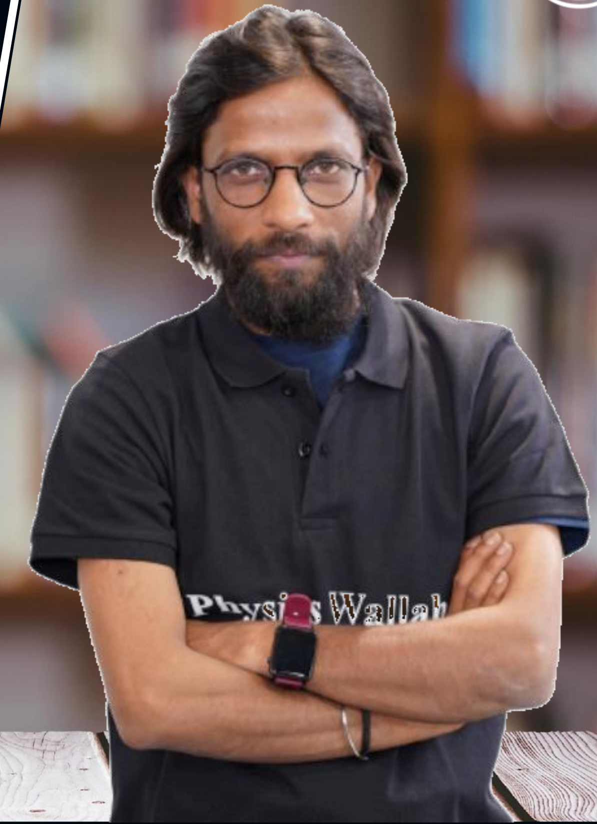
GATE-DATA SCIENCE AND AI

Linear Algebra

Lecture No.- Discussion (02)



By- Rahul sir

$\underline{\underline{\alpha, \beta, \gamma}}$

$(\alpha, 3, -2) = \alpha(0,1,0) + \beta(1,2,-1) + \gamma\underline{\underline{(1,1,-2)}}$

$\underline{\underline{\alpha, \beta, \gamma}}$ ✓

(c) $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix} = \alpha\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} + \beta\begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} \cancel{3} & \cancel{1} \\ \underline{0} & \underline{4} \end{pmatrix} = \begin{pmatrix} \cancel{\alpha} & -\alpha \\ \underline{0} & \underline{2\alpha} \end{pmatrix} + \begin{pmatrix} \cancel{0} & -2\beta \\ \underline{0} & \underline{\beta} \end{pmatrix}$ (2) ✓

Slide

Topic : Linear Algebra

(x,y) $x,y \in R$

#Q. (a) In R^2, express the vector (2,4) as a linear combination of the vector (0,3) and (2,1)

(b) In R^3, express the vector (2, 3, –2) as a linear combination of the vectors (0, 1, 0), (1, 2, –1) and (1,1, –2).

(c) In M2,2, express the matrix $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$ as a linear combination of the matrices $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ and $\begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix}$.

$(2,4) \longrightarrow (0,3)$ & $(2,1)$

Let's say α & β

$(2,4) = \alpha(0,3) + \beta(2,1)$

$2 = \alpha \cdot 0 + 2 \cdot \beta$ —— (I)

$4 = 3 \cdot \alpha + 1 \cdot \beta$ —— (II)

$\boxed{\beta = 1}$

$4 = 3\alpha + 1$

$3 = 3\alpha$

$\boxed{\alpha = 1}$

$(2,4) = 1(0,3) + 1 \cdot (2,1)$

Slide

Topic : Linear Algebra

#Q. (a) In $\mathbb{R}^2$, let $v_1 = (0, 3)$ and $v_2 = (2,1)$. Calculate the linear combination $4v_1 - 2v_2$.

