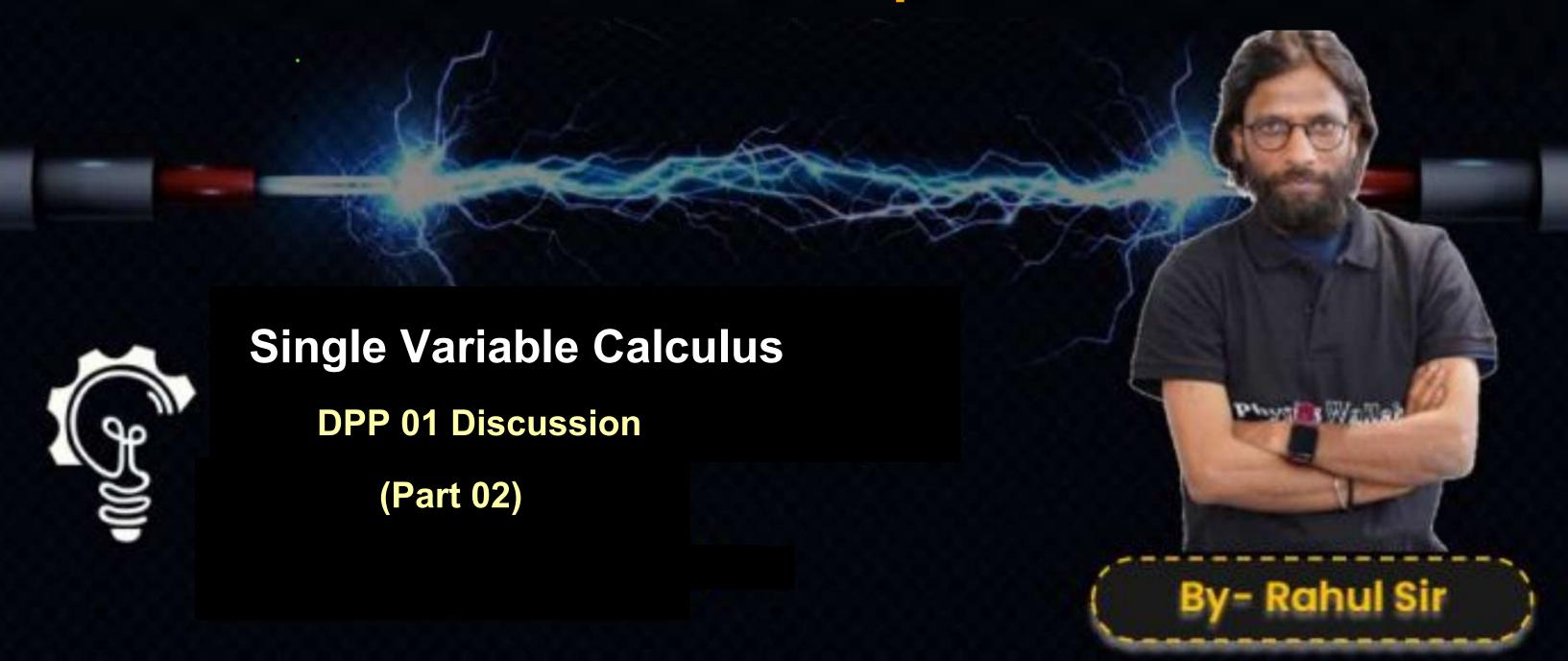
# Data Science & Artificial Intelligence Calculus and Optimization





Q21. If 
$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{x}}{2}$$
, then  $\int_0^\infty e^{-ax^2} dx$  where  $a > 0$  is

$$\int_0^{\infty} e^{-ax^2}$$

Gamma Function

(a) 
$$\frac{\sqrt{\pi}}{2}$$

(b) 
$$\frac{\sqrt{\pi}}{2a}$$

(c) 
$$2\frac{\sqrt{\pi}}{a}$$

(d) 
$$\frac{1}{2}\sqrt{\pi/a}$$

 $T = \int_{0}^{\infty} e^{-\alpha x^{2}}$   $\alpha x^{2} = +$ a.2xdx=dt dx=/dt Function



