UNIT-2

RANDOM VARIABLES

Random variable :- A real variable 'x' whose value is determined by the outcome of a random experiment is called as Random variable.

→ this is also defined as a real valued function on the sample space 's' such that for each coin 'x' of the sample space. the corresponding probability occur.

Eg: when two coins are tossed at a time the sample space consists of $S = \{HH, HT, TH, TT\}$.

→ let 'x' be a random variable which takes the number of heads on 's'. then 'x' takes the values

$\therefore X = \{0, 1, 2\}$

X	0	1	2
$P(X=x_i)$ $= P(x_i)$ $= P_i$	$P(X=0)$ 1/4	$P(X=1)$ 2/4	$P(X=2)$ 1/4

Types of random variables :-

→ Random variables are of two types :-

i) Discrete Random variable

ii) continuous Random variable.

i) Discrete Random variable :- A random variable 'x' which can take only a finite no. of discrete values in any domain is called as a discrete random variable.

→ In other words if the random variable takes the values only on the set $\{0, 1, 2 - - - . n\}$.

Eg: The random variable denoting the number of students in a class i.e., $X(s) = \{x / x$ is a +ve integer$\}$

ii) continuous Random variable :-

A Random variable 'x' which can take values continuously i.e., which takes all the possible (positive) values in a given interval is called a continuous Random Variable.

Eg: the height, age, weight of individuals are examples of continuous Random variable.

probability function of discrete variable (probability mass function (PMF):-

If for discrete value 'x'

The real value function $P(X = x_i) = P(x_i)$ is called the probability function (or) probability mass function of a discrete random variable 'x'.

→ probability function $P(x_i)$ (of a random variabe 'x') posses the following properties:-

→ probability i) $P(x_i) \geq 0$

ii) $\sum_{i=1}^{n} P(x_i) = 1$ i.e., $P(x_i)$ cannot be negative for any value of 'x'.

→ the

'x'

P(x

P(x

Dist

CDF

sup

the

dis

It

Ex

dis

su

x_1

P

a

b

of

P

→ the probability distribution of a random variable 'x' given by the following table :-

X	x_1	x_2 ----	x_n
$P(X=x_i) = P(x_i)$	$P(X=x_1)$	$P(X=x_2)$ ---	$P(X=x_n)$

$$P(X<x_i) = P(X=x_1) + P(X=x_2) + \cdots + P(X=x_{i-1})$$

$$P(X\leq x_i) = P(X=x_1) + P(X=x_2) + \cdots + P(X=x_i)$$

Distribution function (cummulative distribution function CDF) :-

suppose that 'X' is a discrete random variable then the distribution function (or) cummulative distribution function is denoted by $F(x)$ and It is defined as $F(x) = P(X\leq x_i)$

Exceptation mean, variance of a probability distribution :-

suppose a random variable 'x' assumes the values $x_1, x_2, \cdots x_n$ with respective probabilities $P_1, P_2, \cdots P_n$ then the mathematical exceptation are mean are expected value of 'x' denoted by $E(X)$ is is defined as the sum of products of different values of 'x' and the corresponding probabilities i.e., $E(X) = x_1P_1 + x_2P_2 + \cdots + x_nP_n$

$$= \sum_{i=1}^{n} x_iP_i$$

theorem : ①

statement : If 'x' is a discrete Random variable and 'k' is a constant then ((exceptation of))

$$E(x+k) = E(x) + k$$

proof :- By defination of $E(x) = \sum_{i=1}^{x} x_i p_i$

$$E(x+k) = \sum_{i=1}^{n} (x_i + k) p_i$$

$$= \sum_{i=1}^{x} x_i p_i + k \sum_{i=1}^{x} p_i \Rightarrow E(x) + k(1)$$

$$E(x+k) = E(x) + k$$

theorem : ②

statement : 'x' is a discrete Random variable and 'k' is a constant then $E(kx) = kE(x)$

proof :- By defination of $E(x) = \sum_{i=1}^{x} x_i p_i$

$$E(kx) = \sum_{i=1}^{n} (k_i x_i) p_i$$

$$= \sum_{i=1}^{x} (x_i p_i)(k \sum_{i=1}^{x} p_i) \Rightarrow E(x)(k(1))$$

$$E(kx) = E(x)(k)$$

NOTE :- Mean of a discrete random variable is given by $\mu = \sum p_i x_i = E(x)$

Variance :- variance characterizes the variability in the distributions since two distributions with same mean can have different dispersions of data about their means.

→ variance of probability distribution of random variable 'x' is the mathematical exceptation of

$$V(x) = E[x - E(x)]^2 \text{ then}$$

$$= E[x^2 + (E(x))^2 - 2x\,E(x)]$$

$$= E(x^2) + [E(x))^2 - 2[E(x)]$$

$$= E(x^2) - [E(x)]^2$$

$$= \sum_{i=1}^{n} x_i^2 p_i - \left[\sum_{i=1}^{n} x_i p_i\right]^2$$

Standard Deviation:-

It is the positive square root of the variance denoted by 'σ'.

$$\sigma = \sqrt{V(x)}$$

problems:

1) The Random variable 'x' has the following probability function

X	0	1	2	3	4	5	6	7	8
P(X)	$\frac{k}{45}$	$\frac{k}{15}$	$\frac{k}{9}$	$\frac{k}{5}$	$\frac{2k}{45}$	$\frac{6k}{45}$	$\frac{7k}{45}$	$\frac{8k}{45}$	$\frac{4k}{45}$

i) find K

ii) $P(0<x<4)$

iii) $P(X \geq 5)$

iv) Mean

i) $\sum P(x) = 1$

$$\frac{k}{45} + \frac{k}{15} + \frac{k}{9} + \frac{k}{5} + \frac{2k}{45} + \frac{6k}{45} + \frac{7k}{45} + \frac{8k}{45} + \frac{4k}{45} = 1$$

$$\frac{k}{45} + \frac{3k}{45} + \frac{5k}{45} + \frac{9k}{45} + \frac{2k}{45} + \frac{6k}{45} + \frac{7k}{45} + \frac{8k}{45} + \frac{4k}{45} = 1$$

$$= \frac{k+3k+5k+9k+2k+6k+7k+8k+4k}{45} = 1$$

$$= \frac{45k}{45} = 1$$

$$\boxed{k=1}$$

ii) $P(0<x<4)$

$$= P(0<x<4) = P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{k}{15} + \frac{k}{9} + \frac{k}{5}$$

$$= \frac{1}{15} + \frac{1}{9} + \frac{1}{5}$$

$$= 0.377$$

iii) $P(x \geq 5)$

$$\Rightarrow P(x \geq 5) = P(x=5) + P(X=6) + P(X=7) + P(X=8)$$

$$= \frac{6k}{45} + \frac{7k}{45} + \frac{8k}{45} + \frac{4k}{45}$$

$$= \frac{6+7+8+4}{45}$$

$$= 0.556$$

iv) Mean $= \sum x_i p_i$

$$= 0 + \frac{1}{15} + \frac{2}{9} + \frac{3}{5} + \frac{8}{45} + \frac{30}{45} + \frac{42}{45} + \frac{56}{45} + \frac{32}{45}$$

$$= 4.62$$

2) for the following distributions.

x	-3	-2	-1	0	1	2	3
P(x)	0.001	0.01	0.1	-	0.1	0.01	0.001

i) find the missing probability

ii) mean

iii) variance.

$\Sigma P(x) = 1$

i) $P(x) = 0.001 + 0.01 + 0.1 + k + 0.1 + 0.01 + 0.001$

$1 = 0.222 + k$

$\boxed{k = 0.778}$

ii) mean $\Sigma x_i p_i = (-3)(0.001) + (-2)(0.01) + (-1)(0.1) + 0 + 1(0.1) + 2(0.01) + 3(0.001)$

$= -0.003 - 0.02 - 0.1 + 0 + 0.1 + 0.02 + 0.001$

$E(x) = 0$ (or) $\Sigma x P(x)$

iii) Variance $= E(x^2) - (E(x))^2$

$E(x^2) = (9(0.001) + (4(0.01)) + (1(0.1)) + (1(0.1)) + (4 \times 0.01) + 9(0.001)$

$= 0.009 + 0.04 + 0.1 + 0.1 + 0.04 + 0.009$

$\sigma = 0.298$

$\sigma = (0.298) - (0)$
$= 0.298$

3) If 'x' denotes the sum of two numbers that appears when a pair of fair dice is tossed

find: i) distribution function

ii) Mean

iii) variance

x	2	3	4	5	6	7	8	9	10	11	12
P(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

If 2 dice are thrown then the sum x of the two numbers which turn up must be integers between 2 and 12.

For ($x=2$) there is only favourable point (1,1) and Hence probability of $(x=2) = 1/36$

($x=3$) there is only favourable point are (1,2) (2,1) and hence the probability of $(x=3) = 2/36$

Illy we calculate other probabilities. Hence the the table is

$$\text{Mean} = \Sigma(x)(P(x))$$

$$= \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$

$$= \frac{252}{36}$$

$$= 7$$

$$\text{Variance} = E(x^2) - (E(x))^2 \qquad 2^2 \times \frac{1}{36} + 3^2 \times \frac{2}{36} + 4^2 \times \frac{3}{36}$$

$$E(x^2) = \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{252}{36} + \frac{320}{36} + \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36}$$

$$E(x^2) = 54.83$$

variance always +ve

$$\sigma = E(x^2) - (E(x))^2$$

$$= 54.83 - (7)^2 \Rightarrow 54.83 - 49$$

$$= 5.83$$

4)

x	0	1	2	3	4	5	6
$p(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

i) k

ii) $p(x \leq 4)$

iii) $p(3 < x \leq 6)$

iv) find the minimum of a so that $p(x \leq a) > 0.3$

i) $k+3k+5k+7k+9k+11k+13k=1$

$$49k=1$$

$$k=\frac{1}{49}$$

$$k=0.0204$$

ii) $p(3 < x \leq 6)$

$p(x=4)+p(x=5)+p(x=6)$

$= 9k+11k+13k$

$= 33(0.0204)$

$= 0.6732$

iii) $p(x \leq 4)$

$= p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)$

$= k+3k+5k+7k+9k \Rightarrow 25k$

$= 25(0.0204)$

$= 0.51$

iv) $a=0$

$p(x \leq 0)$

$= p(x=0)$

$\Rightarrow k = 0.0204 < 0.3$

a) $P(X \leq 1) = P(X=0) + P(X=1)$

$= k + 3k$

$= 4k$

$= 4(0.0204)$

$= 0.0816 < 0.3$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= k + 3k + 5k$

$= 7k$

$= 7(0.0204)$

$= 0.1836 < 0.3$

c) $P(X \leq 3) = P(X=0) + \cdots + P(X=3)$

$= k + 3k + 5k + 7k$

$= 0.32 > 0.3$

$\therefore \boxed{a = 3}$

Hence the 'a' value is 3.

