# UNIT - III

## DISTRIBUTIONS

→ There are 2 types of theoretical probability distributions:

1) discrete probability distributions
2) continuous probability distributions

1) **Discrete probability distributions :-**

i) Binomial distribution (BD): The conditions for the applicability of a binomial distribution are

Imp 2m for conditions for BD

a) there are 'n' independent trials.
b) Each trial has only 2 possible outcomes.
c) The probabilites of two outcomes remains constant and are denoted by $p, q$.

→ Let the no. of trials be 'n'. The trials be independent i.e., the success (or) failure doesnot effect the outcome of the other trials. Thus, the probability of success remains the same from the trial to trial.

We have $p+q=1$ i.e, total probability of success and failure is always equal to 1.

→ Let the probability of getting 'x' success and $(n-x)$ failures in one trail is $(p^x\ q^{n-x})$

Similary, the total probability of getting 'x' success and $n-x$ failures in $(nCx)$ $^{n}C_x$ no of ways. and the probability of each is $(p^x q^{n-x})$.

thus, if '$x$' is the random variable representing the no. of success and the probability of getting '$x$' success and $(n-x)$ failures is $\boxed{P(X=x) = (^nC_x) p^x q^{n-x}}$

NOTE:

i) $(p+q)^n = \sum_{x=0}^{n} (^nC_x) p^x q^{n-x}$

ii) $p+q=1$

iii) $\sum_{x=1}^{n-1} (^{n-1}C_{x-1}) p^{x-1} q^{n-x} = (p+q)^{n-1}$

properties: (constants of Binomial distributions)

1) Mean of a Binomial distribution is $np$.

proof: $E(X) = \text{Mean} = \mu = \sum_{x=0}^{n} x\, p(x)$

where $p(x) = (^nC_x)\, p^x q^{n-x}$

$$E(X) = \sum_{x=0}^{n} x\, (^nC_x)\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n(n-1)!}{x(x-1)!\,(n-x)!} p^{x-1} \times p\, q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!} p^{x-1} q^{n-x}$$

because $x=0$, $(0-1)! = -1!$ doesnot exist so we put $x=1$

$$= np \sum_{x=1}^{n-1} \left({}^{n-1}C_{x-1}\right) p^{x-1} q^{n-x}$$

$$= np(p+q)^{n-1}$$

$$= np \quad [\because p+q=1]$$

2) Variance of Binomial Distribution is $npq$

$$V(x) = E(X^2) - E(X)^2$$

$$E(X^2) = \sum_{x=0}^{n} x^2 P(x)$$

$$P(x) = ({}^nC_x)\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x^2 \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \left(x(x-1)+x\right) \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{x(x-1)\, n(n-1)(n-2)!}{x(x-1)(x-2)!\,(n-x)!} p^2 p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n-2} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n-2} \left({}^{x-2}C_{x-2}\right) p^{x-2} q^{n-x} + np$$

Binomial distribution $\Rightarrow (nC_x p^x q^{n-x})$

$$= n(n-1)p^2 \sum (p+q)^{n-2} + np$$

$$E(x^2) = n(n+1)p^2 + np \quad [\because p+q=1]$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= \cancel{n^2}p^2 - np^2 + np - \cancel{n^2}p^2$$

$$= np(1-p) = npq$$

**Mode of Binomial Distribution :-**

Mode of the Binomial Distribution is the value of for which $P(x)$ is maximum.

$$\text{Mode} = \begin{cases} \text{in } (n+1)p, \text{ if } (n+1)p \text{ is not an exact integer} \\ (n+1)p-1,\ (n+1)p, \text{ if } (n+1)p \text{ is an exact integer.} \end{cases}$$

**Recurrence relation for probabilities :-**

$$P(x) = (nC_x)p^x q^{n-x}$$

$$P(x+1) = (nC_{x+1})\, p^{x+1} q^{n-(x+1)}$$

$$= (nC_{x+1})\, p^{x+1} q^{n-x-1}$$

$$\frac{P(x+1)}{P(x)} = \frac{(nC_{x+1})\, p^{x+1} q^{n-x-1}}{(nC_x)\, p^x q^{n-x}}$$

$$= \frac{\dfrac{n!}{(x+1)!\,(n-x-1)!}\, p^{x+1} q^{n-x-1}}{\dfrac{n!}{x!\,(n-x)!}\, p^x q^{n-x}}$$

$$= \frac{n!\, p^{x} \cdot p\, q^{n-x} q^{-1}}{x!\,(x+1)\,(n-x-1)!} \times \frac{x!\,(n-x)!}{n!\, p^{x} q^{n-x}}$$

$$= \frac{(n-x-1)!\,(n-x)}{(x+1)\,(n-x-1)!} \times \frac{p}{q}$$

$$\frac{P(x+1)}{P(x)} = \left(\frac{n-x}{x+1}\right)\frac{p}{q}$$

$$\boxed{P(x+1) = \left(\frac{n-x}{x+1}\right)\frac{p}{q}\,P(x)}$$ which is recurrence relation for probabilities.

Problems:

1) 10 coins are tossed simultaneously. find the probability of getting.

i) atleast 7 heads

ii) atleast 6 heads

$n = 10$ ; $p = 1/2$ ; $q = 1/2$

$P(X = x) = P(x) = (nC_x)\, p^x q^{n-x}$

i) at least 7 heads $= P(X \geq 7)$

$$= P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= (10C_7)\left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^3 + (10C_8)\left(\tfrac{1}{2}\right)^8 \left(\tfrac{1}{2}\right)^2 + (10C_9)\left(\tfrac{1}{2}\right)^9 \left(\tfrac{1}{2}\right) + (10C_{10})\left(\tfrac{1}{2}\right)^{10}$$

$$= \left(\tfrac{1}{2}\right)^{10}\left[(10C_7) + (10C_8) + (10C_9) + (10C_{10})\right]$$

$$= \frac{1}{1024}\left[120 + 45 + 10 + 1\right]$$

$= \frac{1}{1024}[176] \Rightarrow 0.1718$

ii) Atleast 6 heads:-

$= P(x \geq 6)$

$= P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= \frac{1}{1024}[176 + 210]$

$= \frac{386}{1024} = \frac{193}{512} = 0.37695$

3) If 10% of the rivets produced by a machine are defective. find the probability that out of 5 rivets choosen at random.

i) Null will be defective

ii) 1 will be defective

iii) Atmost 2 will be defective

iv) Atleast 2 will be defective

$n = 5, \; P(X=x) = (nC_x) p^x q^{n-x}$

Let 'x' be the event of defective rivets

$P$ = probability of defective rivets $= 10\% = 0.1$

$q = 1 - p = 0.9$

i) Null will be defective

$P(X=0) = (5C_0)(0.1)^0 (0.9)^5$

$= (0.9)^5$

$= 0.59049$

$P(x=1) = ({}^5C_1)(0.1)^1(0.9)^4$

$= 0.3280$

iii) atmost two

$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= (0.9)^5 + 5(0.1)(0.9)^4 + ({}^5C_2)(0.1)^2(0.9)^3$

$= 0.5904 + 0.3280 + 0.0729$

$= 0.9913$

iv) at least two

$P(x \geq 2) = 1 - P(x<2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - [(0.9)^5 + 5(0.1)(0.9)^4]$

$= 0.815$

3) The mean and variances of binomial distribution are 4 and 4/3 respectively then find $P(x \geq 1)$.

Mean $= 4$, variance $= \frac{4}{3}$

$P(x \geq 1)$

Mean of Binomial Distribution $= np = 4$

Variance $= npq = \frac{4}{3}$

$$\frac{npq}{np} = \frac{\frac{4}{3}}{4}$$

$q = 1/3 \quad p = 1 - \frac{1}{3} = \frac{2}{3}$

$np = 4$

$n \times \frac{2}{3} = 4$

$2n = 12 \Rightarrow n = 6$

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - (P(X=0))$

$= 1 - ({}^6C_0)(2/3)^0 (1/3)^6$

$= 1 - \left(\frac{1}{3}\right)^6$

$= 0.9963$

4) A discrete random variable 'x' has mean '6' and variance '2'. If it is assumed that the distribution is binomial. find the probability that $P(5 \leq X \leq 7)$. $P(X = x) = ({}^nC_x) p^x q^{n-x}$