## Q22. The expression $\frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ is equal to

$$I_1 = \int_0^{\pi} [x] dx$$

$$I_2 = \int_0^{\pi} [x] dx$$

(where [\*] and {\*} denotes greatest integer function and fractional part function and  $n \in N$ ).

$$= \int_{0}^{3} dx + \int_{1}^{3} dx + \int_{2}^{3} dx + \int_{3}^{3} dx + \dots + \int_{3}^{3} (\eta - 1) dx$$

$$= \frac{1}{3} \int_{0}^{3} dx + \int_{1}^{3} dx + \int_{3}^{3} dx + \dots + \int_{3}^{3} (\eta - 1) dx$$

(c) 
$$n$$
  
(d)  $n-1$ 

$$= 0+|(2-1)+2(3-2)+\cdots+(N-1) turns$$

$$I_{2} = \int_{0}^{n} \{x\} dx = \int_{0}^{n} x - [x] dx$$

$$= \int_{0}^{n} (x - 0) dx + \int_{0}^{2} (x - 1) dx + \dots + (x - [n - 1) dx$$

$$= \frac{\pi}{2} - \frac{\pi(n - 1)}{2}$$

$$= \frac{\pi}{2} - \frac{\pi^{2} + \pi}{2}$$

$$I_{2} = (n - n^{2})$$

$$= (n - n^{2})$$

$$= \frac{\pi}{2} - \frac{\pi^{2} + \pi}{2}$$

$$= \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2}$$



Q23. Let 
$$A = \int_0^1 \frac{e^t dt}{1+t} dt$$
 then  $\int_{a-1}^a \frac{e^{-t}}{t-a-1} dt$  has the value

- (a) Ae-a
- (b) -Ae-a
- (c) -ae-a
- (d) Aea

Mampulate 
$$T = - \begin{cases} 0 & (3-a) \cdot (-d3) \\ 1+3 & = - \end{cases}$$

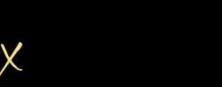
$$= - \begin{cases} 1 & e^{3} \cdot e^{-a} & (-d3) \\ 1+3 & = - \end{cases}$$

$$= - \begin{cases} 1 & e^{3} \cdot e^{-a} & (-d3) \\ 1+3 & = - \end{cases}$$

$$= - \begin{cases} 1 & e^{3} \cdot e^{-a} & (-d3) \\ 1+3 & = - \end{cases}$$

$$f(2a-x)=f(x)$$

$$f(2a-x)=f(x)$$



Q24. 
$$\int_0^{\pi} xf(\sin x)dx$$
 is equal to

$$(T) \rightarrow (T \mid T - x) f(sm \mid T - x)$$

Q24. 
$$\int_{0}^{\pi} xf(\sin x)dx \text{ is equal to}$$

$$\int_{0}^{\pi} (x)dx \text{ is equal to}$$

$$\int_{0}^{\pi} (x)dx = \int_{0}^{\pi} (x)dx$$

$$\pi \int_{0}^{\pi/2} \pi \int_{0}^{\pi/2}$$

(c) 
$$\pi \int_{0}^{\pi/2} f(\cos x) dx$$

(d) 
$$\pi \int_0^{\pi} f(\cos x) dx$$



### Q25. Let $f: R \to R$ be a differentiable function having

f(2) = 6, f'(2) = 
$$\left(\frac{1}{48}\right)$$
. Then  $\lim_{x\to 2} \int_{6}^{f(x)} \frac{4t^3}{x-2} dt$  equals

Newton Leibritz Rule

(a) 18
(b) 12 
$$t_{x} = \frac{1}{(x-2)} \int_{x}^{x} \frac{1}{(x-2)} dx$$

(d)

