Assignment - 3

1 Obtain the rank correlation coefficient between the variables and from the following pair of observed values :-

x	50	55	65	50	55	60	50	65	70	75
y	110	110	115	125	140	115	130	120	115	160

x	y	R_1	R_2	$d = R_1 - R_2$	d^2
50	110	9	9.5	-0.5	0.25
55	110	6.5	9.5	-3	9
65	115	3.5	7	-3.5	12.25
50	125	9	4	5	25
55	140	6.5	2	4.5	20.25
60	115	5	7	-2	4
50	130	9	3	6	36
65	120	3.5	5	-1.5	2.25
70	115	2	7	5	25
75	160	1	1	0	0
					$\Sigma d^2 = 134$

$n = 10$

$\Sigma d^2 = 134$

$m_1 = 2$

$m_2 = 2$

$m_3 = 3$

$m_4 = 3$

$m_5 = 2.$

$$R = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \frac{1}{12}(m_3^3 - m_3) + \frac{1}{12}(m_4^3 - m_4) + \frac{1}{12}(m_5^3 - m_5)\right]}{n(n^2-1)}$$

$$R = 1 - \frac{6\left[134 + \frac{1}{12}(8-2) + \frac{1}{12}(8-2) + \frac{1}{12}(27-3) + \frac{1}{12}(27-3) + \frac{1}{12}(8-2)\right]}{10(100-1)}$$

$$1 - \frac{6\left[134 + \frac{6}{12} + \frac{6}{12} + \frac{24}{12} + \frac{24}{12} + \frac{6}{12}\right]}{990}$$

$$1 - \frac{6\left[134 + 0.5 + 0.5 + 2 + 2 + 0.5\right]}{9}$$

$$1 - \frac{6\left[139.5\right]}{990} = 1 - 0.84$$

$$= 0.16.$$

2) Calculate the correlation coefficient for the following height (in inches) of fathers (x) and their sons (Y):

Fathers height (x)	65	66	67	67	68	69	70	72.
Sons Height (Y)	67	68	65	68	72	72	69	71.

x_i	y_i	$\bar{x} = x_i - \bar{x}$ $x_i - 68$	$\bar{y} = y_i - \bar{y}$ $y_i - 69$	x^2	y^2	xy
65	67	-3	-2	9	4	6
66	68	-2	-1	4	1	2
67	65	-1	-4	1	16	4
67	68	-1	-1	1	1	1
68	72	0	3	0	9	0
69	72	1	3	1	9	3
70	69	2	0	4	0	0
72	71.	4	2	16	4	8
$\Sigma x = 544$	$\Sigma y = 552$			36	44.	24.

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{544}{8} = 68.$$

$$\bar{y} = \frac{\Sigma y_i}{n} = \frac{552}{8} = 9.$$

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}} = \frac{24}{\sqrt{36 \cdot 44}} = \frac{24}{39.79}.$$

$$= 0.60$$

3 For certain x and y series which are correlated the two lines of regression are:-

$5x-6y+90=0$ and $15x-8y-130=0$ Find the means of the two series and the correlation coefficient.

$$5x-6y+90=0$$
$$5x-6y=-90 \rightarrow (1)$$
$$15x-8y=130 \rightarrow (2)$$

$$(1)\times(3) = 15x-18y=-270.$$
$$(2)\times(1) = 15x-8y=130$$
$$-10y=-400$$
$$y=\frac{-400}{-10}=40$$
$$5x-6(40)+90=0$$
$$5x-240+90$$
$$5x-150$$
$$5x=150$$
$$x=30$$

$$6y=5x+90$$
$$y=\frac{5x}{6}+\frac{90}{6}$$
$$y=\frac{5}{6}x+15$$
$$b_{yx}=\frac{5}{6}$$

$$15x=8y+130$$
$$x=\frac{8y}{15}+\frac{130}{15}$$
$$b_{xy}=\frac{8}{15}$$

$$(r)=\sqrt{b_{xy}\times b_{yx}}$$
$$\sqrt{\frac{\cancel{18}}{\cancel{6}_3}\times\frac{\cancel{8}4}{\cancel{18}_3}}$$
$$r=\sqrt{\frac{4}{9}}=2/3$$

4 Use least square regression line to estimate the increase in sale revenue inspected from an increase of 7.5% in advertising expenditure.

Firm	A	B	C	D	E	F	G	H
Annual % increase in advertising expenditure	1	3	4	6	8	9	11	14
Annual % increase in sales revenue	1	2	2	4	6	8	8	9

Advertising expenditure (x_i)	Sales revenue (y)	x^2	y^2	xy
1	1	1	1	1
3	2	9	4	6
4	2	16	4	8
6	4	36	16	24
8	6	64	36	48
9	8	81	64	72
11	8	121	64	88
14	9	196	81	126
56	40.	524.	270	373.

