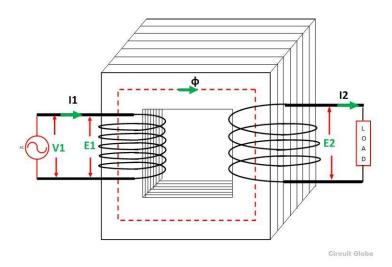
A **Transformer** is a static electrical machine which transfers AC electrical power from one circuit to the other circuit at the constant frequency, but the voltage level can be altered that means voltage can be increased or decreased according to the requirement.

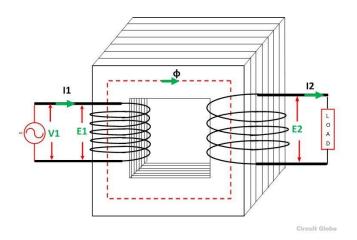
It works on the principle of **Faraday's Law of Electromagnetic Induction** which states that "the magnitude of voltage is directly proportional to the rate of change of flux."



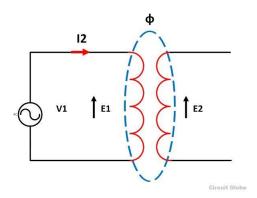
Working Principle of a Transformer

The basic principle on which the transformer works is **Faraday's Law of Electromagnetic Induction** or mutual induction between the two coils. The working of the transformer is explained below. The transformer consists of two separate windings placed over the laminated silicon steel core.

The winding to which AC supply is connected is called primary winding and to which load is connected is called secondary winding as shown in the figure below. It works on the **alternating current only** because an alternating flux is required for mutual induction between the two windings.



When the AC supply is given to the primary winding with a voltage of V_1 , an alternating flux ϕ sets up in the core of the transformer, which links with the secondary winding and as a result of it, an emf is induced in it called **Mutually Induced emf**. The direction of this induced emf is opposite to the applied voltage V_1 , this is because of the Lenz's law shown in the figure below:



Physically, there is no electrical connection between the two windings, but they are magnetically connected. Therefore, the electrical power is transferred from the primary circuit to the secondary circuit through mutual inductance.

The induced emf in the primary and secondary windings depends upon the rate of change of flux linkage that is $(N \ d\phi/dt)$. $d\phi/dt$ is the change of flux and is same for both the primary and secondary windings. The induced emf E_1 in the primary winding is proportional to the number of turns N_1 of the primary windings $(E_1 \infty \ N_1)$. Similarly induced emf in the secondary winding is proportional to the number of turns on the secondary side. $(E_2 \infty \ N_2)$.

Transformer on DC supply

The transformer works on AC supply, and it cannot work not DC supply. If the rated DC voltage is applied across the primary winding, a constant magnitude flux will set up in the core of the transformer and hence there will not be any self-induced emf generation, as for the linkage of flux with the secondary winding there must be an alternating flux required and not a constant flux.

According to Ohm's Law

The resistance of the primary winding is very low, and the primary current is high. So this current is much higher than the rated full load primary winding current. Hence, as a result, the amount of heat produced will be greater and therefore, eddy current loss (I²R) loss will be more.

Because of this, the insulations of the primary windings will get burnt, and the transformer will get damaged.

Turn Ratio

It is defined as the ratio of primary to secondary turns.

Turn ratio =
$$\frac{N_1}{N_2}$$

If $N_2 > N_1$ the transformer is called **Step-up transformer**

If $N_2 < N_1$ the transformer is called **Step down transformer**

Transformation Ratio

The transformation ratio is defined as the ratio of the secondary voltage to the primary voltage. It is denoted by K.

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

As $(E_2 \propto N_2 \text{ and } E_1 \propto N_1)$

Construction of a Transformer

The transformer mainly consists of the Magnetic circuit, electric circuit, dielectric circuit, tanks, and accessories. The main elements of the transformer are the **primary and secondary windings** and the **steel core**. The core of the transformer is made up of silicon steel in order to provide a continuous magnetic path. Usually, the core of the transformer is laminated for minimizing the eddy current loss.

Magnetic circuit

The magnetic circuit of a transformer consists of **core** and **yoke**. The circuit provides the path to the flow of magnetic flux. The transformer consists of a laminated steel core and the two coils. The two coils are insulated from each other and also from the core.

The core of the transformer is constructed from laminations of steel sheet or silicon steel assembled to provide a continuous magnetic path. At usual flux densities, the silicon steel material has low hysteresis losses.

The vertical position on which the coil is wound is called the **limb** while the horizontal position is known as the **yoke**.

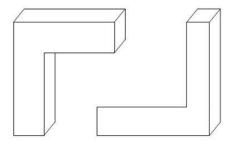
Electric circuit

Construction of the electric circuit of the transformer consists of primary and secondary windings usually made of copper. The Conductors of the rectangular cross-section are generally used for low voltages winding and also for the high voltage winding for large transformers. Conductors of the circular cross-sectional area are used for high voltage winding in the small transformer.

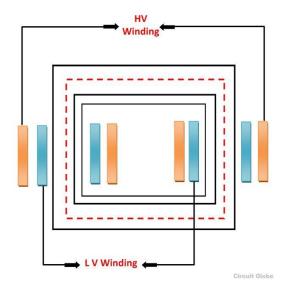
According to the core construction and the manner in which the primary and secondary windings are placed around it, the transformer is named as **core type** and **shell type**.

Core Type Transformer

In a simple core type construction of the transformer, a rectangular frame laminations are formed to build the core of the transformer. The laminations are cut in the form of L-shape strips as shown in the figure below. In order to avoid high reluctance at the joints where laminations are butted against each other, the alternate layers are placed differently to eliminate the continuous joints.



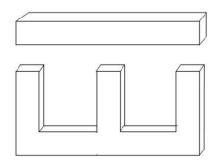
The primary and the secondary windings are interleaved to reduce the leakage flux. Half of each winding are placed side by side or concentrically on either limb of the core.

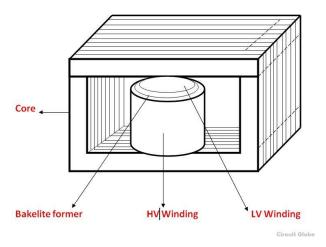


While placing these windings, insulation of Bakelite former is provided between the core and low voltage winding (LV), between the two windings that are between low voltage (LV) and high voltage (HV) windings and also in between coils and yoke. And also in between HV limb and yoke as shown in the figure. To reduce the insulation, the low voltage winding is always placed nearer to the core.

