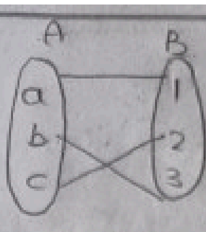
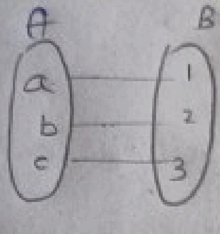
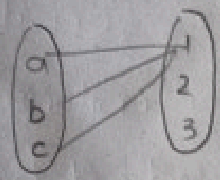
iii) 

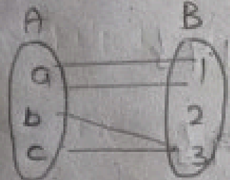

$f : A \to B$ is a function

iv) 


$f : A \to B$ is a function since every element is A having some image in B.

v) 


$A \to B$ is a function since every element of A having some image in B.

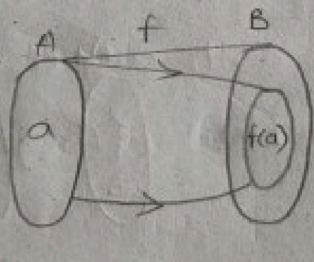
vi) 


$f : A \to B$ is not a function since, in 'A' a is having two image in B

## Range of a function:-

Let $f : A \to B$ is a function defined over the two non-empty sets A and B then the range of F is defined as

Range of $F = \{ f(a) \ \forall a \in A \}$




Eg:- Let $A = \{0, \pm 1, \pm 2, 3\}$ and $f : A \to R$

where 'R' is real no's.

$$f(x) = x^3 - 2x^2 + 3x + 1 \ \forall x \in A$$

find the range of 'f'

$$f(x) = x^3 - 2x^2 + 3x + 1$$

$$f(0) = 1$$

$$f(1) = 1 - 2 + 3 + 1 = 3$$

$$f(-1) = -1 - 2 - 3 + 1 = -5$$

$$f(2) = 2^3 - 2(2)^2 + 3(2) + 1$$

$$= 7$$

$$f(-2) = -21$$

$$f(3) = 19$$

Range of $f = \{1, 3, -5, 7, -21, 19\}$

$$= \{-21, -5, 1, 3, 7, 19\}$$

2) Let $f: R \to R$ is defined by $f(x) = x^2 + 1$. find images of following subsets of 'R'.

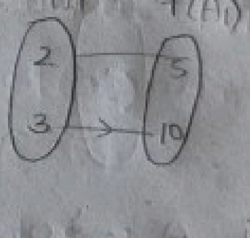
i) $A_1 = \{2, 3\}$ ii) $A_2 = \{-2, 0, 3\}$

iii) $A_3 = \{0, 1\}$ iv) $A_4 = [-6, 3]$

$$f(x) = x^2 + 1$$

i) $f(2) = 2^2 + 1 = 5$

$f(3) = 9 + 1 = 10$




$f(A_1) = \{5, 10\}$

ii) $A_2 = \{-2, 0, 3\}$

$f(-2) = 5$

$f(0) = 1$

$f(3) = 10$

$f(A_2) = \{1, 5, 10\}$

iii) $A_3 = (0,1)$ open    open interval means less than $<$

$f(A_3) = \{f(x) \mid 0<x<1\}$

iv) $A_4 = [-6,3]$ closed    closed interval means $\le$ less than or equal to

$f(A_4) = \{f(x) \mid -6 \le x \le 3\}$

3) Let $A = \{1,2,3,4,5,6\}$

$B = \{6,7,8,9,10\}$

$f: A \to B$ defined as $f = \{(1,7)(2,7)(3,8)(4,6)(5,9)(6,9)\}$

find i) $f^{-1}(6)$    if $B_1 = \{7,8\}$ find $f^{-1}(B_1)$

ii) $f^{-1}(9)$    $B_2 = \{8,9,10\}$ $f^{-1}(B_2)$

$6 = f(9)$    3, 4, 5

i) $f^{-1}(6) = 4$

ii) $f^{-1}(9)$ does not exist

($B_1 = \{7,8\}$

$f^{-1}(7)$ & $f^{-1}(8)$)) wrong

$B_1' = \{1,2\}$

$f^{-1}(1) = 7$

$f^{-1}(2) = 7$

$f^{-1}(B_1) = \{7\}$

iii) $f^{-1}(B_2) = \{6,8,9\}$    $B_2 = \{3,4,5\}$

$f^{-1}(3) = 8$, $f^{-1}(4) = 6$, $f^{-1}(5) = 9$

4) Let $f: R \to R$, $f(x) = \begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x \le 0 \end{cases}$

