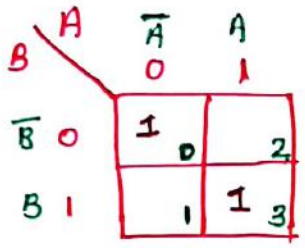
KARNAUGH MAP basics & Key-points. 71

- Developed by Karnaugh in 1953.
- It is used to minimize boolean equations.
- It is build based on gray code.
- Two Variable K-map.

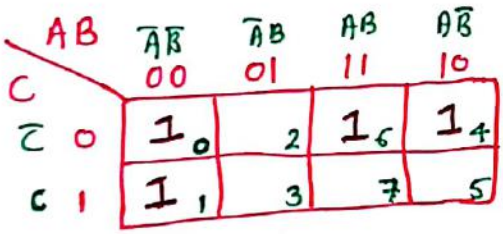
Total Cells = $2^n = 2^2 = 4$



$Y(A,B) = \Sigma m(0,3)$

- Three Variable K-map

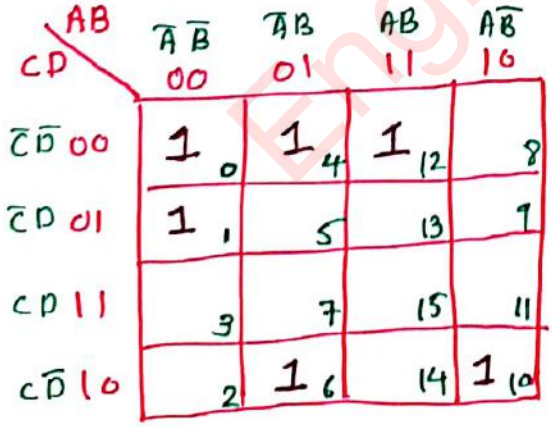
Total Cells = $2^n = 2^3 = 8$



$Y(A,B,C) = \Sigma m(0,1,4,6)$

- Four Variable K-map

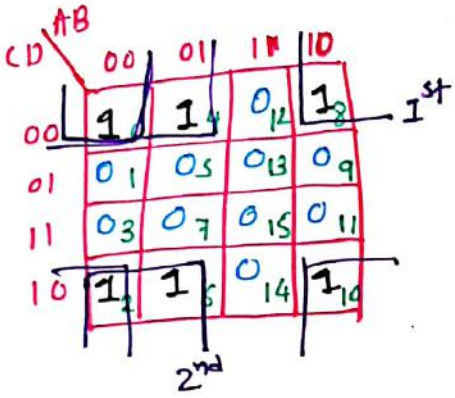
Total Cells = $2^n = 2^4 = 16$



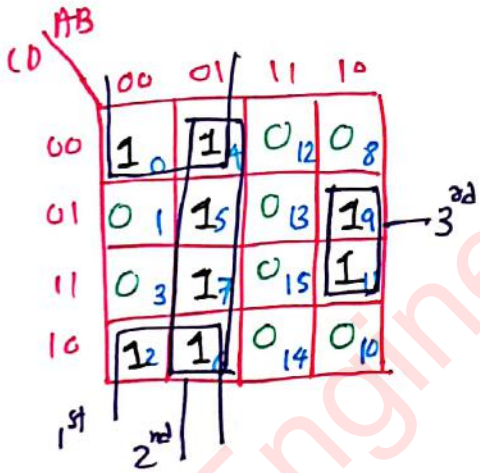
$Y(A,B,C,D) = \Sigma m(0,1,4,6,10,12)$

K-map rules for grouping. 72

1) Group should not contain zero and cells contain '1' must be grouped.
2) We can group 1, 2, 4, 8, ..., 2^n cells.
3) Each group should be as large as possible.
4) Group may overlap.
5) Opposite grouping and corner grouping is allowed.
6) There should be as few groups as possible.



$$Y = \bar{B}\bar{D} + \bar{A}\bar{D}$$



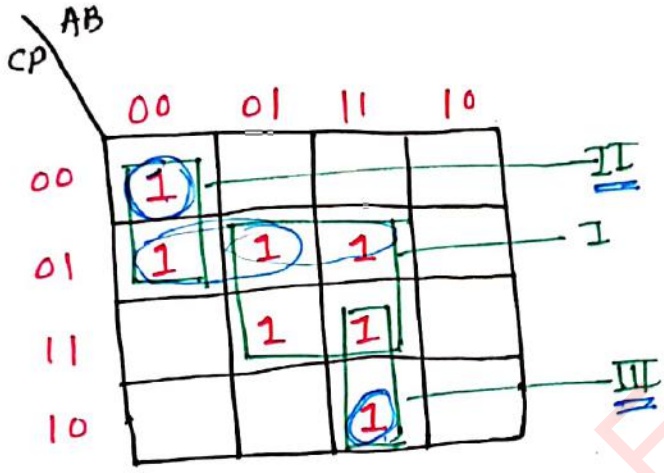
$$Y = \bar{A}\bar{D} + \bar{A}B + A\bar{B}D$$

Implicants, Prime Implicants and Essential Prime Implicants in K-map.