Shell Type Transformer

In a shell-type transformer, the individual laminations are cut in the form of long strips of E and I shape as shown in the figure below. It has two magnetic circuits, and the core has three limbs. The central limb carries the whole of the flux whereas the side limbs carry half of the flux. Therefore, the width of the center is double, to that of the outer limbs.





The leakage flux is reduced by the subdivision of the windings which in return have lesser reactances. Both the primary and the secondary windings are placed on the central limb side by side. The low voltage winding is placed nearer to the core and the high voltage winding is placed outside the low voltage winding.

To reduce the cost of lamination between the core and the low voltage winding, the windings are formed and are wound to the cylindrical shape and then the core laminations are inserted later.

Dielectric Circuit

The dielectric circuit consists of insulations used in different places in the transformer to insulate the conducting parts. The core is laminated to minimize the eddy current losses. The laminations are insulated from each other by a light coating of varnish or by an oxide layer. The thickness of laminations varies from **0.35mm to 0.5mm** for a frequency of **50 Hz**.

Tanks and Accessories

Other different parts and accessories are also fitted on the transformer for its efficient work as well as for longer life and better services of the transformer.

Ideal Transformer: The transformer which is free from all types of losses is known as an ideal transformer. The ideal transformer has the following important characteristic.

- 1. The resistance of their primary and secondary winding becomes zero.
- 2. The core of the ideal transformer has infinite permeability. The infinite permeable means less magnetizing current requires for magnetizing their core.
- 3. The leakage flux of the transformer becomes zero, i.e. the whole of the flux induces in the core of the transformer links with their primary and secondary winding.
- 4. The ideal transformer has 100 percent efficiency, i.e., the transformer is free from hysteresis and eddy current loss.

In an ideal transformer, there is no power loss. Therefore, the output power is equal to the input power.

$$E_2I_2\cos\phi=E_1I_1\cos\phi\quad\text{or }E_2I_2=E_1I_1$$

OR

$$\frac{E_2}{E_1} = \frac{I_1}{I_2}$$

Since $E_1 \propto N_2$ and $E_1 \propto N_1$, also E_1 is similar to V_1 and E_2 is similar to V_2

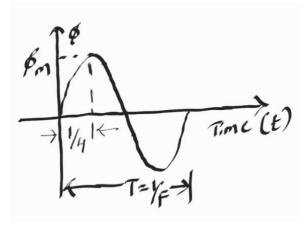
Therefore, the transformation ratio will be given by the equation shown below

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

The primary and the secondary currents are inversely proportional to their respective turns.

EMF EQUATION

When an alternating voltage is applied to primary winding of a transformer, an alternative flux is set up in the iron core which links the both primary and secondary windings of Transformer.



Let $\emptyset_{max} = maximum$ value of flux in Weber

F = supply frequency in hertz

The magnetic flux increases from zero to its maximum value in 1/4 of a cycle (i.e. in 1/4f second)

So average rate of change of flux, $\frac{d\beta}{dt} = \frac{\beta_{max}}{1/4F} = 4F\beta_{max}$

Since the average EMF induced per turn in volts is equal to the average rate of change of flux.

So average EMF induced per turn= $UF \beta_{max}$ (volts)

Since flux \emptyset varies sinusoidally, So EMF induced will be sinusoidal and form factor for sinusoidal wave is 1.11 i.e., the rms or effective value 1.11 times the average value.

。。RMS value of EMF induced or per turn= 1・11 メ 4 チ ゅっこ (volts)

If the number of turns on primary and secondary windings are N_1 and N_2 respectively then,

RMS value of EMF induced in primary E_1 = EMF induced per turn \mathbf{x} number of primary turns

Similarly RMS value of EMF induced in secondary,

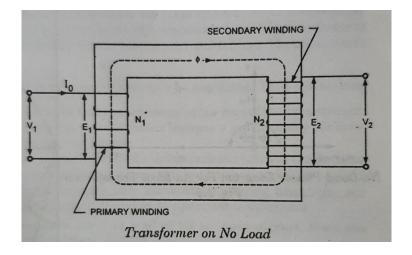
The instantaneous value of sinusoidally varying flux may be given as

$$\emptyset = \emptyset_{max}$$
 sinwt

The maximum value of EMF induced per turn= ω $\beta_{max} = 271$ β_{max} and RMS value of EMF induced per turn= $\frac{1}{\sqrt{2}} \times 271$ $\beta_{max} = 4.44$ β_{max} Hence RMS value of EMF induced in primary $E_1 = 4.44$ $\beta_{max} = 4.$

TRANSFORMER ON NO LOAD:

When the primary winding of a transformer is connected to the source of AC Supply and the secondary is open, then the transformer is said to be at no load condition.



The alternative voltage V_1 applied to the primary winding will cause flow of alternating current in the primary winding since the primary coil is purely inductive and there is no output at the secondary terminal the primary draws the magnetizing current I_m only. The function of this current is to magnetize the core. If the Transformer is truly an ideal Transformer the magnitude of I_m should be 0

The magnetizing current I_m is small in magnitude and lags behind the supply voltage V_1 by 90° . The magnetizing current I_m produces and alternating flux \emptyset , which is at all times proportional to the current and hence in phase with it.