24

Lt 
$$(x)^{3}$$
.  $f'(x)$ 

$$= 4[f(2)]^{3} \cdot f'(2) = 4[f(2)]^{3} \cdot f'(2$$



$$\frac{2 \left( \sin x + \frac{1+s}{\sqrt{1+s}} \right)}{\sqrt{1+s}}$$

$$\sqrt{\frac{1+s}{\sqrt{1+s}}}$$

$$\sqrt{\frac{1+s}{\sqrt{1+s}}}$$

$$\sqrt{1+\sin 2x}$$

$$\sqrt{1+\sin 2x}$$

$$\sqrt{1+\sin 2x}$$

$$\sqrt{1+\sin 2x}$$

$$\sqrt{1+\sin 2x}$$

$$\sqrt{1+\sin 2x}$$

$$\sqrt{1+\cos 2x}$$

$$\sqrt{1+\cos 2x}$$

Q26. The value of 
$$\int_{0}^{\pi/2} \frac{(\sin x + \cos x)^{2}}{\sqrt{1 + \sin 2x}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}} dx \text{ is } = \int_{0}^{\pi} \frac{(\sin x + \cos x)^{2}}{\sqrt{(\sin x + \cos x)^{2}}$$

$$3\int_{0}^{T} (8n \times + (8x)) dx = 2$$

$$(d)$$
 3



Q27. If 
$$f(a + b - x) = f(x)$$
, then  $\int_a^b x f(x) dx$  is equal to using this Parkerty

(a) 
$$\frac{a+b}{2} \int_{a}^{b} f(b-x) dx$$

(a) 
$$\frac{a+b}{2} \int_a^b f(b-x) dx$$
 (b) 
$$\frac{a+b}{2} \int_a^b f(x) dx$$

(c) 
$$\frac{b-a}{2} \int_{a}^{b} f(x) dx$$

(d) 
$$\frac{a+b}{2} \int_a^b f(a+b+x) dx$$

Q28. The value of 
$$\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t}{x \sin x} dt$$
 is

$$\frac{\int_{x}^{\sec^2 t} dt \text{ is}}{\int_{x}^{\cot x} dt} = \frac{\int_{x}^{\cot x} dx}{\int_{x}^{\cot x} dx}$$

(d)

$$\begin{array}{l}
\text{It} \quad \frac{\text{Su} \times 1.2 \times 1}{\text{2 su} \times 10} \\
\text{It} \quad \frac{2 \text{Su} \times 1}{\text{2 su} \times 1} = \text{It} \quad \frac{2 \text{su} \times 1}{\text{2 su} \times 1} \\
\text{It} \quad \frac{2 \text{Su} \times 1}{\text{2 su} \times 1} = \text{It} \quad \frac{2 \text{su} \times 1}{\text{2 su} \times 1} = \text{It}
\end{array}$$

$$= \text{It} \quad \frac{\text{2 su} \times 1}{\text{2 su} \times 1} = \text{It}$$

$$= \text{It} \quad \frac{\text{2 su} \times 1}{\text{2 su} \times 1} = \text{It}$$

Q29. 
$$\int_{\sin x}^{1} t^{2} f(t) dt = 1 - \sin x \ \forall \ x \in (0, \pi/2), \text{ then } f\left(\frac{1}{\sqrt{3}}\right) \text{ is}$$
alphy L-Hospital Rule

(a) 
$$3 = 1 + (1) \cdot 0 + 6m \times + (8m \times) \cos x = 0 + cm \times$$

(d) None of these

$$\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} = \left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$$

Q30. 
$$\lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$$
 is equal to

O 
$$\frac{70}{70}$$
 = It  $\frac{\cos 2x}{2x}$  =  $\frac{1}{x \cos x + \sin x}$  =  $\frac{1}{x \cos x + \sin x}$ 

(d) 
$$-2$$

Q31. If 
$$\int_{\ln 2}^{x} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$$
, then x =

