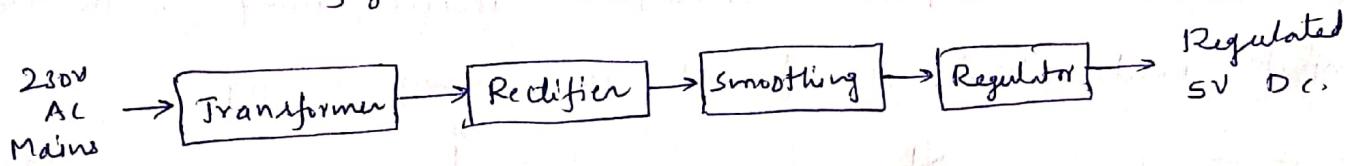


RECTIFIERS AND FILTERS.

①

For the operation of most of the electronic devices and circuits, a d.c. source is required so it is advantageous to convert a.c. supply into d.c. voltages. The process of converting a.c. voltage into d.c. voltage is called as rectification. This is achieved with (i) step-down transformer, (ii) Rectifier, (iii) Filter and (iv) Voltage Regulator circuits. These elements constitute d.c. regulated power supply shown in the fig. below-



Transformer - step down 230V AC mains to low voltage AC.

Rectifier - converts to DC but DC output is varying.

Smoothing - smooth the DC from varying greatly to small ripple.

Regulator - eliminates ripple by setting DC output to a fixed voltage.

P-N junction Diode as a Rectifier:-

A P-N junction diode is a two terminal device that is polarity sensitive. When the diode is forward biased, the diode conducts and allows current to flow through it without any resistance, i.e., the diode is ON.

→ When the diode is reverse biased, the diode does not conduct and no current flows through it, i.e., the diode is OFF, or providing a blocking function.

→ Thus an ideal diode acts as a switch, either open or closed, depending upon the polarity of the voltage placed across it.

→ The ideal diode has zero resistance under forward bias and infinite resistance under reverse bias.

→ Rectifier is defined as an electronic device used for converting ac voltage into unidirectional voltage. A rectifier utilizes unidirectional conduction device like a PN diode.

HALF-WAVE RECTIFIER:-

A halfwave rectifier is one which converts a.c. voltage into a pulsating voltage using only one half cycle of the applied a.c. voltage.

→ The basic halfwave rectifier circuit along with its input and output waveforms is shown in fig.

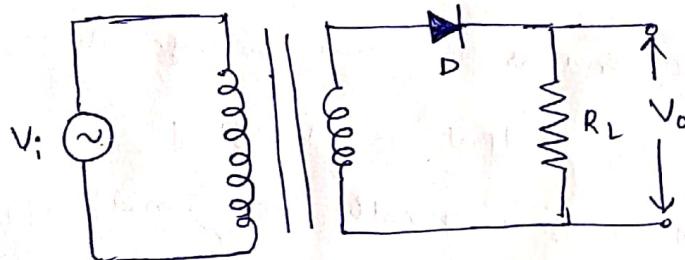


Fig. 1(a)
basic circuit

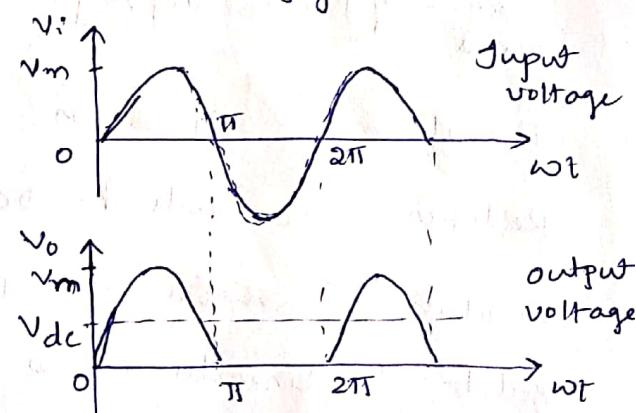


Fig.
half wave rectifier

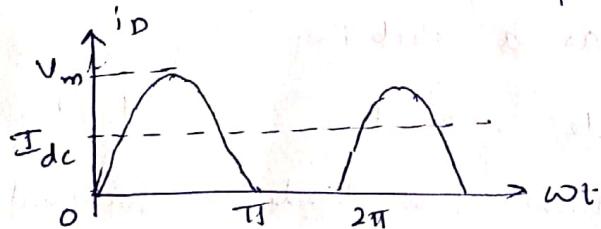


Fig 1(b)-

Fig 1(c) Diode and load current.

The a.c. voltage to be rectified is applied to a single diode connected in series with a load resistor R_L . When it is required to step up or step down the input voltage, a power transformer is used as shown in fig.

Working of Half-wave Rectifier:-

→ For the positive half cycle of input a.c. voltage, the diode D is forward biased and hence it conducts. Now, a current flows in the circuit and there is a voltage drop across R_L .

- This constitutes the output voltage as shown in fig (2). The waveform of diode current or load current is shown in fig (c).
- For the negative half cycle, the diode D is reverse biased, and hence it does not conduct. Now, no current flows in the circuit i.e., $i_D = 0$ and $v_o = 0$. Thus, for the negative half cycle no power is delivered to the load.
- It is obvious from the figure that output is not a steady d.c. but only a pulsating d.c. wave. When it is measured by a d.c. meter, it shows some average positive value both for voltage and current.
- The output waveform can be observed with the help of an oscilloscope by connecting it across the load resistor R_L . Since only half-cycle of the input wave is used, it is called a half-wave rectifier.

Let a sinusoidal voltage V_i be applied to the input of the rectifier. Then

$$V_i = V_m \sin \omega t \rightarrow (1)$$

where V_m is the max. value of supply voltage.

Let the diode be idealized with resistance R_f in the forward direction i.e., in the ON state and $R_r (\approx \infty)$ in the reverse direction i.e., in the OFF state. Now, the current i in the diode or in the load resistance R_L is given by,

$$i = I_m \sin \omega t \text{ for } 0 \leq \omega t \leq \pi \rightarrow (2)$$

$$i = 0 \text{ for } \pi \leq \omega t \leq 2\pi \rightarrow (3)$$

$$\text{where } I_m = \frac{V_m}{R_f + R_L} \rightarrow (4)$$

(i) The d.c. output current:-

The average current I_{dc} is given by

$$I_{dc} = \frac{1}{2\pi} \int_0^{2\pi} i \cdot d(\omega t)$$

$$I_{dc} = \frac{1}{2\pi} \left[\int_0^{\pi} I_m \sin \omega t \cdot d(\omega t) + \int_{\pi}^{2\pi} 0 \cdot d(\omega t) \right]$$

$$I_{dc} = \frac{1}{2\pi} \left[I_m (-\cos \omega t) \Big|_0^{\pi} \right] = \frac{1}{2\pi} \left[I_m [1 - (-1)] \right]$$

$$I_{dc} = \frac{I_m}{\pi} = 0.318 I_m \rightarrow ⑤$$

Substituting the value of I_m , we get

$$I_{dc} = \frac{V_m}{\pi (R_f + R_L)} \rightarrow ⑥$$

If $R_L \gg R_f$, then

$$I_{dc} = \frac{V_m}{\pi R_L} = 0.318 \frac{V_m}{R_L} \rightarrow ⑦$$

(ii) The d.c. output voltage:-

The d.c. output voltage is given by

$$V_{dc} = I_{dc} \times R_L = \frac{I_m}{\pi} \times R_L \rightarrow ⑧$$

$$= \frac{V_m \times R_L}{\pi (R_f + R_L)} = \frac{V_m}{\pi [1 + R_f/R_L]} \rightarrow ⑨$$

If $R_L \gg R_f$, then

$$V_{dc} = \frac{V_m}{\pi} = 0.318 V_m \rightarrow ⑩$$

For a half wave rectifier, there is an output voltage for a period from 0 to π . There is no output for the period from π to 2π . Therefore, the average d.c. value of output voltage is given by,

$$V_{dc} = \frac{\text{area under the curve for full cycle}}{\text{base}}$$

$$V_{dc} = \frac{1}{2\pi} \int_0^{\pi} V_i \cdot d(\omega t) = \frac{1}{2\pi} \int_0^{\pi} V_m \sin \omega t \cdot d(\omega t).$$

$$= \frac{V_m}{2\pi} \left[-\cos \omega t \right]_0^{\pi} = \frac{V_m}{2\pi} [1 - (-1)]$$

$$\boxed{V_{dc} = \frac{V_m}{\pi} = 0.318 V_m}$$

(iii) RMS current and voltage :-

The value of RMS current is given by

$$I_{rms} = \left[\frac{1}{2\pi} \int_0^{2\pi} i^2 \cdot d(\omega t) \right]^{1/2}$$