Mean of B.D $= np = 6$

variance $= npq = 2$

$$\frac{npq}{np} = \frac{2}{6} = 1/3$$

$p = 2/3$

$n \times \frac{2}{3} = 6$

$2n = 18$

$n = 9$

$$P(5 \le x \le 7) = P(x=5) + P(x=6) + P(x=7)$$

$$= ({}^9C_5)\left(\tfrac{2}{3}\right)^5\left(\tfrac{1}{3}\right)^4 + ({}^9C_6)\left(\tfrac{2}{3}\right)^6\left(\tfrac{1}{3}\right)^3 + ({}^9C_7)\left(\tfrac{2}{3}\right)^7\left(\tfrac{1}{3}\right)^2$$

$$= 0.2048 + 0.2731 + 0.2341$$

$$= 0.712$$

5) In a binomial distribution consisting of 5 independent trials. probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameters p and q of the distributions.

$$P(x=x) = ({}^nC_x)\, p^x q^{n-x}$$

given $P(x=1) = 0.4096$

$$P(x=2) = 0.2048$$

$$x = 5$$

$$P(x=1) = ({}^5C_1)\, p q^4 \rightarrow ①$$

$$P(x=2) = ({}^5C_2)\, p^2 q^3 \rightarrow ②$$

$$\frac{①}{②} = \frac{P(x=1)}{P(x=2)} = \frac{({}^5C_1)\, \cancel{p} q^{\cancel{4}}}{({}^5C_2)\, p^{\cancel{2}} \cancel{q^3}}$$

$$\frac{0.4096}{0.2048} = \frac{\cancel{5}q}{\cancel{10}_2\, p}$$

$$2 = \frac{q}{2p}$$

$$4p = q$$

$$4p = 1 - p$$

$$5p = 1$$

$$p = 1/5 \; ; \; q = 4/5$$

6) The overa... examinati... the exa... atleast

Let 'x'...

$p =$

$P(a$

7) the ...

If h...

his ...

$)+P(x=7)$

$(2/3)^6(1/3)^3+(9C_7)(2/3)^7(1/3)^2$

...sting of 5

1 and 2 successes

. Find the

...utions.

5

6) the overall percentage of failure in a certain examination is 20. If 6 candidates appear in the examination what is the probability that atleast 5 pass the examination.

Let 'x' is an event of students appearing in the examination

$p$ = probability of pass % = 80% = 0.8

$q = 1-p = 0.2$

$x=6 \quad P(x=x) = (nC_x)\, p^x q^{n-x}$

P(at least 5 pass)

$P(x \geq 5) = P(x=5) + P(x=6)$

$= (6C_5)(0.8)^5(0.2)^1 + (6C_6)(0.8)^6$

$= 0.6553$

7) the probability of a man hitting a target is $1/3$. If he fires 5 times. what is the probability of his getting the target atleast twice.

$p = 1/3 \,;\, q = 2/3 \,;\, n = 5 \qquad q = 1 - 1/3$

$P(X=x) = (nC_x)\, p^x q^{n-x} \qquad q = 2/3$

P(at least 2 target)

$P(X \geq 2) = P(X=2) + P(X=3)$

$= (5C_2)(1/3)^2(2/3)^3 + (5C_3)(1/3)^3(2/3)^2$

$= (10) \times \frac{1}{9} \times \frac{8}{27} + 10 \times \frac{1}{27} \times \frac{4}{9}$

$= 0.3292 + 0.1646 = 0.4938$

8) In 800 families with 5 children each how many families would you except to have

i) 3 boys

ii) No girl

iii) Atleast one boy

iv) Atmost two girls

v) Two boys and Two girls

Given 'x' is an event of giving a birth boy

$n = 5$ $\quad P(X=x) = (nC_x)\, p^x q^{n-x}$

no. of families $= 800 = N$

$P$ = probability of boy $= 1/2$

$q = 1/2$

i) $P(3\text{ boys}) = P(X=3) = (5C_3)(1/2)^3(1/2)^2$

$= 0.3125$

No. of families of 3 boys = $0.3125 \times 800 = 250$

ii) $P(\text{no girl}) = P(\text{All boys})$

$= P(X=5)$

$= (5C_5)(1/2)^5$

$= (1/2)^5 = 0.03125$

no. of families of no girl $= 0.03125 \times 800$

$= 25$

iii) $P(\text{atleast one boy}) = P(X \geq 1)$

$= 1 - P(X<1)$

$= 1 - (P(X=0))$

$= 1 - (5C_0)(1/2)^5$

$= 1 - 0.03125$

$= 0.9687$

$P(X \geq 1) = 0.9687$

no. of families with atleast one boy $= 0.9687 \times 800$

$= 775$

iv) $P(\text{Atmost two girls}) = P(0G, 5B) + P(1G, 4B) + P(2G, 3B)$

$= P(X=5) + P(X=4) + P(X=3)$

$= (5C_5)(1/2)^5 + (5C_4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 + (5C_3)(1/2)^3(1/2)^2$

$= \left(\frac{1}{2}\right)^5 \left[(5C_5) + (5C_4) + (5C_3)\right]$

$= \frac{1}{32}(16) = 0.5 \times 800 = 400$

v) $P(2\text{ boys}, 2\text{ girls})$

$$P(X=2) = ({}^5C_2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3$$

$$= (10)\left(\frac{1}{4}\right)\left(\frac{1}{8}\right)$$

$$= 0.3125$$

$$= 0.3125 \times 800$$

$$= 250$$

9) ((probability of a man)) the mean of the binomial distribution is 3 and variance is 9/4. find:

i) the value of 'n'.

ii) $P(x \geq 7)$

iii) $P(1 \leq X < 6)$

$$\text{Mean} = np = 3$$

$$\text{variance} = npq = \frac{9}{4}$$

$$\frac{npq}{np} = \frac{\frac{9}{4}}{3} = \frac{9}{4} \times \frac{1}{3} = \frac{3}{4}$$

$$p = 1 - q = 1/4$$

i) $np = 3$

$$n \times \frac{1}{4} = 3$$

$$n = 1/12$$

ii) $P(x \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$$+ P(X=11) + P(X=12)$$

$= ({}^{12}C_7)(1/4)^7(3/4)^5 + ({}^{12}C_8)(1/4)^8(3/4)^4 + ({}^{12}C_9)(1/4)^9(3/4)^3$

$+ ({}^{12}C_{11})(1/4)^{11}(3/4) + ({}^{12}C_{12})(1/4)^{12} + ({}^{12}C_{10})(1/4)^{10}(3/4)^2$

$= 0.0114 + 0.002 + 0.00026 + 0.000035 + 0.0000021$

$+ 0.00000005$

$= 0.014$

iii) $P(1 \le X < 6) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$+ P(X=5)$

$= ({}^{12}C_1)(1/4)^1(3/4)^{11} + ({}^{12}C_2)(1/4)^2(3/4)^{10} + ({}^{12}C_3)(1/4)^3(3/4)^9$

$({}^{12}C_4)(1/4)^4(3/4)^8 + ({}^{12}C_5)(1/4)^5(3/4)^7$

$= 0.1267 + 0.2322 + 0.2581 + 0.1935 + 0.1032$

$= 0.9137$

(29.625)×(0.7494)
= 59.238
like wise we have to calculate

(10) find a Binomial distribution to the following data

observed frequency — expected frequency

| $x$ | $f$ | $fx$ | $\frac{n-x}{x+1}$ | $\left(\frac{n-x}{x+1}\right)\frac{p}{q}$ | $F(x)$ |
|---|---|---|---|---|---|
| 0 | 28 | 0 | 4/1 | 1.9996 | 29.625 |
| 1 | 62 | 62 | 3/2 | 0.7498 | 59.238 |
| 2 | 46 | 92 | 2/3 | 0.3332 | 44.4167 |
| 3 | 10 | 30 | 1/4 | 0.1249 | 14.7996 |
| 4 | 4 | 16 | 0 | 0 | 1.8484 |
| | | 200 | | | 149.9277 |

$\Sigma f = N = 150$

Reccurence for probabilites Binomial distribution is

$$P(x+1) = \left(\frac{n-x}{x+1} \cdot \frac{p}{q}\right) P(x)$$

mean $= \mu = np,\ n=4$

$$\text{Mean} = \mu = \frac{\Sigma fx}{\Sigma f}$$

$$\frac{\Sigma fx}{\Sigma f} = np$$

$$4p = \frac{200}{150}$$

$$p = 0.3333$$

$$q = 1-p$$

$$= 1-0.3333$$

$$= 0.6667$$

$$\frac{p}{q} = \frac{0.3333}{0.6667}$$

$$= 0.4999$$

$$P(x) = (nc_x)\, p^x (q)^{n-x}$$

$$P(0) = (4c_0)\,(0.6667)^4$$

$$= 0.1975$$

$$F(0) = N \times P(0) = 150 \times 0.1975$$

$$= 29.625$$

Sum of the given frequencies is approximately equal to sum of the expecked frequencies.

Hence the given data is fitted.