Implicants - The group's of 1's is Implicants.

Prime Implicants - It largest possible group of 1's.

Essential prime Implicants - At least, There is single 1 which can not be combined in other way.



- group I, II, III are prime Implicants.
- group II & III are Ess. prime Implicants.

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K-map Examples 7h

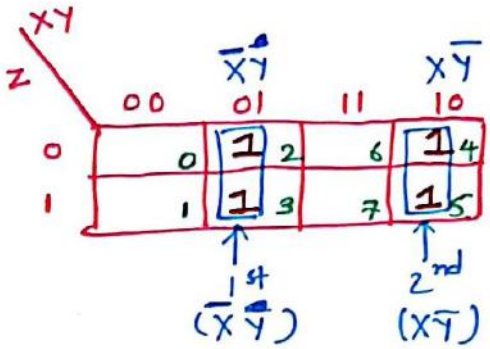
1] In the sum of products function is $f(x,y,z) = \Sigma m(2,3,4,5)$. The prime implicants are

✓ a] xy, $x\bar{y}$

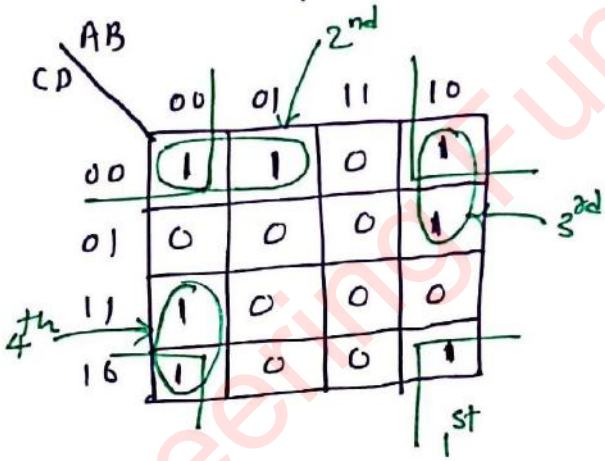
b] $\bar{x}y$, $x\bar{y}\bar{z}$, $x\bar{y}z$

c] $\bar{x}y\bar{z}$, $\bar{x}yz$, $x\bar{y}$

d] $\bar{x}y\bar{z}$, xyz, $x\bar{y}z$



2] The K-map for a boolean function is shown in figure. The number of essential prime implicants for this function is



✓ a] 4

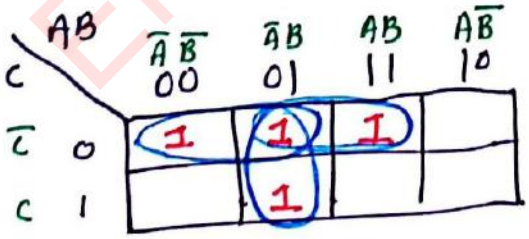
b] 5

c] 6

d] 8

3] Solve given boolean expression using K-map

$y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}$ $= \bar{A}\bar{C} + \bar{A}B + B\bar{C}$



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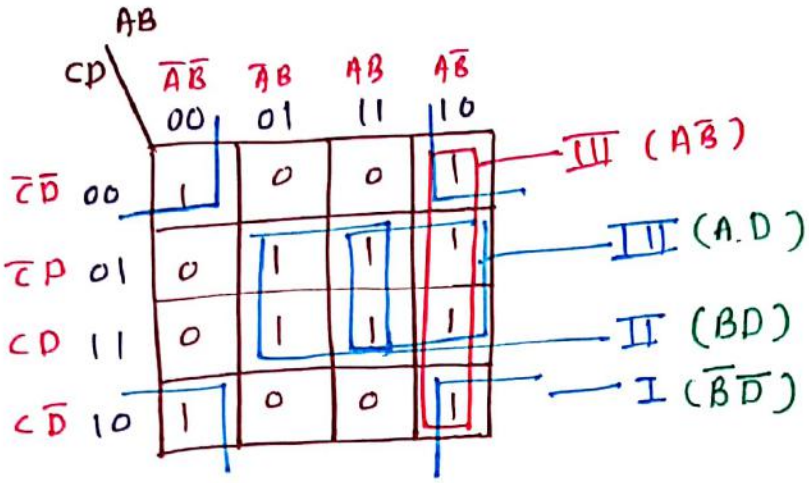
Examples on K-Map

