

# GATE DATA SCIENCE AND AI



## Probability & Statistics

Discrete Probability Distribution & Continuous  
Probability Distribution

DPP Discussion Notes



By- Rahul sir

Distributions  
35 questions

{ Max/min  
10 question }  $\Rightarrow$  Part 02  
{ Order  
Statistics }

Part 1-20 questions

DPP 03 - Part - one





## Topic : Probability & Statistics



Question 03

#Q. Let  $X$  be a poisson random variable and  $P(X=1) + 2P(X=0) = 12P(X=2)$ . Which one of the following statements is TRUE ?

**A**  $0.40 < P(X=0) \leq 0.45$

**B**  $0.45 < P(X=0) \leq 0.50$

**C**  $0.50 < P(X=0) \leq 0.55$

**D**  $0.55 < P(X=0) \leq 0.60$

$X \sim$  Poisson random variable  
Parameter =  $\lambda$   
 $P(X=1) + 2P(X=0) = 12P(X=2)$

We know that Poisson Distribution

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$x =$  No. of success



$$\begin{aligned}
 P(X=1) + 2P(X=0) &= 12P(X=2) \\
 &= \frac{e^{-\lambda} \lambda^1}{1!} + 2 \frac{e^{-\lambda} \lambda^0}{0!} = 12 \frac{e^{-\lambda} \lambda^2}{2!} \\
 &= e^{-\lambda} \lambda + 2e^{-\lambda} = 6e^{-\lambda} \lambda^2 \\
 &= 6\lambda^2 e^{-\lambda} - 2e^{-\lambda} - \lambda e^{-\lambda} = 0 \\
 &= e^{-\lambda} [6\lambda^2 - \lambda - 2] = 0 \\
 &= 6\lambda^2 - \lambda - 2 = 0 \text{ — quadratic eqn} \\
 &\quad \lambda = \frac{2}{3} \quad \lambda = -\frac{1}{2}
 \end{aligned}$$

$$P[X=0] = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-2/3} = \underline{0.51}$$

Poisson random variable:

$$\lambda > 0 \quad \lambda = -\frac{1}{2} \text{ (rejected)}$$

$$\lambda = \frac{2}{3}$$

mean of random vari

$$\text{Parameter } \lambda = \frac{2}{3}$$

$$0.50 \leq P(X=0) \leq 0.55$$

©





## Topic : Probability & Statistics

#Q. A discrete random variable X has the following probability function

$$\text{mean} = \sum_{i=1}^n x_i p_i = 32$$

$$\text{variance } V(X) = \sum_{i=1}^n x_i^2 p_i - \left[ \sum_{i=1}^n x_i p_i \right]^2 = 116$$

x	:	10	20	30	40	50
y	:	0.1	0.1	0.4	0.3	0.1

$$\frac{\mu_x = \text{mean}}{\sigma_x = \text{S.D.}}$$

$$\text{Standard deviation} = 10.77$$

Denote by  $\mu_x$  and  $\sigma_x$  the mean and the standard deviation of X.  
Find  $P(|X - \mu_x| \leq \sigma_x)$ .

**A**

1

$$P[(X - \mu_x) \leq \sigma_x] \\ = P[(X - \mu_x) \leq +\sigma_x]$$

**B**

0.8

**C**

0.7

**D**

0.5

$$P[|X - \mu_x| \leq \sigma_x]$$

$$\mu_x = 32$$

$$\sigma_x = 10.77$$

$$= P[-\sigma_x \leq X - \mu_x \leq \sigma_x]$$

$$= P[\mu_x - \sigma_x \leq X \leq \mu_x + \sigma_x]$$

$$= P[32 - 10.77 \leq X \leq 32 + 10.77]$$

$$= P[\overset{30}{\boxed{21.23}} \leq X \leq \overset{40}{\boxed{42.77}}]$$

$$= P(30) + P(40)$$

$$= 0.3 + 0.4$$

$$= 0.7 \text{ Ans}$$

Ⓑ





## Topic : Probability & Statistics



✓ max/min of random variables

#Q.  $X$  has a uniform distribution on the interval  $(0, 2)$ , and  $Y = \max\{X, 1\}$ . Find  $\text{Var}(Y)$ .

