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UNIT-3

→ Grammar is used to generate the entire language

→ Grammar is an alternative way of specifying language

→ Grammars are useful in describing and analysing a language.

→ Grammar is indicated with the set of statements are called productions

→ Grammar is represented by using 4 tupples

$G = (V, T, P, S)$

V stands for a finite set of Variables or Non-terminals [uppercase letters)

T stands for a finite set of terminals [lower case letters]

P = production rule

S = Start symbol

Ex:→ $S \rightarrow aSB$

$S \rightarrow aB$

$B \rightarrow b$

$V = \{S, B\}$

$T = \{a, b\}$

$S = S$

Derivation:-

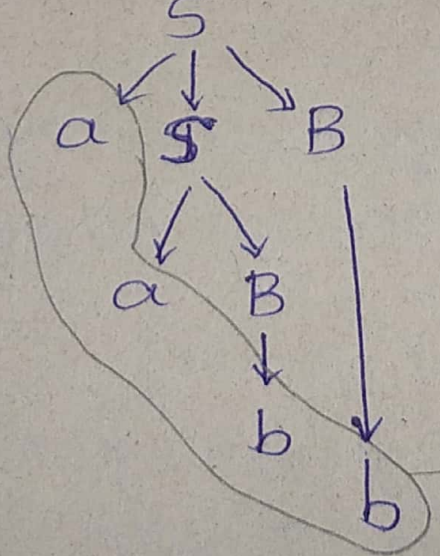
deriving a string from the Grammar
it starts from the start symbol.

$$\left.\begin{aligned} S &\rightarrow aSB \\ &\rightarrow aaBB \\ &\rightarrow aabB \\ &\rightarrow aabb \end{aligned}\right\} \text{These eq's are called as sentential forms}$$

In derivation every non-terminal is replaced with the terminal.

→ We can derive a string from Grammar in two way
i) sentential form ii) Derivation tree

derivation tree:-
We are going to generate a string tree in the form of tree structure.



This is called yield of derivation tree or yield of the parse tree

Regular Grammar:

Regular Grammar is classified into 2 types

1) RLG (Right Linear Grammar)
2) LLG) (Left linear Grammar)

RG

RLG LLG

$A \to \alpha B | \beta$ $A \to B\alpha | \beta$

$V = A, B$
$T = \alpha, \beta$ $S = A$

RLG: A variable is present at right most
LLG: A variable is present at left most

* Combination of LLG, RLG is not a Regular Grammar

→ Regular Grammar is either pure LLG or pure RLG

→ Whatever the language Generated by the type-3 grammar is regular language

Context free Grammar: [Type-2]

A Grammar Which is generated from the context free Language. To accept the context free language we are going to use push Down Automata [PDA]

* $A \to \alpha$ is the context Free Grammar rule

$A \in V$

$\alpha \in \{V \cup T\}^*$

→ Every RG is context free Grammar But every CFG is not RG

→ consider a grammar

→ derivation of the string from the grammar can be done in two ways

i) sentential form ii) derivation tree

* A derivation tree or parse tree is an ordered rooted tree that graphically represents the symmantic information of strings derived from a context free Grammar

Eg:- $G = (V, T, P, S)$ where

$A \to 1AA$ (or) $A \to \epsilon$

$S \to 0B$

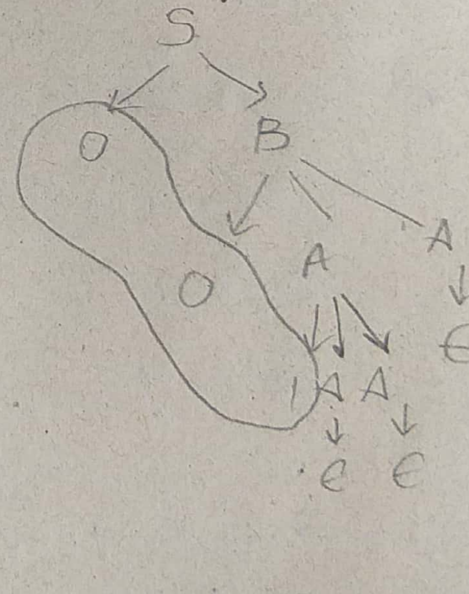
$A \to 1AA \mid \epsilon$

$B \to 0AA$

$V = \{S, B, A\}$

$T = \{0, 1\}$

$S = S$



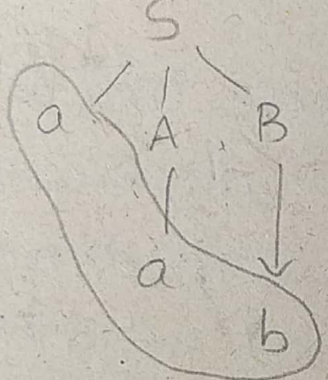
Types of derivation trees :-

i) Left most derivation tree (LMD)
ii) Right " " " [RMD]

→ Left most derivation tree [LMD] :-

A LMD is obtained by applying production to the left most Variable in each step

Ex:-



$S \rightarrow aAB$
$\rightarrow aaB$
$\rightarrow aab$

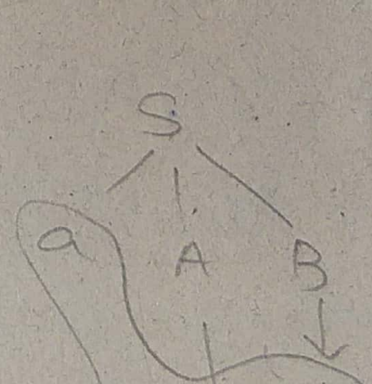
RMD:- A RMD tree is obtained by applying productions to the right most variable in each step.

Ex:- consider a grammar, and generate aab

$S \rightarrow aAB$

$A \rightarrow a$

$B \rightarrow b$



$S \rightarrow aAB$
$\rightarrow aAb$
$\rightarrow aab$

Ex:- consider a grammar $G = \{S \rightarrow aAS \mid ass \mid \epsilon$
$A \rightarrow SbA \mid ba\}$

generate a string aabaa

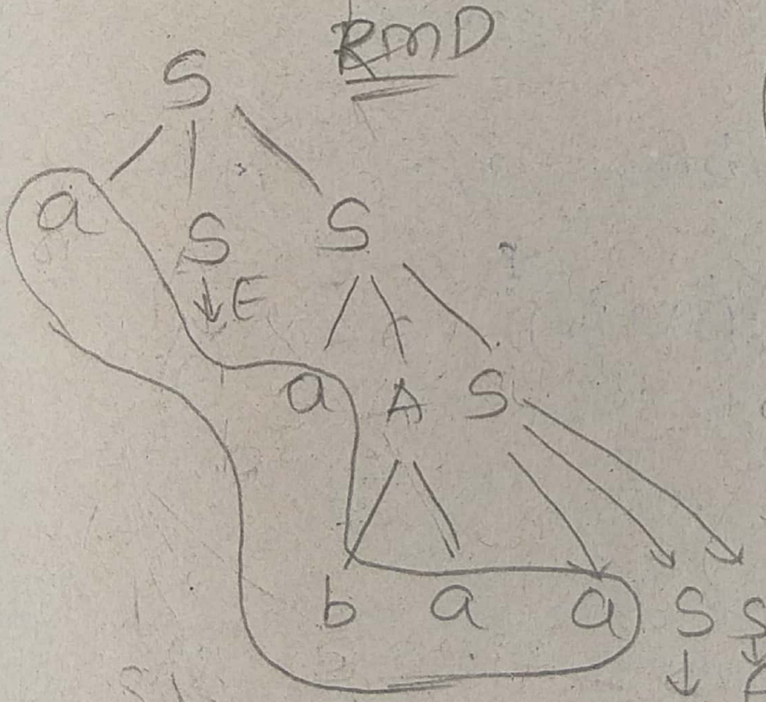
$S \rightarrow aAS \mid ass \mid \epsilon$

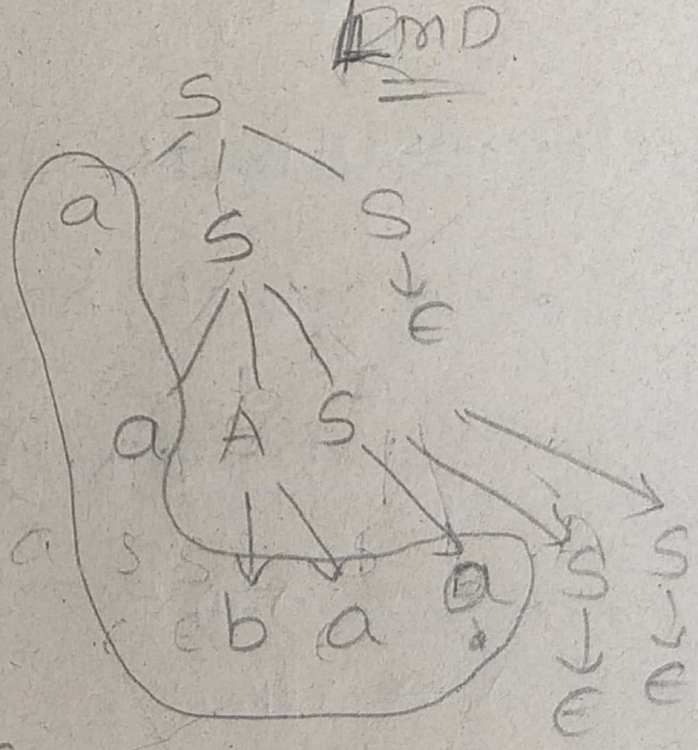
$A \rightarrow SbA \mid ba$

$V = \{S, A\}$

$T = \{a, b, \epsilon\}$

$S = S$





Construct a grammar for set of all Strings over $\{a, b\}$ where length is 2.

$L = \{aa, ab, ba, bb\}$

RE :→ $\underbrace{(a+b)}_{A}\underbrace{(a+b)}_{A}$

$S \rightarrow AA$

$A \rightarrow a|b$

Atleast length is 2.

$L = \{aa, ab, ba, bb, aaa, ---\}$

$(a+b)(a+b)(a+b)^*$

$S \rightarrow AAB$

$A \rightarrow a||b$

$B \rightarrow aB$

$B \rightarrow bB$

$B \rightarrow \epsilon$

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construct a context free Grammar for the language of palindrome over alphabet $\{a, b, c\}$

$L = \{\epsilon, aaa, abcba, a, b, c, aa, bb, cc,$
$aabbaa, aba, aca, --\}$

$S \rightarrow aSa \qquad S \rightarrow cSc$

$S \rightarrow bSb \qquad S \rightarrow \epsilon | a | b | c$

Scanned with CamScanner

Obtain a CFG for the language of even palindrome. Over $\{a, b\}$

$L = \{\epsilon, aa, bb, aabbaa, baab, abba, \ldots\}$

$S \to \epsilon$

$S \to aSa$

$S \to bSb$

odd length:-

$L = \{a, b, aba, bab, \ldots\}$

$S \to a \mid b$

$S \to aSa$

$S \to bSb$

Obtain CFG to generate the language

$L = \{ \omega / n_a(\omega) = n_b(\omega) \}$

$L = \{\epsilon, ab, aabb, baba$

$S \to \epsilon$

$S \to aSb$

$S \to abSa$

$S \to SS$

→ No. of a's > No. of b's

$L = \{b, aba, abaab, \ldots\}$

~~$S \to \epsilon$~~

$S \to XY / \cancel{Y}X / XYX$

$X \to aXb$

$X \to bXa$

$X \to XX$

$X \to \epsilon$

$Y \to aY / a$

Obtain CFG to generate language $L = \{ww^R / w \in (a,b)^*\}$

$L = \{\epsilon, aa, bb, abba, baab, --\}$

$S \to \epsilon$

$S \to aSa$

$S \to bSb$

$L = \{wcw^R \mid w \in (a,b,c\}$

$S \to aSa$

$S \to bSa$

$S \to c$

Obtain CFG for language $L = \{0^n 1^n / n \geq 0\}$

$L = \{\epsilon, 01, 0011, 000111$

$S \to \epsilon$

$S \to 0S1$

$S \to 01$

obtain CFG for $L = \{a^n b^{2n} / n \geq 1\}$

$L = \{abb, aabbbb, ---\}$

$S \rightarrow aSbb$

$S \rightarrow abb$

Generate string aaaabbbbbbbb

aSbb

a(aSbb)bb

aa(aSbb)bbbb

aaa(abb)bbbbbb

Ambiguous Grammar:-

A Grammar is said to be ambiguous grammar if there exists two or more derivation trees (either left or right) for a given string W.

Unambiguous:

A Grammar is said to be unambiguous if there exists only one derivation tree.

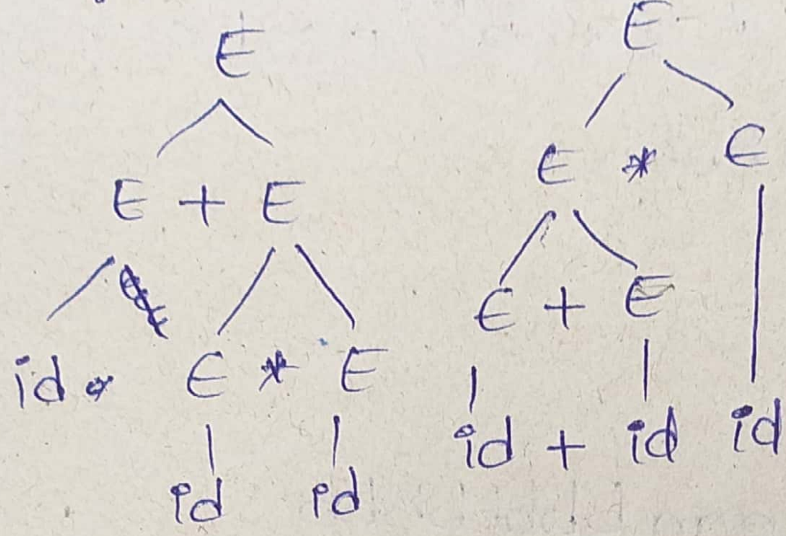
** Ex:- $G = \{\{E\}\}$ show that the given grammar is ambiguous. for given string id+id * id.