5)

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	3k	k^2	$7k^2 + k$

i) k

ii) mean

i) k value

$\Rightarrow 0 + k + 2k + 2k + 3k + 3k + k^2 + 7k^2 + k$

$\Rightarrow 13k + 8k^2 = 1$

$\Rightarrow 8k^2 + 12k$

$\Rightarrow k = 0.0791,\ k = -1.579$

Calculate positive value

ii) mean

$\Sigma x p(x)$

$\Rightarrow 0+k+4k+6k+12k+15k+6k^2+49k^2+7k$

$\Rightarrow 45k+55k^2$

$\Rightarrow 45(0.0791)+55(0.0791)^2$

$= 3.5595+(0.3441)$

$= 3.9036$

5) 5 defective bolt are acciendtly mixed with 20 good once. find the probability distribution of the no of defective bolts. If 4 bolt are drawn at random from this slot and mean also.

$20 + \overset{\text{defective}}{5} = 25 \rightarrow \text{total}$

we have to find $p(X=0), p(X=1) \ldots p(X=4)$

for $p(X=0) = \frac{(20C_4)}{(25C_4)} = \frac{4845}{12650} = 0.3830$

$p(X=1) = \frac{(20C_3)(5C_1)}{25C_4} = \frac{1140\times5}{12650} = 0.4505$

$p(X=2) = \frac{(20C_2)(5C_2)}{25C_4} = \frac{190\times10}{12650} = 0.1501$

$p(X=3) = \frac{(20C_1)(5C_3)}{25C_4} = \frac{20\times10}{12650} = 0.0158$

$p(X=4) = \frac{(5C_4)}{25C_4} = \frac{5}{12650} = 0.000395$

X	0	1	2	3	4
P(X)	0.3830	0.4505	0.1501	0.0158	0.000395

$$Mean = \Sigma x P(x)$$

$$= 0 + 0.4505 + 0.031 + 0.04774 + 1.58 \times 10^{-7}$$

0.000158

$$= 0.7993$$

6) Find mean and various of the uniform probability distribution given by $f(x) = \frac{1}{n}$ for $(x = 1, 2, 3 \cdots n)$

x	1	2	3	4	------	n
P(x)	1/n	1/n	1/n	1/n		1/n

$$mean = \Sigma x P(x)$$

$$= \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \frac{4}{n} + \cdots + \frac{n}{n}$$

$$= \frac{1}{n}[1 + 2 + 3 \cdots n]$$

$$= \frac{1}{n}\left[\frac{n(n+1)}{2}\right]$$

$$= \frac{n+1}{2}$$

Variance :- $E(x^2) = \Sigma x^2 P(x)$

$$= 1^2\left(\frac{1}{n}\right) + 2^2 \times \frac{1}{n} + 3^2 \times \frac{1}{n} + 4^2 \times \frac{1}{n} \cdots n^2 \times \frac{1}{n}$$

$$= \frac{1}{n}[1^2 + 2^2 + 3^2 + \cdots + n^2]$$

$E(x^2) - (E(x))^2$

$$= \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$Variance = \left[\frac{(n+1)(2n+1)}{6}\right] - \left[\frac{(n+1)}{2}\right]^2$$

58×10^{-7}
000158

probability

3---n)

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{(2n+2)(2n+1) - 3(n^2+1+2n)}{12}$$

$$= \frac{(2n+2)(2n+1) - 3n^2 - 3 - 6n}{12}$$

$$= \frac{4n^2 + 2n + 4n + 2 - 3n^2 - 3 - 6n}{12}$$

$$= \frac{n^2 - 1}{12}$$

7) Find the expected value of x and SD of the following distribution

x	8	12	16	20	24
P(x)	1/8	1/6	3/8	1/4	1/12

mean $= \Sigma x P(x)$

$$= \frac{8}{8} + \frac{12}{6} + \frac{48}{8} + \frac{20}{4} + \frac{24}{12}$$

$$= 1 + 2 + 6 + 5 + 2$$

$$= 16$$

variance $= \frac{64}{8} + \frac{144}{6} + \frac{768}{8} + \frac{400}{4} + \frac{576}{12}$

$$= 8 + 24 + 96 + 100 + 48 - (16)^2$$

$$= 276 - 256$$

$$= 20$$

standard deviation $= \sqrt{20}$

$$= 4.4721$$

theorem: prove $E(x+y) = E(x) + E(y)$

proof:- let x assume the values $x_1, x_2 -- x_n$ and y assume the values $y_1, y_2 --- y_m$.

then by defination

$$E(x) = \sum_{i=1}^{n} x_i P_i \,, \quad E(y) = \sum_{j=1}^{m} y_j P_j$$

the sum$(x+y)$ is also a random variable which takes $(n+m)$ values with $(x_i + y_j)$, $i = 1, 2, -- n$, $j = 1, 2, --- m$ with P_{ij}

then by defination

$$E(x+y) = \sum_{i=1}^{n} \sum_{j=1}^{m} (x+y) P_{ij}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i P_{ij} + y_i P_{ij})$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} x_i P_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} y_i P_{ij}$$

$$= \sum_{i=1}^{n} x_i \left(\sum_{j=1}^{m} P_{ij} \right) + \sum_{j=1}^{m} y_j \left(\sum_{i=1}^{n} P_{ii} \right)$$

$$= \sum_{i=1}^{n} x_i P_i + \sum_{j=1}^{m} y_i P_j$$

$$= E(x) + E(y)$$

NOTE:-

i) $E(ax + by) = aE(x) + bE(y)$

ii) $E(x - \bar{x}) = 0$; $\bar{x}$ = mean

iii) $E(ax+b) = aE(x) + b$

theorem :- x, y are two independent random variables then $E(xy) = E(x)\,E(y)$.

proof :- let $x \to x_1, x_2 \ldots x_n$ with probabilites $P_1, P_2 \ldots P_n$

$y \to y_1, y_2 \ldots y_n$ with probability $P_1, P_2, \ldots P_n$

Here x & y are random variable

then By defination

$$E(x) = \sum_{i=1}^{n} x_i P_i$$

also $$E(y) = \sum_{j=1}^{m} y_j P_j$$

x, y are independent variables

$$P(x = x_i \cap y = y_i) = P(x = x_i)\,P(y = y_j)$$

$$P_{ij} = P_i P_j$$

$$E(xy) = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i y_j) P_{ij}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i y_j) P_i P_j$$

$$= \sum_{i=1}^{n} (x_i P_i) \sum_{j=1}^{m} (y_j P_j)$$

$$= E(x) \cdot E(y)$$

Result of variances :-

1) variance of a constant = 0

$$V(k) = 0$$

2) Variance of constant

$$V(x+k) = V(x)$$

3) variance of some constant $V(ax+b) = a^2 V(x)$

theorem:- $V(ax+b) = a^2 V(x)$

proof:- let x is a random variable which takes the values $x_1, x_2, \ldots x_n$ with the probability as $p_1, p_2 \ldots p_n$

let $y = ax+b \rightarrow ①$ (expectation on both sides)

$$E(y) = E(ax+b)$$

$$= aE(x)+b \rightarrow ②$$

Now ① - ②

$$y - E(y) = ax + \cancel{b} - aE(x) - \cancel{b}$$

$$y - E(y) = ax - aE(x)$$

$$y - E(y) = a[x - E(x)]$$

$$E[y - E(y)]^2 = E[a^2(x - E(x))^2]$$

$$V(y) = a^2 E(x - E(x))^2$$

$$V(y) = a^2 V(x)$$

$$V(ax+b) = a^2 V(x)$$

Note:- If x, y are independent random variables then $V(x \pm y) = V(x) \pm V(y)$.

Eg: the probability distribution function of a random variable

x	-2	-1	0	1	2	3
$P(x)$	0.2	0.1	0.3	0.3	0.1	x

find: 1) $E(x)$

2) variance of x

3) $E(2x-3)$

4) $V(2x-3)$

i) $E(X) = \sum_{i=1}^{n} x_i P(x_i)$

$= (-2)(0.2) + (-1)(0.1) + (0)(0.3) + (1)(0.3) + (2)(0.1)$

$= 0$

ii) $V(x) = E(x^2) - (E(x))^2$

$= 4(0.2) + 1(0.1) + (0.3) + 4(0.1)$

$= 0.8 + 0.1 + 0.3 + 0.4$

$= 1.6$

$= 1.6 - 0$

$V(x) = 1.6$

iii) $E(2X-3)$ $[E(aX+b) = aE(X)+b]$

Here $a=2$ $b=-3$

$= (2E(X)-3)$

$= 2 \times (0) - 3$

$= -3$

iv) $V(2X-3) = 2^2 V(X)$ $[V(aX+b) = a^2 V(x)]$

$= 4 \times 1.6$

$= 6.4$

If 3 cars are selected from lot of 6 cars. containing 2 defective. find the probability distribution of the no. of defective cars. Hence find it mean and variance.

out of 6 cars 4 are non-defective and 2 are defective cars non defective

And we are drawing 3 cars that canbe done in $6C_3$ ways

$6C_3 = 20$

$$P(x=0) = \frac{4C_3}{6C_3} = \frac{4}{20} = \frac{1}{5} = 0.2$$

$$P(x=1) = \frac{4C_2 \times 2C_1}{6C_3} = \frac{6\times 2}{20} = \frac{12}{20} = 0.6$$

$$P(x=2) = \frac{4C_1 \times 2C_2}{6C_3} = \frac{4\times 1}{20} = \frac{4}{20} = 0.2$$

distribution table:-

x	0	1	2
$P(x=x_i)$	0.2	0.6	0.2

mean $= 0(0.2) + (0.6) + (0.4)$

$= 1$

variance $= 0 + 0.6 + 0.8$

$= 1.4$

$\Rightarrow 1.4 - (1)^2$

$= 0.4$

2) A fair dice is tossed let the random variable 'x' denote the twice the no appearing on the dice. write probability distribution of x. Hence find it standard deviation.

distribution table:

x	2	4	6	8	10	12
P_i	1/6	1/6	1/6	1/6	1/6	1/6

x is the random variable which takes the twice the no appearing on the dice

$$\text{mean} = \frac{2}{6} + \frac{4}{6} + \frac{6}{6} + \frac{8}{6} + \frac{10}{6} + \frac{12}{6}$$

$= 7$

Varianc

$\Rightarrow$

Results

1) Vari

2) V(

3) V(a

proof:

let

the

$P_1, P_2,$

le

①.

$$\text{Variance} = \frac{4}{6} + \frac{16}{6} + \frac{36}{6} + \frac{64}{6} + \frac{100}{6} + \frac{144}{6}$$

$$= 60.6666$$

$$\Rightarrow 60.6666 - 49$$

$$11.6666 = \text{variance}$$

$$\text{standard deviation} = \sqrt{\text{variance}}$$

$$= \sqrt{11.6666}$$

$$= 3.4156$$

Results on variance :-

1) Variance of constant $= 0 \Rightarrow V(K) = 0$

2) $V(X+K) = V(X)$

3) $V(ax+b) = a^2 V(x)$

↑③ one

proof:-

Let 'x' is a random variable which takes the value $x_1, x_2, \ldots x_n$ with probabilites $p_1, p_2, \ldots p_n$.

