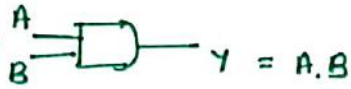
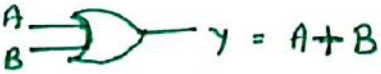
De Morgen's Theorm 40

- Digital Circuit can be implemented by many ways.
- De Morgen's theorm is used to simplify boolean equation.
- Digital expression is simply made by three basic operation

1] Boolean AND → 

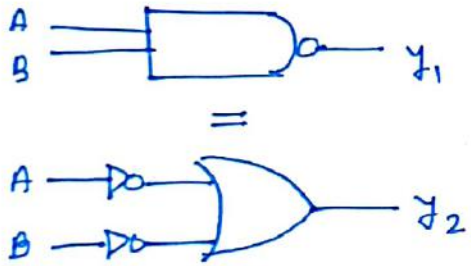
2] Boolean OR → 

3] Boolean NOT → 

- De Morgen's Law

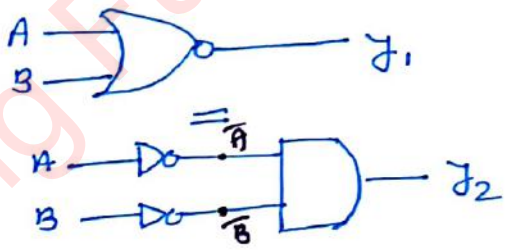
$\overline{A.B} = \overline{A} + \overline{B}$ —(1)

- Compliment of product is Sum of Compliment



$\overline{A+B} = \overline{A}.\overline{B}$ —(2)

- Compliment of Sum is Product of Compliment



Truth Table.

A	B	y_1	$\overline{A}$	$\overline{B}$	y_2
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

A	B	y_1	$\overline{A}$	$\overline{B}$	y_2
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

Ex $\overline{x + \overline{y.z}} = \overline{x}.\overline{P}$

$= \overline{x}.\overline{\overline{y.z}}$

$= \overline{x}.y.z$

Boolean Rules [Boolean Algebra Rules] 41

AND gate.

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

- $A.A = A$
- $A.0 = 0$
- $A.1 = A$
- $A.\bar{A} = 0$

NOT Gate

A	Y
0	1
1	0

- $\bar{\bar{A}} = A$

OR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

- $A + A = A$
- $A + 0 = A$
- $A + 1 = 1$
- $A + \bar{A} = 1$

Distributive Law

- $A.(B+C) = A.B + A.C$
- Imp - $A + (B.C) = (A+B).(A+C)$

Imp
- $A + \bar{A}B = (A+\bar{A}).(A+B) = A+B$
- $\bar{A} + AB = (\bar{A}+A).(\bar{A}+B) = \bar{A}+B$

Commutative Law

$A + B = B + A$

&

$A.B = B.A$

Associative law.

$(A+B)+C = A+(B+C)$

&

$(A.B).C = A.(B.C)$.

De Morgan's Law

$\overline{A+B} = \bar{A}.\bar{B}$ — (1) $\overline{A.B} = \bar{A} + \bar{B}$ — (2)

- Priority → NOT (Highest) → AND → OR (Lowest)

Ex $P = X\bar{Y}\bar{Z} + X\bar{Y}\bar{Z} + Y\bar{Z}$

$= X\bar{Z}(Y+\bar{Y}) + Y\bar{Z}$

$= X\bar{Z} + Y\bar{Z}$

$= (X+Y).\bar{Z}$

- $A.1 = A$

1	0

Boolean Algebra Rules

NOT

$$\overline{\overline{A}} = A$$

AND

$$A.A = A$$
$$A.0 = 0$$
$$A.1 = A$$
$$A.\overline{A} = 0$$

OR

$$A+A = A$$
$$A+0 = A$$
$$A+1 = 1$$
$$A+\overline{A} = 1$$

Distributive Law

$$A+\overline{A}B = A+B$$
$$\overline{A}+AB = \overline{A}+B$$
$$A.(B+C) = A.B + A.C$$

De-Morgen's Law

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$
$$\overline{A.B} = \overline{A} + \overline{B}$$

Boolean Algebra Examples 43

Consensus Theorm 42

Theorm.1 $AB + \bar{A}C + BC = AB + \bar{A}C$

$$LHS = AB + \bar{A}C + BC(A+\bar{A})$$
$$= AB + \bar{A}C + BCA + BC\bar{A}$$
$$= AB(1+C) + \bar{A}C(1+B)$$
$$= AB + \bar{A}C$$
$$= RHS$$

