Then, $A_{mn} = \frac{1}{\Gamma_{mn}} \int_{R_i}^{R_o} \tilde{p}(0,r)\, \psi_{mn}\, r\, dr$

Often, we are more interested in sound power level (PWL) rather than pressure itself.

Intensity is given as

Average $= \frac{1}{Area} \int_{r_i}^{r_o} \square\, 2\pi r\, dr$

$$\underset{\text{time/area-averaged}}{\langle I_z \rangle} = \frac{2\pi}{2 A_z} Re\left[\int_{R_i}^{R_o} \left\{ (1+M^2) P V_z^* + M\left(\frac{|P|^2}{\rho_0 C_0} + \rho_0 C_0 |V_z|^2\right)\right\} r\, dr\right]$$

integration around theta

time-averaging

duct area plane

For $M=0$, $[\ ] \rightarrow P V_z^*$

$\langle I \rangle = \frac{1}{2} Re\{P V_z^*\}$

Let's consider linearized momentum equation

$$\rho_0 \frac{DV_z}{Dt} = -\frac{\partial P}{\partial z} \qquad V_z = V_z e^{-i\omega t + ik_z z},\quad P_z = P_z e^{-i\omega t + i k_z z}$$

$$\rho_0 C_0 \left[-\frac{i\omega}{C_0} + M\, i k_z\right] V_z = -i K_z P, \quad \text{Let } K_z = K_{z,mn}$$

$$V_z = \frac{1}{\rho_0 C_0} \frac{K_{z,mn}}{(\omega/C_0 - K_{z,mn} M)} P$$

Therefore, Modal Power becomes

$$\boxed{W_{mn} = \langle I_z \rangle A_z = \frac{\pi}{\rho_0 C_0} \Gamma_{mn} A_{mn} A_{mn}^* \left[(1+M^2)\, \overset{\text{Conj}}{\left(\frac{K_{z,mn}}{\omega/C_0 - K_{z,mn} M}\right)} + M\left[1 + \left|\frac{K_{z,mn}}{\omega/C_0 - K_{z,mn} M}\right|^2\right]\right]}$$

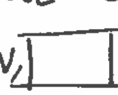
$$\Gamma_{mn} = \int_{R_i}^{R_o} \psi_{mn} \psi_{mn}^* r\, dr$$

Total Power

$$W = \sum_m \sum_n W_{mn} \quad \text{(only for cut-on modes)}$$

Usually m is related to BPF. So for each m, we have multiple ns (radial modes).

The power is only defined at one z plane of interest.

W_1  W_2