Let $y = ax + b \rightarrow ①$

Taking Exceptations on B.S

$$E(Y) = E(ax+b)$$

$$= aE(x) + b \rightarrow ②$$

① - ②

$$y - E(y) = ax + \cancel{b} - aE(x) - \cancel{b}$$

$$Y - E(Y) = a(X - E(X))$$

squaring on B.S and taking exceptation on B.S

$$E(Y - E(Y))^2 = E[a^2(X - E(X))^2]$$

$$V(Y) = a^2 E(X - E(X))^2$$

$$V(Y) = a^2 V(X)$$

$$V(aX + b) = a^2 V(X)$$

NOTE:- If X and Y are independent random variables.

$$V(X \pm Y) = V(X) \pm V(Y)$$

Eg: the probability distribution of a random variable is given by

X	-2	-1	0	1	2
P(x)	0.2	0.1	0.3	0.3	0.1

find: i) $E(X)$

ii) $V(X)$

iii) $E(2X-3)$

iv) $V(2X-3)$

i) $E(X) = \sum_{i=1}^{n} x_i P(x_i)$

$= -2(0.2) + -1(0.1) + 0 + 1(0.3) + 2(0.1)$

$= 0$

ii) $V(X) =$

$E(X$

$V($

iii) $E(2$

iv) $V(2X$

If 3 co

two def

of the

and va

out of

the def

that co

probabi

lly

(ii) $V(X) = E(X^2) - (E(X))^2$

$E(X^2) = \Sigma x_i^2 P(x_i)$

$= (-2)^2 \times (0.2) + 1(0.1) + 0 + 1(0.3) + (-2)^2 \times (0.1)$

$= 1.6$

$V(X) = 1.6 - (0)^2 \Rightarrow V(X) = E(X^2) - (E(X))^2$

$= (1.6) - (0)^2$

$= 1.6$

(iii) $E(2X-3) = 2E(X) - 3$

$= 2(0) - 3$

$= -3$

(iv) $V(2X-3) = 2^2 V(X)$ $a^2 V(x)$

$= 4(1.6)$

$= 6.4$

If 3 cars selected from a lot of 6 cars containing two defective find the probability distribution of the no of defective cars. Hence find its mean, and variance. (repeated prblm)

out of 6 cars 4 are non defective and 2 are the defective cars and we are drawing 3 cars that can be done in $6C_3$ ways.

probability of zero defective cars $P(X=0) = \frac{4C_3}{6C_3}$

$= 0.22$

IIly $P(X=1) = \frac{(4C_2)(2C_1)}{6C_3} = 0.6$

$P(X=2) = \frac{(4C_1)(2C_2)}{6C_3} = 0.2$

distribution table:-

X	0	1	2
$P(X=x_i)$	0.22	0.6	0.2

$$E(x) = \sum_{i=1}^{n} x_i P(x_i)$$

$= 0(0.22) + 1(0.6) + 2(0.2)$

$= 0.6 + 0.4$

$= 1$

$E(x^2) = \sum x_i^2 p(x_i)$

$V(x) = E(x^2) - (E(x))^2$

$= 1.4 - 1$

$= 0.4$

$= 0(0.22) + 1(0.6) + 4(0.2)$

$= 0.6 + 0.8$

$= 1.4$

A fair dice is tossed. Let the random variable 'x' denote twice the number appearing on the x. write the probability distribution of 'x' and hence find the standard deviation. (repeated problm)

(distribution table:-)

'x' is the random variable which takes twice the number appearing on the dice.

distribution table:-

X	2	4	6	8	10	12
$P(X=x_i)$	1/6	1/6	1/6	1/6	1/6	1/6

$$E(x) = \sum_{i=}^{n} x_i P(x_i)$$

$= 2(1/6) + 4(1/6) + 6(1/6) + 8(1/6) + 10(1/6) + 12(1/6)$

$E(x) =$

$E(x^2) = \sum x_i^2$

$= 4(1/16) + 1$

$= 60.666$

$V(x) = E($

$= 7$

$=$

continuous R

when a ran

in an interva

variable (CRV

of 'x'.

Eg: the dist

heights, ar

probability de

f(x) is calle

density func

stasify the

i) $\int_{-\infty}^{\infty} f(x)\, d$

ii) $f(x) \geq 0$

NOTE:

→ $P(a \leq x$

$E(X) = 7$.

$E(X^2) = \sum x_i^2 \cdot P(x_i)$

$= 4(1/6) + 16(1/6) + 36(1/6) + 64(1/6) + 100(1/6) + 144(1/6)$

$= 60.6667$

$V(X) = E(X^2) - (E(X))^2$

$= \cancel{7} - (60.6667) - (49)$

$= 11.6667$

continuous Random variable :-

when a random variable 'x' takes every value in an interval is called continuous random variable (CRV) and it is called continuous distribution of 'x'.

Eg: the distribution defined by variates like temp, heights, and weights are continuous distributions

probability density function (Pdf) :-

$f(x)$ is called the probability density function (or) density function of the variable 'x'. If it has to stasify the following two properties.

i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ii) $f(x) \geq 0$; where $-\infty \leq x \leq \infty$ and $f(x)$ is pdf of 'x'.

NOTE:

→ $P(a \leq x \leq b) = \int_a^b f(x)\,dx$

Cummulative distribution function (cdf) of a continuous random variable :-

If 'x' is a continuous random variable with pdf $f(x)$ then it's cdf is $F(x)$ and it is defined as $F(x) = \int_{-\infty}^{x} f(x)\,dx$

Properties of $F(x)$:-

i) $0 \leq F(x) \leq 1$; where $-\infty < x < \infty$

ii) $F(-\infty) = 0$

iii) $F(\infty) = 1$

iv) $P(a \leq x \leq b) = \int_{a}^{b} f(x)\,dx = F(b) - F(a)$

v) since $F(x) = \int_{-\infty}^{x} f(x)$

$$f(x) = \frac{d}{dx} F(x)$$

Mean, variance of a continuous random variable:-

→ If 'x' is a continuous random variable.

i) Mean $= \mu = E(x)$

$$= \int_{-\infty}^{\infty} x f(x)\,dx$$

ii) variance $= E(x^2) - (E(x))^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$\therefore V(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left[\int_{-\infty}^{\infty} x f(x)\,dx\right]^2$$

problems:

1) the probability density function of a continuous random variable $f(x) = \begin{cases} (1/2)(x+1) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

find : i) Mean

ii) variance

i) Mean $= E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$$= \int_{-\infty}^{-1} x f(x)\, dx + \int_{-1}^{1} x f(x)\, dx + \int_{1}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{-1} 0\, dx + \int_{-1}^{1} \left[\left(\tfrac{1}{2}(x+1)\right) x\right] dx + \int_{1}^{\infty} 0\, dx$$

$$= 0 + \frac{1}{2} \int_{-1}^{1} (x^2 + x)\, dx$$

$$= \frac{1}{2}\left[\left(\frac{x^3}{3}\right)_{-1}^{1} + \left(\frac{x^2}{2}\right)_{-1}^{1}\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{3} + \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{3}\right] = 1/3$$

ii) variance $= E(X^2) - (E(X))^2$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{-1} x^2 f(x)\, dx + \int_{-1}^{1} x^2 f(x)\, dx + \int_{1}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{-1} 0\, dx + \int_{-1}^{1} \left(\tfrac{1}{2}(x+1)\right) x^2 \int_{1}^{\infty} 0\, dx$$

$$= 0 + \frac{1}{2} \int_{-1}^{1} (x^3 + x^2) + \int_{1}^{\infty} 0\, dx$$

$$= 0 + \frac{1}{2}\left[\left[\frac{x^4}{4}\right]_{-1}^{1} + \left[\frac{x^3}{3}\right]_{-1}^{1}\right]$$

$$= 0 + \frac{1}{2}\left[\left(\frac{1}{4} - \frac{1}{4}\right) + \left[\frac{1}{3} + \frac{1}{3}\right]\right]$$

$$= \frac{1}{2}\left(0 + \frac{2}{3}\right)$$

$$= \frac{1}{2}\left(\frac{2}{3}\right)$$

$$= \frac{1}{3} - (\text{mean})^2$$

$$= \frac{1}{3} - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{3} - \frac{1}{9}$$

$$= \frac{3-1}{9}$$

$$= \frac{2}{9}.$$

2) For the continuous probability function

$$f(x) = \begin{cases} k x^2 e^{-x} & x \geq 0 \end{cases}$$

find : i) k

ii) Mean

iii) Variance

i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{\infty} k x^2 e^{-x}\,dx = 1$$

$= k\left[\int_0^\infty x\right.$

$= k\left[x^2(-\right.$

$= k[0+2]$

$2k = 1$

$\boxed{k = 1/2}$

Mean $= E(x) =$

$=$

$= 0$

$=$

$=$

$=$

$= 3$

$E(x^2) =$

$$= k\left[\int_0^\infty x^2 e^{-x}\,dx\right] = 1$$

$$= k\left[x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right]_0^\infty = 1$$

$$= k[0+2] = 1$$

$$2k = 1$$

$$\boxed{k = 1/2}$$

$$\text{Mean} = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_0^\infty x f(x)\,dx$$

$$= 0 + \int_0^\infty x\, k x^2 e^{-x}\,dx \Rightarrow 0 + \int_0^\infty x\left(\tfrac{1}{2}\right) x^2 e^{-x}\,dx$$

$$= \frac{1}{2}\int_0^\infty x^3 e^{-x}\,dx$$

$$= \frac{1}{2}\left[x^3(-e^{-x}) - 3x^2(e^{-x}) + 6x(-e^{-x}) - 6(e^{-x})\right]_0^\infty$$

$$= \frac{1}{2}[0+6]$$

$$= 3$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{-\infty}^{0} x^2 f(x)\,dx + \int_0^\infty x^2 f(x)\,dx$$

$$= 0 + \int_0^\infty x^2 k x^2 e^{-x}\,dx$$

$$= K\int_0^\infty x^4 e^{-x}\,dx$$

$$= K\left[x^4(-e^{-x}) - 4x^3(e^{-x}) + 12x^2(-e^{-x}) - 24x(e^{-x}) + 24(-e^{-x})\right]_0^\infty$$

$$E(X^2) = K[0+24]$$

$$= \frac{1}{2}(24)$$

$$= 12$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= 12 - 3^2$$

$$= 3$$

3) probability density function of a continuous random variable

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & -3 \le x \le -1 \\ \frac{1}{16}(2-6x^2) & -1 \le x \le 1 \\ \frac{1}{16}(3-x)^2 & 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

show that the are under the curve above x-axis is unity also find its mean.

