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NOISE DUE TO TURBULENT FLOW PAST A TRAILING EDGE

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A theoretical method [1] for calculating far field noise from an airfoil in an incident turbulent flow is extended to apply to the case of noise produced by turbulent flow past a trailing edge, and some minor points of the theory in reference [1] are clarified. For the trailing edge noise, the convecting surface pressure spectrum upstream of the trailing edge is taken to be the appropriate input. The noise is regarded as generated almost totally by the induced surface dipoles near the trailing edge and thus equal, but anticorrelated, noise is radiated into the regions above and below the airfoil wake, respectively. The basic assumption of the analysis, from which these concepts of appropriate input and dominance of dipole sources follow, is that the turbulence remains stationary in the statistical sense as it moves past the trailing edge. The results show that such trailing edge noise often is quite small, compared say to that produced by typical oncoming turbulence levels of one percent, but that it might be appreciable for an airfoil with a flow separation, or for a blown flap.

1. INTRODUCTION

In reference [1] a theory was presented for calculating the far field noise produced by an airfoil encountering a turbulent flow. This noise results from the pressure fluctuations on the airfoil, which are calculated by using two-dimensional compressible flow airfoil response functions. The last section of the present note gives clarification to certain minor points of reference [1]. The main body of this note is directed toward an extension of that theory to apply to the case of noise produced by turbulent flow past a trailing edge.

In the recent past there have been several papers [2–8] which have dealt with the trailing edge noise problem. In most of these works the turbulence quadrupole sources have been assumed to be specified, and the noise such sources would produce in the presence of a scattering half-plane has been calculated. This procedure, however, gets involved with difficulties inherent in the jet noise problem. Although the Lighthill–Curle formulation [9, 10] is mathematically exact, in a realistic approximate calculation the proper interpretation of the quadrupole source term can be difficult [11].

In the present note the somewhat different approach is taken of requiring as input the convecting surface pressure spectrum upstream of the trailing edge. This approach is related to the quadrupole source approach: i.e., the surface pressure upstream of the trailing edge could, in principle, be calculated from a known quadrupole distribution plus its mirror image.

Chase [5] also has formulated the problem in terms of airfoil surface pressure. One of the main differences between the present work and that of Chase is in the airfoil response function used. Chase makes use of the Green function for zero mean flow (also used in reference [2]) whereas the present analysis includes the effects of a mean flow velocity, U. Also, Chase concludes that the sound produced by a turbulent flow on one side of a flat plate will radiate differently into the upper and lower half-planes. In contrast, in the present approach the view is taken that the noise is generated almost totally by the induced surface dipoles near the trailing edge and therefore equal (but anticorrelated) radiation into the upper and lower regions is predicted.

The basic assumption of the present analysis is that the turbulent velocity field is unaffected by the presence of the trailing edge: i.e., the turbulence is stationary in the statistical sense as it moves past the trailing edge. This assumption allows the edge noise to be calculated from the spectral characteristics of wall pressure which would exist in the absence of the trailing edge. For the case of a turbulent boundary layer, for example, there have been several studies (see, e.g., references [12–14]) of the wall pressure spectral characteristics needed as input to the present theory.

The validity of the assumption that the turbulence is unaffected by the trailing edge is open to some debate, however: see reference [3], for example. Upstream of the trailing edge the turbulence exists in the presence of its mirror image which must affect the velocity field, especially if a given eddy is less than a correlation length from the surface, whereas downstream of the trailing edge this induced velocity effect does not exist. Even though there is some uncertainty in the accuracy of this basic assumption, the great simplification that it imparts to the problem makes it worth investigating. In the alternative method of working with the volume quadrupole distribution similar difficulties are encountered, since for the method to be useful the strength of the quadrupoles in the trailing edge vicinity must be specified. In any case, the approximation would be expected to give at least correct order of magnitude results.