**A** 0.05

**B** 0.10

**C** 0.15

**D** 0.20



## Topic : Probability & Statistics

- #Q.  $X$  has a Poisson distribution with a mean of 2.  
 $Y$  has a geometric distribution on the integers  $0, 1, 2, \dots$ , also with mean 2.  
 $X$  and  $Y$  are independent.  
Find  $P(X = Y)$

$$X \sim P(\lambda) \rightarrow 2$$

$$Y \sim \text{geo}(\cdot) \text{ mean} = 2$$

$$f(x) = (1-p)^{x-1} p \quad x=1, 2, 3, \dots$$

$$f(x) = p(1-p)^x \quad x=0, 1, 2, 3, \dots$$

$X$  — Poisson  
 $Y$  — geometric

If  $X$  and  $Y$  Are Independent

$$P[X=x]P[Y=y]$$

multiple

**A**

$$\frac{e^{-2/3}}{3}$$

**C**

$$\frac{1}{3}$$

**B**

$$\frac{e^{-1/3}}{3}$$

**D**

$$\frac{e^{1/3}}{3}$$



in given questions { If  $X$  is geometric distribution  $f(x) \rightarrow y$

$$f(y) = p(1-p)^k \quad k = 0, 1, 2, 3, \dots$$

mean  $\text{mean} = E[X] = \frac{q}{p} \quad \text{var}(X) = \frac{1}{p^2}$

mean = 2

$$\frac{q}{p} = 2 \quad \frac{1-p}{p} = 2$$

$$p = \frac{1}{3}$$

Poisson random variable

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad \lambda = 2$$

$$\left( \frac{e^{-2} (2)^x}{x!} \right) = \text{Poisson}$$

If geometric  $k = 1, 2, \dots$

$$f(y) = p(1-p)^{k-1}$$

$$E[X] = \frac{1}{p} \quad \text{var}(X) = \frac{1}{p^2}$$

$$P(Y=y) \quad f(y) = \frac{1}{3} \left( \frac{2}{3} \right)^y \quad y = 0, 1, 2, 3, \dots$$

$$P[X=x] f(x) = \frac{e^{-2} (2)^x}{x!} \quad x = 0, 1, 2, 3, \dots$$



If Poisson + geometric Random Variable both are indep.

$$\underbrace{P[X=x]}_{P(\lambda)} \underbrace{P[Y=y]}_{\text{geo}(p)} \Rightarrow \sum_{\substack{x=0 \\ y=0}}^{\infty} \frac{e^{-2} (2)^x}{x!} \cdot \frac{1}{3} \left(\frac{2}{3}\right)^y$$

$$= \frac{e^{-2}}{3} \sum_{\substack{x=0 \\ y=0}}^{\infty} \frac{(2)^x}{x!} \left(\frac{2}{3}\right)^y$$

$x=y$

$$\Rightarrow \frac{e^{-2}}{3} e^{\frac{4}{3}}$$

$$= \frac{e^{-\frac{6+4}{3}}}{3}$$

$$= \left( \frac{e^{-2/3}}{3} \right) \text{ (A)}$$

$$= \frac{e^{-2}}{3} \sum_{x=0}^{\infty} \frac{2^x}{x!} \left(\frac{2}{3}\right)^x$$

$$= \frac{e^{-2}}{3} \sum_{x=0}^{\infty} \frac{\left(\frac{4}{3}\right)^x}{x!}$$





## Topic : Probability & Statistics



$$\mu_x = 1$$
$$\sigma_x^2 = 4 \quad \sigma_x = 2$$

#Q. If  $X$  has a normal distribution with mean 1 and variance 4, then  
 $P[X^2 - 4X \leq 0] = ?$

$$P(X^2 - 4X \leq 0) = P(X^2 - 4X + 4 \leq 4)$$

$$= P[(X-2)^2 \leq 4]$$

$$= P(-2 \leq X-2 \leq 2)$$

$$= P(0 \leq X \leq 4)$$

$$= P\left(\frac{0-\mu}{\sigma} \leq X \leq \frac{4-\mu}{2}\right)$$

$$= P(-0.5 \leq X \leq 1.5)$$

Using Tables  
= 0.6

**A**

Less than 0.15

**B**

At least 0.15 but less than 0.35

**C**

At least 0.35 but less than 0.55

**D**

At least 0.55 but less than 0.75





## Topic : Probability & Statistics

$$X \rightarrow \theta$$

#Q. X has an exponential distribution with a mean of  $\theta > 0$ , where

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad x \geq 0$$

$$Y = \begin{cases} \frac{X}{2} & \text{if } X \leq \theta \\ X & \text{if } X > \theta \end{cases}$$

Find  $E[Y]$ .