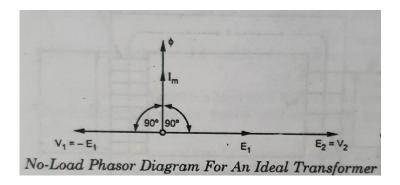
Let the instantaneous linking flux will be given as $\emptyset = \emptyset_{max}$ sinwt

The instantaneous value of induced EMF in primary and secondary winding will be

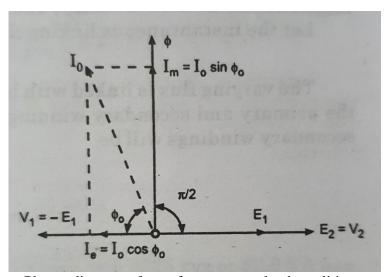
$$e_{1} = -N_{1} \frac{d\phi}{dt} = -N_{1} \frac{d}{dt} (\phi_{\text{max}} \sin \omega t)$$

$$= -N_{1} \omega \phi_{\text{max}} \cos \omega t = N_{1} \omega \phi_{\text{max}} \sin \left(\omega t - \frac{\pi}{2}\right)$$
Similarly, $e_{2} = N_{2} \omega \phi_{\text{max}} \sin \left(\omega t - \frac{\pi}{2}\right)$

Since, primary winding has no ohmic resistance, therefore applied voltage to primary winding is to only oppose the induced emf in primary winding. Hence $V_1 = -e_1$



However, when a varying flux is set up in the magnetic material, there will be power loss called the iron or copper loss. So the input current to the primary under no load condition has also to supply hysteresis and eddy current losses occurring in the core in additional to small amount of copper loss occurring in the primary winding. Hence, the no load primary current I_0 does not lag behind the applied voltage V_1 by 90^0 , but it lags behind the V_1 by angle $\emptyset_0 < 90^0$



Phasor diagram of transformer on no load condition.

Input power on no load $P_0 = V_1 I_0 \cos \emptyset_0$

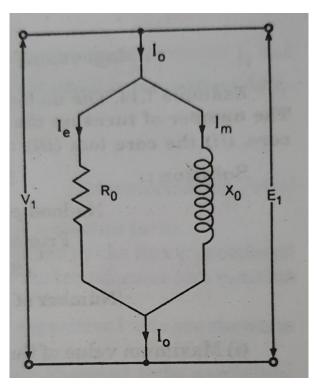
Where $\cos \emptyset_0$ is the primary power factor under no load condition.

No load current I₀, called exciting current has two components,

1) in phase, active or energy component I_e used to meet the iron losses in addition to small amount of copper losses occurring in the primary winding.

2) Quadrature component or wattless component called the magnetizing component, I_m used to create alternative flux in the coil.

The equivalent Circuit of the Transformer on no load is Illustrated in the below figure, I_e and I_m are represented by the current drawn by non-inductive resistance and pure inductive reactance respectively

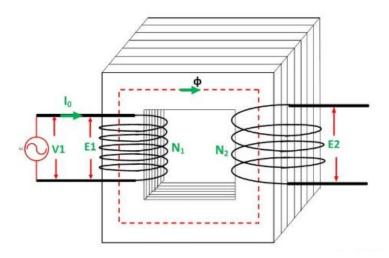


Transformer on Load Condition:

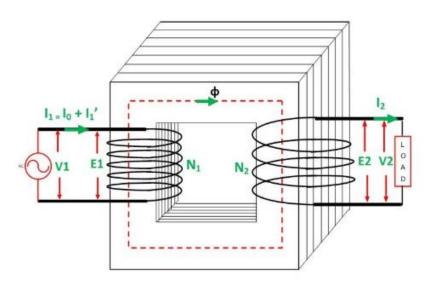
When the transformer is on the loaded condition, the secondary of the transformer is connected to load. The load can be resistive, inductive or capacitive. The current I_2 flows through the secondary winding of the transformer. The magnitude of the secondary current depends on the terminal voltage V_2 and the load impedance. The phase angle between the secondary current and voltage depends on the nature of the load.

Operation of the Transformer on Load Condition:

When the secondary of the transformer is kept open, it draws the no-load current from the main supply. The no-load current induces the magneto motive force N_0I_0 and this force set up the flux Φ in the core of the transformer. The circuit of the transformer at no load condition is shown in the figure below:



When the load is connected to the secondary of the transformer, I_2 current flows through their secondary winding. The secondary current induces the magneto motive force N_2I_2 on the secondary winding of the transformer. This force set up the flux ϕ_2 in the transformer core. The flux ϕ_2 opposes the flux ϕ , according to **Lenz's law**.



As the flux ϕ_2 opposes the flux ϕ , the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1 . Thus, the strength of the V_1 is more than E_1 and an additional primary current I'_1 drawn from the main supply. The additional current is used for restoring the original value of the flux in the core of the transformer so that $V_1 = E_1$. The primary current I'_1 is in phase opposition with the secondary current I_2 . Thus, it is called the **primary counter-balancing current**.

The additional current I'_1 induces the magneto motive force $N_1I'_1$. And this force set up the flux ϕ'_1 . The direction of the flux is the same as that of the ϕ and it cancels the flux ϕ_2 which induces because of the MMF N_2I_2

Now, $N_1I_1'=N_2I_2$ Therefore,

$$I_1' = \left(\frac{N_2}{N_1}\right)I_2 = KI_2$$

(K= Transformation ratio)

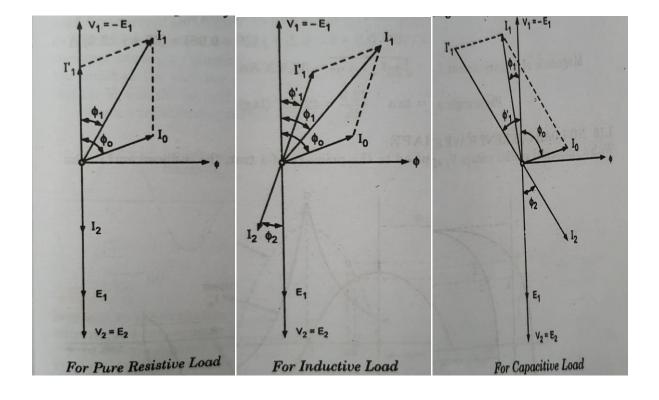
The phase difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.

The power factor of the secondary side depends upon the type of load connected to the transformer.

If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. The total primary current I_1 is the vector sum of the currents I_0 and I_1 '. i.e

$$\overline{I_1} = \overline{I_0} + \overline{I_1'}$$

Phasor Diagram of Transformer on R, L, C Loads:



Turn Ratio:

It is defined as the ratio of primary to secondary turns.

Turn ratio =
$$\frac{N_1}{N_2}$$

$$\frac{T_2}{T_1} = \frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{K}$$

If $N_2 > N_1$ the transformer is called **Step-up transformer**

If $N_2 < N_1$ the transformer is called **Step down transformer**

Transformation Ratio:

The transformation ratio is defined as the ratio of the secondary voltage to the primary voltage. It is denoted by K.