$E \rightarrow E+E$

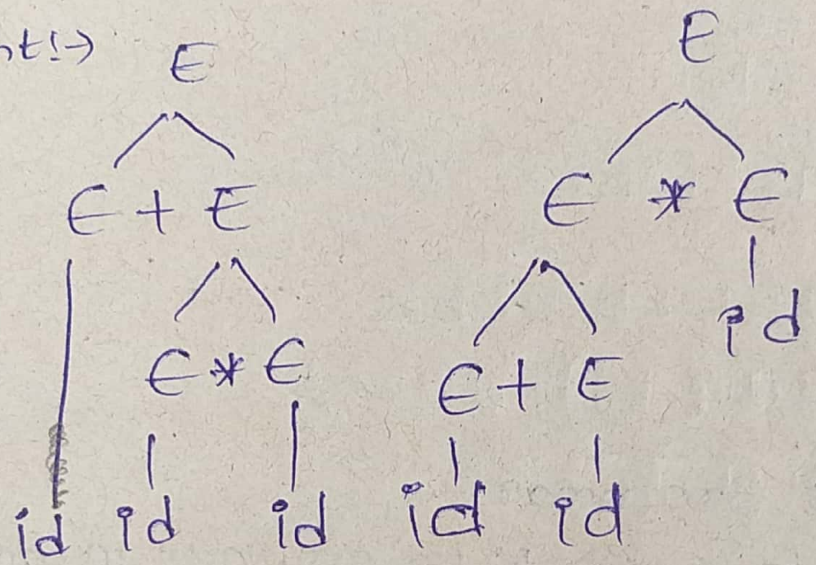
$E \rightarrow E*E$

$E \rightarrow id$

Left:-



Right:->



For the given string id+id * id. has two left most derivation trees (LMDT) & RMDT then the given grammar is Ambiguous.

Ex:-> $G = (\{S\}, \{a, b\}, S, S \to aSb \mid bSa \mid SS \mid \epsilon)$

String is abab

$S \to aSb$

$\Rightarrow abSab \quad (S \to bSa)$

$\Rightarrow abab \quad (S \to \epsilon)$

$S \to SS$

$\Rightarrow aSbS$ ($S \to aSb$)

$\Rightarrow aSbaSb$ ($S \to aSb$)

$\Rightarrow abab$ ($S \to \epsilon$)

Consider a grammar for arithmetic expression $E \to a|b$ and the $w =$ $a-b-a$

$E \to E-E$

$w = b-a-b$

$E \Rightarrow E-E$

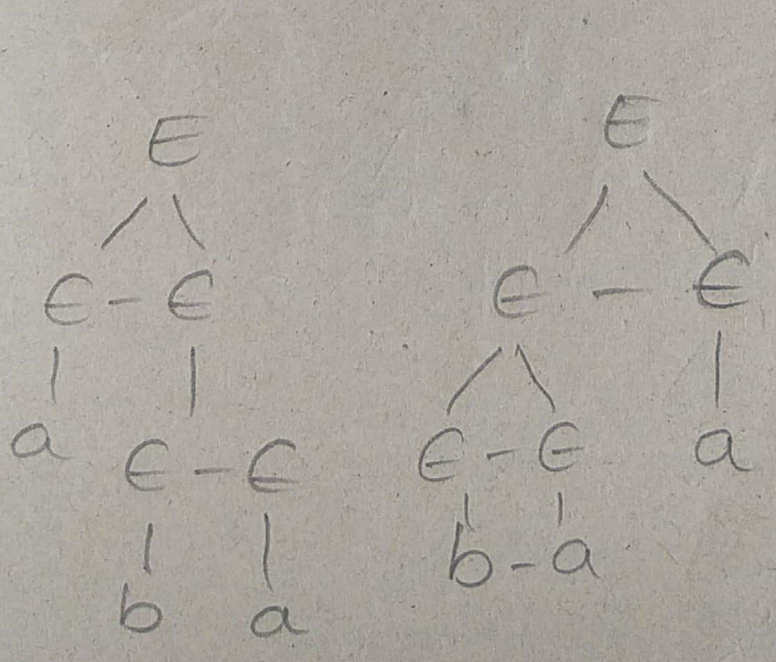
$\Rightarrow a-E-E$ ($E \to a$, $E \to E-E$

$\Rightarrow a-b-a$ ($E \to b$, $E \to a$)

$E \Rightarrow E-E$

$\Rightarrow b-E-E$

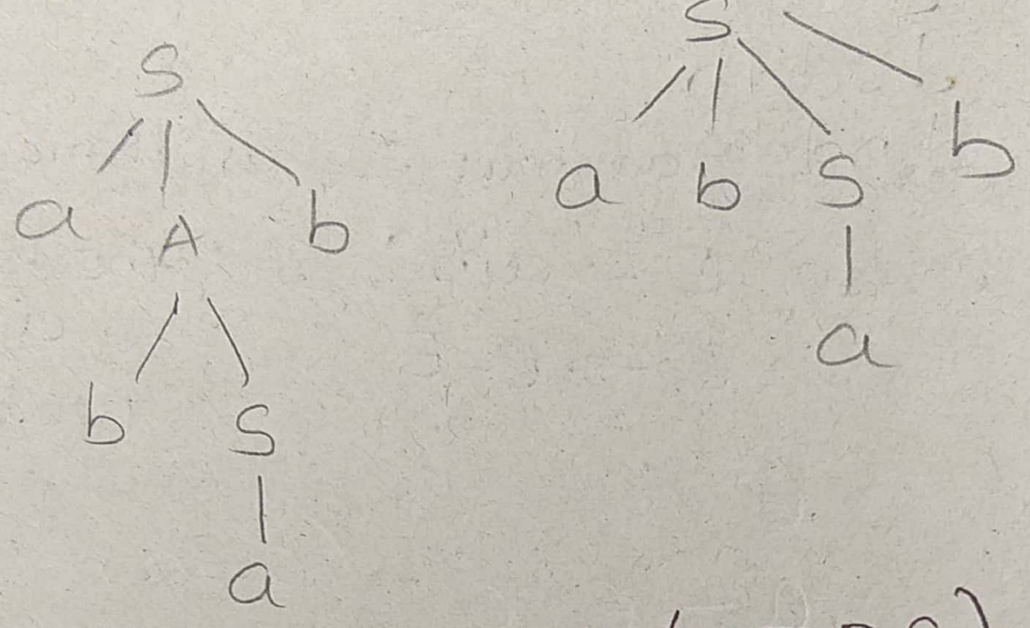
$\Rightarrow b-a-b$



prove that the grammar $S \to a \mid aAb \mid abSb$

$A \to aAAb \mid bS$ is ambiguous

String abab

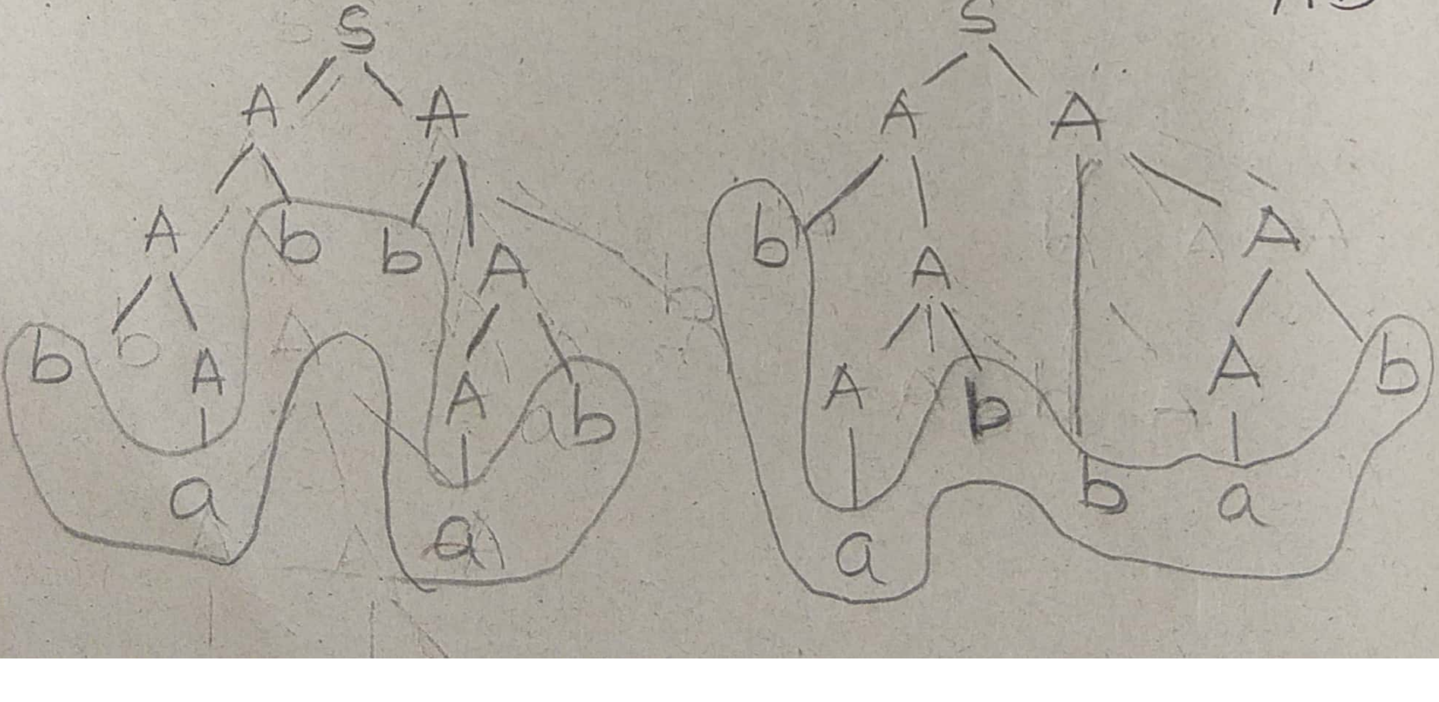


consider grammar $G = (V, T, P, S)$

$V = \{S, A\}$ $T = \{a, b\}$ $S \to AA$

$A \to AAA \mid a \mid bA \mid Ab$

String babbab



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Minimization Of Grammar:-

various languages can effectively represented by context free grammar

→ All the grammars are not always optimized that means grammar may contains some extra symbols [non-terminals & terminals].

having extra symbols unnecessarily increases the length of the grammar.

Simplification of grammar means reduction of grammar by eliminating unnecessary symbols.

Conditions for simplifying of Grammar:-

To reduce the grammar we have to follow 3 procedures.

i) Removal of useless symbols.

ii) Removal of ϵ productions

iii) Removal of unit productions.

Removal of useless Symbols:-

→ Any symbol is useful when it appears on the right hand side in the production rule.

→ and generates some terminal string
If No such derivation exists then it
is supposed to be an useless symbol

1) given $G=(V,T,P,S)$ where $[V=\{S,T,X\}$

$T=\{0,1\}$ $S \to 0T|1T|X|0|1, S]$

$T \to 00$

remove useless symbol from given grammar

Sol:- $S \to 0T$ $S \to 000$ useful

$S \to 1T$ $S \to 100$ useful

$S \to X$ $S \to X$ notuseful

$S \to 0$

$S \to 1$

$T \to 00$ $V=\{S,T\}$

removing symbols :→ $S \to 0T|1T|0|1$

$T \to 00$

2) consider the Grammar $G=(V,T,P,S)$
$V=\{S,A,B\}$ $T=\{0,1\}$ $S \to A11B|11A$

$S \to 1B|11$

$A \to 0$

Sol:→ $B \to BB$

Sol:-

$S \to A11B$ $S \to 011BB$ ×

$S \to 11A$ $S \to 110$

$S \to 1B$ $S \to 1BB$ ×   B is not useful. so remove that production

$S \to 11$ $S \to 11$

$A \to 0$ $A \to 0$

$B \to BB$ $B \to BB$ ×

$V = \{S, A\}$

~~$S \to A11$~~

$S \to 11A$

$S \to 11$

$A \to 0$

3) find CFG with no useless symbol equivalent to $S \to AB \mid CA$

$A \to a$

$B \to BC \mid AB$

$C \to aB \mid b$

Sol

$S \to AB$ $S \to aAB$ $S \to aaBC$ ×

$S \to CA$ $S \to ba$

~~$A \to a$~~ $A \to a$

$B \to BC$ $B \to ABC$ $B \to aBca$ ×

$B \to AB$ $B \to aB$ $B \to aBC$ ×

$C \to aB$ $C \to aBC$ $C \to aBCb$ ×

$C \to b$ $C \to b$

Ans

$S \to CA$

$A \to a$

$C \to b$

4) consider the following grammar $G=(V,T,P,S)$
**
where $V=\{S,X,Y\}$ $T=\{0,1\}$

$P=\{S\to XY/0$
$X\to 1\}$

Sol

$S\to XY$ $S\to 1Y$ ✗

$S\to 0$

$X\to 1$ ✗ (no use with X)

Ans :→ $S\to 0$

5) $G=(V,T,P,S)$ $S\to aA|bB$

$A\to aA|a$

$B\to bB$

$D\to ab|Ea$

$E\to aC|d$

$V=\{S,A,B,C,D,E\}$ $T=\{a,b,d\}$

~~$A\to aA$~~	$S\to aA$	$S\to aa$	Ans
	$S\to bB$	$S\to baB$ ✗	$S\to aA$
	$A\to aA$	$A\to aa$	$A\to aA$
	$A\to a$	$A\to a$	$A\to a$
	$B\to bB$	$B\to bbB$ ✗	
	$D\to ab$	$D\to ab$ ✗	no use
	$D\to Ea$	$D\to aCa$ ✗	with
	$E\to aC$	$E\to aC$ ✗	D, E
	$E\to d$	$E\to d$ ✗	

6) $G=(V,T,P,S)$ $S\to aS|A|C$

$A\to a$

$B\to aa$

$C\to aCb$

sol)

$S\to aS \Rightarrow aA \Rightarrow aa$

$S\to A \Rightarrow S\to a$

$S\to C \Rightarrow S\to aCb$ X

$A\to a$, $A\to a$

$B\to aa$ X

$C\to aCb$ X

Ans

$S\to aS|A$

$A\to a$

7) eliminate the useless symbols from given grammar.

