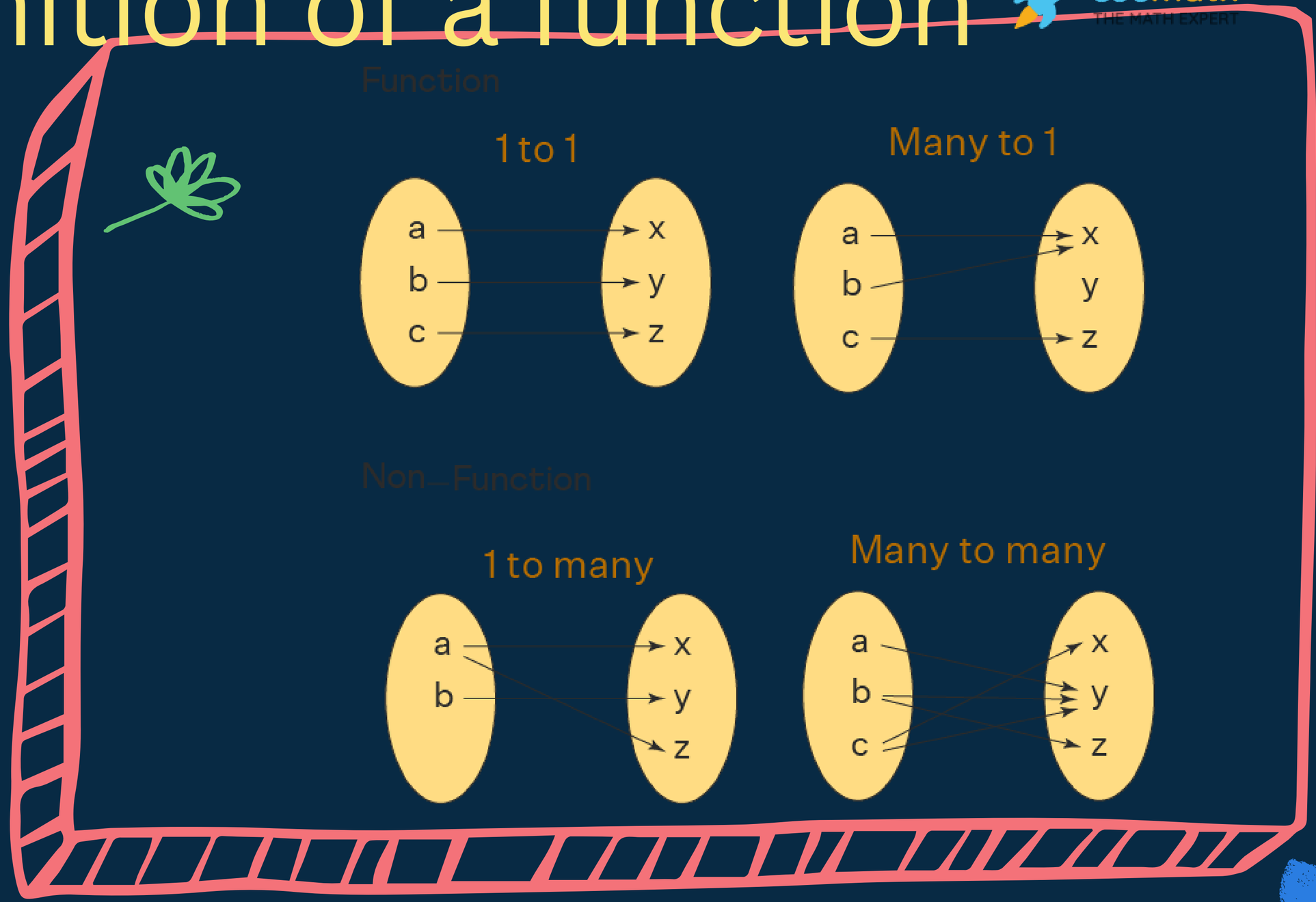
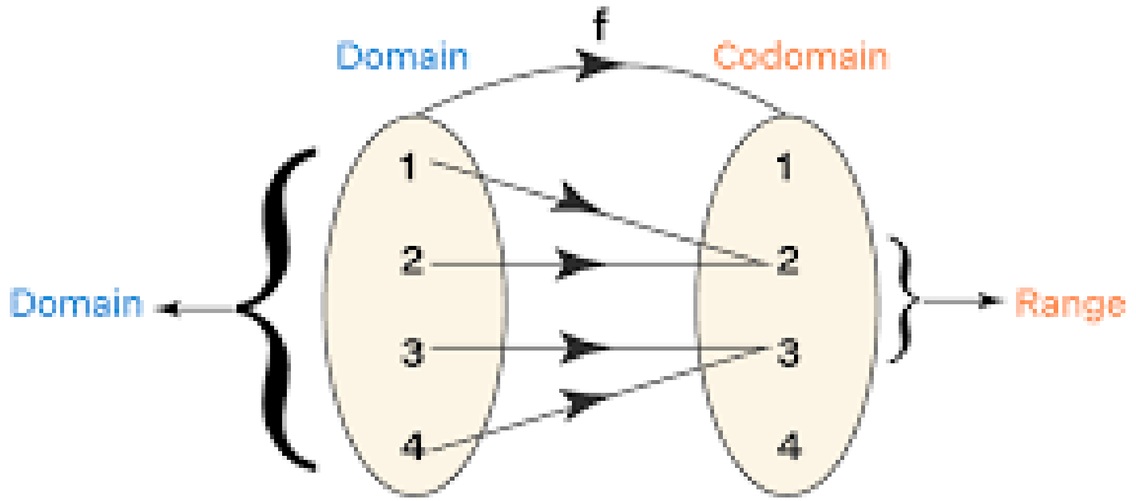
2. Functions

