# ENGINEERING MATHEMATICS

ALL BRANCHES

Linear Algebra

DPP – 03

Part – I Solution

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✓ Cholsky deco –
✓ LU decomo.

✓ DPP01 – Part one ✓ 1-20 question
Part Two – 20 – ( )

✓ Weekend Sunday TEST – Calculus –
7 – 2 marks.
6 – 1 marks.

#Q. Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

SUM = 3
det M = 0

The eigenvalues of M are

(a) 0, 1, 2 ✓

(b) $\boxed{0, 0, 3}$ ✓ sum = 3, det M = 0 ] r(M) = 1

(c) 1, 1, 1

(d) −1, 1, 3

$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ 3X3

Eigen value

sum of Diagonal
$= 1 + 1 + 1$
$= 3$

det M = 0 r(M) = 1

"If det M = 0 atleast one eigenvalue is zero

$\boxed{\text{eigenvalues} = 0, 0, 3}$ ✓

already weekly TEST

#Q. A 3 × 3 matrix M has Tr[M] = 6, Tr[$M^2$] = 26, Tr[$M^3$] = 90. Which of the following can be possible set of eigenvalues of M?

Trace = sum of Diagonal elements = sum of eigen values

$Tr(M) = 6$ $Tr(M^3) = 90$

$Tr(M^2) = 26$

(a) {1, 1, 4}

(b) {–1, 0, 7}

(c) ✓ $\boxed{\{-1, 3, 4\}}$

(d) {2, 2, 2}

$Tr(M) = -1 + 3 + 4 = 6$

$Tr(M^2) = (-1)^2 + (3)^2 + (4)^2 = 26$

$Tr(M^3) = (-1)^3 + (3)^3 + (4)^3 = 90$

#Q. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

det A = 0
r(A) = 1

Eigenvalues = sum of diagonal

= 1 + 4 + 9 = 14

(a) (1, 4, 9)

(b) (0, 7, 7)

(c) (0, 1, 13)

(d) $\boxed{(0, 0, 14)}$ ✓

✓ $\boxed{\text{eigenvalues} = 0, 0, 14}$

Key Pt - det A = 0 r(A) = 1

eigen = (0, 0, sum of eigenvalues) ✓

#Q. The eigenvalue of the anti- symmetric $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ where $n_1, n_2, n_3$ are components of an unit vector, are

X

odd order det = 0

$A = \begin{bmatrix} 0 & n_1 & n_2 \\ -n_1 & 0 & n_3 \\ -n_2 & -n_3 & 0 \end{bmatrix}$

Replace It

wrong-matrix

anti symmetric

eigen value

[A] = skew sym. matrix

characteristic equ$^n$

(a) 0, –i, i

(b) 0, 1, –1

(c) 0, 1 + i, –1 –i

(d) 0, 0, 0

$\lambda^3 - (\text{Trace})\lambda^2 + (A_{11} + A_{22} + A_{33})\lambda - \det A = 0$

$\lambda^3 - (0+0+0)\lambda^2 + (n_1^2 + n_2^2 + n_3^2)\lambda - 0 = 0$

$= \lambda^3 + (n_1^2 + n_2^2 + n_3^2)\lambda = 0$

$= \lambda^3 + \underline{(n_1^2 + n_2^2 + n_3^2)}\lambda = 0$

square It + Add

$\boxed{n_1^2 + n_2^2 + n_3^2 = \dfrac{n_1^2 + n_2^2 + n_3^2}{n_1^2 + n_2^2 + n_3^2} = 1}$ ✓

$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$

put the value $n_1^2 + n_2^2 + n_3^2 = 1$

$\Rightarrow \lambda^3 + 1\lambda = 0$

$\Rightarrow \lambda(\lambda^2 + 1) = 0$

$\Rightarrow \boxed{\lambda = 0 \quad \lambda = \pm i}$

Eigenvalues
$= 0, +i, -i$

If $n_1, n_2, n_3$ Are components of unit vector

Unit vector

square $\hat{n_1} = \dfrac{\vec{n_1}}{|\vec{n_1}|}$

$+ \quad \hat{n_2} = \dfrac{\vec{n_2}}{|\vec{n_2}|}$

$+ \quad \hat{n_3} = \dfrac{n_3}{|\vec{n_3}|}$

#Q. Consider a $n \times n$ $(n > 1)$ matrix A, in which $A_{ij}$ is the product of the indices i and j (namely $A_{ij} = ij$). The matrix A

Degenrate eigen value = Dependent eigen value

$a_{ij} = ij$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}_{3\times 3}$$

(a) Has one degenerate eigenvalue with degeneracy (n-1)

det A = 0

$r(A) = 1$

(b) Has two degenerate eigenvalues with degeneracies 2 and (n-2)

(c) Has one degenerate eigenvalues with degeneracy n

degenrate eigenvalues = Dependent eigen value

(d) Does not have any degenerate eigenvalues

$= [\boxed{0, 0}, (\text{SUM})]$

n eigenvalue
det A = 0
$r(A) = 1$

$\longrightarrow$ (n-1) Degeneracy

#Q. A 2 × 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $^{i\pi/6}$. The smallest value of 'n' such that $\boxed{A^n = I}$

