

ENGINEERING MATHEMATICS





[V Cholsky deco- Part one / 1-20 question Part Two - 20 - ()

Neekend sonday TEST-Calculus-F-2 marks 6-1 marks



#Q. Consider the matrix
$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

The eigenvalues of M are

(a)
$$0, 1, 2$$

(b) $0, 0, 3$ Sum = 3 $\gamma(M) = 1$
(c) $1, 1, 1$

a Which of the

TR(M2)=26

#Q. A 3 × 3 matrix M has Tr[M] = 6, $Tr[M^2] = 26$, $Tr[M^3] = 90$. Which of the

following can be possible set of eigenvalues of M?

Trace = SUM of Diagonal (R/M) = 6 TR(M)=90

$$\frac{T_{R}(M) = -1 + 3 + 4 = 6}{T_{R}[M^{3}] = (-1)^{3} + [3]^{3} + [4]^{2} = 26}$$

$$\frac{T_{R}[M^{3}] = (-1)^{3} + [3]^{3} + [4]^{3} = 90}{T_{R}[M^{3}] = (-1)^{3} + [3]^{3} + [4]^{3} = 90}$$



#Q. The eigenvalues of the matrix
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

Slide-4

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

where n_1 , n_2 , n_3 $A = \begin{bmatrix} D & N_1 & N_2 \\ -N_1 & N_2 \end{bmatrix}$

9)
$$A = \begin{bmatrix} D & N_1 & N_2 \\ -N_1 & D & N_3 \end{bmatrix}$$

Replace It $\begin{bmatrix} -N_2 - N_3 & D \end{bmatrix}$

Wrong-matex anti Symmet

(c)
$$0, 1 + i, -1 - i$$

$$= \frac{\lambda^{3} - (0 + 0 + 0)\lambda^{2} + (\eta^{2} + \eta^{2})\lambda = 0}{\lambda^{3} + (\eta^{2} + \eta^{2} + \eta^{2})\lambda = 0} = 0$$

(d) 0, 0, 0

 $= 1 + (1) + 12 + 13) \lambda = 0$ Square It + Add $\eta_1^2 + \eta_2^2 + \eta_3^2 =$ put the value n2+n2+n Eyen values =) y(yz+1)=D

If 1/12,713 Components o Unt vector Unt Venter

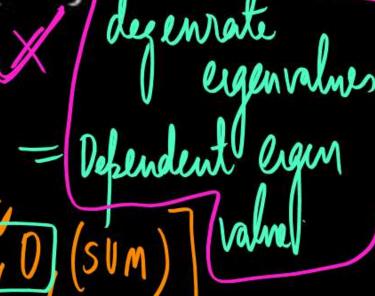


Consider a $n \times n$ (n > 1) matrix A, in which A_{ii} is the product of the

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$
 3×3

Has one degenerate eigenvalue with degeneracy (n-1)

- Has two degenerate eigenvalues with degeneracies 2 and (n-2)
- Has one degenerate eigenvalues with degeneracy n
- (d) Does not have any degenerate eigenvalues





A 2 × 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $i\pi/6$. The smallest value of 'n'

such that
$$A^n = I$$

$$A \rightarrow e^{\frac{i\pi}{5}} e^{\frac{i\pi}{6}}$$

$$A \rightarrow e^{\frac{i\pi}{5}} e^{\frac{i\pi}{6}}$$

$$\lambda_1 = e^{\frac{i\pi}{5}}$$

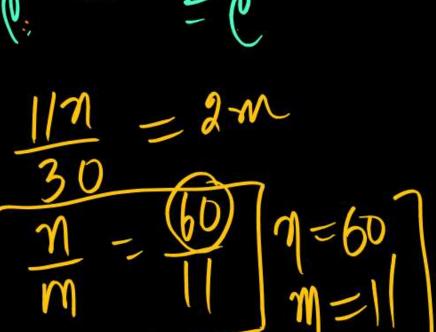
$$\lambda_2 = e^{\frac{i\pi}{5}}$$

$$\lambda_2 = e^{\frac{i\pi}{5}}$$

- 20 (a)
- 30 (b)
- (c)
- 120

```
Pscodnet of eigen values of det A
```

$$0 \frac{11\eta\pii}{30} = 1$$



$$A \rightarrow \lambda_1, \lambda_2$$
 $A^{M} = \lambda_1^{M}, \lambda_2^{M}$
 $1 = 2 \text{ MMTU}$
 $1 = 2 \text{ CMA} + 1 \text{ SMD} = 2 \text{ io}$
 $1 = 2 \text{ CMA} + 1 \text{ SMD} = 2 \text{ io}$
 $1 = 2 \text{ CMA} + 1 \text{ SMD} = 2 \text{ io}$



Given a 2 × 2 unitary matrix satisfying U'U = UU' = I with det U = $e^{i\phi}$, #Q.

one can construct a unitary matrix V(V'V = VV' = I) with det V = 1 from

it by

$$V = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
 ad-bc= $e^{i\beta}$ $(V)^T V = I$

Multiplying U by e-i\psi/2

Multiplying a single element of U by e-io (b)

Multiplying any row or column by e-i\u00f3/2 (c)

Multiplying U by $e^{-i\phi}$ $V = e^{-i\phi/2}$ $V = e^{-i\phi/2}$ V

$$V = Ve^{-i\beta}$$

Slide-7

$$V = Ve^{-\frac{ib}{2}} = \begin{bmatrix} ae^{-\frac{ib}{2}} & be^{-\frac{ib}{2}} \\ ce^{-\frac{ib}{2}} & de^{-\frac{ib}{2}} \end{bmatrix}$$

$$V = e^{-\frac{ib}{2}} e^{-\frac{ib}{2}} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