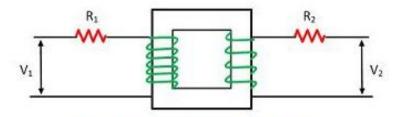
$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

$$K = \frac{I_1}{I_2}$$
(or)

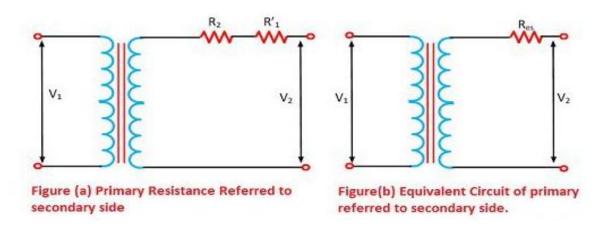
Transformer Winding Resistance

The ideal transformer has no resistance, but in the actual transformer, there is always some resistance to the primary and secondary windings. For making the calculation easy the resistance of the transformer can be transferred to the either side. The resistance is transferred from one side to another in such a manner that the percentage of voltage drop (or) copper losses remains the same when represented on the either side. These resistances are shown external to the windings in the figure below.



Transformer With Winding Resistance

Let the primary resistance R₁ be transferred to the secondary side, and the new value of this resistance be R'₁. The R'₁ is called the equivalent resistance of primary referred to secondary side as shown in the figure below. I₁ and I₂ are the full loads primary and secondary current respectively.



$$Per \ Unit \ Value = \frac{Actual \ value \ in \ any \ unit}{Base \ or \ reference \ value \ in \ the \ same \ unit}$$

Per unit voltage drop before the transfer is equal to the Per unit voltage drop after the transfer.

Then,

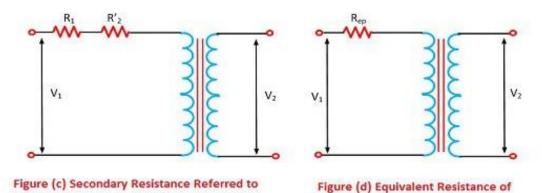
$$\frac{I_2 R_1'}{V_2} \times 100 = \frac{I_1 R_1}{V_1} \times 100$$

$$R_1' = \frac{I_1}{I_2} \times \frac{V_2}{V_1} \times R_1 = K^2 R_1$$

Total equivalent resistance referred to secondary,

$$R_{es} = R_2 + R_1' = R_2 + K^2 R_1$$

Now consider resistance R_2 , when it is transferred to primary, the value of the new resistance is R'_2 . The R'_2 is called the equivalent resistance of the secondary referred to as primary as shown in the figure below.



Similarly

Primary side

Then,

$$\frac{I_1 R_2'}{V_1} \times 100 = \frac{I_2 R_2}{V_2} \times 100$$
$$R_2' = \frac{I_2}{I_1} \times \frac{V_1}{V_2} \times R_2 = \frac{R_2}{K^2}$$

Secondary Referred to Primary side

Total equivalent resistance referred to primary,

$$R_{ep} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2}$$

$$R_{02} = R_2 + R'_1$$

$$R_{01} = R_1 + R_2^{1}$$

$$\frac{I_1 R_{01}}{V_1} = \frac{I_2 R_{02}}{V_2}$$

^{*}Per unit voltage drop referred to primary is equal to the Per unit voltage drop referred to secondary

Transformer with leakage flux:

Leakage flux is a part of a flux which links either primary or secondary winding of the Transformer, but not both the windings of Transformer. The main cause of the production of leakage flux in the secondary Transformer winding is due to the load current. The leakage flux on the primary winding is voltage dependent flux, and it's a constant flux. The leakage flux on the secondary winding is current dependent value and it's a variable flux.

The leakage flux on the transformer can be represented by the element reactance. X_1 and X_2 are imaginary reactance, hence there is no physical appearance in the Transformer. Leakage flux will have the voltage drop, but not the power losses. X_1 Primary reactance which represents leakage flux in primary winding and X_2 Secondary reactance which represents leakage flux in secondary winding

In order to transfer reactance from primary winding to secondary winding. The Per unit reactance drop before the transfer equal to per unit reactance drop after the transfer.

Firon sec to pulmary

F1 F1 F2 F2 -

PU reatance durop BT = p. U reactance durop A.T.

$$x_2' = x_2 \begin{bmatrix} \frac{\epsilon_2}{\epsilon_2} \end{bmatrix} \begin{bmatrix} \frac{\Gamma_2}{\Gamma_1} \end{bmatrix}$$

 $\frac{1}{2} \left(\frac{1}{2} \right) = \frac{x_1}{K^2}$

×d -> Total reactance Ref primary

$$x_{01} = x_1 + x_2$$

 $\frac{1}{2} = \frac{1}{2} = \frac{1}$

Firom primary wdg to see wdg

-3 coon - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 -

P.U readance doop B.T = P.U reactance desop A.T

×02 -> total reaclance ref fec wedy

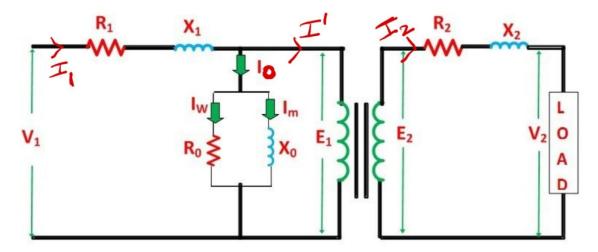
$$x_{02} = x_2 + x_1^{1}$$
 $x_{02} = x_2 + x_1 + x_2^{2}$

observationer 1xxx =1x

- > P.U treactance durop Ref palmany of MF = IXXOI EI
- e) Pu deactance durop Ref Sec of $I|_F = \frac{I_2 \times o_2}{E_2}$ $\times o_2 = \times_2 + \times^2 \times_1$

Equivalent Circuit of a Transformer

The equivalent circuit diagram of any device can be quite helpful in the pre-determination of the behaviour of the device under the various condition of operation. It is simply the circuit representation of the equation describing the performance of the device.



Equivalent Circuit of a Transformer

The simplified equivalent circuit of a transformer is drawn by representing all the parameters of the transformer either on the secondary side or on the primary side.

