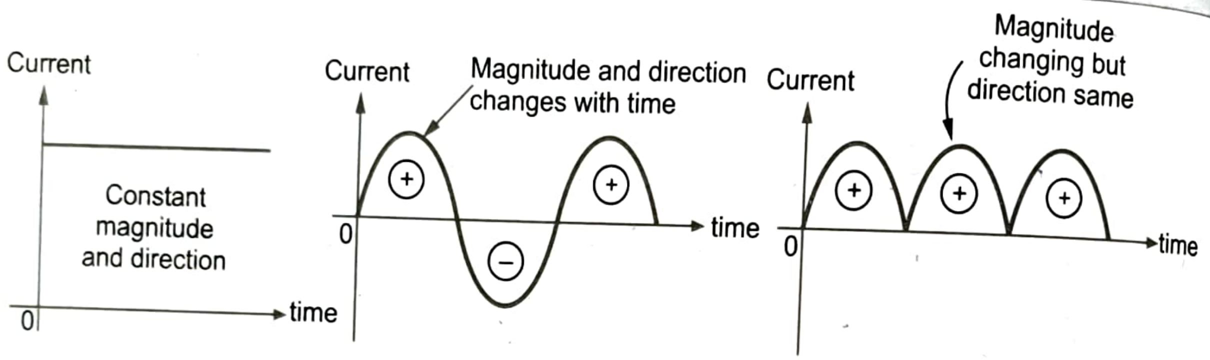
3.1 Introduction

- Electrical supply used for commercial and domestic purposes is alternating.
- The d.c. supply has constant magnitude with respect to time. The Fig. 3.1.1 (a) shows a graph of such current with respect to time.

- *An alternating current (a.c.) is the current which changes periodically both in magnitude and direction.*



(a) Direct current **(b) Alternating current** **(c) Pulsating d.c.**

Fig. 3.1.1

- In alternating waveform there are two half cycles, one positive and other negative. These two half cycles make one cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly.
- The Fig. 3.1.1 (b) shows a graph of alternating current against time.
- In practice some waveforms are available in which magnitude changes but its direction remains same as positive or negative. This is shown in the Fig. 3.1.1 (c). Such waveform is called **pulsating d.c.** The waveform obtained as the output of full wave rectifier is an example of pulsating d.c.

3.1.1 Advantages of A.C.

1. The voltages in a.c. system can be raised or lowered with the help of a device called transformer. In d.c. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible.
3. The a.c. transmission at high voltage is economical and efficient.
4. The construction and cost of alternators is low. High a.c. voltages of about 11 kV can be generated and can be raised up to 220 kV for transmission purpose.
5. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.

6. Whenever it is necessary, a.c. supply can be easily converted to obtain d.c. supply.

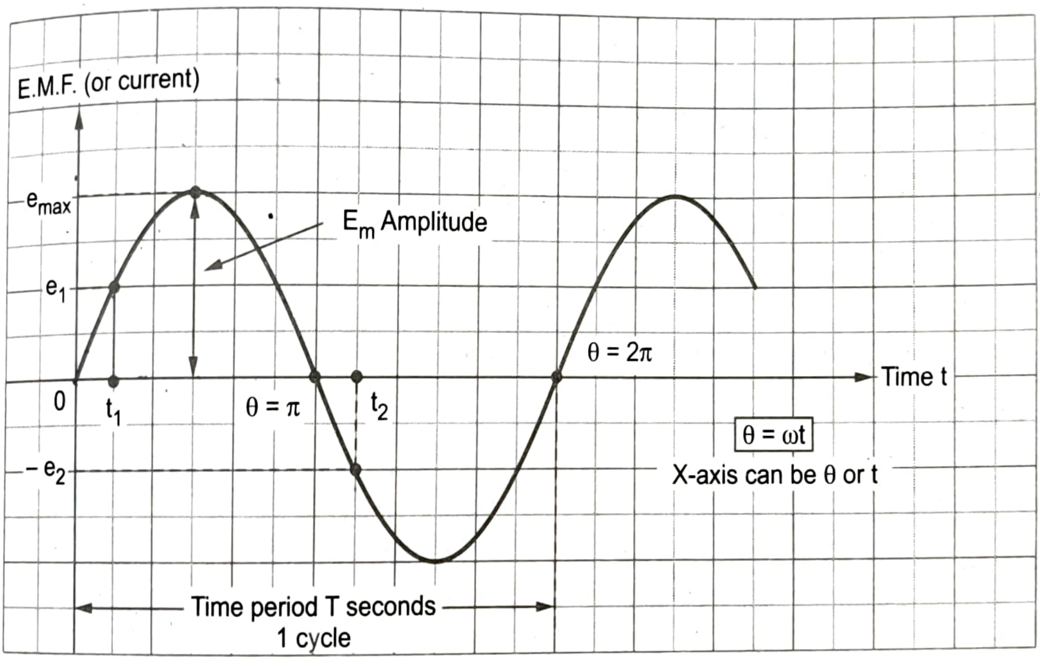
- Due to these advantages, a.c. is used extensively in practice and hence, it is necessary to study a.c. principles.
- The standard waveform used for the a.c. purposes is **purely sinusoidal waveform.**

Review Questions

1. *What is a.c. ? How it differs from d.c. ?*
2. *State the advantages of a.c.*

3.2 Standard Definitions Related to Alternating Quantity

- The Fig. 3.2.1 shows the graphical representation of an alternating quantity.



20ms = T = 2

$f = \frac{1}{20ms} = 50Hz$

Fig. 3.2.1 Representation of an alternating quantity

1. Instantaneous value : The value of an alternating quantity at a particular instant is known as its **instantaneous value.** e.g. e_1 and $-e_2$ are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. 3.2.1.

2. Waveform : The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

3. Cycle : Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle.**

- Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called **periodic waveform.**

- A **cycle** can also be defined as that interval of time during which a complete set of non-repeating events or waveform variations occur (containing positive as well as negative loops).
- One such cycle of the alternating quantity is shown in the Fig. 3.2.1.

One cycle corresponds to 2π radians or 360°.

4. Time period (T) : The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by **T** seconds.

- After every T seconds, the cycle of an alternating quantity repeats. This is shown in the Fig. 3.2.1.

5. Frequency (f) : The number of cycles completed by an alternating quantity per second is known as its **frequency.** It is denoted by **f** and it is measured in **cycles / second** which is known as **Hertz,** denoted as **Hz.**

- As time period T is time for one cycle i.e. seconds/cycle and frequency is cycles/second, we can say that frequency is reciprocal of the time period.

$$\boxed{f = \frac{1}{T} \quad \text{Hz}}$$

- As time period increases, frequency decreases while as time period decreases, frequency increases.
- **In our nation, standard frequency of alternating voltages and currents is 50 Hz.**

6. Amplitude : The maximum value attained by an alternating quantity during positive or negative half cycle is called its **amplitude.** It is denoted as E_m or I_m.

- Thus E_m is called peak value of the voltage while I_m is called peak value of the current.
- The amplitude is also called peak value or maximum value of an alternating quantity.