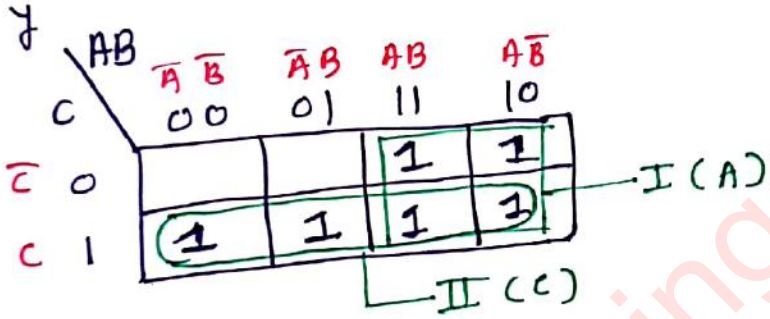
1] Find the Boolean expression for K-map given below.



$= \bar{B}\bar{D} + BD + AD$ or

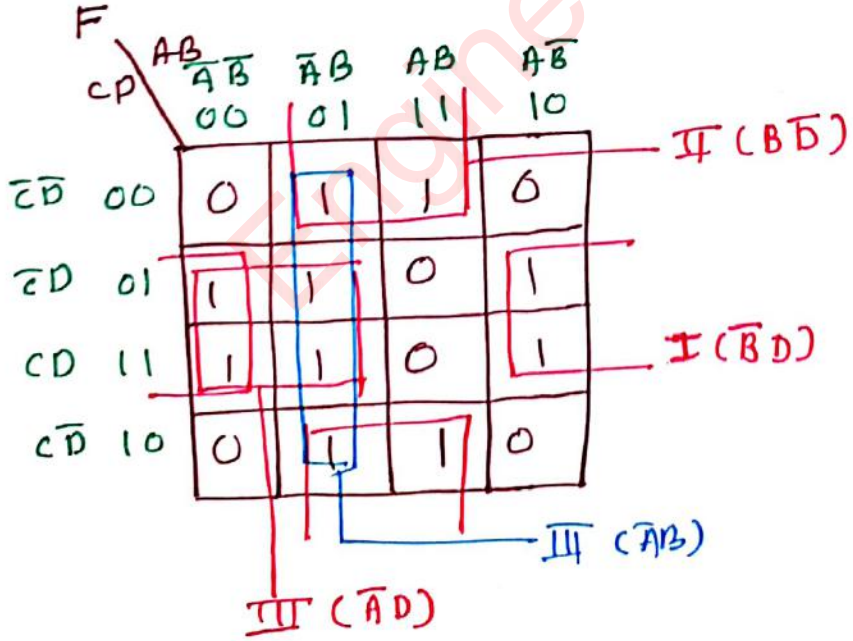
$= \bar{B}\bar{D} + BD + A\bar{B}$

2] $y = \underline{AB} + \underline{\bar{A}BC} + \underline{A\bar{B}} + \underline{C}$. Solve boolean expressim by K-map.



$y = A + C$

3] Find The Boolean expression for K-map given below.