Theorm.2 $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

$$LHS = (A+B)(\bar{A}+C)(B+C)$$
$$= (A\bar{A} + AC + B\bar{A} + BC)(B+C) \quad [A\bar{A} = 0]$$
$$= (AC + B\bar{A} + BC)(B+C)$$
$$= ACB + B\bar{A}B + BCB + ACC + B\bar{A}C + BCC$$
$$= ABC + B\bar{A} + BC + AC + \bar{A}BC + BC$$
$$= ABC + B\bar{A} + BC + AC + \bar{A}BC$$
$$= BC[A + 1 + \bar{A}] + B\bar{A} + AC$$
$$= BC + B\bar{A} + AC$$

$$RHS = (A+B)(\bar{A}+C)$$
$$= A\bar{A} + AC + B\bar{A} + BC$$
$$= AC + B\bar{A} + BC$$

Boolean Algebra Examples 43

1] $\overline{\overline{AB} + \overline{A} + AB}$

$= \overline{\overline{X} + \overline{A} + X}$ $\quad [\because X = AB]$

$= \overline{\overline{A} + 1}$

↓ You may not see that sometimes.

$= \overline{1}$

$= 0$

$= \overline{\overline{AB} + \overline{A} + AB}$

$= \overline{\overline{AB}} \cdot \overline{\overline{A}} \cdot \overline{AB}$

$= AB \cdot A \cdot \overline{AB}$

$= AB \cdot \overline{AB}$

$= 0$

2] $A\,(B + \overline{C}\,\overline{(AB + A\overline{C})})$

$= A.(B + \overline{C}\,[\overline{AB} \cdot \overline{A\overline{C}}])$

$= A.(B + \overline{C}\,[(\overline{A} + \overline{B}).(\overline{A} + \overline{\overline{C}})])$

$= A.(B + \overline{C}\,[(\overline{A} + \overline{B}).(\overline{A} + C)])$

$= A.(B + \overline{C}\,(\overline{A}.\overline{A} + \overline{A}C + \overline{B}\overline{A} + \overline{B}C))$

$= A.(B + \overline{C}\,(\overline{A} + \overline{A}C + \overline{A}\overline{B} + \overline{B}C))$

$= A.(B + \overline{C}\,(\overline{A}(1 + C + \overline{B}) + \overline{B}C))$

$= A.(B + \overline{C}\,(\overline{A} + \overline{B}C))$

$= A.(B + \overline{C}\overline{A} + \overline{B}C\overline{C})$

$= A.(B + \overline{C}\overline{A})$

$= AB + \overline{C}\overline{A}A$

$= AB$

3] If $\overline{A} + AB = 0$, then find values of A & B.

$\Rightarrow \overline{A} + AB = 0$

$\Rightarrow \overline{A} + B = 0$

$\boxed{A = 1}$ & $\boxed{B = 0}$

Boolean Algebra Examples

1] $A\bar{B}(C + BD) + \bar{A}\bar{B}$, Simplify given boolean eq.^{n}

$= A\bar{B}C(C + BD) + \bar{A}\bar{B}$

$= A\bar{B}C + A\bar{B}BC + \bar{A}\bar{B}$

$= A\bar{B}C + \bar{A}\bar{B}$

$= \bar{B}(\bar{A} + AC)$

$= \boxed{\bar{B}(\bar{A} + C)}$

2] $\overline{ABC}(\overline{A + B + C})$

$= (\bar{A} + \bar{B} + \bar{C}).(\bar{A}.\bar{B}.\bar{C})$

$= \bar{A}\bar{A}\bar{B}\bar{C} + \bar{B}\bar{A}\bar{B}\bar{C} + \bar{C}\bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}\bar{C}$