2. TRAILING EDGE NOISE

The model to be used for calculating trailing-edge noise consists of a turbulent flow, with stationary statistical properties, convecting past a trailing edge. Upstream of the edge this produces a convecting pressure pattern on the airfoil surface. Near the trailing edge, in addition to the convecting pressure pattern, a radiating pressure field of comparable magnitude is produced. On the airfoil (assumed to lie in the $z = 0$ plane) a particular spectral component of the convecting surface pressure field with convection velocity U_c and spanwise wavenumber k_y can be written as

$$P = P_0 \, e^{i[\omega(t - x/U_c) - k_y y]}, \tag{1}$$

where x denotes the flow direction, y the spanwise co-ordinate, t the time and $x = 0$ locates the trailing edge. For the purpose of calculating the surface pressure field induced by the convecting pressure pattern, the airfoil is assumed to be semi-infinite with a trailing edge but no leading edge. As discussed in reference [15], this is a good assumption for acoustic wavelengths comparable to or less than a chord. The solution can then be found by application of the Schwartzschild solution [16], which is discussed by Landahl [17]. (A leading edge correction can also be derived by Landahl's method [17], but is not needed for small acoustic wavelengths.) The method of application of the Schwartzschild solution can best be understood by dividing the solution into two parts. Consider first the case where an imaginary extension of the airfoil downstream of the trailing edge is assumed so that the airfoil is infinite in both the upstream and downstream directions. Then the surface pressure component given by equation (1) exists for all x, including the imaginary downstream extension of the airfoil. By using the result of Curle [10], this flow field can be represented by a volume quadrupole distribution plus a surface dipole distribution.

The actual flow field, without the imaginary downstream airfoil extension, consists of the same quadrupole distribution for all x (since the turbulence is assumed statistically stationary), but does not have the dipole distribution downstream. A second solution (consisting of a dipole distribution) is needed which will exactly cancel the dipole distribution on the imaginary airfoil extension, while also satisfying the Kutta condition and the condition of no flow through the airfoil upstream of the trailing edge. The sum of the two solutions then satisfies all the necessary conditions for the problem of stationary turbulence convecting past a trailing edge.

This second solution can be obtained from the general Schwartzschild solution. As shown by Amiet [18] the airfoil surface pressure jump, normalized by P_0, induced by equation (1) with $k_y = 0$ is

$$g(\bar{x}, \omega, U_c) = \{(1+i)E^*[-\bar{x}((1+M)\mu + \bar{K}_x)] - 1\}\mathrm{e}^{-i\bar{K}_x x}, \qquad -2 < \bar{x} < 0, \tag{2}$$

where $K_x \equiv \omega/U_c$, $\mu \equiv M\omega b/U\beta^2$, $\beta^2 \equiv 1 - M^2$, M is the free stream Mach number and the overbar denotes a variable non-dimensionalized by the semichord b. The function

$$E^*(x) = \int_0^x (2\pi\xi)^{-1/2}\,\mathrm{e}^{-i\xi}\,\mathrm{d}\xi \tag{3}$$

is a combination of Fresnel integrals. The solution for $k_y \neq 0$ can also be found by using the Schwartzschild solution. It will be noted that the sum of equations (1) and (2) is zero for $x = 0$: i.e., the Kutta condition is satisfied. Equation (2) makes the assumption that the shed vorticity produced by the unsteady airfoil loading convects downstream with the flow speed U of the free stream. This assumption is standard in airfoil theory, and was also made by Jones [3]. In the present situation this assumption conflicts somewhat with the assumption that the turbulence on the airfoil and in the wake is convecting at a speed smaller than U. Although this is not thought to be a significant defect of the solution, it is perhaps worthy of further investigation. Given the airfoil response function, g, standard spectral techniques can be applied to obtain the far-field noise if the spectral characteristics of the surface pressure field are known.

The surface pressure field can be rather complex, however, and it is worthwhile to make simplifications. In general, the pressure measured at a given point and frequency will be composed of many spectral components of the form of equation (1), for which the axial wavenumber K_x and convection speed U_c are variable, but whose product $K_x U_c = \omega$ is constant. This necessitates a summation of the contributions due to all these spectral components in order to calculate the far field noise at a given frequency. In order to simplify the situation, a given frequency ω will be assumed to be associated with a single value of U_c, and thus K_x. This would appear to be a reasonable approximation, especially since the resulting convection velocity, U_c, appears to be only a weak function of frequency (see, e.g., reference [14], p. 675). Another way of stating this approximation is that the airfoil response, g, for a given frequency is assumed to take an average value appropriate for some mean value of U_c.

