Branch : CSE & IT **Batch : Hinglish**

Algorithms

Introduction to Algorithms and Analysis

DPP

[MCQ]

1. Sort the functions in ascending order of asymptotic(big-O) complexity.

$f_1(n) = n$, $f_2(n) = 80$, $f_3(n) = n^{\log n}$, $f_4(n) = \log\log^2 n$, $f_5(n) = (\log n)^{\log n}$

(a) $f_2(n), f_4(n), f_1(n), f_5(n), f_3(n)$

(b) $f_2(n), f_1(n), f_4(n), f_5(n), f_3(n)$

(c) $f_2(n), f_1(n), f_4(n), f_3(n), f_5(n)$

(d) $f_1(n), f_1(n), f_4(n), f_3(n), f_2(n)$

[MCQ]

2. Consider two function $f(n) = 10n + 2\log n$ and $g(n) = 5n + 2(\log n)^2$, then which of the following is correct option?

(a) $f(n) = \theta(g(n))$

(b) $f(n) = O(g(n))$

(c) $f(n) = \omega(g(n^2))$

(d) None of the above

[MCQ]

3. Consider two function $f(n) = \sqrt{n}$ and $g(n) = n \log n + n$ then $f(n) / g(n)$ is equivalent to how many of the following given below? ________.

(i) $o(n^{-1/2})$

(ii) $O(n^{-1/2})$

(iii) $\Omega(1/\log n)$

(iv) $\theta(n^{-1/2})$

[MCQ]

4. Consider the following C-code

```
void foo (int x)
    {
      int a = 1;
      if (n = = 1)
      return;
      for (; a ≤ n; a++)
        {
          printf("GATEWALLAH");
          break;
        }
    }
```

What is the worst time complexity of above program?

(a) $O(1)$

(b) $O(n)$

(c) $O(\log n)$

(d) $O\sqrt{n}$

[MCQ]

5. Find the time complexity of the following summation, assume that k is constant, $k > 0$

$$\sum_{x=1}^{n} \sum_{y=x+1}^{n} \frac{1}{k}$$

(a) $O(n^2)$

(b) $O(n)$

(c) $O(n^3)$

(d) None of the above

[NAT]

6. How many of the following expressions correctly describes $T(n) = n\log(n^2)$? ______

(a) $\theta(n^2)$

(b) $O(n)$

(c) $\Omega(n)$

(d) $O(n^2)$

[MCQ]

7. Consider two function $f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$ then which of the following is true.

(a) $f_1(n) = (Of_2(n))$

(b) $f_1(n) = \theta(f_2(n))$

(c) $f_1(n) = \omega(f_2(n))$

(d) None of these

Answer Key

1. (a)
2. (a)
3. (2 to 2)
4. (a)
5. (a)
6. (2 to 2)
7. (c)

Hints & Solutions

1. (a)

$80 < n$

$\log \log^2 n < n$

put $n = 10^{100}$

$\log(\log n)^2 = 10^{100}$

$\log(100)^2 < 10^{100}$

$4 < 10^{100}$

$n < n^{\log n}$

taking log on both side

$\log n < \log n \log n$

we know that $(\log n)^2 > \log n$

now, $(\log n)^{\log n} < n^{\log n}$

as we can see that logn in LHS and n on RHS.

$n \quad (\log n)^{\log n}$

taking log on both sides

$\log n < \log n * \log \log n$

From above we conclude that growth of log*logn is higher than 1.

$\therefore$ option (a) is correct.

2. (a)

As we can see in above function, 'n' is the dominating factor in these 2 functions. Which means they also have similar growth rate.

$\therefore f(n) = O(g(n))$

Hence option (a) is correct.

3. (2 to 2)

$$\frac{f(n)}{g(n)} = h(n)$$

Given

$$f(n) = \sqrt{n}, g(n) = n\log n + n$$

$$= \frac{\sqrt{n}}{n\log n + n}$$

$$= \frac{\sqrt{n}}{\sqrt{n}\left(\sqrt{n}\log n + \sqrt{n}\right)}$$

$$= \frac{1}{\sqrt{n}\log n + \sqrt{n}}$$

and clearly $h(n) = O(n^{-0.5})$ and

$h(n) = o(n^{-0.5})$

NOTE: if small 'o' possible then Big 'O' is possible but if Big 'O' possible then small 'o' may or may not possible.

$\therefore$ (i) and (ii) are correct.

Hence 2 expressions are correct.

4. (a)

If we see carefully, loop will execute only one time because of break statement, therefore time complexity will be O(1)

5. (a)

$$\sum_{x=1}^{n}\sum_{y=x+1}^{n}\frac{1}{k} = \frac{1}{k}\sum_{x=1}^{n}\sum_{y=x+1}^{n}(1)$$

$$= \frac{1}{k}\sum_{x=1}^{n}\left[1+1+1+.....n-(x+1)+1 \text{ times}\right]$$

$$= \frac{1}{k}\sum_{x=1}^{n}[n-x]$$

$$= \frac{1}{k}\left[n\sum_{x=1}^{n}(1) - n\sum_{x=1}^{n}x\right] = \frac{1}{k}\left[n.n - \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{k}\left[n^2 - \frac{n^2+n}{2}\right] = \frac{1}{2k}\left[n^2 - n\right]$$

$= O(n^2)$

6. (2 to 2)

Given: $T(n) = n\log(n^2) = 2n\log n$

(i) $T(n) = \theta(n^2)$, which means the value of T(n) is exactly $\theta(n^2)$, but as we can see that T(n) is $n\log(n^2)$ so this is incorrect.

(ii) $T(n) = O(n)$: $T(n) \le k.n$, but value of T(n) is $n\log(n^2)$

So, this is also false.

(iii) $T(n) = \Omega(n)$

$T(n) >= k.n$ and the complexity given for T(n) is $n\log(n^2)$, so it is correct.

(iv) $O(n^2)$

$T(n) = O(n^2)$

$n\log(n^2) \le k.n^2$ which is correct.

Hence 2 expression out of 4 are correct.

7. (c)

$f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$

$n^{2^n} = n^{n^2}$

Taking log on both side

$2^n \log n$ $n^2 \log n$

as we can see that

2^n has more growth rate than

n^2 $\therefore$ we conclude

$f_2(n) < O(f_1(n))$ or

$f_1(n) = \omega(f_2(n))$

$\therefore$ (c) is correct.