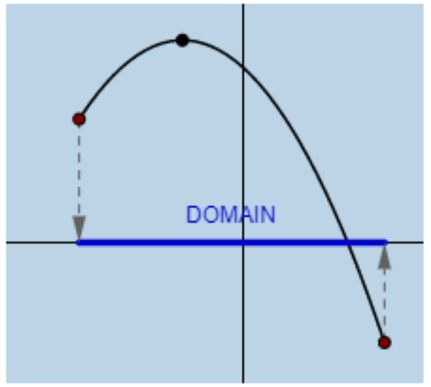
Definition of a function



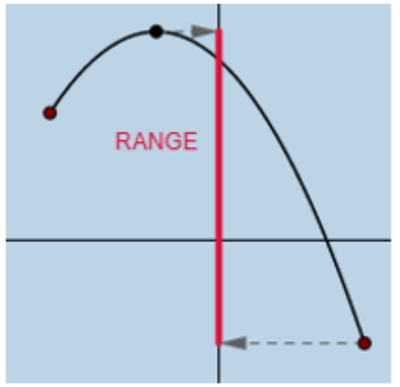
Domain and Range



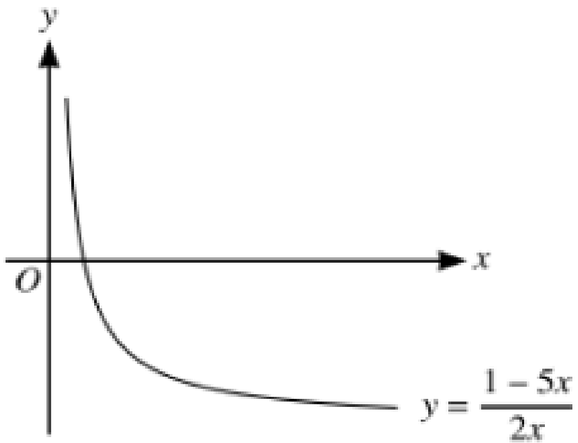
Domain and Range



Domain is all the possible x values of a function.



Range is all the possible y values of a function.



The diagram shows the graph of $y = f^{-1}(x)$, where f^{-1} is defined by $f^{-1}(x) = \frac{1-5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $f(x)$ and state the domain of f. [5]

y=(1-5x)/2x
x=(1-5y)/2y
2yx=1-5y
2yx+5y=1
y(2x+5)=1
y=1/(2x+5)
f(x)=1/(2x+5)

range of f^-1(x): (1-5*0)/2*0 < f^-1(x) ≤ (1-5*2)/2*2
0 < f^-1(x) ≤ -2.25
domain of f(x):0 < x ≤ -2.25