$$\begin{aligned} I_{rms} &= \left[\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \cdot d(\omega t) + \frac{1}{2\pi} \int_{\pi}^{2\pi} 0 \cdot d(\omega t) \right]^{1/2} \\ &= \left[\frac{I_m^2}{2\pi} \int_0^{\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) \cdot d(\omega t) \right]^{1/2} \\ &= \left[\frac{I_m^2}{4\pi} \left\{ (\omega t) - \frac{\sin 2\omega t}{2} \right\}_0^{\pi} \right]^{1/2} \\ &= \left[\frac{I_m^2}{4\pi} \left\{ \pi - 0 - \frac{\sin 2\pi}{2} + \sin 0 \right\} \right]^{1/2} = \left[\frac{I_m^2}{4} \right]^{1/2} \end{aligned}$$

$$\therefore I_{rms} = \frac{I_m}{2} \rightarrow ⑪$$

$$\therefore \boxed{I_{rms} = \frac{V_m}{2(R_f + R_L)}} \rightarrow ⑫$$

RMS voltage across the load is

$$V_{rms} = I_{rms} \times R_L = \frac{V_m \times R_L}{2(R_f + R_L)}$$

$$\text{or } \boxed{V_{rms} = \frac{V_m}{2[1 + (R_f/R_L)]}} \rightarrow ⑬$$

$$\text{If } R_L \gg R_f, \text{ then } V_{rms} = \frac{V_m}{2} \rightarrow ⑭$$

(iv) Rectifier efficiency:-

The rectifier efficiency is defined as the ratio of d.c. output power to the a.c. input power i.e.,

$$\eta = \frac{\text{d.c. power delivered to the load}}{\text{a.c. input power from transformer secondary}} = \frac{P_{dc}}{P_{ac}}$$

$$\text{Now } P_{dc} = (I_{dc})^2 \times R_L = \frac{I_m^2 \cdot R_L}{\pi^2}$$

$$\text{Further } P_{ac} = P_a + P_r$$

where $P_a = \text{power dissipated at the junction of diode}$

$$= I_{rms}^2 \times R_f = \frac{I_m^2}{4} \times R_f$$

and $P_r = \text{power dissipated in the load resistance}$

$$= I_{rms}^2 \times R_L = \frac{I_m^2}{4} \times R_L$$

$$\therefore P_{ac} = \frac{I_m^2}{4} \times R_f + \frac{I_m^2}{4} \times R_L = \frac{I_m^2}{4} (R_f + R_L)$$

$$\eta = \frac{\frac{I_m^2 \cdot R_L}{\pi^2}}{\frac{I_m^2}{4} (R_f + R_L)} = \frac{4}{\pi^2} \cdot \frac{R_L}{(R_f + R_L)}$$

$$\eta = \frac{0.406}{[1 + (R_f/R_L)]}$$

$$\% \text{ of rectifier efficiency} = \frac{0.406 \times 100}{1 + R_f/R_L} = \frac{40.6}{1 + \frac{R_f}{R_L}}$$

Theoretically, the max. value of rectifier efficiency of a half wave rectifier is 40.6%. when $R_f/R_L = 0$.

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{(I_{dc})^2 R_L}{(I_{rms})^2 R_L} = \frac{(V_{dc} R_L)^2 \cdot R_L}{(V_{rms} R_L)^2 \cdot R_L} = \frac{V_{dc}^2}{V_{rms}^2} = \frac{\left(\frac{V_m}{\pi}\right)^2}{\left(\frac{V_m}{2}\right)^2}$$

$$\text{or } \eta = \frac{4}{\pi^2} = 0.406 = 40.6\%$$

(4)

(v) Ripple factor :-

The output of a rectifier consists of d.c. component as well as a.c. component. The a.c. component in the output is called as ripple. This is undesirable. Thus, smaller is the ripple, more effective will be the rectifier. The ripple factor r gives an idea about the waviness of the ~~rectified~~ rectified voltage and is defined as the ratio of the effective value of the a.c. component of voltage or current to the direct or average value. Hence,

$$r = \frac{\text{ripple voltage}}{\text{d.c. voltage}} = \frac{\text{rms value of a.c. component}}{\text{d.c. value of wave}}$$

$$= \frac{(V_r)_{\text{rms}}}{V_{\text{dc}}} = \frac{V_{\text{ac}}}{V_{\text{dc}}} = \frac{(I_r)_{\text{rms}}}{I_{\text{dc}}} = \frac{I_{\text{ac}}}{I_{\text{dc}}}$$

where $(V_r)_{\text{rms}}$ = rms value of a.c. component of output voltage
 V_{dc} = d.c. value of output voltage.

$$\text{Now, } V_{\text{rms}}^2 = \sqrt{V_{\text{dc}}^2 + (V_r)_{\text{rms}}^2} = \sqrt{V_{\text{dc}}^2 + V_{\text{ac}}^2}$$

$$(V_r)_{\text{rms}} = \sqrt{V_{\text{rms}}^2 - V_{\text{dc}}^2}$$

$$\therefore r = \frac{\sqrt{V_{\text{rms}}^2 - V_{\text{dc}}^2}}{V_{\text{dc}}} = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1}$$

$$r = \sqrt{\frac{\left(\frac{V_m}{2}\right)^2}{\left(\frac{V_m}{\pi}\right)^2} - 1} = \sqrt{\left(\frac{11}{2}\right)^2 - 1} = 1.21.$$

The ripple factor is also defined as,

$$r = \frac{(I_r)_{\text{rms}}}{I_{\text{dc}}}$$

The effective or rms value of current is given by

$$I_{rms} = \sqrt{I_{dc}^2 + I_1^2 + I_2^2 + I_3^2 + \dots} = \sqrt{I_{dc}^2 + I_{ac}^2} = \sqrt{I_{dc}^2 + (I_r)_{rms}^2}$$

$$I_{ac} = (I_r)_{rms} = \sqrt{I_1^2 + I_2^2 + I_3^2 + \dots}$$

where $I_1, I_2 \dots$ are the rms value of the ripple components such as fundamental, second harmonic etc, respectively.

The ripple factor, r is given by

$$r = \frac{(I_r)_{rms}}{I_{dc}} = \frac{I_{ac}}{I_{dc}} = \sqrt{\frac{I_{rms}^2 - I_{dc}^2}{I_{dc}^2}} = \sqrt{\left(\frac{(I_r)_{rms}}{I_{dc}}\right)^2 - 1}$$

$$r = \sqrt{\frac{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}{\left(\frac{I_{rms}}{\pi}\right)^2}} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$$

Voltage Regulation: —

It is defined as variation of d.c. output voltage with change in d.c. load current. Thus

$$\% \text{ of Voltage regulation} = \frac{V_{no\text{load}} - V_{full\text{load}}}{V_{full\text{load}}} \times 100\%$$

where $V_{no\text{load}}$ = voltage at no load

$V_{full\text{load}}$ = voltage at full load

For an ideal power supply, the output voltage should be independent of the load current. An ideal power supply has a full-load voltage equal to its no load voltage i.e., it has zero percentage regulation. However is the percentage regulation, better would be the power supply.

$$V_{dc} = \frac{V_m \cdot R_L}{\pi(R_f + R_L)} = \frac{V_m}{\pi} \left[1 - \frac{R_f}{(R_f + R_L)} \right]$$

$$\text{or } V_{dc} = \frac{V_m}{\pi} - I_{dc} \cdot R_f$$

(5)

So half wave rectifier functions as if it were constant voltage source (V_m/π) in series with an internal resistance R_f . Thus $V_{dc} = \frac{V_m}{\pi}$ with no load ($I_{dc} = 0$) and d.c. output voltage decreased linearly with the increase of d.c. output current I_{dc} . By using suitable voltage regulators involving regulator tubes, zener diodes and transistors, the internal resistance can be reduced to a fraction of an ohm. Such regulators will be considered later.

Transformer Utilisation factors:-

The d.c. power to be delivered to the load in a rectifier circuit decides the rating of the transformer used in the circuit.

so, TUF is defined as,

$$TUF = \frac{\text{d.c. power to be delivered to the load}}{\text{a.c. rating of the transformer secondary}} = \frac{P_{dc}}{P_{ac \text{ rated}}}$$

According to theory of transformer, the rated voltage of the secondary will be V_m/π and the actual r.m.s. current flowing through it will be $I_m/\sqrt{2}$.

$$\therefore TUF = \frac{\left(\frac{I_m}{\pi}\right)^2 \times R_L}{\left(\frac{V_m}{\pi}\right) \times \left(\frac{I_m}{\sqrt{2}}\right)}$$

$$\text{But } V_m = I_m (R_f + R_L).$$

$$\therefore TUF = \frac{\left(\frac{I_m}{\pi}\right)^2 \times R_L}{\left(\frac{I_m (R_f + R_L)}{\sqrt{2}}\right) \times \frac{I_m}{\sqrt{2}}} = \frac{2\sqrt{2}}{\pi^2} \times \frac{R_L}{R_f + R_L}$$

$$\text{If } R_L \gg R_f$$

$$\therefore TUF = \frac{2\sqrt{2}}{\pi^2} = 0.287$$

This means that if the transformer rating is 1 kVA (1000VA), then the half wave rectifier can deliver $1000 \times 0.287 = 287$ watt to resistance load.

