

Probability & Statistics

DPP

Random Variables & Bivariate Random Variable

1. For each of the following, determine whether the given values can serve as the probability distribution of a random variable with the given range:

(a) $f(x) = \frac{x-2}{5}$ For $x = 1, 2, 3, 4, 5$;

(b) $f(x) = \frac{x^2}{30}$ For $x = 1, 2, 3, 4$;

(c) $f(x) = \frac{x}{5}$ For $x = 1, 2, 3, 4, 5$;

2. Verify that $f(x) = \frac{2x}{k(k+1)}$ for $x = 1, 2, 3, \dots, k$ can serve as the probability distribution of a random variable with the given range.

3. For each of the following, determine c so that the function can serve as the probability distribution of a random variable with the given range:

(a) $f(x) = cx$ for $x = 1, 2, 3, 4, 5$;

(b) $f(x) = c\left(\frac{5}{x}\right)$ for $x = 1, 2, 3, 4, 5$;

(c) $f(x) = c\left(\frac{1}{4}\right)^x$ for $x = 1, 2, 3, \dots$

(d) $f(x) = cx^2$ for $x = 1, 2, 3, \dots, k$

4. A random variable X has the following probability function:

| | | | | | | | |
|--------|-----|------|------|------|------|-------|-------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $f(x)$ | k | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

- (i) Find k ,
 (ii) Find $P(X \geq 5)$, $P(3 < X \leq 6)$, $P(X < 4)$

5. If $P(x) = \begin{cases} x/15; & x=1,2,3,4,5 \\ 0 & ; \text{ otherwise} \end{cases}$

Find

(i) $P(X = 1 \text{ or } 2)$

(ii) $P\left(\frac{1}{2} < X < \frac{5}{2} / X > 1\right)$

6. For each of the following, determine whether the given values can serve as the values of a distribution function of a random variable with the range $x = 1, 2, 3$ and 4 ;

(a) $F(1) = 0.3$, $F(2) = 0.5$, $F(3) = 0.8$, and $F(4) = 1.2$;

(b) $F(1) = 0.5$, $F(2) = 0.4$, $F(3) = 0.7$, and $F(4) = 1.0$;

(c) $F(1) = 0.25$, $F(2) = 0.61$, $F(3) = 0.83$, and $F(4) = 1.0$;

7. Given that the discrete random variable X has the distribution function

$$f(x) = \begin{cases} x/6; & x=1,2,3 \\ 0 & \text{elsewhere} \end{cases} \quad \text{Find } F(x)$$

8. If X has the distribution function

$$f(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{3} & \text{for } 1 \leq x < 4 \\ \frac{1}{2} & \text{for } 4 \leq x < 6 \\ \frac{5}{6} & \text{for } 6 \leq x < 10 \\ 1 & \text{for } x \geq 10 \end{cases}$$

Find

(a) $P(2 < X \leq 6)$; (b) $P(X = 4)$

(c) $P(X \geq 10)$ (d) $P(X < 4)$

(e) $P(X > 4)$ (f) $P(X \geq 4)$

9. If X has the distribution function

$$f(x) = \begin{cases} 0 & \text{for } x < -1 \\ \frac{1}{4} & \text{for } -1 \leq x < 1 \\ \frac{1}{2} & \text{for } 1 \leq x < 3 \\ \frac{3}{4} & \text{for } 3 \leq x < 5 \\ 1 & \text{for } x \geq 5 \end{cases}$$

Find

- (a) $P(X \leq 3)$; (b) $P(X = 3)$;
 (c) $P(X < 3)$; (d) $P(X \geq 5)$;
 (e) $P(-0.4 < X < 4)$; (f) $P(X = 5)$;
 (g) $P(3 < X < 5)$; (i) $P(3 \leq X < 5)$;
 (j) $P(3 \leq X \leq 5)$

10. Find distribution fx^n of the random variable that has the prob. distribution

$$f(x) = \frac{x}{15}; x = 1, 2, 3, 4, 5$$

11. Let X_1, X_2, \dots, X_n be random sample from the following density function

$$f(x; \theta) = \frac{kx}{\theta^2}; 0 < x < \theta, \theta > 0$$

Find k such that above is a valid density function.