Original function

$f(x) = \sqrt{x+2}$

Domain

$[-2, \infty)$

Range

$[0, \infty)$

Inverse function

$f^{-1}(x) = x^2 - 2$

Domain

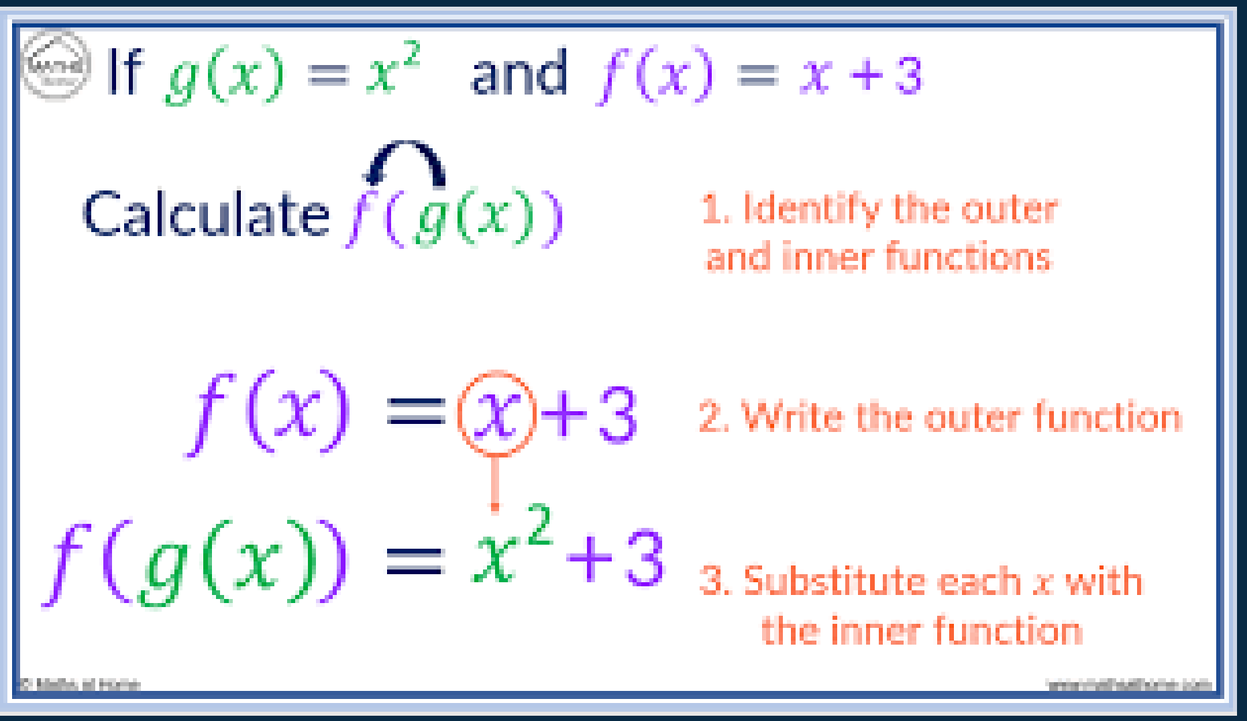
$[0, \infty)$

Range

$[-2, \infty)$

Yup – we got the full switch of the domain & range for the inverse.