Peak Inverse voltage (PIV):-

We have seen that during the negative half cycle of ac input voltage, the diode is reverse biased and does not conduct. Therefore, there is no current in the circuit. As a result, there is no voltage drop across load resistor R_L . So, for the negative voltage at the input, the same amount appears as reverse voltage across the diode. Now, we define peak inverse voltage as the maximum voltage across the diode in the reverse direction, i.e., during non-conducting or off state. Obviously, the peak inverse voltage is, V_m .

Form factor:-

The form factor is defined as,

$$F = \frac{\text{r.m.s. value}}{\text{average value}}$$

$$F = \frac{I_{m/2}}{\overline{I}_{m/1\pi}} = \frac{0.5 I_m}{0.318 I_m} = 1.57.$$

Peak factor:-

It is defined as the ratio of peak value to r.m.s. value

$$\begin{aligned} \text{peak factor} &= \frac{\text{Peak value}}{\text{r.m.s. value}} \\ &= \frac{V_m}{V_{m/2}} = 2. \end{aligned}$$

CENTRE-TAP FULLWAVE RECTIFIER:-

①

In fullwave rectifier, both half cycles of the input are utilized with the help of two diodes working alternately. Therefore, in fullwave rectifier circuit, current flows through load resistor in the same direction for both half cycles of input a.c. voltage.

Fig 11(a) and 11(b) shows the basic circuit of a fullwave rectifier with centre tapped transformer.

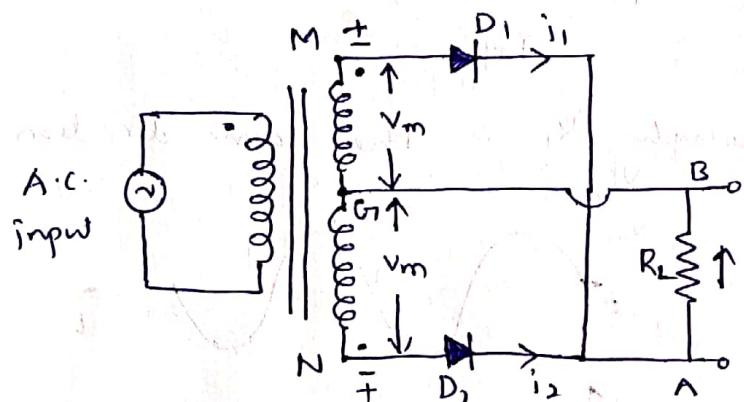


Fig 11(a)

Basic circuit diagram

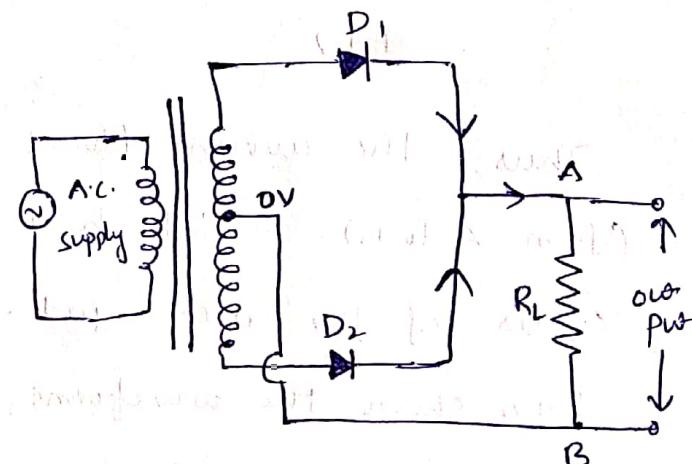


Fig 11(b)

Working of Full wave rectifier:-

→ When input a.c. supply is switched on, the ends M and N of the transformer secondary becomes positive and negative alternately.

→ During positive half of a.c. input, terminal M is positive, G is at zero potential and N is at negative potential.

Now, diode D₁ is forward biased i.e., it conducts and causes a current i₁ in load resistor R_L. Diode D₂ remains non-conducting being reversed biased. This is shown in fig.2.

→ During the negative half cycle, terminal N becomes positive, G is at zero potential and M is at negative potential.

During this cycle, diode D_2 conducts and current i_2 flows in the circuit through load resistor R_L . Diode D_1 is non-conducting. This is shown in fig. 3.

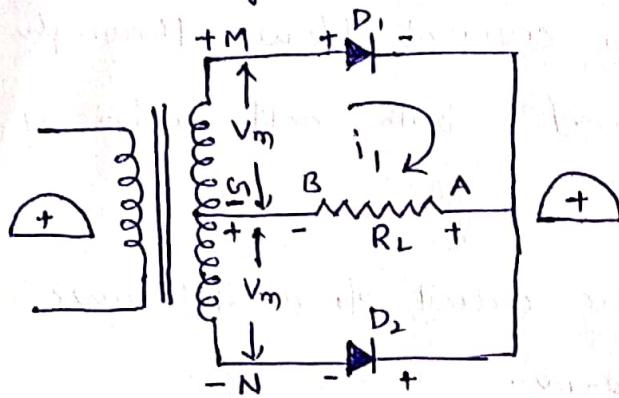


Fig. 2

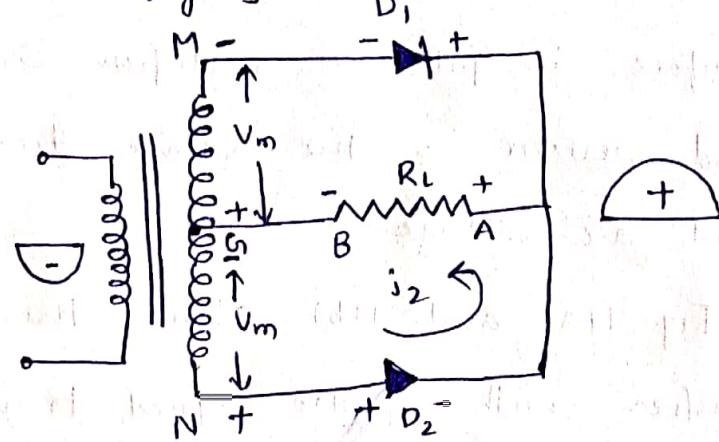


Fig. 3.

Thus, the current flows through R_L in the same direction (from A to B) in both half cycles of the a.c. input.

Fig. 4. shows the waveforms.

It is obvious from fig. 4. that the frequency of the rectified output voltage is twice the supply frequency.

Analysis of Full wave rectifier:-

Let the input voltage V_i is given by

$$V_i = V_m \sin \omega t$$

The current i_1 through diode D_1 and load resistor R_L is given by,

$$i_1 = I_m \sin \omega t \quad \text{for } 0 \leq \omega t \leq \pi$$

$$i_1 = 0 \quad \text{for } \pi \leq \omega t \leq 2\pi$$

$$\text{where } I_m = \frac{V_m}{R_f + R_L}$$

R_f being the diode resistance in ON condition.

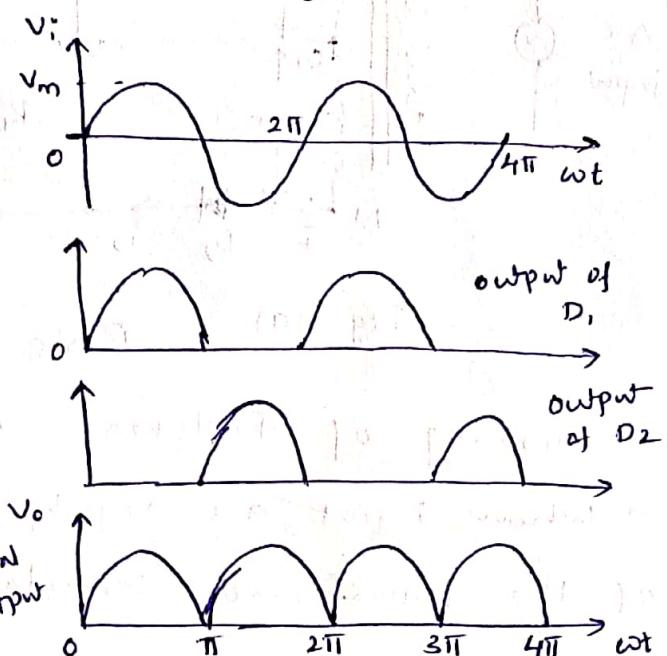


Fig. 4.

