Total: 65/100

Midterm Average: 62/100 (class)

EAE 298 Aeroacoustics, Fall Quarter 2016

Mid-Term Exam
(Closed Note)

20/20

Name : Logan Hakstrom

1. [2 points] What brought the aircraft noise down in 1970s and 80s? Explain the reason.

Introduction of turbofan (high-bypass ratio) engines. Achieve thrust with lower flow velocity and less noise

2. [3 points] Who is the father of aeroacoustics? What is acoustic analogy?

James Lighthill. Acoustic analogy is rearrangement of Navier-Stokes equations to linear acoustic wave equation with Lighthill stress tensor T_{ij} on RHS.

3. [3 points] In FW-H equation, what are the three main sources?

thickness (monopole) due to displacement of fluid particles by moving body
loading (dipole) due to uneven, fluctuating surface pressure on body
Volume (quadrupole) non-linear volume term

4. [2 points] What is the dominant wind turbine noise source? What is the physical mechanism of the noise?

Trailing Edge noise. Hydrodynamic energy in turbulent boundary layer is scattered at sharp trailing edge where upper/lower BLs meet

5. [2 points] What is the blank in the equation, $SPL = 20\log_{10}(blank/P_{ref})$ where SPL is the sound pressure level and P_{ref} is the reference pressure, which is 20μPa?

$$SPL = 20\log_{10}\left(\frac{\overbrace{P_{rms}}^{blank}}{P_{ref}}\right)$$

6. [3 points] What is the 'retarded time'? Do you remember the equation?

Retarded time is time when sound was emitted

$$\tau = t - \frac{|\bar{x}-\bar{y}|}{c}$$

$$|\bar{x}-\bar{y}| = c(t-\tau)$$

$$\tau = t - \frac{|\bar{x}-\bar{y}|}{c}$$

7. [2 points] According to the dimensional analysis of Lighthill's acoustic analogy, how does the acoustic energy vary with respect to Mach number in <u>subsonic</u> case?

M^8

8. [3 points] What is the difference between Lighthill's equation and Lilley's equation?

Lilley moved the mean flow terms to the wave operator so that the mean flow can be solved as part of the solution, rather than adjusting the source term to account for mean flow as in Lighthill

45/80

EAE 298 Aeroacoustics, Fall Quarter 2016

Mid-Term Exam

(Open Note)

Name : Logan Halstrom

1. [15 points] A noise is generated by 80 pure tones, of different frequencies but identical power. Each pure tone has a sound pressure level of 60 dB. Determine the sound pressure level of the noise.

$$SPL = 10\log_{10}\left[\sum_{i=0}^{80} 10^{SPL_i/10}\right]$$

$$= 10\log_{10}\left[80\left(10^{60dB/10}\right)\right] = 10\log_{10} 80$$

$$\boxed{SPL = 10\log_{10}(8\times 10^{7})}$$

I forgot how to do log on my calculator...

45/80

EAE 298 Aeroacoustics, Fall Quarter 2016

Mid-Term Exam
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$$\boxed{SPL = 10 \log_{10}(8\times10^{7})}$$

I forgot how to do $\log_{10}$ on my calculator...

ok.

14/15

L. Halstrom

2. [35 points] Problem about sound intensity and power

2. (a) [20 points] Show that the time-averaged intensity radiated for the various harmonic sound fields whose pressure

source strength

$$p'(\vec{x},t) = p_0 l^{n+1} \frac{\partial^n}{\partial x_1^n}\left\{\frac{\cos\omega\left(t-\frac{r}{c}\right)}{r}\right\}$$

$\cos\theta = \frac{x_1}{r}$

is

p_{ref}

length scale

$$I_{ave} = \frac{p_0^2 l^{2n+2}}{2\rho_0 c r^2}\left(\frac{\omega}{c}\right)^{2n}\cos^{2n}\theta$$

$$\left(\frac{\partial^n}{\partial x_1^n}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right)^2 = \left(\frac{\omega}{c}\right)^{2n}\cos^{2n}\theta = \left(\frac{\omega}{c}\right)^{2n}\left(\frac{x_1}{r}\right)^{2n} = \left(\frac{\omega x_1}{cr}\right)^{2n}$$

for different integer values of n. $p_0 l^{n+1}$ represents the source strength, p_0 being a reference pressure and l an appropriate length scale. Show the detailed derivation and do not jump on the final equation without justification. [Hint: Use a far-field approximation.]

$kr \gg 1$

$r \gg \lambda$

I_{ave} @ farfield, spherical waves behave like plane waves locally

time averaging

$$I_{ave} = \frac{1}{2}\frac{|P_c|^2}{\rho_0 c} = \frac{p_0^2 l^{2n+2}}{2\rho_0 c}\left(\frac{\partial^n}{\partial x_1^n}\left\{\frac{\cos\left[\omega\left(t-\frac{r}{c}\right)\right]}{r}\right\}\right)^2$$

$$\frac{\partial^n}{\partial x_i^n}\{\ \}\frac{\partial^n}{\partial x_i^n}\{\ \}$$

see my solution

$$\frac{\partial}{\partial x_i} = \frac{\partial r}{\partial x_i}\frac{\partial}{\partial r} = \cos\theta\cdot\frac{\partial}{\partial r}$$

$$= \frac{p_0^2 l^{2n+2}}{2\rho_0 c}\frac{\partial^n}{\partial x_1^n}\left\{\frac{\cos\left[\omega\left(t-\frac{r}{c}\right)\right]}{r}\right\}^2$$

$$= \frac{p_0^2 l^{2n+2}}{2\rho_0 c r^2}\left(\frac{\partial}{\partial x_i}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right)^{2n}$$

