



Probability and Statistics

Extra Practice Sheet 01

1. In a regiment of 1000, the mean height of the soldiers is 68.12 units and the standard deviation is 3.374 units. Assuming a normal distribution, how many soldiers could be expected to be more than 72 units? It is given that
 $P(z=1.00) = 0.3413$, $P(z = 1.15) = 0.3749$ and
 $P(z = 1.25) = 0.3944$, where z is the standard normal variable.
2. If the height of 300 students are normally distributed with mean 64.5 inches and standard deviation 3.3 inches, find the height below which 99% of the students lie.
3. The lifetime of radio tubes manufactured in a factory is known to have an average value of 10 years. Find the probability that the lifetime of a tube taken randomly (i) exceeds 15 years, (ii) is less than 5 years, assuming that the exponential probability law is followed.
4. Analysis of past data shows that hub thickness of a particular type of gear is normally distributed about a mean thickness of 2.00 cm with a standard deviation of 0.04 cm.
 - (i) What is the probability that a gear chosen at random will have a thickness greater than 2.06 cm?
 - (ii) How many gears will have a thickness between 1.89 and 1.95 cm?
 Given $\phi(1.5) = 0.4332$, $\phi(2.75) = 0.4970$, $\phi(1.25) = 0.3944$.
5. The breaking strength X of a cotton fabric is normally distributed with $E(x) = 16$ and $\sigma(x) = 1$. The fabric is said to be good if $X \geq 14$. What is the probability that a fabric chosen at random is good. Given that $\Phi(2) = 0.9772$
6. A manufacturer knows from experience that the resistance of resistors he produces is normal with mean $\mu = 140 \Omega$ and standard deviation $\sigma = 5\Omega$. Find the percentage of the resistors that will have resistance between 138Ω and 142Ω . (given $\phi(0.4) = 0.6554$, where z is the standard normal variate).
7. A manufacturing company packs pencils in fancy plastic boxes. The length of the pencils is normally distributed with $\mu = 6''$ and $\sigma = 0.2''$. The internal length of the boxes is $6.4''$. What is the probability that the box would be too small for the pencils (Given that a value of the standardized normal distribution function is $\Phi(2) = 0.9772$).
8. Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 per cent of such fuses are defective.
9. The number of accidents during a year in a factory has the Poisson distribution with mean 1.5. The accidents during different years are assumed independent. Find the probability that only 2 accidents take place during 2 years time.
10. A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantee that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality, [$e^{-5} = 0.006738$]

11. Suppose the number of telephone calls on an operator received from 9.00 to 9.05 follow a Poisson distribution with mean 3. Find the probability that
- the operator will receive no calls in that time interval tomorrow,
 - in the next three days the operator will receive a total of 1 call in that time interval, $[e^{-3} = 0.04978]$
12. In a normal distribution, 7% of the items are under 35 and 89% are under 63. Determine mean and variance of distribution. [Area of z for 0.43 = 1.48. Area of z for 0.39 = 1.23]
13. The length of an item manufactured on an automatic machine tool is a normally distributed random variable with parameters $m(\bar{x}) = 10$, and $\sigma^2 = \frac{1}{200}$. Find the probability of defective production of the tolerance is 10 ± 0.05 .
14. In a mathematics examination, the average grade was 82 and the standard deviation was 5. All the students with grades from 88 to 94 received a grade B. If the grades are normally distributed and 8 students received a B grade, find how many students took the examination.
- Given**
- | | | | | |
|---|--------|--------|--------|--------|
| 6 | 1.20 | 2.00 | 2.40 | 2.45 |
| A | 0.3849 | 0.4772 | 0.4918 | 0.4929 |
15. The income of a group of 10,000 persons was found to be normally distributed with mean Rs. 750 p.m. and standard deviation of Rs. 50. Show that, of this group, about 95% had income exceeding Rs. 668 and only 5% had income exceeding Rs. 832. Also find the lowest income among the richest 100.
16. A continuous type random variable X has probability density $f(x)$ which is proportional to x^2 and X takes values in the interval $[0, 2]$. Find the distribution function of the random variable use this to find $P(X > 1.2)$ and conditional probability $P(X > 1.2/X > 1)$.
17. One bag contains four white and two black beads and another contains three of each colour. A bead is drawn from each bag. What is the probability that one is white and one is black ?
18. The odds that a book will be favorably reviewed by three independent critics are 5 to 2, 4 to 3, 3 to 4 respectively. What is the probability that of the three reviews, a majority will be favorable?
19. Let E and F be independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happen is $\frac{1}{2}$. Then find $P(E)$ and $P(F)$.
20. Given a random variable whose range set is $(1, 2)$ and whose probability is $f(1) = \frac{1}{4}$ and $f(2) = \frac{3}{4}$. Find the mean and variance of the distribution.
21. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of 11 steps, he is just one step away from the starting point.
- Ans.** 0.210677186
22. What would be the expectation of the number of failures preceding the first success in an infinite series of independent trials with the constant probability of success p ?
23. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random
- 1
 - 0
 - At most 2 bolts will be defective.
24. Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or a six ?
25. If the chance that any one of the 10 telephone lines is busy at any instant is 0.2, what is the chance that 5 of the lines are busy? What is the probability that all the lines are busy?