Similarly, the current i_2 flowing through diode D_2 and load resistor R_L is given by,

$$i_2 = 0 \quad \text{for } 0 \leq wt \leq \pi$$

$$i_2 = I_m \sin wt \quad \text{for } \pi \leq wt \leq 2\pi$$

The total current flowing through R_L is the sum of the two currents i_1 and i_2 , i.e.,

$$i = i_1 + i_2$$

(i) D.C. or Average current:-

The average value of output current that a d.c. ammeter will indicate is given by

$$I_{dc} = \frac{1}{2\pi} \int_0^{2\pi} i_1 \cdot d(wt) + \int_0^{2\pi} i_2 \cdot d(wt)$$

$$= \frac{1}{2\pi} \int_0^{\pi} I_m \sin(wt) \cdot d(wt) + 0 + 0 + \frac{1}{2\pi} \int_{\pi}^{2\pi} I_m \sin(wt) \cdot d(wt)$$

$$I_{dc} = \frac{I_m}{\pi} + \frac{I_m}{\pi} = \frac{2I_m}{\pi} = 0.636 I_m.$$

This is double of that of a half wave rectifier

(ii) D.C. output voltage:-

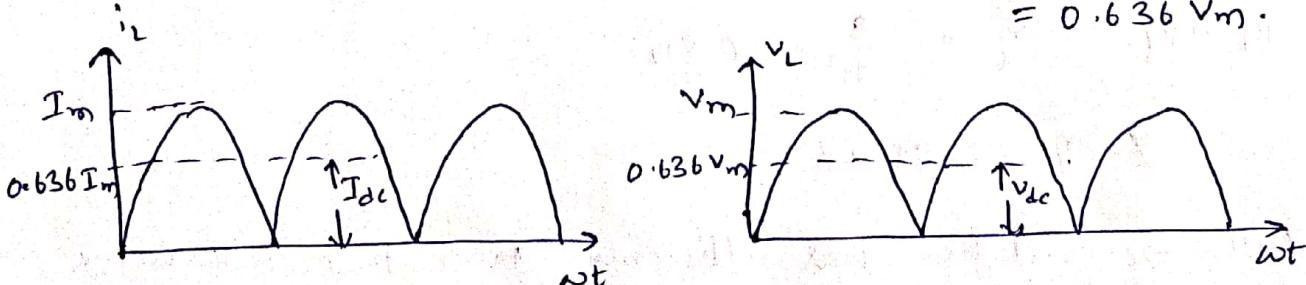
The d.c. output voltage across load is given by

$$V_{dc} = I_{dc} \times R_L = \frac{2I_m \cdot R_L}{\pi} = 0.636 I_m \cdot R_L$$

$$V_{dc} = \frac{2V_m}{\pi(R_f + R_L)} \cdot R_L \quad \text{if } R_L \gg R_f$$

$$\therefore V_{dc} = \frac{2V_m}{\pi}$$

$$= 0.636 V_m.$$



(iii) RMS current:-

The RMS value of the current is given by

$$I_{rms} = \left[\frac{1}{\pi} \int_0^{\pi} i^2 d(\omega t) \right]^{1/2}$$

$$= \left[\frac{I_m^2}{\pi} \int_0^{\pi} \sin^2(\omega t) \cdot d(\omega t) \right]^{1/2} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

(iv) Rectifier efficiency:-

The rectifier efficiency is defined as

$$\eta = \frac{P_{dc}}{P_{ac}}$$

$$\text{Now } P_{dc} = \frac{(V_{dc})^2}{R_L} = \frac{(2V_m)^2}{R_L}$$

$$\text{and } P_{ac} = \frac{(V_{rms})^2}{R_L} = \frac{(V_m)^2}{R_L}$$

$$\eta = \frac{(2V_m/\pi)^2}{(V_m/\sqrt{2})^2} = \frac{8}{\pi^2} = 0.812 = 81.2\%$$

$$\text{The d.c. output power } P_{dc} = I_{dc}^2 \cdot R_L = \frac{4I_m^2}{\pi^2} \cdot R_L$$

$$\text{The a.c. input power } P_{ac} = I_{rms}^2 (R_f + R_L) = \frac{I_m^2}{2} (R_f + R_L)$$

$$\therefore \eta = \frac{\frac{4I_m^2}{\pi^2} \cdot R_L}{\frac{I_m^2}{2} (R_f + R_L)} = \frac{8}{\pi^2} \cdot \frac{R_L}{(R_f + R_L)}$$

$$\text{If } R_L \gg R_f \quad \eta = \frac{8}{\pi^2} = 0.812$$

$$\therefore \eta = 81.2\%$$

Thus, fullwave rectifier has efficiency twice that half wave rectifier.

(v) Ripple factor:-

The form factor of rectified output voltage of a full wave rectifier is given by

$$F = \frac{I_{rms}}{I_{dc}} = \frac{Im/\sqrt{2}}{2Im/\pi} = 1.11$$

The ripple factor r is defined as

$$r = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} = \sqrt{F^2 - 1}$$

$$r = \sqrt{(1.11)^2 - 1} = 0.48$$

(vi) voltage regulation:-

The d.c. output voltage is given by

$$V_{dc} = \frac{2Im \cdot R_L}{\pi} = \frac{2V_m \cdot R_L}{\pi (R_f + R_L)}$$

$$\text{or } V_{dc} = \frac{2V_m}{\pi} \left[1 - \frac{R_f}{R_f + R_L} \right] = \frac{2V_m}{\pi} - I_{dc} \cdot R_f$$

(vii) Peak Inverse voltage:-

PIV is the maximum possible voltage across a diode when it is reverse biased. consider the diode D₁ is in forward biased i.e, conducting and Diode D₂ is reverse biased i.e, non conducting. In this case a voltage V_m is developed across the load resistor R_L . Now the voltage across diode D₂ is the sum of the voltages across load resistor R_L and voltage across the lower half of transformer secondary V_m . Hence PIV of diode D₂ = $V_m + V_m = 2V_m$. Similarly PIV of diode D₁ is $2V_m$.

(viii) Transformer utilisation factor:-

The average TUF in full wave rectifying circuit is determined by considering the primary and secondary winding separately.

There are two secondaries here. Each secondary is associated with one diode. This is just similar to secondary of half wave rectifier. Each secondary has TUF as 0.287. Now,

$$\text{TUF of primary} = \frac{P_{dc}}{\text{volt-amp rating of primary}}$$

$$= \frac{(I_{dc})^2 \cdot R_L}{\left(\frac{V_m}{\sqrt{2}}\right) \left(\frac{I_m}{\sqrt{2}}\right)} = \frac{\left(\frac{2I_m}{\pi}\right)^2 \cdot R_L}{\left(\frac{V_m}{\sqrt{2}}\right) \times \left(\frac{I_m}{\sqrt{2}}\right)}$$

$$= \frac{4I_m^2}{\pi^2} \cdot \frac{2R_L}{I_m^2 (R_f + R_s + R_L)}$$

where R_s is the resistance of secondary

$$\text{TUF of primary} = \frac{8}{\pi^2} \left[\frac{R_L}{R_f + R_s + R_L} \right]$$

$$= \frac{8}{\pi^2} \left[\frac{1}{1 + \left(\frac{R_s + R_f}{R_L} \right)} \right] \quad \because R_L \gg R_s + R_f$$

$$= \frac{8}{\pi^2} = 0.812$$

$$\text{But } (\text{TUF})_{avg} = \frac{P_{dc}}{\text{V-A rating of transformer}}$$

$$= \frac{(\text{TUF})_p + (\text{TUF})_s + (\text{TUF})_s}{3} = \frac{0.812 + 0.287 + 0.287}{3}$$

$$= 0.693$$

FULL-WAVE BRIDGE RECTIFIER:-

A circuit frequently used for electronic d.c. power supplies is a full wave bridge rectifier.

In full wave bridge rectifier four diodes are used as shown in fig.1

→ During the positive input half cycle, terminal M of the secondary of the transformer is positive while the terminal N is negative.

In this situation diodes D_1 and D_3 are forward biased i.e., they conduct whereas diodes D_2 and D_4 are reverse biased i.e., they do not conduct. So a current flows along MABCEFNA as shown by arrows in

fig.1. There will be a voltage drop across R_L .

→ During the negative input half cycle, terminal N of the secondary of transformer becomes positive while the terminal M becomes negative. Under this situation diodes D_2 and D_4 are forward biased i.e., they conduct whereas diodes D_1 and D_3 are reverse biased i.e., they do not conduct. Now a current flows along NFBCFAEM as shown by dotted arrow in fig.1. The current produces a voltage drop across R_L .

