Question no 1

(a) $\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$

$x^2=4y^2 \Rightarrow x=4y$

$y=x/4$

$$\frac{x^2/16 - 2(x)(x/4)}{x^2-4(x^2/16)}$$

$$\frac{\frac{x^2}{16}-\frac{x^2}{2}}{x^2-\frac{x^2}{4}} = \frac{(x^2-8x^2)4}{16(4x^2-x^2)}$$

$$= \frac{-7x^2}{4(3x^2)} \Rightarrow -7/12$$

(B) $\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x}$

$6y=7x$

$y=7x/6$

$$\frac{x-4(7x/6)}{7x+7x}$$

$$\frac{6x-28x}{6(14x)} \Rightarrow \frac{-2-2x}{6(14)x} \Rightarrow \frac{-11}{52}$$

(C) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$

Let $y=mx$

$$\frac{x^2-m^6x^6}{x(m^3x^3)}$$

$$= \frac{x^2(1-m^6x^6)}{x^4m^3} \Rightarrow \frac{1-m^6x^3}{m^3x^2}$$

(d) $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - 7e^{2y}}{6x+2y-3z}$

$$\frac{(-1)^3 - 4(0)e^2}{6(-1)+2(0)-3(4)}$$

$$= \frac{-1}{-16} = \frac{1}{16}$$

Question no 2

(a) $f(x,y) = \cos(x/y)$ $\quad \vec{v} = (3,-4)$

$$f\nabla = \frac{\partial}{\partial x}\cos(x/y)\,i + \frac{\partial}{\partial y}(\cos(x/y))\,j$$

$$= \frac{-1}{y}\sin(x/y)\,i + x(-\sin x/y)(-1/y^2)\,j$$

$$= -\frac{1}{y}\sin\left(\frac{x}{y}\right)i + \frac{x}{y}\sin(x/y)\,j$$

unit vector:

$$\frac{3i - 4j}{\sqrt{9+16}} \Rightarrow \frac{3}{\sqrt{25}}\,i + \frac{4}{\sqrt{25}}\,j$$

$$D\vec{u}f = \frac{-3}{5}\sin(x/y) + \frac{4x}{5y^2}\sin(x/y)$$

$$D\vec{u}f = \frac{1}{5y}\sin(x/y)(4x/y - 3)$$

(b) $f(x,y,z) = x^3y^3 - 4xz$ $\quad \vec{v} = (-1,2,0)$

$$\nabla f = (2y^3x\,i + (-4z)i) + 3y^2x^6\,j\;4x\,k$$

$$v = \frac{-i + 2j + 0k}{\sqrt{1+4}} \Rightarrow \frac{-1}{15}\,i + \frac{2}{\sqrt{5}}\,j + 0k$$

$$D\vec{u}f = \frac{-1}{\sqrt{5}}(2y^3x - 4z) + \frac{2}{\sqrt{5}}(3y^2x^2) + c$$

Question no 3

$f(x,y,z) = 4x - y^2 e^{3xz}$

$\nabla f = 4 - y^2 3xz\,(3z)\,i - 2ye^{3xz}\,j + y^2 e^{3xz} \cdot 3x\,k$

$= (4-0)i - 2j - 9k$

$v = (-1, 4, 2)$

$\Rightarrow \frac{-1}{\sqrt{21}} i + \frac{4}{\sqrt{21}} j + \frac{2}{\sqrt{21}} k$

$\frac{-4}{\sqrt{21}} - \frac{8}{\sqrt{21}} \frac{18}{\sqrt{21}} \Rightarrow \frac{-32}{\sqrt{21}}$

Question no 4

(a) $f(x,y) = \sqrt{x^2+y^2}$ at $(-2, 3)$

$\nabla f = \frac{1}{2}(x^2+y^2)^{-1/2}(2x)i + \frac{1}{2}(x^2+y^2)^{-1/2}(3y^2)j$