11) 7 coins a
heads ob
fit a bino
coins are
i) biased
ii) unbias

| no. of he |
|---|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |

11) 7 coins are tossed at a time, the number of heads observed at each throw is given below. 128 times.
fit a binomial distribution by assuming the coins are :

i) biased

ii) unbiased

| no. of heads | frequency |
| --- | --- |
| 0 | 7 |
| 1 | 6 |
| 2 | 19 |
| 3 | 35 |
| 4 | 30 |
| 5 | 23 |
| 6 | 7 |
| 7 | 1 |

i) unbiased

RR (Recurrence) for probabilites Binomial Distribution is

$$(x+1) = \left(\frac{n-x}{x+1}\,\frac{p}{q}\right) P(x)$$

where the coins are unbiased

$$p = 1/2 = q$$

$$n = 7 \quad \frac{p}{q} = 1$$

$$P(x) = (n|x)\, p^x q^{n-x}$$

$$P(0) = (7C_0)(1/2)^7$$

$$= (1/2)^7$$

$$= \frac{1}{128}$$

$$F(0) = N \times P(0)$$

$$= 128 \times \frac{1}{128}$$

$$= 1$$

| No. of heads (x) | n=7 frequency | $\left(\frac{n-x}{x+1}\right)$ | $\left(\frac{n-x}{x+1}\right)\frac{p}{q}$ | F(x) expected frequency |
|---|---|---|---|---|
| 0 | 7 | 7/1 | 7 | 1 |
| 1 | 6 | 6/2 | 6/2 | 7 |
| 2 | 19 | 5/3 | 5/3 | 21 |
| 3 | 35 | 4/4 | 1 | 35 |
| 4 | 30 | 3/5 | 3/5 | 35 |
| 5 | 23 | 2/6 | 2/6 | 21 |
| 6 | 7 | 1/7 | 1/7 | 7 |
| 7 | 1 | 0 | 0 | 1 |
| | N=128 | | | 128 |

sum of frequencies = sum of the expected frequency

hence the given value is fitted.

ii) unbiased :

Reccurrence for probabilites Binomial Distribution is

$$(x+1) = \left(\frac{n-x}{x+1}\,\frac{p}{q}\right)P(x)$$

Mean of Binomial Distribution $= np = \frac{\Sigma fx}{\Sigma f}$

$$7p = \frac{433}{128}$$

$$= 3.3828$$

$$p = 0.4832$$

$$q = 1-p = 0.5168$$

$$\frac{p}{q} = \frac{0.4832}{0.5168}$$

$$= 0.9349$$

$$P(x) = (n_{C_x}) p^x q^{n-x}$$

$$P(0) = (7_{C_0})(0.5168)^7$$

$$= 0.0098$$

$$F(0) = N \times P(0)$$

$$= 128 \times P(0)$$

$$= 1.2544$$

| $fx$ | No. of heads | frequency | $\left(\frac{n-x}{x+1}\right)$ | $\left(\frac{n-x}{x+1}\right)\frac{p}{q}$ | $F(x)$ |
|---|---|---|---|---|---|
| 0 | 0 | 7 | 7/1 | 6.5443 | 1.2544 |
| 6 | 1 | 6 | 6/2 | 2.8047 | 8.2091 |
| 38 | 2 | 19 | 5/3 | 1.5581 | 23.0240 |
| 105 | 3 | 35 | 4/4 | 0.9349 | 35.8736 |
| 120 | 4 | 30 | 3/5 | 0.5609 | 133.5382<br>20.1215 |
| 115 | 5 | 23 | 2/6 | 0.3116 | 18.8129<br>6.2698 |
| 42 | 6 | 7 | 1/7 | 0.1335 | 5.8626<br>0.8370 |
| 7 | 7 | 1 | 0 | 0 | 0.7826 |
| 433 | | N=128 | | | 127.3574<br>95.5894 |

$$\Sigma f = 128 \qquad \Sigma fx = 433$$

$$P(x) = (nx)\, p^x q^{n-x}$$

$$P(0) = (7c_0)(0.5168)^7$$

$$= (0.5168)^7 = 0.0098$$

$$N \times P(0) = F(0)$$

$$F(0) = 128 \times 0.0098 = 1.2544$$

Recurrence Relation (RR) between probabilites.

$$P(x+1) = \left(\frac{n-x}{x+1}\right)\frac{p}{q}\, P(x)$$

Here sum of the given frequencies approximately equal to sum of expected frequency

here given data is fitted.

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ii) p i

iii) n

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Poisson distribution :-

i) when $n \to \infty$

ii) $p$ is small $p \to 0$

iii) $np = \lambda$

$p(x) = (nc_x)\, p^x q^{n-x}$

the poission distribution can be derived as in limiting case of the binominal distribution under the following conditionals.

condition 1: $p$, the probability of occurs of the even is every small $p \to 0$

condition 2: $n$ is every Large where '$n$' is the no. of trial i.e, $n \to \infty$

condition 3: $np$ is finite quantity say $np = \lambda$, where $\lambda$ is the parameter of poisson distribution.

theorem 1 :- Limiting case of binominal distribution is poission distribution probability of binomial distribution is

$$P(X = x) = P(x)\ (nc_x)\ p^x q^{n-x}$$

with Limiting conditions assume $np = \lambda$

$$p \to 0,\ n \to \infty$$

$$P(x) = \frac{n(n-1)(n-2) \cdots (n-(x-1))}{x!}\, p^x q^{n-x}$$

$$= \frac{\frac{\lambda}{p}\left(\frac{\lambda}{p} - 1\right)\left(\frac{\lambda}{p} - 2\right) \cdots \left(\frac{\lambda}{p} - (x-1)\right)}{x!}\, p^x \frac{(1-p)^n}{(1-p)^x}$$

$$= \frac{\lambda(\lambda - p)(\lambda - 2p) \cdots (\lambda - (x-1)p)}{p^x\, x!}\, p^x \frac{(1-p)^n}{(1-p)^x}$$

$$= \frac{\lambda^x \left(1-\frac{p}{\lambda}\right)\left(1-\frac{2p}{\lambda}\right)\cdots\left(1-\frac{(x-1)p}{\lambda}\right)}{x!} \frac{\left(\left(1-\frac{\lambda}{x}\right)\right)}{(1-p)^x}$$

$$= \frac{\lambda^x}{x!} \underset{\substack{n\to\infty \\ p\to 0}}{\mathrm{Lt}} \left(1-\frac{p}{\lambda}\right)\left(1-\frac{2p}{\lambda}\right)\left(1-\frac{(x-1)p}{\lambda}\right) \frac{\left[\left(1-\frac{\lambda}{x}\right)^{-n/\lambda}\right]^{-\lambda}}{(1-p)^x}$$

$$= \frac{\lambda^x}{x!} \frac{e^{-\lambda}}{1} = \frac{e^{-\lambda}\lambda^x}{x!} \quad \left[\underset{n\to\infty}{\mathrm{Lt}} \left(1-\frac{1}{n}\right)^x = e\right]$$

which the possion distribution ?

Definition :- If 'x' is a discrete Random variable said to follow a possion distribution which assumed to Non-negative values of 'x' with probability mass function given by

$$P(x) < P(X=x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & x = 0,1,2\ldots \\ 0 & \text{otherwise} \end{cases}$$

NOTE: $\Sigma P(X=x) = 1$

$$\Sigma P(x) = \sum_{x=0}^{n} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{n} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= e^{-\lambda} e^{\lambda} = 1$$

$\sin x = x - \frac{x^3}{3!} + \cdots$

$\cos x = x - \frac{x^2}{2} + \cdots$

$(1-x)^{-1} = 1 + x + x^2 - \cdots$

$\therefore e^x = 1 + x + x^2/2 - \cdots$

$\therefore \boxed{P(X) = 1}$

Hence proved

properties :-

1) Mean of the p

P(x)

Mean = e

$= \lambda e^{-}$

$= \lambda$

$= \lambda$

2) variance

E

properties:-

1) Mean of the poisson distribution (P.D) is '$\lambda$'.

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\text{Mean} = E(X) = \mu = \sum_{x=0}^{n} x\,P(x)$$

$$= \sum_{x=0}^{n} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{n} x \frac{e^{-\lambda}\lambda^x}{x(x-1)!}$$

$$= e^{-\lambda}\sum_{x=1}^{n} \frac{\lambda^{x-1}\times\lambda}{(x-1)!}$$

$$= \lambda e^{-\lambda}\sum_{x=1}^{n} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda}\left[1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\cdots\cdots\right]$$

$$= \lambda e^{-\lambda}\times e^{\lambda}$$

$$= \lambda$$

2) variance of poisson distribution is '$\lambda$'.

