ASSIGNMENT-1

1. The proababability that a man will be alive in 25 years is $3/5$, and the probability that his wife will be alive is $2/3$. Find the probability that

ⓐ both will be alive

ⓑ Only ~~she~~ man will be alive

ⓒ Only she will be alive

ⓓ atleast one will be alive

Let H be the event of man living for 25 years

Let W be the event of man's wife living for 25 years

Probability of man will be alive for 25 years is

$$P(H) = 3/5$$

Probability of man's wife will be alive for 25 years is

$$P(W) = 2/3$$

ⓐ Both will be alive

Probability of both husband and wife will be alive for 25 years $= P(H \cap W) = P(H)\,P(W)$

$$= \frac{3}{5} \times \frac{2}{3} = \frac{2}{5} = 0.4 \quad [\because \text{Independent Events}]$$

ⓑ Only man will be alive

probability of wife not alive for 25 years $P(W') = 1 - P(W)$

$$= 1 - 2/3$$

$$= 1/3$$

Probability of only man will be alive = $P(H \cap W')$

$= P(H)\,P(W')$ ($\because$ Independent Events)

$= \frac{3}{5} \times \frac{1}{3} = \frac{1}{5} = 0.2$

(c) Only she will be alive

probability of man will not be alive for 25 years

$P(H') = 2/5$

Probability of only wife will be alive = $P(H' \cap W)$

$= P(H')\,P(W)$ ($\because$ Independent Events)

$= \frac{2}{5} \times \frac{2}{3} = \frac{4}{15} = 0.2667$

(d) atleast one will be alive

probability of atleast one will be alive =

$P(H)P(W') + P(H')P(W) + P(H)P(W)$

$= \frac{3}{5} \times \frac{1}{3} + \frac{2}{5} \times \frac{2}{3} + \frac{3}{5} \times \frac{2}{3}$

$= 0.8667$

Assume that a factory has two machines. Past records show that machine I produces 30% of the items of output and machine II produces 70% of the items. Further, 5% of items produced by machine I were defective and 1% of items produced by machine II were defective. If a defective item is drawn at random, what is the probability that the defective item was produced by machine I or machine II.

let X be the event of defective item is drawn.

~~Proof~~ let A, B be the events of items produced by machine I, II respectively

Probability of machine-I produced item is drawn

$P(A) = 30\% = 0.3$

Probability of item drawn is produced by machine II

$P(B) = 70\% = 0.7$

Probability of item drawn produced by machine I were defective $P(X/A) = 5\% = 0.05$

Probability of item drawn produced by machine II were defective $P(X/B) = 1\% = 0.01$

Probability of drawn item is produced by machine I or machine II is defective

$$= \frac{P(A)\,P(X/A)}{P(A)P(X/A) + P(B)\,P(X/B)} + \frac{P(B)\,P(X/B)}{P(A)P(X/A) + P(B)P(X/B)}$$

[∵ Baye's theorem]

$$= \frac{(0.3)(0.05)}{(0.3)\times(0.05)+(0.7)(0.01)} + \frac{(0.7)(0.01)}{(0.3)(0.05)+(0.7)(0.01)}$$

$= \cancel{0.0224}\ 1.0 = 1.0$

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(3) There are three urns containing 2 white, 3 black balls 3 white, 2 black balls and 4 white, 1 black blacks. respectively. There ~~as~~ is equal probability of each urn being chosen A ball is drawn from an urn chosen at random. Find the probability that white ball is drawn.

let X be the event of drawing a white ball.

let A, B, C be the urns and 2 white, 3 black balls, 3 while, 2 black balls and 4 white, 1 black balls are in them respectively.

Probability of choosing white ball from urn A

$P(A) = 1/3$

Similarly $P(B) = 1/3$

$P(C) = 1/3$

Probability of choosing white ball from urn A

$P(X/A) = 2/5$

Probability of choosing white ball from urn B

$P(X|B) = 3/5$

Probability of choosing white ball from urn C

$P(X|C) = 4/5$

Probability of choosing a white ball

$P(X) = P(A)P(X/A) + P(B)P(X/B) + P(C)P(X/C)$

$= \left(\frac{1}{3}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{4}{5}\right)$

$= 0.6$

(4) A product is assembled from three components x, y, z the probability of these components being defective is respectively 0.01, 0.02, 0.05. What is the probability that the assembled product will not be defective

Given

Probability of component x being defective $P(x) = 0.01$

Probability of component y being defective $P(y) = 0.02$

Probability of component z being defective $P(z) = 0.05$

Probability of component x not being defective $P(x') = 0.99$

lly $P(y') = 0.98$

$P(z') = 0.95$

Probability of assembled component will not be defective

$P(x' \cap y' \cap z') = P(x')\,P(y')\,P(z')$ (∵ Independent Events)

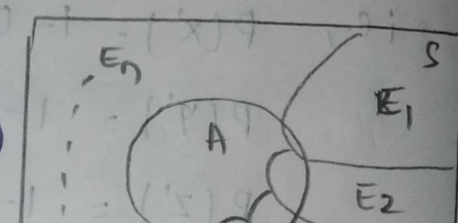
$= 0.99 \times 0.98 \times 0.95$

$= 0.92169$

Baye's Theorem:

Statement: Suppose $E_1, E_2, E_3 \dots E_n$ are mutually exclusive events of a sample space 's' such that $P(E_i) > 0$ $i = 1, 2, 3, \dots, n$ and 'A' is any arbitary event of s. such that the probability of $P(A) > 0$ then

$$P(E_i/A) = \frac{P(A/E_i)(P(E_i))}{\sum_{i=1}^{n} P(A/E_i)P(E_i)}$$



Proof:

Given A is subset of S

$A \subset S$

Where 'A' is any arbitary event in 's'

$A \subset \bigcup_{i=1}^{n} E_i$

where $E_1, E_2, \dots E_n$ are n mutually exclusive events in sample space 'S'.