$= \frac{-2}{\sqrt{13}} i + \frac{27}{2\sqrt{13}} j$

(b) $f(x,y,z) = e^{2x}\cos(y-2z)$ at $(4,-2,0)$

$\nabla f = (e^{2x})2\cos(y-2z)\,i + e^{2x}(-\sin(y-2z)(1))j$

$+ e^{2x}(-\sin(y-2x)(-2)\,k$

$f(4,-2,0) = e^{2x} \cdot 2\cos(-2,-0)\,i + e^{4x}\sin(-2)j$

$+ e^{2x}(-\sin(-2-8)(-2)k$

Question no 5

(a) $f = x^2y\,\hat{i} - (z^3-3x)j + 4y^2\hat{k}$

$f = (x^2y, -(z^3-3x), 4y^2)$

$\nabla \cdot f = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$

$\nabla \cdot f = 2xy\,i - 0j + 0k$

$\Delta f \cdot f = (2xy\,i) \cdot [x^2y\,i - (z^3 - 3x)\,j + 4y^2 k]$

$= 2x^3y^2$

curl $= \nabla f \times f$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & -(z^3 - 3x) & 4y^2 \end{vmatrix}$$

$= (8y + 3z^2)\,i - 0\,j + (2x^2)\,k$

(b) $f = (8x + 2z^2)\,i + \frac{x^3y^2}{2}\,j - (z - 7x)\,k$

div $= \nabla f \cdot f$

$= \left(\frac{\partial}{\partial x}\,i + \frac{\partial}{\partial y}\,j + \frac{\partial}{\partial z}\,k\right) \cdot (2x + 2z) + \frac{x^3}{2} y\,j (-2$

$= 2 + \frac{2x^3}{2}\,y - (1)$

$= 2 + \frac{2x^3}{2}\,y - 1$

curve $= \nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x + 2z^2 & \frac{x^3y^2}{2} & (-z\ 7x) \end{vmatrix}$$

$\frac{x^3y^2}{x^2}\,i - (7 - 4z)\,j + \left(3\,\frac{x^2y^2}{2}\right) k$

Question no 6

(a) $\vec{f} = \left(4y^2 + \frac{3x^2y}{z^2}\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3y}{z^3}\right)$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$, $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$

M N

$f = \left(4y^2 + \frac{3x^2}{z^2}y\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j}$

$+ \left(11 - \frac{2x^3y}{z^3}\right)\hat{k} \Rightarrow B$

$\frac{\partial M}{\partial y} = 8y - 3x^2/z^2$

$\frac{\partial N}{\partial z} = \frac{-2x^3}{z^3}$

$\frac{\partial P}{\partial y} = \frac{-2x^3}{z^3}$

$\frac{\partial M}{\partial z} = \frac{-6x^2y}{z^3}$

$\frac{\partial P}{\partial x} = \frac{-6x^2y}{z^3}$

Hence Proved
vector field is conservative

(b) $f = 6xi + (2x - y^2)j + (6z - x^3)k$

$f = 6xi + (2x - y^2)j + (6z - x^3)k$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(6x) = 0$, $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2x - y^2)$

$= 2$

$\frac{\partial M}{\partial z} = 0$, $\frac{\partial P}{\partial x} = -3x^2$

So Not Proved.

Question no 7

(a) $z = \frac{x^2 - w}{y^4}$, $x = \frac{t^3 + 7}{1}$ $y = \cos(2t)$, $w = 4t$

$\frac{dz}{dt} = ?$

$\frac{dz}{dt} = \frac{2x}{y^4}$, $\frac{dz}{dy} = (x^2 - w)\, y^{-4}$

$\frac{dx}{dt} = 3t^2$, $\frac{dz}{dy} = (x^2 - w) - 4y^{-5}$

$\frac{dx}{dt} = 3t^2$ $\frac{dz}{dy} = \frac{-4(x^2 - w)}{y^5}$

$\frac{dz}{dw} = \frac{-1}{y^4}$ $\frac{dw}{dt} = 4$

(B) $z = x^2y^4 - 2y$, $y = \sin(x^2)$

$\frac{dz}{dx} = ?$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$\frac{dz}{dy} = \frac{\partial}{\partial y}(x^2y^4 - 2y)$$

$$= (4y^3z^2 - 2)$$

$$= (4x^2y^3 - 2)$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin(x^2) = \cos x^2 \cdot 2x$$

$$= 2x\cos x^2$$

$$\frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= (4x^2y^3 - 2)(2x\cos x^2$$

$$= 8x^3y^3\cos x^2 - 4x\cos x^2$$