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Early Development of an "Acoustic Analogy" Approach to Aeroacoustic Theory

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IT is a special pleasure to contribute with this brief historical paper to AIAA's celebrations of its 50th anniversary. I am, also, particularly glad that the paper gives me an opportunity to commemorate the energy and vision shown in 1949 by the late H. B. Irving, in his then position as Assistant Director of Scientific Research (Air) within Britain's government machine. Part of his job was to think about the future of aeronautics, with the object of identifying serious problems likely to arise as part of that future; and to draw attention to new research fields that needed to be opened up without delay if timely solutions to those problems were to be found.

Remarkable vision was shown by Irving in recognizing so clearly, as early as 1949, that jet engines were bound to cause severe noise problems when (as must ultimately happen) they penetrated the civil transport market. This may have made him the first person in the world to identify research on jet noise and means for its reduction as a major new field of research to which effort urgently needed to be devoted.

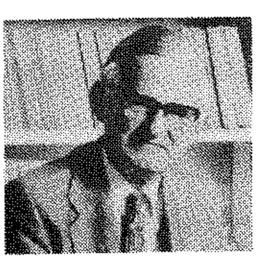
Great energy, too, was shown by Irving when he followed up that insight by a vigorous campaign waged with the object of setting up substantial research groups in this field within British university institutions. These groups (supported, where necessary, by contributions from the modest funds at Irving's disposal) were invited to pursue basic experimental and theoretical studies of the noise of jets; studies that might be usefully complementary to a large-scale program of empirical studies of jet-engine noise and schemes for its reduction initiated at Rolls Royce Ltd.

Irving ensured, too, that the university groups created in this way (especially, the group including G. M. Lilley at the College of Aeronautics in Cranfield, the University of Southampton group that included E. J. Richards and A. Powell, and the University of Manchester group including myself on the theoretical side and J. H. Gerrard on the experimental side) would all take part together in frequent meetings with the outstanding Rolls Royce team under the leadership of F. B. Greatrex. These regular meetings very powerfully influenced the progress of the research work. I remember this in my own case: as the participant mainly concerned with developing theoretical approaches, I was helped by constant contact with several growing bodies of experimental data to adopt the necessary critical and selective approach to my own growing collection of theoretical ideas. The only satisfactory theories are those that simultaneously accord with empirical data and with scientific logic!—a formidable pair of requirements.

I recall Irving coming up to my attic room in the deeply blackened old Owens College building (since revealed by modern stone-cleaning technology as a charming blend of light brown and blue shades!) at Manchester University. In 1949, I was a 25-year-old Senior Lecturer in Applied Mathematics, already copiously involved in compressible-flow theory (especially, high-speed aerodynamics and shock-wave dynamics). Somehow, Irving convinced me that jet noise was an exceptionally exciting theoretical challenge. I recognized that a jet was one of the classical turbulent flows, characterized to be sure by a complex pattern of vorticity; but not (according to views then current) by any dilatation. Compressibility had not been regarded hitherto as a significant factor in vortex motions or turbulence; yet it must be playing a part if any radiated sound were to result.

Next morning I had to go to London, which in 1949 necessitated a four-hour train journey. Once in the train I could not stop thinking about the jet-noise conundrum. It is, furthermore, the literal truth that the only piece of paper I had on me was the proverbial "back of an envelope"! How fortunate that was. Sitting in the impersonal confines of the railway compartment, after having awakened that morning still more clearly conscious of what an exceptionally exciting theoretical problem in mechanics of fluids had been put to me, I had four hours to think about it while prevented from succumbing to the theoretician's besetting weakness; namely, rushing into filling sheets of paper with endless equations....

The essential idea of the acoustic analogy approach came before my journey's end. I remember that the first part of the journey was entirely devoted to considering what would be the appropriate *dependent variable* in a system of equations to describe jet noise. I had found in many problems of compressible flow that the key to producing tractable equations was the correct choice of dependent variable. Often the *pressure* was an optimum choice (for example, in the aerodynamics of disturbances to a nonuniform oncoming stream). However, I soon decided against use of the pressure,



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because I knew (especially, from having attended Sydney Goldstein's advanced lectures on turbulence) about all the complexities of the relationship between velocity fluctuations and pressure fluctuations within a turbulent flow; and I felt that equations which necessarily reflected those complexities would be too difficult to handle.

Finally I decided to use the density as the dependent variable. My reasoning, essentially, was that density variations were not considered to be particularly important within turbulent flows. If, on the other hand, those turbulent flows produced radiated sound fields, such fields would certainly include important density fluctuations. Briefly, the use of density as the dependent variable seemed right for a theoretical treatment aimed at focussing attention on the radiated sound field.

The local rate of change of density, of course, was specified by the equation of continuity as the inward component (minus the divergence) of mass flux. One precious line on the envelope's modest back could safely be filled in! What, however, could be said about the rate of change of mass flux?