Let the equivalent circuit of a transformer having the transformation ratio $K = E_2/E_1$

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

The induced emf E_1 is equal to the primary applied voltage V_1 when the primary voltage drop is less. This voltage causes current I_0 no-load current in the primary winding of the transformer. The no-load current is further divided into two components called **magnetizing current** (I_m) and **working current** (I_w or I_e). These two components of no-load current are due to the current drawn by a non-inductive resistance R_0 and pure reactance X_0 having voltage E_1 or (V_1 – primary voltage drop). The value of no-load current is very small, and thus, it is neglected.

Hence, $I_1 = I_1$ '.

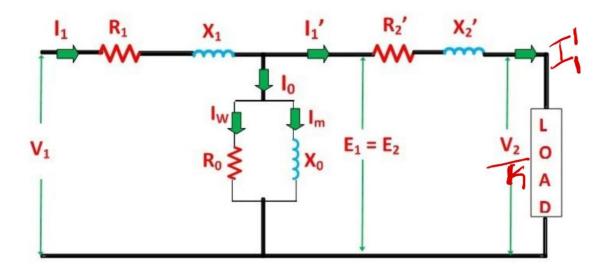
Therefore, $N_1I_1 = N_2I_2$

$$I_1' = \left(\frac{N_2}{N_1}\right)I_2 = KI_2$$

The terminal voltage V_2 across the load is equal to the induced emf E_2 in the secondary winding less voltage drop in the secondary winding.

Equivalent Circuit when all the quantities are referred to Primary side

The equivalent circuit of the transformer all the quantities are to be referred to the primary as shown in the figure below:



Equivalent Circuit of transformer referred to Primary side

The secondary voltage referred to primary i.e $V_2/V_1=K$ therefore $V_1=V_2/K$

Secondary resistance referred to the primary side is given as:

$$R_2' = \frac{R_2}{K^2}$$

The equivalent resistance referred to the primary side is given as:

Secondary reactance referred to the primary side is given as:

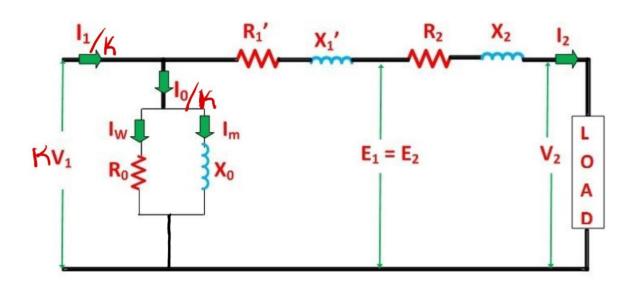
$$X_2' = \frac{X_2}{K^2}$$

The equivalent reactance referred to the primary side is given as:

$$x_{01} = x_1 + x_2^{1}(07) x_{01} = x_1 + x_2/2$$

Equivalent Circuit when all the quantities are referred to Secondary side

The equivalent circuit diagram of the transformer is shown below when all the quantities are referred to the secondary side.



Equivalent circuit of transformer referred to the secondary side.

The primary voltage V1 is referred on secondary i.e $V_2/V_1=K$ therefore $KV_1=V_2$

Primary resistance referred to the secondary side is given as

$$R_1' = K^2 R_1$$

The equivalent resistance referred to the secondary side is given as

Primary reactance referred to the secondary side is given as

$$X_1' = K^2 X_1$$

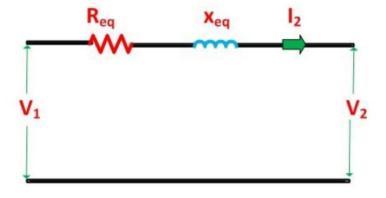
The equivalent reactance referred to the secondary side is given as

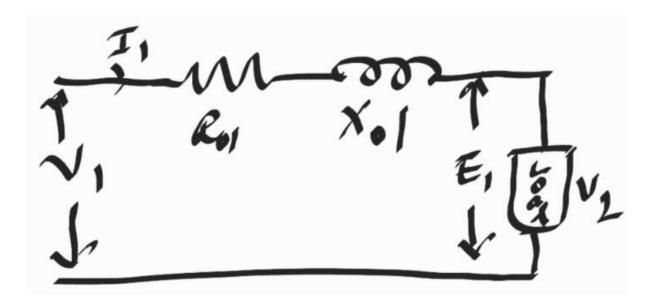
$$x_{02} = x_2 + x_1'(0) x_{02} = x_2 + \kappa^2 x_1$$

No-load current I_0 is hardly 3 to 5% of full load rated current, the parallel branch consisting of resistance R_0 and reactance X_0 can be neglected in the behaviour of the transformer under the loaded condition.

Further simplification of the equivalent circuit of the transformer can be done by neglecting the parallel branch consisting of R_0 and X_0 .

The simplified circuit diagram of the transformer is shown below:





$$R_{01} = R_{1} + R_{2}^{1}(07) R_{01} = R_{1} + R_{2}/2$$

$$X_{01} = X_{1} + X_{2}^{1}(07) X_{01} = X_{1} + X_{2}/2$$

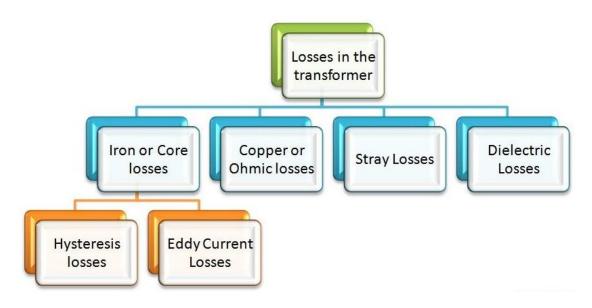
$$R_{02} = R_2 + R_1' \text{ (or)} R_{02} = R_2 + K^2 R_1$$

 $X_{02} = X_2 + X_1' \text{ (or)} X_{02} = X_2 + K^2 X_1$

Types of Losses in a Transformer

There are various types of losses in the transformer such as iron loss, copper loss, hysteresis loss, eddy current loss, stray loss, and dielectric loss. The hysteresis losses occur because of the variation of the magnetization in the core of the transformer and the copper loss occurs because of the transformer winding resistance.

The various types of losses of transformer are:



Iron Losses:

Iron losses are caused by the alternating flux in the core of the transformer as this loss occurs in the core it is also known as **Core loss**. Iron loss is further divided into hysteresis and eddy current loss.

