

MAE 298 Aeroacoustics – Homework #3

Generalized Differentiation and Farassat's Formulation

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Nomenclature

L	Subscript for loading parameters	i	Imaginary number $\sqrt{-1}$
n	Surface normal direction	\exp	Exponential (e)
t	time	E	Exponential term: $kz + n\theta - \omega t$
r	Distance between source and observer	λ	Constant term in Bessel equation
M	Mach number	J	First-order Bessel function
$W(r)$	Radial distribution of mean axial velocity	Y	Second-order Bessel function
c	Speed of Sound	$H^{(n)}$	nth-order Hankel function
$\bar{\rho}$	Mean density	x	Placeholder variable for λr
γ	Specific heat ratio	A, B	Arbitrary Bessel function constants
p	Pressure	C, D	Arbitrary Hankel constants
\tilde{p}	Pressure (Discontinuous across data surface)	\vec{V}	General velocity vector
\bar{p}	Mean pressure	V_r	Velocity component in radial direction
p'	Perturbation pressure	ν	Constant velocity parameter
k	Wavenumber	χ	Constant position parameter
$\bar{\partial}$	Generalized derivative	ζ	Position of vortex sheet dividing inner/outer solution
δ	Delta function	$+/-$	Outer/Inner solution, respectively
ω	Wave oscillating frequency		

Overview

Explore techniques of generalized differentiations and solve FW-H loading term with Farassat's Formulation 1A

$$r = |\vec{x} - \vec{y}|$$

I. Problem 1 – Generalized Differentiation of Wave Equation

The acoustic wave equation without considering the source is expressed as follows:

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0 \quad (1)$$

We can define a new function \tilde{p} using the imbedding technique as follows:

$$\tilde{p} = \begin{cases} p, & f > 0 \\ 0, & f < 0 \end{cases} \quad (2)$$

where $f = 0$ describes the arbitrary moving body. Show that the wave equation whose sound is generated by an arbitrary moving body ($f=0$) can be expressed as follows:

$$\frac{1}{c^2} \frac{\partial^2 \tilde{p}}{\partial t^2} - \nabla^2 \tilde{p} = - \left[\frac{M_n}{c} \frac{\partial p}{\partial t} + p_n \right] \delta(f) - \frac{1}{c} \frac{\partial}{\partial t} [M_n p \delta(f)] - \nabla \cdot [p \vec{n} \delta(f)] \quad (3)$$

where \vec{n} is the unit normal vector on the surface and $p_n = \nabla p \cdot \vec{n}$. Now we can use the Greens function of the wave equation in the unbounded space, the so-called free-space Greens function, to find the unknown function $p(\vec{x}, t)$ everywhere in space. The result is the Kirchhoff formula for moving surfaces.

I.A. Term 1

II. Problem 2 – Farassat Formulation 1A for Loading Noise

Farassats formulation 1 for the loading noise is given as $4p_L$, $=1! + 1! \dots 1! !$ where $L! = \dots = P \cos$. This formulation 1 is difficult to compute since the observer time differentiation is outside the integrals. A much more efficient and practical formulation can be derived by carrying the observer time derivate inside the integrals (formulation 1A). Show that formulation 1A for the loading noise becomes $4p_L$, $=1! \dots 1! ! !$ where $L! = \dots$.

Conclusion

conclude