$A \cap S = A$

$A \cap \left(\bigcup_{i=1}^{n} E_i\right) = A$

$A = A \cap (E_1 \cup E_2 \cup E_3 \dots \cup E_n)$

$A = (A \cap E_1) \cup (A \cap E_2) \dots \cup (A \cap E_n)$ ($\because$ distributive law)

Taking probabilities on both sides

$P(A) = P((A \cap E_1) \cup (A \cap E_2) \dots \cup (A \cap E_n))$

$$P(A) = P((A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n))$$

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)$$

($\because E_1, E_2, \cdots E_n$ are mutually exclusive events)

($A \cap E_1, A \cap E_2 \ldots A \cap E_n$ are also mutually Exclusive events)

~~$\therefore P(A) = P($~~

From 3rd axiom of probability

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

(By multiplication theorem)

$$P(A) = \sum_{i=1}^{n} P(E_i) P(A/E_i) \quad \text{--- (1)}$$

By conditional Probability

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

(By multiplication theorem)

$$= \frac{P(E_i) P(A/E_i)}{P(A)}$$

$$P(E_i/A) = \frac{P(E_i) P(A/E_i)}{\sum_{i=1}^{n} P(E_i) P(A/E_i)} \quad [\text{from (1)}]$$

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5 b) State and prove Addition theorem for n events.

Statement: For n events $A_1, A_2, \cdots A_n$.

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{1 \le i < j \le n} P(A_i \cap A_j) + \cdots + (-1)^{n-1} P(A_1 \cap A_2 \cdots \cap A_n)$$

Proof: We prove the theorem using a mathematical induction method. i.e., To prove that true for $n=2$ events and assumming that the theorem is true for $n=r$ events and to prove it is true for $n=r+1$ events

For two events A_1, A_2 we have

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \quad — ①$$

It is true for $n=2$

let us assume that it is true for $n=r$ events

$$P\left(\bigcup_{i=1}^{r} A_i\right) = \sum_{i=1}^{r} P(A_i) - \sum\sum_{1 \leq i \leq j \leq r} P(A_i \cap A_j) + \cdots + (-1)^{r-1} P(A_1 \cap A_2 \cdots \cap A_r) \quad — ②$$

Consider

$$P\left(\bigcup_{i=1}^{r+1} A_i\right) = P\left(\bigcup_{i=1}^{r} A_i \cup A_{r+1}\right)$$

$$= P\left(\bigcup_{i=1}^{r} A_i\right) + P(A_{r+1}) - P\left(\bigcup_{i=1}^{r} A_i \cap A_{r+1}\right) \quad [\text{from } ①]$$

$$= \sum_{i=1}^{r} P(A_i) - \sum\sum_{1 \leq i \leq j \leq r} P(A_i \cap A_j) + \cdots + (-1)^{r-1} P(A_1 \cap A_2 \cap \cdots A_r) + P(A_{r+1})$$

$$- P\left(\bigcup_{i=1}^{r} A_i \cap A_{r+1}\right) \quad [\text{from } 2]$$

$$\Rightarrow P\left(\bigcup_{i=1}^{r+1} A_i\right) = \sum_{i=1}^{r+1} P(A_i) - \sum\sum_{1 \leq i \leq j \leq r} P(A_i \cap A_j) + \cdots + (-1)^{r-1} P(A_1 \cap A_2 \cap \cdots A_r)$$

$$- \left[P\left(\bigcup_{i=1}^{r} (A_i \cap A_{r+1})\right)\right]$$

$$\left[\bigcup_{i=1}^{r} A_i \cap A_{r+1} = (A_1 \cup A_2 \cup A_3 \cup \dots A_r) \cap A_{r+1}^{r}\right.$$

$$\left.= (A_1 \cap A_{r+1}) \cup (A_2 \cap A_{r+1}) \dots (A_r \cap A_{r+1})\right]$$ [Distributive law]

$$\Rightarrow P\left(\bigcup_{i=1}^{r+1} A_i\right) = \sum_{i=1}^{r+1} P(A_i) - \sum\sum P(A_i \cap A_j) + \dots + (-1)^{r-1}\left(\bigcap_{i=1}^{r} A_i\right) -$$

$$\left[\sum_{i=1}^{r} P(A_i \cap A_{r+1}) - \sum\sum P(A_i \cap A_{r+1} \cap A_j) + \dots + (-1)^{r-1} P\left(\bigcap_{i=1}^{r} A_i \cap A_{r+1}\right)\right]$$

$$\Rightarrow P\left(\bigcup_{i=1}^{r+1} A\right) = \sum_{i=1}^{r+1} P(A_i) - \sum\sum_{1 \le i \le j \le r+1} P(A_i \cap A_j) + \dots + (-1)^{r-1} P\left(\bigcap_{i=1}^{r} A_{r+1}\right)$$

$\therefore$ Hence the theorem is true for $n = r+1$

By the mathematical induction, it is true for all events.