Function  
of Random  
variable.

**A**  $\frac{\theta}{2}(1 + e^{-1})$

**C**  $\frac{\theta}{2}(1 - e^{-1})$

✓ **B**  $\frac{\theta}{2}(1 + 2e^{-1})$

**D**  $\frac{\theta}{2}(1 - 2e^{-1})$



$$Y = \begin{cases} \frac{x}{2} & x \leq \theta \\ x & x > \theta \end{cases}$$

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$$

$$E[Y] = \int_0^{\theta} \left(\frac{x}{2}\right) \frac{1}{\theta} e^{-\frac{x}{\theta}} dx + \int_{\theta}^{\infty} x \cdot \frac{1}{\theta} e^{-\frac{x}{\theta}} dx$$

$$E[Y] = \frac{\theta}{2} [2e^{-1} + 1]$$

(B)

EXPECTATION

Function of random variable

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx \quad f(x) = \lambda e^{-\lambda x}$$

$$\lambda = \frac{1}{\mu}$$

$$E[Y] = \int_{-\infty}^{\infty} Y f(x) dx$$

$$= \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

$$= \frac{1}{\theta} e^{-\frac{x}{\theta}}$$

$$\rightarrow x = g(x)$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) dx$$

Integrating  
by parts





## Topic : Probability & Statistics

Do It

#Q. If  $X$  has exponential distribution with mean 2, then is  $P(X < 1 | X < 2)$ .

$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

$$\mu = 2$$

$$P(X < 1 | X < 2) = P\left(\frac{X < 1 \cap X < 2}{P(X < 2)}\right)$$

**A**  $(1 - e^{-2}) / (1 - e^{-4})$

**B**  $(1 - e^{-0.5}) / (1 - e^{-1})$

**C**  $(1 - e^{-1}) / (1 - e^{-2})$

**D**  $(1 - e^{-0.5}) / (1 - e^{-4})$

$$= \frac{1 - e^{-0.5}}{1 - e^{-1}}$$





## Topic : Probability & Statistics

#Q. In a Poisson distribution, the probability of observing 3 is  $\frac{2}{3}$  times that of observing 4. The mean of the distribution is

Using Poisson Distribution

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0, 1, 2, 3, \dots$$

$$P(3) = \frac{2}{3} P(4)$$

$$\frac{e^{-\lambda} \lambda^3}{3!} = \frac{2}{3} \frac{e^{-\lambda} \lambda^4}{4!}$$

$$\frac{e^{-\lambda} \lambda^3}{3 \times 2 \times 1} = \frac{2}{3} \times \frac{e^{-\lambda} \lambda^4}{4 \times 3 \times 2 \times 1}$$

$$6e^{-\lambda} \lambda^3 = e^{-\lambda} \lambda^4$$

**A** 5

**C** 7

✓ **B** 6

**D** 8

$$6e^{-\lambda}\lambda^3 = e^{-\lambda}\lambda^4$$

$$e^{-\lambda}\lambda^4 - 6e^{-\lambda}\lambda^3 = 0$$

$$\lambda^3 e^{-\lambda} [\lambda - 6] = 0$$

$$\boxed{\lambda = 6}$$

mean of the distribution  
 $\boxed{\lambda = 6}$   
 ✓





## Topic : Probability & Statistics

#Q. The number of misprints per page of a book ( $X$ ) follows the Poisson distribution such that  $(X = 1) = P(X = 2)$ . If the book contains 500 pages, the expected number of pages containing at most one misprint is

$$(2\lambda - \lambda^2)e^{-\lambda} = 0$$

$$(\lambda^2 - 2\lambda)e^{-\lambda} = 0$$

$$\lambda(\lambda - 2)e^{-\lambda} = 0$$

**A**  $5005e^{-2}$   $\lambda = 2$  ✓  $\lambda = 0$  (reject)

**C**  $15006e^{-2}$   $15000e^{-2}$

**B**  $10005e^{-2}$

**D**  $500(1 - 3e^{-2})$

$$P(X=1) = P(X=2)$$

$$\frac{e^{-\lambda} \lambda^1}{1!} = \frac{e^{-\lambda} \lambda^2}{2!}$$

$$2e^{-\lambda} \lambda = \lambda^2 e^{-\lambda}$$



$$\lambda = 2$$

$$\begin{aligned} P(X \leq 1) &= P(X=0) + P(X=1) \\ \text{at most one Page} &= \frac{e^{-\lambda} \lambda^0}{0!} + \frac{e^{-\lambda} \lambda^1}{1!} \\ &= e^{-2} + 2e^{-2} \\ &= \underline{3e^{-2}} \end{aligned}$$