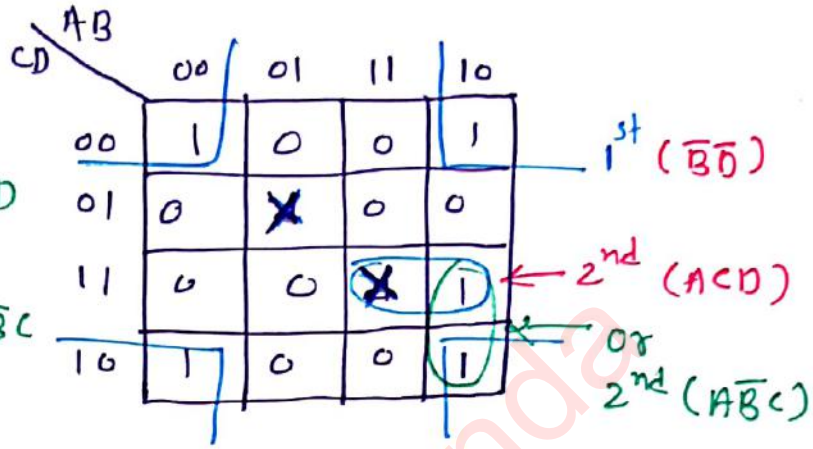
$F = \bar{B}D + B\bar{D} + \bar{A}D$

or

$F = \bar{B}D + B\bar{D} + \bar{A}B$

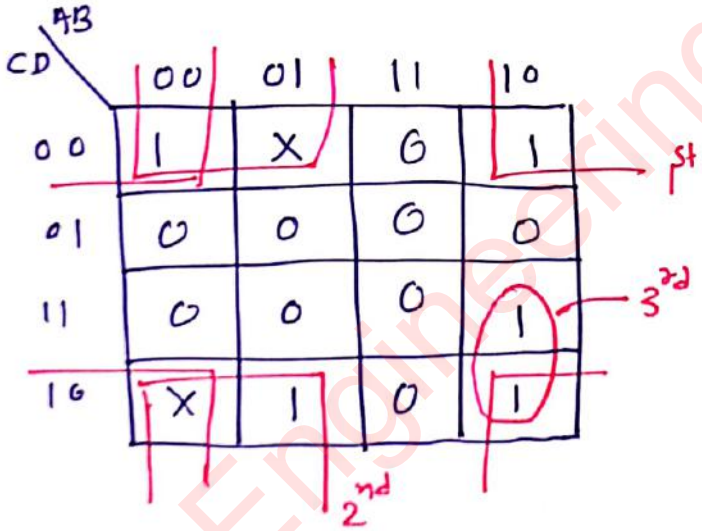
1] The numbers of product term in the minimized Sum of product expression obtained thrugh the following K-map is ____ [x is don't care].

a] 2
b] 3
c] 4
d] 5



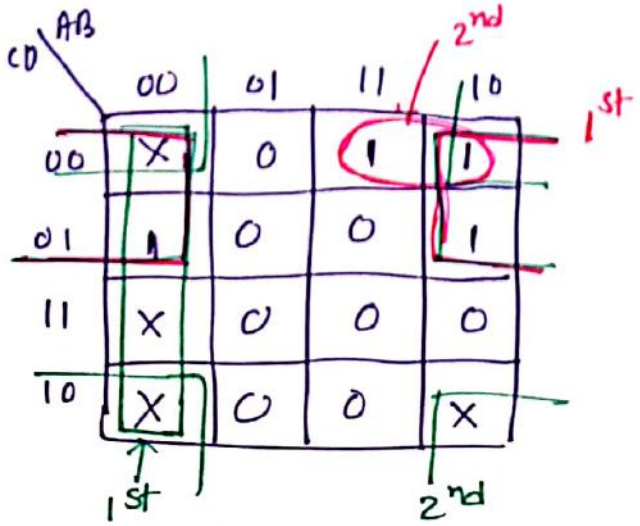
$Y = \bar{B}\bar{D} + ACD$ or $Y = \bar{B}\bar{D} + A\bar{B}C$

2] Solve given boolean K-map [x is don't care]



$Y = \bar{B}\bar{D} + \bar{A}\bar{D} + A\bar{B}C$

3) Solve given boolean K-map [x is don't care]