12. Let X be a continuous random variable with p.d.f:

$$f(x) = \begin{cases} ax; 0 < x < 1 \\ a; 1 \leq x \leq 2 \\ -ax + 3a; 2 \leq x \leq 3 \\ 0; \text{elsewhere} \end{cases}$$

- (i) Determine constant a
 (ii) $P(X \leq 1.5)$

13. The probability density of the random variable Y is given by

$$f(y) = \begin{cases} \frac{1}{8}(y+1) & \text{for } 2 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P(Y < 3.2)$ and $P(2.9 < Y < 3.2)$.

14. The p.d.f of the random variable X is given by

$$f(x) = \begin{cases} \frac{c}{\sqrt{x}} & \text{for } 0 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$$

Find

- (a) The value of c ;
 (b) $P\left(X < \frac{1}{4}\right)$ and $P(X > 1)$

15. The density function of the random variable X is given by

$$g(x) = \begin{cases} 6x(1-x) & \text{for } 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P\left(X < \frac{1}{4}\right)$ and $P\left(X > \frac{1}{2}\right)$

16. (a) Show that $f(x) = 3x^2$ for $0 < x < 1$ represents a density function.
 (b) Calculate $P(0.1 > X < 0.5)$

17. If X has the prob. density fx^n

$$f(x) = \begin{cases} ke^{-3x} & ; x > 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

Find k and $P(0.5 \leq X \leq 1)$

18. The probability density of the continuous random variable X is given by

$$f(x) = \begin{cases} \frac{1}{5} & \text{for } 2 < x < 7 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P(3 < X < 5)$

19. Find the distribution function of the random variable X whose Probability density is given by

$$f(x) = \begin{cases} \frac{1}{3} & \text{for } 0 < x < 1 \\ \frac{1}{3} & \text{for } 2 < x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

20. The distribution fx^n of the random Variable X is given by

$$F(x) = \begin{cases} 1 - (1+x)e^{-x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

Find

- (i) $P(X \leq 2)$
- (ii) $P(1 < X < 3)$
- (iii) $P(X > 4)$

21. Find the distribution function of the random variable X whose probability density is given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

22. Find the distribution function of the random variable X whose probability density is given by

$$f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 < x \leq 1 \\ \frac{1}{2} & \text{for } 1 < x \leq 2 \\ \frac{3-x}{2} & \text{for } 2 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

23. Find a prob. density fx^n for the random variable whose distribution fx^n is given by

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x & \text{for } 0 < x < 1 \\ 1 & \text{for } x > 1 \end{cases}$$

24. The distribution function of the random variable Y is given by

$$f(y) = \begin{cases} 1 - \frac{9}{y^2} & \text{for } y > 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P(Y \leq 5)$ and $P(Y > 8)$

25. A random variable X which can be used in certain circumstances as a model for claim sizes has cumulative distribution function

$$f(x) = \begin{cases} 0 & , x < 0 \\ 1 - \left(\frac{2}{2+x}\right)^3 & , x > 0 \end{cases}$$

Calculate the value of the conditional probability $P(X > 3/X > 1)$

26. The probability density of the random variable Z is given by

$$f(z) = \begin{cases} kze^{-z^2} & \text{for } z > 0 \\ 0 & \text{for } z \leq 0 \end{cases} \quad \text{Find } k$$

27. A random variable X has the following probability distribution

| | | | | | |
|------|-----|----|-----|---|-----|
| X | -2 | -1 | 0 | 1 | 2 |
| P(X) | 1/6 | p | 1/4 | p | 1/6 |

- (i) Find the value of p.
- (ii) Calculate $E(X+2)$, $E(2X^2+3X+5)$

28. If X is the number of points rolled with a balanced die, find the expected value of

$$g(X) = 2X^2 + 1.$$

29. Let X be a random variable with the following probability fx^n

| | | | |
|--------|-----|-----|-----|
| x: | -3 | 6 | 9 |
| P(X=x) | 1/6 | 1/2 | 1/3 |

Find $E(X)$ and $E(X^2)$ and evaluate $E(2X+1)^2$

30. Find the expected value of the random variable Y whose probability density is given by

$$f(y) = \begin{cases} \frac{1}{8}(y+1) & \text{for } -1 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

31. If X has the probability density

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected value of $g(X) = e^{3X/4}$.

32. If the probability density of X is given by

$$f(x) = \begin{cases} 2x^{-3} & \text{for } x > 1 \\ 0 & \text{elsewhere} \end{cases}$$

check whether its mean and its variance exist.