26. An insurance salesman sells policies to 5 men, all of identical age in good health. According to the actuarial tables the probability that a man of this particular age will be alive 30 year hence is $\frac{2}{3}$.

Find the probability that in 30 years.

- (a) All 5 men
(b) At least 3 men
(c) Only 2 men
(d) At least 1 man

Will be alive.

Ans. (a) $\frac{32}{243}$ (b) $\frac{192}{243}$ (c) $\frac{40}{243}$ (d) $\frac{243}{243}$

27. Assuming a Binomial distribution, find the probability of obtaining at least two "six" in rolling a fair die 4 times.
28. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random without replacement. Find the probability that none of the two screws is defective.
29. Out of 800 families with four children each, how many families would be expected to have : (i) 2 boys and 2 girls; (ii) at least one boy; (iii) no girl; (iv) almost two girls ? Assume equal probabilities for boys and girls.
30. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down less than 2 hurdles?
31. An electronic component consists of three parts. Each part has probability 0.99 of performing satisfactorily. The component fails if 2 or more parts do not perform satisfactorily. Assuming that the parts perform independently, determine the probability that the component does not perform satisfactorily.
32. Find the binomial distribution whose mean is 5 and variance is $\frac{10}{3}$.
33. The probability that on, joining Engineering College, a student will successfully complete the course of studies

is $\frac{3}{5}$. Determine the probability that out of 5 students joining the College (i) none and (ii) at least two will successfully complete the course.

34. A carton contains 20 fuses, 5 of which are defective. Three fuses are chosen at random and inspected. What is the probability that at most one defective fuse is found.
35. A bag contains three coins, one of which is coined with two heads, while the other two coins are normal and not biased. A coin is thrown at random from the bag and tossed three times in succession. If heads turn up each time, what is the probability that this is the two-headed coin?
36. In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1.000 such samples, how many would be expected to contain at least 3 defective parts?
37. The incidence of occupational disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers 4 or more will catch the disease ?
38. The following information is obtained concerning an investigation of 50 ordinary shops of small size. Can it be inferred that shops run by women are relatively more in villages than in towns? Use χ^2 test.

	Shops		Total
	In Town	In villages	
Run by men	17	18	35
Run by women	3	12	15
Total	20	30	50

39. Of a group of patients who complained they did not sleep well, some were given sleeping pills while others were given sugar pills (although they all thought they were getting sleeping pills). They were later asked whether the pills helped them or not. The result of their responses are shown in the table given below. Assuming that all patients told the truth, test the hypothesis that there is no difference between sleeping pills and sugar pills at a significance level of 0.05.

	Step well	Did not sleep well
Took sleeping pills	44	10
Took sugar pills	81	35

40. In an experiment on immunization of cattle from tuberculosis the following results were obtained

	Died	Unaffected
Inoculated	12	26
Not inoculated	16	6

Examine the effect of vaccine in controlling susceptibility to tuberculosis.

41. The probability that Nirmal will solve a problem is $\frac{2}{3}$ and the probability that Satyajit will solve it is $\frac{3}{4}$. What is the probability that (a) the problem will be solved (b) neither can solve it.
42. Two persons A and B toss an unbiased coin alternately on the understanding that the first who gets the head wins. If A starts the game, then what are their respective chances of winning ?
43. Four persons are chosen at random from a group containing 3 men, 2 women, and 4 children. Show that the probability that exactly two of them will be children is $\frac{10}{21}$.
44. A five digit number is formed by using the digits 0, 1, 2, 3, 4 and 5 without repetition. Find the probability that the number is divisible by 6.
45. The chances that doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chances of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died, what is the chance that his disease was diagnosed correctly.
46. An anti-aircraft gun can take a maximum of four shots on enemy's plane moving from it. The probabilities of hitting the plane at first, second, third and fourth shots

are 0.4, 0.3, 0.2 and 0.1 respectively. Find the probability that the gun hits the plane.

