ENGINEERING MATHEMATICS

ALL BRANCHES

Linear Algebra

DPP – 03

Part – II Solution

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✓ Weekly TEST

Calculus TEST

✓ ✓ Subject Test

Linear +

Prob +

Calculus.

#Q. One of the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ → eigenvalue = 5

$\lambda_1 = 5$

is 5. The other two eigen values are

$\lambda_1 + \lambda_2 + \lambda_3 =$ Trace of matrix A

(a) 0 and 0 = SUM = 0

(b) 1 and 1 SUM = 2

$5 + (\lambda_2 + \lambda_3) = 2 + 2 + 1$

(c) 1 and –1 SUM = 0

(d) –1 and –1 SUM = –2

$\lambda_2 + \lambda_3 = 5 - 5 = 0$

$\boxed{\lambda_2 + \lambda_3 = 0}$

(c) 1 and –1

Slide-2

(SELF-assessment)

#Q. The normalized eigen vector corresponding to the eigen value 5 is

(DO yourself)

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

(b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Normalized eigen vector

→ eigen values → $\underline{-1, 1, 5}$

→ eigen vector

→ Normalized eigen vector

→ $\underline{-1, 1, 5}$

Slide-3

— 80%

#Q. The eigenvalues of a matrix are i, – 2i and 3i. The matrix is