→ It is obvious from the fig. that current through load resistance R_L

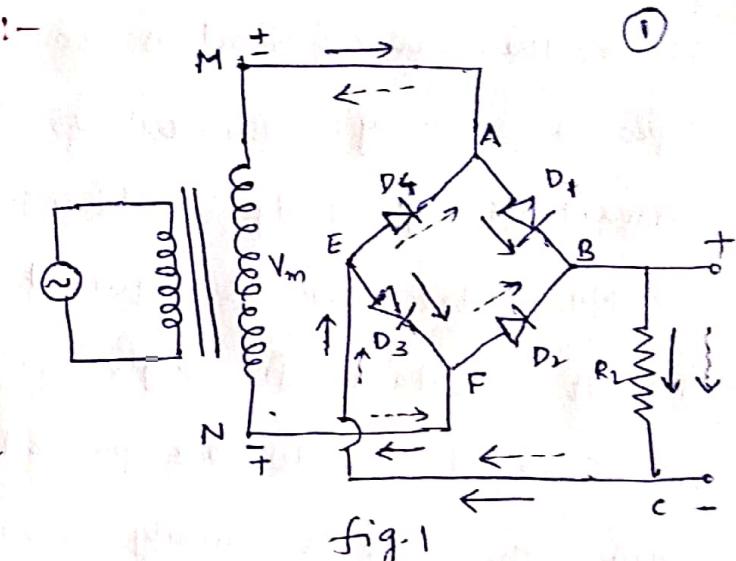


fig.1

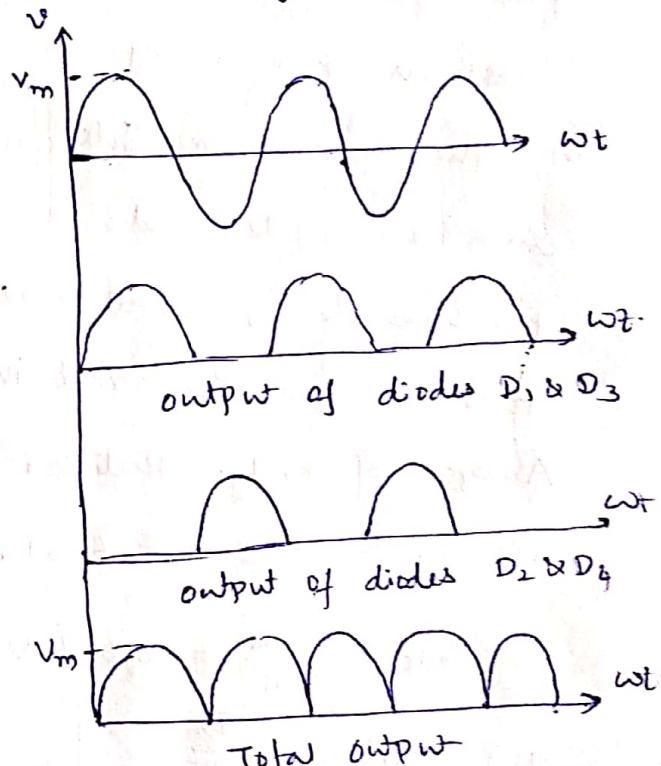


Fig. 2

is in the same direction BC during both half cycle of the input a.c. supply. The output waveforms are shown in fig. 2.

Advantages of Bridge rectifier:-

- (i) No centre-tap is required on transformer.
- (ii) It is suitable for high voltage applications.
- (iii) Since two diodes are present in series in each conduction path, the PIV is equally shared by the two diodes. Thus, it has less PIV rating per diode.
- (iv) The current in both of the supply transformer flows for the entire a.c. cycle and hence for a given power output, power transformer of small size may be used in comparison with that in fullwave rectifier.

Analysis of Bridge Rectifier:-

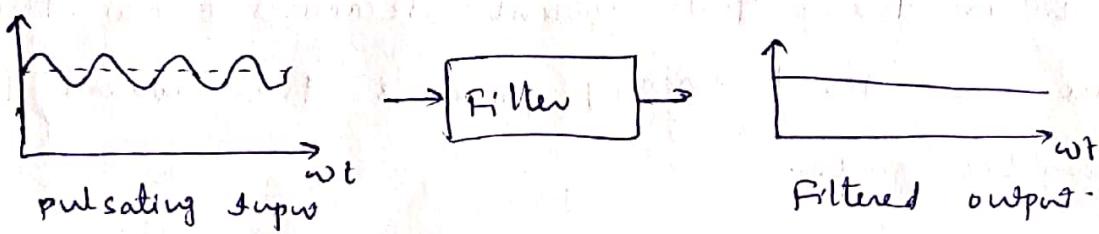
- (i) $I_{dc} = \frac{2I_m}{\pi} = 0.636 I_m$
- (ii) $V_{dc} = \frac{2V_m}{\pi} = 0.636 I_m R_L = 0.636 I_m R_L$
- (iii) $I_{rms} = \frac{I_m}{\sqrt{2}}$
- (iv) efficiency $\eta = \frac{0.812}{1 + R_f/R_L}$ or percentage efficiency = 81.2%
when $R_L \gg R_f$.
- (v) ripple factor $r = 0.48$
- (vi) PIV of a diode in Bridge rectifier: For positive half cycle of input, the diodes D_1 and D_3 are forward biased. These diodes may be regarded as shorted points. On the other hand, the diodes D_2 and D_4 are reverse biased. These diodes have max. reverse voltage equal to the maximum secondary voltage V_m . Therefore, PIV of a diode in a bridge rectifier is given by

$$PIV = V_m$$

(vii) The transformer utilisation factor of primary and secondary will be the same as there is always current through primary and secondary.

$$\text{TFU of secondary} = \frac{P_{dc}}{\text{V.A. rating of secondary}} = \frac{I_{dc} \cdot R_L}{\left(\frac{V_m}{\sqrt{2}}\right) \left(\frac{I_m}{\sqrt{2}}\right)} = 0.812$$

Filters:— Filter is a device that converts the pulsating output of a rectifier into a steady d.c. level. (or) which removes the a.c. component of rectifier output but allows the d.c. component to reach the load.



Fullwave rectifier with series inductor filter :-

A fullwave rectifier with series inductor is shown in fig. The inductor in series with load serves as a filter.

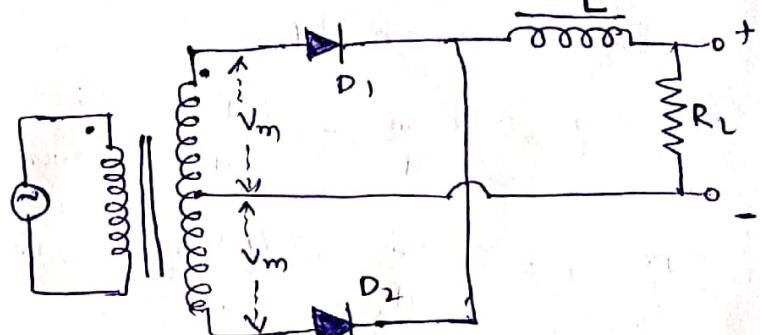


Fig-1

→ The inductor offers high impedance to a.c. variations and none to d.c. current since d.c. resistance of large inductance L is very small. This action is based on the property of inductance to oppose any change of current that may flow through it. Therefore, a.c. component of rectified output is blocked while the d.c. component reaches at the load.

→ Theoretically, the output should contain d.c. voltage but in actual practice, it contains a small a.c. components as well.

Fig. 2. shows the load current waveform with and without filter.

→ When the output current of the rectifier increases

above a certain average value,

magnetic energy is

stored in the Inductor.

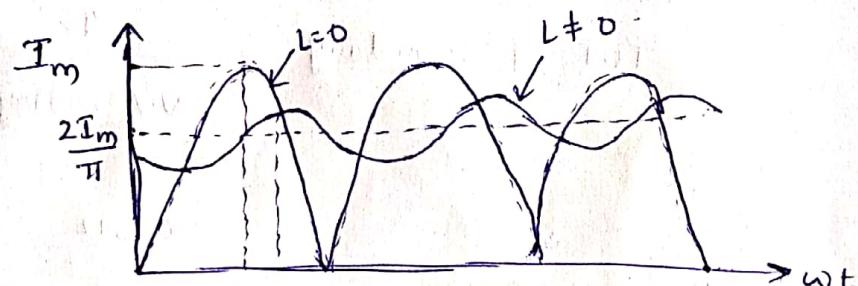


Fig. 2. Load current with and without filtering.

This energy tends to decrease the sudden rise in the current.

→ When the output current decreases below the average value, the stored energy prevents the current from falling down too much.

The current flowing in a full wave rectifier is without filter is given by,

$$i = \frac{2I_m}{\pi} - \frac{4I_m}{\pi} \sum_{k=\text{even no.}} \frac{\cos kwt}{(k-1)(k+1)}$$

$$\text{or } i = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi} \cos 2wt - \frac{4I_m}{15\pi} \cos 4wt \rightarrow ①$$

Since the impedance of an inductor increases with frequency, hence there is a better filtering action for higher harmonic terms. Neglecting the effect of third and higher terms, we have

$$i = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi} \cos 2wt \rightarrow ②$$

Neglecting the diode drop and diode resistance because they introduce a little error.