find $f(0)$, $f(-1)$, $f(5/3)$

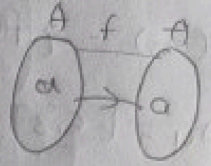
$f(0) = -3x+1$
$= 1$

$f(-1) = -3x+1$
$= 4$

$f(5/3) = 3x-5$
$= 3(5/3)-5$
$= 0$

## Types of functions:-

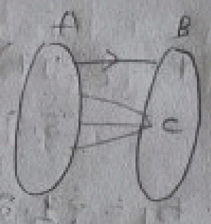
1) Identity function:- $f : A \to A$




i.e., $f(a) = a \; \forall \; a \in A$

2) constant function:-

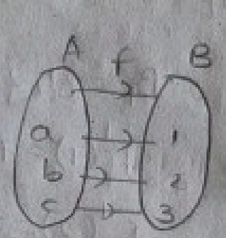
$f : A \to B$ $f(a) = c \; \forall \; a \in A$ where 'c' is the fixed element of 'c'.




3) one-to-one function:-

$F : A \to B$ is said to be one-to-one function if distinct element of 'A' have different images in B.

Eg:

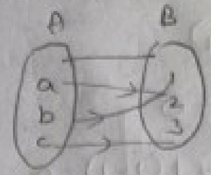



(or)

if $a_1, a_2 \in A$ with $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$
i.e., if wherever $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

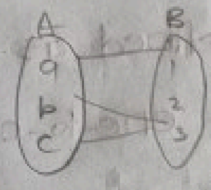
4) on-to function:-

A function $f : A \to B$ is said to be on-to function if every element of B has a pre-image in 'A' (or) if set to be onto function. If range ~~of~~ $(f)$ = codomain




5) **In-to function** :-

Let if $f: A \to B$ be a function if there exist a single element in B having no pre-image 'f'. then f is said to be into function.




6) **Bijective function**:- A function which is both one-to-one and on-to then the function is said to be Bijective function

**problems**:-

1) A function $f: R \to R$ and $g: R \to R$

$f(x) = 3x + 7 \ \forall x \in R$ and $g(x) = x(x^3 - 1) \forall x \in R$

Verify that f is one-to-one but not 'g'.

$f(x) = 3x + 7$

$f(x_1) = 3x_1 + 7$ , $f(x_2) = 3x_2 + 7$

By def of one-to-one function

if $f(x_1) = f(x_2)$

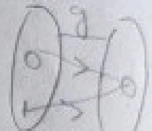
$f(x_1) = f(x_2)$ then $x_1 = x_2$

$3x_1 + \cancel{7} = 3x_2 + \cancel{7}$

$3x_1 = 3x_2$

$x_1 = x_2$

$\therefore$ f is one-to-one function.



$g(x) = x(x^3 - 1)$

let $x = 0$, $g(0) = 0$

$x = 1$, $g(1) = 1(1-1) = 0$

$g: R \to R$ is not one-to-one function since two elements are mapped to single element zero. hence 'g' is not one-to-one function

2) let $f: Z \to Z$ is defined as $f(a) = a+1 \ \forall a \in Z$ find whether $f$ is one-to-one (or) on-to (or) both.

i) one-to-one function:-

if $f(a_1) = f(a_2)$ then $a_1 = a_2$

$a_1 + \cancel{1} = a_2 + \cancel{1}$

$a_1 = a_2$

$\therefore$ f is 1-1 function

ii) onto function:-

If $f(a) = b$ then 'f' is onto

$f(a) = a+1$

$b = a+1$

$a = b-1$

$f(a) = a+1$

$f(b-1) = b - \cancel{1} + \cancel{1}$

$f(a) = b$

$\therefore$ f is onto function

$\therefore$ 'f' is both 1-1 and onto function.

...nction since
...e element
...-one function.
...$-a+1 \ \forall a \in z$
...o) On-to

**Inverse function:-** A function which stastify both one-one function & on-to Function is said to be invertable function i.e., the function is said to have it's inverse only when it is one-to-one and on-to function.