On this matter, careful thought was needed for recognition that it would be essential to use the momentum equation, not in the standard Euler form, but in the less standard form due to Reynolds. He had introduced this equation into turbulence theory with the objective of performing a time averaging (so as to bring out the importance of Reynolds stresses). I had, however, been influenced by remarks of von Kármán at the London International Congress of Applied Mechanics in 1948 to recognize that the Reynolds form of the momentum equation might be useful also in wider contexts.

Here was just what was needed as the second line on the back of the envelope! Much later, thinking about fluxes and densities of fundamental physical quantities, I was to recognize how fortunate I had been: it "just happens" that *flux of mass* (whose inward component determines rate of change of mass density) is identical with *density of momentum* (whose rate of change is given by the inward component of momentum flux).... Thus, this quantity could immediately be eliminated from the two equations to make the double time-derivative of density equal to the double divergence of momentum flux.

I was content on that train to contemplate adiabatic processes only! Accordingly, the momentum flux had just two components: the part due to gradients of pressure was related adiabatically to density distribution, and its double divergence became the Laplacian of the density multiplied by the square of the sound speed; while the remaining part was the momentum *transport* tensor.

It was exciting to see the shape of the resulting equation: above all, it was linear! Density fluctuations (which I was prepared to regard as a by-product of the turbulent flow) satisfied a classical nonhomogeneous wave equation where the "right-hand side" (the forcing term) was the double divergence of the momentum transport tensor. The latter, being quadratic, would be negligible in the sound field itself, and important only in the main region of turbulent flow. I knew, therefore, that Kirchhoff's standard solution of the nonhomogeneous wave equation could be written down as an integral over that region, and applied in its simplified far-field form to give the radiated sound.

Although I am glad in retrospect that an opportunity for an uninterrupted morning of concentrated thought set these researches off along what certainly was the right track at that time (when partial differential equations more complicated than the nonhomogeneous wave equation were hardly lending themselves to mathematical treatment), I am equally glad that I *published* nothing on the subject for another three years. This long interval was devoted to prolonged careful thought about the foundations of the theory, to detailed study of its implications, to dimensional analysis, to progressive comparisons with a growing body of experimental data, and to trying to see my ideas in the context of a careful study of the works of the great early masters of acoustical theory, especially Stokes and Rayleigh. In the meantime, I was learning how to improve various aspects of the theory's presentation by "trying out," one after another, a long sequence of different alternative draft accounts of the theory on a wide variety of friends and colleagues.

Gradually, it became clearer what features of the theory were likely to be of most enduring importance; and I put them into my paper, "On Sound Generated Aerodynamically. I. General Theory," (published in the *Proceedings of the Royal Society A*, Vol. 211, 1952, pp. 564-587). I must confess that, re-reading this paper three decades later, I wish that I had taken equal trouble in the process of composition of all the rest of my published work! The paper was written in the hope that it would continue to be studied in the future as a good introduction to a subject that had not previously been treated, and I believe that it still succeeds in meeting this criterion.

Theoretical developments played, of course, only a small part in the whole process by which Irving's objectives were ultimately achieved. The experimental work was far more influential.

By the mid-1950's the noise level of propeller-driven aircraft had been rising continuously for many years with increase of thrust and propeller tip speed. It was recognized, too, that the possibility of a large discontinuous jump in noise level was threatened by the introduction of civil jet aircraft. By that time, however, jet-noise research had reached the stage where it was possible for large international airports to take effective measures against any such discontinuity. They did this by regulations requiring that noise levels at specified points, representing the populous areas nearest to the airport, should not exceed those measured for the noisier piston-engined aircraft. The aircraft operators, partly by demanding specified noise reductions from the manufacturers, and partly by new techniques of takeoff and climb, were, in general, remarkably successful in meeting the conditions laid down.

Jet-engine manufacturers, including especially Rolls Royce, were initially able to achieve the noise requirements placed on them by adaptations of existing turbojet engines. Greatrex's researches had finally shown how adaptation by fitting special noise-suppressing devices to the jet nozzle could reduce sound energy output by a factor of around 10, while only incurring a modest all-up weight increase and thrust decrease (each of about 0.7% of the takeoff thrust). The influence of theoretical developments on these researches may have been largely of a negative nature, confined to indicating why certain conceivable approaches to suppressor design were unlikely to be effective....

In the longer term, on the other hand, trends in the development of engines to achieve much larger thrust for civil-transport applications may have been influenced very substantially by the essential dimensional analysis derived from the "acoustic analogy" approach and emphasized in my 1952 paper. The fundamental trend (however crudely approximate) to an eighth-power law for noise energy dependence on jet velocity, when compared with the well-known square law for jet thrust dependence, was probably one of the two main factors tending to move the manufacturers of jet engines for civil transport purposes in the 1960's progressively toward seeking to achieve that thrust with turbofan engines of very large diameter and relatively lower jet velocities. These developments have been rather widely beneficial by significantly reducing the cost of air transport in real terms without increasing the resulting noise nuisance.