(a) 20

(b) 30

(c) 60

(d) 120

$A = 2 \times 2$ eigenvalues: $\lambda_1 = e^{\frac{i\pi}{5}}$, $\lambda_2 = e^{\frac{i\pi}{6}}$

$A \longrightarrow e^{\frac{i\pi}{5}}, e^{\frac{i\pi}{6}}$

$A^n \longrightarrow e^{\frac{i\pi n}{5}}, e^{\frac{i\pi n}{6}}$

Product of eigen values = det A

$|A^n| = |I|$

$e^{\frac{i\pi n}{5}} \cdot e^{\frac{i\pi n}{6}} = 1$

$\Rightarrow e^{\frac{11 n\pi i}{30}} = 1$

$e^{\frac{11\cancel{\pi}\cancel{i}n}{30}} = e^{2m\cancel{\pi}\cancel{i}}$

$\frac{11n}{30} = 2m$

$\boxed{\frac{n}{m} = \frac{60}{11}}$

$n = 60$, $m = 11$

$A \to \lambda_1, \lambda_2$

$A^m = \lambda_1^m, \lambda_2^m$

$1 = e^{2m\pi i}$

$\cos\theta + i\sin\theta = e^{i\theta}$

$\cos 2\pi + i\sin 2\pi = e^{i\cdot 2\pi}$

Slide-6

#Q. Given a $2 \times 2$ unitary matrix satisfying $\boxed{U'U = UU' = I}$ with $\boxed{\det U = e^{i\phi}}$, one can construct a unitary matrix V ($V'V = VV' = I$) with $\boxed{\det V = 1}$ from it by

$U = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $ad - bc = e^{i\phi}$

$(\bar{V})^T U = I$

Unitary matrix

$\det U = e^{i\phi}$

(a) Multiplying U by $e^{-i\phi/2}$

(b) Multiplying a single element of U by $e^{-i\phi}$

(c) Multiplying any row or column by $e^{-i\phi/2}$

(d) Multiplying U by $e^{-i\phi}$

$V = Ve^{-\frac{i\phi}{2}}$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} e^{-\frac{i\phi}{2}}$

$= \begin{bmatrix} ae^{-\frac{i\phi}{2}} & be^{-\frac{i\phi}{2}} \\ ce^{-\frac{i\phi}{2}} & de^{-\frac{i\phi}{2}} \end{bmatrix}$

$V = e^{-i\phi/2} U$ (1)

$\boxed{\det V = 1}$ (1)

$V = e^{-i\phi} U$

$\det V = 1$ (2)

$$V = Ve^{-\frac{i\phi}{2}} = \begin{bmatrix} ae^{-\frac{i\phi}{2}} & be^{-\frac{i\phi}{2}} \\ ce^{-\frac{i\phi}{2}} & de^{-\frac{i\phi}{2}} \end{bmatrix}$$

$$V = e^{-\frac{i\phi}{2}} e^{-\frac{i\phi}{2}} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det V = e^{-i\phi} \underbrace{[ad - bc]}_{\det U}$$

$$= e^{-i\phi} e^{i\phi}$$

$$\boxed{\det V = 1}$$

$$\boxed{V = e^{-\frac{i\phi}{2}} U} \rightarrow \det V = 1$$

A

#Q. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are

(a) –5, –2, 7

(b) –7, 0, 7

(c) –4i, 2i, 2i

(d) 2, 3, 6

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}_{3\times 3}$$

[det M] = skew Hermitian matrix

odd order

$\boxed{\det M = 0}$ ✓

$\boxed{\text{eigen values} = -7, 0, 7}$

#Q. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigen vector of

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ if}$$

(a) $b = 0$ or $a = 0$

(b) $\boxed{b = a \text{ or } b = -2a}$

(c) $b = 2a$ or $b = -a$

(d) $b = a/2$ or $b = -a/2$

$AX = \lambda X$. A - matrix given

$\lambda$ = Not given

X = Vector given

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \lambda_1 \begin{bmatrix} a \\ b \\ a \end{bmatrix}$$

$$= \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \lambda_1 \begin{bmatrix} a \\ b \\ a \end{bmatrix} \quad \boxed{\lambda_1 = 1}$$

$$AX = \lambda X$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \lambda_2 \begin{bmatrix} a \\ b \\ a \end{bmatrix}$$

$$= \begin{bmatrix} a+b \\ 2a \\ a+b \end{bmatrix} = \lambda_2 \begin{bmatrix} a \\ b \\ a \end{bmatrix}$$

$$\boxed{\begin{aligned} a+b &= \lambda_2 a \quad — (1) \\ 2a &= \lambda_2 b \quad — (2) \end{aligned}}$$

$$a+b = \lambda_2 a$$
$$2a - \lambda_2 b = 0$$

$$a(1-\lambda_2) + b = 0 \times \lambda_2$$
$$2a - \lambda_2 b = 0 \times 1$$

$$a\lambda_2(1-\lambda_2) + \cancel{b\lambda_2} = 0$$
$$2a - \cancel{\lambda_2 b} = 0$$

$$a\lambda_2(1-\lambda_2) - 2a = 0$$

$$a\lambda_2 - a\lambda_2^2 - 2a = 0$$
$$-a\lambda_2 + a\lambda_2^2 + 2a = 0$$
$$\lambda_2^2 - \lambda_2 + 2 = 0 \qquad \boxed{\lambda_2 = 2, -1}$$