Eigenvalues → i, −2i, 3i

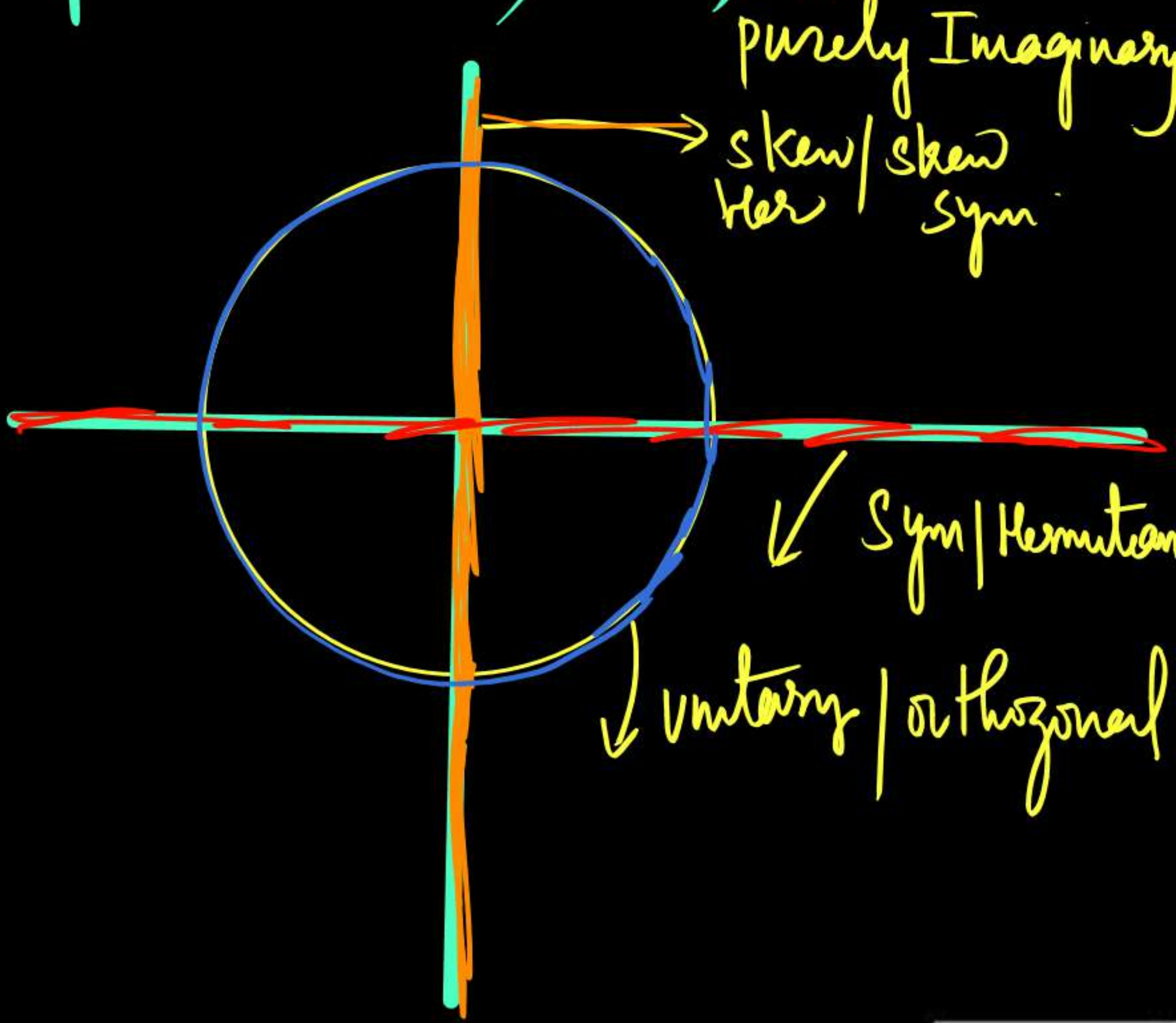
(a) Unitary

(b) Anti-Unitary

(c) Hermitian

(d) Anti-hermitian ✓

Skew-Hermitian $(\bar{A})^T = -A$



Slide-4

#Q. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

$\boxed{\text{SEFF arrrement}}$

Do yourself

→ eigen values

→ eigen vector

(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

SAME

(A) option correct

$\lambda^2 - (5+2)\lambda + (10-4) = 0$

$\lambda^2 - 7\lambda + 6 = 0$

$\lambda^2 - 6\lambda - \lambda + 6 = 0$

$\lambda(\lambda-6) - 1(\lambda-6) = 0$

$\lambda = 1 \quad \lambda = 6$

#Q. Consider a vector $\boxed{\vec{p} = \underset{\downarrow x}{2\hat{\imath}} + \underset{\downarrow y}{3\hat{\jmath}} + \underset{\downarrow z}{2\hat{k}}}$ in the coordinate system $(\hat{\imath}, \hat{\jmath}, \hat{k})$. The axes are rotated anti-clockwise about the $\boxed{\text{Y axis}}$ by an angle of $\boxed{60°}$. The vector $\vec{p}$ in the rotated coordinate system $(\hat{\imath}', \hat{\jmath}', \hat{k}')$ is

