UNIT-IV

ALGEBRA RELATIONS

Cartisean product :- Let A and B be two sets then the set of all ordered pairs of (x, y) where $x \in A$ and $y \in B$ is called the cartisean product (or) cross product (or) product of A and B and it is denoted by $A \times B$.

$$A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$$

$$B \times A = \{(y, x) \mid y \in B \text{ and } x \in A\}$$

$$A \times B \neq B \times A$$

Eg:- let $A = \{-1, 0, 1\}$ $B = \{2, 4\}$

$$A \times B = \{(-1,2)\ (-1,4)\ (0,2)\ (0,4)\ (1,2)\ (1,4)\}$$

$$B \times A = \{(2,-1)\ (2,0)\ (2,1)\ (4,-1)\ (4,0),\ (4,1)\}$$

$$\therefore A \times B \neq B \times A$$

NOTE :- If A and B are finite sets with $|A| = m$ and $|B| = n$ elements then $A \times B$ is a finite set with mn elements.

→ the idea of Cartesian produtt of sets can be extended to any finite no. of sets. For any non empty sets $A_1, A_2 - - - A_k$ then $A_1 \times A_2 \times A_3 \times - - - \times A_k$ is defined as the set of ordered k tuples

Eg: $A = \{1, 0\}$ $B = \{2, -2\}$ $C = \{0, -1\}$ then $A \times B \times C$

$$A \times B \times C = \{(1,2,0)\ (1,+2,-1)\ (0,2,0)\ (0,2,-1)$$
$$(1,-2,0)\ (1,-2,-1)\ (0,-2,0)\ (0,-2,-1)\}$$

Relations : Let A and B be two sets. A relation from A to B is a subset of the cartesian product $A \times B$ where 'R' is set of ordered pair (x, y) such that $x \in A$ & $y \in B$ for every x (or) y (xRy).

$$\therefore R = \{(x, y) \mid x \in A \text{ \& } y \in B,\ xRy\}$$

Eg: Let $A = \{1, 2, 3, 4\}$

$$R = \{(x, y) \mid x \le y,\ \forall\, x, y \in A\}$$

$$R = \{(1,1)\,(1,2)\,(1,3)\,(1,4)\,(2,2)\,(2,3)\,(2,4)\,(3,3)\,(3,4)\,(4,4)\}$$

Domain and Range of relations :-

The set $\{x \mid x \in A, (x,y) \in R\}$ is called a domain of R and is denoted by $D(R)$.

$$\therefore D(R) = \{x \mid x \in A, (x, y) \in R\}$$

Range :- The set $\{y \mid y \in B, (x,y) \in R\}$ is called Range of the relation R and is denoted by $R(R)$.

$$\therefore R(R) = \{y \mid y \in B, (x, y) \in R\}$$

Eg: Let $A = \{2, 3, 5\}$ and $B = \{2, 4, 6, 10\}$

A relation 'R' from A to B is defined as

$$R = \{(x, y) \mid x \text{ divides } y\},\ x \in A,\ y \in B\}$$

$$R = \{(2,2)\,(2,4)\,(2,6)\,(2,10)\,(3,6)\,(5,10)\}$$

Domain of $R = \{2, 3, 5\}$

Range $= \{2, 4, 6, 10\}$

Inverse Relation:- A relation 'R' defined on non empty set 'A' posses it's inverse given by

$R^{-1} = \{(y,x) / x \in A \; \& \; y \in B\}$

for $R = \{(x,y) / x \cdot y \; x \in A \; \& \; y \in B\}$

eg: $A = \{1,2,3,4\}$

$R = \{(x,y) / x < y \; \forall \; x,y \in A\}$

① $R = \{(1,2)(1,3)(1,4)(2,3)(2,4)(3,4)\}$

$R^{-1} = \{(y,x) / xRy \; x \in A \; \& \; y \in B\}$

for $R = \{(x,y) / yRx, \; x \in A \; \& \; y \in R\}$

② $R^{-1} = \{(2,1)(3,1)(4,1)(3,2)(4,2)(4,3)\}$

$R^{-1} = \{(y,x) / x > y \; \forall \; x,y \in A\}$

properties of relations:-

1) **Reflexive relation:-** A relation 'R' on a set 'A' is reflective if $aRa, \forall a \in A$

i.e., $(a,a) \in R, \forall a \in A$

Eg: i) $R_1 = \{(1,1)(2,2)(3,3)(1,2)\}$ is a reflexive relation defined on $A = \{1,2,3\}$

ii) $R_2 = \{(x,y) / x = y, \forall \; x,y \in Z\}$

2) **Irreflexive relation:-** A relation 'R' on a set 'A' is said to be irreflexive if $a \not R a, \forall a \in A$

i.e., $(a,a) \notin R, \forall a \in A$

Eg:- $R_1 = \{(1,2)(2,1)(2,3)\}$ is irreflexive defined on the set $A = \{1,2,3\}$

$R_2 = \{(x,y) \mid x<y,\ \forall\, x,y \in Z\}$

3) **Non-reflexive relation**:- A relation 'R' on a set 'A' is said to be non-reflexive ~~ser~~ if R is neither reflexive nor non-reflexive i.e., if aRa, is true for some 'a' and false for others.

$R = \{(1,2)(2,3)(2,2)(3,1)\}$ is non-reflexive defined on the set $A = \{1,2,3\}$.

Symmetric relation:- A relation 'R' on a set 'A' is symmetric whenever $\underset{}{\overset{(a,b)\in R}{aRb}}$ then $\overset{(b,a)\in R}{bRa}$ i.e., if aRb then bRa.

