Probability & Statistics

Discrete Probability Distribution & Continuous Probability Distribution

DPP

1. Let X and Y be independent exponentially distributed random variables with means $\frac{1}{4}$ and $\frac{1}{6}$ respectively. Let $Z = \min\{x, y\}$. Then E(Z) equal

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$

(c) $\frac{1}{7}$ (d) $\frac{1}{10}$

2. X has an exponential distribution with a mean of 1. Y is defined to be the conditional distribution of $X - 2$ given that $X > 2$, so for instance, for $c > 0$, we have $P[Y > c] = P[X - 2 > c/X > 2]$, What is the distribution of Y ?

(a) Exponential with mean 1

(b) Exponential with mean 2

(c) Exponential with mean $\frac{1}{2}$

(d) Exponential with mean e

3. Let X be a poison random variable and $P(X = 1) + 2P(X = 0) = 12\,P(X = 2)$. Which one of the following statements is TRUE ?

(a) $0.40 < P(X = 0) \le 0.45$

(b) $0.45 < P(X = 0) \le 0.50$

(c) $0.50 < P(X = 0) \le 0.55$

(d) $0.55 < P(X = 0) \le 0.60$

4. A discrete random variable X has the following probability function

x	:	10	20	30	40	50
y	:	0.1	0.1	0.4	0.3	0.1

Denote by μ_x and σ_x the mean and the standard deviation of X. Find $P(|X - \mu_X| \le \sigma_X)$.

(a) 1 (b) 0.8

(c) 0.7 (d) 0.5

5. X has a uniform distribution on the interval (0, 2), and $Y = \max\{X, 1\}$. Find $Var(Y)$.

(a) 0.05

(b) 0.10

(c) 0.15

(d) 0.20

6. X has a Poisson distribution with a mean of 2.

Y has a geometric distribution on the integers 0, 1, 2….., also with mean 2.

X and Y are independent.

Find $P(X = Y)$

(a) $\frac{e^{-2/3}}{3}$ (b) $\frac{e^{-1/3}}{3}$

(c) $\frac{1}{3}$ (d) $\frac{e^{1/3}}{3}$

7. X and Y are independent continuous random variables, with X uniformly distributed on the interval $[0, \theta]$ and Y uniformly distributed on the interval $[0, 2\theta]$. Find $P(Y < 3X)$.

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

8. If X and a normal distribution with mean 1 and variance 4, then $P[X^2 - 4X \le 0] = ?$

(a) Less than 0.15

(b) At least 0.15 but less than 0.35

(c) At least 0.35 but less than 0.55

(d) At least 0.55 but less than 0.75

9. X has an exponential distribution with a mean of $\theta > 0$, where

$$Y = \begin{cases} \dfrac{X}{2} \text{ if } X \le \theta \\ X \text{ if } X > \theta \end{cases}$$

Find $E[Y]$.

(a) $\frac{\theta}{2}(1+e^{-1})$ (b) $\frac{\theta}{2}(1+2e^{-1})$

(c) $\frac{\theta}{2}(1-e^{-1})$ (d) $\frac{\theta}{2}(1-2e^{-1})$

10. If X has exponential distribution with mean 2, then is $P(X < 1 \mid X < 2)$.

(a) $(1 - e^{-2})/(1 - e^{-4})$

(b) $(1 - e^{-0.5})/(1 - e^{-1})$

(c) $(1 - e^{-1})/(1 - e^{-2})$

(d) $(1 - e^{-0.5})/(1 - e^{-4})$

11. In a Poisson distribution, the probability of observing 3 is 2/3 times that of observing 4. The mean of the distribution is

(a) 5 (b) 6

(c) 7 (d) 8

12. The number of misprints per page of a book (X) follows the Poisson distribution such that $(X = 1) = P(X = 2)$. If the book contains 500 pages, the expected number of pages containing at most one misprint is

(a) $5005e^{-2}$ (b) $10005e^{-2}$

(c) $15005e^{-2}$ (d) $500(1 - 3e^{-2})$

13. Security cameras in a railway reservation office indicate that the number of customers arriving before the opening of counters on any given day has the Poisson distribution with mean 3. The minimum number of counters it should open so that there is at least 90% chance of all the waiting customers being immediately served is

(a) 3 (b) 4

(c) 5 (d) 6

14. A thousand candidates appear for an examination. Their scores are independent and have a continuous uniform distribution over the range 0 to 100. The variance of the number of candidates having scores above 30 is

(a) 210 (b) 700

(c) 25/3 (d) 300

15. If X has a normal distribution with mean 1 and variance 4, then $P[X^2 - 2X \le 8] = ?$

(a) 0.13 (b) 0.43

(c) 0.75 (d) 0.86

16. Three individuals are running a one kilometers race. The completion time for each individual is a random variable. X_i is the completion time, in minutes, for person i.