**problems:-**

1) find inverse of the function $f(x) = \frac{x+1}{x}$

$$\text{Let } f(x) = y \Rightarrow x = f^{-1}(y)$$

$$\frac{x+1}{x} = y$$

$$x+1 = xy$$

$$x - xy = -1$$

$$x(1-y) = -1$$

$$x = \frac{-1}{1-y} = \frac{1}{y-1}$$

$$\therefore x = f^{-1}(y)$$

$$\frac{1}{y-1} = f^{-1}(y)$$

$$f^{-1}(y) = \frac{1}{y-1}$$

$$f^{-1}(x) = \frac{1}{x-1}$$

2) find the inverse of $f(x) = 4e^{3x+1}$

$$f(x) = 4e^{3x+1}$$

$$\text{Let } f(x) = y$$

$$y = 4e^{3x+1}$$

$$\log y = \log 4 + (3x+1)$$

$$\log y - \log 4 = 3x+1$$

$$\log \frac{y}{4} = 3x+1$$

$$\log\frac{y}{4}-1=3x$$

$$x=\frac{1}{3}\left[\log\frac{y}{4}-1\right]$$

$$f^{-1}(y)=\frac{1}{3}\left[\log\frac{y}{4}-1\right]$$

$$f^{-1}(x)=\frac{1}{3}\left[\log\frac{x}{4}-1\right]$$

3) find the inverse of $f(x)=\dfrac{10}{\sqrt[5]{7-3x}}$

given, $f(x)=\dfrac{10}{\sqrt[5]{7-3x}}$

let $f(x)=y$

$$y=\frac{10}{\sqrt[5]{7-3x}} \Rightarrow y=\frac{10}{(7-3x)^{1/5}}$$

$y(7-3x)^{1/5}=10$ $\qquad \log y=\log\dfrac{10}{(7-3x)^{1/5}}$

$y^5(7-3x)=10^5$ $\qquad \log y=\log 10-\log(7-3x)^{1/5}$

$7-3x=\left(\dfrac{10}{y}\right)^5$ $\qquad \log y=\log 10-\frac{1}{5}\log(7-3x)$

$3x=7-\left(\dfrac{10}{y}\right)^5$ $\qquad \log y=1-\frac{1}{5}\log(7-3x)$

$x=\dfrac{1}{3}\left[7-\left(\dfrac{10}{y}\right)^5\right]$

$f^{-1}(x)=\frac{1}{3}\left[7-\left(\frac{10}{y}\right)^5\right.$ $\quad \frac{1}{5}\log(7-3x)=1-\log y$ ) wrong

$$f^{-1}(y)=\frac{1}{3}\left[7-\left(\frac{10}{x}\right)^5\right]$$

4) $f: R\to R$ $\quad f(x)=\begin{cases}3x & x>0\\ -3x+1 & x\le 0\end{cases}$

find $f^{-1}(0)$; $f^{-1}(1)$, $f^{-1}(3)$, $f^{-1}[[-5,5]]$

$\frac{0}{\sqrt{7-3x}}$

$\frac{10}{(7-3x)^{1/5}}$

$(7-3x)^{1/5}$

$g(7-3x)$

$g(7-3x)$

$-\log y$ Dumg

$[(-5,5]]$

---

let $y = f(x) \Rightarrow x = f^{-1}(y)$

$$y = \begin{cases} 3x-5 & x>0 \\ -3x+1 & x \le 0 \end{cases}$$

$y = 3x-5,\ x>0$

$y+5 = 3x$

$x = \frac{y+5}{3}$

$f^{-1}(y) = \frac{y+5}{3}$

$f^{-1}(x) = \frac{x+5}{3}$

when $x \le 0$

$y = -3x+1$

$y-1 = -3x$

$x = \frac{1-y}{3}$

$f^{-1}(y) = \frac{1-y}{3} \Rightarrow f^{-1}(x) = \frac{1-x}{3}$

$$f^{-1}(x) = \begin{cases} \frac{x+5}{3} & x>0 \\ \frac{1-x}{3} & x \le 0 \end{cases}$$

$f^{-1}(0) = \frac{1-0}{3} = 1/3$

$f^{-1}(x) = \frac{x+5}{3}\ x>0$

$f^{-1}(1) = \frac{1+5}{3} = 2$

$f^{-1}(3) = \frac{3+5}{3} = 8/3$

$f^{-1}[-5,5]$

$f^{-1}(-5) = \frac{1-x}{3} = \frac{1-(-5)}{3} = 2$

$f^{-1}(5) = \frac{x+5}{3} = \frac{5+5}{3} = 3.3$

$f^{-1}[-5,5] = [2, 3.3]$

5) find inverse of $f(x) = \begin{cases} x^2 & x<0 \\ x & 0<x<1 \\ 1/x & x \ge 1 \end{cases}$

find inverse: i) $3f-6$
ii) $f^2(3)$
iii) $f(-6)$
iv) $f^{-1}(1)$

1) $f(x) = \begin{cases} x^2 & x<0 \\ x & 0<x<1 \\ 1/x & x\geq 1 \end{cases}$

$f^{-1}(x) = \begin{cases} \sqrt{x} & x<0 \\ x & 0<x<1 \\ 1/x & x\geq 1 \end{cases}$

i) $3f-6 = \begin{cases} 3x^2-6 & x<0 \\ 3x-6 & 0<x<1 \\ \frac{3}{x}-6 & x\geq 1 \end{cases}$