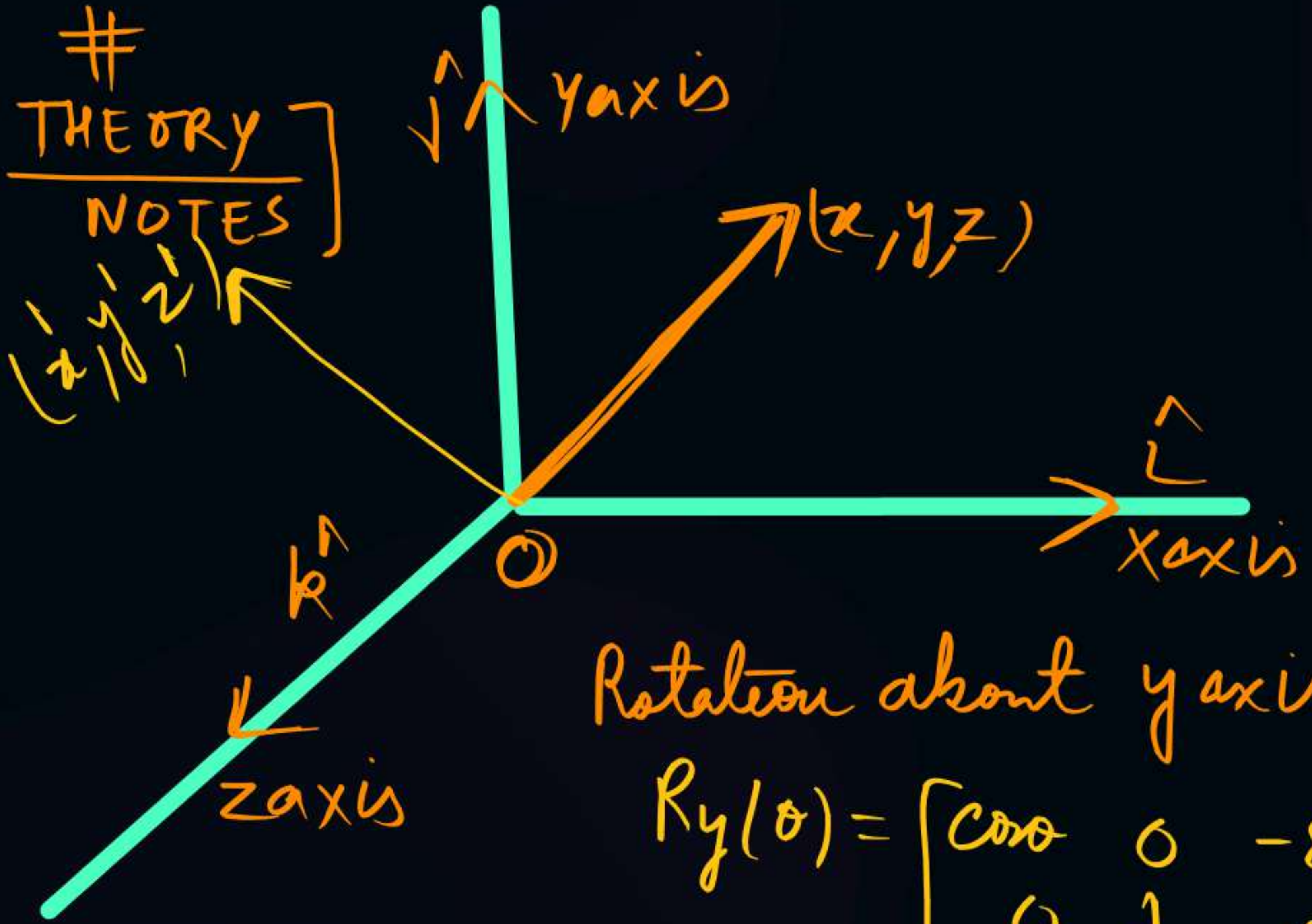
(a) $(1-\sqrt{3})\hat{\imath}' + 3\hat{\jmath}' + (1+\sqrt{3})\hat{k}'$

(b) $(1+\sqrt{3})\hat{\imath}' + 3\hat{\jmath}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{\imath}' + (3+\sqrt{3})\hat{\jmath}' + 2\hat{k}'$