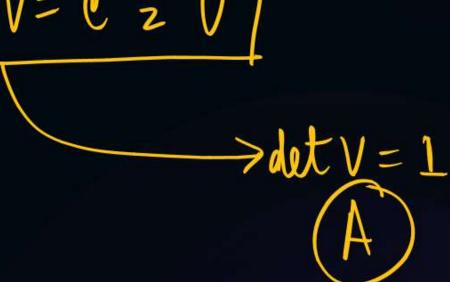
$$det V = e^{-\frac{ib}{2}} \begin{bmatrix} ad - be \\ ad - be \end{bmatrix}$$

$$= e^{-\frac{ib}{2}} \begin{bmatrix} ad - be \\ det V \end{bmatrix}$$

$$= e^{-\frac{ib}{2}} \begin{bmatrix} ad - be \\ det V \end{bmatrix}$$

$$det V = 1$$





Consider the matrix #Q.

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{bmatrix}$$



#Q. The column vector
$$\binom{a}{b}$$
 is a simultaneous eigen vector of

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ if }$$

$$b = 0 \text{ or } a = 0$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ if }$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ b \\ \alpha \end{pmatrix} = \lambda_1 \begin{pmatrix} \alpha \\ b \\ \alpha \end{pmatrix}$$

(a)
$$b = 0$$
 or $a = 0$

(b)
$$b = a \text{ or } b = -2a$$

(c)
$$b = 2a \text{ or } b = -a$$

(d)
$$b = a/2$$
 or $b = -a/2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_1 \begin{bmatrix} \alpha & b & \alpha \\ b & \alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \lambda_1 \begin{bmatrix} a \\ b \\ a \end{bmatrix} \begin{bmatrix} \lambda_1 = 1 \\ \lambda_2 = 1 \end{bmatrix}$$

$$A = \lambda \times$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \lambda_2 \begin{bmatrix} a & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a+b=\lambda_{2}a - 0$$

$$a+b=\lambda_{2}a$$

$$2a-\lambda_{2}b=0$$

$$a(1-\lambda_{2})+b=0\times\lambda_{2}$$

$$2a-\lambda_{2}b=0\times1$$

$$a(1-\lambda_{1})+b\times 1=0$$

$$a(1-\lambda_{1})+b\times 1=0$$

$$a(1-\lambda_{1})+b\times 1=0$$

$$a(1-\lambda_{1})+b\times 1=0$$

$$a(1-\lambda_{1})+b\times 1=0$$

$$a(1-\lambda_{1})+b\times 1=0$$

0/2 (1-/2)-20 = 0

b=a)/

Which one of the following is the inverse of tha matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ #Q.

(a)
$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

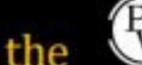
$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

(d)
$$\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d-b \\ -c & a \end{bmatrix}$$



A 3 \times 3 matrix has eigenvalues 0, 2 + i and 2 - i. Which one the following statements is correct?

- The matrix is hermitian
- The matrix is unitary
- The inverse of the matrix exists
- (d) The determinant of the matrix is zero

$$\lambda_1 = D$$

$$\lambda_2 = 2 + i$$

$$\lambda_3 = 2 - i$$



Traceless - D Trace

#Q. A real traceless 4×4 unitary matrix has two eigen values -1 and 1.

The other eigenvalues are

- (a) zero and +2
- (b) -1 and +1
- (c) zero and +1
- (d) +1 and +1

#Q The eigenvalues of the matrix
$$\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

(a)
$$+1$$
 and $+1$

(d)
$$-1$$
 and $+1$

$$A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$

$$= \lambda^{2} - (1+1)\lambda + (1+1^{2}) = 0$$

$$= \lambda^{2} - 2\lambda + 0 = 0$$

$$= \lambda^{2} - 2\lambda = 0$$

$$= \lambda(\lambda - 2) = 0$$

| Ady A | = | A | " (ady A -) = | #Q If A is 2×2 matrix with determinant 2, then the determinant of adj.[adj.[adj(A-1)]] is equal to

- 1/512 (a)
- (b) 1/1024
- 1/128 (c)
- 1/256

$$|ady(ady(A^{-1})| = \left(\frac{1}{2^2}\right)^2$$

$$ady(ady(A^{-1})) = \left(\frac{1}{4^2}\right)^2$$

$$= \frac{1}{2^{56}}$$



#Q The eigenvalues of $(A^4 + 3A - 2I)$, where A is A =

21), where A is
$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$
 are
$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$
 expensations
$$A^{4} + 3A - 2I \longrightarrow \lambda = 1 = (1) + 3 \times 1 - 2$$

$$= 2$$

$$\Rightarrow \lambda = 2 = (2) + 2 \times 3 - 2$$

$$\Rightarrow \lambda = 3 = (3) + 2 \times 3 - 3$$



 $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \end{pmatrix}$

are



A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, Transform

vector \vec{x} from a three dimensional space to a two-dimensional real

space. The transformation matrix T is

(a)
$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$
 (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(b)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

(d)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

space. The transformation matrix T is
$$(a) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \top (b) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \top$$

$$(b) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \top$$

$$(c) \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \top (d) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \top$$

$$(d) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \top$$

$$(d) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \top$$





Thank You!