Thus for d.c. component, the current $I_m = \frac{V_m}{R_L}$. For a.c. component, the impedance of L and R_L will be in series and is given by

$$Z = \sqrt{R_L^2 + (2\omega L)^2}$$

$$\text{frequency of a.c component} = \frac{1}{2\pi} \quad |Z| = \sqrt{R^2 + (\omega L)^2}$$

$$Z = \sqrt{R_L^2 + 4\omega^2 L^2}$$

$$\text{Thus for a.c. component } I_m = \frac{V_m}{\sqrt{R_L^2 + 4\omega^2 L^2}}$$

substituting the value of I_m for d.c. and a.c. in eq-②, we get

$$i = \frac{2V_m}{\pi R_L} - \frac{4V_m}{3\pi \sqrt{R_L^2 + 4\omega^2 L^2}} \cos(2\omega t - \phi)$$

where ϕ is the angle by which the load current lags behind the voltage. $\phi = \tan^{-1} \frac{2\omega L}{R_L}$

(i) Ripple factor:- The ripple factor r is defined as the ratio of rms value of the a.c. component to the d.c. value of the wave i.e., $r = \frac{(I_r)_{rms}}{I_{dc}}$

$$r = \frac{\frac{4V_m}{3\pi \sqrt{2}} \cdot \frac{1}{\sqrt{R_L^2 + 4\omega^2 L^2}}}{2V_m / \pi R_L}$$

$$= \frac{2}{3\sqrt{2}} \cdot \frac{1}{\sqrt{[1 + (4\omega^2 L^2 / R_L^2)]}}$$

$$\text{If } \frac{4\omega^2 L^2}{R_L^2} > 1 \text{ then } r = \frac{1}{3\sqrt{2}} \cdot \frac{R_L}{\omega L} = 0.236 \frac{R_L}{\omega L}$$

This expression shows that ripple factor varies inversely as the magnitude of the inductance. Also the ripple is smaller for smaller values of R_L . i.e., for high currents.

When $R_L \rightarrow \infty$, the value of r is given by

$$r = \frac{2}{3\sqrt{2}} = 0.471$$

Thus, the inductor filter should be used when R_L is consistently small i.e. effective filtering takes place when load current is high.

(iii) Regulation:-

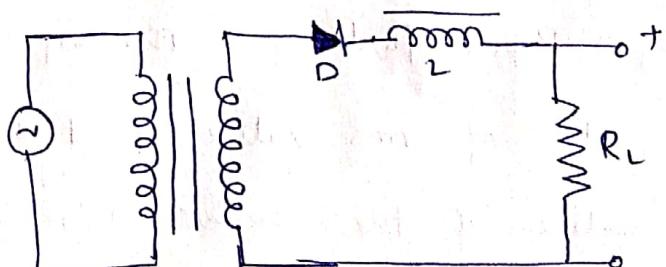
The d.c. output voltage is given by

$$V_{dc} = I_{dc} R_L = \frac{2V_m}{\pi} = 0.636 V_m = 0.636 \cdot \sqrt{2} V_{rms} \\ = 0.90 V_{rms}$$

Here V_{rms} is the transformer secondary voltage measured to the centre tap. So, the output voltage is constant and is independent of the load. Thus, there exists a perfect regulation.

Series Inductor filter for Half wave rectifier:-

The series inductor filter for half wave rectifier is shown in fig. The description of this filter action is same as that for a full wave rectifier.



Expression for ripple factor:- $i = \frac{I_m}{\pi} + \frac{I_m}{2} \sin \omega t - \frac{2I_m}{\pi} \sum_{k=even}^{\infty} \frac{\cos k\omega t}{(k-1)(k+1)}$

For a half-wave rectifier, the output current is given by

$$i = \frac{I_m}{\pi} + \frac{I_m}{2} \sin \omega t - \frac{2I_m}{\pi} \frac{\cos 2\omega t}{3} - \frac{2I_m}{\pi} \frac{\cos 4\omega t}{15} + \dots$$

Neglecting the higher order terms, we have

$$I_{dc} = \frac{I_m}{\pi} = \frac{V_m}{\pi R_L}$$

If I_1 be the r.m.s value of fundamental component of current, then

$$I_1 = \frac{I_m}{2\sqrt{2}} = \frac{\frac{V_m}{2\sqrt{2}(\omega L + R_L)}}{2\sqrt{2}(\omega L + R_L)} = \frac{V_m}{2\sqrt{2}(\omega^2 L^2 + R_L^2)^{1/2}}$$

At operating frequency, the reactance offered by inductance L is very large compared to R_L (i.e., $\omega L \gg R_L$) and hence R_L can be neglected.

$$\therefore I_1 = \frac{V_m}{2\sqrt{2}\omega L}$$

If I_2 be the r.m.s value of second harmonic, then

$$I_2 = \frac{2I_m}{3\sqrt{2}\pi} = \frac{2V_m}{3\sqrt{2}\pi[\sqrt{R_L^2 + 4\omega^2 L^2}]} = \frac{V_m}{3\sqrt{2}\pi\omega L} \quad (\text{as } \omega L \gg R_L)$$

If I_{ac} be the r.m.s value of all current components,

then $I_{ac} = \sqrt{I_1^2 + I_2^2}$

$$\text{Now } r = \frac{V_{ac}}{V_{dc}} \approx \frac{I_{ac} R_L}{I_{dc} R_L} = \frac{I_{ac}}{I_{dc}}$$

$$r = \frac{\sqrt{\left(\frac{V_m}{2\sqrt{2}\omega L}\right)^2 + \left(\frac{V_m}{3\sqrt{2}\pi\omega L}\right)^2}}{\frac{V_m}{\pi R_L}} = \frac{V_m}{\omega L} \sqrt{\left(\frac{1}{8} + \frac{1}{18\pi^2}\right)} \frac{V_m}{\pi R_L}$$

$$r = \frac{\pi R_L}{\omega L} \sqrt{\left(\frac{1}{8} + \frac{1}{18\pi^2}\right)}$$

$$r = \frac{1.13 R_L}{\omega L}$$

Shunt capacitor filter:-

The action of this filter circuit depends upon the fact that a capacitor stores energy when conducting and delivers this energy to load during non-conduction. Through this process, the ripple components are considerably reduced.

Halfwave rectifier with capacitor filter:-

The halfwave rectifier with a capacitor filter is shown in fig. 1.

- During the positive half cycle of a.c input, the diode D is forward biased and hence it conducts. This quickly

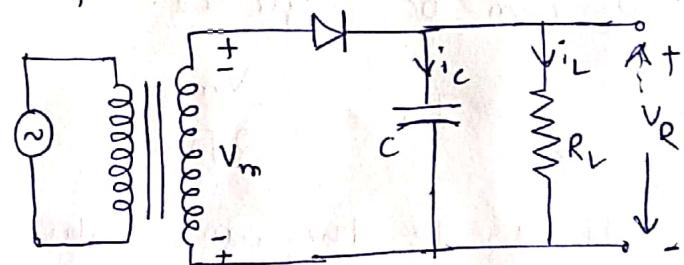
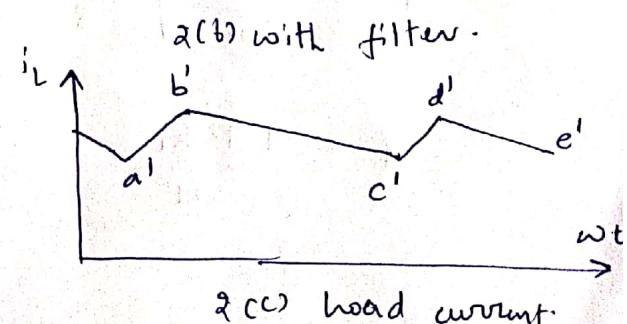
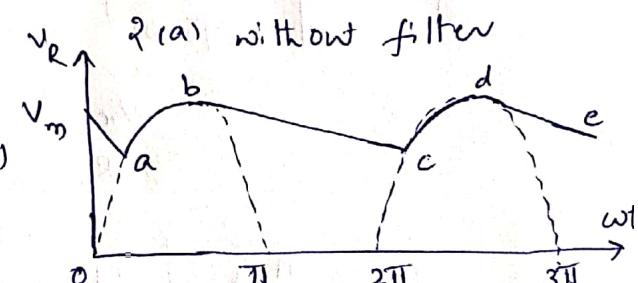
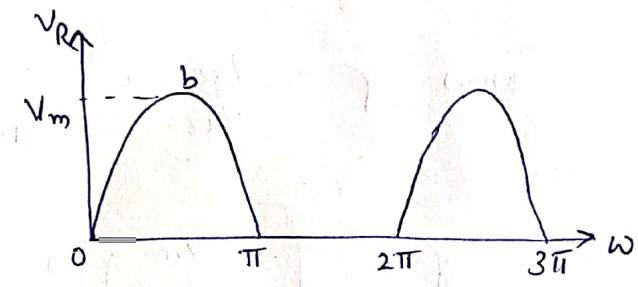


fig. 1.

charges the capacitor C to a voltage V_m because there is no resistance in the charging path except diode forward resistance which is negligible. This is shown by a point 'b' in fig 2(c). When capacitor fully charged, it holds the charge till input a.c. supply to the rectifier goes negative.

- During the negative half cycle, the diode D is reverse biased i.e., it does not conduct. So, the capacitor C discharges through R_L from point 'b' to point 'c' as shown in fig 2(b).
- As the discharging time constant is large, usually 100 times more than charging time, hence the capacitor does not have sufficient time to discharge appreciably.