To prove $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\left(\int_{-\infty}^{-3} + \int_{-3}^{-1} + \int_{-1}^{1} + \int_{1}^{3} + \int_{3}^{\infty}\right) f(x)\,dx$$

$$= 0 + \int_{-3}^{-1} \frac{1}{16}(3+x)^2\,dx + \int_{-1}^{1} \frac{1}{16}(2-6x^2)\,dx + \int_{1}^{3} \frac{1}{16}(3-x)^2\,dx + 0$$

$$= \frac{1}{16}\int_{-3}^{-1}(9+6x+$$

$$= \frac{1}{16}\left[9x + \frac{6x}{2}\right.$$

$$= \frac{1}{16}\left[9(-1+3)+\right.$$

$$= \frac{1}{16}\left[9(2)+3\right.$$

$$= \frac{1}{16}(18-24$$

$$= \frac{1}{16}\left[1\right.$$

$$= \frac{1}{16}$$

$\therefore$ f(x)

4) the frequ

variable i

f(x) =

given th

we k

P(a

P(

$$= \frac{1}{16}\int_{-3}^{-1}(9+6x+x^2) + \frac{1}{16}\int_{-1}^{1}(2-6x^2) + \frac{1}{16}\left[\int_{1}^{3}(9+x^2-6x)\,dx\right]$$

$$= \frac{1}{16}\left[9x + \frac{\cancel{6}^{3}x^2}{\cancel{2}} + \frac{x^3}{3}\right]_{-3}^{-1} + \frac{1}{16}\left(2x - \frac{\cancel{6}^{2}x^3}{\cancel{3}}\right)_{-1}^{1} + \frac{1}{16}\left(9x + \frac{x^3}{3} - \frac{\cancel{6}^{3}x^2}{\cancel{2}}\right)_{1}^{3}$$

$$= \frac{1}{16}\left[9(-1+3) + 3(1-9) + \frac{-1}{3} + \frac{27}{3}\right] + \frac{1}{16}\left(2(1+1) - 2(1+1)\right)$$

$$+ \frac{1}{16}\left[9(3-1) + \frac{27}{3} - \frac{1}{3} - 3(9-1)\right]$$

$$= \frac{1}{16}\left[9(2) + 3(-8) + \frac{26}{3}\right] + \frac{1}{16}\left[2(2) - 2(2)\right] + \frac{1}{16}\left[9(2) + \frac{26}{3} - 3(8)\right]$$

$$= \frac{1}{16}\left(18 - 24 + \frac{26}{3}\right) + \frac{1}{16}(0) + \frac{1}{16}\left[18 - 24 + \frac{26}{3}\right]$$

$$= \frac{1}{16}\left[18 - 24 + \frac{26}{3}\right] + 18 - 24 + \frac{26}{3}\Big]$$

$$= \frac{1}{16}\left(\frac{16}{3}\right) = \frac{1}{3} \neq 1$$

$\therefore$ $f(x)$ is not probability density function.

4) the frequency function of continuous random variable is given by

$$f(x) = \begin{cases} kx(x-1) & 1 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

given that $P(1 \le x \le 3) = \frac{28}{3}$ find the value of k.

we know that

$$P(a \le x \le b) = \int_a^b f(x)\,dx$$

$$P(1 \le x \le 3) = \int_1^3 f(x)\,dx = \frac{28}{3}$$

$$\int_1^3 Kx(x-1)\,dx = \frac{28}{3}$$

$$= K\int_1^3 (x^2 - x)\,dx = \frac{28}{3}$$

doubt (✓)

$$= K\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^3 = \frac{28}{3}$$

$$\left(= K\left[\left(\frac{27}{3} - \frac{1}{3}\right) - \left(\frac{9}{2} - \frac{1}{2}\right)\right] = \frac{28}{3}\right) \text{ wrong}$$

$$= K\left[\frac{26}{3} - 4\right] = \frac{28}{3}$$

$$= K\left[\frac{14}{3}\right] = \frac{28}{3}$$

$$\Rightarrow K = \frac{28}{3} \times \frac{3}{14}$$

$$\boxed{K = 2}$$

$$= K\left[\frac{27}{3} - \frac{9}{2} - \left(\frac{1}{3} - \left(\frac{1}{2}\right)\right)\right]$$

$$= K\left[9 - \frac{9}{2} - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$

$$= K\left(\frac{9}{2} - \left(\frac{-1}{6}\right)\right)$$

$$= K\left(\frac{9}{2} + \frac{1}{6}\right)$$

$$= K\left(\frac{14}{3}\right) = \frac{28}{3}$$

$$K = \frac{28}{3} \times \frac{3}{14}$$

$$\boxed{K = 2}$$

Show that the function

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty \le x \le \infty \text{ is a pdf.}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{(1+x^2)}$$

$$\frac{1}{\pi}\left[\tan^{-1}(x)\right]_{-\infty}^{\infty}$$

$$\frac{1}{\pi}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2} + \frac{\pi}{2}\right]$$

$$= \frac{2\pi}{2} \times \frac{1}{\pi}$$

$$= 1$$

A continuous

distribution fun

$$F(x) = \begin{cases} 0 \\ 1 - \frac{9}{x} \\ \end{cases}$$

find :- i) pdf

ii) Mea

i) pdf

$$f(x) = \frac{d}{dx} F$$

$$= \begin{cases} 0 \\ 0- \end{cases}$$

$$= \begin{cases} 0 \\ \frac{27}{x} \end{cases}$$

$E(x)$

Mean =

$= \frac{2\pi}{2} \times \frac{1}{\pi}$

$= 1$

A continuous random variable 'x' whose distribution function is given by given cdf & find pdf

$$F(x) = \begin{cases} 0 & \text{for } x \leq 3 \\ 1 - \frac{9}{x^3} & \text{for } x > 3 \end{cases}$$

$x^3 = 3x^2$

$\frac{1}{x^3} = \frac{-3}{x^4}$

$\frac{1}{x^2} = \frac{-2}{x^3}$

find :- i) pdf

ii) Mean

i) pdf

$$f(x) = \frac{d}{dx} F(x)$$

$$= \begin{cases} 0 & \text{for } x \leq 3 \\ 0 - 9 \times \left(\frac{-3}{x^4}\right) & x > 3 \end{cases} \quad \frac{x^{-3+1}}{-3+1} \quad x^{-3}$$

$$= \begin{cases} 0 & \text{for } x \leq 3 \\ \frac{27}{x^4} & x > 3 \end{cases}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx \qquad \int_{-\infty}^{\infty} x f(x) dx$$

$$\text{Mean} = E(x) = \int_{3}^{\infty} x \frac{27}{x^4}\, dx$$

$$= \int_{3}^{\infty} \frac{27}{x^3}\, dx$$

$$= 27 \int_{3}^{\infty} x^{-3}\, dx$$

$$= 27\left[\frac{x^{-2}}{-2}\right]_3^{\infty}$$

$$= \frac{-27}{2}\left[x^{-2}\right]_3^{\infty} \Rightarrow \frac{-27}{2}\left(0 - 3^{-2}\right)$$

$$= -\frac{27}{2}\left[-\frac{1}{9}\right] = 3/2$$

the cdf of a CRV 'x' is

$$F(x) = \begin{cases} 1 - e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

find i) f(x)

ii) Mean

iii) variance.

i) $\text{Pdf} = \frac{d}{dx} f(x)$ $\qquad f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

$$\int_{-\infty}^{\infty} f(x)\, dx \qquad \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$= \begin{cases} 0 - (-e^{-2x} \times 2) & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases} \qquad 0 + \int_0^{\infty} 1 - e^{-2x}\, dx = 1$$

$$(1 - e^{-2x})\,dx = 1$$

$$= \begin{cases} 2x e^{-2x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases} \qquad 0 - (-2)e^{-2x} = 2e^{-2x}$$

ii) Mean $= E(X) = \int_{-\infty}$

$= ($

$= 2$

$= 2$

$=$

$=$

iii) variance $E(X^2) =$

$[E(X^2) - (E(X))^2)$

$=$

A continuous

following prob

function $F(x$

$f(x) =$

find: i) $F(x$

i) $F(x) = $ C.d

ii) Mean $= E(X) = \int_{-\infty}^{\infty} x f(x) dx$

$$= \left(\int_{-\infty}^{0} + \int_{0}^{\infty}\right) x f(x) dx$$

$$= 2\int_0^\infty x e^{-2x} dx$$

$$= 2\left[x\left(\frac{e^{-2x}}{-2}\right) - 1\left(\frac{e^{-2x}}{-2x-2}\right)\right]_0^\infty \Rightarrow 2\left[(0-0) - \frac{1}{4}(0-1)\right]$$

$$= 2\left(\frac{1}{4}\right) = 1/2$$

$$= 2\left[0 + \frac{1}{4} \times 1\right] \Rightarrow 2\left(\frac{1}{4}\right)$$

$$= 1/2$$

iii) variance $E(X^2) = \int_0^\infty x^2 2e^{-2x} dx$

$[E(X^2) - (E(X))^2]$

$$= \left[\frac{x^3}{3} \cdot 2 \cdot e^{-2x}\right]_0^\infty$$

$$= \left[(0-0) - 1\left(\frac{e^{-2x}}{-2(-2)}\right)_0^\infty\right]$$

$$= \left((0-0) - \frac{1}{4}(0-1)\right)$$

$$= \frac{1}{4}$$

A continuous random variable 'x' has the following probability function. find the distribution function $F(x)$ and $P(1 \le x \le 2)$

$$f(x) = \begin{cases} Kx^2 & \text{in } 0<x<3 \\ 0 & \text{otherwise} \end{cases}$$

find: i) $F(x)$ ii) $P(1 < x \le 2)$

i) $F(x) = \text{c.d.f} = \int_{-\infty}^{x} f(x) dx$

$$F(x) = \int_{-\infty}^{x} kx^2\,dx$$

To find k :-

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\left(\int_{-\infty}^{0} + \int_{0}^{3} + \int_{3}^{\infty}\right) f(x)\,dx = 1$$

$$0 + \int_{0}^{3} kx^2\,dx + 0 = 1$$

$$k\left(\frac{x^3}{3}\right)_0^3 = 1$$

$$\frac{k}{3}(27-0) = 1$$

$$27k = k$$

$$k = \frac{3}{27} = \frac{1}{9} \qquad \boxed{k = \frac{1}{9}}$$

$$F(x) = \int_{-\infty}^{x} kx^2\,dx$$

$$= \int_{-\infty}^{x} 0\,dx + \int_{0}^{x} kx^2\,dx$$

$$= 0 + k\left(\frac{x^3}{3}\right)_0^x$$

$$= \frac{1}{27}(x^3 - 0) = \frac{x^3}{27}$$

i) $P(1 < x \leq 2)$

$$= \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx$$

$$= 0 + \int_{1}^{2} kx^2\,dx \ldots$$

$$= k\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{27}[8-1]$$

$$= 7/27$$

A continuous ran...

function is given ...

$$f(x) = \begin{cases} \frac{1}{4} \ldots \\ 0 \ldots \end{cases}$$

find : mean & va...