$S\to aA|Bb|cC|a$

$A\to aB$

$B\to a|Aa$

$C\to cCD$

$D\to ddd$

sol)

$S\to a$ ✓

$S\to aA \Rightarrow aaB \Rightarrow aaa$ ✓

$S\to Bb \Rightarrow ab$ ✓

$S\to cC \Rightarrow ccCD$ X

$A\to aB \Rightarrow aa$ ✓

$B\to a$ ✓

$B\to Aa \Rightarrow aBa \Rightarrow aaa$ ✓

$C\to cCD$ X

$D\to ddd$ X

Ans

$S\to aA|Bb|a$

$A\to aB$

$B\to a|Aa$

8Q) $G = (V, T, P, S)$; $V = \{S, A, B\}$, $T = \{a\}$ and

P is $S \to a / AB$

$A \to a$

Sol $S \to a$

$S \to AB \Rightarrow aB$ ×

$A \to a$ ×

Ans.

$S \to a$

9Q) $S \to AB | CA$

$B \to BC | AB$

$A \to a$

$C \to aB | b$

Sol $S \to AB \Rightarrow aBC$ ×

$S \to CA \Rightarrow ba$ ✓

$A \to a$ ✓

$C \to aB \Rightarrow aBC$ ×

$C \to b$ ✓

Ans

$S \to CA$

$A \to a$

$C \to b$

10Q) $S \to aB | bX$

$A \to BAd | bSX | a$

$B \to aSB | bBX$

$X \to SBD | aBX | ad$

Sol $S \to aB$ ⇒ $\acute{a}(aSB)$ ×
$S \to bX$ ⇒ bad ✓
$A \to BAd$ ⇒ aSBad ×
$A \to bSX$ ⇒ bbXX ⇒ bbadad ✓
$A \to a$ ⇒ $A \to a$ ✓
$B \to aSB$ ⇒ abXbBX ×
$B \to bBX$ ⇒ baSBX ×
$X \to SBD$ ⇒ bXbBXD ×
$X \to aBX$ ⇒ aBad ×
$X \to ad$ ⇒ $X \to ad$ ✓

Ans

$S \to bX$
$A \to bSX$
$A \to a$
$X \to ad$

11Q) $S \to aAa$
$A \to bBB$
$B \to ab$
$C \to ab$

12Q) $A \to xyz / Xyzz$
$X \to Xz / xYx$
$Y \to yYy / XZ$
$Z \to Zy / z$

13Q) $S \to 0 / A$
$A \to AB$
$B \to 1$

11A)

S→aAa ⇒ abBBa ⇒ abababa ✓

A→bBB ⇒ babab ✓

B→ab ✓

C→ab ✗

Ans S→aAa

A→bBB

B→ab

12A) A→xyz ✓

A→Xyzz ⇒ Xzyzz or xYxyzz ✗

X→Xz ✗

X→xYx ✗

Y→yYy ✗

Y→XZ ✗

Z→Zy ⇒ zy ✓

Z→z ✓

Sol

A→xyz

13A) S→0 ✓

S→A ⇒ AB ⇒ A1 ✗

A→AB ✗

B→1

Ans S→0

Removing ϵ productions

→ In FA & RE that ϵ indicates a string with no value.

→ Even in CFG If there is any ϵ-productions we can remove it without changing the meaning of the grammar thus ϵ-productions are not necessary in a grammar

procedure for removal

1) To remove $A \rightarrow \epsilon$ look for all productions whose right side contains A

2) replace each occurences of A with ϵ.

3) Add the resultant productions to the grammar {After replacing A with ϵ}

1Q) Remove null productions from the following Grammar

$S \rightarrow ABAC$
$A \rightarrow aA \mid \epsilon$
$B \rightarrow bB \mid \epsilon$
$C \rightarrow c$

Sol→

replacing on A with ϵ

$S \rightarrow BAC$
$S \rightarrow ABC$
$A \rightarrow a$

replacing B with ϵ

$S \rightarrow AAC$
$A \rightarrow aA$
$B \rightarrow b$
$C \rightarrow c$

Ans

$S \rightarrow ABAC \mid BAC \mid ABC \mid AAC \mid BC \mid AC \mid C$

$A \rightarrow aA \mid a$

$B \rightarrow bB$

$C \rightarrow c$

2) remove ϵ-production from given grammar

$S \rightarrow 0S \mid 1S \mid \epsilon$

sol $S \rightarrow 0 \mid 1$ placing ϵ in S

Ans $S \rightarrow 0S \mid 1S \mid 0 \mid 1$

3Q) remove ϵ- production from given grammar & preserve meaning of given grammar

$S \rightarrow XYX$

$X \rightarrow 0X \mid \epsilon$

$Y \rightarrow 1Y \mid \epsilon$

sol $S \rightarrow XYX$ $\quad S \rightarrow XY \mid YX \mid Y \mid XX$

$X \rightarrow 0X \mid \epsilon$ $\quad X \rightarrow 0X \mid 0$

$X \rightarrow 1Y \mid \epsilon$ $\quad Y \rightarrow 1Y \mid 1$

Ans $S \rightarrow XYX \mid XY \mid YX \mid Y \mid XX \mid X \mid 0 \mid 1$

4Q) $S \to aSa \mid bSb \mid \epsilon$

Sol $S \to aa$ $S \to bb$

$S \to aSa \mid bSb \mid aa \mid bb$

5Q) $A \to 0B1 / 1B1$

$B \to 0B \mid 1B \mid \epsilon$

$A \to 0B1 \Rightarrow A \to 01$

$A \to 1B1 \Rightarrow A \to 11$

$B \to 0B \Rightarrow B \to 0$

$B \to 1B \Rightarrow B \to 1$

$B \to \epsilon$

$A \to 0B1 \mid 1B1 \mid 01 \mid 11$

~~A~~ | ~~0B~~ $B \to 0B \mid 0 \mid 1B \mid 1$

6Q) $S \to a \mid Ab \mid aBa$

$A \to b \mid \epsilon$

$B \to b \mid \epsilon$

$S \to a$

$S \to Ab$

$S \to aBa$

$A \to b \mid \epsilon$

$B \to b \mid \epsilon$

$S \to a$

$S \to b$

$S \to aa$

$S \to a \mid b \mid aa \mid Ab \mid aBa$

$A \to b$

$B \to b$

7Q) $S \to XY$

$X \to Zb$

$Y \to bW$

$Z \to AB$

$W \to Z$

$A \to aA \mid bA \mid \epsilon$

$B \to Ba \mid Bb \mid \epsilon$

$S \to XY$

$X \to Zb$

$Y \to bW$

$Z \to A \mid B \mid AB$

$W \to Z$

$A \to a \mid b \mid aA \mid bA$

$B \to a \mid b \mid Ba \mid Bb$

$S \to XY$

$ZbbW$

$AbbZ$

$abbb$

8Q) $S \to ACB \mid cbB \mid Ba$

$A \to da \mid BC$

$B \to gc \mid \epsilon$

$C \to ha \mid \epsilon$

11Q) $S \to AdA$

$A \to sb \mid bcc \mid \epsilon$

$C \to cc \mid abb$

10Q) $S \to ABaC$

$A \to BC$

$B \to b \mid \epsilon$

$C \to D \mid \epsilon$

$D \to d$

9Q) $S \to aS \mid AB$

$A \to \epsilon$

$B \to \epsilon$

$D \to b$

8A) $S \to ACB$ $\quad S \to AC \mid AB \mid A$

$S \to cbB$ $\quad S \to cb \mid bB \mid b$

$S \to Ba$ $\quad S \to a$

$A \to da$ $\quad A \to da$

$A \to BC$ $\quad A \to BC \mid B \mid C$

$B \to gC$ $\quad B \to g$

$B \to \epsilon$ $\quad C \to ha$

$C \to ha$

$C \to \epsilon$

$S \to ACB \mid AC \mid AB \mid A \mid cbB \mid cb \mid bB \mid b \mid a$

$A \to da \mid BC \mid B \mid C$

$B \to g$

$C \to ha$

9A) $S \to aS$ $\quad A \to \epsilon$ $\quad B \to \epsilon$

$S \to AB$ $\quad S \to B$ $\quad S \to A$

$A \to \epsilon$

$B \to \epsilon$

$D \to b$

sol $S \to aS$

$S \to AB \mid A \mid B$

$D \to b$

10A) $S \to ABaC$ $\quad B \to \epsilon$ $\quad C \to \epsilon$

$A \to BC$ $\quad S \to AaC$ $\quad S \to ABa$

$B \to b \mid \epsilon$ $\quad A \to C$ $\quad A \to B$

$C \to D \mid \epsilon$ $\quad S \to Aa$

$D \to d$

Ans $S \to ABaC \mid AaC \mid ABa \mid Aa$. $\quad C \to D, \; D \to d$

$A \to BC, \; B \to b,$

11A) $S \to AaA$

$A \to Sb$

$A \to bCC$

$A \to \epsilon$

$C \to cC$

$C \to abb$

$A \to \epsilon$

$S \to Aa$

$S \to aA$

$S \to a$

Ans

$S \to AaA \mid Aa \mid aA \mid a$

$A \to Sb \mid bCC$

$C \to cC \mid abb$

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3) Unit productions elimination:

The unit productions are the productions in which one non-terminal gives another non-terminal

Ex:→ $X \to Y$.

$Y \to Z$

procedure for removal:-

1) find out the unit productions in the given grammar

2) To remove unit productions $A \to B$ add productions $A \to x$ & $B \to x$

here x is the terminal.

3) delete unit production $A \to B$

4) repeat Step-2 untill all unit productions are removed.

1) remove unit productions from given grammar

$S \rightarrow Aa \mid B$

$B \rightarrow A \mid bb$

$A \rightarrow a \mid bc \mid B$

unit productions

$S \rightarrow B$	$S \rightarrow Aa \mid bb \mid a \mid bc$
$B \rightarrow A$	$B \rightarrow a \mid bc \mid bb$
$A \rightarrow B$	$A \rightarrow a \mid bc \mid bb.$

useless symbols:→ B is useless

$S \rightarrow Aa \mid bb \mid a \mid bc$

$A \rightarrow a \mid bc \mid bb$

2) $S \rightarrow XY$

$X \rightarrow a$

$Y \rightarrow Z \mid b$

$Z \rightarrow M$

$M \rightarrow N$

$N \rightarrow a$

unit productions:-

$Y \rightarrow Z$	$Y \rightarrow b \mid a$
$Z \rightarrow M$	$Z \rightarrow a$
$M \rightarrow N$	$M \rightarrow a$

$S \rightarrow XY$

$X \rightarrow a$

$Y \rightarrow b \mid a$

$Z \rightarrow a$

$M \rightarrow a$

$N \rightarrow a$

useless symbols:→

N, Z, M are useless

$S \rightarrow XY$

$X \rightarrow a$

$Y \rightarrow b \mid a$

3) $S \to 0A \mid 1B \mid C$

$A \to 0S \mid 00$

$B \to 1 \mid A$

$C \to 01$

unit productions:-

$S \to C$ $\quad$ $S \to 0A \mid 1B \mid 01$

$B \to A$ $\quad$ $B \to 1 \mid 0S \mid 00$

$C \to 01$

$A \to 0S \mid 00$

useless :-> $S \to 0A \mid 1B \mid 01$

$B \to 1 \mid 0S \mid 00$

$A \to 0S \mid 00$

4) $S \to A \mid 0C1$

$A \to B \mid 01 \mid 10$

$C \to \epsilon \mid CD$

unit productions

$S \to A$ $\quad$ $S \to 0C1 \mid 01 \mid 10$

$A \to B$

$\cancel{C \to \epsilon}$

i) $S \to 0C1 \mid 01 \mid 10$

$C \to \epsilon \mid CD$

C, D is useles

ii) $S \to 0C1 \mid 01 \mid 10$

$C \to \epsilon$

ϵ- production

$S \to 01 \mid 10 \mid 0C1$

$C \to \epsilon$

iii) $S \to 01 \mid 10$

5) $S \to AB$

$A \to a$

$B \to C \mid b$

$C \to D$

$D \to E \mid bc$

$E \to d \mid Ab$

unit productions

$B \to C$

$C \to D$

$D \to E$

$S \to AB$

$B \to b \mid bc \mid d \mid Ab$

$C \to bc \mid d \mid Ab$

$D \to bc \mid d \mid Ab$

$A \to a$ $\quad$ $E \to d \mid Ab$

useless $\Rightarrow$ $S \to AB$

$A \to a$

$B \to b \mid bc \mid d \mid Ab$

$C \to bc \mid d \mid Ab$

Scanned with CamScanner

Push Down Automata [PDA]:-

→ PDA is a way to implement context free grammar is a similar way to design finite Automata for regular Grammar.

→ PDA is more powerful than FA.

→ FA has a very limited memory but PDA has a unlimited memory.