$n = 8$

$\bar{x} = \frac{\Sigma x}{n} = \frac{56}{8} = 7$

$n = 8$

$\bar{y} = \frac{\Sigma y}{n} = \frac{40}{8} = 5.$

$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n}$

$373 - \frac{(40)(56)}{8} = 93$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$

$= 524 - \frac{(56)^2}{8} = 132.$

$b = \frac{S_{xy}}{S_{xx}} = \frac{93}{132} = 0.70$

$a = 5 - 0.70(7)$

$a = 5 - 4.9$

$a = 0.1.$

when $x = 7.5$

Regression on y on x.

$y = a + bx.$

$y = 0.1 + 0.70(7.5)$

$y = 5.25 + 0.1$

$y = 5.35.$

5 Marks in statistics and mathematics for 450 students at a certain examination is given below.

Mean marks in statistics.	40
Mean marks in mathematics.	48
Standard deviation of marks in statistics.	12
The variance of marks in mathematics	256
Sum of the product of deviations of marks from their respective means.	42075

Given $n = 450$

Let the marks in statistics and economics be x, y

$\bar{x} = 40, \bar{y} = 48.$

$\sigma_x = 12, \sigma_y^2 = 256, \sigma_y = \sqrt{256} = 16$

$\sum dxdy = 42075.$

$r = \dfrac{\sum dxdy}{n \times \sigma x \times \sigma y}$

$\sum dxdy = 42075.$

$r = \dfrac{42075}{450 \times 16 \times 12} = 0.45$

Regression equation, y on x.

$y - \bar{y} = r \cdot \dfrac{\sigma y}{\sigma x}(x - \bar{x})$

$y - 48 = 0.48\left(\dfrac{\cancel{16}^{\,4}}{\cancel{12}_{\,3}}\right)(x - 40)$

$y - 48 = 0.48\left(\dfrac{4}{3}\right)(x - 40)$

$y - 48 = 0.48(1.3)(x - 40)$

$y - 48 = 0.624(x - 40)$

$y - 48 = 0.624x - 24.96$

$y = 0.624(50) - 24.96 + 48.$

$y = 31.2 + 23.04.$

$y = 54.06.$

6 The following table shows the scores given by three judges A, B and C for ten participants in a competition.

A	25	32	38	18	43	48	45	30	40
B	45	42	38	40	25	28	12	16	22
C.	20	30	35	15	39	37	40	38	33.

which pair of judges have the nearest opinion?

Judge 1	Judge 2	Judge 3	$d_1 = R_1 - R_2$	d_1^2	$d_2 = R_2 - R_3$	d_2^2	$d_3 = R_3 - R_1$	d_3^2
25	45	20	-20	400	25	625	-5	25
32	42	30	-10	100	12	144	-2	4
38	38	35	0	0	3	9	-3	9
18	40	15	-22	484	25	625	-3	9
22	20	19	2	4	1	1	-3	9
43	25	39	18	324	-11	121	-4	16
39	28	37	11	121	-9	81	-2	4
45	12	40	33	1089	-28	784	-5	25
30	16	38	14	196	-22	484	-8	64
40	22	33	18	324	4	16	-7	49
				3042		2890		214

$n = 10$

$R_{12} = 1 - \frac{6\sum d_1^2}{n(n^2-1)}$

$R_{12} = 1 - 6(200)$

$1 - \frac{6(3042)}{10(100-1)} = 1 - \frac{18252}{990} = 1 - 18.4$

$= -17.4$

$R_{23} = 1 - \frac{6\sum d_2^2}{n(n^2-1)} = 1 - \frac{6(2890)}{990}$

$= 1 - 17.51$

$= -16.51$

$R_{31} = 1 - \frac{6\sum d_3^2}{990}$

$1 - \frac{6 \times 214}{990}$

$1 - 1.2$

$= -0.2$

7 The ranks of 15 students in two subjects A and B are given below. The two numbers with brackets denote the ranks of a student in A and B subjects respectively. (1,10) (2,7) (3,2) (4,6) (5,4), (6,8) (7,3) (8,1) (9,11) (10,15) (11,9) (12,5) (13,14) (14,12) (15,13) Find spearman's rank correlation coefficient.

Rank A (R_1)	Rank(B) R_2	$d = R_1 - R_2$	d^2
1	10	-9	81
2	7	-5	25
3	2	1	1
4	6	-2	4
5	4	1	1
6	8	-2	9
7	3	4	16
8	1	7	49
9	11	-2	4
10	15	-5	25
11	9	2	4
12	5	7	49
13	14	-1	1
14	12	2	4
15	13	2.	4

$\Sigma d^2 = 272$

$$R = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

$$1 - \frac{6 \times 272}{15(225-1)} = 1 - \frac{6 \times 272}{15(224)} = 1 - \frac{1632}{3360} = 1 - 0.48.$$

$R = 0.52$