$$\begin{aligned} \text{How much Pages} & \\ &= 500 \times 3e^{-2} \\ &= \underline{1500e^{-2}} \end{aligned}$$

(C)





## Topic : Probability & Statistics



$$\lambda = 3$$

#Q. Security cameras in a railway reservation office indicate that the number of customers arriving before the opening of counters on any given day has the Poisson distribution with mean 3. The minimum number of counters it should open so that there is at least 90% chance of all the waiting customers being immediately served is

Minimum No. of counters  
= 3 ✓

**A** 3 ✓

**B** 4

**C** 5

**D** 6



$$\lambda = 3$$

$$P[X=0] = \frac{e^{-\lambda} \lambda^0}{0!} \quad P[X=1] = \frac{e^{-\lambda} \lambda^1}{1!}$$

$$P[X=2] = \frac{e^{-\lambda} \lambda^2}{2!} \quad P[X=3] = \frac{e^{-\lambda} \lambda^3}{3!}$$

$$P[X=x] = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\frac{e^{-\lambda} \lambda^0}{0!} + \frac{e^{-\lambda} \lambda^1}{1!} + \frac{e^{-\lambda} \lambda^2}{2!} + \frac{e^{-\lambda} \lambda^3}{3!} \geq 0.90$$

$$= \frac{e^{-3} (3)^0}{0!} + \frac{e^{-3} (3)^1}{1!} + \frac{e^{-3} (3)^2}{2!} + \frac{e^{-3} (3)^3}{3!} \geq 0.90$$

0 counter  
1 counter  
2 counter  
3 counter

0.90

$N=3$

$n=3$

0+1+2+3

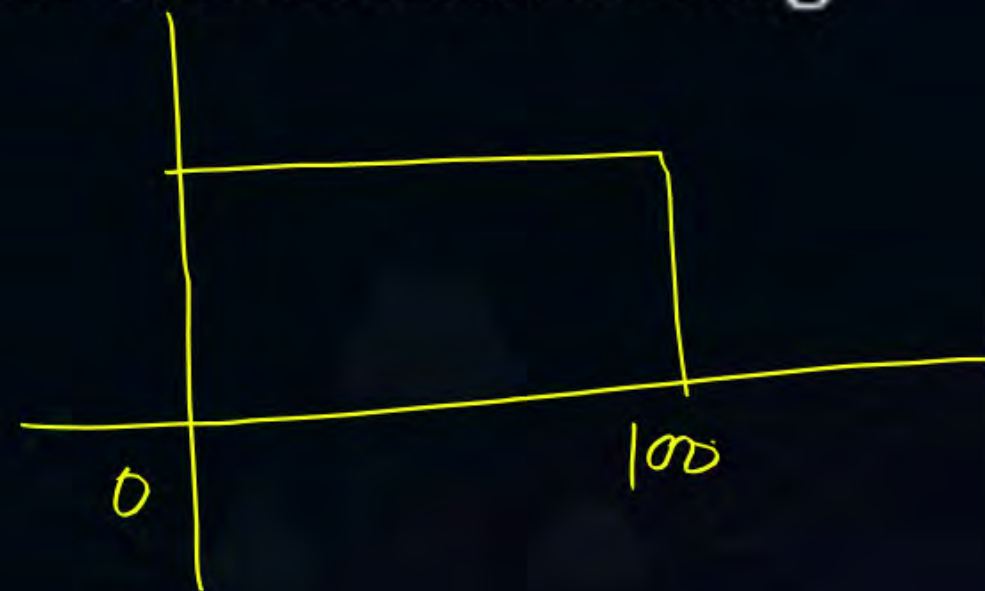




## Topic : Probability & Statistics



#Q. A thousand candidates appear for an examination. Their scores are independent and have a continuous uniform distribution over the range 0 to 100. The variance of the number of candidates having scores above 30 is