33. If the probability density of X is given by

$$f(x) = \begin{cases} 2(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Show that $E(X^r) = \frac{2}{(r+1)(r+2)}$

(b) and use this result to evaluate $E[(2X+1)^2]$

34. A continuous random variable X has the *p.d.f.*,

$$f(x) = \begin{cases} a(1-x^2) & 2 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Find a
(ii) Find $E(X)$

35. Certain coded measurements of the pitch diameter of threads of a fitting have the probability density

$$f(x) = \begin{cases} \frac{4}{\pi(1+x^2)} & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected value of this random variable.

36. Let X be a random variable denoting the hours of life in electric light bulb. Suppose X is distributed with density function

$$f(x) = \frac{1}{1000} e^{-x/1000} \quad \text{for } x > 0$$

Find the expected life time of such a bulb.

37. $f(x) = \frac{1}{2}(x+1); -1 < x < 1$. Find the variance of X.

38. $f(x) = \lambda e^{-\lambda x}, 0 < x < \infty$. Find the variance of X.

39. A continuous random variable has PDF:

$$\frac{k}{(x+5)^4} \quad x > 0$$

Calculate:

- (i) k
(ii) $F(10)$
(iii) $E[X]$

40. A discrete random variable X has the following probability distribution:

| | | | | | |
|----------|------|------|------|-----|------|
| X | 1 | 2 | 3 | 4 | 5 |
| P(X = x) | 0.05 | 0.15 | 0.35 | 0.4 | 0.05 |

Calculate:

- (i) $F(3)$ (ii) $E[X]$
(iii) $\text{var}[X]$

41. If the probability density of X is given by

$$f(x) = \begin{cases} 6x(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability of $Y = X^3$.

42. Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X. Which of the following statements is always true?

- (a) $F(x) + F(-x) = 1$ for all $x \in \mathbb{R}$
(b) $F(x) - F(-x) = 0$ for all $x \in \mathbb{R}$
(c) $F(x) + F(-x) = 1 + P(X = x)$ for all $x \in \mathbb{R}$
(d) $F(x) + F(-x) = 1 - P(X = x)$ for all $x \in \mathbb{R}$

43. The cumulative distribution function of a random variable X is given by

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{4}{9}, & \text{if } 0 \leq x < 1 \\ \frac{8}{9}, & \text{if } 1 \leq x < 2 \\ 1, & \text{if } x \geq 2 \end{cases}$$

Which of the following statements is (are) TRUE?

- (a) The random variable X takes positive probability one at two points
- (b) $P(1 \leq X \leq 2) = \frac{5}{6}$
- (c) $E(X) = \frac{2}{3}$
- (d) $P(0 < X < 1) = \frac{4}{9}$

44. Consider the function $f(x)$ defined as $f(x) = ce^{-x^4}$, $x \in \mathbb{R}$. For what value of c is f a probability density function?

- (a) $\frac{2}{\Gamma(1/4)}$ (b) $\frac{4}{\Gamma(1/4)}$
- (c) $\frac{3}{\Gamma(1/4)}$ (d) $\frac{1}{\Gamma(1/4)}$

45. Let $F: [0, 2] \rightarrow \mathbb{R}$ be the function defined by

$$F(x) = \int_{x^2}^{x+2} e^{x[t]} dt, \text{ Where } [t] \text{ denotes the greatest integer less than or equal to } t. \text{ Then the derivative of } F \text{ at } x = 1 \text{ equals}$$

- (a) $e^3 + 2e^2 - e$ (b) $e^3 - e^2 + 2e$
- (c) $e^3 - 2e^2 + e$ (d) $e^3 + 2e^2 + e$
46. Let X be a discrete random variable with the probability mass function

$$P(X = n) = \begin{cases} \frac{-2c}{n} & n = -1, -2, \\ d & n = 0, \\ cn & n = 1, 2, \\ 0 & \text{otherwise} \end{cases}$$

Where c and d are positive real numbers. If $P(|X| \leq 1) = \frac{3}{4}$, then $E(X)$ equal

- (a) $\frac{1}{12}$ (b) $\frac{1}{6}$
- (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

47. Let X be a continuous random variable with the probability density function

$$f(x) = \frac{e^x}{(1+e^x)^2}, -\infty < x < \infty.$$

Then $E(X)$ and $P(X > 1)$, respectively, are

- (a) 1 and $(1+e)^{-1}$ (b) 0 and $2(1+e)^{-2}$
- (c) 2 and $(2+2e)^{-1}$ (d) 0 and $(1+e)^{-1}$

48. Let X be the number of heads obtained in a sequence of 10 independent tosses of a fair coin. The fair coin is tossed again X number of times independently, and let Y be the number of heads obtained in these X number of tosses. Then $E(X + 2Y)$ equals _____.

49. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} ax^2, & 0 < x < 1, \\ bx^{-4}, & x \geq 1, \\ 0, & \text{otherwise,} \end{cases}$$

Where a and b are positive real numbers. If $E(X) = 1$, then $E(X^2)$ equals _____.

50. Let X be a continuous random variable with distribution function

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ ax^2, & \text{if } 0 \leq x < 2, \\ 1, & \text{if } x \geq 2, \end{cases}$$

For some real constant a . Then, $E(X)$ is equal to

- (a) $4/3$ (b) $1/4$
(c) 1 (d) 0

51. Let X be a random variable having the probability density function

$$f(x) = \frac{1}{8\sqrt{2\pi}} \left(2e^{-\frac{x^2}{2}} + 3e^{-\frac{x^2}{8}} \right), -\infty < x < \infty$$

Then, $4E(X^4)$ is equal to _____.

52. For $k = 1, 2, \dots, 10$, let the probability density function of the random variable X_k be

$$f_{X_k}(x) = \begin{cases} \frac{e^{-x/k}}{k}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Then $E\left(\sum_{k=1}^{10} kX_k\right)$ is equal to _____

53. If random variable X assumes only positive integral values, with the probability

$$P(X=x) = \frac{2}{3} \left(\frac{1}{3} \right)^{x-1}, x=1, 2, 3, \dots, \text{ then } E(X) \text{ is}$$

- (a) $2/9$ (b) $2/3$
(c) 1 (d) $3/2$

54. A fair coin is tossed. If a head occurs, 1 fair dice is rolled; if a tail occurs, 2 fair dice are rolled. If Y is the total on the dice or die, then $P[Y=6] =$

- (a) $1/9$ (b) $5/36$
(c) $11/72$ (d) $1/6$

55. Let X be a random variable with probability density function

$$f(x) = \begin{cases} 4x^k, & \text{if } 0 < x < 1 \\ x - \frac{x^2}{2}, & \text{if } 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Where k is a positive integer. Then

$$P\left(\frac{1}{2} < X < \frac{3}{2}\right) =$$

- (a) $85/96$ (b) $75/95$

- (c) $65/96$ (d) $85/95$

56. A random Variable X has a probability mass of 0.2 at $X=0$ and a probability mass of 0.1 at $X=1$. For all other values, X has the following density function:

$$f(x) = \begin{cases} 0 & x < 0 \\ x & 0 < x < 1 \\ 2x & 1 < x < c \\ 0 & x \geq c \end{cases}, \text{ where } c \text{ is a constant.}$$

Find $P(X < 1/X > 0.5)$

- (a) Less than 0.6
(b) At least 0.6 less than 0.7
(c) At least 0.8 but less than 0.8
(d) At least 0.8 but less than 0.9

57. Let X be a continuous random variable with pdf

$$f_x(x) = \begin{cases} cx^2 & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

For some positive constant c , the value of

$$P\left(X \leq \frac{2}{3} \mid X > \frac{1}{3}\right) \text{ is}$$

- (a) $3/26$ (b) $5/26$
(c) $7/26$ (d) $11/26$

58. Let X be a random variable with probability mass function.

$$f(x) = \begin{cases} \frac{x}{15}; & x=1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements are correct ?

- (a) $P(X=2) = \frac{2}{15}$
(b) $P(X=1 \text{ or } 2) = \frac{1}{2}$
(c) $P\left(\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right) = 1/7$
(d) $P\left(\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right) = 2/7$

59. Let X be a random variable with distribution function.

$$F(x) = \begin{cases} 0, & \text{for } -\infty \leq x < 0 \\ \frac{x^2}{4}, & \text{for } 0 \leq x < 1 \\ \frac{2x-1}{4}, & \text{for } 1 \leq x < 2 \\ -\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4}, & \text{for } 2 \leq x < 3 \\ 1, & \text{for } 3 \leq x < \infty \end{cases}$$

Which of the following statements is correct?