3] $\overline{(A + \bar{B}C)}(A\bar{B} + \overline{ABC})$

$= (\bar{A}.\overline{\bar{B}C})(A\bar{B} + \bar{A} + \bar{B} + \bar{C})$

$= (\bar{A}.BC)(A\bar{B} + \bar{A} + \bar{B} + \bar{C})$

$= (\bar{A}.BC)(\bar{B}(A + 1) + \bar{A} + \bar{C})$

$= \bar{A}BC(\bar{A} + \bar{B} + \bar{C})$

$= \bar{A}\bar{A}BC + \bar{B}\bar{A}BC + \bar{C}\bar{A}BC$

$= \bar{A}BC$

4] $A + \bar{B}C(A + \overline{\bar{B}C})$

$= A + \bar{B}C(A + \bar{\bar{B}} + \bar{C})$

$= A + \bar{B}C(A + B + \bar{C})$

$= A + \bar{B}CA + \bar{B}CB + \bar{B}C\bar{C}$

$= A + \bar{B}CA$

$= A(1 + \bar{B}C)$

$= A$

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Boolean Algebra Examples 45

1] If $x=1$ in the logic eq.^{n}

$$[x + z[\bar{y} + [\bar{z} + x\bar{y}]]][\bar{x} + \bar{z}(x+y)] = 1$$

then

A] $y = z$ B] $y = \bar{z}$ C] $z = 1$ D] $z = 0$

$\Rightarrow [x + z[\bar{y} + [\bar{z} + x\bar{y}]]][\bar{x} + z(x+y)] = 1$

$\Rightarrow [\underbrace{1}_{[1+A]=1} + z[\bar{y} + [\bar{z} + 1.\bar{y}]]][\underbrace{0 + \bar{z}(1+y)}_{[0+A=A]}] = 1$

$\Rightarrow 1.[\bar{z}.1] = 1$

$\Rightarrow \bar{z} = 1$

$\Rightarrow \boxed{z = 0}$

2] If we have 3 variables A, B & C. Find the output $Y = 1$ for majority of "1" in A, B & C. also minimise the function How "0"

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	← 1
1	0	0	0
1	0	1	← 1
1	1	0	← 1
1	1	1	← 1

$Y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$= \bar{A}BC + A\bar{B}C + AB(\bar{C} + C)$

$[\bar{C} + C = 1]$

$= \bar{A}BC + A\bar{B}C + AB$

$= \bar{A}BC + A[\bar{B}C + B]$

[As distributive law $\bar{B}C + B = B + C$]

$= \bar{A}BC + A(B + C)$

$= \bar{A}BC + AB + AC$

$= B[\bar{A}C + A] + AC$

$[\bar{A}C + A = A + C]$

$= B(A + C) + AC$

$= \boxed{BA + BC + AC}$

Dual and Self Dual of Boolean expression

- To get dual of given expression, we need to replace
  - * OR with AND
  - * AND with OR
  - * 1 with 0
  - * 0 with 1

e.g.1 Find out dual of

⇒ $(A+B).(\bar{A}+C).(B+C) = (A+B).(\bar{A}+C)$

⇒ $\boxed{A.B + \bar{A}.C + B.C = A.B + \bar{A}.C}$

e.g.2 $F = AB + A\bar{B}C + A\bar{C}$, Then find dual of F.

$F_d = (A+B).(A+\bar{B}+C).(A+\bar{C})$

- Self Dual - If dual of function is same function then it is refered as self dual.

e.g.3 $F = AB + BC + AC$

Find given function F is self dual or not.

$F_d = (A+B).(B+C).(A+C)$

$= (B + A.C).(A+C)$ [As per distributive rule $(A+B)(B+C) = (B+AC)$]

$= BA + BC + A.C.A + A.C.C$

$= BA + BC + AC + AC$

$= BA + BC + AC$

$\boxed{F_d = F}$ ← So function F is self dual.

- For n number of variable, total self dual $= 2^{2^{n-1}}$

e.g.4 For $n = 2$ variable find out total self dual.

$= 2^{2^{n-1}} = 2^{2^{2-1}} = 2^2 = 4$

→ we have variable A & B.

$A \longleftrightarrow A$

$B \longleftrightarrow B$

$\bar{A} \longleftrightarrow \bar{A}$

$\bar{B} \longleftrightarrow \bar{B}$

e.g.5 If $n = 5$ variables, then find total self dual.

$= 2^{2^{n-1}} = 2^{2^{5-1}} = 2^{2^4} = 2^{16} = 65536$

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SOP, POS & Canonical Form of Boolean function Representation

SOP - Sum of Product [DNF - Disjunctive Normal Form]

- It is a summation of product terms.

Eg $Y = A.B + A.\bar{B}.C + A\bar{B}\bar{C}$

(Summation; Product terms.)

POS - Product of sum [CNF - Conjuctive Normal Form].

- It is a product of sum terms.

Eg. $Y = (A+B).(A+C).(\bar{A}+\bar{B})$

(Product. ; Summation terms.)

Canonical Form

- Standard SOP (SSOP)
- Each product term contains all the variables of the function.