7. Angular frequency (ω) : It is the frequency expressed in electrical radians per second.

- As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as ($2\pi \times$ cycles/sec.). It is denoted by 'ω' and its unit is radians/second. The relation between frequency 'f' and angular frequency 'ω' is,

$$\boxed{\omega = 2\pi f \quad \text{radians/sec.} \quad \text{or} \quad \omega = \frac{2\pi}{T} \quad \text{radians/sec.}}$$

- The angle θ and the angular frequency ω are related to each other through time as,

$$\boxed{\theta = \omega t \quad \text{radians} \quad \text{or} \quad \theta = 2\pi f t \quad \text{radians}}$$

- Thus the waveform of an alternating quantity can be shown with respect to time 't' or angle 'θ' as 'ω' is constant.

8. Peak to Peak value : The value of an alternating quantity from its positive peak to negative peak is called its peak to peak value. It is denoted as I_{p-p} or V_{p-p}.

$$\boxed{\text{Amplitude} = \frac{\text{Peak to Peak Value}}{2}}$$

Review Question

1. *Sketch the sinusoidal alternating current waveform and define the following terms :*
   i) Instantaneous value ii) Waveform iii) Time period iv) Cycle v) Frequency
   vi) Amplitude or peak value or maximum value.

3.3 Equation of an Alternating Quantity

- As the standard waveform of an alternating quantity is purely sinusoidal, the equation of an alternating voltage can be expressed as,

$$\boxed{e = E_m \sin\theta \text{ volts}}$$

where E_m = Amplitude or maximum or peak value of the voltage.

e = Instantaneous value of an alternating voltage

- Similarly equation of an alternating current can be expressed as,

$$\boxed{i = I_m \sin\theta}$$

where I_m = Amplitude or maximum or peak value of the current.

i = Instantaneous value of an alternating current

The equation can be expressed in various forms as,

$$\text{Now, } \theta = \omega t \text{ radians} \qquad \text{... (3.3.1)}$$

$$\therefore \quad \boxed{e = E_m \sin(\omega t)}$$

$$\text{But, } \omega = 2\pi f \text{ rad/sec.} \qquad \text{... (3.3.2)}$$

$$\therefore \quad \boxed{e = E_m \sin(2\pi f t)}$$

$$\text{But, } f = \frac{1}{T} \text{ seconds}$$

$$\therefore \quad \boxed{e = E_m \sin\left(\frac{2\pi}{T} t\right)} \qquad \text{... (3.3.3)}$$

Important Note : *In all the above equations, the angle θ is expressed in* **radians**. *Hence, while calculating the instantaneous value of the e.m.f., it is necessary to calculate the sine of the angle expressed in radians.*

> *Mode of the calculator should be* **converted to radians, to calculate the sine of the angle** *expressed* **in radians,** *before substituting* **in** *any of* **the above equations.**

Example 3.3.1 *An alternating current of frequency 60 Hz has a maximum value of 12 A : i) Write down the equation for instantaneous values. ii) Find the value of the current after 1/360 second. iii) Time taken to reach 9.6 A for the first time.*

In the above cases assume that time is reckoned as zero when current wave is passing through zero and increasing in the positive direction.

Solution : $f = 60$ Hz, $I_m = 12$ A, $\omega = 2\pi f = 2\pi \times 60 = 377$ rad/sec

i) Equation of instantaneous value is, $i = I_m \sin \omega t = \mathbf{12 \sin 377\, t}$

ii) $t = \frac{1}{360}$ sec i.e. $i = 12 \sin 377 \frac{1}{360} = 12 \sin 1.0472 = \mathbf{10.3924\ A}$

Note : sin of 1.0472 must be calculated in **radian** mode.

iii) $i = 9.6$ A i.e. $9.6 = 12 \sin 377\, t$ i.e. $\sin 377\, t = 0.8$

$\therefore$ $377\, t = 0.9272$ i.e. $t = \mathbf{2.459 \times 10^{-3}\ sec}$ ... $\sin^{-1}$ in **radian** mode

Example 3.3.2 *A sinusoidal voltage of 50 Hz has a maximum value of $200\sqrt{2}$ volts. At what time measured from a positive maximum value will the instantaneous voltage be equal to 141.4 volts ?*

Solution : $f = 50$ Hz, $V_m = 200\sqrt{2}$ V, $v_1 = 141.4$ V

The equation of the voltage is, $v = V_m \sin(2\pi ft) = 200\sqrt{2}\sin(2\pi \times 50\, t)$ V

For $v = v_1 14\ 1.4 = \ = 200\sqrt{2}\sin(2\pi \times 50 \times t_1)$

$\therefore$ $t_1 = 1.666 \times 10^{-3}$ sec ... Use **radian mode** for sin

But this time is measured from $t = 0$. At positive maximum, time is $\frac{T}{4} = \frac{1}{4f} = 5 \times 10^{-3}$ sec so $t = t_1 = 1.666 \times 10^{-3}$ sec is before positive maximum.

From Fig. 3.3.1.

$t_m - t_1 = 5 \times 10^{-3} - 1.666 \times 10^{-3}$

$= \mathbf{3.314 \times 10^{-3}\ sec}$

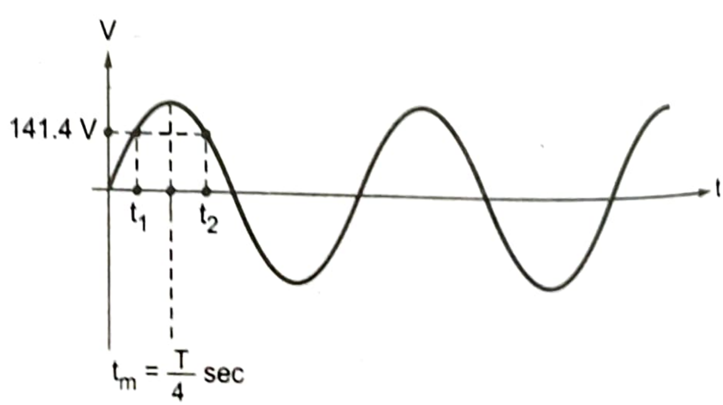


Fig. 3.3.1

As the waveform is symmetrical, at the time of 3.314×10^{-3} sec measured after positive maximum value, the instantaneous voltage will be again 141.4 V.

Example 3.3.2 *A sinusoidal wave of frequency 50 Hz has its maximum value of 9.2 Amps. What will be its value at (a) 0.002 sec after the wave pass through zero in positive direction, (b) 0.0045 sec after the wave passes through positive maximum. Show the values of current in a neat sketch of the waveform.*

Solution : The waveform is shown in the Fig. 3.3.2.

Now $I_m = 9.2$ A and

$f = 50$ Hz

$\therefore\ i = I_m \sin 2\pi ft$

$= 9.2 \sin 100\pi t$ A

a) At $t = 0.002$ sec,

$i = 9.2 \sin (100\pi \times 0.002)$

$= \mathbf{5.4076\ A}$

... Use sin in radians

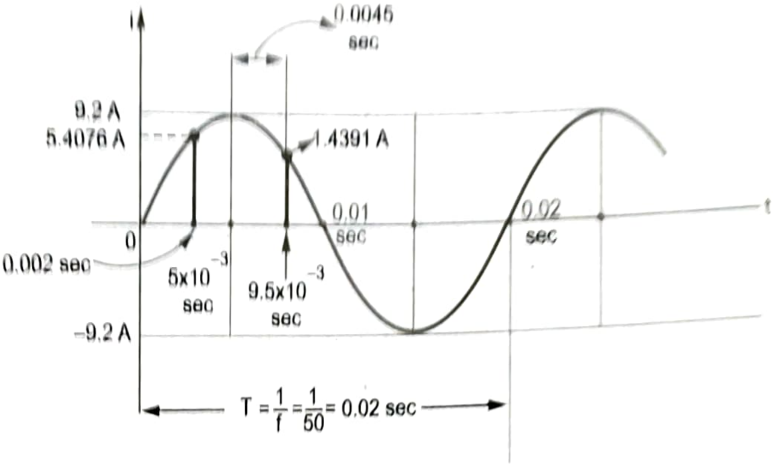


Fig. 3.3.2

b) At $t = 0.0045$ sec

after positive maximum as shown, time period $T = \frac{1}{f} = 0.02$ sec so positive maximum occurs at $t = \frac{T}{4} = \frac{0.02}{4} = 5 \times 10^{-3}$ sec. After this 0.0045 sec means value of i at $t = 5 \times 10^{-3} + 0.0045 = 9.5 \times 10^{-3}$ sec from t = 0.