(b) In $\mathbb{R}^4$, let $v_1 = (1, 2, 1, 3)$ and $v_2 = (2, 1, 0, -1)$. Calculate the linear combination $3v_1 + 2v_2$.

$$4(0,3) - 2(2,1)$$

$$(4, 12) - (4, 2)$$

$$(0, 10)$$

R^4

$$3\begin{pmatrix}1 & 0 & 2\\ 0 & -1 & 3\end{pmatrix} - 2\begin{pmatrix}2 & -1 & 0\\ 0 & 3 & -4\end{pmatrix} + \begin{pmatrix}-1 & 0 & 0\\ 0 & 2 & 1\end{pmatrix}$$

#Q. For each of the following vector spaces V and vectors v_1, v_2 and v_3 in V, form the linear combination $3v_1 - 2v_2 + v_3$.

(a) $V = P_3$, $v_1 = 1 + x + x^2$, $v_2 = 1 - x$, $v_3 = x + x^2$.

(b) $V = M_{2,3}$, $v_1 = \begin{pmatrix}1 & 0 & 2\\ 0 & -1 & 3\end{pmatrix}$, $v_2 = \begin{pmatrix}2 & -1 & 0\\ 0 & 3 & -4\end{pmatrix}$, $v_3 = \begin{pmatrix}-1 & 0 & 0\\ 0 & 2 & 1\end{pmatrix}$.

$3v_1 - 2v_2 + v_3$

P_3

$(1, 0, 0)$ $(0, 1, 0)$ $(0, 0, 1)$

x^2 x 1

$V_1 = \underline{\underline{1+x+x^2}}$

$\underline{\underline{x^2, x, 1}}$

$\underline{\underline{(1, 1, 1)}}$; $(0, -1, 1)$; $\underline{\underline{(1, 1, 0)}}$

$3V_1 - 2V_2 + V_3$

$3(1, 1, 1) - 2(0, -1, 1) + (1, 1, 0)$

$(3, 3, 3) + (0, 2, -2) + (1, 1, 0)$

$(4, 6, 1) \longrightarrow \underline{\underline{4x^2 + 6x + 1}}$

Slide

#Q. (a) Given the basis $E = \{(1,2), (-3, 1)\}$ for $\mathbb{R}^2$, determine the standard coordinate representation of $(2, 1)_E$.

(b) Given the basis $E = \{(1, 0, 2), (-1, 1, 3), (2, -2, 0)\}$ for $\mathbb{R}^3$, determine the standard coordinate representation of $(1, 1, -1)_E$.

$$(2,1)_E = 2(1,2) + 1(-3,1)$$
$$= (2,4) + (-3,1)$$
$$= \underline{\underline{(-1,5)}}$$

$$1(1,0,2) + 1(-1,1,3) - 1(2,-2,0)$$
$$= (-2,3,5)$$

Topic : Linear Algebra

$x \in V$
$y \in V$

$x+y$

#Q. Show that neither of the following sets is a real vector space.

(a) $V = \{(x, y) \in \mathbb{R}^2 : y = 2x + 1\}$

(b) $V = \left\{\begin{pmatrix} 0 & a \\ b & c \end{pmatrix} : a, b, c \in \mathbb{Z}\right\}$

$y = 2x + 1$

$(x_1, y_1) \in V$

$(x_2, y_2) \in V$

$(x_1 + x_2, y_1 + y_2) \in V$

$y_1 = 2x_1 + 1$ — (I)

$y_2 = 2x_2 + 1$ — (II)

$y_1 + y_2$
$= 2(x_1 + x_2) + 1$

zero element $\boxed{0}$

$y_1 + y_2 = 2(x_1 + x_2) + 2$ 1 $\cancel{1}$

closed

V.S

$V = \begin{pmatrix} 0 & a \\ b & c \end{pmatrix}$ $a, b, c \in \mathbb{Z}$

Multy   Identity

$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 1, 1   1

Slide

Real

#Q. Show that neither of the following sets is a real vector space.

(a) V = {all polynomials of degree equal to 5}

(b) $V = \{a + bi \in \mathbb{C} : a \geq 0\}$

v.s

identity element under add^n

identity

0

=

equal to 5

$a+bi \quad ; \quad a \geq 0$

V.S

$\alpha \in F$

$v \in V.S$

-1

$\alpha = -1$

$\times \quad \boxed{\alpha v \in V.S}$

$v = -a - bi \quad \times$

$a \geq 0$

$-a \leq 0$

Slide

Topic : Linear Algebra

L.I subset

#Q. In each case, determine whether set S of matrices is a linearly independent subset of $M_{2,2}$.

