EAE 298 Aeroacoustics
Fall Quarter 2016
Homework #3

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Problem 1. [50 points]

The acoustic wave equation without considering the source is expressed as follows:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0$$

We can define a new function $\widetilde{p}$ using the imbedding technique as follows:

$$\begin{aligned} \widetilde{p} &= p, \quad f > 0 \\ \widetilde{p} &= 0, \quad f < 0 \end{aligned}$$

where $f = 0$ describes the arbitrary moving body. Show that the wave equation whose sound is generated by an arbitrary moving body (=0) can be expressed as follows:

$$\frac{1}{c^2}\frac{\partial^2 \widetilde{p}}{\partial t^2} - \overline{\nabla^2}\widetilde{p} = -\left[\frac{M_n}{c}\frac{\partial p}{\partial t} + p_n\right]\delta(f) - \frac{1}{c}\frac{\partial}{\partial t}\left[M_n p \delta(f)\right] - \nabla \cdot \left[p\vec{n}\delta(f)\right]$$

where $\vec{n}$ is the unit normal vector on the surface and $p_n = \nabla p \cdot \vec{n}$. Now we can use the Greens function of the wave equation in the unbounded space, the so-called free-space Greens function, to find the unknown function $p(\vec{x}, t)$ everywhere in space. The result is the Kirchhoff formula for moving surfaces.

Problem 2. [50 points]

Farassats formulation 1 for the loading noise is given as

$$4\pi p'_L(\vec{x}, t) = \frac{1}{c}\frac{\partial}{\partial t}\int_{f=0}\left[\frac{L_r}{r(1-M_r)}\right]_{ret} dS + \int_{f=0}\left[\frac{L_r}{r^2(1-M_r)}\right]_{ret} dS$$

where $L_r = \Delta P \vec{n} \cdot \hat{r} = \Delta P \cos\theta$. This formulation 1 is difficult to compute since the observer time differentiation is outside the integrals. A much more efficient and practical formulation can be derived by carrying the observer time derivate inside the integrals (formulation 1A). Show that formulation 1A for the loading noise becomes

$$\begin{aligned} 4\pi p'_L(\vec{x}, t) =& \frac{1}{c}\int_{f=0}\left[\frac{\dot{L}_r}{r(1-M_r)^2}\right]_{ret} dS \\ &+ \int_{f=0}\left[\frac{L_r - L_M}{r^2(1-M_r)^2}\right]_{ret} dS + \frac{1}{c}\int_{f=0}\left[\frac{L_r(r\dot{M}_r + c(M_r - M^2))}{r^2(1-M_r)^3}\right]_{ret} dS \end{aligned}$$

where $L_M = \vec{L} \cdot \vec{M}$.