$$\therefore \quad i = 9.2 \sin \left(100\pi \times 9.5 \times 10^{-3}\right) = \mathbf{1.4391\ A}$$

Review Questions

1. *State the equation of an alternating quantity. State its various forms.*
2. *An alternating current varying sinusoidally with a frequency of 50 Hz has a peak value of current as $20\sqrt{2}$ Amp. At what time, measured from negative maximum value, instantaneous current will be $10\sqrt{2}$ Amp. ?* **[Ans. : 6.666×10^{-3} sec.]**

3.4 R.M.S. and Average Values of a Sinusoidal Quantity

June-09, May-04, 07, 08, 12, Dec.-03, 04, Aug.-08, Nov.-03, 05, March-06

- As alternating sinusoidal quantity varies with time in magnitude and direction so for its paractical analysis its r.m.s. (root mean square) and average values are considered. The r.m.s. value is also called an effective value of a sinusoidal quantity.

3.4.1 Effective Value or R.M.S. Value

- An alternating current varies from instant to instant, while the direct current is constant, with respect to time.
- For the comparison of the two, a common effect to both the type of currents can be considered. Such an effect is heat produced by the two currents flowing through the resistance. The heating effect can be used to compare the alternating and direct current. From this, r.m.s. value of an alternating current can be defined as,

*The **effective or r.m.s. value** of an alternating current is given by that steady current (D.C.) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.*

3.4.1.1 Analytical Method of Obtaining R.M.S. Value

Steps to find r.m.s. value of an a.c. quantity :

1. Write the equation of an a.c. quantity. Observe its behaviour during various time intervals.
2. Find square of the a.c. quantity from its equation.
3. Find average value of square of an alternating quantity as,

$$\text{Average} = \frac{\text{Area of curve over one cycle of squared waveform}}{\text{Length of the cycle}}$$

4. Find square root of average value which gives r.m.s. value.

- Consider sinusoidally varying alternating current and square of this current as shown in the Fig. 3.4.1.

Step 1 : The current $i = I_m \sin \theta$

Step 2 : Square of current $i^2 = I_m^2 \sin^2 \theta$

- The area of curve over half a cycle can be calculated by considering an interval $d\theta$ as shown.

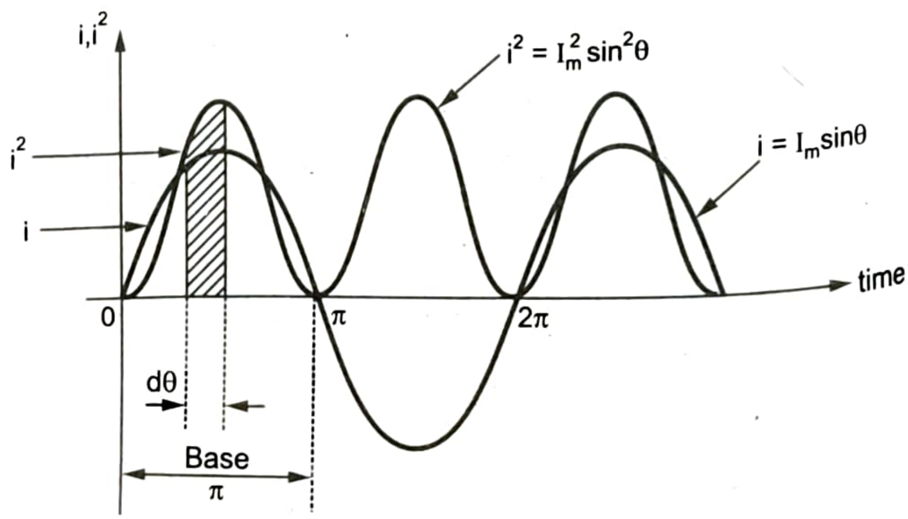


Fig. 3.4.1 Waveform of current and square of the current

- Area of square curve over half cycle = $\int_0^{\pi} i^2 \, d\theta$ and length of the base is π.

Step 3 : $\therefore$ Average value of square of the current over half cycle is,

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i^2 \, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta = \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{I_m^2}{2\pi}[\pi] = \frac{I_m^2}{2}$$

Step 4 : Root mean square value i.e. r.m.s. value can be calculated as,

$$I_{r.m.s.} = \sqrt{\text{Mean or average of square of current}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad \boxed{I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m}$$

- **The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.**

*The **instantaneous** values are denoted by **small** letters like i, e etc. while **r.m.s.** values are represented by **capital** letters like I, E etc.*

The above result is also applicable to **sinusoidal** alternating voltages.

$$\therefore \quad \boxed{V_{r.m.s.} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m}$$

The r.m.s. values are used for specifying alternating quantities. The given values such as 230 V, 110 V are r.m.s. values of alternating quantities unless and otherwise specified to be other than r.m.s.

In practice, everywhere, r.m.s. values are used to analyze alternating quantities.

3.4.2 Average Value

The **average value** of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

For a symmetrical a.c., the average value over a complete cycle is zero *as both positive and negative half cycles are exactly identical. Hence, the average value is defined for half cycle only.*

Average value can also be expressed by that steady current which transfers across any circuit, the same amount of charge as is transferred by that alternating current during the same time.

3.4.2.1 Analytical Method of Obtaining Average Value

Consider sinusoidally varying current, $I = I_m \sin\theta$

Consider the elementary interval of instant 'dθ' as shown in the Fig. 3.4.2 The average instantaneous value of current in this interval is say, 'i' as shown.

The average value can be obtained by taking ratio of area under curve over **half cycle** to length of the base for half cycle.

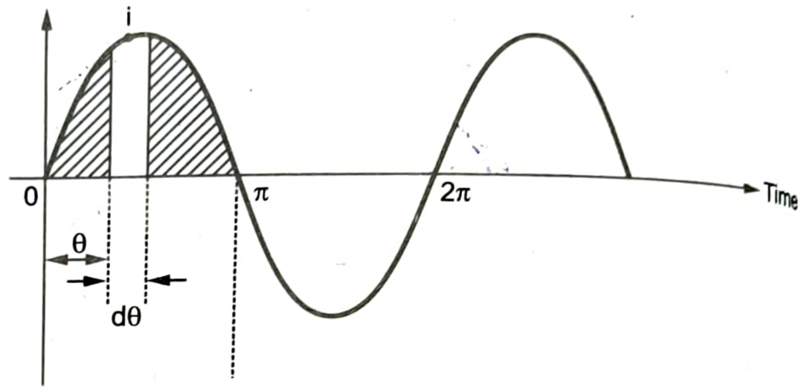


Fig. 3.4.2 Average value of an alternating current

$$\therefore \quad \boxed{I_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}}$$

$$I_{av} = \frac{\int_0^{\pi} i\, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i\ d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta = \frac{I_m}{\pi}\int_0^{\pi}\sin\theta = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}[-\cos\pi + \cos 0] = \frac{I_m}{\pi}[2] = \frac{2 I_m}{\pi} = 0.637\ I_m$$

For a purely sinusoidal waveform, the average value is expressed in terms of its maximum value as,

$$\therefore \quad \boxed{I_{av} = 0.637\ I_m \quad \text{and} \quad V_{av} = 0.637\ V_m}$$

3.4.3 Form Factor (K_f)

The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value,

Form factor, $\boxed{K_f = \dfrac{\text{r.m.s. value}}{\text{Average value}}}$

The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\ I_m}{0.637\ I_m} = \mathbf{1.11} \quad \text{for sinusoidally varying quantity}$$

3.4.4 Crest or Peak Factor (K_p)

The peak factor of an alternating quantity is defined as ratio of maximum value to the r.m.s. value.