**A** 210

**B** 700

**C**  $25/3$

**D** 300



$$f(x) = \begin{cases} \frac{1}{100} & [0, 100] \\ 0 & \text{otherwise} \end{cases}$$

$$P(X > 30) = \int_{30}^{100} \frac{1}{100} dx = \frac{70}{100} = 0.7$$

$$P(30 \text{ above}) = 0.7$$

$$P(\overline{30} \text{ above}) = 0.3$$

Bernoulli  
Trials

$$f(x) = \frac{1}{(b-a)}$$

0

$$p = 0.70$$

$$q = 0.30$$

100

$$\text{variance} = n p q \longrightarrow \text{binomial mean} = n p$$

$$= 1000 \times 0.7 \times 0.3$$

$$= 210 \text{ students}$$

$$\text{var} = n p q$$

A





## Topic : Probability & Statistics

$$X \sim N(\mu, \sigma^2)$$

mean = 1    var(X) = 4  
S.D = 2

#Q. If X has a normal distribution with mean 1 and variance 4, then  $P[X^2 - 2X \leq 8] = ?$

$$\begin{aligned} &= P(-2 \leq X \leq 4) \\ &= P\left(\frac{-2 - \mu}{\sigma} \leq Z \leq \frac{4 - \mu}{\sigma}\right) \\ &= P\left(\frac{-2 - 1}{2} \leq Z \leq \frac{4 - 1}{2}\right) \\ &= P(-1.5 \leq Z \leq 1.5) \end{aligned}$$

$$\begin{aligned} P(X^2 - 2X \leq 8) &= P((X^2 - 2X + 1) \leq 8 + 1) \\ &= P((X - 1)^2 \leq 9) \\ &= P((X - 1)^2 \leq 3^2) \end{aligned}$$

$$\Rightarrow P(-3 \leq (X - 1) \leq 3)$$

$$\Rightarrow P(-3 + 1 \leq X \leq 3 + 1)$$

①

**A** 0.13    Z SCORE  
= 0.86

**C** 0.75

**B** 0.43

✓ **D** 0.86





## Topic : Probability & Statistics



$$\begin{array}{ccc} \text{Person 1} & \text{Person 2} & \text{Person 3} \\ X_1 & X_2 & X_3 \end{array}$$

- #Q. Three individuals are running a one kilometers race. The completion time for each individual is a random variable.  $X_i$  is the completion time, in minutes, for person  $i$ .
- $X_1$  : uniform distribution on the interval  $[2.9, 3.1]$  *Uniformly*
- $X_2$  : uniform distribution on the interval  $[2.7, 3.1]$  *Uniformly*
- $X_3$  : uniform distribution on the interval  $[2.9, 3.3]$  *Uniformly*
- The three completion times are independent of one another.
- Find the expected latest completion time (nearest .1).