$Y = \bar{B}\bar{C} + A\bar{C}\bar{D}$

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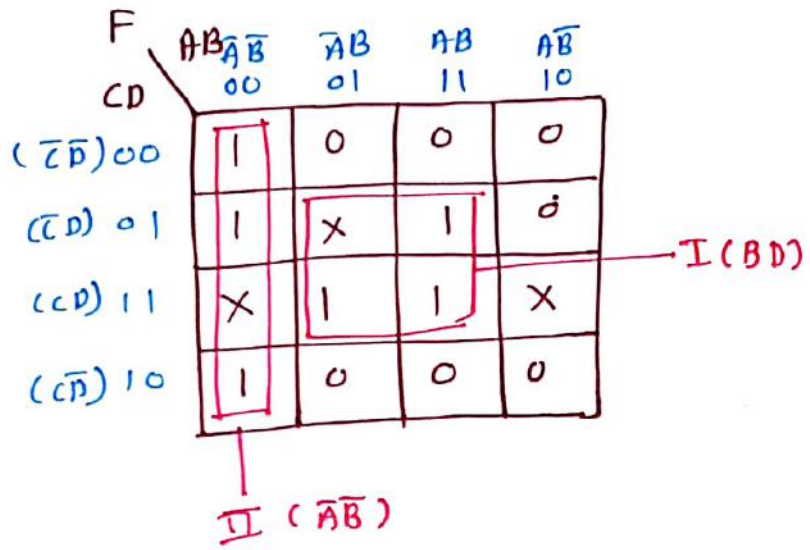
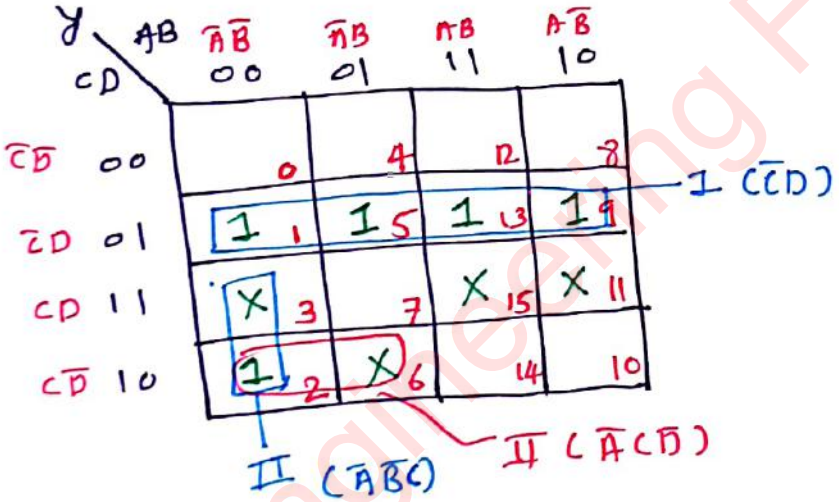
K-map Examples with don't care.

1] Solve K-map



$F = BD + \bar{A}\bar{B}$

2] $Y_{ABCD} = \Sigma_m(1, 2, 5, 9, 13) + \Sigma_d(3, 6, 11, 15)$

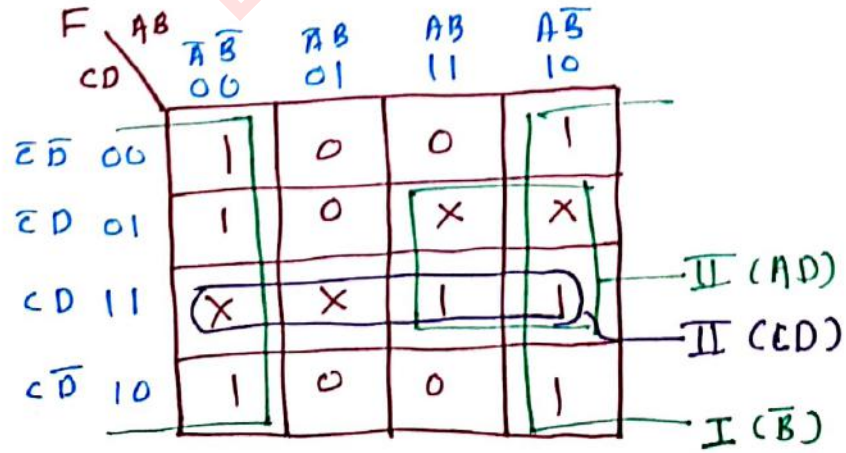


$Y = \bar{C}D + \bar{A}\bar{B}C$

or

$Y = \bar{C}D + \bar{A}C\bar{D}$

3] Solve K-map



$F = \bar{B} + AD$

or

$F = \bar{B} + CD$

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K-Map for POS expression

Steps for POS expression

- take grouping of 0
- find function (f_d)
- Put compliment of all variables ($\bar{f}$)

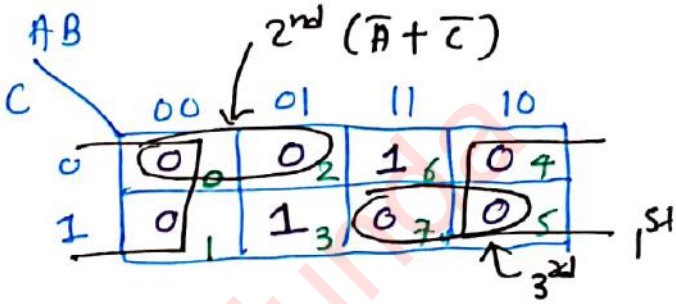
★ If boolean function is given by $y = \Sigma m(3,6)$ then