Peak factor $$K_p = \frac{\text{Maximum value}}{\text{r.m.s. value}}$$

The peak factor for sinusoidally varying alternating currents and voltages can be obtained as,

=1.414

$$K_p = \frac{I_m}{0.707\ I_m} = \mathbf{1.414} \quad \text{for sinusoidal waveform}$$

Example 3.4.1 *The equation of an alternating current is given by i = 42.42 sin 628t. Calculate its i) Maximum value ii) Frequency iii) RMS value iv) Average value v) Form factor*

Solution : Compare given equation with $i = I_m \sin(\omega t)$

i) $I_m = \mathbf{42.42\ A}$ ii) $f = \frac{\omega}{2\pi} = \frac{628}{2\pi} = \mathbf{100\ Hz}$ iii) $I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = \mathbf{30\ A}$

iv) $I_{av} = 0.637\ I_m = \mathbf{27.0215\ A}$ v) $K_f = \frac{\text{r.m.s.}}{\text{Average}} = \frac{30}{27.0215} = \mathbf{1.11}$

Example 3.4.2 *Find average value and RMS value of the resultant current in a wire which carries simultaneously a direct current of 10 A and sinusoidal alternating current with a peak of 10 A.* **June-09, Set-1, Marks 10**

Solution : i) The resultant is shown in the Fig. 3.4.3

ii) For d.c., $I_{dc} = 10$ A

For a.c. $i = I_m \sin\theta = 10 \sin\theta$

So the resultant is,

$$i_R = I_{dc} + i = \mathbf{10 + 10 \sin\theta}$$

This is the expression for the resultant wave.

iii) Now $i_R = 10 + 10 \sin\theta$

The average value can be obtained as,

$$i_R(\text{average}) = \frac{1}{2\pi}\int_0^{2\pi} i_R \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}[10+10\sin\theta]d\theta$$

$$= \frac{1}{2\pi}[10\theta-10\cos\theta]_0^{2\pi}$$

$$= \frac{1}{2\pi}[10(2\pi-0)-10(\cos 2\pi-\cos 0)]$$

$$= \mathbf{10\ A}$$

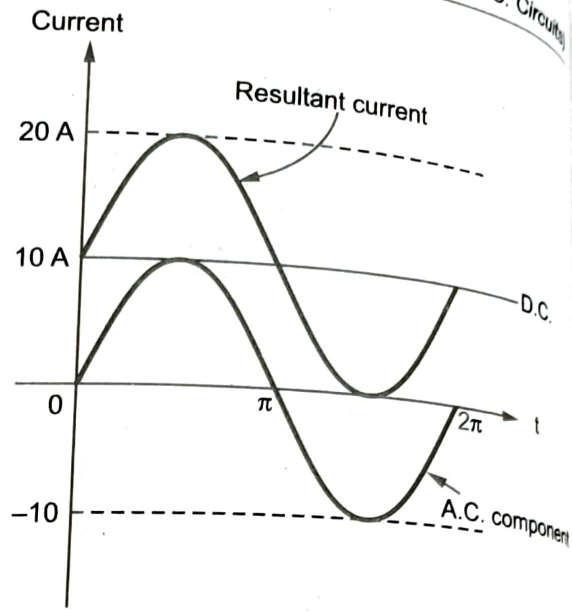


Fig. 3.4.3

iv) The r.m.s. value is given by,

$$i_R\ (\text{r.m.s.}) = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_R^2 \, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(10+10\sin\theta)^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}[100+200\sin\theta+100\sin^2\theta]\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[100+200\sin\theta+100\left(\frac{1-\cos 2\theta}{2}\right)\right]d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[100\,\theta-200\cos\theta+100\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)\right]_0^{2\pi}}$$

$$= \sqrt{\frac{1}{2\pi}\left[100(2\pi-0)-200(\cos 2\pi-\cos 0)+\frac{100}{2}(2\pi-0)-\frac{100}{4}(\sin 4\pi-\sin 0)\right]}$$

$$= \sqrt{\frac{1}{2\pi}[300\times\pi]} = \sqrt{150} = \mathbf{12.2474\ A}$$

v) Form factor $= \dfrac{\text{R.M.S.}}{\text{Average}} = \dfrac{12.2474}{10} = \mathbf{1.2247}$

Peak factor $= \dfrac{\text{Maximum}}{\text{R.M.S.}} = \dfrac{20}{12.2474} = \mathbf{1.633}$

Example 3.4.3 Find the form factor for the following waveform. **May-04, Set-1, Marks 8**

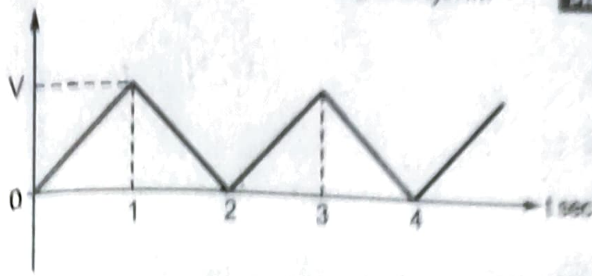


Fig. 3.4.4

Solution : The time period of the cycle is T = 2 sec.

For t = 0 to 1, Slope $= \frac{V-0}{1-0} = V$ i.e. $v(t) = Vt$

For t = 1 to 2, the equation of v(t) is $v(t) = mt + C$

The two points are (1, V) and (2, 0) hence slope $m = \frac{0-V}{2-1} = -V$

$$\therefore \quad v(t) = -Vt + C$$

Putting 2^{nd} point, $0 = -2V + C$ i.e. $C = 2V$

$$\therefore \quad v(t) = -Vt + 2V = V[2-t]$$

$$\text{Average value} = \frac{\text{Area under curve}}{\text{Base}} = \frac{\int_0^2 v(t)\,dt}{2} = \frac{1}{2}\left\{\int_0^1 Vt\,dt + \int_1^2 V(2-t)\,dt\right\}$$

$$= \frac{V}{2}\left\{\left(\frac{t^2}{2}\right)_0^1 + \left(2t - \frac{t^2}{2}\right)_1^2\right\} = \frac{V}{2}\left\{\frac{1}{2} + 2\times 2 - \frac{2^2}{2} - 2 + \frac{1^2}{2}\right\}$$

$$= \frac{V}{2}\left\{\frac{1}{2} + 4 - 2 + \frac{1}{2} - 2\right\} = \frac{V}{2} = \mathbf{0.5\ V}$$