(d) $(1-\sqrt{3})\hat{\imath}' + (3-\sqrt{3})\hat{\jmath}' + 2\hat{k}'$

THEORY NOTES



Rotation about y axis

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$\theta = 60^0$

Rotation

→ x axis Rotation
→ y axis "
→ z axis Rotation

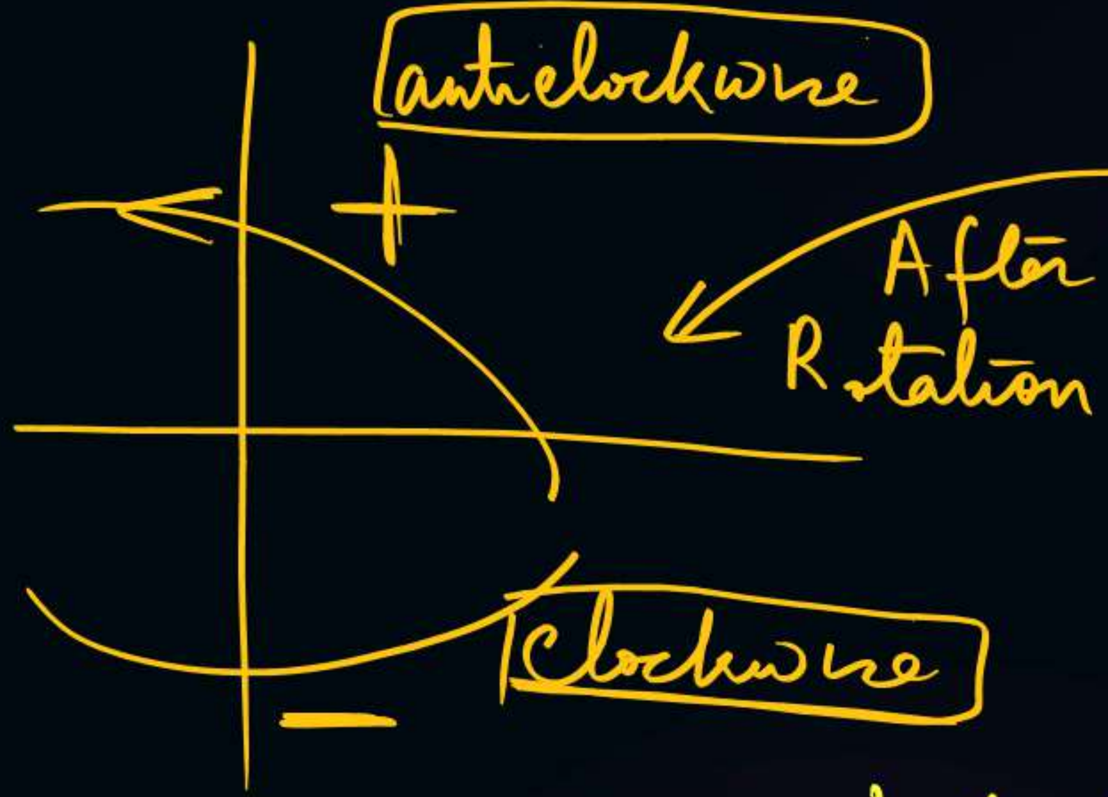
x-axis – along $\hat{i}$
y axis – along $\hat{j}$
z axis – along $\hat{k}$

$(x, y, z) \rightarrow (2, 3, 2)$

$= 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\theta = 60^0$ Rotate
about y axis

(x', y', z')



Rotation about y axis $\theta = 60^\circ$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}}_{y\text{ axis}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos 60^\circ & 0 & -\sin 60^\circ \\ 0 & 1 & 0 \\ \sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\sqrt{3} \\ 3 \\ 1+\sqrt{3} \end{bmatrix}$$

$x' = (1-\sqrt{3})$

$y' = 3$

$z' = (1+\sqrt{3})$

$$\boxed{\text{vector} = (1-\sqrt{3})\hat{i} + 3\hat{j} + (1+\sqrt{3})\hat{k}}$$

#Q. For arbitrary matrices E, F, G and H, if $\boxed{EF - FE = 0}$, then Trace (EFGH) is equal

Notes

(a) Trace (HFEG)

(b) Trace (E), Trace(F), Trace(G), Trace(H)

(c) Trace (GFEH)

(d) Trace (EGHF)

$$\left.\begin{aligned}\text{Trace}(1234) &= \text{Trace}(4123)\\ &= \text{Trace}(3412)\\ &= \text{Trace}(2341)\end{aligned}\right]\ \text{cycle}$$

$$\left.\begin{aligned}\text{Trace}(EFGH) &= \text{Trace}(HEFG)\\ &= \text{Trace}(GHEF)\\ &= \text{Trace}(FGHE)\end{aligned}\right]$$

cycles

Ref

Permutations

$$\text{Trace}(HEFG) = \text{Trace}(HFEG)$$

$$EF = FE$$

#Q. An unitary matrix $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ is given , where a, b, c, d, α and β are real . The inverse of the matrix is

$\Rightarrow \dfrac{1}{ade^{i\alpha} - bce^{i\beta}}[\;U\;]$ $U = \begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$

(a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$

(b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$

(c) $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$

(d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$ ✓

If U is Unitary matrix

$U(\bar{U})^T = I$

U^{-1} multiply

$\boxed{(\bar{U})^T = U^{-1}}$

$(\bar{U})^T = \begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix}^T = \begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$ ✓

Slide-8

#Q. The eigenvalue of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are

$2\times 2 \rightarrow \lambda^2 - (0+0)\lambda + (0 - i^2) = 0$

$\Rightarrow \lambda^2 - 0 + (+1) = 0$

$\lambda^2 + 1 = 0$

$\boxed{\lambda = \pm i}$

eigen values Are Complex and Distinct

(a) Real and distinct

(b) Complex and distinct

(c) Complex and coinciding

(d) Real and coinciding

#Q. The eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

Eigen values of the matrix

$\boxed{\lambda = 5, 1, -1}$

det = product of eigen value = -5

(a) 5, 2, – 2

(b) –5, –1, 1

(c) 5, 1, –1

(d) –5, 1, 1

#Q. Two matrices A and B are said to be similar if $\boxed{B = P^{-1} AP}$ for some invertible matrix P. Which of the following statements is NOT TRUE?