Eg:- $R = \{(1,2)(2,1)(2,3)(3,2)(1,1)\}$ defined on the set $A = \{1,2,3\}$

(**Asymmetric relation**:-)) wrong

$$R = \{(x,y) \mid x^2+y^2 = 1,\ \forall\, x,y \in R\}$$

real numbers

$$\left[\begin{array}{l} xRy \Rightarrow x^2+y^2 = 1 \\ yRx \Rightarrow y^2+x^2 = 1 \end{array}\right]$$

Asymmetric relation:- A relation 'R' on a set 'A' is asymmetric if whenever $(a,b) \in R$ then $(b,a) \notin R$ i.e., aRb then $b \not R a$.

Eg:- $R = \{(1,2)(2,3)(3,2)\}$ is asymmetric on $A = \{1,2,3\}$.

Anti-symmetric relation:-

A relation 'R' on a set 'A' is anti-symmetric if $aRb,\ bRa \Rightarrow a=b$ i.e., $(a,b) \in R, (b,a) \in R \Rightarrow a=b$

Eg: the relation less than or equal to, defined on the set of real numbers is an anti-symmetric relation:

$R = \{(x,y) \mid x \leq y, \forall x, y \in \text{Real numbers}\}$

Eg: Let $A = \{1,2,3\}$

$R = \{(1,1)(2,2)\}$ is antisymmetric defined on A.

Transitive relation:- A relation R on a set A is transitive if $aRb, bRc \Rightarrow aRc$

i.e., if $(a,b) \in R, (b,c) \in R$ then $(a,c) \in R$

Eg: $R = \{(1,2)(2,1)(1,1)(2,3)(3,2)(2,2)\}$ is transitive defined on the set $A = \{1,2,3\}$

Eg:- $R = \{(x,y) \mid$ if $x \leq y, y \leq z$ then $x \leq z, \forall x,y,z \in R\}$

1) Reflexive $\rightarrow aRa, \forall a \in A$

2) irreflexive $\rightarrow a \not R a, \forall a \in A$

3) non-reflexive $\rightarrow aRa$, for some a

4) symmetric $\rightarrow$ if aRb then bRa

5) asymmetric $\rightarrow$ if aRb then $b \not R a$

6) anti-symmetric $\rightarrow aRb, bRa \Rightarrow a=b$

7) transitive $\rightarrow aRb, bRc \Rightarrow aRc$

Types of relations :-

1) Equivalence relation :- A relation `R` on a set `A` is called an equivalence relation if it is reflexive, symmetric and transitive i.e., R is equivalence relation on `A`. if it has the following properties.

i) $(a,a) \in R \ \forall a \in A$ (reflexive)

ii) if $(a,b) \in R$ then $(b,a) \in R \ \forall a,b \in A$ (symmetric)

iii) if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$, $\forall a,b,c \in A$ (transitive)

problems :-

1) Give an example of a relation

i) reflexive and transitive but not symmetric

ii) symmetric and transitive but not reflexive

iii) reflexive and symmetric but not transitive

Let $A = \{1,2,3\}$ and

R is a relation defined on A

i) $R_1 = \{(1,1)(2,2)(3,3)(1,2)(2,3)\}$

R_1 is reflexive and transitive but not symmetric.

since $(1,2) \in R$ but $(2,1) \in R_1$

ii) $R_2 = \{(1,1)(1,2)(2,1)(2,3)(3,2)(2,2)\}$

R_2 is symmetric and transitive but not reflexive since $(3,3) \notin R_2$

iii) $R_3 = \{(1,1)(2,2)(3,3)\}(1,2)(2,1)(2,3)(3,2)\}$

$(1,2) \in R_3$, $(2,3) \in R_3$ but $(1,3) \notin R_3$

2) Consider the following relation on the set $A = \{1, 2, 3, 4, 5, 6\}$ and R is defined as $R = \{(x,y) / |x-y| = 2\}$. Is R reflexive, symmetric, transitive?

i) R is not $A = \{1,2,3,4,5,6\}$

$$R = \{(x,y) / |x-y| = 2\}$$

$R = \{(1,3)(3,1)(2,4)(4,2)(3,5)(5,3)(4,6)(6,4)\}$

i) R is not reflexive since $(1,1)(2,2) \cdots (6,6) \notin R$

ii) R is symmetric since $\forall_{x,y \in A}\, x, y\ |x-y| = 2$

i.e., $x-y = +2$ i.e $(x,y) \in R$

$y-x = -2$ i.e, $(y,x) \in R$

$\therefore$ R is symmetric

iii) R is not transitive

since $(1,3)(3,1) \in R$ but $(1,1) \notin R$

3) If R be a relation defined on the set of integers with $R = \{(x,y) / (x-y)$ is divisible by $6\}$ then prove that R is an equivalence relation

i) Reflexive :-

$R = \{(x,y) / x-y$ is divisible by $6\}$

i) Reflexive :- let $x \in Z$

$x - x = 0$ and 0 is divisible by 6

$\therefore$ R is reflexive

ii) symmetric : let $x-y$ divisible by 6

$\Rightarrow y-x$ is always divisible by 6

i.e., if $(x,y) \in R$ then $(y,x) \in R$

$\therefore$ R is symmetric

iii) Transitive : If $x-y$ divisible by 6,

$\Rightarrow y-x$ is divisible by 6

then $(x-y)+(y-z) = x-z$, is divisible by 6

hence 'R' is transitive.

$\therefore$ R is an equivalence relation.