$$\text{R.M.S. value} = \sqrt{\frac{\text{Area of curve over a squared waveform}}{\text{Length of base over a cycle}}}$$

$$= \sqrt{\frac{\int_0^2 v^2(t)\,dt}{2}} = \frac{1}{\sqrt{2}}\sqrt{\int_0^1 v^2(t)\,dt + \int_1^2 v^2(t)\,dt}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\int_0^1 V^2 t^2\,dt + \int_1^2 V^2(2-t)^2\,dt} = \frac{V}{\sqrt{2}}\sqrt{\int_0^1 t^2\,dt + \int_1^2 \left(4 - 4t + t^2\right) dt}$$

$$= \frac{V}{\sqrt{2}}\sqrt{\left(\frac{t^3}{3}\right)_0^1 + \left[4t - \frac{4t^2}{2} + \frac{t^3}{3}\right]_1^2} = \frac{V}{\sqrt{2}}\sqrt{\frac{1}{3} + 8 - 8 + \frac{8}{3} - 4 + 2 - \frac{1}{3}}$$

$$= \frac{0.8164\ V}{\sqrt{2}} = \mathbf{0.5773\ V}$$

$$\therefore \text{ Form factor } = \frac{\text{r.m.s}}{\text{average}} = \frac{0.5773}{0.5V} = \mathbf{1.1546}$$

Example 3.4.4 *Find R.M.S. and average value of the following waveform.*

Dec.-03, Set-1, May-04, Set-2, Marks 10

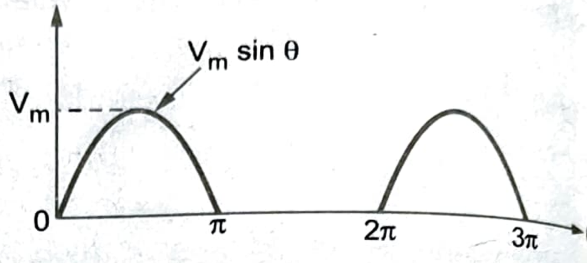


Fig. 3.4.5

Solution : The average value is,

$$V_{av} = \frac{\text{Area of curve over a cycle}}{\text{Length of base over a cycle}}$$

... Cycle is from 0 to 2π

$$= \frac{\int_0^{2\pi} v(\theta)\, d\theta}{2\pi} \quad \left.\begin{matrix} \ldots\ v(\theta) = V_m \sin\theta, \text{ for } 0 < \theta < \pi \\ \ldots\ v(\theta) = 0, \quad \text{for } \pi < \theta < 2\pi \end{matrix}\right\}$$

$$\therefore \quad V_{av} = \frac{\int_0^{\pi} V_m \sin\theta\, d\theta + \int_0^{2\pi} 0\, d\theta}{2\pi} = \frac{-V_m[\cos\theta]_0^{\pi}}{2\pi} = \frac{-V_m[\cos\pi - \cos 0]}{2\pi}$$

$$= \frac{-V_m[-1-1]}{2\pi} = \frac{V_m}{\pi} = \mathbf{0.318\ V_m}$$

$$V_{r.m.s.} = \sqrt{\frac{\text{Area under the squared wave cycle}}{\text{Length of base over a cycle}}}$$

$$\therefore \quad v^2(d\theta) = V_m^2 \sin^2\theta\, d\theta, \quad \text{for } 0 < \theta < \pi$$

$$= 0, \qquad \text{for } \pi < \theta < 2\pi$$

$$\therefore \quad V_{r.m.s.} = \sqrt{\frac{\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta + \int_0^{2\pi} 0\, d\theta}{2\pi}} = \sqrt{\frac{V_m^2 \int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}{2\pi}}$$

$$= \frac{V_m}{\sqrt{2\pi}}\sqrt{\frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}} = \frac{V_m}{\sqrt{2\pi}}\sqrt{\frac{1}{2}\left[\pi - 0 - \frac{\sin 2\pi}{2} + \frac{\sin 0}{2}\right]}$$

$$= \frac{V_m}{\sqrt{2\pi}} \times \sqrt{\frac{\pi}{2}} = \frac{V_m}{2} = \mathbf{0.5\ V_m}$$

Example 3.4.5 *The current of the following wave form is passed through 5 ohms resistance. Find the power consumed.*

Dec.-04, Set-4, Marks 8

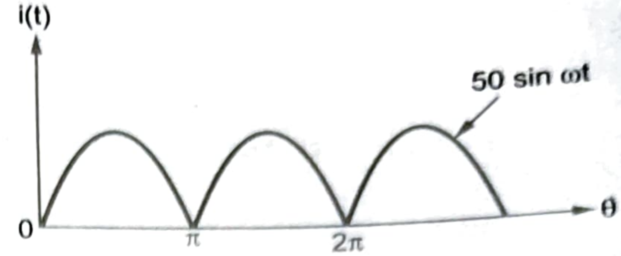


Fig. 3.4.6

Solution : Let us find the r.m.s. value of the current,

$$i(t) = 50 \sin \omega t \quad \text{i.e} \quad i^2(t) = 2500 \sin^2 \omega t$$

$$I_{r.m.s.} = \sqrt{\frac{\text{Area of curve over a squared wave cycle}}{\text{Length of base over a cycle}}}$$

$$= \sqrt{\frac{\int_0^{\pi} i^2 d\theta}{\pi}} = \sqrt{\frac{\int_0^{\pi} 2500 \sin^2 \theta \, d\theta}{\pi}} \quad \ldots \theta = \omega t$$

$$= \frac{50}{\sqrt{\pi}} \sqrt{\int_0^{\pi} \sin^2\theta \, d\theta} = \frac{50}{\sqrt{\pi}} \sqrt{\int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \frac{50}{\sqrt{2\pi}} \sqrt{\int_0^{\pi} (1-\cos 2\theta) \, d\theta} = \frac{50}{\sqrt{2\pi}} \sqrt{\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \frac{50\sqrt{\pi}}{\sqrt{2\pi}} = \frac{50}{\sqrt{2}} = 35.3553 \text{ A}$$

$$\therefore \quad P = I_{rms}^2 \times R = (35.3553)^2 \times 5 = \mathbf{6250 \text{ W}}$$

Example 3.4.6 *Find the form factor of the given waveform.*

Aug.-08, Set-4, May-04, Set-3, May-08, Set-4, Nov.-05, Set-3, March-06, Set-1, Marks 15

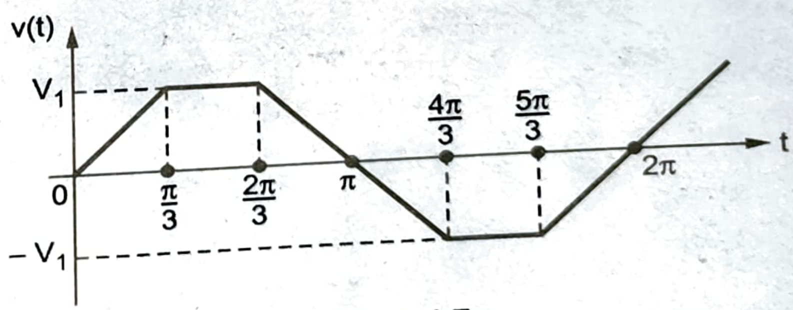


Fig. 3.4.7

Solution : To find r.m.s. value

$$V_{rms} = \sqrt{\frac{\int_0^{\pi} v^2(t) \, dt}{\pi}}$$

$$= \frac{1}{\sqrt{\pi}} \left[\int_0^{\pi/3} v^2(t) \, dt + \int_{\pi/3}^{2\pi/3} v^2(t) \, dt + \int_{2\pi/3}^{\pi} v^2(t) \, dt\right]^{1/2}$$