- Due to this fact, the voltage of capacitor C decreases slightly. Thus, during negative half cycle the capacitor maintains sufficient large voltage across R_L as shown in fig.
- During the next positive half cycle, the capacitor voltage increases from point c to point d to V_m . The process is then repeated. Fig. 2(b) shows the load voltage variation with input a.c. voltage. It is obvious from fig. that nearly constant d.c. voltage appears across R_L at all times.
- In case of a purely resistive load, the wave shape of load current i_L is the same as that of V_R . This is shown in fig 2(c).

During periods 'a'b' and 'c'd' the current is supplied by diode and during periods 'b'c' and 'd'e' etc., by the capacitor. Let us consider the diode current. The instant at which the conduction starts is called cut-in-point and the instant at which conduction stops is called the cut-out point. Fig. 3 shows the cut-in and cut-out points.

During cut-in and cut-out points, the diode current flows and diode output voltage is greater than the capacitor voltage.

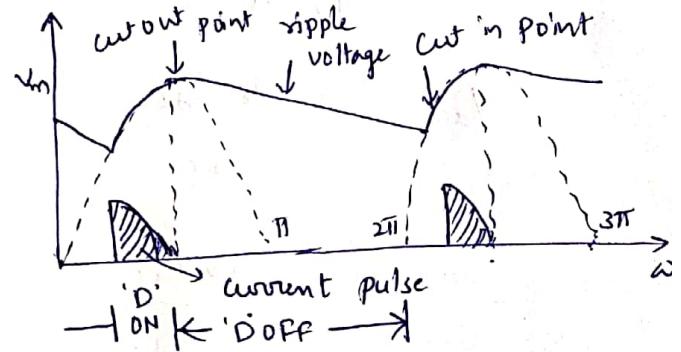


Fig. 3

From Fig. 3, the diode current is short duration pulses i.e. a surging current. Hence, the diode acts as a switch which permits charge to flow in capacitor when the transformer exceeds the capacitor voltage and then disconnects the power source when the transformer voltage falls below that of the capacitor.

Expression for Ripple factor:-

From Fourier analysis,

$$i = I_{dc} + 2I_{dc} \cos \omega t + 2I_{dc} \cos 2\omega t + 2I_{dc} \cos 3\omega t + \dots$$

$$V_{ac} = \sqrt{V_1^2 + V_2^2 + V_3^2 + \dots}$$

where $V_1 = \frac{2I_{dc}}{\sqrt{2}} \cdot X_C$ [$\because X_C \ll R_L$, R_L can be treated as open circuited].

$$X_C = \frac{1}{\omega C}$$

Now $V_1 = \frac{2I_{dc}}{\sqrt{2}} \times \frac{1}{\omega C}$ fundamental component of a.c. ripple

$$V_2 = \frac{2I_{dc}}{\sqrt{2}} \times \frac{1}{2\omega C} \text{ second harmonic ripple}$$

$$V_3 = \frac{2I_{dc}}{\sqrt{2}} \times \frac{1}{3\omega C} \text{ third harmonic ripple}$$

$$\therefore V_{ac} = \sqrt{\left(\frac{2I_{dc}}{\sqrt{2}} \times \frac{1}{\omega C}\right)^2 + \left(\frac{2I_{dc}}{\sqrt{2}} \times \frac{1}{2\omega C}\right)^2 + \left(\frac{2I_{dc}}{\sqrt{2}} \times \frac{1}{3\omega C}\right)^2 + \dots}$$

$$V_{ac} = \frac{2I_{dc}}{\sqrt{2} \omega C} \left[\sqrt{1 + \frac{1}{4} + \frac{1}{9} + \dots} \right] = \frac{\sqrt{2} I_{dc}}{\omega C} \cdot \sqrt{1.36}$$

$$V_{ac} = \frac{1.64 I_{dc}}{\omega C}$$

$$\text{so } r = \frac{V_{ac}}{V_{dc}}$$

$$r = \frac{\frac{1.64 I_{dc}}{\omega C}}{I_{dc} R_L} = \frac{1.64}{\omega C R_L}$$

$$r = \frac{1}{2\sqrt{3} f_C R_L}$$

Fullwave Rectifier with Capacitor filter:-

(6)

The analysis of a fullwave rectifier with a capacitor filter is simply extension of halfwave circuit.

Fig. 1 shows the circuit of a fullwave rectifier with capacitor filter.

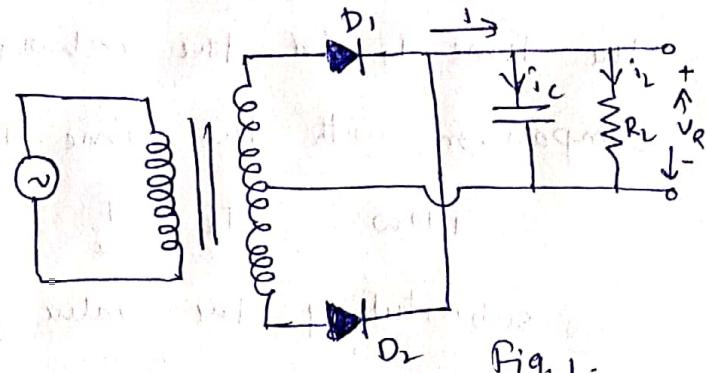


Fig. 1.

and fig. 2 shows, the output voltage waveform and diode current pulses.

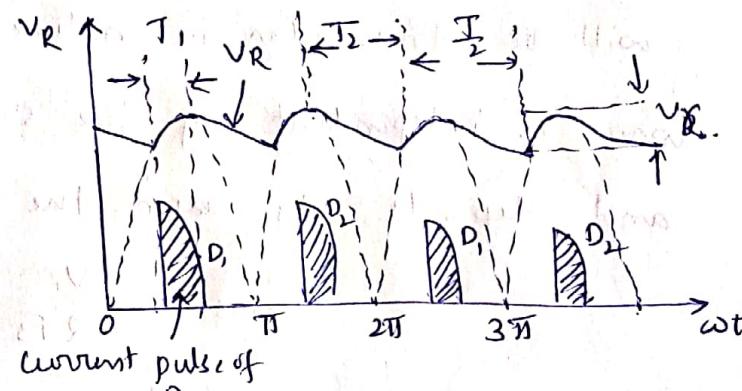


Fig. 2. Output voltage waveform and diode current pulses

The ripple factor may be calculated as:

Let T_2 = Total non-conducting time of capacitor

I_{dc} = average value of the capacitor discharge current over an interval of T_2 .

charge lost by capacitor during interval T_2 is given by

$$(q)_{\text{discharge}} = I_{dc} \times T_2 \quad \rightarrow ①$$

This charge lost is replaced during interval T_1 when the voltage across capacitor changes by an amount equal to peak to peak voltage of ripple V_r . so the charge in the capacitor is given by

$$(q)_{\text{charge}} = \text{voltage} \times \text{capacity} = V_r \times C \rightarrow ②$$

$$\text{But } (q)_{\text{charge}} = (q)_{\text{discharge}}$$

$$V_r C = I_{dc} T_2$$

$$\therefore V_r = \frac{I_{dc} \times T_2}{C} \rightarrow ③$$

Now, we assume that load is light, ripple is small, and the time T_1 for the recharge of the capacitor is small in comparison with the time T_2 of the discharge i.e., $T_2 \gg T_1$.

$$\text{Now } T_2 \approx \frac{T}{2} = \frac{1}{2f} \rightarrow ④$$

substituting the value of T_2 from eq-④ in eq-③, we get

$$V_r = \frac{I_{dc}}{2fC} \rightarrow ⑤$$

According to the assumptions made above, the ripple waveform will be triangular in nature. The rms value of this triangular wave is independent of the slopes or lengths of straight lines and depends only upon the peak value.

$$(V_r)_{rms} = \frac{V_r}{2\sqrt{3}} \rightarrow ⑥$$

$$\therefore (V_r)_{rms} = \frac{I_{dc}}{4\sqrt{3}f \cdot C} = \frac{V_{dc}}{4\sqrt{3}fC \cdot R_L} \quad (\because I_{dc} = \frac{V_{dc}}{R_L})$$

$$\text{Now } r = \frac{(V_r)_{rms}}{V_{dc}} = \frac{1}{4\sqrt{3}fC \cdot R_L}$$

Thus, ripple may be decreased by increasing C or R_L or both.

The output voltage V_{dc} is given by

$$V_{dc} = V_m - \frac{V_r}{2}$$

$$V_{dc} = V_m - \frac{I_{dc}}{4fC}$$

$$V_{dc} = V_m - \frac{V_{dc}}{4fCR_L}$$

$$V_{dc} + \frac{V_{dc}}{4fCR_L} = V_m$$

$$V_{dc} = V_m \cdot \left(\frac{4fCR_L}{1 + 4fCR_L} \right)$$

7

CHOKE INPUT Filter or L-C Filter

In inductor filter, the ripple factor is directly proportional to load resistance while in capacitor filter inversely proportional to the load resistance.