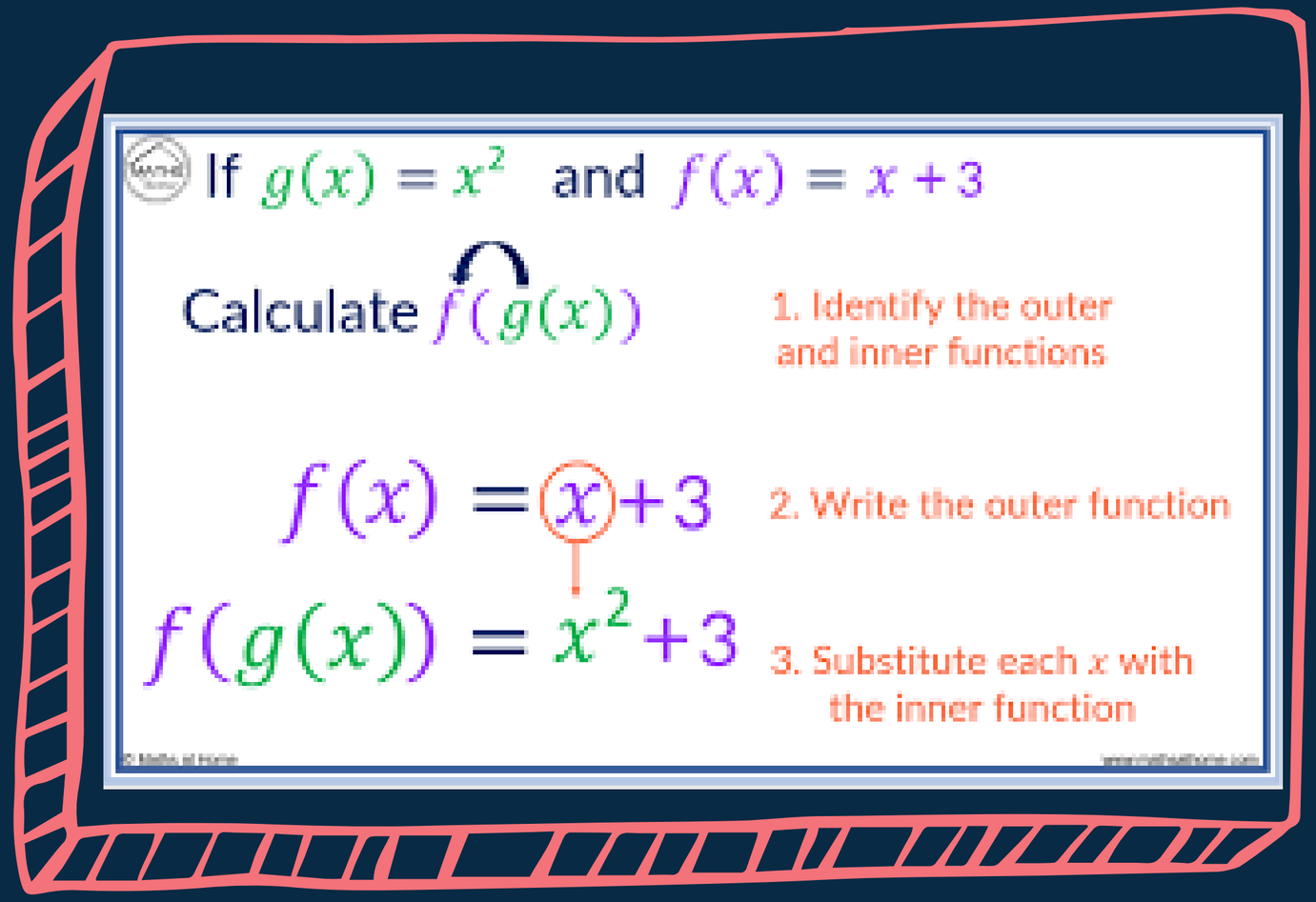
2.2 Composite functions



2.2 Composite functions

fg only exists if the range of g is contained within the domain of f.

In general, fg(x) ≠ gf(x).



Functions f and g are defined by

$$f : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R}, \ x \neq \frac{3}{2}.$$

Solve the equation $ff(x) = gf(2)$.

ff(x) = 10 - 3(10-3x)

ff(x)=10-30+9x

ff(x)=9x-20

gf(x)=10/(3-2(10-3x))

gf(x)=10/(3-20+6x)

gf(x)=10/(6x-17)

9x-20=10/(6x-17)