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$E(X^2) = \sum_{x=0}^{n} x^2 P(x)$$

$$= \sum_{x=0}^{n} x^2 \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{n}\left[x(x-1)+x\right]\frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{n} x(x-1)\frac{e^{-\lambda}\lambda^{x}}{x!} + \sum_{x=0}^{n} x\,\frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=0}^{n} x(x-1)\,\frac{e^{-\lambda}\lambda^{x-2}\times\lambda^{2}}{x(x-1)(x-2)!} + \lambda$$

$$= e^{-\lambda}\lambda^{2}\sum_{x=2}^{n}\frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= e^{-\lambda}\lambda^{2}\left[1+\lambda+\frac{\lambda^{2}}{2!}+\cdots\right]+\lambda$$

$$= e^{-\lambda}\lambda^{2}\times e^{\lambda}+\lambda$$

$$E(x^2) = \lambda^2+\lambda$$

$$V(x) = E(x)^2-(E(x))^2$$

$$= \cancel{\lambda^2}+\lambda-\cancel{\lambda^2}$$

$$= \lambda$$

3) Mode of the poission distribution

Mode is the value of 'x' for which $P(x)$ is maximum. therefore i) $P(x) \geq P(x+1) \rightarrow ①$

ii) $P(x-1) \leq P(x) \rightarrow ②$

Case-(i): $P(x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$

$P(x-1) \leq P(x)$

$$① \Rightarrow \frac{e^{-\lambda}\lambda^{x}}{x!} \geq \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$$

$(x+1)$

$$1 \geq \frac{\lambda}{x+1}$$

$$x+1 \geq \lambda$$

$$x \geq \lambda - 1$$

$$\lambda - 1 \leq x \rightarrow (3)$$

$(2) \Rightarrow P(x-1) \leq P(x)$

$$\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!} \leq \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$\frac{\lambda^{x} \times \lambda^{-1}}{(x-1)!} \leq \frac{\lambda^{x}}{(x-1)! \times x}$$

$$\frac{1}{\lambda} \leq \frac{1}{x}$$

$$\lambda \geq x$$

$$x \leq \lambda \rightarrow (4)$$

from (3) & (4)

$$\therefore \lambda - 1 \leq x \leq \lambda$$

Hence, mode of the poisson distribution lies between $\lambda-1$ and $\lambda$

case-(1): If $\lambda$ is an integer then $\lambda - 1$ is also an integer so, we have two maximum values and distribution is bimodal and two modes are $\lambda - 1$, $\lambda$

case-(2): If $\lambda$ is not an integer then the mode of the poisson distribution is integral part of '$\lambda$'.

Recurrence relation of probabilities poisson distribution:-

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$$

$$\frac{P(x+1)}{P(x)} = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} \times \frac{x!}{e^{-\lambda}\lambda^{x}}$$

$$= \frac{\lambda^{x} \times \lambda \; x!}{x!\,(x+1)\,\lambda^{x}}$$

$$= \frac{\lambda}{x+1}$$

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

1) The average no. of accidents on a national highway is 1.8 perday. Determine the probability that no. of accidents are:

i) atleast one

ii) atmost one

Given, $\lambda = 1.8$

Average no of accidents on a national highway = 1.8 perday

i) at least one, i.e mean of the P.D $\lambda = 1.8$

Probability that accident per day

$P(x \geq 1) = 1 - P(x < 1)$ $\quad P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-1.8}(1.8)^{x}}{x!}$

$= 1 - P(x = 0)$

$= 1 - \frac{e^{1.8}(1.8)^{0}}{0!}$

$= 1 - e^{1.8}$

$= -5.0496$

ii) at most one

$P(x \leq 1) = P(x=0) + P(x=1)$

$= \frac{e^{-\lambda}\lambda^{0}}{0!} + \frac{e^{-\lambda}\lambda^{1}}{1!}$

$$= e^{-1.8} + e^{-1.8}(1.8)$$

$$= 0.1652 + 0.1652(1.8)$$

$$= 0.1652 + 0.2973$$

$$= 0.4625$$

2) A distributer of bean seeds determine from extensive test that 5% of large batch of seeds will not guarntee he setts the seeds in packets of 200 and guarntees that 90% of germination determine that a particular packet will voilate the guarntees packet.

Let $p$ = the probability of seed not germinating

$= 5\% \Rightarrow 0.05$

$\lambda$ = mean no. of seeds of sample of 200

$\lambda = np$

$= 200 \times 0.05$

$= 10$

the packet will violates the guarntees if it contains more than 20 germination seeds

therefore the probability that the

$P(x>20) = 1 - P(x \leq 20)$ [90% of 200 → 180 Voilate the guarntee =20]

$= 1 - [P(x=0) + P(x=1) + \cdots + P(x=20)]$

$= 1 - e^{-10}\left[1 + 10 + \frac{10^2}{2!} + \frac{10^3}{3!} + \cdots + \frac{10^{20}}{20!}\right]$

3) An isurance company found that only 0.01%. population is involved in a certain type of accident each year if it's 1000 policy holders were randomly selected from the population what is the probability that not more than two of it's clients are involved in such an accident next year.

P = probability of percentage of population is involved in certain type of accident

n = 1000 (randomly selected holders

average no. of accidents to the next year

$\lambda = np$

$= 1000 \times 0.0001$

$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$\lambda = 0.1$

probability not more than 2

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-0.1} + e^{-0.1} \times 0.1 + \frac{e^{-0.1} \times (0.1)^2}{2!}$

$= e^{-0.1}\left[1 + 0.1 + \frac{(0.1)^2}{2!}\right]$

$= 0.9048\left[1 + 0.1 + \frac{0.01}{2!}\right]$

$= 0.9048[1 + 0.1 + 0.0005]$

$= 0.9048[1.105]$

$= 0.9998$

4) suppose X has probability distribution if $P(X=2) = \frac{2}{3}P(X=1)$

find i) $P(X=0)$

ii) $P(X=3)$

iii) Mean

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X=2) = \frac{2}{3}P(X=1)$$

$$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3} \times \frac{e^{-\lambda}\lambda}{1!}$$

$$\frac{\lambda}{2} = \frac{2}{3}$$

$$3\lambda = 4$$

$$\lambda = 4/3$$

$$\lambda = 1.3333$$

i) $P(X=0) = e^{-1.3333} - 0.2636$

ii) $P(X=3) = \frac{e^{-1.3333}(1.3333)^3}{3!}$ 2.3701

$$= \frac{(0.2636).(0.0182)}{6}$$

$$= \frac{0.0064}{6}$$ 0.6247

$$= 0.000$$ 0.1041

iii) Mean of P.D is $\lambda = 1.3333$

5) If x is a possion variate such that $P(X=0) = P(X=2) + 3P(X=4)$

find i) Mean ii) $P(X \leq 2)$

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X=0) = P(X=2) + 3P(X=4)$$

$$e^{-\lambda} = \frac{e^{-\lambda}\lambda^2}{2!} + \frac{3e^{-\lambda}\lambda^4}{4!}$$

$$1 = \frac{\lambda^2}{2} + \frac{3\lambda^4}{24}$$

$$1 = \frac{\lambda^2}{2} + \frac{\lambda^4}{8}$$

$$\lambda^4 + 4\lambda^2 - 8 = 0 \qquad \text{put } \lambda^2 = x$$

$$x^2 + 4x - 8 = 0$$

$x = 1.464$ , $-5.464$ (does not exist)

$\lambda^2 = 1.4641$, $\lambda^2 = -5.4641$

$$\lambda = \pm 1.21$$

$$\lambda = 1.21, -1.21$$

i) mean of $x$ is $\lambda = 1.21$, $\lambda = -1.21$

ii) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

where $\lambda = 1.21$ $\quad \because P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$$P(X \le 2) = e^{-1.21} + e^{-1.21}(1.21) + e^{-1.21}\frac{(1.21)^2}{2}$$

$$= e^{-1.21}\left[1 + 1.21 + \frac{(1.21)^2}{2}\right]$$

$$= e^{-1.21}\left[2.21 + \frac{1.4641}{2}\right]$$

$$= e^{-1.21}\left[2.21 + 0.73205\right]$$

$$= 0.2981\,[2.94205]$$

$$= 0.8770$$

If $\lambda = -1.21$

$$P(X \le 2) = e^{1.21}\left[1 + (-1.21) + \left(\frac{-1.21}{2}\right)^2\right]$$

$= e^{1.21}\left[1 - 1.21 + \frac{1.4641}{2}\right]$

$= e^{1.21}\left[1 - 1.21 + 0.73205\right]$

$= (3.3534)(0.52205)$

$= 1.750671$

$\therefore$ mean of x is 1.21

6) 2% of the items of factory are defective the items are packed in boxes what is probability that there will be in a box of 100 items.