47. An electronic component consists of three parts. Each part has probability 0.99 of performing satisfactorily. The component fails if two or more parts do not perform satisfactorily. Assuming that the parts perform independently, determine the probability that the component does not perform satisfactorily.
48. The face cards are removed from a full pack. Out of the remaining 40 cards, 4 are drawn at random. What is the probability that they belong to different suits ?
49. Of the cigarette smoking population, 70% are men and 30% women, 10% of these men and 20% of these women smoke 'WILLS.' What is the probability that a person seen smoking a 'WILLS' will be a man.
50. The distribution of typing mistakes committed by a typist is given below. Assuming a Poisson model, find out the expected frequencies:

Mistakes per page	0	1	2	3	4	5
No. of pages	142	156	69	27	5	1

51. Let x be the number of cars per minute passing a certain crossing of roads between 5.00 P.M. and 7.00 P.M. on a holiday. Assume x has a Poisson distribution with mean 4. Find the probability of observing almost 3 cars during any given minute between 5.00 P.M. and 7 P.M. (given $e^{-4} = 0.0183$)
52. Let x be the number of cars, passing a certain point, per minute at a particular time. Assuming that x has a poisson distribution with mean 0.5, find the probability of observing 3 or fewer cars during any given minute.
53. Number of customers arriving at a service counter during a day has a Poisson distribution with mean 100. Find the probability that at least one customer will arrive on each day during a period of five days. Also find the probability that exactly 3 customers will arrive during two days.
54. The random variable X has a Poisson distribution. If $P(X = 1) = 0.01487$, $P(X = 2) = 0.04461$. Then find $P(X = 3)$.

55. A source of water is known to contain bacteria with mean number of bacteria per cc equal to 2. Five 1 cc test tubes were filled with water. Assuming that Poisson distribution is applicable, calculate the probability that exactly 2 test tubes contain at least 1 bacterium each.
56. In a normal summer, a truck driver gets on an average one puncture in 1000 km. Applying Poisson distribution, find the probability that he will have
(i) no puncture
(ii) two punctures in a journey of 3000 kms.
57. Wireless sets are manufactured with 25 soldered joints each. On the average, 1 joint in 500 is defective. How many sets can be expected to be free from defective joints in a consignment of 10000 sets ?
58. In a class of 12 students, 5 are boys and the rest are girls. Find the probability that a student selected will be a girl.
59. A bag contains 7 red and 8 black balls. Find the probability of drawing a red ball.
60. Three of the six vertices of a regular hexagon are chosen at random. Find the probability that the triangle with three vertices is equilateral.
61. What is the probability that a leap year, selected at random, will contain 53 Sundays.
62. Choose the correct answer:
(a) In solving any problem, odds against A are 4 to 3 and odds in favour of B in solving the same problem are 7 to 5. The probability that the problem will be solved is-
(i) $\frac{5}{21}$ (ii) $\frac{16}{21}$ (iii) $\frac{15}{84}$ (iv) $\frac{69}{84}$
(b) In a given race, the odds in favour of horses A, B, C, D are 1 : 3, 1 : 4, 1 : 5, 1 : 6 respectively. The probability that horse C wins the race is
(i) $\frac{1}{4}$ (ii) $\frac{1}{5}$ (iii) $\frac{1}{6}$ (iv) $\frac{1}{7}$
(c) In tossing a fair die, the probability of getting an odd number or a number less than 4 is
(i) 2 (ii) $\frac{1}{2}$ (iii) $\frac{2}{3}$ (iv) $\frac{3}{4}$

- (d) An unbiased coin is tossed 3 times. The probability of obtaining two heads is

(i) $\frac{1}{2}$ (ii) $\frac{3}{8}$ (iii) 1 (iv) $\frac{1}{8}$

63. Fill in the blanks with appropriate correct answer
(a) Chance of throwing 6 at least once in four throws with single dice is....
(b) A pair of fair dice is thrown and one die shows a four. The probability that the other die shows 5 is
64. The table shows the relation between the performance in mathematics and electronics, using a (a) 0.05 (d) 0.01 significance level.