4) Let 'I' be the set of non-zero integers and the relation 'R' on it is defined as xRy if $x^y = y^x \; \forall x,y \in I$ then show that R is an equivalence relation.

i) reflexive :- $x^x = x^x \; \forall x \in I$

$\therefore xRx$

$\therefore$ R is reflexive

ii) symmetric :- $xRy \rightarrow x^y = y^x \; \forall x,y \in I$

$y^x = x^y \rightarrow yRx$

$\therefore$ R is symmetric

iii) transitive :-

$xRy \rightarrow x^y = y^x \Rightarrow x^{y/x} = y$

$yRz \rightarrow y^z = z^y \Rightarrow z-y$

consider $y^z = z^y$

$(x^{y/x})^z = z^y$

$x^{\frac{zy}{x}} = z^y$

$$x^{2y} = z^{xy} \Rightarrow x^{z} = z^{\frac{xy}{y}}$$

$$x^{z} = z^{x}$$

$$\Rightarrow xRz$$

$\therefore$ R is transitive

$\therefore$ R is equivalence relation

5) Determine the nature of the relation R on the set defined by aRb if and only if, a,b∈Z $ab \geq 0$,

i) reflexive :-

$$aRa \text{ iff } a \cdot a = a^2 \geq 0$$

if and only if

$\therefore$ R is reflexive

ii) symmetric :-

$$aRb = ab \geq 0 \Rightarrow ba \geq 0$$

$$ab = ba \Rightarrow bRa$$

$\therefore$ R is symmetric

iii) transitive :-

$$aRb \Rightarrow ab \geq 0$$

$$bRc \Rightarrow bc \geq 0$$

To prove aRc

consider $(ab)(bc) = ab^2c \geq 0$

$$\Rightarrow ac \geq 0 \;(\text{provided } b^2 = 0,\ b = 0)$$

R is not transitive when $ac < 0$

for eg:- $a = 2, b = 0, c = -6$

$$ab^2c = 2 \times 0 \times -6 \geq 0$$

but $ac = 2 \times (-6) = -12 < 0$

R is not an equivalence relation

N×N

4) Let 'R' be relation of N×N which is defined as $(a,b)\,R\,(c,d)$ iff $ad=bc$ then poove that 'R' is an equivalence relation.

1) reflexive :- $(a,b)\,R\,(a,b) \Rightarrow ab=ba$

$\Rightarrow ba=ab$

$\Rightarrow (a,b)\,R\,(a,b)$

ii) symmetric :- $(a,b)\,R\,(c,d) \Rightarrow ad=bc$

$\Rightarrow bc=ad$

$\Rightarrow cb=ad$

$\Rightarrow (c,d)\,R\,(a,b)$

$\therefore$ R is symmetric

iii) transitive :- $(a,b)\,R\,(c,d) \Rightarrow ad=bc$

$(c,d)\,R\,(e,f) \Rightarrow cf=de$

To poove $(a,b)\,R\,(e,f)$

given $ad=bc$, $cf=dc$

$ad(cf)=bc(de)$

$a\cancel{d}\cancel{c}f=b\cancel{c}\cancel{d}e$

$af=be$

$(a,b)\,R\,(e,f)$

$\therefore$ R is transitive and equivalence relation

5) Show that the relation 'R' defined on N×N by $(a,b)\,R\,(c,d)$ iff $a+d=b+c$ then poove that 'R' is an equivalence relation.

1) seflexive :- $(a,b)\,R\,(a,b) \Rightarrow a+b=b+a$

$\Rightarrow b+a=a+b$

$\Rightarrow (a,b)\,R\,(a,b)$

$\therefore$ R is seflexive.

ii) symmetric :- $(a,b) R (c,d)$ $a+d = b+c$

$$\Rightarrow b+c = a+d$$

$$\Rightarrow c+b = a+d$$

$$\Rightarrow (c,d) R (a,b)$$

$\therefore$ R is symmetric

iii) transitive :- $(a,b) R (c,d) \Rightarrow a+d = b+c$

$(c,d) R (e,f) \Rightarrow c+f = d+e$

To prove $(a,b) R (e,f)$

given $a+d = b+c$, $c+f = b+c$

$$(a+d)+(c+f) = (b+c)+(d+e)$$

$$a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$$

$$(a+f) = (b+e)$$

$$(a+f) R (b+e)$$

$\therefore$ R is transitive and equivalence relation.

7) Let 'R' be a relation defined as $R = \{(a,b) / a+b \text{ is even}, \forall a,b \in N\}$ show that 'R' is an equivalence relation.

Reflexive :-

$\forall a,b \in N$, $aRb \Rightarrow a+b$ is even

$a+a = 2a$ is an even number

$aRa \ \forall a \in N$

$\therefore$ R is reflexive

Symmetric:- $aRb \Rightarrow a+b$ is even

To prove $b+a$ is even

since $a+b$ is even

$b+a$ is also even

bRa

$\Rightarrow$ 'R' is symmetric

Transitive:- $aRb \rightarrow a+b$ is even

$bRc \rightarrow b+c$ is even

To prove $a+c$ is even

$a+b+b+c = a+2b+c$

$\Rightarrow$ even [$\because$ $a+c$ is also even and $2b$ is also even]

$\therefore aRc$

$\therefore$ R is transitive

imp

8) Let 'A' be the set of integers and n fixed positive integer. Define the relation R on aRb iff $n/a-b$ [n divides $(a-b)$] (or) aRb iff $a-b=kn$ (some k) this is also written as $a \equiv b \pmod n$. show that 'R' is an equivalence relation.

Reflexive:- $aRb \Rightarrow a-b=kn \Rightarrow n/a-b$

$\Rightarrow a-a = 0 \times n$

$\Rightarrow n/a-a$

$\Rightarrow$ 'R' is reflexive

Symmetric :-

$aRb \rightarrow n/a-b$

$\Rightarrow n/-(a-b)$

$\Rightarrow n/b-a$

$\Rightarrow bRa$

$\therefore$ R is symmetric

Transitive :-

$aRb \Rightarrow n/a-b$

$bRc \Rightarrow n/b-c$

To prove aRc i.e $n/a-c$

given

$n/a-b, \; n/b-c$

$n/a-b+b-c$

$n/a-c$

aRc

$\therefore$ R is transitive

Representation of relation :-

There are 2 methods to represent a relation

1) Matrix method

2) Di-graph method / directed graph method

1) Matrix method :- A binary relation 'R' with 'n' elements is represented by an $n \times n$ matrix called the relation matrix denoted by $M_R = [a_{ij}]$ where

$$a_{ij} = \begin{cases} 1 & \text{if } a_i R a_j \\ 0 & \text{otherwise} \end{cases}$$

Eg: let $A = \{1,2,3\}$

$$R = \{(x,y) \mid x \le y,\ \forall x,y \in A\}$$

$$R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (1,3)\ (2,3)\}$$

$$M_R = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

properties:-

1) A relation matrix is reflexive if all its diagonal element entities are unity.