Hysteresis Loss

The core of the transformer is subjected to an alternating magnetizing force, and for each cycle of emf, a hysteresis loop is traced out. Power is dissipated in the form of heat known as hysteresis loss and given by the equation shown below:

$$P_h = K \Pi B_{max}^{1.6} f V$$
 watts

Where

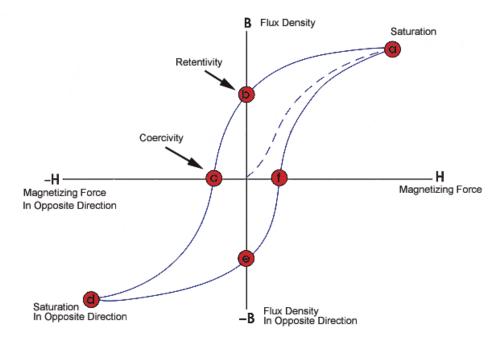
- Kn is a proportionality constant which depends upon the volume and quality of the material of the core used in the transformer,
- f is the supply frequency(Hz)
- B_{max} is the maximum or peak value of the flux density.(wb/m²)
- V is the volume of the material (m³)

The iron or core losses can be minimized by using silicon steel material for the construction of the core of the transformer.

Hysteresis Loss

Hysteresis loss is caused by the magnetization and demagnetization of the core as current flows in the forward and reverse directions. As the magnetizing force (current) increases, the magnetic flux increases. But when the magnetizing force (current) is decreased, the magnetic flux doesn't decrease at the same rate, but less gradually. Therefore, when the magnetizing force reaches zero, the flux density still has a positive value. In order for the flux density to reach zero, the magnetizing force must be applied in the negative direction.

The relationship between the magnetizing force, H, and the flux density, B, is shown on a hysteresis curve, or loop. The area of the hysteresis loop shows the energy required to complete a full cycle of magnetizing and de-magnetizing, and the area of the loop represents the energy lost during this process.



Residual flux: Presence of flux in the transformer core even after the magnetising force is made to zero.

Retentivity: The process of production of residual flux is known as retentivity. **Coercivity:** Force required in the negative manner to bring the residual flux to zero.

Eddy Current Loss:

When the flux links with a closed circuit, an emf is induced in the circuit and the current flows, the value of the current depends upon the amount of emf around the circuit and the resistance of the circuit.

Since the core is made of conducting material, these EMFs circulate currents within the body of the material. These circulating currents are called **Eddy Currents**. They will occur when the conductor experiences a changing magnetic field. As these currents are not responsible for doing any useful work, and it produces a loss (I²R loss) in the magnetic material known as eddy current losses.

The eddy current loss is minimized by making the core with thin laminations.

The equation of the eddy current loss is given as:

$$P_e = K_e B_m^2 t^2 f^2 V$$
 watts

Where,

- K_e coefficient of eddy current. Its value depends upon the nature of magnetic material like volume and resistivity of core material, the thickness of laminations
- B_m maximum value of flux density in wb/m²
- T thickness of lamination in meters
- F frequency of reversal of the magnetic field in Hz
- V the volume of magnetic material in m³

Copper Loss (or) Ohmic Loss

These losses occur due to ohmic resistance of the transformer windings. If I_1 and I_2 are the primary and the secondary current. R_1 and R_2 are the resistance of primary and secondary winding then the copper losses occurring in the primary and secondary winding will be $I_1^2R_1$ and $I_2^2R_2$ respectively.

Therefore, the total copper losses will be

$$P_c = I_1^2 R_1 + I_2^2 R_2$$

These losses varied according to the load and known hence it is also known as variable losses. Copper losses vary as the square of the load current.

Stray Loss:

The occurrence of these stray losses is due to the presence of leakage field. The percentage of these losses is very small as compared to the iron and copper losses so they can be neglected.

Dielectric Loss:

Dielectric loss occurs in the insulating material of the transformer that is in the oil of the transformer, or in the solid insulations. When the oil gets deteriorated or the solid insulation gets damaged, or its quality decreases, and because of this, the efficiency of the transformer gets affected.

Transformer Efficiency

The Efficiency of the transformer is defined as the ratio of useful output power to the input power. The input and output power are measured in the same unit. Its unit is either in Watts (W) or KW. Transformer efficiency is denoted by ' Π '.

$$\begin{split} \eta &= \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}} \\ \eta &= \frac{\text{output power}}{\text{output power} + \text{iron losses} + \text{copper losses}} \\ \eta &= \frac{V_2 I_2 \text{Cos}\phi_2}{V_2 I_2 \text{Cos}\phi_2 + P_i + P_c} \end{split}$$

Where,

- V₂ Secondary terminal voltage
- I₂ Full load secondary current
- $\cos \phi_2$ power factor of the load
- P_i Iron losses = hysteresis losses + eddy current losses
- P_c Full load copper losses = $I_2^2 R_{es}$

Consider, the x is the fraction of the full load. The efficiency of the transformer regarding x is expressed as

$$\eta_x = \frac{x \times \text{V}_2 I_2 \cos \varphi_2}{x \times \text{Output} + P_i + x^2 P_c} = \frac{x \times V_2 I_2 \cos \varphi_2}{x \times V_2 I_2 \cos \varphi_2 + P_i + x^2 I_2^2 R_{es}}$$

The copper losses vary according to the fraction of the load.

Maximum Efficiency Condition of a Transformer

The efficiency of the transformer along with the load and the power factor is expressed by the given relation:

$$\eta = \frac{V_2 I_2 Cos\phi_2}{V_2 I_2 Cos\phi_2 + P_i + I_2^2 R_{es}} = \frac{V_2 Cos\phi_2}{V_2 Cos\phi_2 + P_i / I_2 + I_2 R_{es}} (1)$$

The value of the terminal voltage V_2 is approximately constant. Thus, for a given power factor the Transformer efficiency depends upon the load current I_2 . In equation (1), the numerator is constant and the transformer efficiency will be maximum if the denominator with respect to the variable I_2 is equated to zero.

$$\frac{d}{dI_2} = \left(\begin{array}{ccc} V_2 \; \text{Cos} \phi_2 + \frac{P_i}{I_2} + \; I_2 R_{es} \end{array}\right) = 0 \qquad \text{or} \qquad 0 - \frac{P_i}{I_2^2} + \; R_{es} = 0$$

$$\text{Or}$$

$$I_2^2 R_{es} = P_i \; \dots \dots \dots (2)$$

i.e Copper losses = Iron losses

Thus, the transformer will give the maximum efficiency when their copper loss is equal to the iron loss.