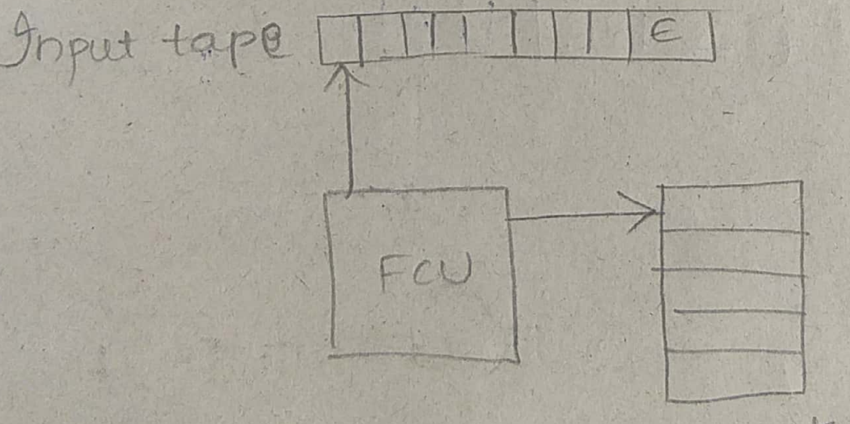
→ PDA = FA + stack

→ PDA has 3 components.

i) An input tape

ii) A finite control unit

iii) A stack with infinite memory



→ PDA can be accepted two types

i) final state

ii) Empty stack

Formal definition :→ It is defined as collection of 7 tuples

$(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q \rightarrow$ set of all states

$\Sigma \rightarrow$ input symbols

$\Gamma \rightarrow$ finite set of Stack alphabets

$\delta \rightarrow$ transition function

$$\delta : Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma^*$$

$q_0 \rightarrow$ initial state

$Z_0 \rightarrow$ bottom of the stack or initial stack symbol

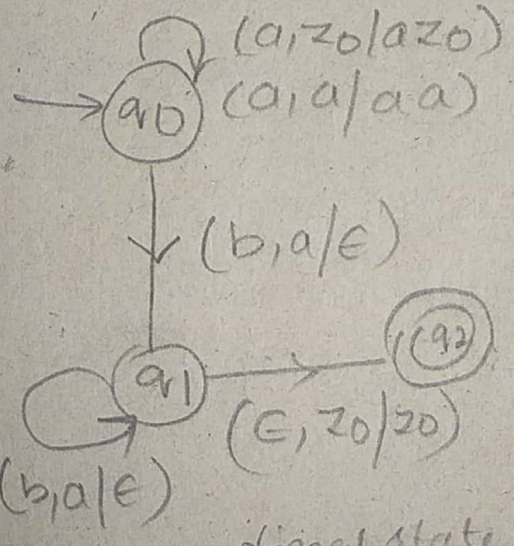
$Z_0 \in \Gamma$

$F \rightarrow$ Final state

1) construct PDA for the given language

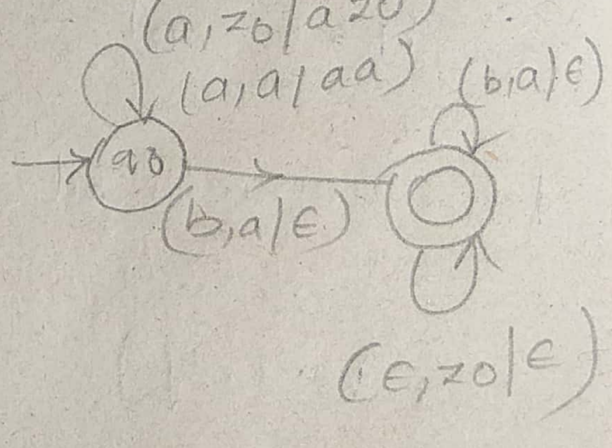
$L = \{a^n b^n \mid n \geq 1\}$

aabb



final state

(or)



Empty state

$(a, z_0 / a z_0)$

$a \downarrow$ input to PDA

$z_0 \downarrow$ top of stack

$a z_0$ ↳ a is pushed to stack

$(a, a / aa)$

$(b, a / \epsilon)$

$\epsilon \downarrow$ a popped from stack

Transition function:-

$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$ final state

or

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$ empty state

Instantaneous description of PDA

aaa bbb

(q, x, y)

$q \rightarrow$ initial state

$x \rightarrow$ input string

$y \rightarrow$ top of stack

ID = $\vdash (q_0, aaabbb, z_0)$

Turnstile

$\vdash (q_0, aabbb, az_0)$

$\vdash (q_0, abbb, aa)$

$\vdash (q_0, bbb, aa)$

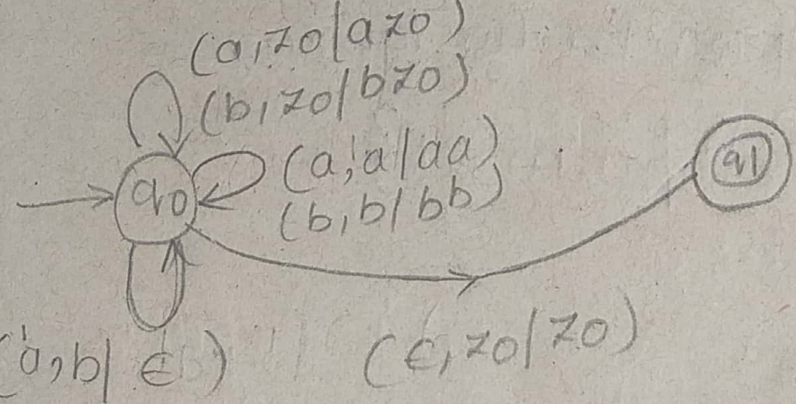
$\vdash (q_1, bb, aa)$

$\vdash (q_1, b, az_0)$

$\vdash (q_1, \epsilon, z_0)$ → final state (or)