Mean = E(x...

$E(x) = \Big[\big(\ldots$

= 0 ...

= 1/...

= 1/4 ...

= 1/4

= 1/...

= 4 ...

$$= 0 + \int_1^2 kx^2\,dx + 0$$

$$= k\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{27}[8-1]$$

$$= 7/27$$

$$\int f(x)\,dx = 1$$

$$\int_1^2 kx^2\,dx = 1$$

$$k\left(\frac{x^3}{3}\right)_1^2$$

$$= \frac{8}{9} - \frac{1}{3}$$

$$= \frac{7}{27}$$

A continuous random variable probability density function is given by

$$f(x) = \begin{cases} \frac{1}{4} e^{-x/4} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

find: mean & variance.

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} x\, f(x)\,dx$$

$$E(X) = \left[\left(\int_{-\infty}^{0} + \int_{0}^{\infty}\right) x\, f(x)\,dx\right]$$

$$= 0 + \int_0^\infty x \frac{1}{4} e^{-x/4}\,dx$$

$$e^{-ax} = \frac{e^{-ax}}{-a}$$

$$= \frac{1}{4}\int_0^\infty x e^{-x/4}\,dx$$

$$= \frac{1}{4}\left[x\left(\frac{e^{-x/4}}{-1/4}\right) - 1 \times \left(\frac{e^{-x/4}}{-1/4 \times -1/4}\right)\right]_0^\infty$$

$$= \frac{1}{4}\left[0 - \left(0 - \frac{1}{1/16}\right)\right]$$

$$= \frac{1}{4}(16)$$

$$= 4$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \left(\int_{-\infty}^{0} + \int_{0}^{\infty}\right) x^2 f(x)\,dx$$

$$= 0 + \int_{0}^{\infty} x^2 \frac{1}{4} e^{-x/4}\,dx$$

$$= \frac{1}{4}\left[x^2\left(\frac{e^{-x/4}}{-1/4}\right) - 2x\left(\frac{e^{-x/4}}{-1/4 \times -1/4}\right) + 2\left(\frac{e^{-x/4}}{-1/4^3}\right)\right]_0^{\infty}$$

$$= \frac{1}{4}\left[0 - (-2 \times 4^3)\right]$$

$$= \frac{1}{4}\left(2 \times 4^3\right)$$

$$= 32$$

$$\therefore V(x) = E(x^2) - (E(x))^2$$

$$= 32 - (4)^2$$

$$= 32 - 16$$

$$= 16$$

If 'x' is a continuous random variable function

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{Otherwise} \end{cases}$$

i) mean
ii) variance

find: $E(25x^2 + 30x - 5)$

$$E(25x^2 + 30x - 5) = 25E(x^2) + 30E(x) - 5$$

$$\left[\text{formula: } E(ax + b) = aE(x) + b\right]$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \left[\int_{-\infty}^{0} + \int_{0}^{1} + \int_{1}^{2} + \int_{2}^{\infty}\right] x f(x)\,dx$$

$$= 0 + \int_{0}^{1} x f(x)\,dx + \int_{1}^{2} x f(x)\,dx + 0$$

$$= \int_{0}^{1} x \cdot x\,dx + \int_{1}^{2} x(2-x)\,dx$$

$$= \int_{0}^{1} x^2\,dx + \int_{1}^{2} 2x - x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + 2\left[\frac{x^2}{2}\right]_1^2 + \left[\frac{x^3}{3}\right]_1^2$$

$$= \left[\frac{1}{3} - 0\right] + 2\left[\frac{4}{2} - \frac{1}{2}\right] - \left[\frac{8}{3} - \frac{1}{3}\right]$$

$$= \frac{1}{3} + 3 - \frac{7}{3}$$

$$= \frac{1+9-7}{3} = \frac{10-7}{3} = 1$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \left(\int_{-\infty}^{0} + \int_{0}^{1}\right) x^2 f(x)\,dx$$

variance:

$E(x^2) - (E(x))^2$

$= 1.166 - (1)^2$

$= 0.1666$

iii) $E(25x^2 + 30x - 5)$

$25E(x^2) + 30E(x) - 5$

$25(1.1666) + 30(1) - 5$

$29.165 + 30 - 5$

54.165

cdf (discrete random variables):-

probability mass function of discrete random variables is given by

$$f(x) = 1/4 \quad x = 0, 1, 2, 3$$

Find $F(x)$ sketch its graph

0	1	2	3
1/4	1/4	1/4	1/4

$F(x) = 0 \leq f(x) \leq 1$

$-\infty \leq x < \infty$

$F(-\infty) = 0$

$F(\infty) = 1$

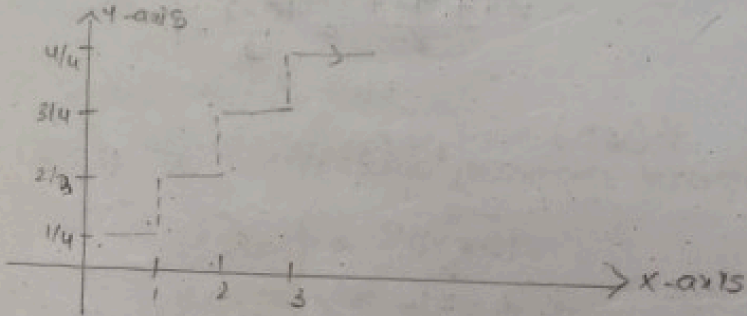
i) $F(x) = 0 \quad x < 0 \; (0, \infty)$

$= 1/4 \quad 0 \leq x < 1 \; [0, 1)$

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = 1 \leq x < 2 \; [1, 2)$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} = 2 \leq x < 3 \; [2, 3)$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1 \quad x \geq 3 \; [3, \infty)$



2) A Discrete Random variable as the following Distribution function:

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ 1/3 & \text{for } 1 \leq x < 4 \\ 1/2 & \text{for } 4 \leq x < 6 \\ 5/6 & \text{for } 6 \leq x < 10 \\ 1 & \text{for } x \geq 10 \end{cases}$$

find i) $P(2 < x \leq$

ii) $P(X = 5)$

iii) $P(X \leq 6)$

iv) $P(X = 6)$

v) draw the

Given that

$F(x) = 1 \quad x < 1$

i) $P(2 < x \leq 6)$

$P(X=3) \; P(X=4)$

$= F(6) -$

$= 5/6 -$

$= \frac{1}{2} =$

ii) $P(X = 5) = P($

$= F$

2

iii) $P(X \leq 6) =$

$=$

iv) $P(X = 6) =$

$= 5/$

$= 1$

find i) $P(2 < x \leq 6)$

ii) $P(X = 5)$

iii) $P(X \leq 6)$

iv) $P(X = 6)$

v) draw the graph of $f(x)$

Given that

$F(x) = 1 \quad x < 1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$F(x) = P(X \leq x)$$

i) $P(2 < x \leq 6)$

$P(X=3)\ P(X=4)\ P(X=5)\ P(X=6)$

$= F(6) - F(2)$

$= 5/6 - \frac{1}{3}$

$= \frac{1}{2} = 0.5$

ii) $P(X=5) = P(X \leq 5) - P(X < 5)$

$= F(5) - P(X < 5)$

$= 1/2 - \frac{1}{2}$

$= 0$

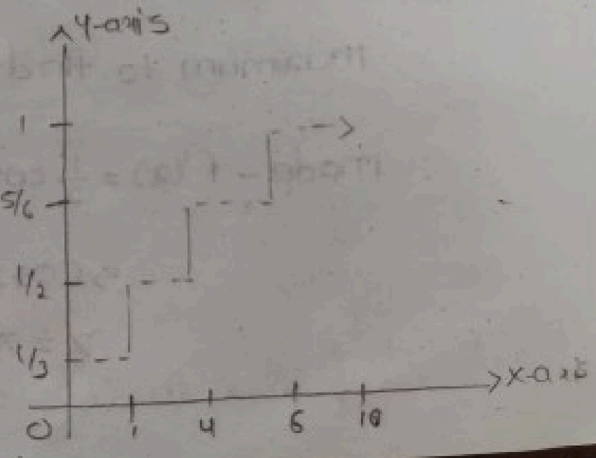
iii) $P(X \leq 6) = F(6)$

$= 5/6$

iv) $P(X=6) = P(X \leq 6) - P(X < 6)$

$= 5/6 - \frac{1}{2}$

$= 1/3.$



3) probability density function of Random variable

$$\begin{cases} f(x) = \frac{1}{2}\sin x & 0 \le x \le \pi \\ 0 & \text{else or otherwise} \end{cases}$$

Find mean and mode of the distribution.

