

ENGINEERING MATHEMATICS

EE & ECE




Linear Algebra

DPP – 01 Discussion

Part – I



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 ✓ DPP01 - Part one ✓ 1-20 question
 Part Two - 20 - ()
 [✓ Cholsky decomp -
 ✓ LV decomp.]

✓ Weekend Sunday TEST - Calculus -

7 - 2 marks

6 - 1 marks

#Q.

Consider the matrix $M =$

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$\sum M = 3$
 $\det M = 0$

The eigenvalues of M are

(a) 0, 1, 2 ✓

(b) 0, 0, 3 ✓

(c) 1, 1, 1

(d) -1, 1, 3

$$\left. \begin{array}{l} \sum M = 3 \\ \det M = 0 \end{array} \right\} \chi(M) = 1$$

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3 \times 3}$$

$$\det M = 0 \quad \chi(M) = 1$$

Eigen value
 sum of diagonal
 $= 1 + 1 + 1$
 $= 3$

"If $\det M = 0$ at least one eigen value is zero"

$$\boxed{\text{eigen values} = 0, 0, 3} \quad \checkmark$$

#Q. A 3×3 matrix M has $\text{Tr}[M] = 6$, $\text{Tr}[M^2] = 26$, $\text{Tr}[M^3] = 90$. Which of the following can be possible set of eigenvalues of M ?

(a) $\{1, 1, 4\}$

(b) $\{-1, 0, 7\}$

(c) $\{-1, 3, 4\}$

(d) $\{2, 2, 2\}$

Trace = sum of diagonal elements
= sum of eigen values.

$$\text{Tr}(M) = -1 + 3 + 4 = 6$$

$$\begin{aligned} \text{Tr}(M) &= 6 & \text{Tr}(M^3) &= 90 \\ \text{Tr}(M^2) &= 26 \end{aligned}$$

$$\text{Tr}(M^2) = (-1)^2 + (3)^2 + (4)^2 = 26$$

$$\text{Tr}(M^3) = (-1)^3 + (3)^3 + (4)^3 = 90$$

#Q. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

$\det A = 0$
 $r(A) = 1$
 Eigen values
 = sum of diagonal

$$= 1 + 4 + 9 = 14$$

✓ eigen values = 0, 0, 14

(a) (1, 4, 9)

(b) (0, 7, 7)

(c) (0, 1, 13)

(d) (0, 0, 14) ✓

Key Pt - $\det A = 0$ & $r(A) = 1$
 eigen = (0, 0, sum of eigen values) ✓

#Q. The eigenvalue of the anti-symmetric
are components of a unit vector, are

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

~~X~~ odd order
 $\det = 0$ where n_1, n_2, n_3

$$A = \begin{pmatrix} 0 & n_1 & n_2 \\ -n_1 & 0 & n_3 \\ -n_2 & -n_3 & 0 \end{pmatrix}$$

Replace it
Wrong-matrix

anti-symmetric
eigenvalue

- (a) ☒ 0, -i, i
- (b) 0, 1, -1
- (c) 0, 1 + i, -1 - i
- (d) 0, 0, 0

[A] = skew sym matrix
Characteristic eqn

$$\lambda^3 - (\text{Trace})\lambda^2 + (A_{11} + A_{22} + A_{33})\lambda - \det A = 0$$

$$\lambda^3 - (0 + 0 + 0)\lambda^2 + (n_1^2 + n_2^2 + n_3^2)\lambda - 0 = 0$$

$$= \lambda^3 + (n_1^2 + n_2^2 + n_3^2)\lambda = 0$$

$$= \lambda^3 + (\eta_1^2 + \eta_2^2 + \eta_3^2) \lambda = 0$$

square it + Add

$$\eta_1^2 + \eta_2^2 + \eta_3^2 = \frac{\eta_1^2 + \eta_2^2 + \eta_3^2}{\eta_1^2 + \eta_2^2 + \eta_3^2} = 1$$

$$\rightarrow \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

put the value $\eta_1^2 + \eta_2^2 + \eta_3^2 = 1$

$$\Rightarrow \lambda^3 + 1\lambda = 0$$

$$\Rightarrow \lambda(\lambda^2 + 1) = 0$$

$$\Rightarrow \boxed{\lambda = 0 \quad \lambda = \pm i}$$

Eigen values:
 $= 0, +i, -i$

If η_1, η_2, η_3 are components of unit vector

Unit vector square:

$$\hat{\eta}_1 = \frac{\vec{\eta}_1}{|\vec{\eta}_1|}$$

$$+ \hat{\eta}_2 = \frac{\vec{\eta}_2}{|\vec{\eta}_2|}$$

$$+ \hat{\eta}_3 = \frac{\vec{\eta}_3}{|\vec{\eta}_3|}$$

#Q. Consider a $n \times n$ ($n > 1$) matrix A , in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$). The matrix A