(9x-20)(6x-17)=10

54x^2-120x-153x+340=10

54x^2-273x+340=10

54x^2-273x+330=0

18x^2-91x+110=0

$$x = \frac{-(-273) + 57}{2 \cdot 54} : \frac{55}{18}$$

$$x = \frac{-(-273) - 57}{2 \cdot 54} : 2$$

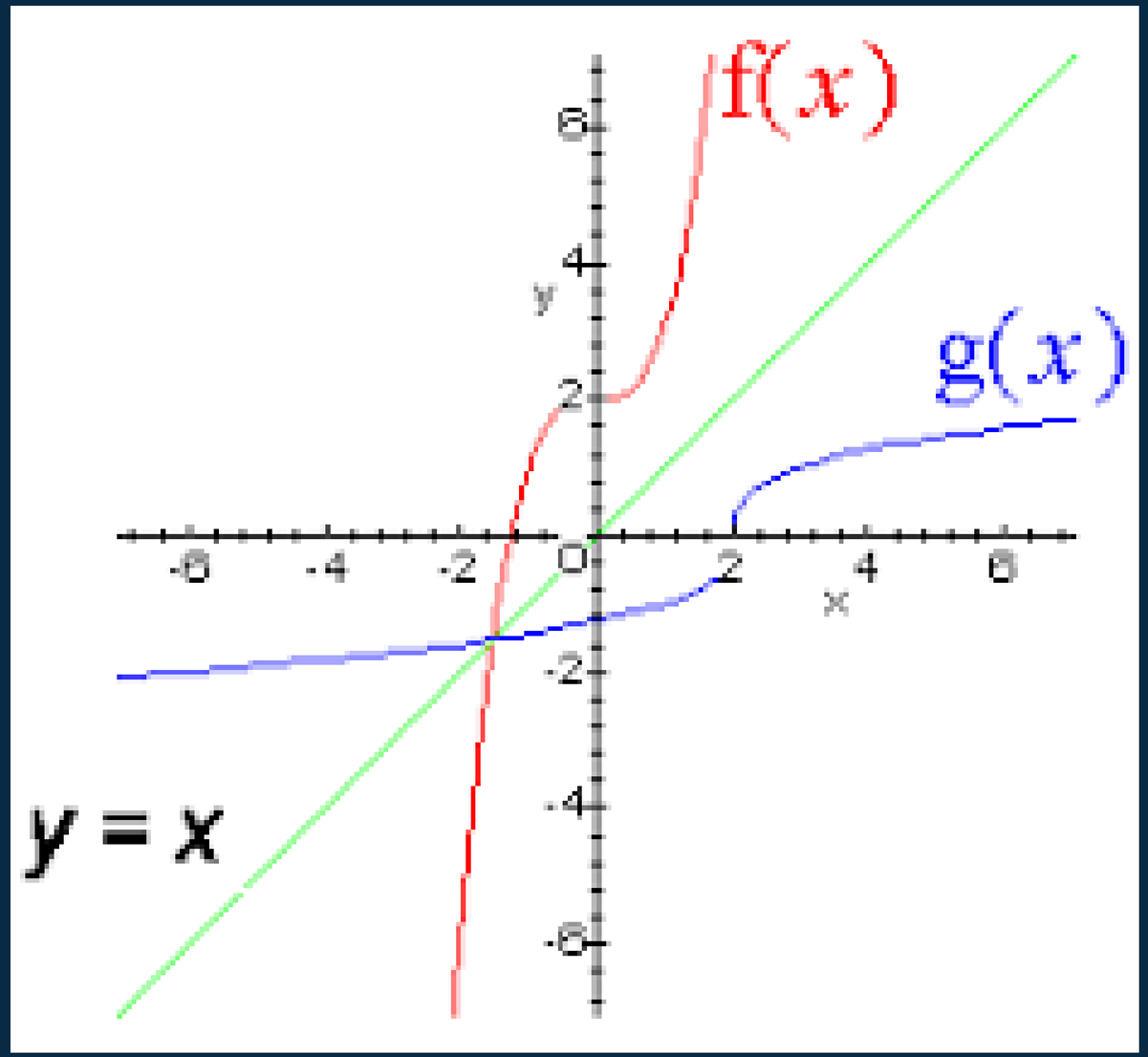
The graph of a function and its inverse

- remember that not every function has an inverse
- An inverse function f ^-1(x) exists if, and only if, the function f(x) is a **one-one mapping**.
- If f and f^−1 are the same function, then f is called a **self-inverse function**.

https://www.google.com/search?q=self+inverse+function&source=lmns&bih=625&biw=1366&rlz=1C1BNSD_enUA978UA978&hl=en&sa=X&ved=2ahUKEwjHgvaCp5z7AhUs0IsBHRRfBHEQ_AUoAHoECAEQAA&safe=active&ssui=on#kpvalbx=_rRlpY4rVKtO0oASSi6ngBQ_28

The graph of a function and its inverse

symmetrical about the line y = x .