With the above approximation an expression for the far field sound can be obtained by analogy with reference [1]. From equation (20) therein, accounting for the difference in the normalization of g, the far field sound spectrum for an observer in the $y = 0$ plane is

$$S_{pp}(x, 0, z, \omega) = \left(\frac{\omega b z}{2\pi c_0 \sigma^2}\right)^2 l_y(\omega) d |\mathscr{L}|^2 S_{qq}(\omega, 0), \tag{4a}$$

where c_0 is the sound speed,

$$|\mathscr{L}| = \left| \int_{-2}^{0} g(\xi, \omega, U_c)\,\mathrm{e}^{-i\mu\xi(M - x/\sigma)}\,\mathrm{d}\xi \right|, \tag{4b}$$

$$l_y(\omega) = \frac{1}{S_{qq}(\omega, 0)} \int_0^\infty S_{qq}(\omega, y)\,\mathrm{d}y, \tag{4c}$$

$\sigma^2 = x^2 + \beta^2 z^2$ and $S_{qq}(\omega, y)$ represents the spanwise cross-spectrum of surface pressure. Equation (4a) gives the sound produced by a boundary layer on one side of the airfoil; for statistically identical boundary layers on both sides, the results should be multiplied by 2. The integral in equation (4b) with g given by equation (3) can readily be evaluated in closed form giving

$$|\mathscr{L}| = \frac{1}{\Theta}\left|(1+i)\left\{\sqrt{\frac{(1+M+\bar{K}_x/\mu)}{(1+x/\sigma)}}\,E^*[2\mu(1+x/\sigma)]\mathrm{e}^{-i2\Theta} - E^*[2((1+M)\mu+\bar{K}_x)]\right\} + 1 - \mathrm{e}^{-i2\Theta}\right|, \tag{5}$$

where $\Theta \equiv \bar{K}_x + \mu(M - x/\sigma)$.

The above results can be applied to any case of turbulent flow past a trailing edge, such as a blown flap or flow separation, so long as the assumption of stationary turbulence convecting past the edge is reasonably well satisfied. At present, however, the only case for which the airfoil surface pressure characteristics are sufficiently well documented is that of a turbulent boundary layer, which will be used here as an example.

For a turbulent boundary layer in a uniform flow with no mean pressure gradient, Corcos [12] states that the normalized cross-spectrum, $S_{qq}(\omega, y)/S_{qq}(\omega, 0)$, can be represented by a function $B(\omega y/U_c)$ depending only on a single dimensionless variable. (This has been called into question by Willmarth and Roos [13] for certain ranges of frequency and separation, but it should be adequate for present illustrative purposes.) Graphical integration of the curve for B given in Figure 13 of reference [12] gives, for the spanwise correlation length,

$$l_y(\omega) \approx 2{\cdot}1\, U_c/\omega. \tag{6}$$

The surface pressure spectrum data of Willmarth and Roos [13] (also presented in Figures (7–39) of reference [14]) for a turbulent boundary layer flow of density ρ_0 and free stream speed U can be approximated by the empirical expression

$$S_{qq}(\omega, 0) = (\tfrac{1}{2}\rho_0 U^2)^2(\delta^*/U)2\times 10^{-5}/(1 + \tilde{\omega} + 0{\cdot}217\tilde{\omega}^2 + 0{\cdot}00562\tilde{\omega}^4), \qquad 0{\cdot}1 < \tilde{\omega} < 20, \tag{7}$$

where $\tilde{\omega} \equiv \omega\delta^*/U$. The convection velocity U_c has a small variation with frequency, as noted previously, but for lack of detailed information this variation will be ignored and an average value $U_c = 0{\cdot}8\,U$ used. The turbulent boundary layer displacement thickness, δ^*, at the trailing edge is approximately [19]

$$\delta^*/c \approx 0{\cdot}047\, Re_c^{-1/5} \tag{8}$$

where Re_c is the Reynolds number based on chord, c.

Introducing these approximate surface pressure results into equation (4a) gives an estimate of the far-field sound. (Equation (4a) must first be multiplied by 8π to account for (i) a boundary layer on both upper and lower surfaces, (ii) to convert to a single sided $(0 < \omega < \infty)$ spectrum and (iii) to convert to a 1 Hz bandwidth.) Figure 1 shows the results for an airfoil with chord $2b = 5$ meters, span $2d = 40$ meters, Mach number $M = 0{\cdot}3$ and an observer distance of 200 meters above the retarded position of the airfoil: i.e., $x = M\sigma$ in equation (5). For comparison, the noise generated by the same airfoil interacting with isotropic turbulence with a level of 1 % and an integral scale of one meter, calculated by the method of reference [1], is also shown. The trailing edge noise produced by the turbulent boundary layer is seen to be quite small, an observation also made by others: e.g., Meecham [20]. However, the possibility remains that an alternate source of convecting pressure fluctuations, such as a blown flap or an airfoil with a flow separation, could produce greater pressure spectrum levels on the airfoil, and thus greater noise as the turbulence moves past the trailing edge. Some work