- (a) It is the distribution function of continuous random variable
 (b) It is the distribution function of discrete random variable
 (c) It is distribution function of both discrete and continuous random variable.
 (d) It is not distribution function of discrete and continuous random variable.
60. Let X be a continuous random variable with the probability density function.
- $$f(x) = \frac{e^x}{(1+e^x)^2}, -\infty < x < \infty$$
- The $E(X)$ and $P(X > 1)$, respectively, are
- (a) 1 and $(1+e)^{-1}$
 (b) 0 and $2(1+e)^{-2}$
 (c) 2 and $(2+2e)^{-1}$
 (d) 0 and $(1+e)^{-1}$
61. Let X be random variable with mean μ_x and variance $\sigma_x^2 > 0$, then $\text{Var}(aX + b)$ is
- (a) $a\sigma_x^2$ (b) $a^2\sigma_x^2$
 (c) $a\sigma_x^2 + b$ (d) $a^2\sigma_x^2 + b$
62. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} ax^2 & 0 < x < 1 \\ bx^{-4} & x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Where a and b are positive real numbers. If $E(X) = 1$, then $E(X^2)$ equals _____.

63. The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \leq x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \leq x < \frac{3}{4} \\ \frac{x+3}{5} & \frac{3}{4} \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

Then $P\left(\frac{1}{4} \leq X \leq 1\right)$ is

- (a) $\frac{1}{20}$ (b) $\frac{11}{20}$
 (c) $\frac{7}{20}$ (d) $\frac{13}{20}$

64. Let X be a continuous random variable with PDF

$$f(x) = \begin{cases} ax & 0 \leq x < 1 \\ a & 1 \leq x \leq 2 \\ -ax + 3a & 2 < x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

What is the value of the constant a ?

- (a) 1 (b) $\frac{1}{2}$
 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

65. If X is a random variable with density

$$f(x) = \frac{1}{4} e^{-\frac{|x|}{2}}, -\infty < x < \infty.$$

Then $E(|X|)$ _____.

76. The probability density function of a random variable X is given by.

$$f(x) = \begin{cases} \frac{1}{4} & , \text{ if } |x| \leq 1 \\ \frac{1}{4x^2} & , \text{ otherwise} \end{cases}$$

Then $P\left(-\frac{1}{2} \leq X \leq 2\right) =$ _____.

67. Consider the function

$$f(x) = \begin{cases} k(x - [x]) & 0 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Where $[x]$ is the integral part of x . The value of k for which the above function is a probability density function of some random variable is

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

68. Let X be a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Then $P\left(\frac{1}{4} < X < 1\right)$ is equal to _____.

69. Let the probability density function of a random variable X be

$$f(x) = \begin{cases} x & 0 \leq x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the value of c is equal to _____.

70. Let the random variable X have the distribution function.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \leq x < 1 \\ \frac{3}{5} & 1 \leq x < 2 \\ \frac{1}{2} + \frac{x}{8} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

Then $P(2 \leq X < 4)$ equal to _____.

71. Let X and Y be continuous random variables with the joint probability density function.

$$f(x, y) = \begin{cases} cx(1-x) & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Where c is a positive real constant Then $E(X)$ equals

- (a) $\frac{1}{5}$ (b) $\frac{1}{4}$
(c) $\frac{2}{5}$ (d) $\frac{1}{3}$

72. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x}{8} & \text{if } 0 < x < 2 \\ \frac{k}{8} & \text{if } 2 \leq x \leq 4 \\ \frac{6-x}{8} & \text{if } 4 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

Where k is a real constant Then $P(1 < X < 5)$ equals _____.

73. Let X be a random variable having the distribution function.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < 1 \\ \frac{1}{3} & 1 \leq x < 2 \\ \frac{1}{2} & 2 \leq x < \frac{11}{3} \\ 1 & x \geq \frac{11}{3} \end{cases}$$

Then $E(X)$ is equal to _____.