2) A relation matrix is symmetric if it is symmetric matrix i.e., $(A^T = A)$.

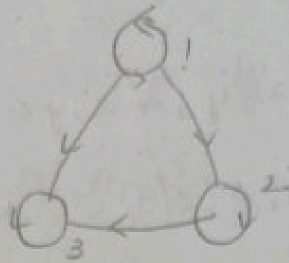
2) Di-graph representation:- A relation can also be represented pictorically by drawing its digraph. All elements of the non-empty sets are considered to be the vertices and arrow is drawn from the vertex to another vertex is called an directed edge.

→ If a vertex is connected to itself i.e., if $a \in a$ then there exist a self loop at that vertex.

Eg: let $A = \{1,2,3\}$

$$R = \{(x,y) \mid x \le y,\ \forall x,y \in A\}$$

$$R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (1,3)\ (2,3)\}$$



properties:-

1) If a relation is reflexive there is a self loop at each vertex.

2) If a relation is symmetric then there exist 2 edges between the vertexs one must be written to other.

problems:

1) Let $A = \{1, 2, 3, 4\}$ and $R = \{(x, y) / x > y\}$. Find relation matrix and also draw its graph

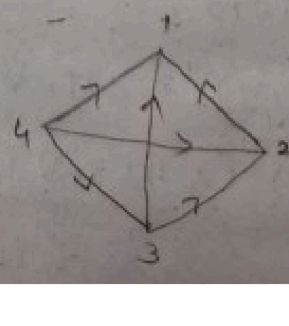
Relation matrix:-

$$a_{ij} = \begin{cases} 1 & a_i R a_j \\ 0 & \text{otherwise} \end{cases}$$

$$R = \{(4,1)\ (4,2)\ (4,3)\ (3,1)\ (3,2)\ (2,1)\}$$

$$M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

di-graph:-



is a self loop

there exist

e must be

$>y\}$. Find

ph

)}

2) Draw the graph for the following relations.

i) on $x = \{1, 2\}$

$R = \{(1,1)\ (2,2)\ (1,2)\}$

ii) on $x = \{1, 2, 3\}$

$R = \{(1,1)\ (1,3)\ (1,2)\ (2,2)\ (2,3)\ (3,2)\ (2,1)\ (3,3)\}$

i) $R = \{(1,1)\ (2,2)\ (1,2)\}$

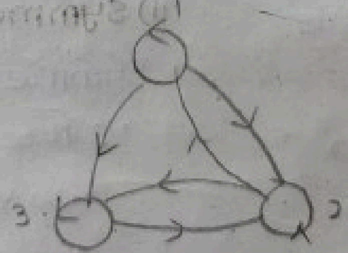
$$M_R = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 1 & 1 \\ 2 & 0 & 1 \end{array}$$

di-graph:-

ii) $R = \{(1,1)\ (1,3).\ (1,2)\ (2,2)\ (2,3)\ (3,2)\ (2,1)\ (3,3)\}$

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 3 & 0 & 1 & 1 \end{array}$$

digraph:-

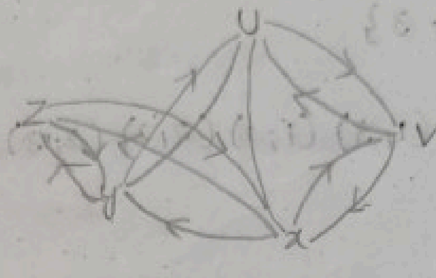


3) let $A = \{u, v, x, y, z\}$ and R be relation on A given by relational matrix.

$$M_R = \begin{array}{c|ccccc} & u & v & x & y & z \\ \hline u & 0 & 1 & 1 & 1 & 0 \\ v & 1 & 0 & 1 & 0 & 0 \\ x & 1 & 1 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 & 1 \\ z & 0 & 0 & 1 & 1 & 0 \end{array}$$

find the relation and draw its graph.

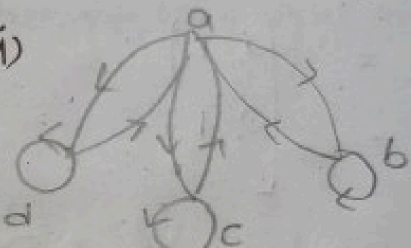
$R = \{(u,v)(u,x),(u,y)(v,u)(v,x)(x,y)(x,v)(x,$
$(y,u)(y,z)(z,x)(z,y)\}$



4) Find the relation from the following graphs:

i)

ii)



i) $R = \{(a,a)(a,b)(b,b)$
$(b,c)(c,b)(c,a)(c,c)\}$

i) Reflexive :- 'R' is reflexive.
∵ there exist a self loop at each vertex

ii) Symmetric :- 'R' is not symmetric.
∵ there exist an edge from a to b but no return edge from b to a

iii) transitive :- 'R' is not transitive
since aRb, bRc but a$\not R$c
there exist (∃)

ii) $R = \{(a,b)(a,c)(a,d)(b,a)(b$
$(c,a)(c,c)(d,a)(d,d)\}$

i) Reflexive :- 'R' is not reflexive because at vertex 'a' does not contain iself loop.

ii) symmetric :- 'R' is symmetric.
∵ for every edge between two vertexes there exist a return edge.

iii) transitive :- 'R' is not transitive
since there exist an edge between a to d also from 'd' to 'a' but no self loop at a

