Subject: Calculus and optimization
Chapter: Single Variable Calculus

DPP-01

1. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at

(a) 0 (b) 1/2
(c) 1 (d) 1/4

2. The product of minimum value of x^x and maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) e (b) e^{-1}
(c) 1 (d) e^2

3. The minimum value of the function defined by $f(x) = \max(x, x+1, 2-x)$ is

(a) 0 (b) 1/2
(c) 1 (d) 3/2

4. The greatest and the least values of the function, $f(x) = 2 - \sqrt{1+2x+x^2}$, $x \in [-2, 1]$ are

(a) 2, 1 (b) 2, –1
(c) 2, 0 (d) None of these

5. The difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $[-\pi/2, \pi/2]$ is

(a) $\frac{\sqrt{3}+\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}+\sqrt{2}}{2}+\frac{\pi}{6}$
(c) $\frac{\pi}{2}$ (d) π

6. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is

(a) 2 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

7. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(a) 41 (b) 1
(c) 17/7 (d) 1/4

8. The maximum value $x^3 - 3x$ in the interval [0, 2] is

(a) 1 (b) 2
(c) 0 (d) –2

9. Minimum value of $\frac{1}{3\sin\theta - 4\cos\theta + 7}$ is

(a) $\frac{7}{12}$ (b) $\frac{5}{12}$
(c) $\frac{1}{12}$ (d) $\frac{1}{6}$

10. The number of values of x where $f(x) = \cos x + \cos\sqrt{2}x$ attains its maximum value is

(a) 1 (b) 0
(c) 2 (d) infinite

11. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ in [0, 1] is

(a) 1 (b) 2
(c) 3 (d) $2^{1/3}$

12. if $\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{6}$, then x can be equal to

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$
(c) 2 (d) None of these

13. if $f(x) = \begin{cases} x; & x < 1 \\ x-1; & x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x)dx$ is equal to

(a) 1 (b) 4/3
(c) 5/3 (d) 5/2

14. $\int_0^{\pi} |1+2\cos x|\, dx$ equal to :

(a) $2\pi/3$ (b) π

(c) 2 (d) $\frac{\pi}{3}+2\sqrt{3}$

15. The value of $\int_{-1}^{3} (|x-2|+[x])dx$ is equal to (where [*] denotes greatest integer function)

(a) 7 (b) 5

(c) 4 (d) 3

16. If $\int_{-1}^{3/2} |x\sin\pi x|dx = \frac{k}{\pi^2}$, then the value of k is

(a) $3\pi + 1$ (b) $2\pi + 1$

(c) 1 (d) 4

17. $\int_{\log\pi-\log 2}^{\log\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}$ dx is equal to

(a) $\sqrt{3}$ (b) $-\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{3}}$

18. If $I_1 = \int_e^{e^2} \frac{dx}{\ln x}$ and $I_2 = \int_1^2 \frac{e^x}{x}dx$, then

(a) $I_1 = I_2$ (b) $2\, I_1 = I_2$

(c) $I_1 = 2\, I_2$ (d) None of these

19. $\int_{2-\log 3}^{3+\log 3} \frac{\log(4+x)}{\log(4+x)+\log(9-x)}dx$

(a) Cannot be evaluated

(b) Is equal to 5/2

(c) is equal to 1+2 log 3

(d) Is equal to ½ + log 3

20. $\int_0^{\infty} [2e^{-x}]\, dx$ is equal to

(where [*] denotes the greatest integer function)

(a) 0 (b) ln 2

(c) e^2 (d) $2e^{-1}$

21. If $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{x}}{2}$, then $\int_0^{\infty} e^{-ax^2}dx$ where a > 0 is

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\frac{\sqrt{\pi}}{2a}$

(c) $2\frac{\sqrt{\pi}}{a}$ (d) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

22. The expression $\frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ is equal to

(where [*] and {*} denotes greatest integer function and fractional part function and $n \in N$).