$M_{2\times 2}$

subset

$A \subseteq B$

L.I (a) $S = \left\{\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix}\right\}$

(b) $S = \left\{\begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 0 & -4 \end{pmatrix}\right\}$

$\alpha - \beta = 0$

$\boxed{\alpha = \beta}$

$\alpha\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix} + \beta\begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$2\alpha + \beta = 0$

$\alpha = 0$

$\begin{pmatrix} 0 & \cancel{\alpha} \\ 0 & \cancel{2\alpha} \end{pmatrix} + \begin{pmatrix} 0 & \cancel{-\beta} \\ -2\beta & \cancel{\beta} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$v_1 = \begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} -1 & 2 \\ 0 & -4 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix} + \beta \begin{pmatrix} -1 & 2 \\ 0 & -4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\alpha \\ 0 & 2\alpha \end{pmatrix} + \begin{pmatrix} -\beta & 2\beta \\ 0 & -4\beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$-\alpha + 2\beta = 0$$

$$\underline{\underline{\beta = 0}}$$

$$\boxed{\alpha = 0}$$

$$\underline{\underline{\beta = 0}}$$

Slide

#Q. Show that each of the following is a spanning set for $\mathbb{R}^2$.

(a) $\{(1, 1), (-1, 2)\}$

(b) $\{(2, -1), (3, 2)\}$

R^2

span set

(1, 1) (-1, 2)

element R^2

Each vector (x, y)

(x, y) $\{(1,1) ; (-1,2)\}$

α & β

(x, y) in R^2

$(x, y) = \alpha(1,1) + \beta(-1,2)$

$(x, y) = (\alpha - \beta, \alpha + 2\beta)$

$\alpha - \beta = x$ — (I)

$\alpha + 2\beta = y$ — (II)

$\alpha = \frac{1}{3}(2x+y)$

$\beta = \frac{1}{3}(y-x)$

R^2 $(1,1)$ & $(-1,2)$ α & β

Slide

α, β, γ

$\mathbb{R}^3$

#Q. Show that $\{(1, 0, ,0), (1, 1, 0)(2, 0, 1)\}$ is a spanning set for $\mathbb{R}^3$.

~~The following worked exercise shows that Strategy C6 can be used for vector spaces other than $\mathbb{R}^2$ and $\mathbb{R}^3$.~~

R^2 & R^3 ✗ α, β, γ

$$(x, y, z) = \alpha(1, 0, 0) + \beta(1, 1, 0) + \gamma(2, 0, 1)$$

$$(\alpha + \beta + 2\gamma, \beta, \gamma) = (x, y, z)$$

$\gamma = z, \quad \beta = y$

$$\boxed{\alpha = x - 2z - y}$$

$1 \cdot a + x^2$

$1 \cdot x + x^2$

a^2

x = S

#Q. For each of the following, determine whether the set S is a subspace of the vector space V.

V.S

(a) $V = P_3$, $S = \{a + bx \ : a, b \in \mathbb{R}\}$

(b) $V = P_3$, $S = \{x + ax^2 \ : a, b \in \mathbb{R}\}$ =

S V

(c) $V = M_{2,2}$, $S = \left\{\begin{pmatrix} a & 1 \\ 0 & d \end{pmatrix} : a, b \in \mathbb{R}\right\}$ =

S V

P_3 $a + bx$

$S \rightarrow$ v.s

under v.s

$P_3 = \quad 0 + 0x + 0x^2 = 0$

$0 + 0 \cdot x + 0x^2$

$a + ax^2$

$a + bx$

$a = b = 0$

$1 \cdot x + \cancel{ax^2}$

$\boxed{a > 0}$

$1 \cdot x = 1 = 0$

S

$\boxed{P_3 = ax^2 + x}$

0

Slide

#Q. In each of the following cases, determine whether S is a linearly independent subset of the vector space V.