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}_{3 \times 3}$$

degenerate eigen value = Dependent eigen value
 $a_{ij} = ij$

(a) ✓ Has one degenerate eigenvalue with degeneracy $(n-1)$

$\det A = 0$
 $\rho(A) = 1$

(b) ✗ Has two degenerate eigenvalues with degeneracies 2 and $(n-2)$

(c) ✗ Has one degenerate eigenvalues with degeneracy n

(d) ✗ Does not have any degenerate eigenvalues

✓ degenerate eigenvalues
 = Dependent eigen value
 = $\boxed{0, 0} \text{ (sum)}$

n eigen value
 $\det A = 0$
 $\rho(A) = 1$] $\rightarrow (n-1)$
degeneracy

#Q. A 2×2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = I$

- (a) 20
- (b) 30
- (c) 60
- (d) 120

$A = 2 \times 2$ eigenvalues

$$\lambda_1 = e^{\frac{i\pi}{5}}, \lambda_2 = e^{\frac{i\pi}{6}}$$

$$A \rightarrow e^{\frac{i\pi}{5}}, e^{\frac{i\pi}{6}}$$

$$A^n \rightarrow e^{\frac{i\pi n}{5}}, e^{\frac{i\pi n}{6}}$$

Product of eigenvalues = det A

$$|A^n| = |I|$$

$$e^{\frac{i\pi n}{5}} \cdot e^{\frac{i\pi n}{6}} = 1$$

$$\Rightarrow e^{\frac{11n\pi i}{30}} = 1$$

$$e^{\frac{11\pi i n}{30}} = e^{2m\pi i}$$

$$\frac{11n}{30} = 2m$$

$\frac{n}{m} = \frac{60}{11}$

$$\left[\begin{array}{l} n = 60 \\ m = 11 \end{array} \right]$$

$A \rightarrow \lambda_1, \lambda_2$

$$A^m = \lambda_1^m, \lambda_2^m$$

$$1 = e^{2m\pi i}$$

$$\cos \theta + i \sin \theta = e^{i\theta}$$

$$\cos 2\pi + i \sin 2\pi = e^{i \cdot 2\pi}$$

#Q. Given a 2×2 unitary matrix satisfying $U'U = UU' = I$ with $\det U = e^{i\phi}$, one can construct a unitary matrix V ($V'V = VV' = I$) with $\det V = 1$ from it by

$$U = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad ad - bc = e^{i\phi} \quad (\bar{V})^T U = I$$

Unitary matrix
 $\det V = e^{i\phi}$

- (a) Multiplying U by $e^{-i\phi/2}$
- (b) Multiplying a single element of U by $e^{-i\phi}$
- (c) Multiplying any row or column by $e^{-i\phi/2}$
- (d) Multiplying U by $e^{-i\phi}$

\checkmark ① $V = e^{-i\phi/2} U$ $\det V = 1$ ①

$V = e^{-i\phi} U$ $\det V = 1$ ②

$$= \begin{bmatrix} a e^{-\frac{i\phi}{2}} & b e^{-\frac{i\phi}{2}} \\ c e^{-\frac{i\phi}{2}} & d e^{-\frac{i\phi}{2}} \end{bmatrix}$$



$$V = U e^{-\frac{i\theta}{2}} = \begin{bmatrix} a e^{-\frac{i\theta}{2}} & b e^{-\frac{i\theta}{2}} \\ c e^{-\frac{i\theta}{2}} & d e^{-\frac{i\theta}{2}} \end{bmatrix}$$

$$V = e^{-\frac{i\theta}{2}} e^{-\frac{i\theta}{2}} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det V = e^{-i\theta} \underbrace{[ad - bc]}_{\det U}$$

$$= e^{-i\theta} e^{i\theta} \det U$$

$$\boxed{\det V = 1}$$

$$\boxed{V = e^{-\frac{i\theta}{2}} U}$$

$$\rightarrow \det V = 1$$

(A)

#Q. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are

- (a) -5, -2, 7
- (b) ✓ -7, 0, 7
- (c) -4i, 2i, 2i
- (d) 2, 3, 6

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}_{3 \times 3}$$

[det M] = skew Hermitian matrix

odd order

$$\boxed{\det M = 0} \checkmark$$

$$\boxed{\text{eigen values} = -7, 0, 7}$$

#Q. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigen vector of

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ if}$$

- (a) $b = 0$ or $a = 0$
- (b) $b = a$ or $b = -2a$
- (c) $b = 2a$ or $b = -a$
- (d) $b = a/2$ or $b = -a/2$

