Data Science and Artificial Intelligence

Data Science & AI

Linear Algebra
Discussion

DPP No.- 01

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Topics to be Covered

Topic Questions

Topic Discussion

Topic : Linear Algebra

Q1. Let A be a 5×8 matrix, then each column of A contains.

A. 5 elements

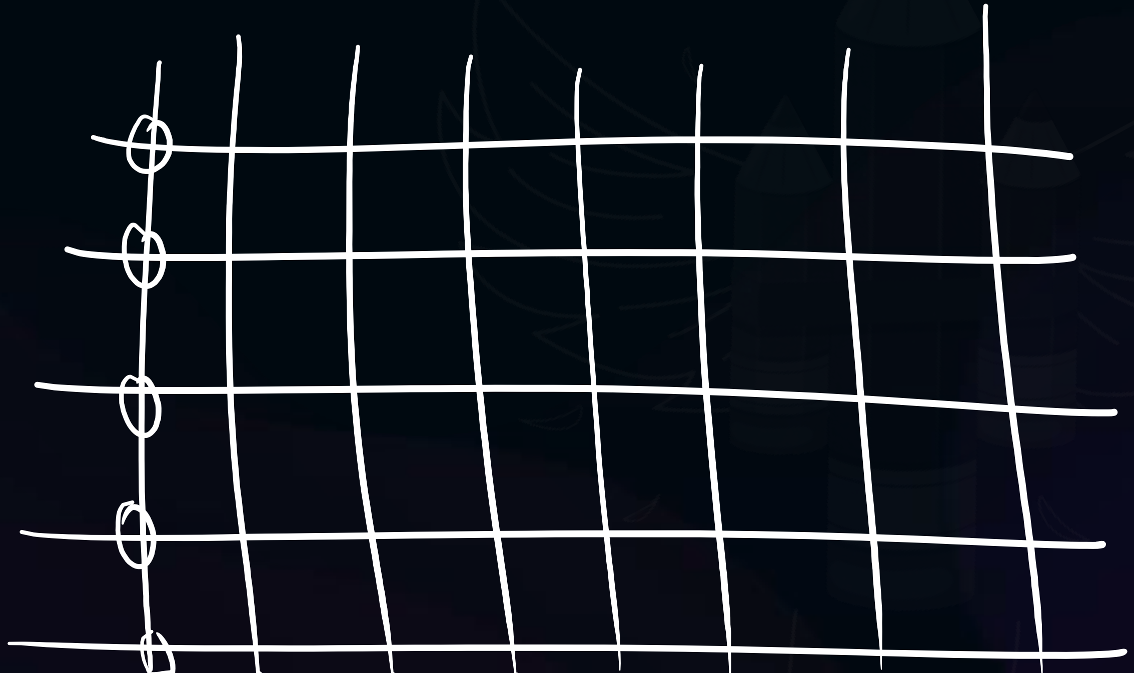
B. 8 elements

C. 40 elements

D. 13 elements

5 rows

8 columns



Topic : Linear Algebra

10×5

Q2. If A is a matrix of order 10 × 5, then each row of A contains-

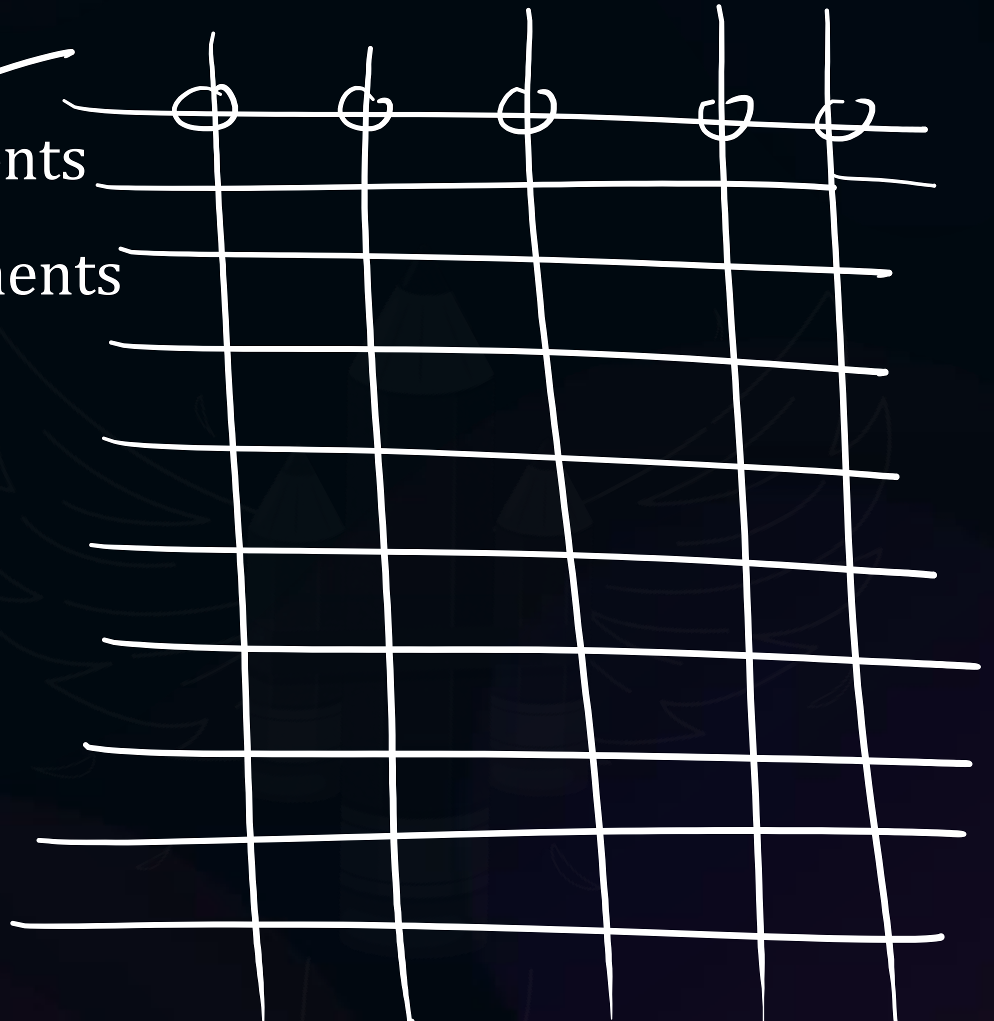
A. 25 elements

B. 05 elements

C. 10 elements

D. 150 elements

05 elements



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Q3. The number of all possible matrices of order 2×3 with each entry 1 or -1 is-

A. 32

B. 12

C. 6

D. 64

2×3

1 $\quad$ -1 $\quad$ 1 or -1

$$A = \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix}_{2 \times 3}$$

6 elements

$$= 2^{m \times n} = 2^{2 \times 3} = 2^6 = 64$$

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Q4. If A is of order m × n and B is of order p × q, then AB is defined only if

A. m = q

B. m = p

C. ~~n = p~~

D. n = q

$m \times n = p \times q$

$\boxed{n = p}$

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Q5. If P is of order 2×3 and Q is of order 3×2, then PQ is of order

A. 2×3

B. 2×2

C. 3×2

D. 3×3

$P \rightarrow 2 \times 3$

$Q \rightarrow 3 \times 2$

$PQ = 2 \times 2$

col^m =

rows

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Q6. If $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then

A. $A^2 = 0$

B. $A^2 = A$

C. $A^3 = A$

D. $A^2 = 2A$

$\underline{\underline{A^2, A^3}}$

$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

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Q7. If A = [x, y, z] B = $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$, C = [α, β, γ]^T , then ABC is-

$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = C =$

3×1

$1\times 3 \quad 3\times 3 \quad 3\times 1$

1×1

$1\times 3 \quad 3\times 1$

1×1

A. Not defined

B. is a 3 × 3 matrix

C. is a 1 × 1 matrix

D. is a 3 × 3 matrix

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Q8. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $= P$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ $= Q$, then A is equal to

A. $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$\begin{matrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{matrix}$

B. $\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A + B = P$

$A - 2B = Q$

$\underline{- \quad + \quad -}$

$3B = P - Q$

$3B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} -\frac{2}{3} & +\frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} \end{bmatrix}$ $B = \begin{bmatrix} 2/3 & -\frac{1}{3} \\ \frac{1}{3} & 2/3 \end{bmatrix}$