**A** 2.9

**B** 3.0

**C** 3.1

**D** 3.2



$$E[X_1] = \frac{a+b}{2} = \frac{2.9+3.1}{2}$$

$$E[X_1] = 3$$

$$E[X_2] = \frac{2.7+3.1}{2} = \frac{5.8}{2} = 2.9$$

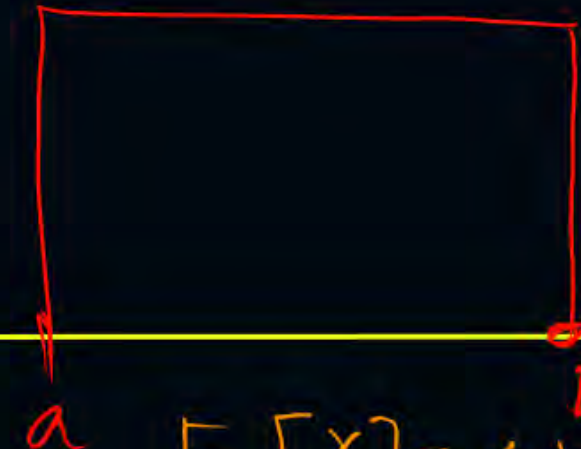
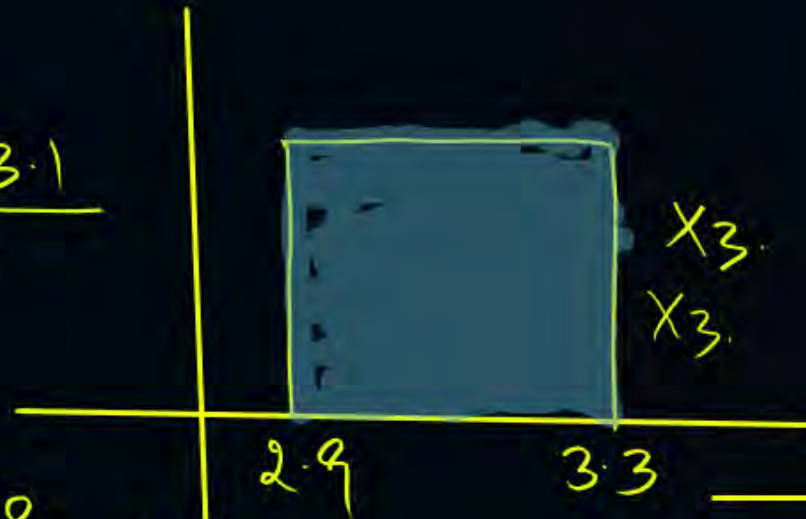
$$E[X_3] = \frac{2.9+3.3}{2} = \frac{6.2}{2} = 3.1$$

Completion Time

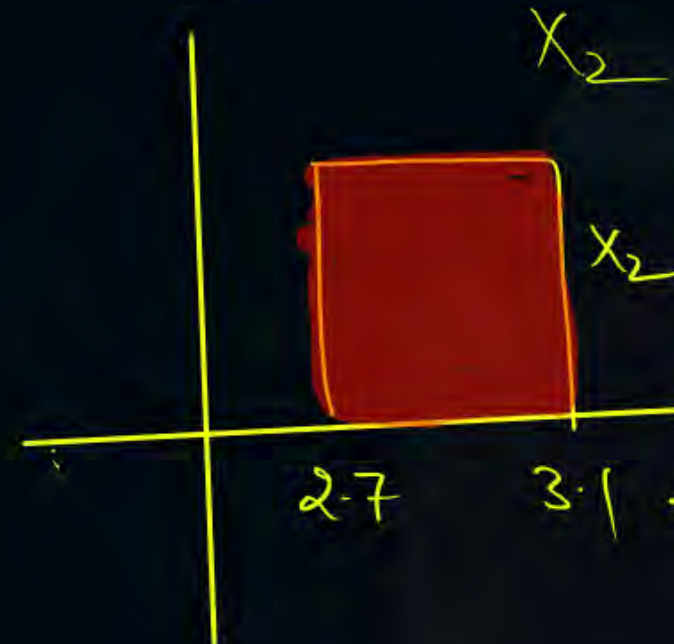
$$E[X] = E[X_1] + E[X_2] + E[X_3]$$

$$= \frac{3 + 2.9 + 3.1}{3}$$

$$E[X] = 3$$



$$E[X] = \frac{a+b}{2}$$



$$P(X_1) = \frac{1}{3}$$

$$P(X_2) = \frac{1}{3}$$

$$P(X_3) = \frac{1}{3}$$







## Topic : Probability & Statistics

#Q. Customers arrive randomly and independently at a service window, and the time between arrivals as an exponential distribution with a mean of 12 minutes. Let  $X$  equal the number of arrivals per hour. What is  $P[X = 10]$ ?

$$\lambda = 12 \text{ min}$$

$$\mu = \frac{60}{12} = 5$$

$$P[X=10] = \frac{e^{-\lambda} \lambda^x}{x!}$$

Poisson  
random  
variable

$$= \frac{e^{-5} (5)^{10}}{10!}$$

**A**  $\frac{10e^{-12}}{10!}$

**B**  $\frac{10^{-12} e^{-10}}{10!}$

**C**  $\frac{12^{-10} e^{-10}}{10!}$

**D**  $\frac{5^{10} e^{-5}}{10!}$





## Topic : Probability & Statistics

#Q. Which one of the following is not possible for a binomial distribution?

Do yourself

**A**

Mean = 2, variance =  $\frac{3}{2}$

**B**

Mean = 5, variance = 9

**C**

Mean = 10, variance = 5

**D**

Mean = 4, variance =  $\frac{8}{3}$





## Topic : Probability & Statistics

#Q. A coin is twice as likely to turn up tails as heads. If the coin is tossed independently, what is the probability that the third head occurs on the fifth toss?