$$\text{mean} = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \left[\int_{-\infty}^{0} + \int_{0}^{\pi} + \int_{\pi}^{\infty} x f(x)\right] dx$$

$$= 0 + \int_{0}^{\pi} x \times \frac{1}{2}\sin x\,dx + 0$$

$$\int f(x).g(x)\,dx = f(x)\int g(x)\,dx - \int\left(d/dx\, f(x) \int g(x)\,dx\right)dx$$

$$= \frac{1}{2}\int_{0}^{\pi} \underset{f(x)}{x}\,\underset{g(x)}{\sin x}\,dx + 0$$

$$= \frac{1}{2}\left[x(-\cos x) - 1(-\sin x)\right]_{0}^{\pi}$$

$$= \frac{1}{2}\left[\pi(-\cos\pi) - 1(-\sin\pi)\right] - 0\left[-\cos 0 - 1(-\sin 0)\right]$$

$$= \frac{1}{2}\left[\pi(-(-1)) + 0 - 0\right]$$

$$= \frac{\pi}{2}$$

Mode:- Mode is the value of x for which $f(x)$ is maximum to find the maximum value

$$\text{mode} - f'(x) = \frac{1}{2}\cos x = 0$$

$$\Rightarrow \cos x = 0$$

$$x = \pi/2$$

f

f''(

f(x) is h

4) Let x be

with prob

f(x)

find i) F

ii) P(

iii) P

iv) f

i) P(2 ≤

=

=

ii) P(3

$$f''(x) = \frac{-\sin x}{2}$$

$$f''(\pi/2) = \frac{-\sin \pi/2}{2}$$

$$= \frac{-1}{2} < 0$$

$f(x)$ is having maximum value at $x = \pi/2$

$\therefore$ mode of $f(x) = \pi/2$

4) Let x be a continuous random variable with probability density function.

$$f(x) = \begin{cases} 1/8 & 0 \le x \le 8 \\ 0 & \text{else where} \end{cases}$$

find i) $P(2 \le x \le 5)$

ii) $P(3 \le x \le 7)$ $\quad f(x) = \frac{1}{8}$

iii) $P(X \le 6)$

iv) $f(x)$

i) $P(2 \le x \le 5)$

$$= \int_2^5 f(x)\,dx$$

$$= \int_2^5 1/8\,dx$$

$$= 1/8 \int_2^5 dx$$

$$= \frac{1}{8}[5-2]$$

$$= 3/8$$

ii) $P(3 \le x \le 7)$

$$= \int_3^7 f(x)\,dx \Rightarrow \int_3^7 1/8\,dx = 1/8 \int_3^7 dx = \frac{1}{8}[x]_3^7$$

$$= \frac{1}{8}[7-3]$$

$$= \frac{4}{8}$$

$$= 1/2$$

iii) $\left[\int_{-\infty}^{0} 0 \int_{0}^{6} \int_{6}^{\infty} 0 + \frac{1}{8} dx\right]$

$$= \int_0^6 f(x)\,dx$$

$$= \int_0^6 1/8\, dx$$

$$= 1/8 \int_0^6 dx$$

$$= 1/8[6-0]$$

$$= 6/8$$

$$= 3/4$$

iv) $f(x)\,dx$

$$\int F(x)\,dx$$

$$= \int \frac{1}{8}\, dx$$

$$= \frac{1}{8}x$$

Multiple Random variable :-

Introduction :- In multiple Random variables techniques we use varies Random variables occuring at a time. In many practical problems Several Random variables inter act with each other. from these idea interact more

than one f
of multiple
definition :-
are two d
outcome of
called Mu
We
is (x, y)
Example-①
two pair
Experimen
Let $x =$
$y =$
define
where (
two dic
joint pro
Let (x
Sample
$f(x,y)$
Probab
joint
the jo

than one Random variable is extended to study of multiple Random variable (x, y).

definition :- consider two Random variable (x, y) are two dimensional Random variable (x, y). The outcome of a trail for the pair $(x = x, y = y)$ is called Multiple Random variable.

We call (x, y) and it's distribution is discrete is (x, y) can assume only finite no. of pairs.

example-①:- 1) Consider the experiment of tossing two pair dices the sample space for this experiment as 36 equally likely coins.

Let x = sum on dices two dices

y = difference on two dices thus, we can define by variate Random variable (x, y) where (x = sum on 2 dices, y = difference on two dices)

joint probability function:-

Let (x, y) be a discrete Random variable on a sample space 's' then the probability function $f(x, y) = P(X = x, Y = y)$ is called joint probability function or joint pmf

joint probability distribution:-

The joint probability distribution two Random variable (x,y) is usually represented with following table:-

$x \backslash y$	y_1	y_2	...	y_j	...	y_n	Total
x_1	P_{11}	P_{12}	...	P_{1j}	...	P_{1m}	P_1
x_2	P_{21}	P_{22}	...	P_{2j}	...	P_{2m}	P_2
$\vdots$							
x_i	P_{i1}	P_{i2}	...	P_{ij}	...	P_{im}	P_i
$\vdots$							
x_n	P_{n1}	P_{n2}	...	P_{nj}	...	P_{nm}	P_n
Total	P_1	P_2	...	P_j	...	P_m	1

$$\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j) = 1$$

Marginal probability function:-

suppose that the joint distribution of two random variable x and y is given then the probability distribution of x is determined by

$$P(X=x_i) = P_X(x_i) = P(X=x_i \cap Y=y_i)$$

$$= P(X=x_i \cap Y=y_1) + P(X=x_i \cap Y=y_2) + \cdots + P(X=x_i \cap Y=y_m)$$

$$= P_i$$

$$= \sum_{j=1}^{m} P_{ij}$$

$$= \sum_{j=1}^{m} P(x_i, y_j)$$

discrete - mass function

continuous - probability function

lly $P(Y = y_j) = P_Y(y_j)$

$$= P(x = x_1 \cap Y = y_j) + P(x = x_2 \cap y = y_j) + \cdots$$

$$= P_j$$

$$= \sum_{i=1}^{n} P_{ij}$$

→ $P_{i\cdot}$ & $P_{\cdot j}$ are the marginal probability functions of X and Y.

Conditional probability function:-

Conditional probability of X given Y

$$P(X = x_i / Y = y_j) = \frac{P(X = x_i \cap Y = y_j)}{P(Y = y_j)} = \frac{P(x_i, y_j)}{P(y_j)}$$

Conditional probability of Y given X

$$P(Y = y_j / X = x_i) = \frac{P(Y = y_j \cap X = x_i)}{P(X = x_i)} = \frac{P(x_i, y_j)}{P(x_i)}$$

→ Two random variables X and Y are said to be independent.

$$P(X = x_i \cap Y = y_j) = P(X = x_i)\, P(Y = y_j)$$

→ Joint probability distribution functions (CDF):-

Let (X, Y) be a two dimensional random variable then these joint distribution is denoted by $F_{XY}(x, y)$ and it represents the probability that simultaneously the observation (x, y) will have the probability for discrete random variables have the property.

i) For Discrete random variables

$$F_{XY}(x,y) = P(X \le x, Y \le y) = P(-\infty \le X \le x, -\infty \le Y \le y)$$

ii) For continuous random variables

$$F_{XY}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(x,y)\, dy\, dx$$

where $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$

properties:

i) $F(\infty, \infty) = 1 \quad -\infty \le x \le \infty$

ii) $F(-\infty, -\infty) = 0 \quad -\infty \le y \le \infty$

iii) $F_{XY}(-\infty, y) = F_{XY}(x, -\infty) = 0$

iv) $0 \le F_{XY}(x,y) \le 1$

v) $P(x_1 \le X \le x_2, Y \le y_1) = F(x_2, y_1) - F(x_1, y_1)$

vi) $F_X(x) = F_{XY}(x, \infty) = P(X \le x, Y \le \infty) \Rightarrow P(X \le x)$

vii) $F_Y(y) = F_{XY}(\infty, y) = P(X \le \infty, Y \le y) \Rightarrow P(Y \le y)$

viii) $\cancel{I}$ density function $f(x,y)$ is continuous then

$$f(x,y) = \frac{\partial^2 F}{\partial x\, \partial y}$$

problems:

1) for the following bi-variable distribution of (X,Y) find:

i) $P(X \le 1, Y = 2)$

ii) $P(X \le 1)$

iii) $P(Y = 3)$

iv) $P(y \leq 3)$

v) $P(x < 3, y \leq 4)$

$x \backslash y$	1	2	3	4	5	6	total
0	0	0	$1/32$	$2/32$	$2/32$	$3/32$	$\frac{1}{4}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$\frac{1}{8}$
total	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{3}{16}$	$\frac{1}{4}$	1

i) $P(x \leq 1, y = 2)$

$P(x=0, y=2) + P(x=1, y=2)$

$= 0 + \frac{1}{16}$

$= \frac{1}{16}$

ii) $P(x \leq 1)$

$P(x=0) + P(x=1) = \frac{1}{4} + \frac{5}{8} = \frac{7}{8}$

iii) $P(y=3) = \frac{11}{64}$

iv) $P(y \leq 3) = P(y=1) + P(y=2) + P(y=3)$

$= \frac{3}{32} + \frac{3}{32} + \frac{11}{64}$

$= \frac{23}{64}$

v) $P(x < 3, y \leq 4)$

$= P(x=0, y \leq 4) + P(x=1, y \leq 4) + P(x=2, y \leq 4)$

$= \left(0 + 0 + \frac{1}{32} + \frac{2}{32}\right) + \left(\frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{32} + \frac{1}{32} + \frac{1}{64} + \frac{1}{64}\right)$

$= 9/16$

2) Given the following bivariant distribution obtain probability

find:

i) Marginal distribution of 'x' & 'y'

ii) the conditional distribution of 'x' given $y=2$

$x \backslash y$	0	1	2	Total
-1	1/15	3/15	2/15	6/15
0	2/15	2/15	1/15	5/15
1	1/15	1/15	2/15	4/15
Total	4/15	6/15	5/15	1

Marginal distribution of x is

$$P_X(x) = P_i = \sum_{j=1}^{m} P_{ij}$$

x	$P_{i.}$
-1	6/15
0	5/15
1	4/15

Marginal distribution of y

$$P_Y(y) = P_{ij}$$

y	P_{ij}
0	4/15
1	6/15
2	5/15

Conditional distribution of 'x' given $y=2$

$$P(x=x_i/y=2) = \frac{P(x=x_i \cap y=2)}{P(y=2)}$$

whe

$= \frac{P(x=}{}$

$=$

3) Two b

contain

x an

balls

draw

i) Joi

ii) M

iii) C

ion obtain

n $y=2$

where $x_i = -1, 0, 1$

$$= \frac{P(x=-1 \cap y=2)}{P(y=2)} + \frac{P(x=0 \cap y=2)}{P(y=2)} + \frac{P(x=1 \cap y=2)}{P(y=2)}$$

$$= \frac{\frac{2}{15}}{\frac{5}{15}} + \frac{\frac{1}{15}}{\frac{5}{15}} + \frac{\frac{2}{15}}{\frac{5}{15}} = 1$$

3) Two balls are drawn at random from a box containing 3 red, 2 green and 4 white balls. If x and y are the no. of red balls and green balls respectively included among the two balls drawn from the box. find: doubt

i) Joint probability of x and y

ii) Marginal probability of x and y

iii) conditional distibution of x given $y=2$.