Diagonalization

If A and B matrix ARE SIMILAR

(a) Det A = Det B

(b) Trace of A = Trace of B

(c) A and B have the same eigenvectors

(d) A and B have the same eigenvalues

det A = det B

$r(A) = r(B)$

eigen values SAME

eigen vector Are Not SAME

Trace A = Trace B

#Q. A 3 × 3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalues of the matrix is:

(a) 18

(b) 12

(c) 9

(d) 6

$\lambda_1 + \lambda_2 + \lambda_3 = 11$

$\lambda_1 \lambda_2 \lambda_3 = 36$

$\lambda_1 = 6$

$\lambda_2 = 3$

$\lambda_3 = 2$

$6 \times 3 \times 2 = 36$

$6 + 3 + 2 = 11$

Trace = 11

det A = 36

Largest eigenvalue = 6 ✓ Ans

#Q. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

eigenvalues of the matrix $= 0, -\sqrt{2}, \sqrt{2}$

(a) 0, 1, 1

(b) 0, – √2, √2

(c) 1/ √2 , 1/ √2 , 0

(d) √2 , √2, 0

#Q. The degenerate eigenvalues of the matrix $M = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ = degenerate eigen value OR repeated eigen values

is ___5___

Eigen values = $2, \boxed{5, 5}$

Degenerate eigen value = 5

#Q. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is

(a) Orthogonal

(b) Symmetric

(c) Anti-symmetric

(d) Unitary

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}.$$

$$\boxed{A(\bar{A})^T = I}$$ Unitary matrix

D

#Q. Which of the following is INCORRECT for the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Weekly TEST 06

(a) It is its own inverse

(b) It is its own transpose

Already Discuss

(c) It is non-orthogonal

(d) It has eigen values ± 1

#Q. The symmetric pair of $P = \begin{pmatrix} a \\ b \end{pmatrix}(a - 2b)$ is :

(a) $\begin{pmatrix} a^2 - 2 & ba - 1 \\ ba - 1 & b^2 - 2 \end{pmatrix}$

(b) $\begin{pmatrix} a(a-1) & b \\ b & b^2 \end{pmatrix}$

(c) $\begin{pmatrix} a(a-1) & b(a-1) \\ b(a-1) & b^2 \end{pmatrix}$

(d) $\begin{pmatrix} a(a-2) & b(a-1) \\ b(a-1) & b^2 \end{pmatrix}$

SEE sol^n →

$\left\{ \begin{array}{l} \underline{\text{Weekly TEST 06}} \\ \text{already discuss} \end{array} \right.$

$P = \frac{P+P^T}{2} + \frac{P-P^T}{2}$

Slide-17

#Q. $(x \quad y)\begin{pmatrix}5 & -7\\ 7 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = 15$

SEE The sol^n ✓

[Weekly TEST 06: already Discuss]

The matrix equation above represents

(a) A circle of radius $\sqrt{15}$

(b) An ellipse of semi major axis $\sqrt{5}$ ✓

(c) A ellipse of semi major axis 5

(d) A hyperbola

#Q. The product PQ of any two real symmetric matrices P and Q is:

If product PQ of Any Two real sym matrix P and Q is

(a) Symmetric for all P and Q

(b) Never symmetric

(c) Symmetric if PQ = QP

(d) Antisymmetric for all P and Q

Symmetric If $\boxed{PQ = QP}$

multiplication is commutative

#Q. A matrix given by $M = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$ The eigenvalues of the M are