i) Two Defective items

ii) Atleast three defective items

$np = \lambda \quad 100 \times 0.02 = 2 = \lambda$

Given $n = 100$ and the probability of Defective items is $2\% = 0.02$ since P is small and 'n' is large we use possion distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

i) probability of two defective items

$$P(X = 2) = \frac{e^{-2}\lambda^{2}}{2!} \quad (\because \lambda = 2)$$

$$= \frac{e^{-2}(2)^2}{2!} = \frac{e^{-2}(4)}{2!}$$

$$= \frac{(0.1353)(4)}{2}$$

$$= \frac{0.5412}{2}$$

$$= 0.2706$$

ii) Atleast 3 defective items

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad (\because \lambda = 2)$$

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - P(X=0) + P(X=1) + P(X=2)$$

$$= 1 - \frac{e^{-2}\lambda^0}{0!} + \frac{e^{-2}\lambda^1}{1!} + \frac{e^{-2}\lambda^2}{2!}$$

$$= 1 - \frac{e^{-2}(2)^0}{0!} + \frac{e^{-2}(2)^1}{1!} + \frac{e^{-2}(2)^2}{2!}$$

$$= 1 - \frac{e^{-2}(1)}{1} + \frac{e^{-2}(2)}{(1)} + \frac{e^{-2}(4)}{2}$$

$$= 1 - e^{-2}[1+2+2]$$

$$= 1 - e^{-2(5)} \Rightarrow 1 - e^{-10}$$

$$= 1 - 0.41$$

$$= 0.59$$

7) The Average number of phone calls per minute coming into a switch board between 2pm and 4pm is 2.5. Find the probability that during one particular minute:

i) 4 are fewer

ii) more than 6 calls

Average phone calls per minute is $\lambda = 2.5$

probability of P.D is $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

i) $P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$+ P(X=4)$

$$= \frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!} + \frac{e^{-2.5}(2.5)^2}{2!} + \frac{e^{-2.5}(2.5)^3}{3!}$$

$$+ \frac{e^{-2.5}(2.5)^4}{4!}$$

$$= \frac{e^{-2.5}(1)}{1} + \frac{e^{-2.5}(2.5)}{1} + \frac{e^{-2.5}(6.25)}{2} + \frac{e^{-2.5}(15.625)}{6} + \frac{e^{-2.5}(39.0625)}{24}$$

$e^{-2.5} = 0.0820$

$$= 0.0820 + 0.205 + \frac{0.5125}{2} + \frac{1.28125}{6} + \frac{3.203125}{24}$$

$$= 0.0820 + 0.205 + 0.25625 + 0.2135 + 0.1334$$

$$= 0.89015$$

(ii) more than 6 calls

$$P(X \geq 6) = 1 - P(X \leq 6)$$

$$1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)]$$

$$= 1 - e^{-2.5}\left[1 + 2.5 + \frac{(2.5)^2}{2} + \frac{(2.5)^3}{6} + \frac{(2.5)^4}{24} + \frac{(2.5)^5}{120} + \frac{(2.5)^6}{6!}\right]$$

$$= 1 - e^{-2.5}\left[1 + 2.5 + 3.125 + \frac{15.625}{6} + \frac{39.0625}{24} + \frac{97.65625}{120} + \frac{244.1406}{720}\right]$$

$$= 1 - e^{-2.5}[1 + 2.5 + 3.125 + 2.6041 + 1.6276 + 0.8138 + 0.3390]$$

$$= 1 - e^{-2.5}[12.0095]$$

$$= 1 - 0.0820(12.0095)$$

$$= 1 - 0.9847$$

$$= 0.0153$$

Fit a poisson distribution to the following data.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| f | 305 | 365 | 210 | 80 | 28 | 9 | 2 | 1 |

| $x$ | f | $fx$ | $\frac{\lambda}{x+1}$ |
|---|---|---|---|
| 0 | 305 | 0 | 1.201 |
| 1 | 365 | 365 | 0.6005 |
| 2 | 210 | 420 | 0.4003 |
| 3 | 80 | 240 | 0.3002 |
| 4 | 28 | 112 | 0.2402 |
| 5 | 9 | 45 | 0.2001 |
| 6 | 2 | 12 | 0.1715 |
| 7 | 1 | 7 | 0.1501 |
| | N=1000 | 1201 | |

Mean of P.D $\lambda = \frac{\Sigma fx}{\Sigma f}$

$= \frac{1201}{1000}$

$\lambda = 1.201$

Recurrence relation b/w probabilities is

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \frac{e^{-1.201}(1.201)^{8}}{8!}$$ wrong

=0.

1) Fit a possian distribution with a following data and find expected values.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f$ | 125 | 95 | 49 | 20 | 8 | 3 |

| $x$ | $f$ | $fx$ | $\lambda/x+1$ | $f(x)$ Expected frequency |
|---|---|---|---|---|
| 0 | 125 | 0 | $1/1=1$ | 110.34 |
| 1 | 95 | 95 | $1/2=0.5$ | 110.34 |
| 2 | 49 | 98 | $1/3=0.333$ | 55.17 |
| 3 | 20 | 60 | $1/4=0.25$ | 18.3881 |
| 4 | 8 | 32 | $1/5=0.2$ | 4.5970 |
| 5 | 3 | 15 | $1/6=0.1666$ | 0.9194 |
| | 300 | 300 | | 299.7545 |

mean of $\lambda = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{300}{300} = 1$

RR of P.D

$$P(x+1) = \frac{\lambda}{(x+1)} P(x)$$

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(0) = \frac{e^{-\lambda}\lambda^0}{0!}$$

$$\Rightarrow e^{-\lambda} = e^{-1} = 0.36787$$

$$F(0) = N \times P(0)$$

$$= 300 \times 0.3678$$

$$= 110.34$$

Fit a poisson distribution to the following data

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $f$ | 305 | 365 | 210 | 80 | 28 | 9 | 2 | 1 |

| $x$ | $f$ | $fx$ | $\lambda/x+1$ | $F(x)$ expected frequency |
|---|---|---|---|---|
| 0 | 305 | 0 | 1.201 | 300.8931 × 1.201 = 361.3726 |
| 1 | 365 | 365 | 0.6005 | 361.3726 × 0.6005 = 217.0041 |
| 2 | 210 | 420 | 0.4003 | 217.0042 |
| 3 | 80 | 240 | 0.3002 | 86.8667 |
| 4 | 28 | 112 | 0.2402 | 26.0817 |
| 5 | 9 | 45 | 0.2001 | 6.2648 |
| 6 | 2 | 12 | 0.1715 | 1.2535 |
| 7 | 1 | 7 | 0.1501 | 0.2149 |
| | N=1000 | 1201 | | 999.9515 |

Mean of P.D $\lambda = \frac{\Sigma fx}{\Sigma f} = \frac{1201}{1000} = 1.201$

Recurrence relation

RR of P.D

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-\lambda} = e^{-1.201} = 0.3008931$$

$$F(0) = N \times P(0)$$

$$= 1000 \times 0.3008$$

$$= 300.8931$$

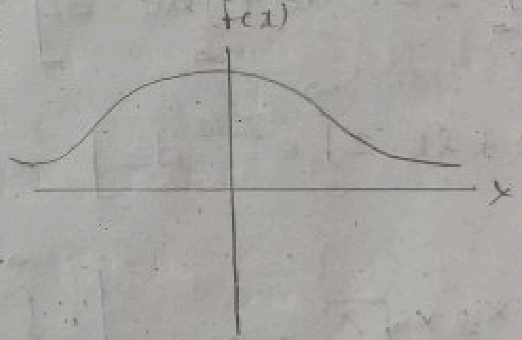
## Normal Distribution :-

A random variable 'x' is said to have normal continuous distribution with PDF

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty$$
$$-\infty < \mu < \infty$$
$$\sigma > 0$$

where '$\mu$' is the mean and '$\sigma$' is the standard deviation of 'x'.

→ A random variable 'x' is said to be a normal random variable having the normal curve representing the normal distribution by considering 'x' value on x-axis and f(x) values on y-axis. The shape of the normal curve is bell-shaped curve.