Coin  $\begin{cases} H \\ T \end{cases}$

$$\Rightarrow {}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{4-2}$$

$$= 6 \times \frac{1}{9} \times \frac{4}{9}$$

$$= \frac{24}{81} = \frac{8}{27}$$

$\boxed{\begin{matrix} 5^{th} \\ \text{Tail} \end{matrix}}$   
 $\downarrow$   
 $3(H)$

$$\downarrow p = \frac{1}{3}$$

$$P(H) = p$$

$$P(\text{Tail}) = 2p$$

$$p + 2p = 1$$

$$\boxed{p = \frac{1}{3}}$$

$$q = \frac{2}{3}$$

$$= \frac{8}{27} \times \frac{1}{3} = \frac{8}{81}$$

**A**  $\frac{8}{81}$

**C**  $\frac{16}{81}$

**B**  $\frac{40}{243}$

**D**  $\frac{80}{243}$



$P(\text{third HEAD in fifth Toss})$

$$P(\text{SUCCESS}) = \frac{1}{2} \quad P(F) = \frac{1}{2}$$

$$\text{SUCCESS} = 2$$

Using binomial dist

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=2) = {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}$$

$$= 6 \times \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{6}{16} = \frac{3}{8}$$

$\textcircled{H} \textcircled{H} \textcircled{T} \textcircled{T} \textcircled{H}$

$$\text{third HEAD} = \frac{1}{2} \checkmark$$

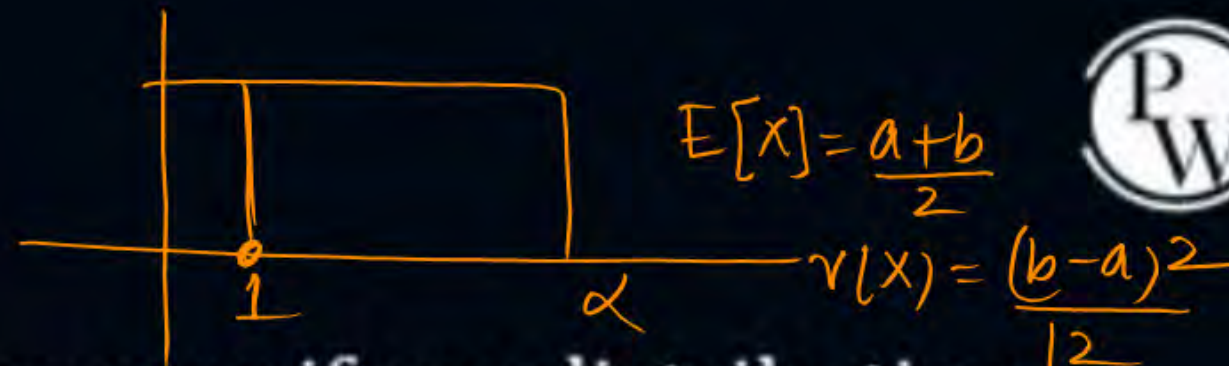
$$= \frac{3}{8} \times \frac{1}{2} = \left(\frac{3}{16}\right)$$

$n=4$   
(No. of trials)





## Topic : Probability & Statistics



#Q. Let  $X$  be a random variable with a continuous uniform distribution on the interval  $(1, \alpha)$  where  $\alpha > 1$ . If  $E[X] = 6$ ,  $Var[X]$ , then  $\alpha =$

$$E[X] = 6$$
$$\frac{\alpha+1}{2} = 6$$
$$\frac{\alpha-1}{2} = \frac{(\alpha-1)^2}{12}$$

$$\frac{\alpha+1}{2} = \frac{(\alpha-1)^2}{12}$$

$$\alpha+1 = \frac{(\alpha-1)^2}{2}$$

$$\alpha+3\alpha+1-1-\alpha^2 = 0$$

$$\alpha^2-3\alpha = 0$$

$$\alpha(\alpha-3) = 0$$

$$\alpha = 0$$
$$\alpha = 3$$

Parameter  $\alpha = 3$

**A** 2

**C** 4

☒ **B** 3

**D** 7





## Topic : Probability & Statistics



H.W

#Q. Suppose that  $X$  has uniform distribution on the interval  $[0, 100]$ . Let  $Y$  denote the greatest integer smaller than or equal to  $X$ . Which of the following is true?