For $0 < t < \frac{\pi}{3}$, slope $= \dfrac{V_1 - 0}{\frac{\pi}{3} - 0} = \dfrac{3V_1}{\pi}$ i.e. $v(t) = \dfrac{3V_1}{\pi} t$

$$\therefore \int_0^{\pi/3} v^2(t)\,dt = \int_0^{\pi/3} \frac{9V_1^2}{\pi^2} t^2 dt = \frac{9V_1^2}{\pi^2}\left[\frac{t^3}{3}\right]_0^{\pi/3} = \frac{9V_1^2}{\pi^2} \times \frac{\pi^2}{27 \times 3} = \frac{V_1^2 \pi}{9}$$

For $\frac{\pi}{3} < t < \frac{2\pi}{3}$, $v(t) = V_1$

$$\therefore \int_{\pi/3}^{2\pi/3} V_1^2\,dt = V_1^2 [t]_{\pi/3}^{2\pi/3} = V_1^2\left[\frac{2\pi}{3} - \frac{\pi}{3}\right] = \frac{V_1^2 \pi}{3}$$

For $\frac{2\pi}{3} < t < \pi$, slope $= \dfrac{0 - V_1}{\pi - \frac{2\pi}{3}} = \dfrac{-3V_1}{\pi}$

$$\therefore \quad v(t) = \frac{-3V_1 t}{\pi} + C$$ ... as does not pass through origin

At point $(\pi, 0)$, $0 = \dfrac{-3V_1}{\pi} \times \pi + C$ i.e. $C = 3V_1$

$$\therefore \quad v(t) = \frac{-3V_1 t}{\pi} + 3V_1$$

$$\therefore \int_{2\pi/3}^{\pi} v^2(t)\,dt = \int_{2\pi/3}^{\pi}\left(\frac{-3V_1 t}{\pi} + 3V_1\right)^2 dt = \int_{2\pi/3}^{\pi}\left[\frac{9V_1^2}{\pi^2}t^2 - \frac{18V_1^2 t}{\pi} + 9V_1^2\right] dt$$

$$= 9V_1^2\left[\frac{t^3}{3\pi^2} - \frac{2}{\pi}\frac{t^2}{2} + t\right]_{2\pi/3}^{\pi} = 9V_1^2\left[\frac{\pi}{3} - \pi + \pi - \frac{8\pi}{81} + \frac{4\pi}{9} - \frac{2\pi}{3}\right] = \frac{V_1^2 \pi}{9}$$

$$\therefore \quad V_{r.m.s.} = \frac{1}{\sqrt{\pi}} \times \sqrt{\frac{V_1^2 \pi}{9} + \frac{V_1^2 \pi}{3} + \frac{V_1^2 \pi}{9}} = \frac{\sqrt{5}\,V_1}{3} = \mathbf{0.7453\ V_1}$$

The average value is also to be obtained over half cycle.

$$\therefore \quad V_{av} = \frac{\int_0^{\pi} v(t)\,dt}{\pi} = \frac{1}{\pi}\left[\int_0^{\pi/3} v(t)dt + \int_{\pi/3}^{2\pi/3} v(t)dt + \int_{2\pi/3}^{\pi} v(t)\,dt\right]$$

$$\int_0^{\pi/3} v(t)\,dt = \int_0^{\pi/3} \frac{3V_1 t}{\pi} dt = \frac{3V_1}{\pi}\left[\frac{t^2}{2}\right]_0^{\pi/3} = \frac{V_1 \pi}{6}$$

$$\int_{\pi/3}^{2\pi/3} v(t)\,dt = \int_{\pi/3}^{2\pi/3} V_1 dt = V_1[t]_{\pi/3}^{2\pi/3} = \frac{V_1\pi}{3}$$

$$\int_{2\pi/3}^{\pi} v(t)\,dt = \int_{2\pi/3}^{\pi}\left[\frac{-3V_1 t}{\pi}+3V_1\right]dt = 3V_1\left[\frac{-t^2}{2\pi}+t\right]_{2\pi/3}^{\pi}$$

$$= 3V_1\left[\frac{-\pi}{2}+\pi+\frac{4\pi}{18}-\frac{2\pi}{3}\right] = \frac{V_1\pi}{6}$$

$$\therefore \quad V_{av} = \frac{1}{\pi}\left[\frac{V_1\pi}{6}+\frac{V_1\pi}{3}+\frac{V_1\pi}{6}\right] = \frac{2V_1}{3} = \mathbf{0.666\ V_1}$$

$$K_f = \text{Form factor} = \frac{\text{R.M.S.}}{\text{Average}} = \frac{0.7453\ V_1}{0.6666\ V_1} = \mathbf{1.1179}$$

Example 3.4.7 *A current wave is represented by the equation i = 20 sin 251t. Calculate the maximum and R.M.S. value of current and its frequency.*

June-09, Set-1, Marks 6

Solution : $i = 20 \sin 251t$ A

Compare with, $i = I_m \sin \omega t$

$\therefore \quad I_m = 20$ A ... Maximum value

For sinusoidal current, $I_{RMS} = \dfrac{I_m}{\sqrt{2}} = \dfrac{20}{\sqrt{2}} = \mathbf{14.1421\ A}$

$\omega = 251$ i.e. $2\pi f = 251$

$\therefore \quad f = \mathbf{39.948\ Hz}$ ... Frequency

Review Questi...

3.7 Concept of Phase and Phase Difference

May-08, June-09

- In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant.
- It is represented in terms of angle θ in radians or degrees, measured from certain reference.

Phase : The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) traveled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

- Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. 3.7.1 at the instant A is $\phi = 0°$.