a) $y = B(A+C)(\bar{A}+\bar{C})$ ✓

b) $y = B(A+\bar{C})(\bar{A}+\bar{C})$

c) $y = \bar{B}(A+\bar{C})(\bar{A}+C)$

d) $y = \bar{B}(A+C)(\bar{A}+\bar{C})$



$Y_d = \bar{B}.(\bar{A}+\bar{C}).(A+C)$

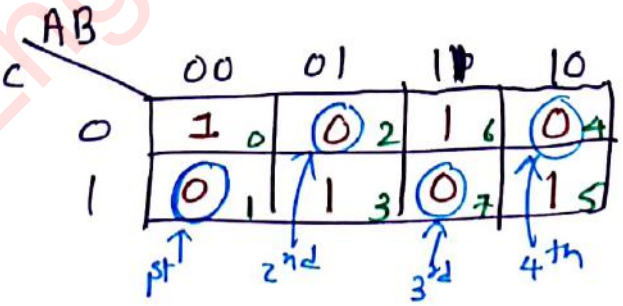
$\boxed{Y = B.(A+C).(\bar{A}+\bar{C})}$

★ If boolean function is given by.

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

→ function of y in terms of POS

$Y = $ _____



$Y_d = (\bar{A}+\bar{B}+C).(\bar{A}+B+\bar{C}).(A+B+C).(A+\bar{B}+\bar{C})$

$\boxed{Y = (A+B+\bar{C}).(A+\bar{B}+C).(\bar{A}+\bar{B}+\bar{C}).(\bar{A}+B+\bar{C})}$

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5 Variable K-Map

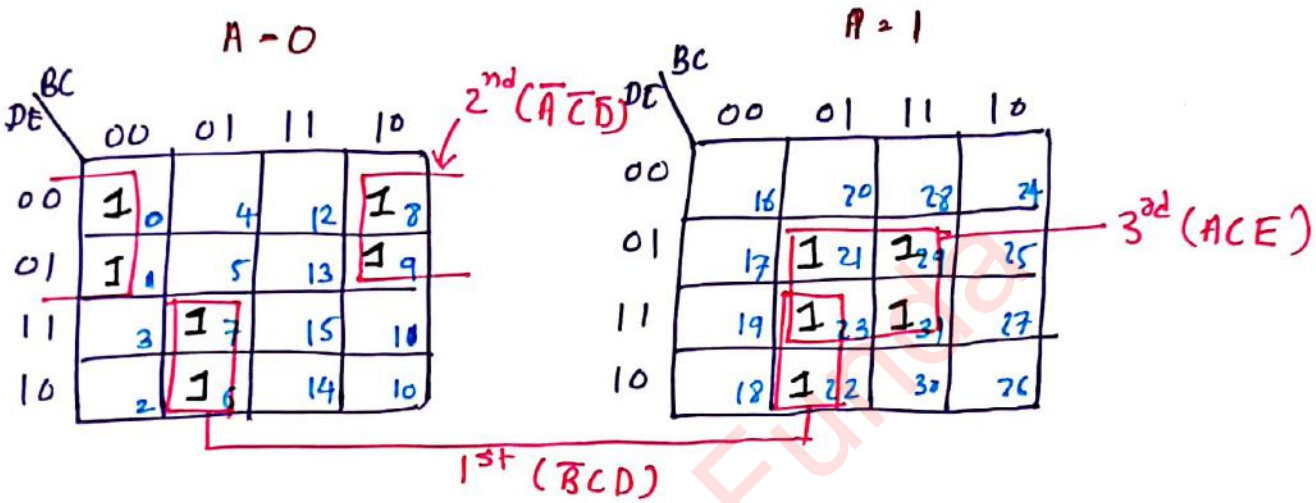
$f(A, B, C, D, E) = \Sigma m(0, 1, 6, 7, 8, 9, 21, 22, 23, 29, 31)$