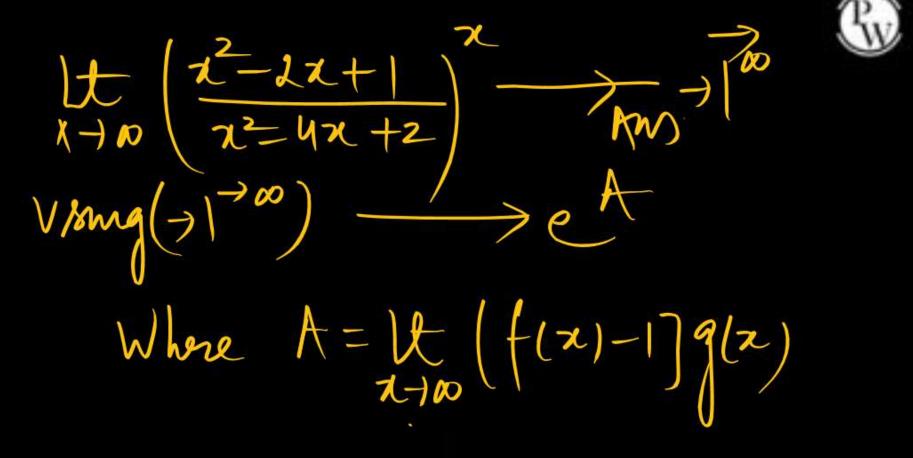
$$= \frac{2t}{(t^2+1)} \frac{4t}{4t}$$

$$= \frac{1}{(2\pi)^{2}} = \frac{$$

$$2[tan' \sqrt{e^2-1} - tan'] = \pi$$
= 2 [tan'  $\sqrt{e^2-1} - \pi$ ] =  $\pi$ 

Q32. 
$$\underset{x\to\infty}{Limit} \left( \frac{x^2 - 2x + 1}{x^2 - 4x + 2} \right)^x =$$

(c) 
$$e^2$$





Q33. If  $\alpha$  and  $\beta$  be the roots of  $ax^2 + bx + c = 0$  then

$$\lim_{x \to \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x - \alpha}}$$
is
$$\lim_{x \to \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x - \alpha}}$$

(b) 
$$\ln |a(\alpha - \beta)|$$

(c) 
$$e^{a(\alpha - \beta)}$$

(d) 
$$e^{a\alpha - \beta}$$

(a)