Let $A = \{1, 2, 3\}$

$R = \{(1,1)(2,2)(3,1)(2,1)(3,2)\}$

$R^2 = \{(1,1)(2,2)(3,1)(2,1)(3,2)\} \circ \{(1,1)(2,2)(3,1)(2,1)(3,2)\}$

$= \{(1,1)(2,2)(3,1)(2,1)(3,2)\}$

$R^3 = R \circ R^2$

$= \{(1,1)(2,2)(3,1)(2,1)(3,2)\} \circ \{(1,1)(2,1)(3,1)(3,2)(2,1)\}$

$= \{(1,1)(2,2)(3,1)(2,1)(3,2)\}$

$R^+ = \{R \cup R^2 \cup R^3 \cup \text{------}\}$

$= \{(1,1)(2,2)(3,1)(2,1)(3,2)\}$

$\because$ It is a transitive Relation

$\therefore$ Given R it sets is the transitive relation so $\boxed{R^+ = R}$

Partitions and Blocks :-

Let 's' be a given set and $A = \{A_1, A_2, \text{--} A_n\}$ where each A_i is a subset of 's' and $S = \bigcup_{i=}^{n} A_i$ then the set 'A' is called covering of 's' and the sets $A_1, A_2, \text{---} A_n$ are said to power 's'. If in addition to this elements of 'A' which are subsets of 's' are mutually disjoined then 'A' is called partioned of 's' and the sets $A_1, A_2 \text{--} A_n$ are called the blocks of the partioned

Equivalence class :-

let 'R' be a equivalence relation defined on non-empty set 'A' and $a \in R$ then equivalence class of 'A' is defined as $[a] = \{x / aRx, \forall (a,x) \in R\}$ and it is d»

Induced class/quotient class :-

the collection of all equivalence classes which are defined on non-empty set 'A' is called induced class and it is (induced) denoted by A/R

problems:

1) Let $A = \{1,2,3,4,5\}$ and $R = \{(1,1)(2,2)(3,3)(4,4)(5,5)(1,2)(2,1)(5,4)(4,5)\}$ find equivalence classes of A and A/R.

$[1] = \{1,2\}$

$[2] = \{1,2\}$

$[3] = \{3\}$

$[4] = \{4,5\}$

$[5] = \{4,5\}$

$A/R = \{[1],[3],[4]\}$

$A/R = \{[2],[3],[5]\}$

partition $= \{\{1,2\},\{3\},\{4,5\}\}$

2) If $A = \{1,2,3 --- 12\}$ and $R = \{(x,y)/x-y$ is a multiple of 5, $x,y \in A\}$. Find (cartesian) partition of A and A/R

$R = \{(1,1)(2,2) \ldots \ldots (12,12)(1,6)(6,1)(2,7)(7,2)$
$(3,8)(8,3)(4,9)(9,4)(5,10)(10,5)(6,11)(11,6)$
$(7,12)(12,7)(1,11)(11,1)(2,12)(12,2)\}$

2nd Equivalence relation - 1 question 5M

$[1] = \{1, 6, 11\}$ $\quad$ $[7] = \{2, 7, 12\}$

$[2] = \{2, 7, 12\}$ $\quad$ $[8] = \{3, 8\}$

$[3] = \{3, 8\}$ $\quad$ $[9] = \{4, 9\}$

$[4] = \{4, 9\}$ $\quad$ $[10] = \{5, 10\}$

$[5] = \{5, 10\}$ $\quad$ $[11] = \{1, 6, 11\}$

$[6] = \{1, 6, 11\}$ $\quad$ $[12] = \{2, 7, 12\}$

partition $= \{\{1,6,11\}\{2,7,12\}\{3,8\}\{4,9\}\{5,10\}\}$

$A/R = \{[1][2][3][4][5]\}$

$A/R = \{[11][12][9][10][8]\}$

3) let $A = \{1, 2, 3, 4\}$, and partition $P = \{\{1,2,3\},\{4\}\}$ find equivalence relation determined by 'p'.

$R = \{(1,1)(2,2)(3,3)(1,2)(2,1)(2,3)(3,2)(1,3)(3,1)(4,4)\}$

4) If $P = \{\{1,3\}\{2,4,5\}\}$ is a partition of $A = \{1, 2, 3, 4, 5\}$ find the corresponding equivalence relation.

$R = \{\{1,3\}\times\{1,3\}\ \{2,4,5\}\times\{2,4,5\}\}$

$= \{(1,1)(1,3)(3,1)(3,3)(2,2)(2,4)(2,5)(4,2)(4,4)(5,5)(5,2)(5,4)(5,5)\}$

5) Let 'z' be a set of integers and R be the relation defined on z has congruence module of 'p'. Determine the equivalence classes

generated by the elements of z also find z/R?

$R = \{(x,y) / x \equiv y \pmod 3\}$

$= \{(x,y) / x-y$ is divisible by $3 ; \forall x,y \in z\}$

$z = \{ \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \}$

$[0] = \{ -6, -3, 0, 3, 6, 9 \ldots \}$

$[1] = \{ \ldots -8, -5, -2, 1, 4, 7, 10 \ldots \}$

$[2] = \{ \ldots -7, -4, -1, 2, 5, 8, 11 \ldots \}$

$A/R = \{ [0], [1], [2] \}$

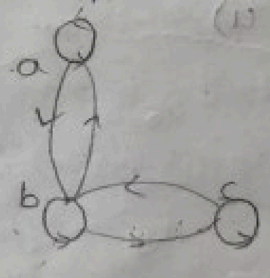
compability Relations :- for 2m we should write def and also def of reflexive and symmetric

A relation 'R' is said to be compability relation if it is reflexive and symmetric.

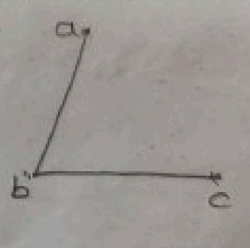
→ All equivalence relations are compability relations.