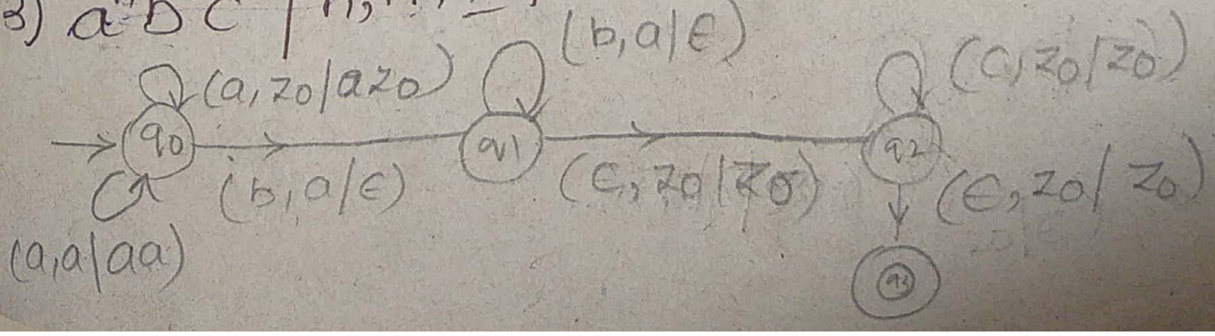
$\vdash (q_1, \epsilon)$ Emptystack

2) construct PDA $n_a(w) = n_b(w)$



L = {abc, aabbc, aaabbbc

3) $a^n b^n c^m \mid n,m \geq 1$



aabbccc

$ID = \vdash (q_0, aabbccc, z_0)$

$\vdash (q_0, abbccc, az_0)$

$\vdash (q_0, bbccc, aa)$

$\vdash (q_1, bccc, az_0)$

$\vdash (q_2, ccc, z_0)$

$\vdash (q_2, cc, z_0)$

$\vdash (q_2, c, z_0)$

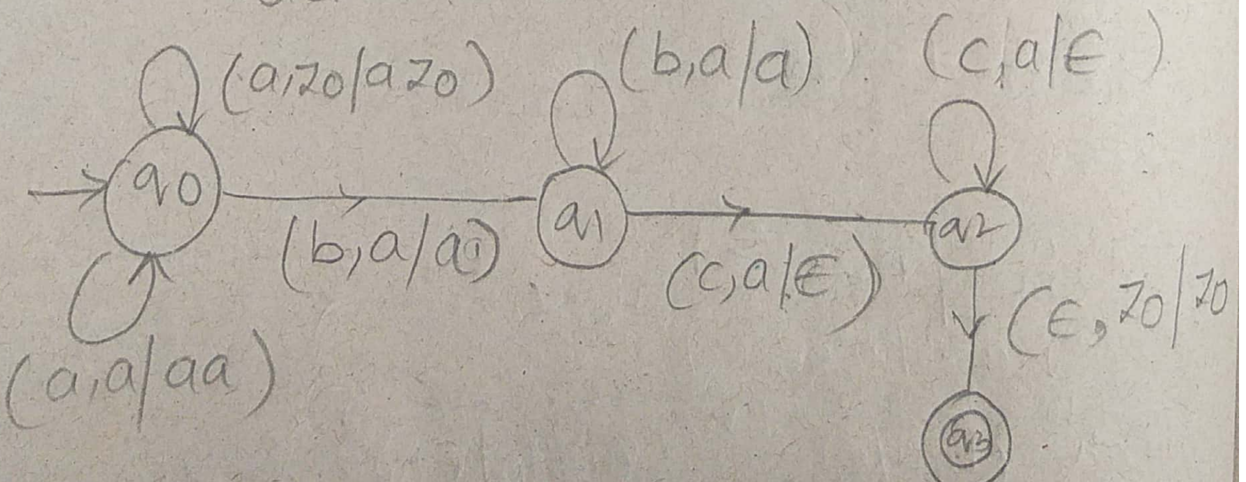
$\vdash (q_2, \epsilon, z_0)$

$\vdash (q_3, z_0)$

4) construct PDA for the language

$L = \{a^n b^m c^n \mid n, m \geq 1\}$

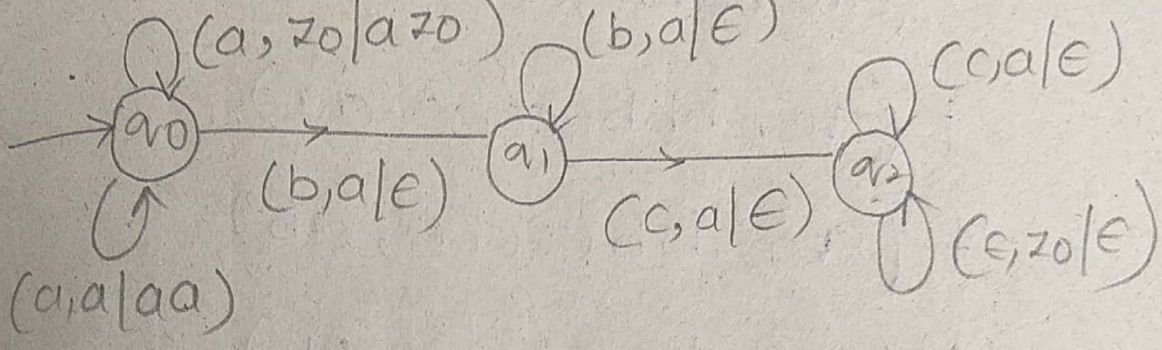
aabbbcc



5) $L = \{ a^{m+n} b^m c^n \mid n, m \geq 1 \}$

$n=2 \quad n=2$

$a^4 b^2 c^2$

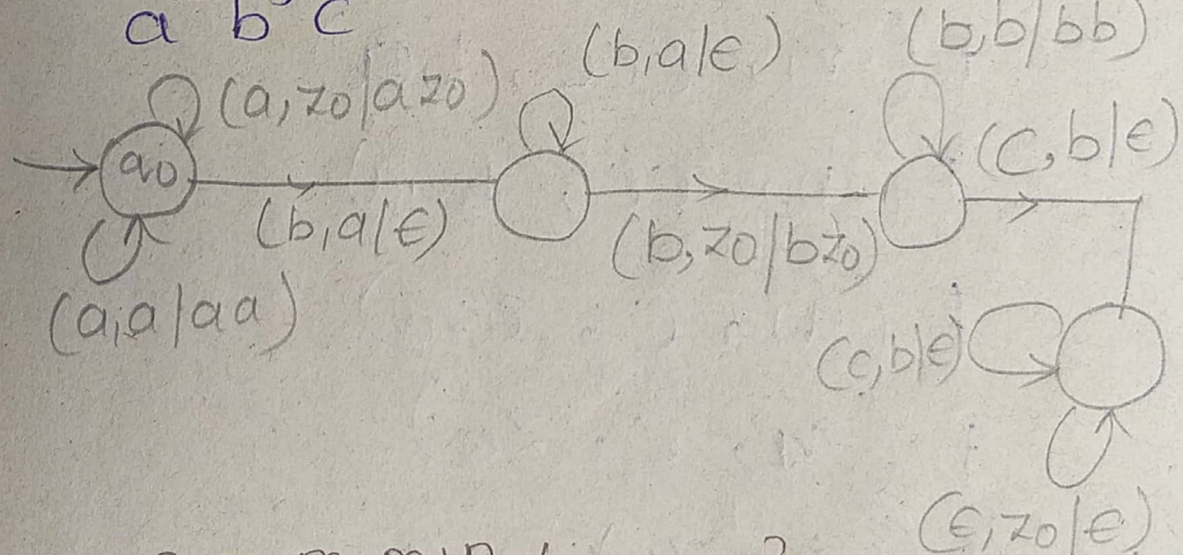


6) $L = \{ a^n b^{m+n} c^m \mid n, m \geq 1 \}$

$n=2 \quad m=3 \qquad a^n b^n . b^m c^m.$

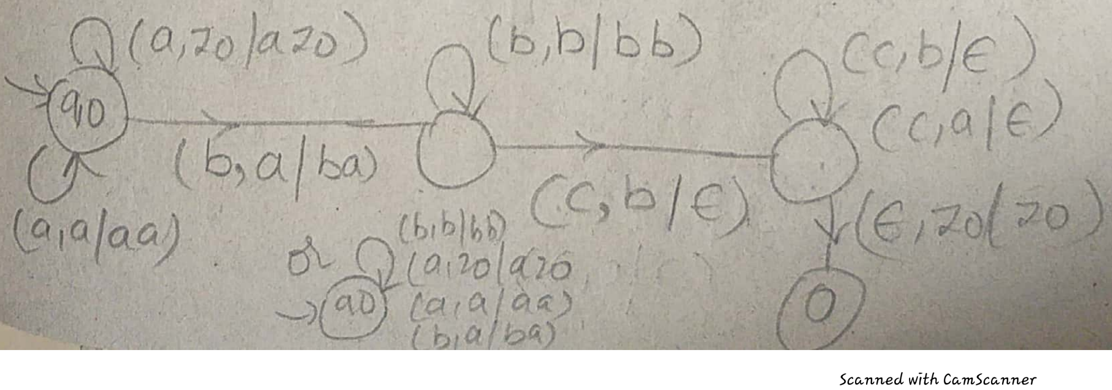
$a^2 b^2 b^3 c^3$

$a^2 b^5 c^3$



7. $L = \{ a^n b^m c^{m+n} \mid n, m \geq 1 \}$

$n=2 \quad m=3 \qquad a^2 b^3 c^3 c^2$



(8) aabbbccccc

ID $\Rightarrow \vdash (q_0, aabbbccccc, Z_0)$

$\vdash (q_0, abbbccccc, aZ_0)$

$\vdash (q_0, bbbccccc, aa)$

$\vdash (q_0, bbccccc, ab)$

$\vdash (q_0, bccccc, bb)$

$\vdash (q_0, ccccc, bb)$

$\vdash (q_1, cccc, bb)$

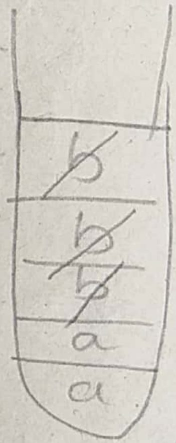
$\vdash (q_1, ccc, ba)$

$\vdash (q_1, cc, baa)$

$\vdash (q_1, c, aZ_0)$

$\vdash (q_2, \epsilon, Z_0)$

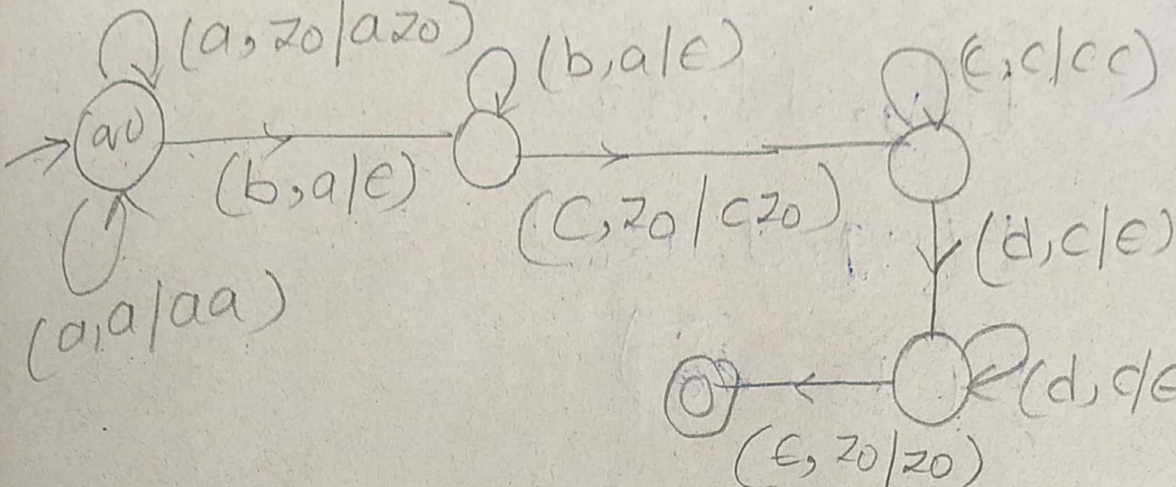
$\vdash (q_2, Z_0)$



8) $L = \{a^n b^n c^m d^m \mid n, m \geq 1\}$

$n=2 \quad m=3$

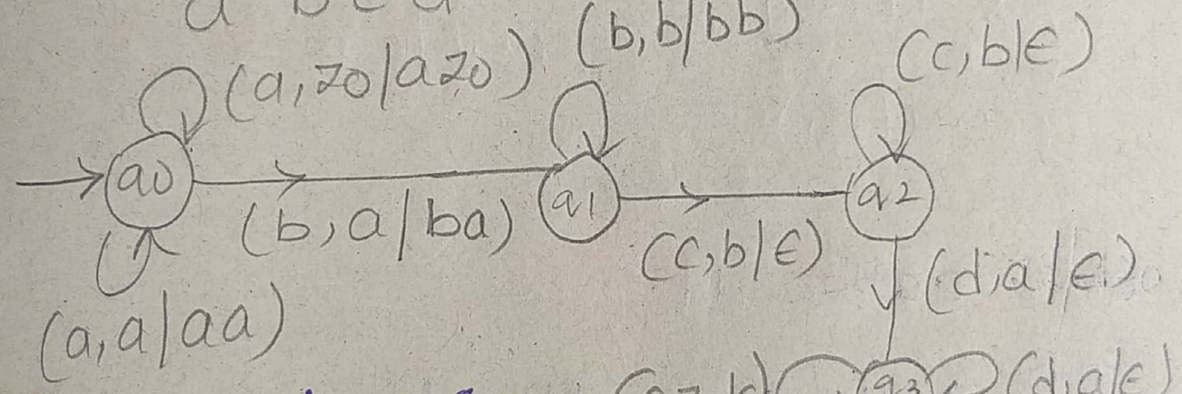
$a^2 b^2 c^3 d^3$



9) $L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$

$n = 3 \quad m = 2$

$a^3 b^2 c^2 d^3$



aaabbccddd ⇒ string

ID ⇒ $\vdash (q_0, aaabbccddd, z_0)$

$\vdash (q_0, aabbccddd, az_0)$

$\vdash (q_0, abbccddd, aa)$

$\vdash (q_0, bbccddd, aa)$

$\vdash (q_1, bccddd, ba)$

$\vdash (q_1, ccddd, bb)$

$\vdash (q_2, cddd, ba)$

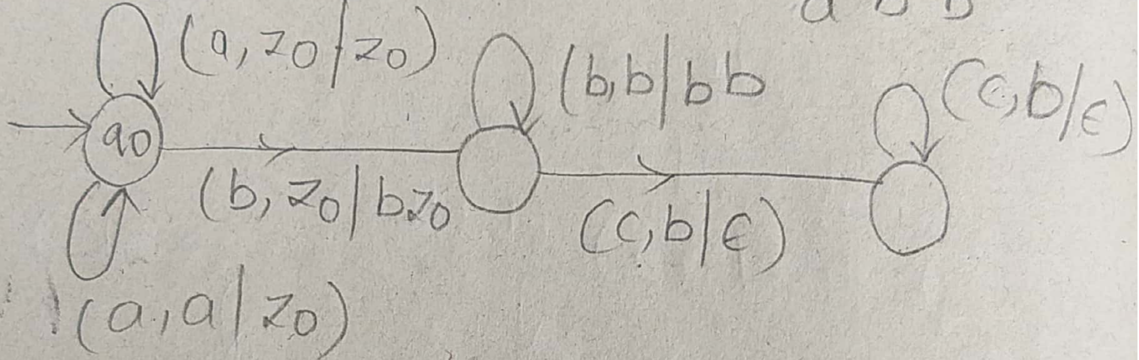
$\vdash (q_2, ddd, aa)$

$\vdash (q_2, dd, aa)$

$\vdash (q_2, d, aZ_0)$

$\vdash (q_3, \overset{\epsilon}{Z_0})$

$L = \{ a^n b^m c^m \mid n, m \geq 1 \}$ $n=2$ $m=3$
$a^2 b^3 b^3$



8Ans) aabbccdd.

ID:→ $\vdash (q_0, aabbccdd, Z_0)$

$\vdash (q_0, abbccdd, aZ_0)$ ⊣

$\vdash (q_0, bbccdd, aa)$

$\vdash (q_1, bccdd, aZ_0)$

$\vdash (q_1, ccdd, Z_0)$

$\vdash (q_2, cdd, cZ_0)$

$\vdash (q_2, dd, cc)$

$\vdash (q_3, d, cZ_0)$

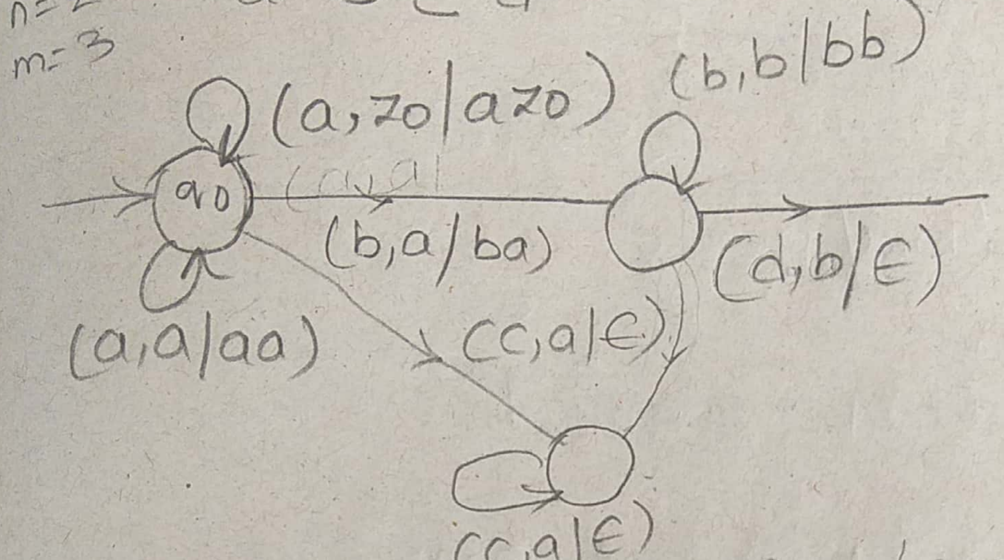
$\vdash (q_3, \epsilon, Z_0)$

$\vdash (q_4, Z_0)$

24|2|20

10) $L = \{a^n b^m c^n d^m \mid n, m \geq 1\}$

$n=2$ $a^2 b^3 c^2 d^3$
$m=3$

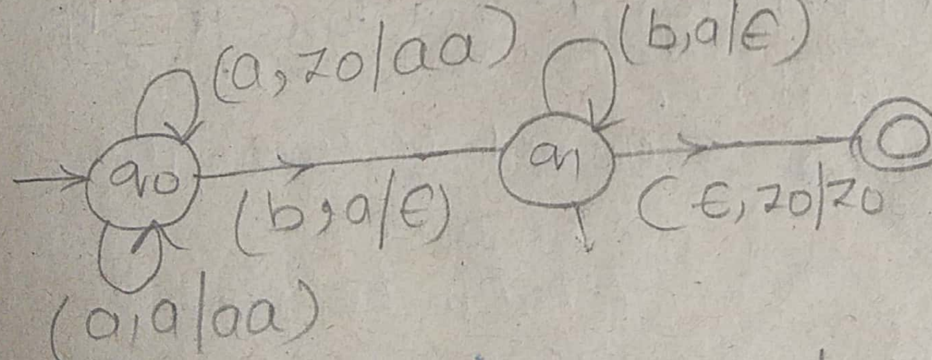


It is not a context free language
we cannot draw PDA

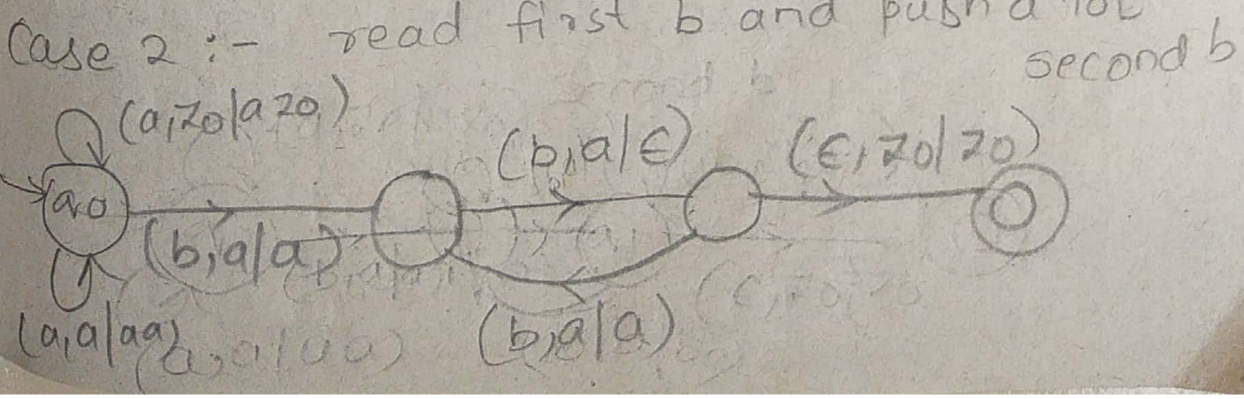
11) construct a PDA for $L = a^n b^{2n} \mid n \geq 1$

$n = 2$ $a^2 b^4$

Case 1 :- a — 2a's (push 2a's for single a



Case 2 :- read first b and push a for second b

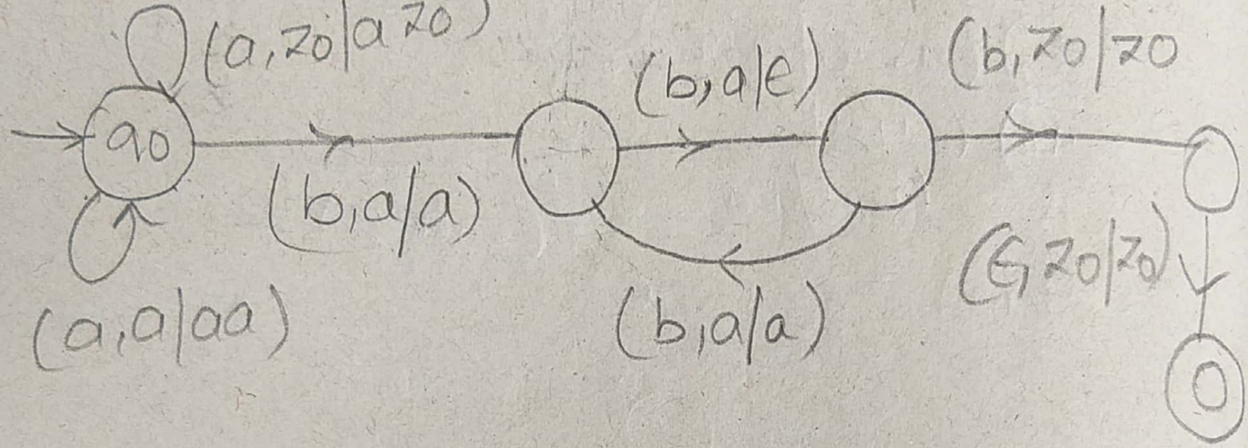


12) $L = \{a^n b^{2n+1} \mid n \geq 1\}$

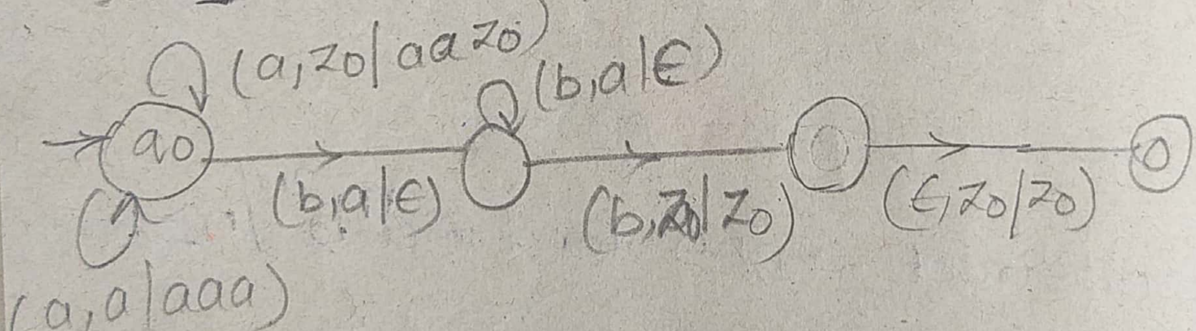
b
b pop(a)
b read
b pop(a)
b read

case:-2 $n=2$

$a^2 b^5$ aabbbbb



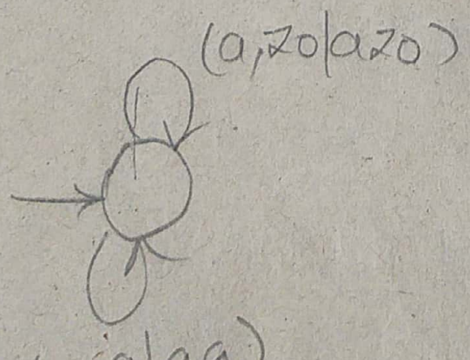
case:-1 :-



25|2|20

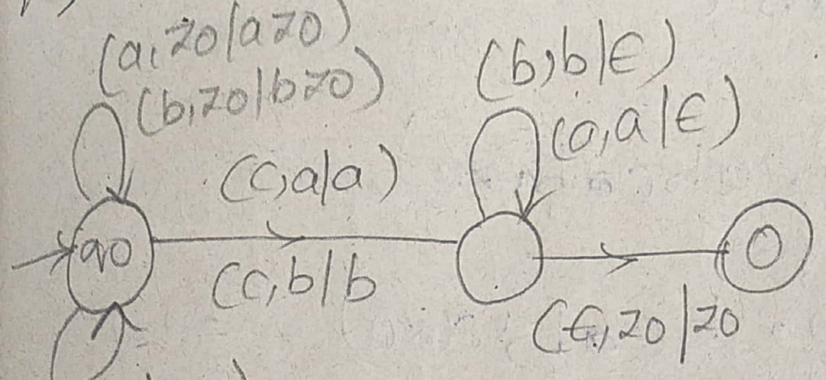
13) construct a PDA for $L = \{a^n b^n c^n \mid n \geq 1\}$