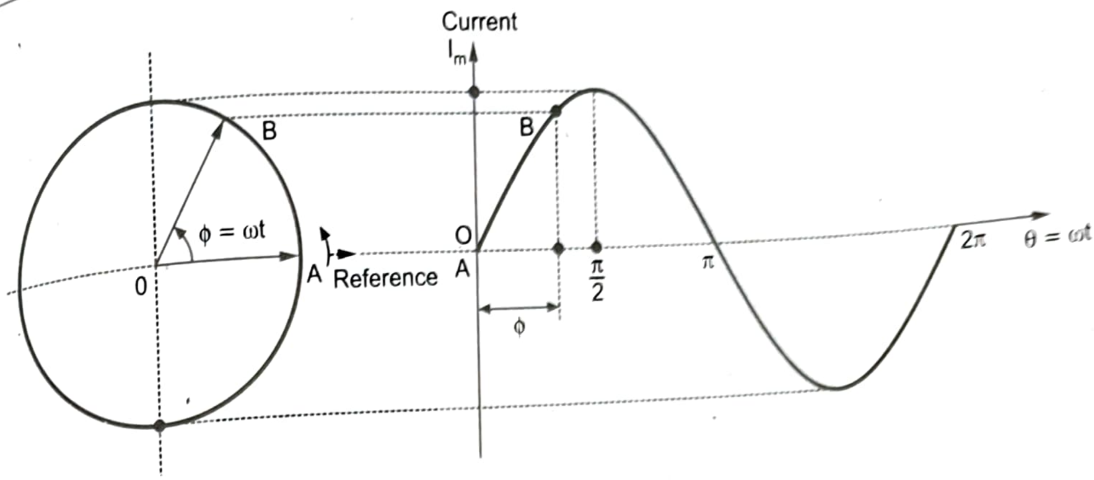


Fig. 3.7.1 Concept of phase

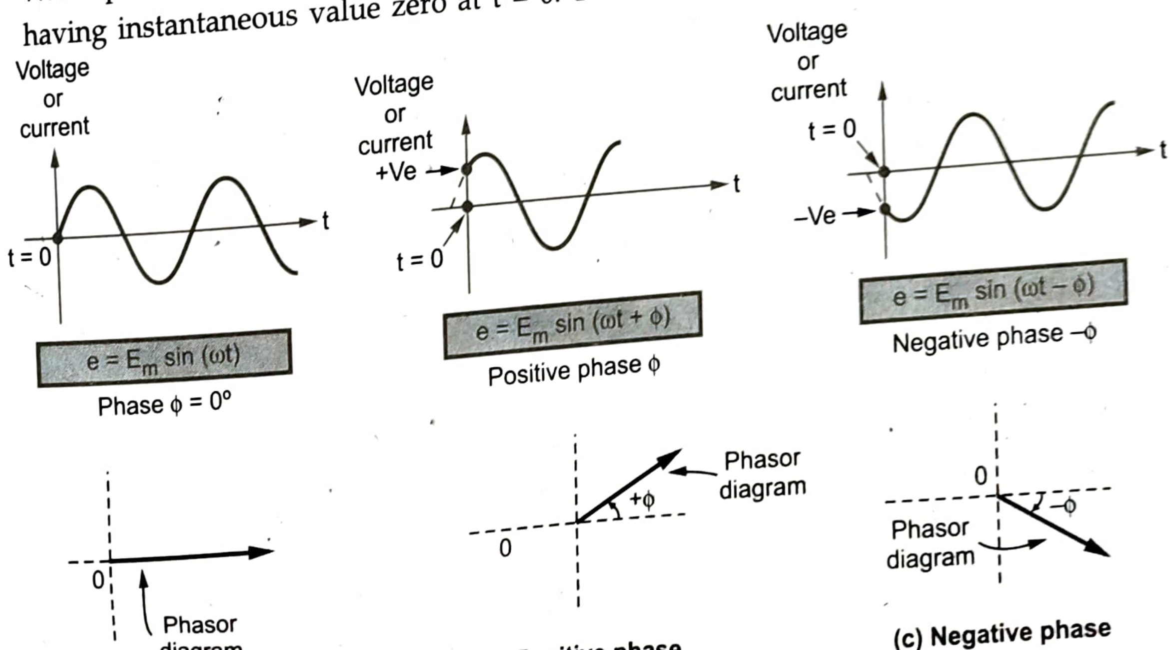
- While the phase of the current at the instant B is the angle ϕ through which the phasor has traveled, measured from the reference axis i.e. X-axis.
- In general, the phase ϕ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0^\circ$ to 360°.
- In terms of phase, the equation of an alternating quantity can be modified as,

$$e = E_m \sin(\omega t \pm \phi) \qquad \text{where } \phi = \text{Phase of the alternating quantity}$$

- Let us consider three cases;

Case 1 : $\phi = 0^\circ$:

- When phase of an alternating quantity is zero, it is standard pure sinusoidal quantity having instantaneous value zero at t = 0. This is shown in the Fig. 3.7.2 (a).



(a) Zero phase (b) Positive phase (c) Negative phase

Fig. 3.7.2 Concept of phase

Case 2 : Positive phase ϕ :

- When phase of an alternating quantity is positive it means that quantity has some positive instantaneous value at $t = 0$. This is shown in the Fig. 3.7.2 (b).

Case 3 : Negative phase ϕ :

When phase of an alternating quantity is negative it means that quantity has some negative instantaneous value at $t = 0$. This is shown in the Fig. 3.7.2 (c).

1. The **phase** is measured with respect to **reference direction** i.e. positive X-axis direction.
2. The **phase** measured in **anticlockwise** direction is **positive** while the phase measured in **clockwise direction** is **negative**.

- **The difference between the phases of the two alternating quantities is called the phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities.

1. **Zero Phase Difference :** Consider the two alternating quantities having same frequency f Hz having different maximum values.

$$e = E_m \sin(\omega t) \quad \text{and} \quad i = I_m \sin(\omega t) \qquad \text{where} \quad E_m > I_m$$

- The phasor representation and waveforms of both the quantities are shown in the Fig. 3.7.3.

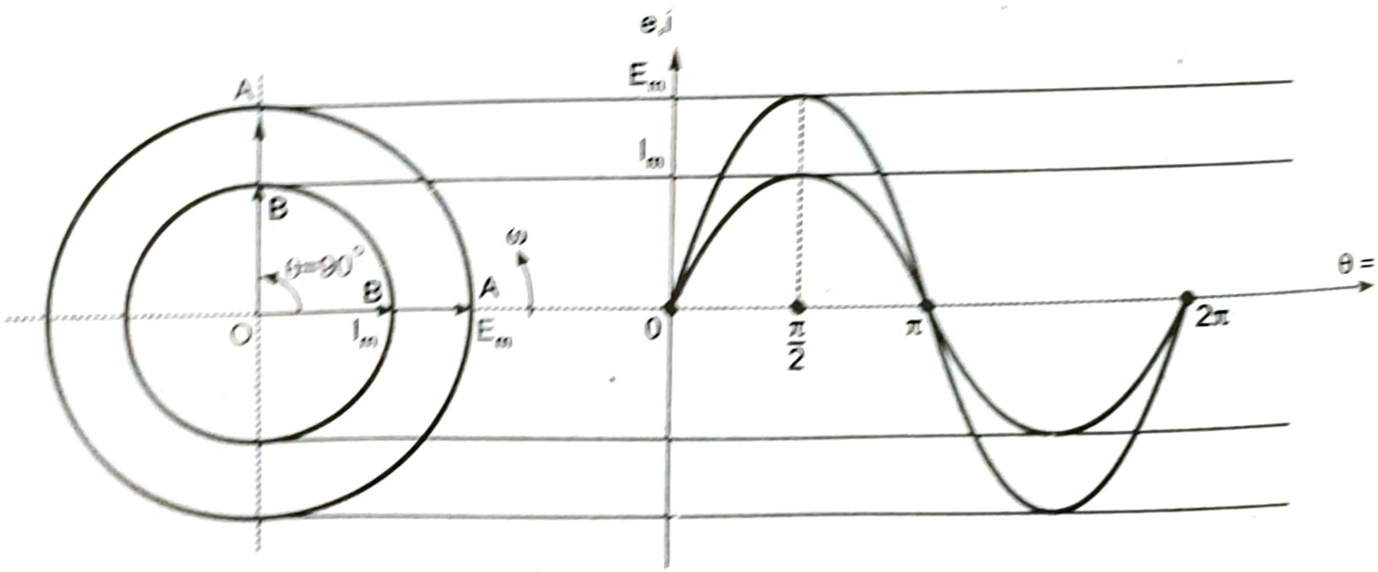


Fig. 3.7.3 In phase alternating quantities

- So, at any instant, we can say that the phase of voltage e will be same as phase of i. The difference between the phases of the two quantities is zero at any instant.