Transformation Rules for Functions

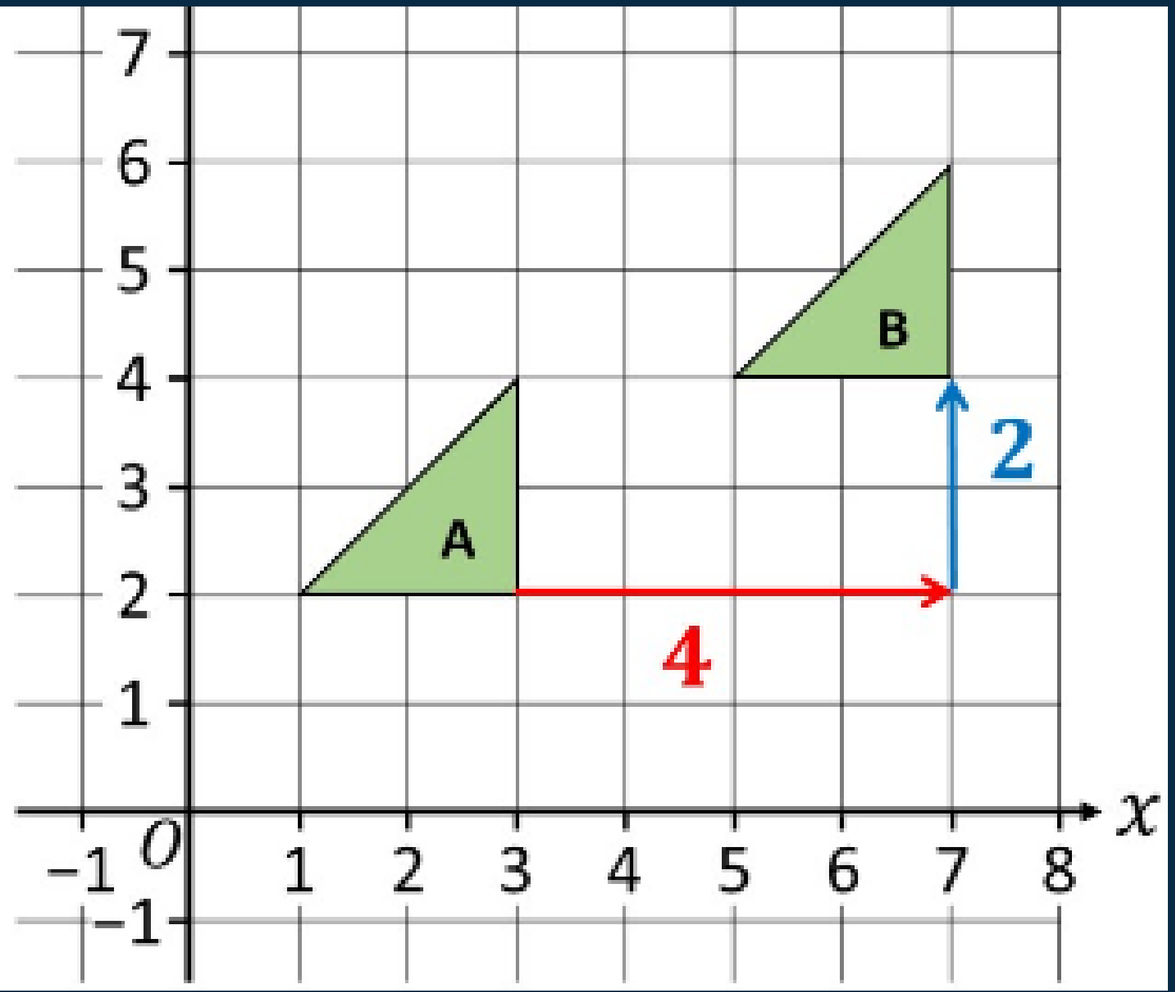
Function Notation	Type of Transformation	Change to Coordinate Point
$f(x) + d$	Vertical translation up d units	$(x, y) \rightarrow (x, y + d)$
$f(x) - d$	Vertical translation down d units	$(x, y) \rightarrow (x, y - d)$
$f(x + c)$	Horizontal translation left c units	$(x, y) \rightarrow (x - c, y)$
$f(x - c)$	Horizontal translation right c units	$(x, y) \rightarrow (x + c, y)$
$-f(x)$	Reflection over x-axis	$(x, y) \rightarrow (x, -y)$
$f(-x)$	Reflection over y-axis	$(x, y) \rightarrow (-x, y)$
$af(x)$	Vertical stretch for $\lvert a\rvert > 1$	$(x, y) \rightarrow (x, ay)$
	Vertical compression for $0 < \lvert a\rvert < 1$	
$f(bx)$	Horizontal compression for $\lvert b\rvert > 1$	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
	Horizontal stretch for $0 < \lvert b\rvert < 1$	