X_1 : uniform distribution on the interval [2.0, 3.1]

X_2 : uniform distribution on the interval [2.7, 3.1]

X_3 : uniform distribution on the interval [2.9, 3.3]

The three completion times are independent of one another.

Find the expected latest completion time (nearest .1).

(a) 2.9 (b) 3.0

(c) 3.1 (d) 3.2

17. Customers arrive randomly and independently at a service window, and the time between arrivals has an exponential distribution with a mean of 12 minutes. Let X equal the number of arrivals per hour. What is $P[X = 10]$?

(a) $\frac{10e^{-12}}{10!}$ (b) $\frac{10^{-12}e^{-10}}{10!}$

(c) $\frac{12^{-10}e^{-10}}{10!}$ (d) $\frac{5^{10}e^{-5}}{10!}$

18. Which one of the following is not possible for a binomial distribution?

(a) Mean = 2, variance = $\frac{3}{2}$

(b) Mean = 5, variance = 9

(c) Mean = 10, variance = 5

(d) Mean = 4, variance = $\frac{8}{3}$

19. A coin is twice as likely to turn up tails as heads. If the coin is tossed independently, what is the probability that the third head occurs on the fifth toss?

(a) $\frac{8}{81}$ (b) $\frac{40}{243}$

(c) $\frac{16}{81}$ (d) $\frac{80}{243}$

20. Let X be a random variable with a continuous uniform distribution on the interval $(1, \alpha)$, Where $\alpha > 1$. If $E[X] = 6$. $Var[X]$, then $\alpha =$

(a) 2 (b) 3

(c) 4 (d) 7

21. Suppose that X has uniform distribution on the interval [0, 100]. Let Y denote the greatest integer smaller than or equal to X. Which of the following is true?

(a) $P(Y \le 25) = \frac{1}{4}$ (b) $P(Y \le 25) = \frac{26}{100}$

(c) $E(Y) = 50$ (d) $E(Y) = \frac{101}{2}$

22. Let X be a Geom(0.4) random variable. Then $P(X = 5 \mid X \ge 2)$ equals_____.

23. Let X be a $Bin(2, p)$ random variable, and Y be a $Bin(2, p)$, $0 < p < 1$.

If $P(X \ge 1) = \frac{5}{9}$, then $P(Y \ge 1) =$ _______.

24. If X is a $U(0, 1)$ random variable, then $P\left(\min(X, 1-X) \le \frac{1}{4}\right)$ _______.

25. Let the random variable X have uniform distribution on the interval $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$.

Then $P(\cos X > \sin X)$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

26. Let X be a random variable with the probability density function

$$f(x \mid r, \lambda) = \frac{\lambda^r}{(r-1)!} x^{r-1} e^{-\lambda x}, \; x > 0, \lambda > 0, r > 0.$$

If $E(X) = 2$ and $Var(X) = 2$, then $P(X < 1)$ equals ______.

27. Let X_1 and X_2 be independent geometric random variables with the same probability mass function given by $P(X = k) = P(1 - p)^{k-1}$, $k = 1,2, \ldots$.. Then the value of $P[X_1 = 2 / X_1 + X_2 = 4]$ correct up to three decimal places is ______.

28. Let X be a Poisson random variable with mean $\frac{1}{2}$. Then, $E[(X + 1)!]$ equals

(a) $2e^{-\frac{1}{2}}$ (b) $4e^{-\frac{1}{2}}$

(c) $4e^{-1}$ (d) $2e^{-1}$

29. Let $\{X_j\}$ be a sequence of independent Bernoulli random variables with

$P(X_j = 1) = \frac{1}{4}$ and let $Y_n = \frac{1}{n}\sum_{j=1}^{n} X_j^2$. Then Y_n converges, in probability to _______ .

30. Let X_1, X_2, X_3, X_4 be independent exponential random variables with mean $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Then, $Y = \min(X_1, X_2, X_3. X_4)$ has exponential distribution with mean equal to ______.

31. X, Y are independent exponential random variables with means 4 and 5, respectively. Which of the following statements is true?

(a) $X + Y$ is exponential with mean 9

(b) XY is exponential with mean 20

(c) Max (X, Y) is exponential

(d) Min (X, Y) is exponential

32. If the moment generating function of a random variable X is given by

$$M(t) = \frac{5}{15}e^t + \frac{4}{15}e^{2t} + \frac{3}{15}e^{3t} + \frac{2}{15}e^{4t} + \frac{1}{15}e^{5t}$$

Then what is the probability mass function of X?