(a) V = P_3, S = $\{\cancel{1}, \cancel{x}, \cancel{x^2}, \cancel{x^3}, 1 + x + x^2 + x^3\}$ =

(b) V = $M_{2,2}$, $S = \left\{\begin{pmatrix}1 & 2\\ 0 & -1\end{pmatrix}, \begin{pmatrix}1 & 0\\ -1 & 2\end{pmatrix}\right\}$ =

(c) V = $M_{2,2}$, $S = \left\{\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}, \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right\}$ =

(d) V = $\mathbb{C}$, S = $\{1 + i, 1 - i\}$

$(1 + x + x^2 + x^3)$

$= 1 + 1\cdot x + 1\cdot x^2 + 1\cdot x^3$

LD

$v_5 = v_1 + v_2 + v_3 + v_4$

$$\alpha\begin{pmatrix}1 & 2\\ 0 & -1\end{pmatrix}+\beta\begin{pmatrix}1 & 0\\ -1 & 2\end{pmatrix}$$

$$\begin{pmatrix}\alpha+\beta & 2\alpha\\ -\beta & -\alpha+2\beta\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

LI

$2\alpha=0$

$\alpha=0$

$\alpha+\beta=0$

$$\boxed{\alpha=0=\beta}$$

$(1+i, 1-i)$ LOI

Slide

(x, y)

#Q. Show that each of the following is a spanning set for $\mathbb{R}^2$.

(a) $\{(1, 2), (2, -3)\}$

(b) $\{(1,0), (1,1), (1, -2)\}$

α, β, γ

x, y, z

α, β, γ

$(x, y) = \alpha(1, 2) + \beta(2, -3)$

α & β $\Rightarrow$ x & y

#Q. Show that $\{1 + x, 1 + x^2, 1 + x^3, x\}$ is a spanning set for P_4.

P_4

P_4 $\quad a + bx + cx^2 + dx^3$

a, b, c, d

$a + bx + cx^2 + dx^3 = \alpha(1+x) + \beta(1+x^2) + \gamma(1+x^3) + \delta(x)$

$\alpha, \beta, \gamma, \delta \rightarrow a, b, c, d$

x, y, z

#Q. (a) Verify that {(3,4,0), (8,–6,0), (0,0,5)} is an orthogonal basis for $\mathbb{R}^3$.

$v_1 \quad v_2 \quad v_3$

(b) Express the vector (10,0, 4) in terms of this basis.

$v_1 \cdot v_2 = 0$

$v_2 \cdot v_3 = 0$

$v_1 \cdot v_3 = 0$

Basis

LI — span $\mathbb{R}^3$

$\alpha v_1 + \beta v_2 + \gamma v_3 = 0$

$\boxed{\alpha = \beta = \gamma = 0}$

$(10, 0, 9) = \alpha_1 \underline{\quad v_1 \quad} + \alpha_2 \underline{\quad v_2 \quad} + \alpha_3 \underline{\quad v_3 \quad}$

$$\boxed{\alpha_1 = \frac{v_1 \cdot \vartheta}{v_1 \cdot v_1}} \qquad \boxed{\alpha_2 = \frac{v_2 \cdot \vartheta}{v_2 \cdot v_2}}$$

$$\vartheta = (10, 0, 9) \qquad \boxed{\alpha_3 = \frac{v_3 \cdot \vartheta}{v_3 \cdot v_3}}$$

$\alpha_1 = 6/5 \qquad \alpha_2 = 4/5 \qquad \alpha_3 = 4/5$

Slide

$M_{2\times 2}$

#Q. For each of the following vector spaces V and sets of vector S in V, determine ⟨S⟩.

(a) V = $\mathbb{R}^3$, S = {(1, 0, 0)}.

(b) $V = M_{2,2}, S = \left\{\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\right\}$

$V = \mathbb{R}^3$

$S = \{(1,0,0)\}$

$\langle S \rangle = \{\alpha(1,0,0) \quad \alpha \in \mathbb{R}\}$

$\langle S \rangle = \alpha\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} + \beta\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$

$\alpha, \beta \in \mathbb{R}\}$

$\langle S \rangle = \left\{\begin{matrix} 2\alpha-\beta & 0 \\ 0 & 3\alpha+2\beta \end{matrix}\right\}$

$\alpha, \beta \in \mathbb{R}$

#Q. (a) Find the E-coordinate representation of the vector (5,–4) with respect to the basis E = {(1,2), (–3, 1)} for $\mathbb{R}^3$.