**A**  $P(Y \leq 25) = \frac{1}{4}$

**B**  $P(Y \leq 25) = \frac{26}{100}$

**C**  $E(Y) = 50$

**D**  $E(Y) = \frac{101}{2}$





## Topic : Probability & Statistics

#Q. Let  $X$  be a Geom (0.4) random variable. Then  $P(X = 5 | X \geq 2)$  equals \_\_\_\_.

$X$  is geometric random variable  
 $p = 0.4$

$X$  is geometric Random variable

$$f(x) = (1-p)^{x-1} p \quad x = 0, 1, 2, 3, \dots$$

$$f(x) = (1-p)^{x-1} p \quad x = 1, 2, 3, \dots$$

$$P(X=1) = (1-p)^0 \cdot p$$

$$\begin{aligned} & P(X=5 | X \geq 2) \\ &= \frac{P(X=5 \cap X \geq 2)}{P(X \geq 2)} \\ &= \frac{P(X=5)}{1 - P(X=1)} \end{aligned}$$

$$\begin{aligned} &= \frac{(1-p)^4 \cdot p}{1 - 0.4} = \frac{(0.6)^4 \cdot (0.4)}{(0.6)} = 0.0864 \\ &= 0.4 \times (0.6)^3 \\ &\text{Ans } \checkmark \end{aligned}$$





## Topic : Probability & Statistics

#Q. Let  $X$  be a  $\text{Bin}(2, p)$  random variable, and  $Y$  be a  $\text{Bin}(4, p)$ ,  $0 < p < 1$ .

If  $P(X \geq 1) = \frac{5}{9}$ , then  $P(Y \geq 1) = \underline{\frac{65}{81}}$ .

$$P(X \geq 1) = 1 - P(X=0) = \frac{5}{9}$$

$$P(X=0) = \frac{4}{9}$$

$$= {}^nC_0 p^0 q^{n-0} = \frac{4}{9}$$

$$= q^n = \frac{4}{9}$$

$$= q^2 = \frac{4}{9}$$

$$q = \frac{2}{3}$$

$$p = 1 - \frac{2}{3} = \frac{1}{3}$$

$$X \rightarrow (2, p) \quad 0 < p < 1$$

$$Y \rightarrow (4, p)$$

$$P(X \geq 1) = \frac{5}{9}$$

$$P(X \geq 1)$$

$$p = \frac{1}{3} \quad q = \frac{2}{3}$$

$$\text{Bin}(2, p) \\ n=2$$





$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) & n=4 & \quad X = B(4, p) \\ &= 1 - \left[ {}^n C_0 p^0 q^{n-0} \right] & & \quad \underline{n=4} \\ &= 1 - \left[ q^n \right] = 1 - (1-p)^n & & \quad = 1 - \left(1 - \frac{1}{3}\right)^4 = 1 - \left(\frac{2}{3}\right)^4 \\ & & & \quad = 1 - \frac{16}{81} \\ & & & \quad = \frac{65}{81} \end{aligned}$$





## Topic : Probability & Statistics

#Q. Let  $X$  be a random variable with the probability density function

$$f(x|r, \lambda) = \frac{\lambda^r}{(r-1)!} x^{r-1} e^{-\lambda x}, x > 0, \lambda > 0, r > 0.$$

— gamma distribution  
mean =  $\frac{r}{\lambda}$   
variance  $\frac{r}{\lambda^2}$

If  $E(X) = 2$  and  $\text{Var}(X) = 2$ , then  $P(X < 1)$  equals \_\_\_\_\_.

$$\frac{r}{\lambda} = 2 \quad \frac{r}{\lambda^2} = 2$$

$r = 2$   
 $\lambda = 1$

$$P(X < 1) = \int_0^1 \frac{1^2}{1!} x^{2-1} e^{-x} dx$$
$$= \int_0^1 x e^{-x} dx$$

Ans



# THANK - YOU

Topics to be Covered

- Introduction
- The Role of the Project Manager
- Project Management Process
- Project Management Tools
- Project Management Software
- Project Management Communication
- Project Management Risk Management
- Project Management Quality Management
- Project Management Cost Management
- Project Management Time Management
- Project Management Resource Management
- Project Management Stakeholder Management
- Project Management Procurement Management
- Project Management Integration Management
- Project Management Change Management
- Project Management Closure Management