→ The total area bounded by the curve and x-axis is '1' i.e., $\int_{-\infty}^{\infty} f(x)dx = 1$

→ The area under the curve between the ordinates $x=a$ and $x=b$ represents the probability that x lies between a and b

$$P(a<x<b) = \int_a^b f(x)\,dx$$

1 property will come compulsory in exam
prepare for 5 properties

## Constants of normal distribution

1) Property-①: Mean of the normal distribution is $\mu$

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$E(X) = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Put $z = \frac{x-\mu}{\sigma}$

$z\sigma = x-\mu$

$z\sigma + \mu = x$

$dz = \frac{dx}{\sigma}$

$\sigma dz = dx$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu) e^{-z^2/2} \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \sigma z e^{-z^2/2} dz + \int_{-\infty}^{\infty} \mu e^{-z^2/2} dz\right]$$

where $x \to -\infty$ $z \to \infty$

$x \to \infty$ $z \to \infty$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + \mu \cdot 2\int_{0}^{\infty} e^{-z^2/2} dz\right]$$

$$= \frac{1}{\sqrt{2\pi}} \mu \times 2 \times \frac{\sqrt{\pi}}{\sqrt{2}} = \mu \qquad \left[\because \int_0^{\infty} e^{-z^2/2} dz = \sqrt{\frac{\pi}{2}}\right]$$

2) Variance of normal distribution:-

$$V(x) = E(X^2) - (E(X))^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z+\mu)^2 e^{-z^2/2} \cancel{\sigma}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \int_{-\infty}^{\infty} \sigma^2 z^2 e^{-z^2/2} dz + \int_{-\infty}^{\infty} \mu^2 e^{-z^2/2} dz + \int_{-\infty}^{\infty} 2\sigma\mu z e^{-z^2/2} dz \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ 2\sigma^2 \int_0^{\infty} z^2 e^{-z^2/2} dz + 2\mu^2 \int_0^{\infty} e^{-z^2/2} dz + 0 \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[ \sigma^2 \int_0^{\infty} z^2 e^{-z^2/2} dz + \mu^2 \times \sqrt{\frac{\pi}{2}} \right]$$

$$= \sqrt{\frac{2}{\pi}}\, \sigma^2 \int_0^{\infty} z^2 e^{-z^2/2} dz + \mu^2$$

put $\frac{z^2}{2} = t \Rightarrow z^2 = 2t$, $z = \sqrt{2t}$

$\frac{\cancel{2} z\, dz}{\cancel{2}} = dt$

$dz = dt/z$

$$= \sigma^2 \int_0^{\infty} 2t\, e^{-t} \frac{dt}{\sqrt{2t}} + \mu^2$$

$$= \sqrt{\frac{2}{\pi}} \times \frac{2}{\sqrt{2}}\, \sigma^2 \int_0^{\infty} t^{1/2} e^{-t} dt + \mu^2$$

$\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx$

$\Gamma(n+1) = n\Gamma(n)$, $\Gamma(1/2) = \sqrt{\pi}$

$\Gamma(3/2) = \Gamma(1/2 + 1) = \frac{1}{2}\Gamma(1/2) = 1/2\sqrt{\pi}]$

$$= \frac{2}{\sqrt{\pi}}\, \sigma^2 \int_0^{\infty} e^{-t} t^{3/2-1} dt + \mu^2$$

$$= \frac{2}{\sqrt{\pi}}\, \sigma^2\, \Gamma(3/2) + \mu^2$$

$$= \frac{2}{\sqrt{\pi}}\, \sigma^2 \frac{1}{2}\sqrt{\pi} + \mu^2 = \sigma^2 + \mu^2$$

$$E(x^2) = \sigma^2 + \mu^2$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \sigma^2 + \mu^2 - \mu^2$$

$$= \sigma^2.$$

3) Mode of Normal Distribution:-

Mode is the value of 'x' for which $f(x)$ is maximum to find maximum value of 'x'.

find $f'(x)$ and solve the equation $f'(x)$ is zero also for some value of $x$ $f''(x) < 0$

probability density function (pdf) of normal distribution is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$

limits for ND
$-\infty < x < \infty$
$-\infty < \mu < \infty$
$\sigma > 0$

$$f'(x) = \frac{1}{\sigma\sqrt{2\pi}}\left[e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \times -\frac{1}{2} \times 2\left(\frac{x-\mu}{\sigma}\right) \times \frac{1}{\sigma}\right]$$

$$f'(x) = -\frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \left(\frac{x-\mu}{\sigma^2}\right)$$

$$f'(x) = -f(x)\left(\frac{x-\mu}{\sigma^2}\right)$$

at $x=\mu$ $f'(x)=0$

$$f''(x) = -\left[f(x)\frac{1}{\sigma^2} + f'(x)\left(\frac{x-\mu}{\sigma^2}\right)\right]$$

at $x=\mu$

$$f''(x) = -\frac{f(x)}{\sigma^2}$$

$f''(x) < 0$ at $x=\mu$

$\mu$ is maximum value of 'x'

$\therefore$ $x=\mu$ is the Mode of N.D

4) Median of Normal Distribution:-

W.k. that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Let 'M' is the median of Normal Distribution

$$\int_{-\infty}^{M} f(x)\,dx + \int_{M}^{\infty} f(x)\,dx = 1$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

consider $\int_{-\infty}^{M} f(x)\,dx = 1/2$

$$\int_{-\infty}^{\mu} f(x)\,dx + \int_{\mu}^{M} f(x)\,dx = 1/2 \rightarrow (1)$$

only consider $\int_{-\infty}^{\mu} f(x)\,dx$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

put $\frac{x-\mu}{\sigma} = z$

$x = \sigma z + \mu$

$dx = \sigma\,dz$

when $x \to -\infty$

$z \to \infty$

$x = \mu \quad z = 0$

$$\int_{-\infty}^{\mu} f(x)\,dx = \int_{-\infty}^{\mu} \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{0} e^{-z^2/2} \cancel{\sigma}\,dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-z^2/2}\,dz$$

$\left[\because e^{-z^2/2} \text{ is even function} \quad \int_{-\infty}^{0} e^{-z^2/2}\,dz = \int_{0}^{\infty} e^{-z^2/2}\,dz\right]$

$$= \frac{1}{\sqrt{2\cancel{\pi}}} \sqrt{\frac{\cancel{\pi}}{2}}$$

$$= \frac{1}{2}$$

sub $\int_{-\infty}^{\mu} f(x)\,dx = 1/2$ in (1)

$$① \Rightarrow \frac{1}{2} + \int_{\mu}^{M} f(x)\,dx = 1/2$$

$$\Rightarrow \int_{\mu}^{M} f(x)\,dx = 0$$

$$\Rightarrow M = \mu$$

i.e., Median of Normal distribution is $\mu$.

NOTE: from the above results we notice that for N.D mean, median, mode coincide i.e., mean = median = mode. Hence, the distribution is symmetrical.

5) Mean deviation from mean of N.D :-

from defination M.D from mean is given by

$$\int_{-\infty}^{\infty} |x-\mu| f(x)$$

where f(x) is the pdf of N.D

$$\therefore M.D = \int_{-\infty}^{\infty} |x-\mu| f(x)\,dx$$

$$= \int_{-\infty}^{\infty} |x-\mu| \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} |x-\mu| e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $\frac{x-\mu}{\sigma} = z$

$dx = \sigma dz$

$x-\mu = \sigma z$

$x \to -\infty \quad z \to -\infty$

$x \to \infty \quad z \to \infty$

$e^{-z^2/2}$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} |\sigma z|\, e^{-z^2/2}\, \cancel{\sigma}\, dz$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_{0}^{\infty} z\, e^{-z^2/2}\, dz \Rightarrow \frac{2\sigma}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-z^2/2} (z\, dz)$$

put $\frac{z^2}{2} = t$

$\frac{\cancel{2} z\, dz}{\cancel{2}} = dt$

$z = 0 \quad t = 0$

$z \to \infty \quad t \to \infty$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t}\, dt$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \left[-e^{-t}\right]_{0}^{\infty}$$

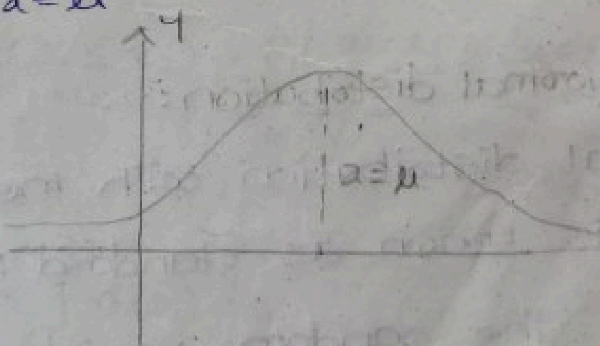
$$= \frac{2\sigma}{\sqrt{2\pi}} \left[-(0-1)\right]$$

$$= \frac{\sqrt{2}\,\sqrt{2}\,\sigma}{\sqrt{2}\,\sqrt{\pi}}$$

$$= \sqrt{\frac{2}{\pi}}\, \sigma$$

## chief characteristics of ND:-

→ the graph of the N.D $y = f(x)$ in $xy$ plane is known as normal curve which is symmetrical about the line $x = \mu$




→ the curve is bell shaped curve, and the

→ the two tails on the right and left sides of the mean extends to ∞.

→ The area under the Normal curve represents the total population.