$$\underset{3}{R} + \underset{2}{G} + \underset{4}{W} = 9$$

x (red balls) $= \{0, 1, 2, 3\}$

y (green balls) $= \{0, 1, 2\}$

$$P(0,0) = \frac{(3C_0)(2C_0)(4C_2)}{9C_2} = \frac{1}{6}$$

$$P(0,1) = \frac{(3C_0)(2C_1)(4C_1)}{9C_2} = \frac{2}{9}$$

$$P(0,2) = \frac{(3C_0)(2C_2)(4C_0)}{(9C_2)} = \frac{1}{36}$$

$P(1,0) = \frac{(3C_1)(2C_0)(4C_1)}{(9C_2)} = \frac{1}{3}$

$P_{(1,1)} = \frac{(3C_1)(2C_1)(4C_0)}{(9C_2)} = \frac{1}{6}$

$P(1,2) =$ doesnot exist

$(P_{2,2}) = 0$, $(P_{2,0}) = \frac{(3C_2)(2C_0)(4C_0)}{(9C_2)} = \frac{1}{12}$

$P(3,0) = 0$, $P(3,1) = 0$, $P(3,2) = 0$

x \ y	0	1	2	total
0	$\frac{1}{16}$	$\frac{2}{9}$	$\frac{1}{36}$	$\frac{5}{12}$
1	$\frac{1}{3}$	$\frac{1}{6}$	0	$\frac{1}{2}$
2	$\frac{1}{12}$	0	0	$\frac{1}{12}$
3	0	0	0	0
total	$\frac{7}{12}$	$\frac{7}{18}$	$\frac{1}{36}$	1

Marginal probability of x

x	$P_{i.}$
0	5/12
1	1/2
2	1/12
3	0

Marginal probability of y

y	$P_{.j}$
0	$\frac{7}{12}$
1	$\frac{7}{18}$
2	1/36

conditional probability of x given $y=2$

$$P(x=x_i / y=2) = \frac{P(x=x_i \cap y=2)}{P(y=2)}$$

$$= \frac{P(x=0 \cap y=2)}{P(y=2)} + \frac{P(x=1 \cap y=2)}{P(y=2)} + \frac{P(x=2 \cap y=2)}{P(y=2)} + \frac{P(x=3 \cap y=2)}{P(y=2)}$$

$$= \frac{\frac{1}{36}}{1/36} + 0 + 0 + 0$$

$$= 1$$

4) The joint probability mass function of x & y given by $P(x,y) = k(2x+3y)$, $x=0,1,2$ and $y=1,2,3$

find: i) find 'k'

ii) Marginal probability function of x and y

iii) conditional probability of x given $y=1$

iv) conditional probability of y given $x=2$

v) probability distibution of $x+y$

i)

x \ y	1	2	3	total
0	3k	6k	9k	18k
1	5k	8k	11k	24k
2	7k	10k	13k	30k
total	15k	24k	33k	72k

$72k = 1$

$\boxed{k = 1/72}$

i) Marginal probability of x

x	P_i
0	$\frac{18}{72}$
1	$\frac{24}{72}$
2	$\frac{30}{72}$

Marginal probability of y

y	P_j
1	$\frac{15}{72}$
2	$\frac{24}{72}$
3	$\frac{33}{72}$

iii) conditional probability of x given y=1

$$P(x=x_i / y=1) = \frac{P(x=x_i \cap y=1)}{P(y=1)}$$

$$= \frac{P(x=0 \cap y=1)}{y=1} + \frac{P(x=1 \cap y=1)}{y=1} + \frac{P(x=2 \cap y=1)}{y=1}$$

$$= \frac{3/72}{3/72} + \frac{5/72}{3/72} + \frac{7/72}{3/72}$$

$$= \frac{3}{15} + \frac{6}{15} + \frac{9}{15}$$

$$= \frac{15}{15}$$

$$= 1$$

iv) conditional probability of y given x=2

$$P(y=y_i / x=2) = \frac{P(y=y_i \cap x=2)}{P(x=2)}$$

$$= \frac{P(y=1 \cap x=2)}{x=2} + \frac{P(y=2 \cap x=2)}{x=2} + \frac{P(y=3 \cap x=2)}{x=2}$$

$= \frac{7/72}{30/72}$

v) let

prob

5) verify

In continuous variable

Is

i) P(

To

$= \frac{7/72}{30/72} + \frac{10/72}{30/72} + \frac{13/72}{30/72} \Rightarrow \frac{7}{30} + \frac{10}{30} + \frac{13}{30} = \frac{30}{30} = 1$

v) let $Z = X+Y$

probability function of $X+Y$

$X = 0, 1, 2 \qquad Y = 1, 2, 3$

$Z = 1, 2, 3, 4, 5$

probabilities = 1, 2, 3, 4, 5

$X=0+Y=1 = 0+1 = 1$

$X=1+Y=1 = 1+1 = 2$

$X=2+Y=1 = 2+1 = 3$

$X=1+Y=2 = 1+2 = 3$

$X=0+Y=2 = 0+2 = 2$

$X=2+Y=2 = 2+2 = 4$

$Z = X+Y$	$P(Z)$
$Z=1$	$P_{01} = \frac{3}{72}$
$Z=2$	$P_{02} + P_{11} = 6k + 5k = 11k = \frac{11}{72}$
$Z=3$	$P_{03} + P_{12} + \cancel{P_{21}} = 9k + 8k + \cancel{7k} = 24/72$
$Z=4$	$P_{13} + P_{22} + \cancel{P_{31}} + \cancel{P_{31}} = 11k + 10k = 21/72$
$Z=5$	$P_{23} + \cancel{P_{41}} + \cancel{P_{05}} + \cancel{P_{32}} = 13k = 13/72$

5) verify that $f(x,y) = \begin{cases} 2/5\,(2x+3y), & 0 \le x \le 1 \\ & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$

In continuous random variable if there is $\le$ (or) $<$ both are equal

Is it Joint probability function? If so find

i) $P(X,Y) \in A$ where A is the region $\{(x,y) \mid 0 < x < 1/2,\ 1/4 < y < 1/2\}$

To prove

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\left[\int_{x=\infty}^{0} \int_{y=\infty}^{0} + \int_{x=0}^{1} \int_{y=0}^{1} + \int_{x=1}^{\infty} + \int_{y=1}^{\infty} \right] f(x,y)$$

$$= 0 + \int_{x=0}^{1}\int_{y=0}^{1} \frac{2}{5}(2x+3y)\,dy\,dx + 0$$

$$= \frac{2}{5}\int_{x=0}^{1} 2x(y)_0^1 + \left(\frac{3y^2}{2}\right)_0^1 dx$$

$$= \frac{2}{5}\int_{x=0}^{1}\left[2x(1-0) + \frac{3}{2}(1^2-0)\right]dx$$

$$= \frac{2}{5}\int_{x=0}^{1}\left(2x+\frac{3}{2}\right)dx$$

$$= \frac{2}{5}\left(\cancel{2}\frac{x^2}{\cancel{2}} + \frac{3}{2}(x)\right)_0^1$$

$$= \frac{2}{5}\left[1+\frac{3}{2}\right]$$

$$= \frac{2}{5}\times\frac{5}{2}$$

$$= 1$$

Given function is joint probability function.

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0\le x\le 1,\ 0\le y\le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P\left((x,y)\,/\,0<x<1/2,\ \frac{1}{4}<y<1/2\right) = \int_{0}^{1/2}\int_{y=1/4}^{1/2} \frac{2}{5}(2x+3y)\,dy\,dx$$

$$= \frac{2}{5}\int_{x=0}^{1/2} 2x(y)_{1/4}^{1/2} + \left(\frac{3y^2}{2}\right)_{1/4}^{1/2} dx$$

$$= \frac{2}{5}\int_{x=0}^{1/2} 2x\left(\frac{1}{2}-\frac{1}{4}\right) + \frac{3}{2}\left(\frac{1}{4}-\frac{1}{16}\right)dx$$

$$= \frac{2}{5}\int_{x=0}^{1/2} \left(2x(1/4) + \frac{3}{2}(1/16)\right)dx$$

$$= \frac{2}{5}\left[\left[\frac{\cancel{2}x^2}{\cancel{2}}\right]_0^{1/2} + \frac{3}{32}(x)_0^{1/2}\right] \quad 1.2473$$

$$= \frac{2}{5}\left[\frac{1}{4} + \frac{3}{32}(1/2)\right] = \frac{2}{5}\left[\frac{1}{4} + \frac{3}{64}\right]$$

$$= \frac{19}{160}$$

Marginal pdf of continuous random variable:-

Joint pdf if $f(x,y)$ then marginal probability function of x is denoted by

$$f_X(x) = \int_y f(x,y)\,dy$$

$$((f_Y(y) = \int_x f(x,y)\,dx\,.)) \text{wrong}$$

~~Joint~~ marginal pdf of y is

$$f_Y(y) = \int_x f(x,y)\,dx$$

Joint conditional probability:-

$$f(x/y) = \frac{f(x,y)}{f_Y(y)}$$

$$f(y/x) = \frac{f(x,y)}{f_X(x)}$$

Stastical Independence (stostatic):-

let x and y be two random variables discrete or continuous with joint pdf $f(x,y)$ and marginal pdf $f_{XY}(x,y) = f_X(x) f_Y(y)$ respectively. The random variables X and Y are said to be statistical independent if and only if $f_X(x) f_Y(y)$.

1) Given joint density function $f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0<x<2,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$

find: i) $f(x/y)$

ii) verify whether X, Y are statistical independent or not

iii) Marginal pdf

iii) Marginal probability function of X is

$$f_X(x) = \int_y f(x,y)\,dy$$

$$= \int_0^1 \frac{x(1+3y^2)}{4}\,dy$$

$$= \frac{x}{4}\int_0^1 (1+3y^2)\,dy$$

$$= \frac{x}{4}\left[y + \frac{3y^3}{3}\right]_0^1$$

$$= \frac{x}{4}\left[(1-0)+(1-0)\right] \Rightarrow \frac{x}{4}(2)$$

$$= \frac{x}{2}$$

$$f_Y(y) = \int_x f(x,y)\,dx$$

$$= \int_{x=0}^{2} \frac{x(1+3y^2)}{4}\,dx$$

(x, y) and

(y) respectively.

be said to be

if $f_X(x) f_Y(y)$.

$$\begin{cases} \frac{x(1+3y^2)}{4} & 0<x<2,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

is

(2)

$$= \int_{x=0}^{2} \left(\frac{x}{4} + \frac{3y^2}{4} \right) dx$$

$$= \left[\frac{1}{4} \times \frac{x^2}{2} + \frac{3y^2}{4} (x)_0^2 \right]$$

$$= \frac{1}{8}(4-0) + \frac{3y^2}{4}(2-0)$$

$$= \frac{1}{2} + \frac{3}{2}y^2 \Rightarrow \frac{1+3y^2}{2}$$

iii) $f(x/y) = \dfrac{f(x,y)}{f_Y(y)}$

$$= \frac{\frac{x(1+3y^2)}{4}}{\frac{1+3y^2}{2}}$$

$$= \frac{x(1+3y^2)}{\cancel{4}_2} \times \frac{\cancel{2}^1}{1+3y^2}$$

$$= \frac{x}{2}$$

ii) $f(x,y) = \dfrac{x(1+3y^2)}{4}$

$$f_X(x) = \frac{x}{2}$$

$$f_Y(y) = \frac{1+3y^2}{2}$$

$$f_X(x) f_Y(y) = \frac{x(1+3y^2)}{4}$$

$$= f(x,y)$$

∴ X, Y are statistical independent.