$$\eta_{\text{max}} = \frac{V_2 I_2 \text{Cos} \varphi_2}{V_2 I_2 \text{Cos} \varphi_2 + 2P_i} \qquad as (P_c = P_i)$$

From equation (2) the value of output current I_2 at which the transformer efficiency will be maximum is given as

$$I_2 = \sqrt{\frac{P_i}{R_{es}}}$$

If x is the fraction of full load KVA at which the efficiency of the transformer is maximum then,

Copper losses = x^2P_c (where P_c is the full load copper losses)

Iron losses = P_i

For maximum efficiency: $\mathbf{x}^2 \mathbf{P_c} = \mathbf{P_i}$

Therefore

$$x = \sqrt{\frac{P_i}{P_c}} \dots \dots \dots (3)$$

Thus, output KVA corresponding to maximum efficiency

$$\eta_{\text{max}} = x \text{ X full load KVA (4)}$$

Putting the value of x from the above equation (3) in equation (4) we will get,

$$\eta_{\text{max}} = \sqrt{\frac{P_i}{P_c}} X \text{ full load KVA}$$

$$\eta_{\text{max}} = \text{Full load KVA X} \sqrt{\frac{\text{iron losses}}{\text{copper losses at full load}}} \dots \dots \dots \dots (5)$$

The above equation (5) is the maximum efficiency condition of the transformer

All Day Efficiency of a Transformer

All day efficiency means the power consumed by the transformer throughout the day. It is defined as the **ratio of output power to the input power** in kWh or wh of the transformer over 24 hours. Mathematically, it is represented as

All day efficiency,
$$\eta_{\text{all day}} = \frac{\text{output in kWh}}{\text{input in kWh}}$$
 (for 24 hours)

All-day efficiency of the transformer depends on their load cycle. The load cycle of the transformer means the repetitions of load on it for a specific period.

The ordinary or commercial efficiency of a transformer is defined as the ratio of the output power to the input power.

$$\eta = \frac{\mathsf{output}\;\mathsf{power}}{\mathsf{input}\;\mathsf{power}} = \frac{\mathsf{output}\;\mathsf{power}}{\mathsf{output}\;\mathsf{power} + \mathsf{losses}}$$

What is the need for All Day Efficiency?

Some transformer efficiency cannot be judged by simple commercial efficiency as the loads on certain transformer fluctuate throughout the day.

For example, the distribution transformers are energized for 24 hours, but they deliver very light loads for the major portion of the day, and they do not supply rated or full load, and most of the time the distribution transformer has 50 to 75% load on it.

As we know, there are various losses in the transformer such as iron and copper loss. The iron loss takes place at the core of the transformer. Thus, the iron or core loss occurs for the whole day in the distribution transformer.

The second type of loss known as a copper loss and it takes place in the windings of the transformer and is also known as the **variable loss**. It occurs only when the transformers are in the loaded condition.

Hence, the performance of such transformers cannot be judged by the commercial or ordinary efficiency, but the efficiency is calculated or judged by All Day Efficiency also known as **operational efficiency** or **energy efficiency** which is computed by the energy consumed for 24 hours.

Voltage Regulation of a Transformer

Definition: The voltage regulation is defined as the change in terminal voltage from no load voltage to full load voltage in the fraction of no load or full load voltage. The voltage regulation determines the ability of the transformer to provide the constant voltage for variable loads.

When the transformer is loaded with continuous supply voltage, the terminal voltage of the transformer varies. The variation of voltage depends on the load and its power factor.

Mathematically, the voltage regulation is represented as:

Voltage Regulation =
$$\frac{E_2 - V_2}{E_2}$$
 % Voltage Regulation =
$$\frac{E_2 - V_2}{E_2} \times 100$$

where,

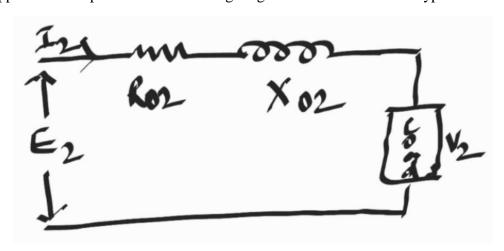
E₂ – secondary terminal voltage at no load

 V_2- secondary terminal voltage at full load

voltage regulation of transformer is always expressed in % practically, it is order of 5-10%

% voltage reg=
$$(V_{NL}-V_{FL}/\ V_{NL})*100 =>$$
 voltage reg down % voltage reg= $(V_{NL}-V_{FL}/\ V_{FL})*100 =>$ voltage reg up Since $V_{NL}>V_{FL}$

The approximate expression for the voltage regulation for the different types of the load is:



1. For inductive load (lagging power factor)

$$E_{2} = I_{2}R_{02}Cos\phi_{2} + I_{2}X_{02}Sin\phi_{2} + V_{2}$$

$$OR$$

$$E_{2} - V_{2} = I_{2}R_{02}Cos\phi_{2} + I_{2}X_{02}Sin\phi_{2}$$

$$V \cdot g = \frac{E_2 - V_2}{E_2} = \frac{I_2 R_{02} \cos \theta_2 + I_2 X_{02} \sin \theta_2}{E_2}$$

$$V \cdot g = \frac{I_2 R_{02} \cos \theta_2 + I_2 X_{02} \sin \theta_2}{X_{100}}$$

$$E_2$$

o For Capacitive load (leading power factor)

$$\begin{split} E_2 &= \ I_2 R_{02} \text{Cos} \phi_2 - \ I_2 X_{02} \text{Sin} \phi_2 + V_2 \\ \text{OR} \\ E_2 &- V_2 &= \ I_2 R_{02} \text{Cos} \phi_2 - \ I_2 X_{02} \text{Sin} \phi_2 \end{split}$$

$$\frac{E_{2}^{-V_{2}}}{E_{2}} = \frac{I_{2}Ro_{2}cosb_{2} - I_{2}Xo_{2}shnb_{2}}{E_{2}}$$

$$\frac{E_{2}^{-V_{2}}}{E_{2}} = \frac{I_{2}Ro_{2}cosb_{2} - I_{2}Xo_{2}shnb_{2}}{E_{2}}$$

$$\frac{V_{3}V_{3}g}{E_{2}} = \frac{I_{2}Ro_{2}cosb_{2} - I_{2}Xo_{2}shnb_{2}}{E_{2}} \times 1000$$