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Q9. If $\underset{2\times 2}{x} + \underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then 'X' is equal to

A. $\begin{bmatrix} 0 & 1 \\ 0 & 6 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -1 \\ 0 & -6 \end{bmatrix}$

C. $\cancel{\begin{bmatrix} -1 & 0 \\ -6 & 0 \end{bmatrix}}$

D. $\begin{bmatrix} 1 & 0 \\ 6 & 0 \end{bmatrix}$

$X = ?$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -6 & 0 \end{bmatrix}$

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$\rightarrow \underline{\underline{Q}}$

Q10. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 5 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{bmatrix}$, then B is equal to

A. $\frac{1}{3}\begin{bmatrix} -2 & -1 & 15 \\ 5 & 8 & -11 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 1 & -15 \\ 5 & -8 & -11 \end{bmatrix}$

C. $\begin{bmatrix} 2 & -1 & 15 \\ 5 & 8 & 11 \end{bmatrix}$

D. $\cancel{-\frac{1}{3}\begin{bmatrix} 2 & 1 & -15 \\ 5 & -8 & -11 \end{bmatrix}}$

$2A = \begin{bmatrix} 2 & 4 & 6 \\ -4 & 10 & 14 \end{bmatrix}$

$3B = 2A - Q$

$= \begin{bmatrix} +2 & +1 & -15 \\ +5 & -8 & -11 \end{bmatrix}$

$= \begin{bmatrix} -2 & -1 & 15 \\ -5 & 8 & 11 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ -4 & 10 & 14 \end{bmatrix} + \begin{bmatrix} -4 & -5 & +9 \\ -1 & -2 & -3 \end{bmatrix}$

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Q11. If $\begin{bmatrix} x & 1 \\ -1 & -y \end{bmatrix} + \begin{bmatrix} y & 1 \\ 3 & x \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then

A. $x = -1, y = 0$

B. ~~$x = 1, y = 0$~~

C. $x = 0, y = 1$

D. $x = 1, y = 1$

$$\begin{bmatrix} x+y & 2 \\ 2 & x-y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$x+y=1$

$x-y=1$

$2x=2$

$\boxed{x=1}$

$y=0$

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Q12. Let $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ and A + B – 4l = 0 , then B is equal to

3×3

I_3

A. $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -5 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -4 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

C. $\cancel{\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -1 \end{bmatrix}}$

D. $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 4 & -2 \\ 3 & 4 & 1 \end{bmatrix}$

$A + B = 4I$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} - A$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$

$\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -1 \end{bmatrix}$

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$B = 0$ or $a = b$

$bC = Ca \rightarrow C = 0$ or $a = b$

Q13. If a diagonal matrix is commutative with every matrix of the same order then it is necessarily

A. If a diagonal matrix is commutative with every matrix of the same order then it is necessarily

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

B. A scalar matrix

C. A unit matrix

D. A diagonal matrix with exactly two diagonal elements different. $\times$

$Bb = aB$

$$\begin{bmatrix} aA & \cancel{aB} \\ \cancel{bC} & bD \end{bmatrix} = \begin{bmatrix} Aa & \cancel{Bb} \\ \cancel{Ca} & Db \end{bmatrix}$$

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Q14. $\underset{2\times 3}{\underset{A}{\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}}} \underset{3\times 1}{\underset{B}{\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}}} + 2\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ is equal to

2×1

A. $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$

B. $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$

C. $\begin{bmatrix} 44 \\ 43 \end{bmatrix}$

D. $\begin{bmatrix} 43 \\ 50 \end{bmatrix}$

$AB = \begin{bmatrix} 21+4+10 \\ 27+8+5 \end{bmatrix}$

$\begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 10 \end{bmatrix}$

$\begin{bmatrix} 43 \\ 50 \end{bmatrix}$

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Q15. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then f(A) is equal to-

$A^2 \rightarrow 2\times2$

$4A \rightarrow 2\times2$

$5I \rightarrow 2\times2$

$f(A) = A^2 + 4A - 5I$

A. $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

2×2

$\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

$\begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + 4A - 5I$

$-2I$

Q16. If A is a square matrix such that $A^2 = I$, then $A - I$ is equal to-

A. I

B. 0

C. A

D. I + A

$A^2 - I = 0$

$(A+I)(A-I) = 0$

$A - I = 0$

$A + I = 0$

$A - I = 0$

$A - I$

$-2I$

$A + I = 0$

$A = -I$

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Q17. Let A $\begin{bmatrix} 1 & x/n \\ -x/n & 1 \end{bmatrix}$, then $\lim\limits_{n\to\infty} A^n$ is-

A. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

C. $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

D. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^n =$

A^2

(1, 2)

(2, 1)

1

$\frac{x}{n} \to 0$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\frac{x}{n} \to 0$

$\begin{bmatrix} 1 - \frac{x^2}{n^2} & \frac{2x}{n} \\ -\frac{2x}{n} & 1 \end{bmatrix} = A^2$

$\begin{bmatrix} 1 - \frac{x^2}{n^2} & \frac{2x}{n} \\ -\frac{2x}{n} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{x}{n} \\ -\frac{x}{n} & 1 \end{bmatrix} = 1 + \frac{x^3}{n^3} \quad \frac{3x}{n} - \frac{x^3}{n^3}$

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$Q = PAP^T$

$P^T Q^{2013} P$

Q18. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix}$ and $Q = PAP^T$, then $P^TQ^{2013}P$

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

A. $\begin{bmatrix} 1 & 2013 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 2013 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \frac{\sqrt{3}+1}{2} & -\frac{1}{2}+\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

C. $\begin{bmatrix} 2013 & 0 \\ 0 & 2013 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 2013 \\ 2013 & 0 \end{bmatrix}$

$\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}+1}{2} & \frac{-1+\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$\frac{3}{4} + \frac{\sqrt{3}}{4} + \frac{1}{4}$

$\frac{1+\sqrt{3}}{4}$

Q19. If A is square matrix, then A is symmetric, iff

A. $A^2 = A$

B. $A^2 = I$

C. $A^T = A$

D. $A^T = -A$

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Q20. If A is a square matrix, then A is skew symmetric iff-

A. $A^2 = A$

B. $A^2 = I$

C. $A^T = A$

D. $A^T = -A$

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Q21. If A is any square matrix, then

A. $A + A^T$ is ~~skew~~ symmetric sym

B. $A - A^T$ is ~~symmetric~~ skew

C. AA^T is symmetric

D. AA^T is skew symmetric

$(A + A^T)^T$

$A^T + (A^T)^T$

$A^T + A$

$A + A^T$

$=$

$(A - A^T)^T$

$A^T - (A^T)^T$

$A^T - A$

$-(A - A^T)$

$(AA^T)^T$

$(A^T)^T A^T$

AA^T

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Q22. Each diagonal element of a skew symmetric matrix is-

A. Zero

B. Positive and equal

C. Negative and equal

D. Any real number

$$a_{ii} = -a_{ii}$$

$$2a_{ii} = 0$$

$$\boxed{a_{ii} = 0}$$

$$-A = A^T$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

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Q23. Let A be the set of all 3 × 3 matrices which are symmetric with entries 0 or 1. If there are five I's and four 0's then number of matrices in A

A. 6

~~B. 12~~ ✓

C. 9

D. 58

let x be the no of 1's on main diagonal & y be the no of 1's above main diag^n

$x + 2y = 5$

$x = 1, y = 2$ or $x = 3, y = 1$

case $x = 1, y = 2$

3 3 → 9

case 2

1 → 3

$\boxed{9 + 3 = 12}$

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Q24. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals.

A. 52

B. 103

C. 201

D. 205

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 9 & 1 \end{bmatrix} = P$$

$$Q = [q_{ij}]$$

$$\boxed{P^{50} - I = Q}$$

$$\boxed{\frac{q_{31} + q_{32}}{q_{21}}}$$

$$P^{50}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 \cdot 2 & 1 & 0 \\ 16 \cdot 3 & 4 \cdot 2 & 1 \end{bmatrix}$$

$$P^3$$

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70 - 100

Q25

Q25. If A is a singular matrix, then adj A is

A. ~~Singular~~

B. Non-singular

C. Symmetric

D. Non defined

adj A = ?

$|A| = 0$

$|\text{adj } A| = 0$

A = → cofactor

$$\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} = \Delta$$

$|\Delta| = 0$

THANK - YOU