→ If a relation is compability relation then it is not neccessary to draw loops at each vertex and parallel edges between the vertexes

Eg:



graph of the compability relation



Simplified graph

$R = \{ (a,a)(b,b)(c,c)$
$(a,b)(b,a)(b,c)$
$(c,b)\}$

orda question will come

Maximal compability blocks:-

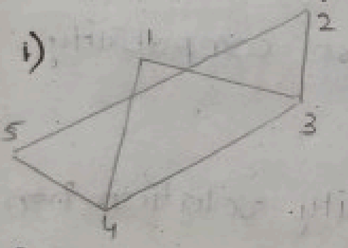
→ Draw a simplified graph of a compability relation
→ pick from this graph the largest complete polygons. By largest complete polygon we need a polygon in which any vertex is connected to every other vertex.

→ Any two elements which are compatable to one another is also forms maximal compability blocks.

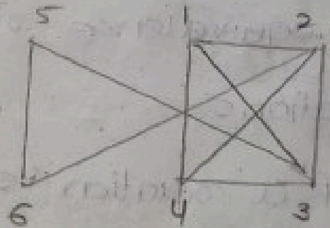
→ Blocks of the compability relation need not to be disjoined.

1) Find the maximal compability blocks of the simplified graphs :-

i)

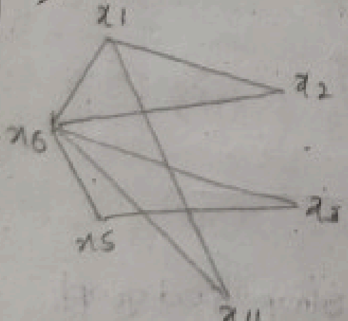


2)

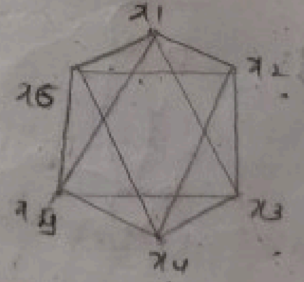


{{1,3,4}, {2,5} {2,3} {4,5}} {{1,2,3,4} {5,6} {3,5} {2,6}}

3)



4)

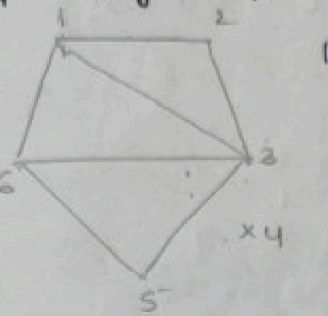


{x1, x2, x6} {x1, x4, x6}
{x3, x5, x6}

{x1, x2, x3, x4, x5, x6}

1) find the relation matrix to the following compability graph and also find its block.

1)



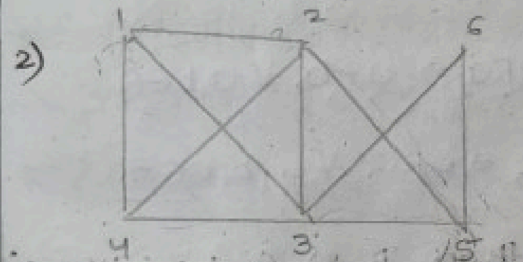
Blocks :- {1,2,3} {1,3,6} {3,5,6}

Relation matrix :-

$$\begin{array}{c|ccccc} 2 & 1 & & & & \\ 3 & 1 & 1 & & & \\ 4 & 0 & 0 & 0 & & \\ 5 & 0 & 0 & 1 & 0 & \\ 6 & 1 & 0 & 1 & 0 & 1 \\ \hline & 1 & 2 & 3 & 4 & 5 \end{array}$$

R = {(1,1) (2,2) (3,3) (4,4) (5,5) (6,6) (1,2)(2,1)(2,3) (3,2) (1,3)(3,1) (1,6) (6,1) (3,6) (6,3) (5,6)(6,5) (3,5) (5,3)}

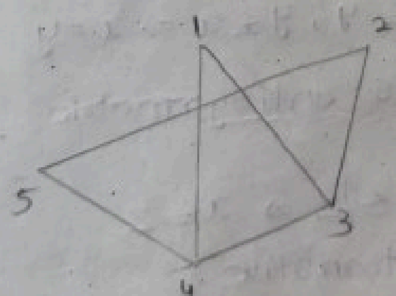
2)



Blocks :- {1,2,3,4} {2,3,5} {3,5,6}

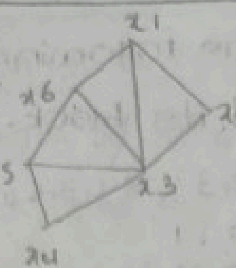
$$\begin{array}{c|ccccc} 2 & 1 & & & & \\ 3 & 1 & 1 & & & \\ 4 & 1 & 1 & 1 & & \\ 5 & 0 & 1 & 1 & 0 & \\ 6 & 0 & 0 & 1 & 0 & 1 \\ \hline & 1 & 2 & 3 & 4 & 5 \end{array}$$

3)



Blocks :- {2,3,4,5} {1,3,4}

$$\begin{array}{c|cccc} 2 & 0 & & & \\ 3 & 1 & 1 & & \\ 4 & 1 & 0 & 1 & \\ 5 & 0 & 1 & 0 & 1 \\ \hline & 1 & 2 & 3 & 4 \end{array}$$



Blocks :- $\{\{x_1, x_2, x_3, x_6\} \{x_3, x_5, x_6\}$
$\{x_3, x_4, x_5\}\}$

	x_1	x_2	x_3	x_4	x_5
x_2	1				
x_3	1	1			
x_4	0	0	1		
x_5	0	0	1	1	
x_6	1	0	1	0	1

partial ordering relations :-

A Binary relation are in a non-empty set 'A' is called partial ordinary (ordering) relations if it is

i) reflexive $aRa, \forall a \in A$

ii) anti symmetric $aRb, bRa \Rightarrow a=b, \forall a,b \in A$

iii) transitive $aRb, bRc \Rightarrow aRc, \forall a,b,c \in A$

eg:① If $A = \{1, 2, 3\}$ then $R = \{(1,1)(2,2)(3,3)\}$ is a partial ordering relation.

eg:② :- Let $R = \{(x,y) \mid x \leq y, y \leq x; \forall x,y \in R\}$

i) reflexive :- $x \leq x, \forall x \in R$

$\therefore \leq$ is reflexive

ii) anti symmetric :- $x \leq y, y \leq x \Rightarrow x = y$

$\therefore \leq$ is antisymmetric

iii) transtive :- $x \leq y, y \leq z \Rightarrow x \leq z$

$\therefore \leq$ is transtive

$(Z, \leq)$ is a partial ordering relation

po-set :- A set 'P' on which partial ordering relation is defined is called partial ordered set

In general $(P, \leq)$ $(P, \subseteq)$ are posets.