$n=2$ aabbcc



cannot be drawn PDA
because
c has to be popped
but there is no equivalent
variable to c

*14) Construct a PDA for $L = \{ w c w^R \mid w \in (a,b)^+ \}$



aa
ab
ba
bb

(a, a|aa)
(a, b|ab)
(b, a|ba)
(b, b|bb)

ababa cababa

ID :- $\vdash (q_0, ababacababa, z_0)$

$\vdash (q_0, babacababa, az_0)$

$\vdash (q_0, abacababa, aba)$

$\vdash (q_0, bacababa, aaba)$

$\vdash (q_0, acababa, aba)$

$\vdash (q_0, cababa, ab)$

$\vdash (q_1, ababa, aba)$

$\vdash (q_1, baba, ab)$

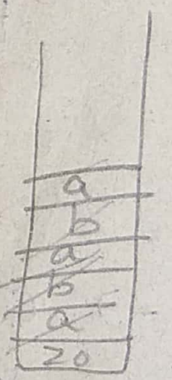
$\vdash (q_1, aba, aba)$

$\vdash (q_1, ba, ab)$ $\vdash (q_2, z_0)$

$\vdash (q_1, a, az_0)$

$\vdash (q_1, \epsilon, z_0)$

$\vdash (q_1,$

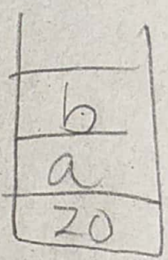


odd palindrom
wcw^R

abcaba

even palindrome ww^R

ID :- $\vdash (q_0, abcaba, Z_0)$



$\vdash (q_0, bcaba, aZ_0)$

$\vdash (q_0, caba, ba)$

$\vdash (q_1, aba, ba)$

$\vdash (q_1, ab, abZ_0)$

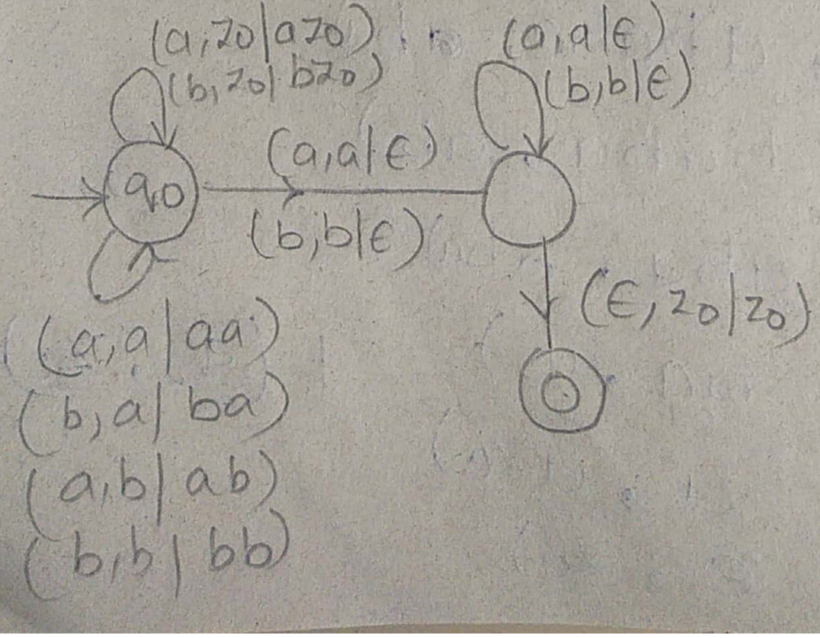
$\vdash (q_2, \epsilon, Z_0)$

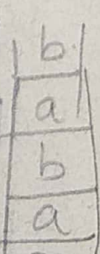
$\vdash (q_2, Z_0)$

→ abcab is rejected by our PDA drawn above.

*15) $L = \{ ww^R \mid w \in (a,b) \}$

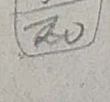
ababbaba





aa
ba
ab
bb

abba



aaaa

$(q_0, aaaa, z_0)$

$\downarrow$

(q_0, aaa, az_0)

No centre | centre

(q_0, aa, aaz_0) $\qquad$ (q_1, aa, z_0)

Nocentre | centre

$(q_0, a, aaaz_0)$ $\qquad$ (q_1, a, az_0)

No | centre $\qquad$ $\downarrow$

(q_1, a, az_0)

$\downarrow$

$(q_0, \epsilon, aaaaz_0)$ stopped $\qquad$ (q_1, ϵ, aaz_0) stopped $\qquad$ (q_1, ϵ, z_0)

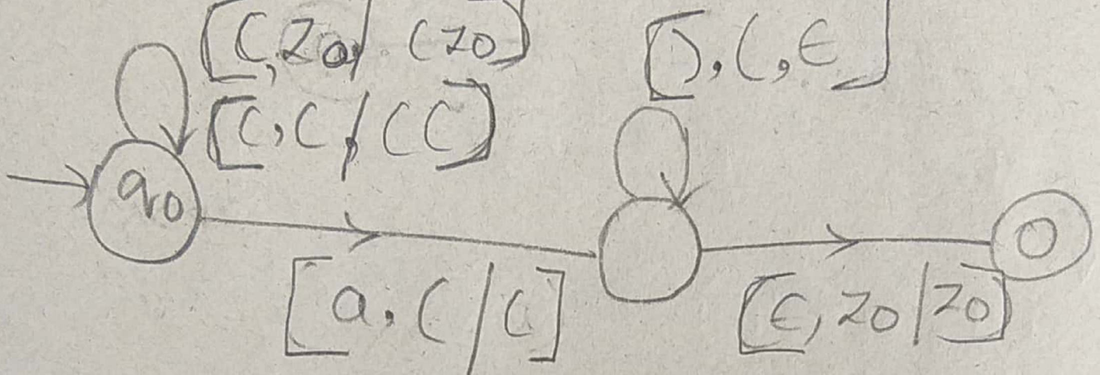
$\downarrow$

(q_2, z_0).

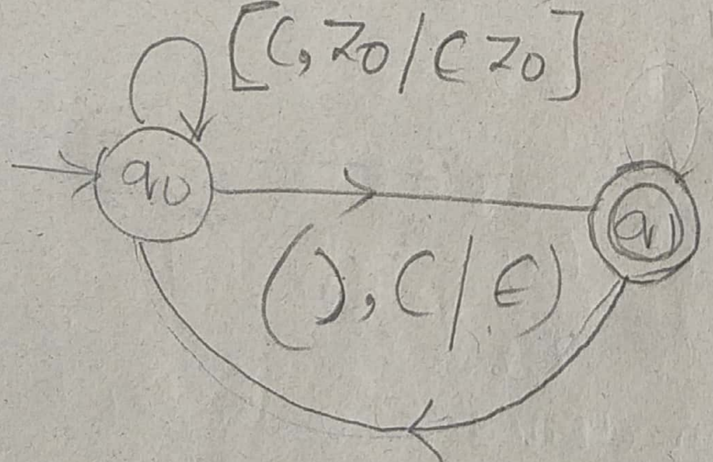
In previous problem there is a centre but the given problem we don't have any centre thats why we don't know when to push and when to pop. we can think if the top of stack is equivalent to input symbol it may be the centre of the word but this is not to be exactly correct thats why we can't draw the DPDA for this we are able to draw only NPDA.

16)** construct a PDA for balanced paranthesis

(((a)))



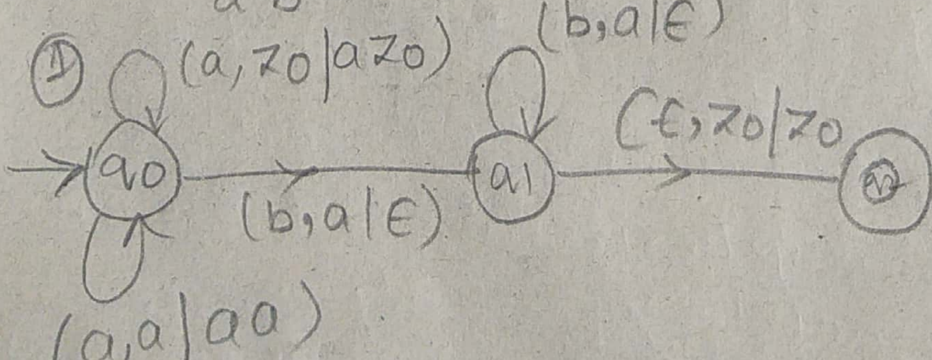
( ) ( )



(17) $L = \{a^n b^n / n \geq 1\} \cup \{a^n b^{2n} / n \geq 1\}$

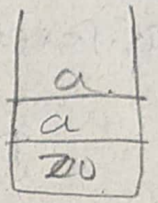
$a^2 b^2$

①



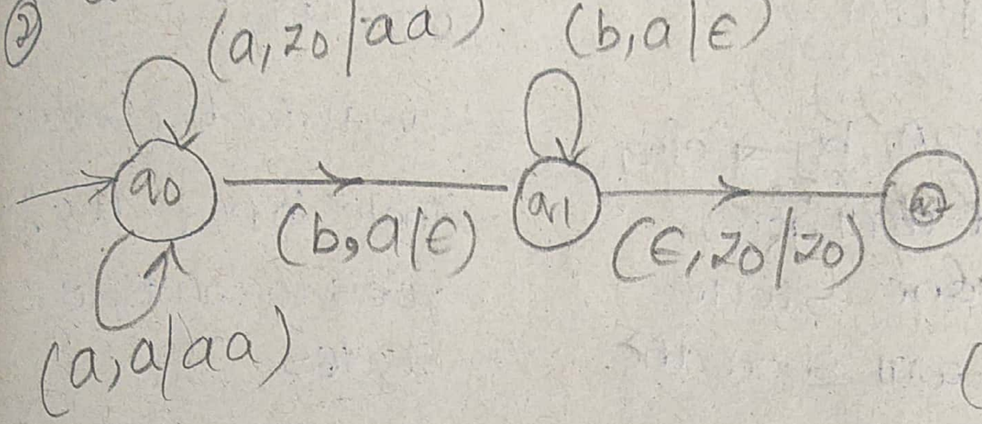
$a^n b^{2n}$

Case:-1 pushing 2 a's instead of 1



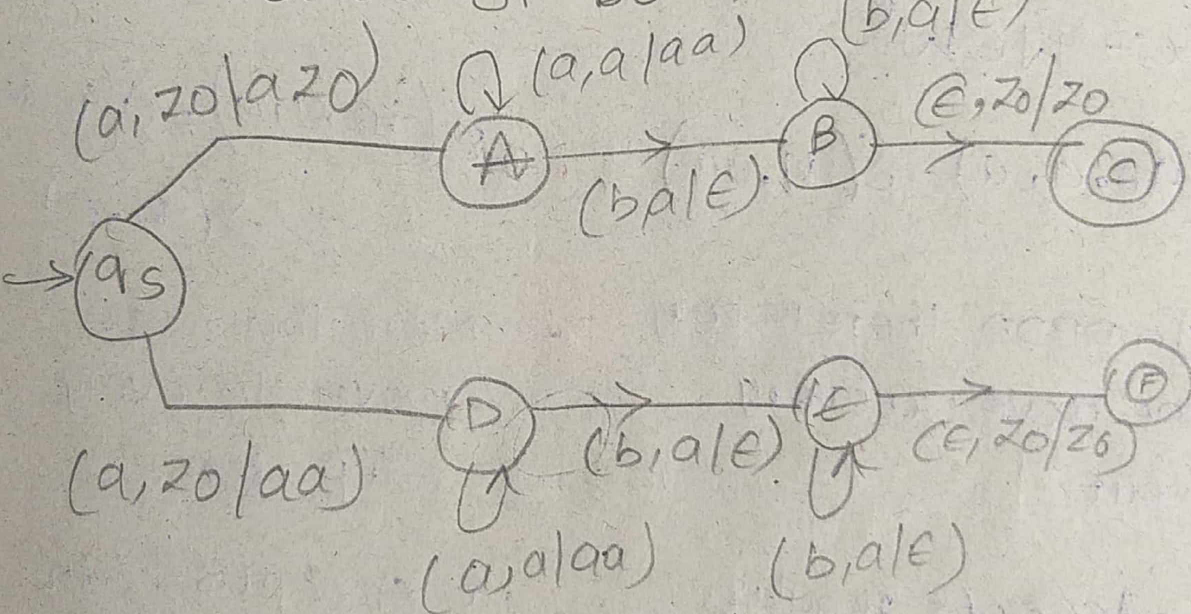
$a^2 b^4$

(2)



(NPDA)

now union of both.



aabb

(1)	(2)
$(q_0, aabb, Z_0)$	$(q_0, aabb, Z_0)$
(q_0, abb, aZ_0)	(q_0, abb, aaZ_0)
(q_0, bb, aaZ_0)	$(q_0, bb, aaaaZ_0)$
(q_0, b, aZ_0)	$(q_0, b, aaaZ_0)$
(q_1, ϵ, Z_0)	(q_1, ϵ, aaZ_0)
(q_2, Z_0)	Stopped

Difference between DPDA & NPDA

DPDA	NPDA
→ In DPDA by seeing input it will goes to only one state and they will show the determinism	→ In this by seeing single input it will goes to multiple states
Ex:- WCW^R $\delta(q_0, a, a) = \delta(q_1, aa)$	Ex:- WW^R $\delta(q_0, a, a) = \delta(q_0, aa)$ $\delta(q_1, \epsilon)$
→ In DPDA there is only one move on every input	→ More than one moves for every input
→ Accepts regular language, context free language and in between RL & CFL	→ Accepts both RL & CFL
→ less powerful	→ more powerful
→ All languages are not accepted by DPDA	→ All languages are accepted by NDPDA

27/2/20

Conversion from CFG to PDA:

Whenever we are converting CFG to PDA. the default PDA is acceptance by empty stack.