- *When such phase difference between the two alternating quantities is **zero**, the two quantities are said to be **in phase**.*

- In the a.c. analysis, it is not necessary that all the alternating quantities must be always in phase. It is possible that if one is achieving its zero value and at the same instant the other is having some negative value or positive value then such two quantities are said to have **phase difference** between them.

2. Lagging Phase Difference : Consider an e.m.f. having maximum value E_m and current having maximum value I_m.

- Now, when e.m.f. 'e' is at its zero value, the current 'i' has some negative value as shown in the Fig. 3.7.4.

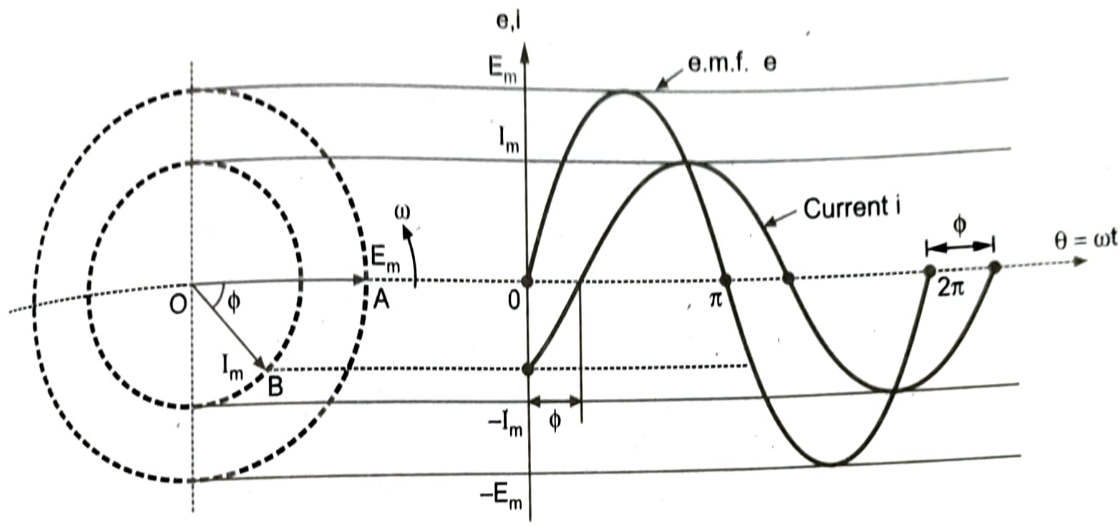


Fig. 3.7.4 Concept of phase difference (Lag)

- Thus, there exists a phase difference ϕ between the two phasors.
- Now, as the two are rotating in anticlockwise direction, we can say that current is falling back with respect to voltage, at all the instants by angle ϕ. This is called **lagging phase difference.** The current i is said to lag the voltage e by angle ϕ.
- The current i achieves its maximum and zero values, ϕ angle later than the corresponding maximum and zero values of voltage.
- The equations of the two quantities are written as,

$e = E_m \sin \omega t$ and $i = I_m \sin (\omega t - \phi)$ and 'i' is said to lag 'e' by angle ϕ.

3. Leading Phase Difference :

- It is possible in practice that the current 'i' may have some positive value when voltage 'e' is zero. This is shown in the Fig. 3.7.5
- It can be seen that there exists a phase difference of ϕ angle between the two. But in this case, current 'i' is ahead of voltage 'e', as both are rotating in anticlockwise direction with same speed.

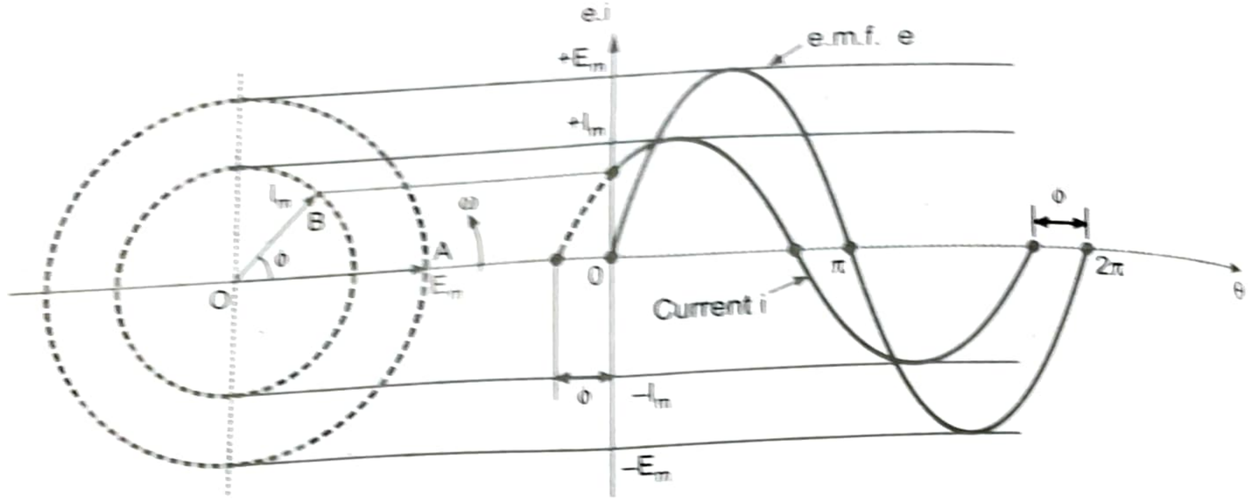


Fig. 3.7.5 Concept of phase difference (Lead)

- Thus, current is said to be leading with respect to voltage and the phase difference is called **leading phase difference.**
- At all instants, current i is going to remain ahead of voltage 'e' by angle 'ϕ'.
- The equations of such two quantities are written as

$e = E_m \sin \omega t$ and $i = I_m \sin (\omega t + \phi)$ and 'i' is said to lead 'e' by angle ϕ

3.7.1 Phasor Diagram

- The diagram in which different alternating quantities of the same frequency sinusoidal in nature are represented by individual phasors indicating exact phase interrelationships is known as **phasor diagram.**
- All phasors have a particular fixed position with respect to each other .
- *Phasor diagram can be considered as a still picture of these phasors at a particular instant.*

Example 3.7.1 *Two sinusoidal currents are given by, $i_1 = 10 \sin (\omega t + \pi/3)$ and $i_2 = 15 \sin (\omega t - \pi/4)$. Calculate the phase difference between them in degrees.*

Solution : The phase of current i_1 is $\pi/3$ radians i.e. 60° while the phase of the current i_2 is $-\pi/4$ radians i.e. $-45°$. This is shown in the Fig. 3.7.6.

Hence the phase difference between the two is,

$$\phi = \theta_1 - \theta_2 = 60° - (-45°) = 105°$$

And i_2 lags i_1.

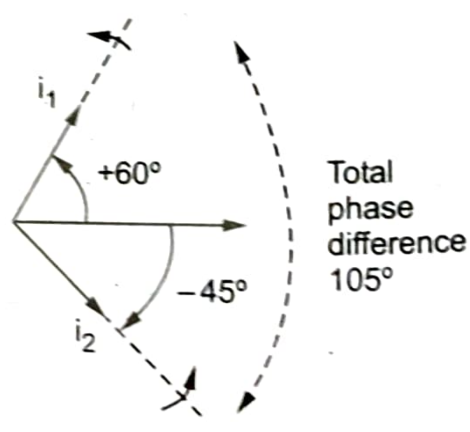


Fig. 3.7.6