Eg:- $(P, \subseteq)$ is a poset

i) reflexive :- $A \subseteq A \ \forall \ A \in P(A)$

ii) Anti symmetric :- $A \subseteq B, B \subseteq A \Rightarrow A = B \ \forall \ A, B \in P(A)$

iii) transitive :- $A \subseteq B, B \subseteq C \Rightarrow A \subseteq C \ \forall \ A, B, C \in P(A)$

$\therefore$ $(P, \subseteq)$ is a poset)

Hasse diagram (poset diagram):-

the diagramatic representation of poset is called Hasse diagram.

→ The partial ordering less than (or) equal to on a set 'P' can be represented by hasse diagram (or) poset diagram

→ The procedure for drawing hasse diagram for a poset 'P' has follows :-

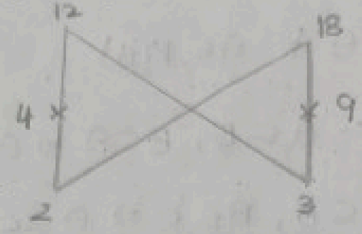
i) Each element is represented by a small circle / dot.

ii) The element $x \in P$ is drawn below the other element for $y \in P$ if $x < y$.

iii) An edge is drawn between the elements x and y if y^* covers x and if $x < y$ but y does not cover 'x' then 'x' & 'y' are not connected directly.

Problems:

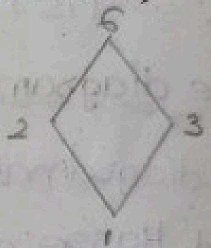
1) Let $P = \{2,3,4,9,12,18\}$ draw the hase diagram of P.



2) Draw the Hasse diagram $(D_6, /)$

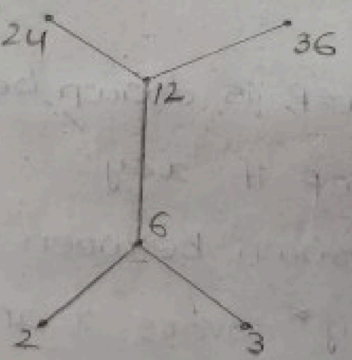
$D_6 = \{1,2,3,6\}$

$(D_6, /) = (\{1,2,3,6\} - /)$



3) Let $P = \{2,3,6,12,24,36\}$ and R is relation $R = \{(x,y) / x \text{ divides } y\}$. Write its partial order Relation hasse diagram.

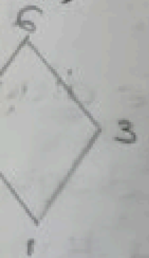
$R = \{(2,2)\ (3,3)\ (6,6)\ (12,12)\ (24,24)\ (36,36)$
$(2,6)\ (2,12)\ (2,24)\ (2,36)\ (3,6)\ (3,12)\ (3,24)\ (3,36)$
$(6,12)\ (6,24)\ (6,36)\ (12,24)\ (12,36)\}$



4) Let A be any set and $P(A)$ is power set of A. $\subseteq$ is the relation. Draw the Hasse diagrams of $(P(A), \subseteq)$ for

hase diagram

(6,1)



is relation its partial

(36,36)

(3,24) (3,36)

36)}

power set

the Hasse

i) $A = \{a\}$

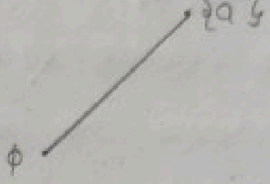
ii) $A = \{a, b\}$

iii) $A = \{a, b, c\}$

iv) $A = \{a, b, c, d\}$

v) ~~A~~ - i) $A = \{a\}$

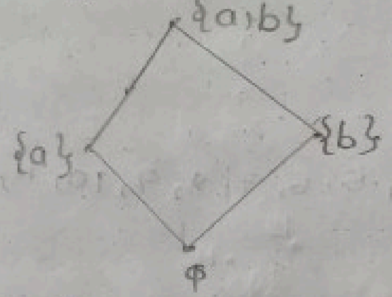
$P(A) = \{\phi, \{a\}\}$



ii) $A = \{a, b\}$

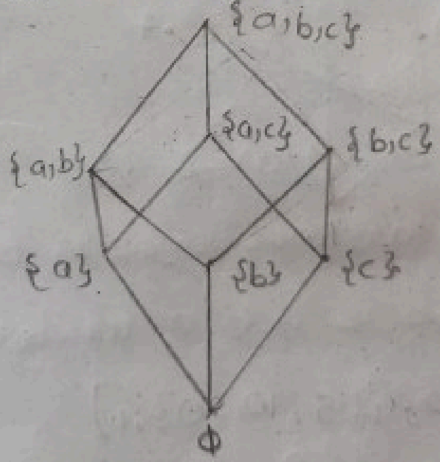
$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