Mathematics	Electronics		
	High marks	Medium marks	Low marks
High marks	56	71	12
Medium marks	47	163	38
Low Marks	14	42	85

65. The results of a survey made to determine whether the age of a driver 21 years of age and older has any effect on the number of automobile accidents in which he is involved (including minor accidents) are given in the table below. At a level of significance of (a) 0.05 and (b) 0.01, test the hypothesis that number of accidents is independent of the age of the driver.

		Age of the driver				
		21-30	31-40	41-50	51-60	61-70
Number of accidents	0	748	821	786	720	672
	1	74	60	51	66	50
	2	31	25	22	16	15
	More than 2					

66. A die thrown 60 times with the following results.

Face	1	2	3	4	5	6
Frequency	8	7	12	8	14	11

67. Fit a Binomial Distribution of the data

x	0	1	2	3	4	5
f	38	144	342	287	164	25

and test for goodness of fit at the level of significance 0.05.

Answer Key

1. 125
2. 68.7295 inches
3. (i) 0.2231, (ii) 0.3935.
4. (i) 0.068 (ii) 62
5. 0.9772
6. 31.08%
7. 0.0228.
8. 0.785
9. 0.224
10. 0.0136875
11. (i) e^{-3} (ii) $3 \times (e^{-3})^2 (e^{-3}.3)$
12. $\mu = 50.29, \sigma^2 = 106.73$
13. 0.04798
14. 75 students
15. Rs. 866
16. ??
17. $1/2$
18. $\frac{209}{343}$
19. ??
20. Mean = $\frac{7}{4}$. Var = $\frac{3}{16}$

21. 0.210677186

22. ??

23. (a) 0.4096. (b) 0.4096, (c) 0.9728.

24. 233

25. $10C_5 (0.2)^5 (0.8)^5, (0.2)^{10}$

26. (a) $\frac{32}{243}$ (b) $\frac{192}{243}$ (c) $\frac{40}{243}$ (d) $\frac{243}{243}$

27. $\frac{171}{1296}$

28. $\frac{7}{15}$

29. (i) 300, (ii) 750, (iii) 50, (iv) 550.

30. $\frac{8}{3} \left(\frac{5}{6} \right)^9$

31. 0.000298

32. ${}^{15}C_r \left(\frac{1}{2} \right)^r \left(\frac{2}{3} \right)^{15-r}$

33. (i) $\frac{32}{3125}$ (ii) $\frac{2853}{3125}$

34. $\frac{27}{32}$

35. $\frac{4}{5}$

36. 324

37. $\frac{53}{3125}$

38. $x^2 = 3.57$, Hypothesis is wrong.

39. The hypothesis cannot be rejected at the 0.05 level.

40. $x^2 = 9.367$, vaccine is effective

41. (a) $\frac{11}{12}$ (b) $\frac{1}{12}$

42. 4 ; 1

43. ???

44. $\frac{4}{25}$

45. $\frac{6}{13}$

46. 0.6976.

47. 0.000298

48. $\frac{1000}{9139}$

49. $\frac{7}{13}$

50. 147, 147, 74, 25, 6, 1 pages.

51. 0.4331

52. 0.998

53. $(1-e^{-100})^5, e^{-200} \times \frac{4(100)^3}{3}$

54. 0.08922

55. $\frac{2}{5}(1 - e^{-2}) = 0.3459$

56. (i) e^{-3} (ii) $4.5 e^{-3}$

57. 9512

58. $\frac{7}{12}$

59. $\frac{7}{15}$

60. $\frac{1}{10}$

61. $\frac{2}{7}$

62.

(a)

Ans. (ii)

(b)

Ans. (iii)

(c)

Ans. (iii)

(d)

Ans. (ii)

63.

(a)

Ans. $\frac{671}{1296}$

(b)

Ans. $\frac{1}{36}$

64. The hypothesis can be rejected at both levels.

65. The hypothesis cannot be rejected at either level.
66. Test at 5% level of significance if the die is honest, assuming that $F(x^2 > 11.1) = 0.05$ with 5 d.f.
67. $x^2 = 7.97$, Binomial distribution gives fit at 5% level.
-



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