Eg. $F(A,B,C) = A.B.C + A.\bar{B}.C + A.B.\bar{C}$

SSOP ✓
SOP ×

Standard POS (SPOS)

- Each sum terms contains all the variables of the function.

E.g. $F(A,B,C) = (A+B+\bar{C}).(\bar{A}+B+C)$

SPOS ✓
POS ×

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Minterms and Maxterms in Boolean function Representation

Minterms

- Each individual term in SSOP is called as minterms

Maxterms

- Each individual term in SPOS is called as Maxterm

For 3 variables

F	A B C	minterms.	Maxterms
0	0 0 0	$\bar{A}\bar{B}\bar{C} \rightarrow m_0$	$(A+B+C) \rightarrow M0$
1	0 0 1	$\bar{A}\bar{B}C \rightarrow m_1$	$(A+B+\bar{C}) \rightarrow M1$
0	0 1 0	$\bar{A}B\bar{C} \rightarrow m_2$	$(A+\bar{B}+C) \rightarrow M_2$
1	0 1 1	$\bar{A}BC \rightarrow m_3$	$(A+\bar{B}+\bar{C}) \rightarrow M3$
1	1 0 0	$A\bar{B}\bar{C} \rightarrow m_4$	$(\bar{A}+B+C) \rightarrow M4$
0	1 0 1	$A\bar{B}C \rightarrow m_5$	$(\bar{A}+B+\bar{C}) \rightarrow M5$
1	1 1 0	$AB\bar{C} \rightarrow m_6$	$(\bar{A}+\bar{B}+C) \rightarrow M6$
1	1 1 1	$ABC \rightarrow m_7$	$(\bar{A}+\bar{B}+\bar{C}) \rightarrow M7$

$F = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

$= \Sigma m\,(1, 3, 4, 6, 7)$

$F = (A+B+C).(A+\bar{B}+C)(\bar{A}+B+\bar{C})$

$= \Pi M\,(0, 2, 5)$

- Max-terms and Min-terms are compliment to each other.

$F_{ABC} = \Sigma m\,(0, 2, 3, 5) \rightarrow F_{ABC} = \Pi M\,(1, 4, 6, 7)$

↑ 3 variables → total min & max terms = $2^3 = 8$ (0, 1, 2, 3, 4, 5, 6, 7)

SOP to SSOP Conversion 49

Step-1 - Indentify the missing variables in product terms.

$$F(A,B,C) = \underbrace{A.B}_{\uparrow C \text{ is missing}} + A.\bar{B}.C + \underbrace{A.C}_{\uparrow B \text{ is missing}}$$

Step-2 - Multiply [missing Variable + it's complement]

$$F(A,B,C) = A.B.[C+\bar{C}] + A\bar{B}.C + A.C.[B+\bar{B}]$$

$$= A.B.C + A.B.\bar{C} + A.\bar{B}.C + \underbrace{A.B.C}_{\times} + \underbrace{A.\bar{B}.C}_{\times}$$

Step-3 - Neglect the repeated terms.

$$F(A,B,C) = \underbrace{A.B.C}_{m_7} + \underbrace{A.B.\bar{C}}_{m_6} + \underbrace{A\bar{B}.C}_{m_5}$$

$$= \Sigma m(5,6,7)$$

$$F(A,B,C,D) = \underbrace{A.B}_{\substack{\downarrow \\ C \& D \text{ is missing}}} + \underbrace{A.C}_{\substack{\downarrow \\ B \& D \text{ is missing.}}} + ABC\bar{D}$$

$$= A.B.(C+\bar{C})(D+\bar{D}) + A.C.(B+\bar{B})(D+\bar{D}) + ABC\bar{D}$$

$$= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + \underbrace{ABCD}_{\times} + \underbrace{ABC\bar{D}}_{\times} + A\bar{B}CD + AC\bar{B}\bar{D} + \underbrace{ABC\bar{D}}_{\times}$$

$$= \underbrace{ABCD}_{m_{15}} + \underbrace{ABC\bar{D}}_{m_{14}} + \underbrace{AB\bar{C}D}_{m_{13}} + \underbrace{AB\bar{C}\bar{D}}_{m_{12}} + \underbrace{A\bar{B}CD}_{m_{11}} + \underbrace{A\bar{B}C\bar{D}}_{m_{10}}$$

$$= \Sigma m(10,11,12,13,14,15)$$

HW $F(A,B,C,D) = A + CD$

↑ Solve this in SSOP

POS to SPOS Conversion SO

Step-1 - Identify the missing Varible.