Hasse diagram for $(P(A), \subseteq)$



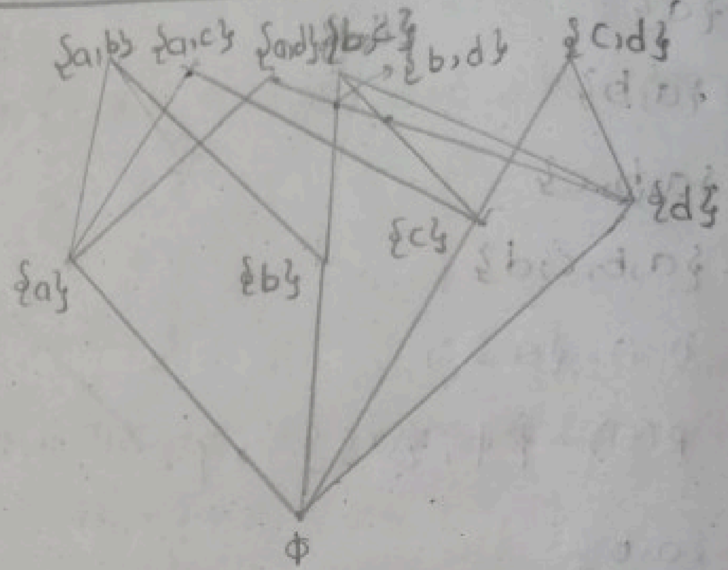
iii) $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$



iv) $A = \{a, b, c, d\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\},$
$\{c, d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}$
$\{a, b, d\}, \{b, c, d\} \{a, c, d\}, \{a, b, c, d\}\}$



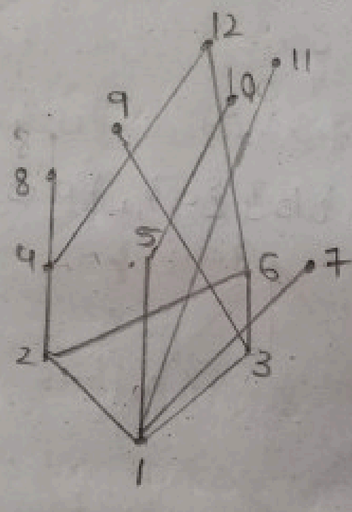
2) Draw poset diagram to each of the followin

i) $(I_{12}, |)$

ii) $(D_{20}, |)$

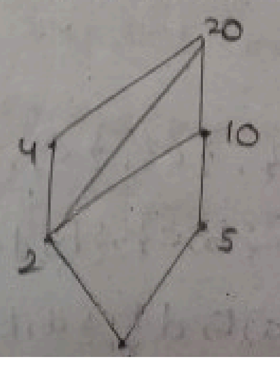
iii) $(D_{30}, |)$

i) $I_{12} = (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}, |)$



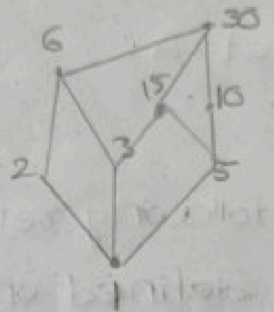
ii) $(D_{20}, |)$

$D_{20} = (\{1, 2, 4, 5, 10, 20\}, |)$



iii) $(D_{30}, |)$

$D_{30} = (\{1, 2, 3, 5, 6, 10, 15, 30\}, |)$



Functions:-

A function is a special case of relation. Let A and B be two non-empty sets and R is the relation from A to B. then, R may not relate one element of A to more than one element of A to unique element of B. Hence, every function is a relation But, a relation need not be a relation.

Function:-

let A, B be any two sets the a relation from A to B is said to be a function. if for every elements a belongs to A. there is a unique image in B.

$$s.t \ (a, b) \in f$$

$$i.e \ f(a) = b$$

1) b is called image of a.

2) a is called pre-image of b.

3) If $A \to B$ is a function then, A is called domain, B is called co-domain of 'f'.

4) If domain and co-domain of function are same i.e., $F: A \rightarrow A$ then, f is called operator on the set A.

problems:-

1) State whether the following relation are functions are not defined on the Relation on set $A = \{a, b, c\}$ $B = \{1, 2, 3\}$

i) $R_1 = \{(a,1)(a,3)(b,3)(c,1)\}$

ii) $R_2 = \{(a,2)(b,3)\}$

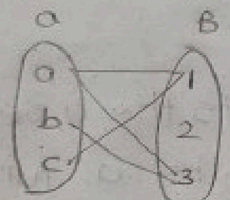
iii) $R_3 = \{(a,1)(b,3)(c,1)\}$

iv) $R_4 = \{(a,1)(b,2)(c,3)\}$

v) $R_5 = \{(a,1)(b,1)(c,1)\}$

vi) $R_6 = \{(a,1)(a,2)(b,3)(c,3)\}$

i)

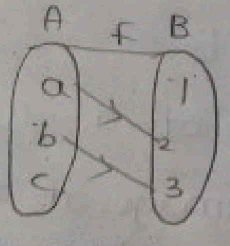


$f(a) = 1$ also, $f(a) = 3$

a passes two messages in B

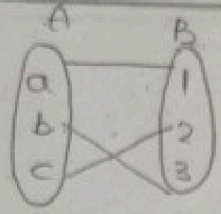
$f: A \rightarrow B$ is not a function

ii)



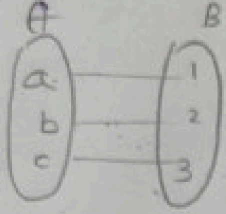
c does not passes any image in B.

iii)



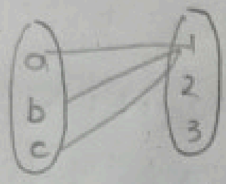
f : A → B is a function

iv)



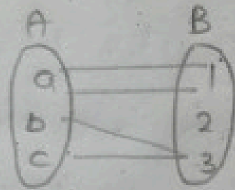
f : A → B is a function. since every element is A having some image in B.

v)



A → B is a function since every element of A having some image in B.

vi)



f : A → B is not a function since, in 'A' a is having two image in B