(a) $1/n - 1$ (b) $1/n$

(c) n (d) $n - 1$

23. Let $A = \int_0^1 \frac{e^t dt}{1+t}dt$ then $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1}$ dt has the value

(a) Ae^{-a} (b) $-Ae^{-a}$

(c) $-ae^{-a}$ (d) Ae^{a}

24. $\int_0^{\pi} xf(\sin x)dx$ is equal to

(a) $\pi\int_0^{\pi} f(\sin x)dx$

(b) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$

(c) $\pi\int_0^{\pi/2} f(\cos x)dx$

(d) $\pi\int_0^{\pi} f(\cos x)dx$

25. Let f : R $\rightarrow$ R be a differentiable function having f(2) = 6, f '(2) = $\left(\frac{1}{48}\right)$. Then $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2}$dt equals

(a) 18 (b) 12

(c) 36 (d) 24

26. The value of $\int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is

(a) 0 (b) 1

(c) 2 (d) 3

27. If $f(a+b-x)=f(x)$, then $\int_a^b xf(x)dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(b-x)dx$

(b) $\frac{a+b}{2}\int_a^b f(x)dx$

(c) $\frac{b-a}{2}\int_a^b f(x)dx$

(d) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$

28. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(a) 3 (b) 2

(c) 1 (d) –1

29. $\int_{\sin x}^{1} t^2 f(t)dt = 1-\sin x \;\forall x\in(0,\pi/2)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

(a) 3 (b) $\sqrt{3}$

(c) 1/3 (d) None of these

30. $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2 dt}{x\sin x}$ is equal to

(a) –1 (b) 1

(c) 2 (d) –2

31. If $\int_{\ln 2}^{x}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$, then x =

(a) 4

(b) ln 8

(c) ln 4

(d) None of these

32. $\underset{x\to\infty}{\text{Limit}}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x =$

(a) 1 (b) 2

(c) e^2 (d) e

33. If α and β be the roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha}(1+ax^2+bx+c)^{\frac{1}{x-\alpha}}$ is

(a) $a(\alpha-\beta)$ (b) $\ln|a(\alpha-\beta)|$

(c) $e^{a(\alpha-\beta)}$ (d) $e^{a|\alpha-\beta|}$

34. $\lim_{x\to\pi/2}\frac{2^{-\cos x}-1}{x(x-\pi/2)} =$

(a) $\frac{2\ln 2}{\pi}$ (b) ln2

(c) $2/\pi$ (d) Does not exists

35. $\underset{x\to\pi/2}{\text{Limit}}\frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$ is

(a) 1/16 (b) –1/16

(c) 1/32 (d) –1/32

36. $\lim_{x\to 0}(\cos mx)^{n/x^2}$

(a) $e^{-m^2n/4}$ (b) $e^{-m^2n/2}$

(c) $e^{-mn^2/2}$ (d) $e^{-mn^2/4}$

37. $\underset{x\to 0}{\text{Limit}}\frac{(4^x-1)^3}{\sin\left(\frac{x}{p}\right)\ln\left(1+\frac{x^2}{3}\right)} =$

(a) 9 p (log 4) (b) 3 p $(\log 4)^3$

(c) 12 p $(\log 4)^3$ (d) 27 p $(\log 4)^2$

38. Evaluate

$$\underset{n\to\infty}{\text{Limit}}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots+\frac{1}{\sqrt{n^2+2n}}\right)$$

(a) 1 (b) 1/2

(c) 0 (d) 2

39. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^x$ is equal to

(a) e^4 (b) e^2

(c) e^3 (d) e

40. Let α and β be the distinct roots of $ax^2 + bx + c = 0$,

Then $\lim\limits_{x\to\alpha}\dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) $\dfrac{1}{2}(\alpha-\beta)^2$ (b) $-\dfrac{a^2}{2}(\alpha-\beta)^2$

(c) 0 (d) $\dfrac{a^2}{2}(\alpha-\beta)^2$

41. $\lim\limits_{x\to0}\left(\cot\left(\dfrac{\pi}{4}+x\right)\right)^{\text{cosec}x}$ =

(a) e^{-1} (b) e^2

(c) e^{-2} (d) e^1

42. $\lim\limits_{x\to\infty}\left(\sin\dfrac{1}{x}+\cos\dfrac{1}{x}\right)^x$ is

(a) e (b) e^2

(c) 1/e (d) does not exist

43. $\lim\limits_{x\to\infty}\dfrac{\sin(6x^2)}{\ln\cos(2x^2-x)}$ =

(a) 12 (b) –12

(c) 6 (d) –6

44. $\lim\limits_{x\to0}\dfrac{e^{-x^2/2}-\cos x}{x^3\sin x}$ =

(a) 1/4 (b) 1/6

(c) 1/12 (d) 1/8

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Answer Key

1. (d)
2. (c)
3. (d)
4. (c)
5. (d)
6. (d)
7. (a)
8. (b)
9. (c)
10. (a)
11. (b)
12. (a)
13. (c)
14. (d)
15. (a)
16. (a)
17. (a)
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20. (b)
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34. (a)
35. (c)
36. (b)
37. (b)
38. (d)
39. (a)
40. (d)
41. (c)
42. (a)
43. (b)
44. (c)

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