→ Mean, Median and mode of the normal distribution coincides at $x = \mu$. So, the normal curve is uni-modal.

→ $x$-axis is the asymptote (line which meets the curve at $\infty$) to the curve.

→ The probability that the normal variate 'x' with mean $\mu$ and S.D($\sigma$) lies between $x_1$ and $x_2$ is

$$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(x)\,dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

→ Area under the normal curve is distributed as

i) Area of normal curve between $\mu-\sigma$ and $\mu+\sigma$ is 68.27%.

ii) Area of normal curve between $\mu-2\sigma$ and $\mu+2\sigma$ is 94.3%.

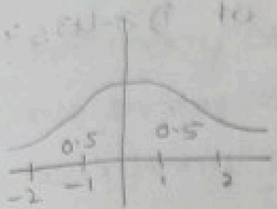
iii) Area of normal curve between $\mu-3\sigma$ and $\mu+3\sigma$ is 99.73%.

Standard Normal distribution:-

The Normal distribution with mean $\mu=0$ and $\sigma=1$ is known as standard Normal Distribution. The random variables that follows this distribution is denoted by "z" and it is given by

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

The shape of this distribution is also bell shaped curve.




$\mu-\sigma$ to $\mu+\sigma$ 68%

$\mu-2\sigma$ to $\mu+2\sigma$ 95%

$\mu-3\sigma$ to $\mu+3\sigma$ 99%

the probability density function of normal variate

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad -\infty < z < \infty$$

Here $z$ is SN varient with $\mu=0, \sigma=1$

Area under Normal probability curve :-

(probability integral) :-

Let $x$ is a R-D follows normal distribution with $x \sim N(\mu \sigma^2)$ and let $z = \frac{x-\mu}{\sigma}$ is a SN varient follows normal distribution with $\mu=0$ and $\sigma=1$

$z \sim N(0,1)$

And probability of $p(x_1 \leq X \leq x_2)$ where $X \sim N(\mu, \sigma^2)$

$$z_1 = \frac{x_1-\mu}{\sigma}, \quad z_2 = \frac{x_2-\mu}{\sigma}$$

$$\therefore p(x_1 \leq X \leq x_2) = P\left(\frac{x_1-\mu}{\sigma} \leq \frac{x-\mu}{\sigma} \leq \frac{x_2-\mu}{\sigma}\right)$$

$$= p(z_1 \leq z \leq z_2)$$

$$= \int_{z_1}^{z_2} \phi(z)\, dz$$

where $\phi(z)$ is pdf is of $z$ given by

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

Problems:

1) For a normally distributed variate with mean 1 and $\sigma = 3$. find the probabilities of i) $3.43 \le x \le 6.19$

ii) $-1.43 \le x \le 6.19$

given : $\mu = 1$, $\sigma = 3$

i) $P(3.43 \le x \le 6.19)$

when $x_1 = 3.43$ $\quad z_1 = \frac{x_1 - \mu}{\sigma} = \frac{3.43 - 1}{3}$

$= 0.81$

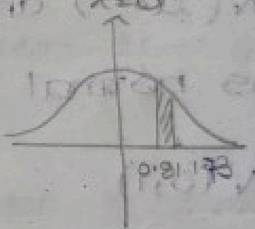
when $x_2 = 6.19$ $\quad z_2 = \frac{x_2 - \mu}{\sigma} = \frac{6.19 - 1}{3} = 1.73$

$P(3.43 \le x \le 6.19) = P(0.81 \le z \le 1.73)$

$= A(1.73) - A(0.81)$

$= 0.4582 - 0.2910$

$= 0.1672$




ii) $P(-1.43 \le x \le 6.19)$

when $x_1 = -1.43$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{-1.43 - 1}{3} = -0.81$

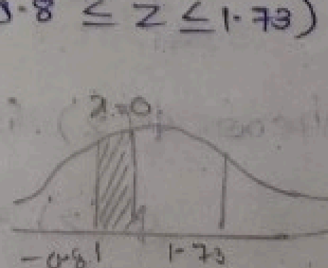
when $x_2 = 6.19$ $\quad z_2 = 1.73$

$P(-1.43 \le x \le 6.19) = P(-0.8 \le z \le 1.73)$

$= A(1.73) + A(0.81)$

$= 0.4582 + 0.2910$

$= 0.7492$



2) thousand students have written an examination the mean of the test is 35, $\mu = 35, \sigma = 5$ assuming that the distribution is Normal find

i) How many students marks lies b/w 25 & 40

ii) How many students get more than 40.

iii) How many students below 20 marks

given:

$$\text{Mean} = \mu = 35$$

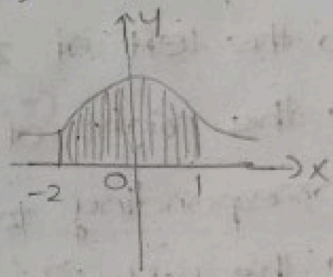
$$\text{S.D} = \sigma = 5$$

i) $P(25 \le x \le 40)$

when $x_1 = 25$, $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{25-35}{5} = -2$

when $x_2 = 40$, $z_2 = \frac{x_2 - \mu}{\sigma} = \frac{40-35}{5} = 1$

$P(25 \le x \le 40) = P(-2 \le z \le 1)$




$= A(1) + A(2)$

$= 0.3413 + 0.4772$

$= 0.8185$

$\therefore$ no. of students $= 0.8185 \times 1000$

$= 818$

ii) students more than 40

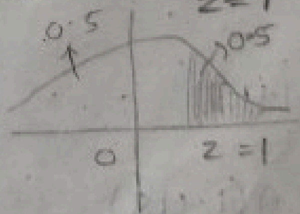
$z = \frac{x - \mu}{\sigma}$

$= \frac{40-35}{5}$

$z = 1$

when we consider z the value lies b/w -3 to 3

$P(x \ge 40) = P(z \ge 1)$




$= 0.5 - A(1)$

$= 0.5 - 0.3413$

$= 0.1587$

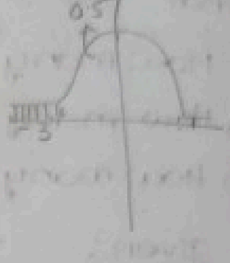
no. of students whose marks are $> 40$

$= 1000 \times 0.1587 = 159$ students

iii) Student below 20

probability of x below 20 marks

$P(x \leq 20)$.




when $x_1 = 20$ $z_1 = \frac{x_1 - \mu}{\sigma}$

$= \frac{20-35}{5} = -3$

$P(x \leq 20) = P(z \leq -3)$

$= 0.5 - A(3)$

$= 0.5 - .4987$

$= 0.0013$

No. of students are $1000 \times 0.0013 = 1$

3) If 'x' is a normal variate then find the area

i) To the left of $z = -1.78$

ii) to the right of $z = -1.45$

iii) corresponding to $-0.8 \leq z \leq 1.53$

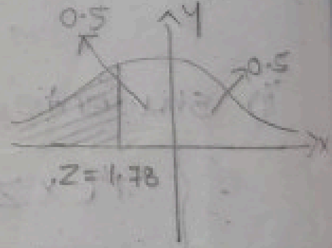
iv) to the left of $z = -2.52$ and to the right of $z = 1.83$

i) $P(z \leq -1.78)$

$= 0.5 - A(1.78)$

$= 0.5 - 0.4625$

$= 0.0375$

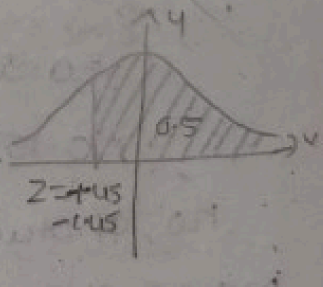



ii) $P(z \geq -1.45)$

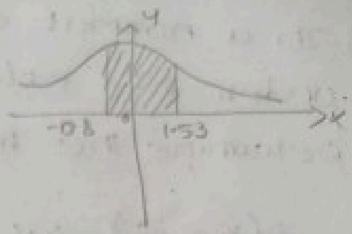
$= 0.5 + A(1.45)$

$= 0.5 + 0.4265$

$= 0.9265$



iii) $P(0.8 \le z \le 1.53)$

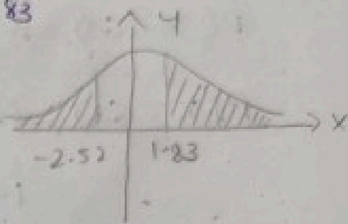



$A(1.53) + A(0.8)$

$= 0.4372 + 0.2881$

$= 0.7251$

iv) to the left of $-2.524$ to right of $1.83$




$0.5 - A(2.52) + 0.5 - A(1.83)$

$= 0.0395$

4) the mean height of 500 students is 151cm and the S.D is 15cm. Assuming that the heights are normally distributed. find how many students heights lies between 120 and 155 cm

Mean height of 500 students $(\mu) = 151\text{cm}$

$S.D(\sigma) = 15\text{cm}$

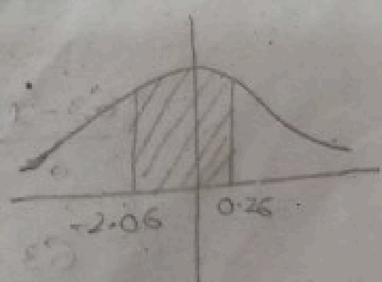
Assuming i) $P(120 \le x \le 155)$

when $x_1 = 120$ $\quad z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{120-151}{15} = -2.06$

when $x_2 = 155$ $\quad z_2 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{155-151}{15} = 0.26$

$\therefore P(120 \le x \le 155) = P(-2.06 \le z \le 0.26)$

$= A(2.06) + A(0.26)$




$= 0.4803 + 0.1026$

$= 0.5829$

no. of students $= 500 \times 0.5829$

$= 291$

5) In a normal distribution 7% of the items are under 35, 89% of the items are under 63. Determine the mean and S.D.