$AX = \lambda X$. A - matrix given
 $\lambda =$ Not given
 $X =$ vector given

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \lambda_1 \begin{bmatrix} a \\ b \\ a \end{bmatrix}$$

$$= \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \lambda_1 \begin{bmatrix} a \\ b \\ a \end{bmatrix} \quad \boxed{\lambda_1 = 1}$$

$$AX = \lambda X$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \lambda_2 \begin{bmatrix} a \\ b \\ a \end{bmatrix}$$

$$= \begin{bmatrix} a+b \\ 2a \\ a+b \end{bmatrix} = \lambda_2 \begin{bmatrix} a \\ b \\ a \end{bmatrix}$$

$$a\lambda_2 - a\lambda_2^2 - 2a = 0$$

$$-a\lambda_2 + a\lambda_2^2 + 2a = 0$$

$$\lambda_2^2 - \lambda_2 + 2 = 0$$

$$\lambda_2 = 2, -1$$

$$a+b = \lambda_2 a \quad \text{--- (1)}$$

$$2a = \lambda_2 b \quad \text{--- (2)}$$

$$a+b = \lambda_2 a$$

$$2a - \lambda_2 b = 0$$

$$a(1-\lambda_2) + b = 0 \times \lambda_2$$

$$2a - \lambda_2 b = 0 \times 1$$

$$a\lambda_2(1-\lambda_2) + b\lambda_2 = 0$$

$$2a - \lambda_2 b = 0$$

$$a\lambda_2(1-\lambda_2) - 2a = 0$$

$$\text{Put } \lambda_2 = -1$$

$$a+b = 2a$$

$$b = a$$

$$2a = -b$$

#Q. Which one of the following is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

(b) ✓ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

$$\begin{aligned}
 A &= \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2} \\
 A^{-1} &= \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\
 &= \frac{1}{1-0} \begin{bmatrix} 1 & +1 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ (c)}
 \end{aligned}$$

#Q. A 3×3 matrix has eigenvalues 0, $2 + i$ and $2 - i$. Which one the following statements is correct?

$$\lambda_1 = 0$$

$$\lambda_2 = 2 + i$$

$$\lambda_3 = 2 - i$$

- (a) The matrix is hermitian
- (b) The matrix is unitary
- (c) The inverse of the matrix exists
- (d) ✓ The determinant of the matrix is zero

det of matrix A
 = product of eigen values
 = $0 \times (2 + i) \times (2 - i)$
 = 0 ✓

$$\text{Traceless} = \underline{0} \text{ Trace}$$

#Q. A real traceless 4×4 unitary matrix has two eigen values -1 and 1 .

The other eigenvalues are

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = \underbrace{-1 + 1}_{0} + \underbrace{\lambda_3 + \lambda_4}_{-1 + 1} = 0$$

(a) zero and $+2$

(b) ☒ -1 and $+1$

(c) zero and $+1$

(d) $+1$ and $+1$

$$\left. \begin{array}{l} \lambda_3 = -1 \\ \lambda_4 = +1 \end{array} \right\}$$

$$= 0 \\ = \text{Traceless}$$

#Q The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

- (a) +1 and +1
- (b) Zero and +1
- (c) ✓ Zero and +2
- (d) -1 and +1

$$A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} \quad i^2 = -1$$

$$= \lambda^2 - (1+1)\lambda + (1+i^2) = 0$$

$$= \lambda^2 - 2\lambda + 0 = 0$$

$$= \lambda^2 - 2\lambda = 0$$

$$= \lambda(\lambda - 2) = 0$$

$$= \boxed{\lambda = 0 \quad \lambda = 2}$$

(c)

$\rightarrow 3 \times 3 = 3 \times 3 \checkmark$
 #Q If A is 2×2 matrix with determinant 2, then the determinant of $\text{adj}[\text{adj}[\text{adj}(A^{-1})]]$ is equal to

- (a) $1/512$
- (b) $1/1024$
- (c) $1/128$
- (d) $1/256$

$$| \text{Adj } A | = |A|^{n-1} (\text{adj } A^{-1}) = \left[\frac{1}{|A|} \right]^{n-1}$$

$$| \text{Adj}(A^{-1}) | = \left(\frac{1}{2} \right)^{3-1}$$

$$| \text{adj}(\text{adj}(A^{-1})) | = \left(\frac{1}{2^2} \right)^2$$

$$| \text{adj}(\text{adj}(\text{adj}(A^{-1}))) | = \left(\frac{1}{4^2} \right)^2 = \frac{1}{256}$$

#Q The eigenvalues of $(A^4 + 3A - 2I)$, where A is $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ are eigenvalues = 1, 2, 3

(a) 2, 20, 88

(b) 1, 2, 3

(c) 2, 20, 3

(d) 1, 20, 88

$$\underline{A^4 + 3A - 2I} \rightarrow \lambda = 1 = (1)^4 + 3 \times 1 - 2 = 2$$

$$\rightarrow \lambda = 2 = (2)^4 + 2 \times 3 - 2$$

$$\rightarrow \lambda = 3 = (3)^4 + 2 \times 3 - 3 = \underline{88}$$

#Q. Eigen value of matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are

eigen value =
 sum of eigen value = Trace
 = 0

(a) -2, -1, 1, 2 ✓

(b) -1, 1, 0, 2

(c) 1, 0, 2, 3

(d) -1, 1, 0, 3

$$\lambda_1 = -2$$

$$\lambda_2 = -1$$

$$\lambda_3 = 2$$

$$\lambda_4 = 1$$

#Q. A linear transformation T , defined as $T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, Transform vector \vec{x} from a three dimensional space to a two-dimensional real space. The transformation matrix T is

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}^T$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^T$

(c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}^T$

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}^T$

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \end{bmatrix}$$



Thank You !