→ Grammar contains 4 tupples

$G = (V, T, P, S)$

V = Non-terminal   P = production rule

T = Terminals   S = Start symbol.

→ PDA formal definition contains 7 tuppes

$= \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$

we are going to convert these 4 tupples into 7 tupples

$Q = \{q\}$

Σ = Terminal symbols

$\Gamma = (T \cup V)$

$q_0 = \{q\}$

$Z_0 = S$ [start symbol].

$F = \phi$ or $\epsilon(\Lambda)$

where δ is defined by:

1) for start symbol:

$$\delta(q, \epsilon, Z_0) = (q, SZ_0)$$

2) for Non-terminals:

$$\delta(q, \epsilon, S) = \{(q, \alpha) \mid S \to \alpha \text{ is in production } P\}$$

3) for terminals:

$$\delta(q, a, a) = \{(q, \epsilon) \mid a \in \Sigma\}$$

4) for final transition:

$$\delta(q, \epsilon, Z_0) = (q, \epsilon)$$

1) construct a PDA equivalent to the following CFG

$S \to 0BB$

$B \to 0S \mid 1S \mid 0$

$G = (V, T, P, S)$

$V = \{S, B\}$ $\quad S = S$

$T = \{0, 1\}$

$P: S \to 0BB$

$B \to 0S \mid 1S \mid 0$

$(Q, \Sigma, \Gamma, q_0, \delta, z_0, F)$

$Q = \{q\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{S, B, 0, 1\}$

$q_0 = \{q\}$

$z_0 = S$

$F = \phi$

δ:

for start symbol:-

$\delta(q, \epsilon, z_0) = \delta(q, Sz_0)$

for Non-terminals:

$\delta(q, \epsilon, S) = (q, 0BB)$

$\delta(q, \epsilon, B) = (q, 0S)$

$\delta(q, \epsilon, B) = (q, 1S)$

$\delta(q, \epsilon, B) = (q, 0)$

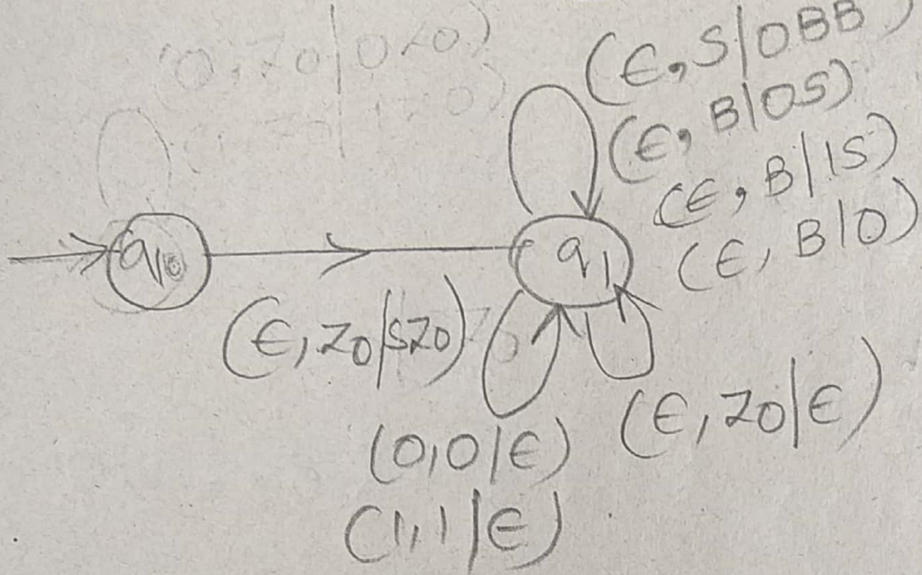
for terminals:

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 0, 0) = (q, \epsilon)$

for final state

$\delta(q_r, \epsilon, Z_0) = (q_r, \epsilon)$



010000

ID :- $\delta(q_0, 010000, \overset{S}{}Z_0)$

$\delta(q_r, 010000, 0BB)$

$\delta(q_r, 010000, 01SB)$

$\delta(q_r, 010000, 010BBB)$

$\delta(q_r, 010000, 0100BB)$

$\delta(q_r, 010000, 01000B)$

$\delta(q_r, 010000, 010000)$

$\delta(q_r, \epsilon, Z_0)$

$\delta(q_r, \epsilon)$

2) consider following grammar and construct equivalent PDA

$S \to aSa$

$S \to bSb$

$S \to c$

$G = (V, T, P, S)$ PDA $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$V = \{S\}$

$T = \{a, b, c\}$

$P: \to S \to aSa$

$S \to bSb$

$S \to c$

$S = S$

$Q = \{q\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{S, a, b, c\}$

$q_0 = \{q\}$

$Z_0 = S$

$F = \phi.$

δ:

for start symbol.

$\delta(q, \epsilon, Z_0) = \delta(q, S Z_0)$

for non-terminals:-

$\delta(q, \epsilon, S) = (q, aSa)$

$\delta(q, \epsilon, S) = (q, bSb)$

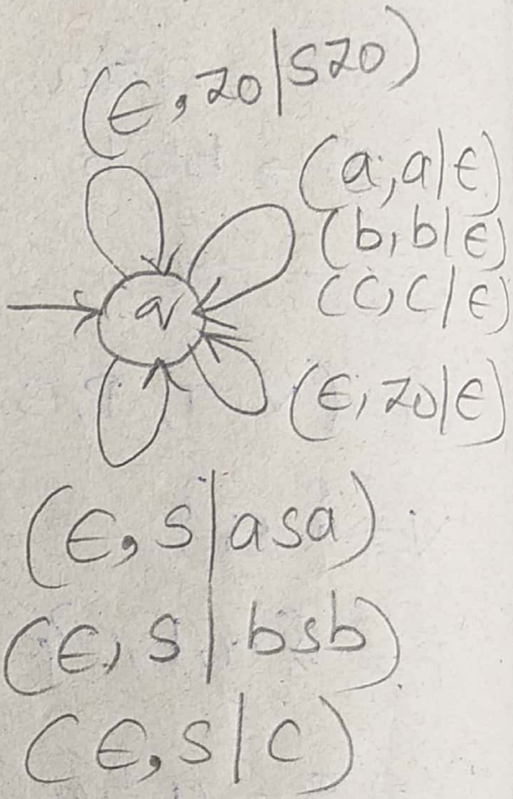
$\delta(q, \epsilon, S) = (q, c)$

for terminals:-

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\delta(q, c, c) = (q, \epsilon)$



for final state

$\delta(q, \epsilon, Z_0) = (q, \epsilon)$

abbcbba

$\delta(q, abbcbba, SZ_0)$

$\delta(q, abbcbba, aSaZ_0)$

$\delta(q, abbcbba, abSba)$

$\delta(q, abbcbba, abbSbba)$

$\delta(q, abbcbba, abbcbba)$

$\delta(q, \epsilon, Z_0)$

$\delta(q, \epsilon)$

3) $S \to 0S1 / A$

$A \to 1A0 | S | \epsilon$

$G = (V, T, P, S)$

$V = \{S, A\}$

$T = \{\epsilon, 0, 1\}$

P:- $S \to 0S1 / A$

$A \to 1A0 | S | \epsilon$

$S = S$

PDA:- $(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q\}$

$\Sigma = \{\epsilon, 0, 1\}$

$\Gamma = \{S, A, 0, 1\}$

$q_0 = \{q\}$

$z_0 = S$

$F = \phi$

δ:

for start symbol.

$\delta(q, \epsilon, z_0) = (q, Sz_0)$

for non-terminals:

$\delta(q, \epsilon, S) = (q, 0S1)$

$\delta(q, \epsilon, S) = (q, A)$

$\delta(q, \epsilon, A) = (q, 1A0)$

$\delta(q, \epsilon, A) = (q, S)$

$\delta(q, \epsilon, A) = (q, \epsilon)$

~~$\delta(q$~~

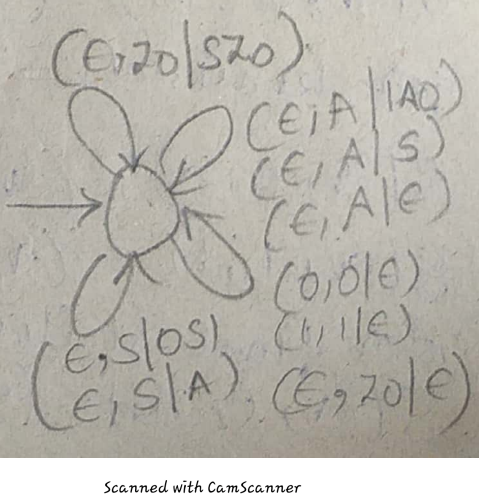
for terminals:-

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

for final state:-

$\delta(q, \epsilon, z_0) = (q, \epsilon)$



4) $S \to aABB \mid aAA$

$A \to aBB$

$B \to bBB \mid A$.

$G = \{V, T, P, S\}$

$V = \{S, A, B\}$

$T = \{a, b\}$

$S = S$

PDA:- $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q\}$

$\Sigma = \{a, b\}$

$\Gamma = \{S, A, B, a, b\}$

$q_0 = q$.

$Z_0 = S$ $\quad F = \phi$

δ:

for start symbol:-

$\delta(q, \epsilon, Z_0) = \delta(q, SZ_0)$

for non-terminals:-

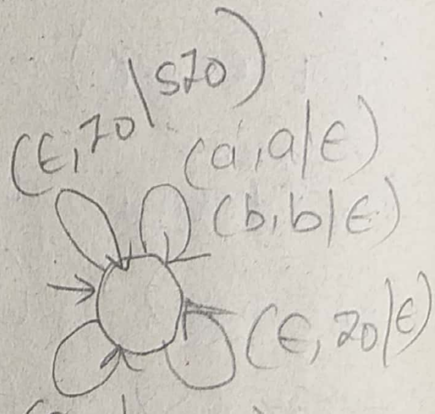
$\delta(q, \epsilon, S) = (q, aABB)$

$\delta(q, \epsilon, S) = (q, aAA)$

$\delta(q, \epsilon, A) = (q, aBB)$

$\delta(q, \epsilon, B) = (q, bBB)$

$\delta(q, \epsilon, B) = (q, A)$



(ε, S | aABB)

(ε, S | aAA)

(ε, A | aBB)

(ε, B | bBB)

(ε, B | A)

for terminals:-

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

for final state:-

$\delta(q, \epsilon, Z_0) = (q, \epsilon)$

5) $E \to E+E$

$E \to E*E$

$E \to a$

$G=(V,T,P,S)$

$V=\{E\}$

$T=\{a, +, *\}$

$S=S$

PDA:- $\{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$

$Q=\{q\}$ $q_0=q$

$\Sigma=\{a\}$ $Z_0=E$

$\Gamma=\{E, a, +, *\}$ $F=\phi$

δ:-

for start symbol

$\delta(q, \epsilon, Z_0) = (q, EZ_0)$

for non-terminals:-

$\delta(q, \epsilon, E) = (q, E+E)$

$\delta(q, \epsilon, E) = (q, E*E)$

$\delta(q, \epsilon, E) = (q, a)$

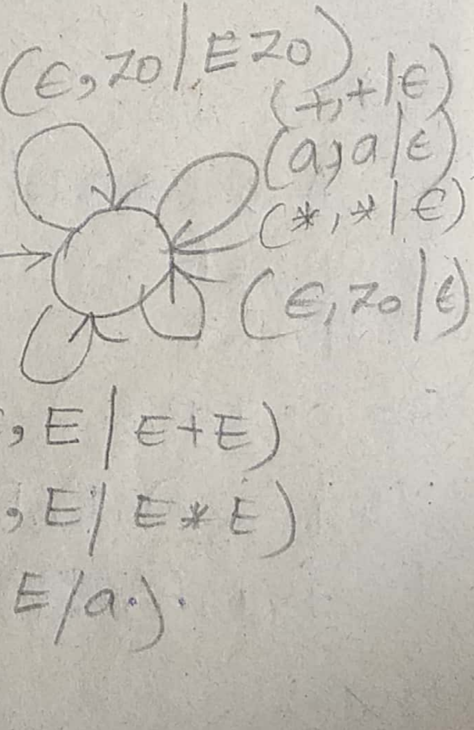
for terminals:-

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, +, +) = (q, \epsilon)$, $\delta(q, *, *) = (q, \epsilon)$

for final state:-

$\delta(q, \epsilon, z_0) = (q, \epsilon)$



6) $S \rightarrow aAA$

$A \rightarrow aS \mid bS \mid a$

$G = (V, T, P, S)$

$V = \{S, A\}$

$T = \{a, b\}$

$S = S$

PDA :- $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q\}$ $\quad q_0 = \{q\}$

$\Sigma = \{a, b\}$ $\quad Z_0 = S$

$\Gamma = \{S, A, a, b\}$ $\quad F = \phi$.