(a) $f(x) = \frac{6-x}{15}; x = 1,2,3,4,5$

(b) $f(x)=\frac{7-2x}{15}$; $x=1,2,3,4,5$

(c) $f(x)=\frac{4x+1}{15}$; $x=1,2,3,4,5$

(d) $f(x)=\frac{4+x}{15}$; $x=1,2,3,4,5$

33. The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively.

Then $P(X \geq 1)$ is equal to

(a) $\left(\frac{1}{3}\right)^6$ (b) $\left(\frac{2}{3}\right)^6$

(c) $1-\left(\frac{1}{3}\right)^6$ (d) $1-\left(\frac{2}{3}\right)^6$

34. If the independent random variable X and Y are binomially distributed, respectively with $n = 3$, $p=\frac{1}{3}$ and $n=5, p=\frac{1}{3}$. What is $P(X+Y \geq 1)$?

(a) $\left(\frac{1}{3}\right)^8$ (b) $\left(\frac{2}{3}\right)^8$

(c) $1-\left(\frac{1}{3}\right)^8$ (d) $1-\left(\frac{2}{3}\right)^8$

35. X is random variable with Poisson distribution, $P(X = 1) = 2P(X = 0)$, then $P(X = 2)$ is

(a) Equal to $P(X = 1)$ (b) Twice $P(X = 1)$

(c) Equal to $P(X = 0)$ (d) 4 times $P(X = 0)$

36. An unbiased coin is tossed until a head is obtained. If N denotes the number of tosses required, what is $P(N > 1)$?

(a) 1/2 (b) 1

(c) 1/4 (d) 1/8

37. Let X be a random variable such that $P(X=i)=\frac{1}{2n+1}$ for $i = -n, -n + 1, \ldots, -1, 0, 1, \ldots, n$ what is $V(X)$?

(a) $\frac{n(n+1)}{3}$ (b) $\frac{n^2(n+1)}{3}$

(c) $\frac{2^n(n+1)}{3}$ (d) $\frac{n^2(n+1)^2}{3}$

38. X and Y are random variables satisfying $\log Y = X \sim N(0, 1)$, then $E(Y)$ is

(a) e^2

(b) e

(c) $e^{1/2}$

(d) None

39. Let X be non-negative integer valued random variable with mean equal to 1 and variance equal to $\frac{5}{6}$. Then which of the following can be distribution of X?

(a) Binomial with $n = 6$ and $p=\frac{1}{6}$

(b) Poisson with $\lambda = 1$

(c) Geometric with $p=\frac{1}{6}$

(d) Discrete uniform

40. Let X be a random variable taking values in a set E. Let $P(X > a + b \mid X > a) = P(X > b)$ for all $a, b \in E$. Then which of the following is a possible distribution of X?

(a) Poisson

(b) Geometric

(c) Long-normal

(d) Exponential

41. The moment generating function of a random variable is $(0.7+0.3e^t)^{10}$. The mean and variance of the random variable are respectively.

(a) (7, 2.1) (b) (3, 2.1)

(c) (2.1, 7) (d) (2.1, 3)

42. If X follows binomial distribution with parameters n and p, then variance of X/n is

(a) $\frac{p(1-p)}{n}$

(b) $np(1-p)$

(c) $p(1-p)$

(d) $\dfrac{p(1-p)}{n^2}$

43. Let a random variable X follow the chi-square with parameter 6. The $E[X]^2$ is

$p(x) = q^{x-1}p$; $x = 1, 2, 3, \ldots,$

(a) 48

(b) 12

(c) 36

(d) 24

44. Suppose that X_1 and X_2 follow binomial distribution with parameters n_1, p_1 and n_2, p_2 respectively. It is given that $E(X_1) = E(X_2)$. Then

(a) $\dfrac{n_1}{n_2} = \dfrac{p_1}{p_2}$

(b) $V(X_1) = V(X_2)$

(c) $\dfrac{n_1}{n_1} = \dfrac{p_2}{p_2}$

(d) $\dfrac{n_1}{n_2} = \dfrac{p_2}{p_1}$

45. Let X be uniformly distributed over the interval $[a, b]$, where $0 < a < b$.

If $E(X) = 2$, $V(X) = \dfrac{4}{3}$ then $P[X < 1]$ is

(a) $\dfrac{3}{4}$

(b) 1

(c) $\dfrac{1}{2}$

(d) $\dfrac{1}{4}$

Answer Key

1. (d)
2. (a)
3. (c)
4. (c)
5. (b)
6. (a)
7. (d)
8. (d)
9. (b)
10. (b)
11. (b)
12. (c)
13. (c)
14. (a)
15. (d)
16. (c)
17. (d)
18. (b)
19. (a)
20. (b)
21. (a)
22. (0.084 to 0.086)
23. (0.80 to 0.81)
24. (0.20721)
25. (d)
26. (0.25 to 0.27)
27. (0.332 to 0.334)
28. (b)
29. (0.25)
30. (0.1)
31. (d)
32. (a)
33. (c)
34. (d)
35. (a)
36. (c)
37. (a)
38. (c)
39. (a)
40. (d)
41. (b)
42. (a)
43. (a)
44. (d)
45. (d)