Translation by vector

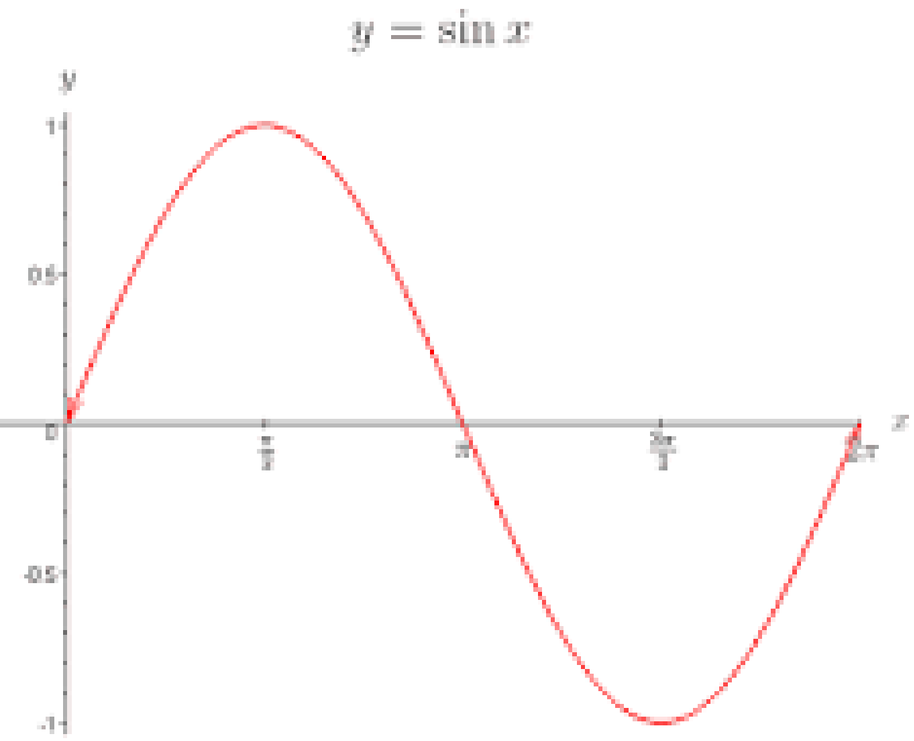
Describe the translation of **A** to **B** with a vector.

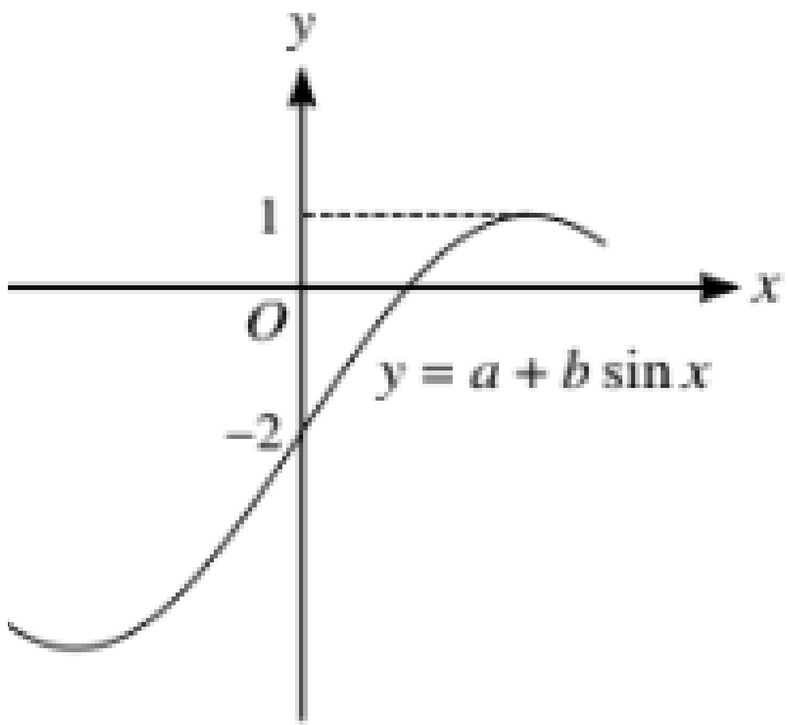
$$\begin{pmatrix} \textbf{Horizontal} \\ \textbf{Vertical} \end{pmatrix} \begin{matrix} \leftrightarrow \\ \updownarrow \end{matrix}$$

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$$



- positive numbers mean right or upward
- negative numbers mean left or downward





The diagram shows part of the graph of $y = a + b \sin x$. Find the values of the constants a and b.

[2]

a - translation --> a= -2

b - horizontal stretch --> b = 3