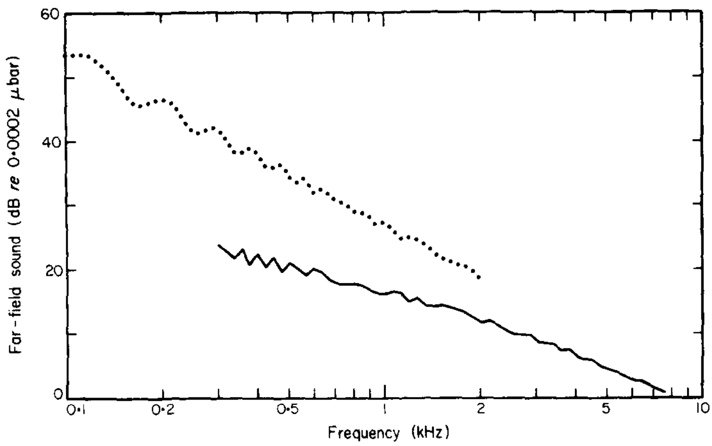

Figure 1. Trailing edge noise compared with incident turbulence noise. Span = 40 meters, chord = 5 meters, $M = 0{\cdot}3$, observer position = 200 meters directly above airfoil retarded position. ——, Trailing edge noise; · · · ·, incidence fluctuation noise with integral scale = 1 meter and intensity = 1 %.

has been done on these types of turbulence but further experimental results, including detailed measurements of the surface pressure cross-spectral characteristics, are needed for verification of the present theory.

3. FURTHER COMMENT ON INCIDENT TURBULENCE NOISE

The limitation given in reference [1] for the validity of the theory of airfoil-turbulence noise presented therein is that

$$\Lambda \equiv MK_x d \gg 1. \tag{9}$$

This criterion is sufficient, but not always necessary. In physical terms equation (9) requires that the acoustic wavelength be much smaller than the airfoil span. This restriction permits the theory to be applied to an airfoil of arbitrary aspect ratio, AR, since the airfoil lift tends to concentrate toward the leading edge (within the order of an acoustic wavelength) as the wavelength tends to zero. Thus, the effective aspect ratio tends toward infinity and the end effects to zero. However equation (9) appears to exclude the small Mach number problem, which need not be the case. It was not clearly pointed out in reference [1] that another valid criterion for the applicability of the theory is that both the following conditions hold:

$$AR \gg 1, \qquad K_x d \gg 1. \tag{10}$$

For this case the end effects are limited to a distance of the order of a chord from the airfoil ends and can be neglected. Equation (10) allows elimination of the integral over spanwise wavenumber, k_y, as for equation (9) with the same final results.

Because of the small Mach number, the theory and experiment of reference [21] for the lift spectrum of an airfoil in turbulence are closer to the limit given by equation (10) than equation (9). Even though the AR of the lift sensing element was only 2·67 for that experiment, for large $K_x d$ the exact calculations of Graham do not differ to a significant extent from the solution for the limiting case as shown by Figure 5 of reference [1]. If AR were to be increased, the agreement should be closer. It is interesting to note that the limits given by equations (9) and (10) lead to exactly the same results for the airfoil lift spectrum as given by the approximate "strip theory" taken in the same limit: e.g., see references [21, 22]. Further discussion of the spectral approach applied to airfoil lift calculations can be found, for example, in reference [23].

One minor correction should be made to the results given in Appendix 2 of reference [1]. A factor

$$\exp\{i(\bar{\omega}/\beta^2)[(1-\beta)\ln M + \beta\ln(1+\beta) - \ln 2]\} \tag{11}$$

was accidentally omitted from the airfoil response function given by equation (40). The factor is not needed for the calculations given in the paper since only the absolute value of lift is needed. However, for comparison with numerical lift results such as those of Graham [24], the phase is important and the factor, which has been a source of some confusion in the past, should be included as discussed in references [15, 25–27].

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