ii) $f^2(3) = 1/3$

$f^2(3) = (1/3)^2 = 1/9$

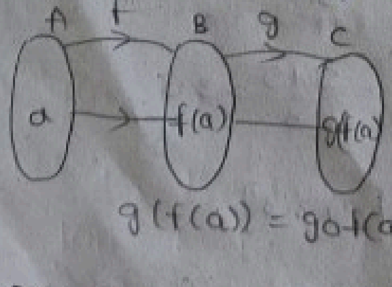
iii) $f(-6) = (-6)^2 = 36$

iv) $f^{-1}(1) = 1/1 = 1$

Composition of function :-

If $f: A\to B$, $g: B\to C$ be any two matrix then the composition mapping from $f\to g$ is denoted by $gof$ and it is denoted by

$$gof(x) = g(f(x))$$




$g(f(a)) = gof(a)$

Eg: Let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$ $\forall x \in R$

find: i) $fog$ ii) $gof$ iii) $fogoh$ iv) $fof$ v) $hog$

o matrix then

$\to g$ is denoted

$h(x) = 3x \; \forall x \in R$

iv) fof v) hog

i) $fog(x) = f(g(x))$

$= f(x-2)$

$= x-2+2 = x$

ii) $gof(x) = g(f(x))$

$= g(x+2)$

$= x+2-2 = x$

iii) $fogoh(x) = f(g(h(x)))$

$= f(g(3x))$

$= f(3x-2)$

$= 3x-2+2 = 3x$

iv) $fof = f(f(x))$

$= f(x+2)$

$= x+2+2 = 4+x$

v) $hog = h(g(x))$

$= h(x-2)$

$= 3x-2$

2) $f: R \to R$, $f(x) = x^2$ and $g(x) = x/x-2$ find fog is gof is defined.

$fog = f(g(x))$

$= f\left(x/_{x-2}\right)$

$= \left(\frac{x}{x-2}\right)^2$

$= \frac{x^2}{(x-2)^2} = \frac{x^2}{x^2+4-2x}$

$gof = g(f(x))$

$= g(x^2)$

$= \frac{x^2}{x^2-2}$

3) Let $A = B = \{x / -1 \leq x \leq 1\}$ for the each of the functions from $A \to B$. find whether following

functions are i) one-to-one

ii) on-to

iii) ~~Bijunction~~ Bijective

find : i) $f(x) = \frac{x}{2}$

ii) $g(x) = |x|$

iii) $h(x) = x^2$

iv) $l(x) = \sin \pi x$

i) a) $f(x) = \frac{x}{2}$

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\frac{x_1}{x_2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$

$f(x)$ is one-to-one

b) on-to:- $f(x) = y \Rightarrow \frac{x}{2} = y \Rightarrow x = 2y$

when $y = -1$ $f(-2) \notin [-1, 1]$

there 'f' is not onto function

c) $y = -1$ has pre-image in A

$\therefore f(x)$ is not Bi-jective function

ii) a) $g(x) = |x|$

$a_1, a_2 \in A$

$g(a_1), g(a_2) \in B$

$g(a_1) = g(a_2)$

$|a_1| = |a_2|$

$\Rightarrow a_1 = \pm a_2$

since '$a_1$' is having two images $a_2$ $-a_2$

$\therefore$ 'g' is not one-to-one function.

All the elements of 'B' having the preimages in B.

$\therefore$ 'g' is on-to function

Hence 'g' is not a bijective function.

iii) $h(x) = x^2$

$$x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$$

$\therefore$ 'h' is not one-to-one function

$$h(x) = y$$

$$x^2 = y$$

$$\therefore x = \sqrt{y}$$

$$h(\sqrt{y}) = y$$

if $y \in B = [-1, 1]$

Let $y = -1 \in B$ $\sqrt{y} = \sqrt{-1} \notin A$

$\therefore$ h is not on-to function

Hence 'h' is not a bijective function

iv) $l(x) = \sin \pi x$

$$\sin \pi x_1 - \sin \pi x_2 \Rightarrow \pi x_1 = \pi x_2 \rightarrow x_1 = x_2$$

$$(\sin \pi x_1 - \sin \pi x_2 = 0)$$

when $x_1 = -1$

$\sin(-\pi) = 0$

$\therefore$ $l(x)$ is not one-to-one function $x_1 = 1$ $\sin \pi = 0$

On to:- $\sin \pi x = y$

$$f(x) = y$$

$$\pi x = \sin^{-1} y$$

$$x = \frac{\sin^{-1} y}{\pi}$$

$$f(x) = y$$

$$f\left(\frac{\sin^{-1} y}{\pi}\right) = y$$

If $y \in B = [-1, 1]$

$\frac{1}{\pi}\sin^{-1} y \notin A$ [since inverse trignometric function may or may not $\in [-1, 1]$

$\therefore$ f is not on-to and bijective function