$$Any = e^{a(\alpha-\beta)}$$

$$ax^{2}+bx+c=0$$
 $a(x-d)(x-p)=0$ 
This Resta

$$\frac{1}{2^{-\cos x}-1}$$

$$\lim_{x \to \pi/2} \frac{2^{-\cos x} - 1}{x(x - \pi/2)} =$$

$$\lim_{x \to \pi/2} \frac{2^{-\cos x} - 1}{x(x - \pi/2)} =$$

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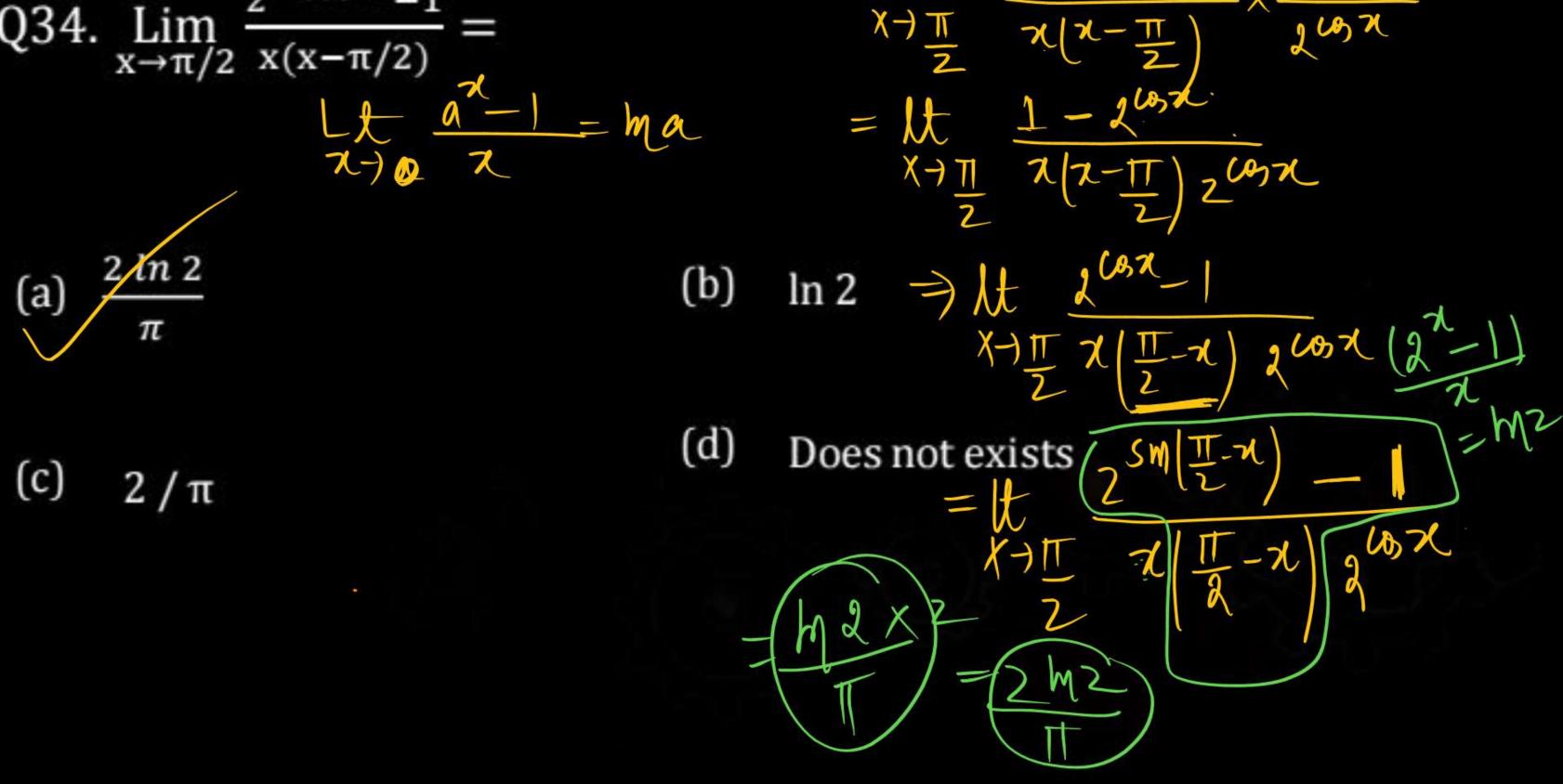
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$$\lim_{x \to \pi/2} \frac{2^{-\cos x} - 1}{x(x - \pi/2)} =$$

$$=\frac{1}{2} + \frac{1}{2} + \frac{1$$





Q35. Limit 
$$\frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$$
 is

- (a) 1/16
- (b) -1/16
- (c) /1/32
- (d) -1/32

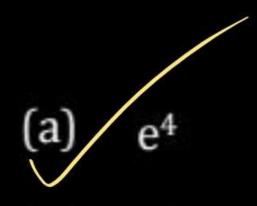
Q36. 
$$\lim_{x\to 0} (\cos m x)^{\frac{n}{x^2}}$$
 $\lim_{x\to 0} (\cos m x)^{\frac{n}{x^2}}$ 
 $\lim_{x\to 0} (\cos m x)^{\frac{n}{x^2}} = \lim_{x\to 0} \lim_$ 

Q37. 
$$\lim_{x\to 0} \frac{(4^x-1)^3}{\sin(\frac{x}{p})\ln(1+\frac{x^2}{3})} =$$

(b) 
$$3 p (log 4)^3$$



Q39. 
$$\lim_{x\to\infty} \left(\frac{x^2+5x+3}{x^2+x+3}\right)^x$$
 is equal to



- (b) e<sup>2</sup>
- (c)  $e^3$
- (d) e

$$\begin{aligned}
\text{It} \left(\frac{\chi^2 + 5\chi + 3}{\chi^2 + \chi + 3}\right)^{\chi} & = 0 \\
\text{Where} &= A = \text{It} \left[f(\chi) - 1\right]g(\chi) \\
(Am) &= e^{4}
\end{aligned}$$