Therefore, the capacitor filter has low ripple at heavy loads while inductor filter at small loads. A combination of these

two filters may be selected to make the ripple independent of load resistance. The resulting filter is called L-type choke input filter.

- The capacitor shunting at the load bypasses the harmonic currents because it offers very low reactance to a.c. ripple current while it appears as an open circuit to d.c. current
- on the other hand the inductor offers a high impedance to the harmonic terms. In this way, most of the ripple voltage is eliminated from the load voltage.

Regulation: - The output of the rectifier is given by

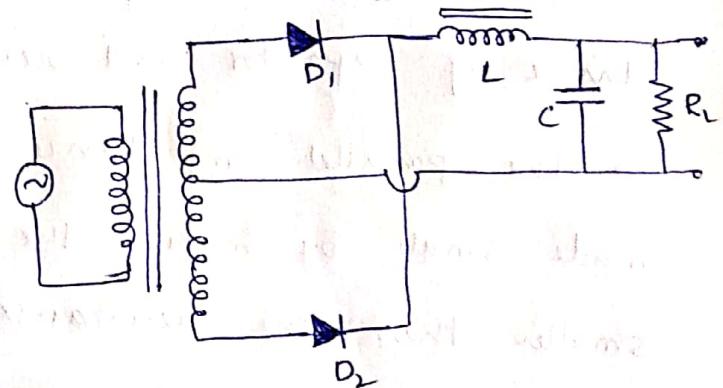
$$v = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi} \cos 2\omega t \rightarrow ①$$

considering that the inductor has no d.c. resistance, the d.c. output voltage is

$$V_{dc} = \frac{2V_m}{\pi} + \cancel{\frac{4V_m}{3\pi} \cos 2\omega t} \rightarrow ②$$

Let R be the series resistance of transformer, diode and inductor then d.c. output voltage is

$$V_{dc} = \frac{2V_m}{\pi} - I_{dc} R \rightarrow ③$$



Ripple factor:- The main aim of the filter is to suppress the harmonic components. So, the reactance of the choke must be large as compared with the combined parallel impedance of capacitor and resistor.

→ The parallel impedance of capacitor and resistor can be made small by making the reactance of the capacitor much smaller than the resistance of load.

→ Now, the ripple current which has passed through L will not develop much ripple voltage across R_L because the reactance of C at the ripple frequency is very small as compared with R_L . Thus for LC filter

$$X_L \gg X_C \text{ at } 2\omega = 4\pi f \quad \text{and} \quad R_L \gg X_C$$

Under these conditions, the a.c. current through L is determined primarily by $X_L = 2\omega L$.

The rms value of the ripple current is

$$(I_r)_{rms} = \frac{4V_m}{3\pi\sqrt{2}} \cdot \frac{1}{X_L} = \frac{2}{3\sqrt{2} X_L} \left(\frac{2V_m}{\pi} \right) = \frac{\sqrt{2}}{3 X_L} (V_{dc}) \rightarrow (4)$$

Although it was stated that X_C is small as compared with R_L but it is not zero. The a.c. voltage across the load is the voltage across the capacitor. Hence

$$(V_r)_{rms} = (I_r)_{rms} \times X_C = \left(\frac{\sqrt{2}}{3 X_L} V_{dc} \right) (X_C) \rightarrow (5)$$

$$\text{ripple factor } r = \frac{(V_r)_{rms}}{V_{dc}} = \frac{\frac{\sqrt{2}}{3 X_L} \cdot V_{dc} \cdot X_C}{V_{dc}} = \frac{\sqrt{2} \cdot X_C}{3 X_L}$$

$$\text{but } X_C = \frac{1}{2\omega C} \quad \text{and} \quad X_L = 2\omega L.$$

$$r = \frac{\sqrt{2}}{3(2\omega L)} \times \frac{1}{2\omega C} = \frac{1}{6\sqrt{2}\omega^2 LC}$$

$$r = \frac{1}{6\sqrt{2}\omega^2 LC}$$

This shows that r is independent of R_L .
The critical inductance: — The current i consists of the following two components
(i) d.c. component i.e., $I_{dc} = V_{dc}/R_L$ and
(ii) a.c. component of peak value = $\frac{4V_m}{3\pi \cdot X_L}$
Since I_{dc} should not exceed the negative peak of the a.c. component, we have

$$\frac{V_{dc}}{R_L} \geq \frac{4V_m}{3\pi \cdot X_L}$$

$$\frac{V_{dc}}{R_L} \geq \frac{2V_{dc}}{3X_L} \quad (\because V_{dc} = \frac{2V_m}{\pi})$$

$$X_L \geq 2 \cdot \frac{R_L}{3}$$

But $X_L = 2\omega \cdot L$ here

$$\therefore L_c \geq \frac{R_L}{3\omega}$$

The CLC or PI (Π) filter: —

A very smooth output may be obtained by a filter consisting of one inductance and two capacitors connected across its each end as

shown in fig. As the three components are arranged in the form of letter Π , the

filter is called as Π filter.

→ This filter is used when, for a given transformer, higher voltage and lower ripple is required than that obtained by

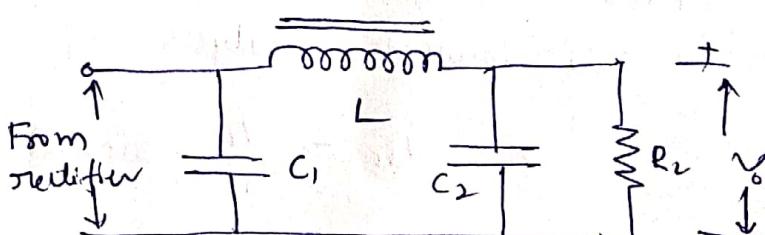


Fig. Π filter

L-section filter

→ In this filter, C_1 is selected to offer very low reactance to the ripple frequency. Thus, major part of the filtering is done by C_1 . Most of the remaining ripple is removed by L-section filter consisting of choke L and capacitor C_2 .

The filtering action of each component is described below:

(i) **Capacitor C_1 :** We know that a capacitor offers a low reactance path to a.c. component while offers an infinite resistance to d.c. component. Therefore, capacitor C_1 bypasses an appreciable amount of a.c. component to ground while allows the d.c. component to pass through it.

(ii) **Inductor L :** We know that the inductor ~~offers~~ offers a high reactance to a.c. component of rectifier output while zero resistance to d.c. component. As a result, it allows the d.c. component to pass through it and blocks the a.c. component which could not be bypassed by capacitor C_1 .

(iii) **Capacitor C_2 :** It further bypasses the a.c. component of rectifier output which could not be blocked by an inductor L .

Ripple factor:-

The Fourier analysis of a triangular wave is given by

$$v = V_{dc} - \frac{V_r}{\pi} \left(\sin 2wt - \frac{\sin 4wt}{2} + \frac{\sin 6wt}{3} - \dots \right) \quad \text{--- (1)}$$

In case of full wave rectifier with capacitor filter,

$$V_r = \frac{I_{dc}}{2fC} = \frac{I_{dc}}{2fC_1} \quad (\because C = C_1 \text{ here})$$

The rms of second harmonic voltage is

(9)

$$(V_r)_{rms} = \frac{V_r}{\pi f \sqrt{2}}$$

$$\therefore (V_r)_{rms} = \frac{I_{dc}}{2\pi f C_1 \sqrt{2}} = \sqrt{2} I_{dc} \cdot X_{C_1}$$

where $X_{C_1} = \frac{1}{2\pi f C_1} = \frac{1}{4\pi f C_1}$ = reactance of C_1 at second harmonic frequency.

The voltage $(V_r)_{rms}$ is impressed on 2-section. Now, the ripple voltage $(V_r)'_{rms}$ can be obtained by multiplying $(V_r)_{rms}$

by $\frac{X_{C_2}}{X_L}$, i.e., $(V_r)'_{rms} = (V_r)_{rms} \times \frac{X_{C_2}}{X_L}$

$$\text{or } (V_r)'_{rms} = \sqrt{2} \cdot I_{dc} \cdot X_{C_1} \cdot \frac{X_{C_2}}{X_L}$$

Hence the ripple factor is given by

$$r = \frac{(V_r)'_{rms}}{V_{dc}} = \frac{\sqrt{2} \cdot I_{dc} \cdot X_{C_1} \cdot \frac{X_{C_2}}{X_L}}{I_{dc} \cdot R_L}$$

$$r = \frac{\sqrt{2} \cdot X_{C_1} \cdot X_{C_2}}{R_L \cdot X_L}$$

Here, all reactances are calculated at second harmonic frequency

$$\therefore r = \boxed{\frac{\sqrt{2}}{8\omega^3 C_1 C_2 L R_L}}$$

$$X_{C_1} = \frac{1}{2\pi f C_1}, \quad X_{C_2} = \frac{1}{2\pi f C_2}$$

$$X_L = 2\pi f L$$

$$\text{At } f = 50\text{Hz} \quad r = \frac{5700}{L C_1 C_2 R_L}$$

where C_1, C_2 are in MF, L in Henry,
 R_L in Ohms.