Put $\lambda_2 = 2$, $-1$

$$a+b = 2a$$
$$b = a$$
$$\boxed{2a = -b}$$

#Q. Which one of the following is the inverse of tha matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{1-0}\begin{bmatrix} 1 & +1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad c$$

Slide-14

#Q. A 3 × 3 matrix has eigenvalues 0, 2 + i and 2 − i. Which one the following statements is correct?

$\lambda_1 = 0$

$\lambda_2 = 2+i$

$\lambda_3 = 2-i$

(a) The matrix is hermitian

(b) The matrix is unitary

(c) The inverse of the matrix exists

(d) The determinant of the matrix is zero ✓

det of matrix A

= product of eigen values

$= 0 \times (2+i) \times (2-i)$

$= 0$ ✓

Traceless = 0 Trace

#Q. A real traceless 4 × 4 unitary matrix has two eigen values –1 and 1. The other eigenvalues are

$$\lambda_1+\lambda_2+\underbrace{\lambda_3+\lambda_4} = \underbrace{-1+1}_{0}+\underbrace{\lambda_3+\lambda_4}_{\underbrace{-1+1}_{0}}$$

$= 0$

$=$ Traceless

(a) zero and +2

(b) –1 and +1

(c) zero and +1

(d) +1 and +1

$$\left.\begin{array}{l}\lambda_3 = -1\\ \lambda_4 = +1\end{array}\right]$$

Slide-16

#Q The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

(a) +1 and +1

(b) Zero and +1

(c) Zero and +2

(d) –1 and +1

$A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ $\quad i^2 = -1$

$= \lambda^2 - (1+1)\lambda + (1 + i^2) = 0$

$= \lambda^2 - 2\lambda + 0 = 0$

$= \lambda^2 - 2\lambda = 0$

$= \lambda(\lambda - 2) = 0$

$= \boxed{\lambda = 0 \quad \lambda = 2}$ $\quad$ C

$3\times3 = 3\times3$ ✓

$|\text{Adj}\,A| = |A|^{n-1}$ $\quad (\text{adj}\,A^{-1}) = \left(\frac{1}{|A|}\right)^{n-1}$

#Q If A is 2×2 matrix with determinant 2, then the determinant of adj.[adj.[adj($A^{-1}$)]] is equal to

$|\text{Adj}(A^{-1})| = \left(\frac{1}{2}\right)^{3-1}$

$|\text{adj}(\text{adj}(A^{-1}))| = \left(\frac{1}{2^2}\right)^2$

$|\text{adj}(\text{adj}(\text{adj}(A^{-1})))| = \left(\frac{1}{4^2}\right)^2$

$= \frac{1}{256}$

(a) 1/512

(b) 1/1024

(c) 1/128

(d) 1/256

#Q The eigenvalues of $(A^4 + 3A - 2I)$, where A is $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ are

eigenvalues $= 1, 2, 3$

(a) 2,20,88

(b) 1,2,3

(c) 2,20,3

(d) 1,20,88

$A^4 + 3A - 2I \longrightarrow \lambda = 1 = (1)^4 + 3\times 1 - 2$

$= 2$

$\rightarrow \lambda = 2 = (2)^4 + 2\times 3 - 2$

$= 20$

$\rightarrow \lambda = 3 = (3)^4 + 2\times 3 - 3$

$= 88$

#Q. Eigen value of matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are

(a) –2, –1, 1, 2

(b) –1, 1, 0, 2

(c) 1, 0, 2, 3

(d) –1, 1, 0, 3

eigen value =

sum of eigen value = Trace $= \underline{0}$

$\lambda_1 = -2$

$\lambda_2 = -1$

$\lambda_3 = 2$

$\lambda_4 = 1$

Slide-20

#Q. A linear transformation T, defined as $T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1+x_2\\x_2-x_3\end{pmatrix}$, Transform vector $\vec{x}$ from a three dimensional space to a two-dimensional real space. The transformation matrix T is

$$T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{bmatrix}x_1+x_2\\x_2-x_3\end{bmatrix}$$

(a) $\begin{pmatrix}1 & 1 & 0\\0 & 1 & -1\end{pmatrix}$ T

(b) $\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\end{pmatrix}$ T

$$\begin{bmatrix}1 & 1 & 0\\0 & 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}x_1+x_2\\x_2-x_3\end{bmatrix}$$

(c) $\begin{pmatrix}1 & 1 & 1\\-1 & 1 & 1\end{pmatrix}$ T

(d) $\begin{pmatrix}1 & 0 & 0\\0 & 0 & 1\end{pmatrix}$ T

# Thank You !