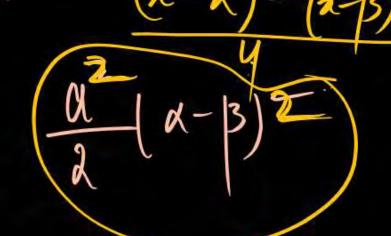
#### Q40. Let $\alpha$ and $\beta$ be the distinct roots of $ax^2 + bx + c = 0$ ,

Then 
$$\lim_{x\to\alpha} \frac{1-\cos(\alpha x^2+bx+c)}{(x-\alpha)^2}$$
 is equal to 
$$\lim_{x\to\alpha} \frac{1-\cos(\alpha x^2+bx+c)}{(x-\alpha)^2}$$

$$\frac{1-(x-x)^2}{(x-x)^2}$$

(a) 
$$\frac{1}{2}(\alpha-\beta)^2$$

(b) 
$$-\frac{a^2}{2}(\alpha - \beta)^2$$

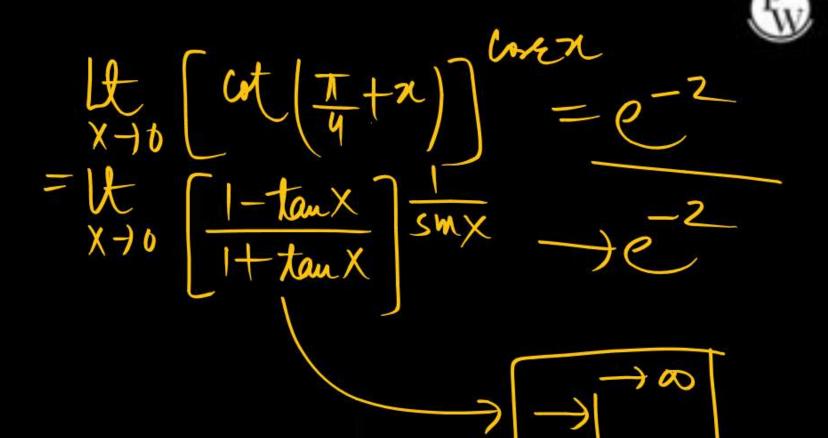


(d) 
$$\frac{a^2}{2}(\alpha-\beta)^2$$

$$Q41.\lim_{x\to 0} \left(\cot\left(\frac{\pi}{4} + x\right)\right)^{\cos ecx} =$$

(b) 
$$e^2$$

(c) 
$$/e^{-2}$$



$$60 \qquad 770 \qquad 471 \qquad x70$$

Q42. 
$$\lim_{x\to\infty} \left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)^{\otimes}\right)$$
 is
$$= \lim_{y\to 0} \left[\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)^{\otimes}\right] + \lim_{y\to 0} \left[\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x$$

$$Ans = e^A$$
 $A = 1$ 

Rv

Q43. 
$$\lim_{\substack{x \to \infty \\ \text{Incos}(2x^2-x)}} \frac{\sin(6x^2)}{\ln(6x^2)} = \frac{\text{L-Hospital Rule}}{\ln(6x^2)} - \text{Two times}$$

$$L = \lim_{x \to \infty} \frac{\sin(6x^2)}{\ln(6x^2)} = -|2|$$

$$x \to 0 \quad \text{mes}(2x^2-x) = -|2|$$

- (a) 12
- (b) -12
- (c) 6
- (d) -6



$$Q44.\lim_{x\to 0}\frac{e^{-x^2/2}-\cos x}{x^3\sin x}=\text{Two Times} \quad \text{Vsing $L$-Hospital} \\ L=\frac{1}{12}$$

- (a) 1/4
- (b) 1/6
- (c) / 1/12
- (d) 1/8

Verbritz Rule:  $f(x) = \begin{pmatrix} y(x) \\ f(t)dt \end{pmatrix}$ The Integral  $f(x) = \begin{cases} x \\ \text{sunt } dt \end{cases}$  $= 8m\chi^{3}.3x^{2}-8m\chi^{2}.2x$ 

dafin)=(d) f(t) dt > f(4/2) イルリス)ー  $f[p(x)]\frac{d}{dx}[p(x)]$ The Integral