2) Joint pdf (x,y) is given by $f(x,y)=\begin{cases} 8xy & 0<x<y<1 \\ 0 & \text{otherwise} \end{cases}$

find i) Marginal probability

ii) conditional distribution

iii) Are x & y are independent

i) Marginal probability function of x is

$$f_X(x) = \int_y f(x,y)\,dy$$

$$= \int_0^1 8xy\,dy$$

$$= 8x\left[\frac{y^2}{2}\right]_0^1$$

$$= 8x\left(\frac{1}{2}-\frac{0}{2}\right)$$

$$= \overset{4x}{\cancel{8x}}\left(\frac{1}{\cancel{2}}\right)$$

$$= 4x$$

Marginal probability function of y is

$$f_Y(y) = \int_x f(x,y)\,dx$$

$$= \int_0^1 8xy\,dx \qquad = \int_0^1 8xy\,dx$$

$$= y\left(\frac{8x^2}{2}\right)_0^1 \qquad = 8y\int_0^1 x\,dx$$

$$= y(4 \qquad = 8y\int_0^1 \frac{x^2}{2}$$

$$= 8y\left(\frac{1}{2}-\frac{0}{2}\right) = 4y$$

ii) conditional dist

$f(x|y) =$

(i) $f(x,y) =$

$f_X(x$

f

f

iii) ∴ x and

3) Random va

$f(x,y) =$

find i) Ma

ii) Exa

i) $f_X(x)$

$f(x,y) = \begin{cases} 8xy & 0<x<y<1 \\ 0 & \text{otherwise} \end{cases}$

dent

of x is

& y is

dx

$\int_0^1 x\,dx$

$\frac{x^2}{2}$

$-\frac{0}{2}) = 4y$

ii) conditional distribution.

$$f(x/y) = \frac{f(x,y)}{f_y(y)}$$

$$= \frac{8^2xy}{4y}$$

$$= 2xy$$

(i) $f(x,y) = 8xy$

$$f_x(x) = 4x$$

$$f_y(x) = 4y$$

$$f_x(x)\,f_y(y) = \cancel{8}\;16xy \neq f(x,y)$$

iii) $\therefore$ x and y are not stastical independent

3) Random variables x & y has joint distribution

$$f(x,y) = \begin{cases} 6(1-x-y) & \text{for } x>0,\ y>0,\ x+y<1 \\ 0 & \text{otherwise} \end{cases}$$

$y = 1-x$
$x = 1-y$

find i) Marginal pdf of x & y

ii) Examine 'x' & 'y' are independent

i) $f_x(x) = \int_y f(x,y)\,dy$

$$= \int_{y=0}^{1-x} 6(1-x-y)\,dy$$

$$= 6\left[y - xy - \frac{y^2}{2}\right]_0^{1-x}$$

$$= 6\left[(1-x) - x(1-x) - \frac{(1-x)^2}{2}\right]_0^{1-x}$$

$$= 6(1-x)\left[1-x-\frac{(1-x)^2}{2}\right]_0^{1-x}$$

$$= 6(1-x)\left[1(1-x)-1(0)-x(1-x)-x(0)-\frac{(1-x)}{2}x(1-x)-\frac{(1-x)}{2}(0)\right]$$

$$= 6(1-x)\left[1-x-x+x^2+\frac{(1+2x-x^2)}{2}\right]$$

$$= \cancel{6}^{3}(1-x)\left[\frac{2-2x-\cancel{2x}+2x^2-1+\cancel{2x}-x^2}{\cancel{2}}\right]$$

$$= 3(1-x)\left[x^2-2x+3\right]$$

$$= (3-3x)(x^2-2x+3) \Rightarrow 3x^2-6x+9-3x^3+6x^2-9x$$

The two random variables x and y have joint pdf $f(x,y)=\frac{1}{2x^2y}$ $1\le x<\infty,\ \frac{1}{x}<y<x$

i) ~~Margir~~ Derive the marginal distribution of x and y also, find conditional distribution of y for x and x for y

i) Marginal pdf of x

$$f_X(x)=\int_y f(x,y)\,dy$$

$$= \int_{1/x}^{x}\frac{1}{2x^2y}$$

$$= \frac{1}{2x^2}$$

Marginal pd

$$= \int_{1/x}^{x} \frac{1}{2x^2 y}\, dy$$

$$= \frac{1}{2x^2} \int_{y=1/x}^{x} \frac{1}{y}\, dy$$

$$= \frac{1}{2x^2} \left(\log y\right)_{1/x}^{x}$$

$$= \frac{1}{2x^2} \left[\log x - \log \frac{1}{x}\right]$$

$$= \frac{1}{2x^2} \left[\log \frac{x}{1/x}\right]$$

$$= \frac{1}{2x^2} \left[\log x^2\right]$$

$$= \frac{1}{\cancel{2}x^{\cancel{2}}} (\cancel{2}\log x) = \frac{\log x}{x^2}$$

Marginal pdf of y

$$f_Y(y) = \int_{x} \frac{1}{2x^2 y}\, dx$$

$$= \frac{1}{2y} \int_{x=1}^{\infty} \frac{1}{x^2}\, dx$$

$$= \frac{1}{2y} \left[\frac{-1}{x}\right]_{1}^{\infty}$$

$$= \frac{-1}{2y} \left[0 - \frac{1}{1}\right]$$

$$= \frac{1}{2y}$$ ″

Conditional distibution of x for y

$$f(x|y) = \frac{f(x,y)}{f_y(y)}$$

$$= \frac{\frac{1}{2x^2y}}{\frac{1}{2y}} \rightarrow \frac{1}{x^2}$$

find k for that $f(x,y) = kxy \quad 1 \le x \le y \le 2$.

i) Will be the probability density function.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\int_{x=1}^{2}\int_{y=1}^{2} kxy\,dy\,dx = 1$$

$$kx\int_{x=1}^{2}\left[\frac{y^2}{2}\right]$$

$$kx\left[\frac{y^2}{2}\right]_1^2 \quad kx\left[\frac{4}{2} - \frac{1}{2}\right] \Rightarrow$$

$$kx\left[\frac{4-1}{2}\right]$$

$$= \int\left(\frac{3kx}{2}\right)dx$$

$$= \frac{3k}{2}$$

$$= \frac{3k}{2}\left[\frac{x^2}{2}\right]_1^2 = \frac{3}{2}k\left[\frac{4}{2} - \frac{1}{2}\right]$$

$$= \frac{3}{2}k\left[\frac{3}{2}\right]$$

$$\Rightarrow \frac{k9}{4} = 1 \quad k = \frac{4}{9}$$

If the joint co

$$F(x,y) = \begin{cases} (1-e^{-x})(1 \\ 0 \quad \text{othe} \end{cases}$$

find i) joint p

ii) $P(1 < x <$

i) joint pdf $f(x$

$f($

$P(1 < x < 3, 1$

If the joint cdf of x and y is given by

$$F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}) & \text{for } x>0, y>0 \\ 0 & \text{otherwise} \end{cases}$$

find i) joint pdf $f(x,y)$

ii) $P(1<X<3, 1<Y<2)$

i) joint pdf $f(x,y) = \frac{\partial}{\partial x}(1-e^{-x}) \frac{\partial}{\partial y}(1-e^{-y})$

$$= e^{-x} \times e^{-y}$$

$$f(x,y) = \begin{cases} e^{-(x+y)} & x>0, y>0 \\ 0 & \text{Otherwise} \end{cases}$$

$$P(1<x<3, 1<y<2) = \int_{x=1}^{3} \int_{y=1}^{2} f(x,y)\, dy\, dx$$

$$= \int_{x=1}^{3} \int_{y=1}^{2} e^{-x} e^{-y}\, dy\, dx$$

$$= \int_{x=1}^{3} e^{-x} \left(e^{-y}\right)_1^2 dx$$

$$= \int_{x=1}^{3} -e^{-x}(e^{-2} - e^{-1})\, dx$$

$$= (e^{-1} - e^{-2}) \int_{x=1}^{3} e^{-x}\, dx$$

$$= \left(\frac{1}{e} - \frac{1}{e^2}\right)\left(-e^{-x}\right)_1^3$$

$= \left(\frac{1}{e} - \frac{1}{e^2}\right)\left(-\left(e^{-3} - e^{-1}\right)\right)$

$= \left(\frac{1}{e} - \frac{1}{e^2}\right)\left(\frac{1}{e} - \frac{1}{e^3}\right)$

x and y are two discrete RV having joint probability function $F(x,y) = \frac{1}{27}(2x-y)$ where x and y can assumed the values of 0, 1, 2. find marginal distribution & conditional distribution of y for x

x \ y	0	1	2	total
0	0	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{3}{27}$
1	$\frac{2}{27}$	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{9}{27}$
2	$\frac{4}{27}$	$\frac{5}{27}$	$\frac{6}{27}$	$\frac{15}{27}$
total	$\frac{6}{27}$	$\frac{9}{27}$	$\frac{12}{27}$	1

$f(x,y) = \frac{1}{27}(2x+y) \qquad x = 0,1,2$

$\qquad\qquad\qquad\qquad y = 0,1,2$

conditional distribution y to x

$$P(y = y_i / x = x_i) = \frac{P(y = y_i \cap x = x_i)}{P(x = x_i)}$$

$$\frac{P(y = 0 \cap x = x_1)}{P(x = x_i)}$$

conditional distribu

x \ y	y=0	y
x=0	0	
x=1	$\frac{2}{9}$	
x=2	4/15	

conditional distribution table

$x \backslash y$	$y=0$	$y=1$	$y=2$
$x=0$	0	$1/3$	$2/3$
$x=1$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$
$x=2$	$4/15$	$5/15$	$6/15$