$= \bar{B}CD + \bar{A}C\bar{D} + ACE$

Total cell $= 2^n$

$= 2^5$

$= 32$



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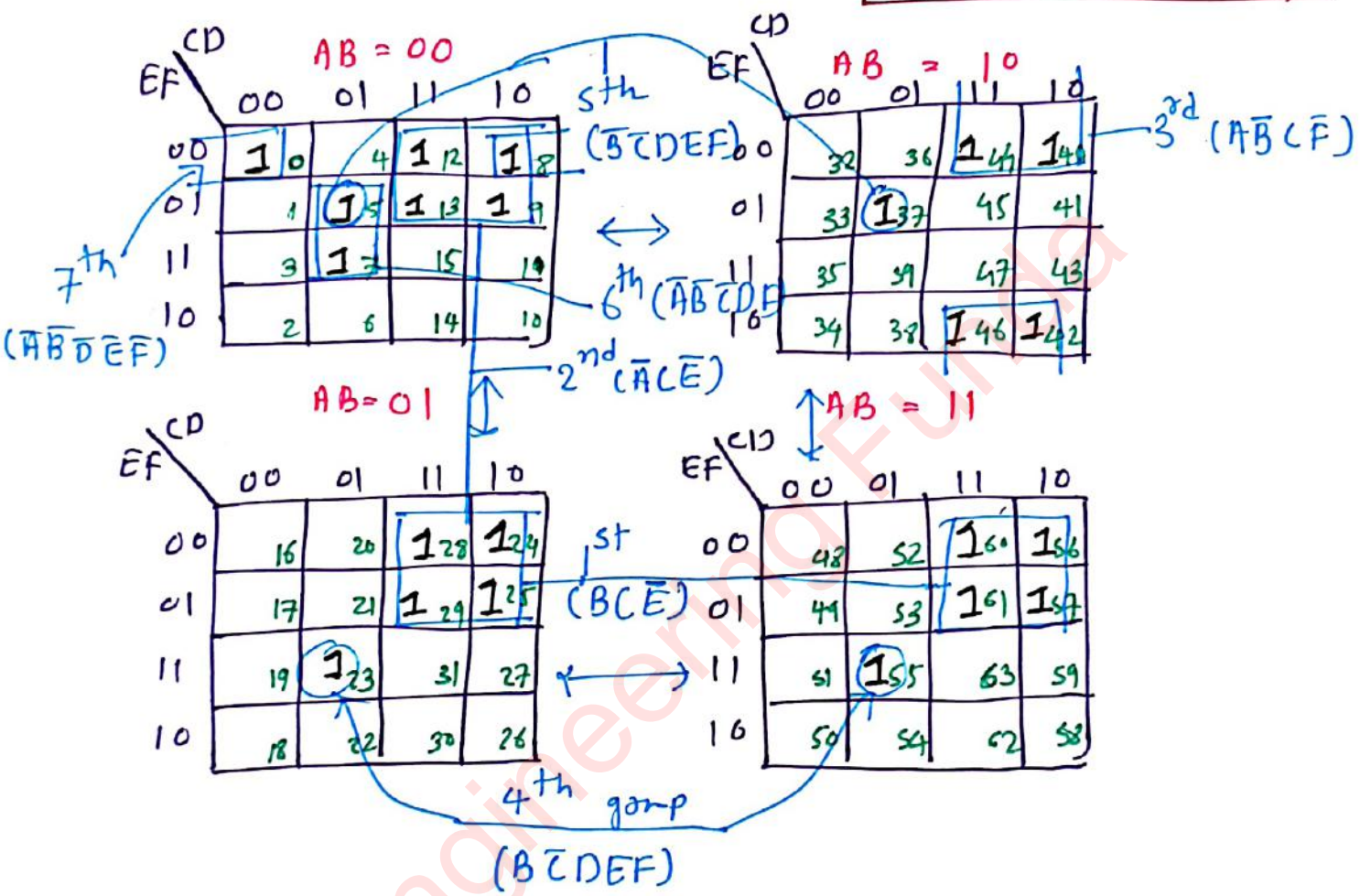
6 Variable K-Map.

$f(A,B,C,D,E,F) = \Sigma m(0,5,7,8,9,12,13,23,24,25,28,29,37,40,42,44,46,55,56,57,60,61)$.

$n = 6$

$\rightarrow$ total $= 2^n = 2^6 = 64$

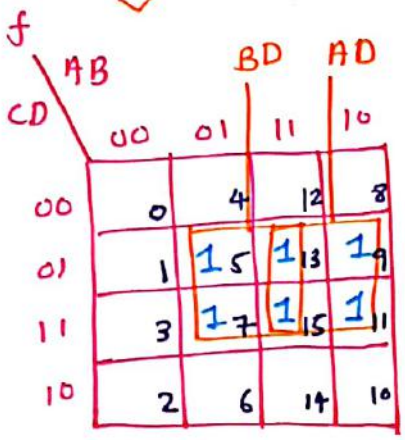
$= \bar{B}C\bar{E} + \bar{A}C\bar{E} + A\bar{B}C\bar{F} + B\bar{C}DEF + \bar{B}\bar{C}DEF + \bar{A}\bar{B}\bar{C}DF + \bar{A}\bar{B}\bar{D}\bar{E}\bar{F}$