(a) Real and positive ✗

(b) Purely imaginary with modulus 1 ✗

(c) ✓ Complex with modulus 1

(d) Real and negative ✗

$$M = \frac{1}{\sqrt{2}}\begin{bmatrix} i & 1 \\ 1 & i \end{bmatrix}$$

eigen values

$$\Rightarrow \frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}$$

$$\boxed{\begin{gathered}\text{\# Modulus: } z = a + ib \\ |z| = \sqrt{a^2+b^2}\end{gathered}}$$

$$\lambda_1 = \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}} = \lambda_2$$

complex

$$|\lambda_1| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$|\lambda_2| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

#Q. The inverse of the matrix $M = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is

Inverse of matrix

$\Rightarrow \underline{M^2 - I}$

Characterstic eqn

(a) $M - I$

(b) $M^2 - I$

step ② $\lambda = M$

step ③ Multiply with M^{-1}

(c) $I - M^2$

(d) $I - M$

$$\boxed{M^{-1} = M^2 - I}$$

#Q The normalized eigenvectors of the matrix $\boxed{N = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}}$ are β_1 and β_2 with the eigenvalues λ_1 and λ_2 respectively and $\lambda_1 > \lambda_2$. If the eigenvector $\alpha = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is expressed as $\alpha = P\beta_1 + Q\beta_2$. Find the constant P and Q

(a) $\frac{1+i}{2}, \frac{1-i}{2}$ ✓

(b) $\frac{2+i}{2}, \frac{1+i}{2}$

(c) $\frac{1+i}{3}, \frac{1-i}{4}$

(d) $\frac{i}{2}, \frac{1+i}{2}$

$N = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ β_1, β_2

λ_1, λ_2

$\alpha = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\alpha = P\beta_1 + Q\beta_2$

$$N = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\lambda = 1, -1$$

$$\beta_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$\beta_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix}$$

Eigen values

$$\lambda^2 - (0)\lambda + (0 + i^2) = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$

linear combination

$$\boxed{p = \alpha x + \beta y}$$

$$\boxed{\alpha = \beta_1 P + Q \beta_2}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{P}{\sqrt{2}}\begin{bmatrix} -i \\ 1 \end{bmatrix} + Q \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\boxed{P = \frac{1+i}{2} \quad Q = \frac{1-i}{2}}$$

#Q. The trace of a 2 × 2 matrix is 4 and its determinant is 8. If one of the eigenvalues is 2(1 + i), the other eigenvalue is

(a) 2(1 - i)

(b) 2(1 + i)

(c) (1 + 2i)

(d) (1 - 2i)

$\left.\begin{array}{l}\lambda_1+\lambda_2=4\\ \lambda_1\lambda_2=8\end{array}\right]$ eigenvalue $\lambda_1 = 2(1+i)$

$\boxed{\text{other Eigen value} = 2(1-i)}$ ✓

#Q. The eigenvalues of the matrix representing the following pair of linear equation $x + iy = 0$, $ix + y = 0$ are

$x + iy = 0$ $\quad A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$

$ix + y = 0$

(a) $1 + i, 1 + i$

(b) $1 - i, 1 - i$

(c) $1, i$

(d) $1 + i, 1 - i$

$\lambda^2 - (1+1)\lambda + (1 - i^2] = 0$ $\quad i^2 = -1$

$\lambda^2 - 2\lambda + 2 = 0$

$\boxed{\lambda = 1+i, 1-i}$ ✓

#Q. For the given set of equations:

$$x + y = 1$$

$$y + z = 1$$

$$x + z = 1$$

$\underline{AX = B}$

$\left\{ \begin{array}{l} x = \frac{1}{2} \\ y = \frac{1}{2} \\ z = \frac{1}{2} \end{array} \right]$ Non Trivial sol^n

$C = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right]$

rank A = 3
rank C = 3
No. of variables = 3.

This is condition consistent

$r(A) = r(C) = n$ = Unique sol^n

or

$\underline{\text{Non Trivial sol}^n}$

Which one of the following statements is correct ?

(a) Equation are inconsistent

(b) Equations are consistent and a single non-trivial solution exists

(c) Equation are consistent and many solutions exist

(d) Equation are consistent and only a trivial solution exists

Trivial = (0, 0, 0)
Non trivial = (a, b, c)

Thank You !