δ:

for start symbol

$\delta(q, \epsilon, Z_0) = (q, SZ_0)$

for non-terminals

$\delta(q, \epsilon, S) = (q, aAA)$

$\delta(q, \epsilon, A) = (q, aS)$

$\delta(q, \epsilon, A) = (q, bS)$

$\delta(q, \epsilon, A) = (q, a)$

for terminals :-

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

for final state :-

$\delta(q, \epsilon, Z_0) = (q, \epsilon)$

7) $S \rightarrow aA$

$A \rightarrow aABD \mid bB \mid a$

$B \rightarrow b$

$D \rightarrow d$

$G = (V, T, P, S)$

$V = \{S, A, B, D\}$

$T = \{a, b, d\}$

$S = S$

PDA :- $(Q, \Sigma, \Gamma, q_0, \delta, Z_0, F)$

$Q = \{q\}$

$\Sigma = \{a, b, d\}$

$\Gamma = \{S, A, B, D, a, b, d\}$

$q_0 = q$

$Z_0 = S$

$F = \phi$

δ:

for start symbol

$\delta(q, \epsilon, Z_0) = (q, SZ_0)$

for Non-terminals

$\delta(q, \epsilon, S) = (q, aA)$

$\delta(q, \epsilon, A) = (q, aABD)$

$\delta(q, \epsilon, A) = (q, bB)$

$\delta(q, \epsilon, A) = (q, a)$

$\delta(q, \epsilon, B) = (q, b)$

$\delta(q, \epsilon, D) = (q, d)$

for terminal

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\delta(q, d, d) = (q, \epsilon)$

for final state:-

$\delta(q, \epsilon, Z_0) = (q, \epsilon)$

8) $S \rightarrow aSbb \mid a$

$G = (V, T, P, S)$ PDA $= \{Q, \Sigma, \Gamma, \delta, Z_0, q_0, F\}$

$V = \{S\}$ $Q = \{q\}$ $Z_0 = S$

$T = \{a, b\}$ $\Sigma = \{a, b\}$ $q_0 = q$

$S = S$ $\Gamma = \{S, a, b\}$ $F = \phi$

δ:

for start symbol

$\delta(q, \epsilon, Z_0) = (q, SZ_0)$

for non-terminals

$\delta(q, \epsilon, S) = (q, aSbb)$

$\delta(q, \epsilon, S) = (q, a)$

for terminals:

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

for final state:

$\delta(q, \epsilon, Z_0) = (q, \epsilon)$

9) $S \to aABC$

$A \to aB|a$

$B \to bA|b$

$C \to a$

$G = (V, T, P, S)$

$V = \{S, A, B, C\}$

$T = \{a, b\}$

$S = S$

PDA :- $\{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$

$Q = \{q\}$ $\quad q_0 = q$

$\Sigma = \{a, b\}$ $\quad z_0 = S$

$F = \phi$

$\Gamma = \{S, A, B, C, a, b\}$

δ:

for start symbol:

$\delta(q, \epsilon, z_0) = (q, Sz_0)$

for Non-terminals

$\delta(q, \epsilon, S) = (q, aABC)$

$\delta(q, \epsilon, A) = (q, aB|a)$

$\delta(q, \epsilon, B) = (q, bA)$

$\delta(q, \epsilon, B) = (q, b)$

$\delta(q, \epsilon, C) = (q, a)$

$\delta(q, \epsilon, A) = (q, a)$

for terminal:-

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

for final state:-

$\delta(q, \epsilon, z_0) = (q, \epsilon)$

10) $S \to aABB | aAA$

$A \to aBB|a$

$B \to bBB|A$

$C \to a$

$G = (V, T, P, S)$

$V = \{S, A, B, C\}$

$T = \{a, b\}$

$S = S$

PDA :- $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q\}$ $\Gamma = \{S, A, B, C, a, b\}$

$\Sigma = \{a, b\}$ $q_0 = q$ $Z_0 = S$ $F = \phi$

δ:

for start symbol:

$\delta(q, \epsilon, Z_0) = (q, SZ_0)$

for Non-terminals:-

$\delta(q, \epsilon, S) = (q, aABB)$

$\delta(q, \epsilon, S) = (q, aAA)$

$\delta(q, \epsilon, A) = (q, aBB)$

$\delta(q, \epsilon, A) = (q, a)$

$\delta(q, \epsilon, B) = (q, bBB)$

$\delta(q, \epsilon, B) = (q, A)$

$\delta(q, \epsilon, C) = (q, a)$

for terminals:-

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

for final state:-

$\delta(q, \epsilon, Z_0) = (q, \epsilon)$

29/2/20

Conversion from PDA to CFG:

If $A = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$ is a PDA, when CFG is defined as $G = \{V, T, P, S\}$

1) construction of set of Non-terminals:

$V = \{S\} \cup \{[q, Z, q'] \mid q, q' \in Q, Z \in \Gamma\}$

2) construction of production:

i) S - production

$S \to [q_0, Z_0, q] \quad q \in Q$

ii) For pop-operation:

$\delta(q, a, Z) \to (q', \epsilon)$

$\boxed{(q, Z, q') \to a}$

iii) for push and NO operation (Read)

$\delta(q, a, Z) \to (q, Z_1, Z_2, \ldots Z_n)$

$[q, Z, q'] \to a[q_1, Z_1, q_2][q_2, Z_2, q_3] \ldots$
$\ldots [q_m, Z_m, q']$

Where $q', q_1, q_2, q_3 \ldots q_m \in Q$

problems:-

1) construct CFG from PDA

$A = [\{q_0, q_1\}, \{a, b\}, \{Z_0, Z\}, \delta, q_0, Z_0, \phi\}$

$\delta: \delta(q_0, b, Z_0) = (q_0, ZZ_0) \quad \delta(q_0, b, Z) = (q_0, ZZ)$

$\delta(q_1, b, Z) = (q_1, \epsilon) \quad \delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$

$\delta(q_0, a, Z) = (q_1, Z) \quad \delta(q_1, a, Z_0) = (q_0, Z_0)$

1) construction of set of non-terminals

$V = \{S \cup [q_0, z_0, q_0][q_0, z_0, q_1]$
$[q_0, z, q_0][q_0, z, q_1]$
$[q_1, z_0, q_0][q_1, z_0, q_1]$
$[q_1, z, q_0][q_1, z, q_1]\}$

2) construction of production

i) S - production

$S \rightarrow [q_0, z_0, q_0]$
$S \rightarrow [q_0, z_0, q_1]$

ii) for push operation

$$\delta(\underset{q}{q_0}, \underset{a}{b}, \underset{z}{z_0}) = (\underset{q_1}{q_0}, \underset{z_1}{z}\underset{z_2}{z_0})$$

$[q_0, z_0, q_0] \rightarrow b[q_0, z, q_0][q_0, z_0, q_0]$
$[q_0, z_0, q_1] \rightarrow b[q_0, z, q_0][q_0, z_0, q_1]$
$[q_0, z_0, q_0] \rightarrow b[q_0, z, q_1][q_1, z_0, q_0]$
$[q_0, z_0, q_1] \rightarrow b[q_0, z, q_1][q_1, z_0, q_1]$

$$\delta(\underset{q}{q_0}, \underset{a}{b}, \underset{z}{z}) = (\underset{q_1}{q_0}, \underset{z_1}{z}\underset{z_2}{z})$$

$[q_0, z, q_0] \rightarrow b[q_0, z, q_0][q_0, z, q_0]$
$[q_0, z, q_1] \rightarrow b[q_0, z, q_0][q_0, z, q_1]$
$[q_0, z, q_0] \rightarrow b[q_0, z, q_1][q_1, z, q_0]$
$[q_0, z, q_1] \rightarrow b[q_0, z, q_1][q_1, z, q_1]$

iii) pop - operation

$$\delta(\underset{q}{q_1}, \underset{a}{b}, \underset{z}{z}) = (\underset{q'}{q}, \epsilon)$$

$[q_1, z, q_1] \rightarrow b$

$\delta(\underset{q}{\underline{q_0}}, \underset{a}{\underline{\epsilon}}, \underset{Z}{\underline{z_0}}) = (\underset{q'}{\underline{q_0}}, \epsilon)$

$[q_0, z_0, q_0] \rightarrow \epsilon$

iv) No operation

$\delta(\underset{q}{\underline{q_0}}, \underset{a}{\underline{a}}, \underset{Z}{\underline{z}}) = (\underset{q_1}{\underline{q_1}}, \underset{Z_1}{\underline{z}})$

$[q_0, z, q_0] \rightarrow a[q_1, z, q_0]$

$[q_0, z, q_1] \rightarrow a[q_1, z, q_1]$

$\delta(\underset{q}{\underline{q_1}}, \underset{a}{\underline{a}}, \underset{Z}{\underline{z_0}}) = (\underset{q_1}{\underline{q_0}}, \underset{Z_1}{\underline{z_0}})$

$[q_1, z_0, q_0] \rightarrow a[q_0, z_0, q_0]$

$[q_1, z_0, q_1] \rightarrow a[q_0, z_0, q_1]$

These are the CFG is $\{V, T, P, S\}$

$V = \{S \cup [q_0, z_0, q_0] [q_0, z_0, q_1]$
$[q_0, z, q_0] [q_0, z, q_1]$
$[q_1, z_0, q_0] [q_1, z_0, q_1]$
$[q_1, z, q_0] [q_1, z, q_1] \}$

$T = \{a, b\}$

$S \rightarrow$ start symbol

production rule:

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

$[q_0, z_0, q_0] \rightarrow b[q_0, z, q_0][q_0, z_0, q_0]$

$[q_0, z_0, q_1] \rightarrow b[q_0, z, q_0][q_0, z_0, q_1]$

$[q_0, z_0, q_0] \rightarrow b[q_0, z, q_1][q_1, z_0, q_0]$

$[q_0, Z_0, q_1] \rightarrow b[q_0, Z, q_1][q_1, Z_0, q_1]$

$[q_0, Z, q_0] \rightarrow b[q_0, Z, q_0][q_0, Z, q_0]$

$[q_0, Z, q_1] \rightarrow b[q_0, Z, q_0][q_0, Z, q_1]$

$[q_0, Z, q_0] \rightarrow b[q_0, Z, q_1][q_0, Z, q_0]$

$[q_0, Z, q_1] \rightarrow b[q_0, Z, q_1][q_0, Z, q_1]$

$(q_1, Z, q_1) \rightarrow b$

$(q_0, Z_0, q_0] \rightarrow \epsilon$

$[q_0, Z, q_0] \rightarrow a[q_1, Z, q_0]$

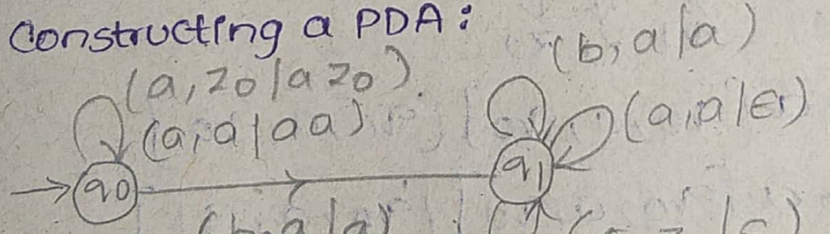
$[q_0, Z, q_1] \rightarrow a[q_1, Z, q_1]$

$[q_1, Z_0, q_0] \rightarrow a[q_0, Z_0, q_0]$

$[q_1, Z_0, q_1] \rightarrow a[q_0, Z_0, q_1]$

9) construct a PDA accepting language $L = \{a^n b^m a^n \mid m, n \geq 1\}$ by null storne and construct the CFG accepting same.

Constructing a PDA:



$\delta(q_0, a, Z_0) = (q_0, aZ_0)$ $\delta(q_1, b, a) = (q_1, a)$

$\delta(q_0, a, a) = (q_0, aa)$ $\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_0, b, a) = (q_1, a)$ $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

Constructing the CFG :- $\{\{q_0, q_1\}, \{a, b\}, \{Z_0\}, \delta, q_0, Z_0, \phi\}$

1) Construction of set of Non-terminals

$V = \{S \cup [q_0, Z_0, q_0][q_0, Z_0, q_1]$
$[q_1, Z_0, q_0][q_1, Z_0, q_1]$

$[q_0, a, q_0][q_0, a, q_1][q_1, a, q_0][q_1, a, q_1]\}$

2) Construction of production:-

i) S-production

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

ii) for push operation

$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\quad\; q \;\; a \;\; z \qquad q_1 \;\; z_1, z_2$

$[q_0, z_0, q_0] \rightarrow a[q_0, a, q_0][q_0, z_0, q_0]$

$[q_0, z_0, q_1] \rightarrow a[q_0, a, q_0][q_0, z_0, q_1]$

$[q_0, z_0, q_0] \rightarrow a[q_0, a, q_1][q_1, z_0, q_0]$

$[q_0, z_0, q_1] \rightarrow a[q_0, a, q_1][q_1, z_0, q_1]$

$\delta(q_0, a, a) = (q_0, aa)$

$\quad\; q \;\; a \;\; z \qquad q_1 \;\; z_1 \;\; z_2$

$[q_0, a, q_0] \rightarrow a[q_0, a, q_0][q_0, a, q_0]$

$[q_0, a, q_1] \rightarrow a[q_0, a, q_0][q_0, a, q_1]$

$[q_0, a, q_0] \rightarrow a[q_0, a, q_1][q_1, a, q_0]$

$[q_0, a, q_1] \rightarrow a[q_0, a, q_1][q_1, a, q_1]$

iii) pop operation

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\quad\; q \;\; a \;\; z \qquad q'$

$[q_1, a, q_1] \rightarrow a$

Scanned with CamScanner

$$\delta(\underset{q}{\underline{q_1}}, \underset{a}{\underline{\epsilon}}, \underset{z}{\underline{Z_0}}) = (\underset{q'}{\underline{q_1}}, \epsilon)$$

$$[q_1, Z_0, q_1] \rightarrow \epsilon$$

(iv) No operation

$$\delta(\underset{q}{\underline{q_0}}, \underset{a}{\underline{b}}, \underset{z}{\underline{a}}) = (\underset{q_1}{\underline{q_1}}, \underset{z_1}{\underline{a}})$$

$$[q_0, a, q_0] \rightarrow b[q_1, a, q_0]$$

$$[q_0, a, q_1] \rightarrow b[q_1, a, q_1]$$

$$\delta(\underset{q}{\underline{q_1}}, \underset{a}{\underline{b}}, \underset{z}{\underline{a}}) = (\underset{q_1}{\underline{q_1}}, \underset{z_1}{\underline{a}})$$

$$[q_1, a, q_0] \rightarrow b[q_1, a, q_0]$$

$$[q_1, a, q_1] \rightarrow b[q_1, a, q_1]$$