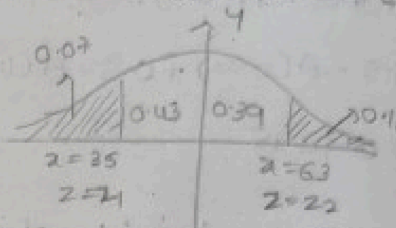
$P(x < 35) = 7\% = 0.07$

we have subtract from 0.5 value

$0.07 - 0.5 = 0.43$

$0.11 + 0.5 = 0.39$

$P(x < 63) = 89\% = 0.89$




$P(x \geq 63) = 1 - P(x < 63)$

$= 1 - 0.89$

$= 0.11$

$P(x < 35) = P\left(z < \frac{35-\mu}{\sigma}\right) = P(z < z_1) = 0.07$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{35-\mu}{\sigma} \Rightarrow z_1 = 1.4$

$A(0.43)$

$\frac{35-\mu}{\sigma} = -1.48$

$35 - \mu = -1.48\sigma$

$\mu - 1.48\sigma = 35 \rightarrow ①$

$P(x > 63) = 0.11$

$P(x > 63) = P\left(z_2 > \frac{63-\mu}{\sigma}\right) = 0.11 \Rightarrow z_2 = 1.23$

$P(z > z_2) = 0.11 \Rightarrow z_2 = 1.23$

where $z_2 = \frac{63-\mu}{\sigma}$

$A(0.39)$

$\frac{63-\mu}{\sigma} = 1.23$

$63 - \mu = 1.23\sigma$

$1.23\sigma + \mu = 63 \rightarrow ②$

solve ① & ②

$\mu - 1.48\sigma = 35$

$\mu + 1.23\sigma = 63$

$-2.71\sigma = -28$

$\sigma = \frac{28}{2.71}$

$\sigma = 10.3321$

$\mu - 1.48(10.3321) = 35$

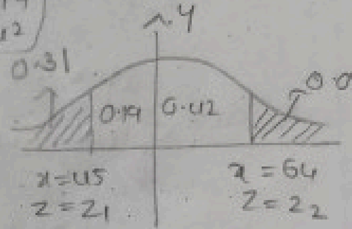
$\mu - 15.2915 = 35$

$\mu = 35 + 15.2915$

$\mu = 50.2915$

6) find the mean and S.D of a normal distribution in which 31% of item are under 45 & 8% of items are over 64.

$\boxed{\begin{array}{l} 0.31 - 0.5 = 0.19 \\ 0.08 - 0.5 = 0.42 \end{array}}$




$P(x<45) = 31\% = 0.31$

$P(x \geq 64) = 8\% = 0.08$

$P(x<45) = P\left(z < \frac{45-\mu}{\sigma}\right) = P(z<z_1) = 0.31$

$\Rightarrow z_1 =$

$z_1 = \frac{x_1-\mu}{\sigma} = \frac{45-\mu}{\sigma}$

$P(x<45) = 0.31$

$P\left(z < \frac{45-\mu}{\sigma}\right) = 0.31$

$z_1 = \frac{45-\mu}{\sigma}$

$P(z<z_1) = 0.31$

$z_1 = \frac{45-\mu}{\sigma}$

$P(x>64) = 0.08$

$P\left(z > \frac{64-\mu}{\sigma}\right) = 0.08$

$z_1 = \frac{64-\mu}{\sigma}$

$$z_1 = \frac{45-\mu}{\sigma} = -0.5$$

$$45-\mu = -0.5\sigma$$

$$\mu - 0.5\sigma = 45 \rightarrow (1)$$

$$\frac{64-\mu}{\sigma} = z_2 =$$ value equivalent to the area 0.42 on the right. $z = z_2$

$$\frac{64-\mu}{\sigma} = 1.41$$

$$64-\mu = 1.41\sigma$$

$$\mu + 1.41\sigma = 64$$

Solving (1) & (2) $\mu = 49.97$
$\sigma = 9.94$

$$\mu - 0.5\sigma = 45$$
$$(1)\ \mu + 1.41\sigma = 64$$

$$-1.91\sigma = 19$$
$$\sigma = 9.64$$

Normal appro

the normal distribution is used to approximate the binomial distribution for large values of 'n'. the calculation of binomial probabilites in such case binomial can be replaced by normal and the required probability can be calculated. Here we consider two cases.

case-(i): when $p = q = 1/2$

mean of the binomial distribution $\mu = np$ and

S.D ($\sigma$) = $\sqrt{npq}$

hence the corresponding normal distribution with $\mu$ & $\sigma$ can be calculated using binomial distribution and it is given by

$$p(x_1 \leq x \leq x_2) = p(z_1 \leq z \leq z_2)$$

where $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{x_1 - np}{\sqrt{npq}}$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{x_2 - np}{\sqrt{npq}}$$

case-(ii): when $p \neq q$

for large values of 'n' we consider the real class interval as $(x - 1/2, x + 1/2)$ where '$x$' is the no. of success and $z_1$ for the lower limit '$x$' is given by

$$z_1 = \frac{(x_1 - 1/2) - np}{\sqrt{npq}}$$

$$z_2 = \frac{(x_2 + 1/2) - np}{\sqrt{npq}}$$

$\therefore$ probability of $p(x_1 \leq x \leq x_2) = p(z_1 \leq z \leq z_2)$

problems:- 1) find the probability of getting atleast 55 heads when 100 coins are tossed using normal distribution.

$p = 1/2,\ q = 1/2,\ n = 100$

$P(x \geq 55)$

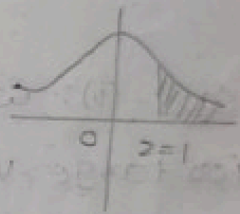
$\mu = np = 100 \times 1/2 = 50$

$\sigma = \sqrt{npq} = \sqrt{100 \times \frac{1}{2} \times 1/2} = 5$

$P(x \geq 55) = P(z \geq z_1)$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{55-50}{5} = 1$

$P(x \geq 55) = P(z \geq 1)$




$= 0.5 - A(1)$

$= 0.5 - 0.3413$

$= 0.1587$

2) find the probability that by guess work a student can correctly answer 25 to 30 questions in a multiple choice questions of 80. Assuming that in each question with 4 choices only one choice (answer) is correct. find the p

$n = 80\ ;\ p = 1/4\ ;\ q = 3/4$

$\mu = np = 80 \times 1/4 = 20$

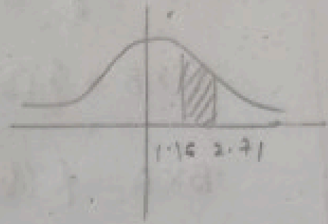
$\sigma = \sqrt{npq} = \sqrt{80 \times \frac{1}{4} \times \frac{3}{4}} = 3.87$

$$P(25 \le x \le 30) = P(25 - 0.5 \le x \le 30 + 0.5)$$

when $x_1 = 25$

$$z_1 = \frac{(x_1 - 0.5) - np}{\sqrt{npq}}$$

$$z_1 = \frac{(25-0.5)-20}{3.87} \Rightarrow \frac{24.5-20}{3.87} = 1.16$$




$$z_2 = \frac{(x_2 + 0.5) - np}{\sqrt{npq}}$$

$$= \frac{(30+0.5)-20}{3.87} = 2.71$$

$$P(25 \le x \le 30) = P(1.16 \le z \le 2.71)$$

$$= A(2.71) - A(1.16)$$

$$= 0.4966 - 0.3770$$

$$= 0.1196.$$