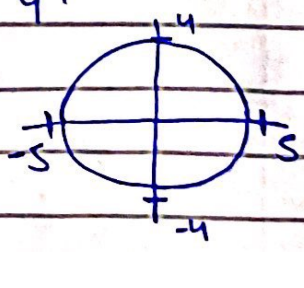
$\boxed{x = -8}$

(e)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^4}$

$a = 5$

$b = 4$



$c = \sqrt{a^2 - b^2} = \sqrt{25-16} = \pm 3$

$f_1 = (3, 0)$
$f_2\ (-3, 0)$

Length of major axis = $2a$
$= 2(5) = 10$

(f)

major axis = 10
minor axis = 8
$2a = 10$
$a = 5$
$2b = 8$
$b = 4$

equation of ellipse

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

Q2

(a) $AB = 4i - j + k$
$CD = 0i - j + (3-\lambda)k$
L_1 : line $AB = OA + \lambda AB$
L_2 = line $CD = OC + \mu CD$

$$L_1 : r_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$L_2 : r_2 = \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -1 \\ 3-\lambda \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2)\cdot(a_2 - a_1)}{(b_1 \times b_2)} \Rightarrow b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & -1 & 3-k \end{vmatrix}$$

$$= \sqrt{17\lambda - 26\lambda + 169}$$

$$a_2 - a_1 = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$$3 = \frac{(-3+\lambda)5 + 2(12-4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$$= 49\lambda^2 + 1 - 24\lambda$$

$\lambda =$ Then $\lambda^2 - 5\lambda + 4 = 0$

(B) Plane ABD when $\lambda = 1$

$$D = 2i + 7j + k$$

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix}$$

$$= i(-2-3) - j(8+5) + k(12-5)$$

$$= -5i - 13j + 7k \Rightarrow -10 - 91s + 7t$$

for π_2

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 5 & -3 & 5 \end{vmatrix}$$

$$= -8i - 15j + 17k \Rightarrow 8x + 15y - 17z - 13J$$

(C) $\theta = \cos^{-1}\left|\frac{|n_1 . n_2|}{|n_1||n_2|}\right|$

$$n_1 . n_2 = 4 + 195 + 1.19$$

$$\cos^{-1}\left(\frac{318}{(15.58)(24.04)}\right)$$

$$\cos^{-1}\left(3.8/374.56\right)$$

$$\theta = 31.89^\circ$$