Where,

$$\frac{I_2R_{02}}{E_2}$$
 x 100 is a percentage resistance drop $\frac{I_2X_{02}}{E_2}$ x 100 is a percentage reactance drop

$$\frac{\% v \cdot g = I_{2}Ro_{2}(os\phi_{2} \pm I_{2}X_{2}sin\phi_{2})}{E_{2}} \times 100}{E_{2}}$$

$$\frac{\% v \cdot g = \left[\frac{I_{2}Ro_{2}}{E_{2}}\cos\phi_{2} \pm \frac{I_{2}X_{02}}{E_{2}}\sin\phi_{2}\right]}{(os\phi_{2} \pm \% \times sin\phi_{2})}$$

$$\frac{\% v \cdot g = \% R(os\phi_{2} \pm \% \times sin\phi_{2})}{E_{2}}$$

1.15.1 Condition For Zero Regulation. Regulation =

$$\frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{E_2}$$

Regulation will be zero if the numerator will be equal to zero

or
$$I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi = 0$$

or tan
$$\phi = \frac{-R_{02}}{X_{02}}$$
 ... (1.26)

The -ve sign indicates that zero regulation occurs at a leading power factor.

1.15.2. Condition For Maximum Regulation.

Regulation will be maximum if $\frac{d}{d\phi}$ (regulation) = 0

or
$$\frac{d}{d\phi} \frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{E_2} = 0$$

or $-\frac{I_2 R_{02}}{E_2} \sin \phi + \frac{I_2 X_{02}}{E_2} \cos \phi = 0$
or $\tan \phi = \frac{X_{02}}{R_{02}} \dots (1.27)$

i.e. Maximum regulation occurs at lagging power factor.

Auto Transformer

An **Auto Transformer** is a transformer with only one winding wound on a laminated core. An auto transformer is similar to a two winding transformer but differ in the way the primary and secondary winding are interrelated. A part of the winding is common to both primary and secondary sides.

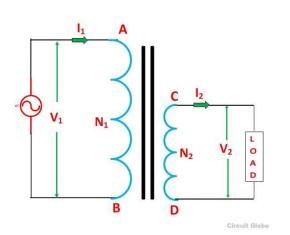
On load condition, a part of the load current is obtained directly from the supply and the remaining part is obtained by transformer action. An Auto transformer works as a **voltage regulator.**

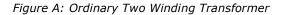
Auto Transformer with Circuit Diagram

In an ordinary transformer, the primary and the secondary windings are electrically insulated from each other but connected magnetically as shown in the figure below. While in auto transformer the primary and the secondary windings are connected magnetically as well as electrically. In fact, a part of the single continuous winding is common to both primary and secondary.

There are two types of auto transformer based on the construction. In one type of transformer, there is continuous winding with the taps brought out at convenient points determined by the desired secondary voltage. However, in another type of auto transformer, there are two or more distinct coils which are electrically connected to form a continuous winding. The construction of Auto transformer is shown in the figure below.

The primary winding AB from which a tapping at C is taken, such that CB acts as a secondary winding. The supply voltage is applied across AB, and the load is connected across CB. The tapping may be fixed or variable. When an AC voltage V_1 is applied across AB, an alternating flux is set up in the core, as a result, an emf E_1 is induced in the winding AB. A part of this induced emf is taken in the secondary circuit.





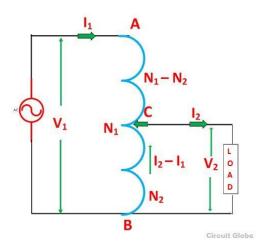


Figure B: Auto - Transformer

Let,

- V₁ primary applied voltage
- V₂ secondary voltage across the load
- I₁ primary current
- I₂ load current
- N_1 number of turns between A and B
- N₂ number of turns between C and B

Neglecting no-load current, leakage reactance and losses,

$$V_1 = E_1$$
 and $V_2 = E_2$

Therefore, the transformation ratio:

$$K = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

As the secondary ampere-turns are opposite to primary ampere-turns, so the current I_2 is in phase opposition to I_1 . The secondary voltage is less than the primary. Therefore current I_2 is more than the current I_1 . Therefore, the resulting current flowing through section BC is $(I_2 - I_1)$.

The ampere-turns due to section BC = current x turns

$$\text{Ampere turns due to section BC} = (I_2 - \ I_1) N_2 = \left(\frac{I_1}{K} - \ I_1\right) x \ N_1 K = \ I_1 N_1 \ (1 - K) \ ... \ ... \ (1)$$

$$\mbox{Ampere turns due to section AC} = \ \mbox{I}_1(\mbox{N}_1 - \mbox{ N}_2) = \ \mbox{I}_1\mbox{N}_1 \Big(1 - \frac{\mbox{N}_2}{\mbox{N}_1}\Big) = \ \mbox{I}_1\mbox{N}_1(1 - \mbox{K}) \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \ ... \$$

Equation (1) and (2) shows that the ampere-turns due to section BC and AC balance each other which is characteristic of the transformer action.

Advantages of Auto transformer

- Less costly
- Better regulation
- Low losses as compared to ordinary two winding transformer of the same rating.

Disadvantages of Auto transformer

There are various advantages of the auto transformer, but then also one major disadvantage, why auto transformer is not widely used, is that

- The secondary winding is not insulated from the primary winding. If an auto transformer is used to supply low voltage from a high voltage and there is a break in the secondary winding, the full primary voltage comes across the secondary terminal which is dangerous to the operator and the equipment. So the auto transformer should not be used for interconnecting high voltage and low voltage systems.
- Used only in the limited places where a slight variation of the output voltage from input voltage is required.

Applications of Auto transformer

- It is used as a starter to give up to **50 to 60%** of full voltage to the stator of a squirrel cage induction motor during starting.
- It is used to give a small boost to a distribution cable, to correct the voltage drop.
- It is also used as a voltage regulator
- Used in power transmission and distribution system and also in the audio system and railways.