Linear Algebra
DPP-02

1. (a) In $\mathbb{R}^2$, express the vector (2,4) as a linear combination of the vector (0,3) and (2,1)

(b) In $\mathbb{R}^3$, express the vector (2, 3, –2) as a linear combination of the vectors (0, 1, 0), (1, 2, –1) and (1,1, –2).

(c) In M2, 2, express the matrix $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$ as a linear combination of the matrices $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ and $\begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix}$.

2. (a) In $\mathbb{R}^2$, let v_1 = (0, 3) and v_2 = (2,1). Calculate the linear combination $4v_1 - 2v_2$.

(b) In $\mathbb{R}^4$, let v_1 = (1, 2, 1, 3) and v_2 = (2, 1, 0, –1). Calculate the linear combination $3v_1 + 2v_2$.

3. For each of the following vector spaces V and vectors v_1, v_2 and v_3 in V, form the linear combination $3v_1 - 2v_2 + v_3$.

(a) V = P_3, $v_1 = 1 + x + x^2$, $v_2 = 1 - x$, $v_3 = x + x^2$.

(b) V = $M_{2,3}$, $v_1 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & -4 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix}$.

4. (a) Given the basis E = {(1,2), (–3, 1)} for $\mathbb{R}^2$, determine the standard coordinate representation of $(2, 1)_E$.

(b) Given the basis E = {(1, 0, 2), (–1, 1, 3), (2, –2, 0)}for $\mathbb{R}^3$, determine the standard coordinate representation of $(1, 1, -1)_E$.

5. Show that neither of the following sets is a real vector space.

(a) V = {(x, y)$\in \mathbb{R}^2$: y = 2x +1}

(b) $V = \left\{ \begin{pmatrix} 0 & a \\ b & c \end{pmatrix} : a, b, c \in \mathbb{Z} \right\}$

6. Show that neither of the following sets is a real vector space.

(a) V = {all polynomials of degree equal to 5}

(b) V = {a + bi $\in \mathbb{C}$: a ≥ 0}

7. In each case, determine whether set S of matrices is a linearly independent subset of $M_{2,2}$.

(a) $S = \left\{ \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix} \right\}$

(b) $S = \left\{ \begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 0 & -4 \end{pmatrix} \right\}$

8. Show that each of the following is a spanning set for $\mathbb{R}^2$.

(a) {(1, 1), (–1, 2)}

(b) {(2, –1), (3, 2)}

9. Show that {(1, 0, ,0), (1, 1, 0)(2, 0, 1)} is a spanning set for $\mathbb{R}^3$.

10. For each of the following, determine whether the set S is a subspace of the vector space V.

(a) V = P_3, S = {$a + bx$: a, b $\in \mathbb{R}$ }

(b) V = P_3, S = {$x + ax^2$: a, b $\in \mathbb{R}$ }

(c) V = $M_{2,2}$, $S = \left\{ \begin{pmatrix} a & 1 \\ 0 & d \end{pmatrix} : a, b \in \mathbb{R} \right\}$

11. In each of the following cases, determine whether S is a linearly independent subset of the vector space V.

(a) V = P_4, S = $\{1, x, x^2, x^3, 1 + x + x^2 + x^3\}$

(b) $V = M_{2,2},\ S = \left\{\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}\right\}$

(c) $V = M_{2,2},\ S = \left\{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right\}$

(d) V = $\mathbb{C}$, S $\{1 + i, 1 - i\}$

12. Show that each of the following is a spanning set for $\mathbb{R}^2$.

(a) $\{(1, 2), (2, -3)\}$

(b) $\{(1,0), (1,1), (1, -2)\}$

13. Show that $\{1 + x, 1 + x^2, 1 + x^3, x\}$ is a spanning set for P_4.

14. (a) Verify that {(3,4,0), (8,–6,0), (0,0,5)} is an orthogonal basis for $\mathbb{R}^3$.

(b) Express the vector (10,0, 4) in terms of this basis.

15. For each of the following vector spaces V and sets of vector S in V, determine $\langle S \rangle$.

(a) V = $\mathbb{R}^3$, S = {(1, 0, 0)}.

(b) $V = M_{2,2}, S = \left\{\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\right\}$

16. (a) Find the E-coordinate representation of the vector (5,–4) with respect to the basis E = {(1,2), (–3, 1)} for $\mathbb{R}^3$.

(b) Find the E-coordiante representation of the vector (–3, 5, 7) with respect to the basis E = {(1, 0, 2), (–1, 1, 3), (2, –2, 0)} for $\mathbb{R}^3$.

17. If A and B are two matrices and if AB exists, then BA exists-

(a) Only if A has as many rows as B has columns

(b) Only if both A and B are square matrices

(c) Only if A and B are skew matrices

(d) Only if both A and B are sysmmetirc.

18. Determine whether each of the following sets is a basis for $\mathbb{R}^3$.

(a) {(0, 1, 2), (0, 2, 3), (0, 6, 1)}

(b) {(1,2,1), (1,0,–1), (0, 3, 1)}

(c) {(1, 0, 0), (0,1, 0), (0, 0, 1), (1,1,1)}

19. Determine whether {(1.2, –1, –1),(–1,5,1,3)} is a basis for $\mathbb{R}^4$.

20. Determine whether each of the following sets of vectors is a linearly independent subset of V.

(a) V = $\mathbb{R}^2$, {(1, 0,), (–1, –1)}.

(b) V = $\mathbb{R}^2$, {(1, –1,), (1, 1), (2, 1)}.

(c) V = $\mathbb{R}^3$, {(1,1, 0,), (–1, ,1,1)}.

(d) V = $\mathbb{R}^3$, {(1, 0, 0,), (1, 1, 0), (1, 1, 1)}.

(e) V = $\mathbb{R}^4$, {(1, 2, 1,0), (0, –1, 1, 3)}.

21. (a) Show that (2,1,1) and (1, –4, 2) are orthogonal.

(b) Determine which of the following vectors are orthogonal:

$v_1 = (-2, 6, 1)$, $v_2 = (9, 2, 6)$, $v_3 = (4, -15, -1)$

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