K-map examples of GATE, DRDO & ISRO Examinations

1] The function $f(A, B, C, D) = \Sigma_m(5, 7, 9, 11, 13, 15)$ is independent of Variables

a] B b] C c] A & C d] D



$f = BD + AD$

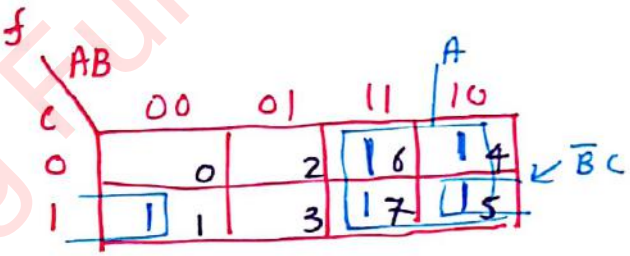
2] The Standard Sum of Product of the function $f = A + \bar{B}C$ is expressed as

a) $\Sigma_m(1, 4, 5, 6, 7) + d(2, 3)$

b) $\Sigma_m(1, 4, 5, 6, 7)$

c) $\Sigma_m(0, 2, 3) + d(1, 4, 5, 6, 7)$

d) $\Pi M(1, 4, 5, 6, 7)$



$f = \Sigma_m(1, 4, 5, 6, 7)$

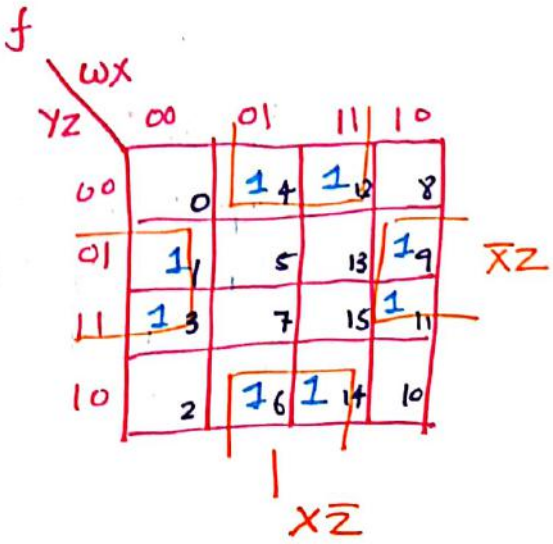
3) Consider the following Boolean function of four variables $f(w, x, y, z) = \Sigma_m(1, 3, 4, 6, 9, 11, 12, 14)$, the function

a) Independent of One Variable

b) Independent of two Variable

c) Independent of three Variable

d) Dependent of all Variable.



$f = \bar{X}Z + \bar{Z}X$

$f = X \oplus Z$

Quine McCluskey Minimization Technique

$Y(A, B, C, D) = \Sigma m(0, 1, 3, 7, 8, 9, 11, 15)$

Step-1 – Represent given Number in Binary.

N	A	B	C	D	
0	0	0	0	0	✓
1	0	0	0	1	✓
3	0	0	1	1	✓
7	0	1	1	1	✓
8	1	0	0	0	✓
9	1	0	0	1	✓
11	1	0	1	1	✓
15	1	1	1	1	

Step-2 – Form a group based on Number of 1's

group	Term	A	B	C	D	
group-0	(0)	0	0	0	0	✓
group-1	(1)	0	0	0	1	✓
	(8)	1	0	0	0	✓
group-2	(3)	0	0	1	1	✓
	(9)	1	0	0	1	✓
group-3	(7)	0	1	1	1	✓
	(11)	1	0	1	1	✓
group-4	(15)	1	1	1	1	✓

Step-3 Find Matched Pair with One bit Difference & Mark location

group	Pair	A	B	C	D	
group-0	(0-1)	0	0	0	–	✓
	(0-8)	–	0	0	0	✓
group-1	(1-3)	0	0	–	1	✓
	(1-9)	–	0	0	1	✓
	(8-9)	1	0	0	–	✓
group-2	(3-7)	0	–	1	1	✓
	(3-11)	–	0	1	1	✓
	(9-11)	1	0	–	1	✓
group-3	(7-15)	–	1	1	1	✓
	(11-15)	1	–	1	1	✓

Step-4 – Repeat Step 3 till end.

group	Pair	A	B	C	D
group-0	(0-1-8-9)	–	0	0	–
$\bar{B}\bar{C}$	(0-8-1-9)	–	0	0	–
group-1	(1-3-9-11)	–	0	–	1
$\bar{B}D$	(1-9-3-11)	–	0	–	1
group-2	(3-7-11-15)	–	–	1	1
CD	(3-11-7-15)	–	–	1	1

Step-5 Implicant Table.

PI	Minterms	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	0-1-8-9	⊗	×			⊗	×		
$\bar{B}D$	1-3-9-11		×	×			×	×	
$C\bar{D}$	3-7-11-15			×	⊗			×	⊗

$Y = \bar{B}\bar{C} + C\bar{D}$