$F(A,B,C) = (\bar{A}+\bar{B}).A.(A+B+\bar{C})$

C is missing — B & C is missing

Step-2 - Add with that Varaible & it's complement Seperatly.

$F(A,B,C) = (\bar{A}+\bar{B}+C).(\bar{A}+\bar{B}+\bar{C}).$

$(A+B+C).(A+B+\bar{C}).(A+\bar{B}+C).(A+\bar{B}+\bar{C})$

$(A+B+\bar{C}).$ ×

Step-3 - Neglect repeated terms.

$F(A,B,C) = \underset{M_6}{(\bar{A}+\bar{B}+C)}\ \underset{M_7}{(\bar{A}+\bar{B}+\bar{C})}\ \underset{M_0}{(A+B+C)}.\ \underset{M_1}{(A+B+\bar{C})}$

$\underset{M_2}{(A+\bar{B}+C)}\ \underset{M_3}{(A+\bar{B}+\bar{C})}$

$= \Pi M(0,1,2,3,6,7)$

Eg $F(A,B,C) = (A+\bar{B})(\bar{A}+C)(\bar{A}+\bar{B}+C)$

C is missing — B is missing.

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+C)$

$M_2 \quad M_3 \quad M_4 \quad M_6$

$= \Pi M(2,3,4,6)$

HW $F(A,B,C) = B(A+C)$

↑ Convert into SPOS.

SSOP to SPOS Conversion & SPOS to SSOP Conversion

5]

1] $F(A,B,C) = \Sigma m(0,1,4,7)$. Convert SSOP to SPOS.

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

$n = 3$, [0, 1, 2, 3, 4, 5, 6, 7]

$F(A,B,C) = \Pi m[2,3,5,6]$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

2] $F(A,B,C) = \Pi m(1,2,6)$. Convert SPOS to SSOP.

$= (A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$

$n = 3$, [0, 1, 2, 3, 4, 5, 6, 7]

$F(A,B,C) = \Sigma m(0,5,4,5,7)$

$= \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$

3] $F(A,B,C) = ABC + AB\bar{C} + \bar{A}BC$, Convert SSOP to SPOS.

m_7 m_6 m_3

$= \Sigma m(3,6,7)$

$n = 3$, [0, 1, 2, 3, 4, 5, 6, 7]

$F(A,B,C) = \Pi m(0,1,2,4,5)$

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

4] $F(A,B,C) = (A+\bar{B}+C)(\bar{A}+B+\bar{C})(A+B+\bar{C})$, Convert SPOS to SSOP.

M_2 M_5 M_1

$F(A,B,C) = \Pi m(1,2,5)$

$n = 3$, [0, 1, 2, 3, 4, 5, 6, 7].

$F(A,B,C) = \Sigma m(0,3,4,6,7)$

$= \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

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Examples on SOP & POS S2

1) If $n = 3$ variables then total minterms is 8
total maxterms is 8
total terms is 256

- total minterms = $2^n = 2^3 = 8$
- total maxterms = $2^n = 2^3 = 8$
- total terms = $2^{2^n} = 2^{2^3} = 2^8 = 256$

- total self dual = $2^{2^{n-1}}$

2) $y(A,B,C) = A + \bar{B}C$. Find SSOP & SPOS.

A → B & C is missing
$\bar{B}C$ → A is missing.

$= A.(B+\bar{B})(C+\bar{C}) + \bar{B}.C(A+\bar{A})$

$= A.B.C + A.B.\bar{C} + A.\bar{B}.C + A.\bar{B}.\bar{C} + A.\bar{B}.C + \bar{A}.\bar{B}.C$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$

$m_7 \quad m_6 \quad m_5 \quad m_4 \quad m_1$

$= \Sigma m(1,4,5,6,7)$

$n = 3$, [0, 1, 2, 3, 4, 5, 6, 7]

$y(A,B,C) = \Pi m[0,2,3]$

$= (A+B+C).(A+\bar{B}+C).(A+\bar{B}+\bar{C})$

3) $y = (A+B)(A+C)$. find total minterms & maxterms.
(5) (3)

C is missing, B is missing.

$= (A+B+C).(A+B+\bar{C}).(A+B+C)(A+\bar{B}+C)$

$= (A+B+C).(A+B+\bar{C}).(A+\bar{B}+C).$

$m_0 \quad m_1 \quad m_2.$

$= \Pi m(0,1,2)$

$n = 3$, [0, 1, 2, 3, 4, 5, 6, 7].

$y = \Sigma m(3,4,5,6,7)$

$= \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$