(b) Find the E-coordinate representation of the vector (–3, 5, 7) with respect to the basis E = {(1, 0, 2), (–1, 1, 3), (2, –2, 0)} for $\mathbb{R}^3$.

$(5,-4) = \alpha(1,2) + \beta(-3,1)$

$\alpha - 3\beta = 5$

$2\alpha + \beta = -4$

$\alpha = -1, \beta = -2$

$(5,-4) = -1(1,2) + (-2)(-3,1) = (-1,-2)_E$

$= x$

$\alpha \quad \beta$

Topic : Linear Algebra

A $m \times n$

B $n \times m$

$m \times n \quad n \times m$

AB = BA =

#Q. If A and B are two matrices and if AB exists, then BA exists-

(a) Only if A has as many rows as B has columns

(b) Only if both A and B are square matrices

(c) Only if A and B are skew matrices

(d) Only if both A and B are sysmmetirc.

A B

$m \times n \quad n \times p$

Row col^n

$p = m$

BA

$n \times p \quad m \times n$

$n \times m \quad m \times n$

Row A = col^n b

Row

col^n

Topic : Linear Algebra

$(1,1,1) = 1\cdot(1,0,0) + 1\cdot(0,1,0) + 1\cdot(0,0,1)$

#Q. Determine whether each of the following sets is a basis for $\mathbb{R}^3$.

Span (a) {(0, 1, 2), (0, 2, 3), (0, 6, 1)} ✗

LI (b) {(1,2,1), (1,0,–1), (0, 3, 1)} ✓

(c) {(1, 0, 0), (0,1, 0), (0, 0, 1), (1,1,1)}

LD

$(1,1,1)$

$\alpha(1,2,1)+\beta(1,0,-1)+\gamma(0,3,1) = (0,0,0)$

$\boxed{\alpha=\beta=\gamma=0}$ ✓

$R^3 = (x,y,z)$

$\alpha(1,2,1)+\beta(1,0,-1)+\gamma(0,3,1)$

Span

#Q. Determine whether $\{(1,2,-1,-1),(-1,5,1,3)\}$ is a basis for $\mathbb{R}^4$.

$(1,2,-1,-1)$ & $(-1,5,1,3)$

LI

2

(x,y,z,w)

$= \alpha(1,2,-1,-1) + \beta(-1,5,1,3)$

$\alpha - \beta = x$

$2\alpha + 5\beta = y$

$-\alpha + \beta = z$

$-\alpha + 3\beta = w$

$x + z = 0$

x, y, z & w can take any real value

$(1, 2, -1, -1)$ & $(-1, 5, 1, 3)$

is not a basis ✗

is not spaning set $\mathbb{R}^4$

Slide

#Q. Determine whether each of the following sets of vectors is a linearly independent subset of V.

(a) $V = \mathbb{R}^2$, $\{(1, 0,), (-1, -1)\}$.

(b) $V = \mathbb{R}^2$, $\{(1, -1,), (1, 1), (2, 1)\}$.

(c) $V = \mathbb{R}^3$, $\{(1,1, 0,), (-1, 1,1)\}$.

(d) $V = \mathbb{R}^3$, $\{(1, 0, 0,), (1, 1, 0), (1, 1, 1)\}$.

(e) $V = \mathbb{R}^4$, $\{(1, 2, 1,0), (0, -1, 1, 3)\}$.

α β

$\alpha = \beta = 0$

#Q. (a) Show that (2,1,1) and (1, –4, 2) are orthogonal.

(b) Determine which of the following vectors are orthogonal:

$V_1 = (-2, 6, 1)$, $v_2 = (9, 2, 6)$, $v3 = (4, -15, -1)$

$v_1 \cdot v_2 = -18 + 12 + 6$

$v_2 \cdot v_3 = 36 - 30 - 6$
$= 0$

$v_1 \cdot v_3 = -8 - 90 - 1 \neq 0$

$v_1 \cdot v_2$

$2 - 4 + 2 = 0$

$v_1 \,\&\, v_3 \neq 0$

THANK - YOU