74. Let $P(X = n) = \frac{\lambda}{n^2(n+1)}$, where λ is an appropriate constant.

Then $E(X)$ is _____

- (a) $2\lambda + 1$ (b) λ
 (c) ∞ (d) 2λ



Answer Key

1. (a) No (b) Yes (c) No

2. (Yes)

3. (A) $c = \frac{1}{15}$ (b) $c = \frac{12}{137}$

4. $(\frac{1}{49}, \frac{24}{49}, \frac{33}{49}, \frac{16}{49})$

5. (i) $1/5$ (ii) $1/7$

6. (a) No (b) No (c) Yes

7.
$$\begin{cases} 0 & x < 1 \\ \frac{1}{6} & 1 \leq x < 2 \\ \frac{3}{6} & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

8. (a) $3/6$ (b) $1/6$ (c) $1/6$

(d) $1/3$ (e) $3/6$ (f) $1/2$

9. (a) $3/4$ (b) $1/4$ (c) $1/2$

(d) $3/4$ (e) $1/2$ (f) $1/4$

(g) 0 (h) $1/4$ (i) $1/2$

10. (1)

11. ($k=2$)

12. (i) $a = \frac{1}{2}$ (ii) $1/2$

13. (0.54, 0.15187)

14. (a) $1/4$ (b) 0.5

15. 0.15625, 0.5

16. 0.124

17. $K = 3$ & $P(0.5 \leq x \leq 1) = 0.173$

18. $2/5$

19.
$$f_y(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{3}y & 0 < y < 1 \\ \frac{1}{3} & 1 \leq y < 2 \\ \frac{y-1}{3} & 2 \leq y < 4 \\ 1 & y \geq 4 \end{cases}$$

20. (i) $1-3^{e-2}$ (ii) $\frac{2}{e}(1-2e^{-2})$ (iii) 5^{e-4}

21.
$$F_y(y) = \begin{cases} 0 & y < 0 \\ \frac{y^2}{2} & 0 \leq y < 1 \\ \frac{4y-2-y^2}{2} & 1 \leq y \leq 2 \\ 1 & y \geq 2 \end{cases}$$

22.
$$F_y(y) = \begin{cases} 0 & x < 1 \\ x^2/4 & 0 \leq x < 1 \\ \frac{2x-1}{4} & 1 \leq x < 2 \\ \frac{6x-x^2-2}{4} & 2 \leq x < 2 \\ 1 & x \geq 3 \end{cases}$$

23.

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } 0 < x < 1, f(x) = 27, 5/24, 2, 17/2 \\ 0 & \text{for } x > 1 \end{cases}$$

24. $16/25, 9/64$

25. $(3/5)^3$

26. $k = 2$

27. (i) $5/24$, (ii) $2, 51/6$

28. $94/3$

29. $E(x) = \frac{11}{2}, E(x^2) = \frac{93}{2}, E(2x+1)^2 = 209$

30. $1/12$

31. 4

32. $E(x) = 2$

33. 3

34. (i) $1/42$ (ii) $31/8$

35. $\frac{2}{\pi} \log 2$

36. 1000 hrs

37. $2/9$

38. $1/\lambda^2$

39. (i) $k = 375$ (ii) $F(10) = 0.963$

(iii) $E(Y) = 2.5$

40. (i) 0.55 (ii) 3.25

(iii) 0.8875

41. $f(y) = 2(y^{-1/3} - 1); 0 < y < 1$

- | | |
|-----------------|--------------------|
| 42. (c) | 58. (a,b,c) |
| 43. (b, c) | 59. (a) |
| 44. (a) | 60. (d) |
| 45. (a,b, c, d) | 61. (b) |
| 46. (a) | 62. (1.4) |
| 47. (d) | 63. (b) |
| 48. (10) | 64. (b) |
| 49. (1.40) | 65. (2) |
| 50. (a) | 66. (0.5) |
| 51. (147) | 67. (c) |
| 52. (385) | 68. (0.6875) |
| 53. (d) | 69. (21/4) |
| 54. (c) | 70. (0.4) |
| 55. (a) | 71. (c) |
| 56. (a) | 72. (0.87 to 0.88) |
| 57. (c) | 73. (2.25) |
| | 74. (b) |



Any issue with DPP, please report by clicking here:- <https://forms.gle/t2SzQVvQcs638c4r5>

For more questions, kindly visit the library section: Link for web: <https://smart.link/sdfez8ejd80if>

Telegram Link: <https://t.me/mathandaptitudes>



PW Mobile APP: <https://smart.link/7wwosivoicgd4>