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$$= \frac{p_0^2 l^{2n+2}}{2\rho_0 c r^2}\left\{\left(-\sin\omega\left(t-\frac{r}{c}\right)\right)\frac{\partial}{\partial x_1}\left(\omega t-\frac{\omega r}{c}\right)\right\}^{2n}$$

$$= \left\{\left(-\sin\left(\omega\left(t-\frac{r}{c}\right)\right)\right)\left(0-\frac{\omega}{c}\right)\right\}^{2n}$$

$$\boxed{= \frac{p_0^2 l^{2n+2}}{2\rho_0 c r^2}\left(\frac{\omega}{c}\right)^{2n}\sin^{2n}\left[\omega\left(t-\frac{r}{c}\right)\right]}\ ?$$

L. Halstrom

2. (b) [15 points] Show that the power radiated by the source from 2.(a) is

$$P = \frac{2\pi p_0^2 l^2}{\rho_0 c(2n+1)}\left(\frac{\omega l}{c}\right)^{2n}$$

Show the detailed derivation and do not jump on the final equation without justification. [Hint: the surface integration for a sphere can be calculated by $2\pi \int_0^\pi A(r,\theta) r^2 \sin\theta \, d\theta$.]

$$W_{ave} = \iint_{S_c} \bar{I}_{ave} \bar{n}\, dS_c = \iint_{S_c} \frac{p_0^2 l^{2n+2}}{2\rho_0 c r^2}\left(\frac{\omega}{c}\right)^{2n} \cos^{2n}\theta \, dS_c$$

$$= \iint \frac{p_0^2 l^2}{2\rho_0 c r^2}\left(\frac{\omega l}{c}\right)^{2n} \cos^{2n}\theta \, dS$$

$$W_{ave} = \frac{2\pi p_0^2 l^2}{2\rho_0 c}\left(\frac{\omega l}{c}\right)^{2n}\int_0^\pi \frac{1}{\cancel{r^2}} \cos^{2n}\theta \, \cancel{r^2} \sin\theta \, d\theta$$

$$= \frac{\pi p_0^2 l^2}{\rho_0 c}\left(\frac{\omega l}{c}\right)^{2n}\int_0^\pi (\cos^2\theta)^n \sin\theta \, d\theta = \int_0^\pi (1-\sin^2\theta)^n \sin\theta \, d\theta$$

$u = \cos\theta$ $\qquad \theta = 0 \quad u = 1$

$du = -\sin\theta \, d\theta$ $\qquad \theta = \pi \quad u = -1$

$$\int_{-1}^1 u^{2n}\, du = \frac{u^{2n+1}}{2n+1}\Big]_{-1}^1 = \frac{2}{2n+1}$$

missing 2

$$\int_0^\pi -u^{2n}\, du = \left[\frac{-1}{2n+1}(\cos\theta)^{2n+1}\right]_0^\pi = \frac{1}{2n+1}$$

$$\boxed{W_{ave} = P = \frac{\pi p_0^2 l^2}{\rho_0 c(2n+1)}\left(\frac{\omega l}{c}\right)^{2n}}$$

11/15 ~~10/15~~

can't get 2 using time-averaged intensity from 2a

L. Halstrom

2. (b) [15 points] Show that the power radiated by the source from 2.(a) is

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Show the detailed derivation and do not jump on the final equation without justification. [Hint: the surface integration for a sphere can be calculated by $2\pi \int_0^\pi A(r,\theta) r^2 \sin\theta \, d\theta$.]

$$W_{ave} = \iint_S \bar{I}_{ave} \, \bar{\eta} \, dS_c = \iint_{S_c} \frac{P_0^2 \, l^{2n+2}}{2\rho_0 c r^2}\left(\frac{\omega}{c}\right)^{2n} \cos^{2n}\theta \, dS_c$$

$$= \iint \frac{P_0^2 l^2}{2\rho_0 c \, r^2}\left(\frac{\omega l}{c}\right)^{2n} \cos^{2n}\theta \, dS$$

$$W_{ave} = \frac{2\pi P_0^2 l^2}{2\rho_0 c}\left(\frac{\omega l}{c}\right)^{2n} \int_0^\pi \frac{1}{r^2} \cos^{2n}\theta \, r^2 \sin\theta \, d\theta$$

$$= \frac{\pi P_0^2 l^2}{\rho_0 c}\left(\frac{\omega l}{c}\right)^{2n} \int_0^\pi \left(\cos^2\theta\right)^n \sin\theta \, d\theta = \int_0^\pi \left(1-\sin^2\theta\right)^n \sin\theta \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$\theta = 0 \quad u = 1$

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$$\int_{-1}^{1} u^{2n} \, du = \left.\frac{u^{2n+1}}{2n+1}\right]_{-1}^{1} = \frac{2}{2n+1}$$

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$$\int_0^\pi -u^{2n} \, du = \left[\frac{-1}{2n+1}(\cos\theta)^{2n+1}\right]_0^\pi = \frac{1}{2n+1}$$

$$\boxed{W_{ave} = P = \frac{\pi P_0^2 l^2}{\rho_0 c (2n+1)}\left(\frac{\omega l}{c}\right)^{2n}}$$

11/15 ~~10/15~~

can't get 2 using time-averaged intensity from 2a

L. Halstrom

3. [30 points] Problem about a moving source

3. (a) [10 points] Show that the retarded time τ^* for a moving source with the constant velocity of U in the x_1 direction and the source location $x_2 = l$ and $x_3 = 0$ is

$$\tau^* = \left(t - \frac{|\vec{x}|}{c} + \frac{x_2 l}{|\vec{x}|c}\right)\frac{1}{1 - M\cos\theta} \quad for\ |\vec{x}| \gg l, |U\tau^*|$$

Show the detailed derivation and do not jump on the final equation without justification.

$$c(t-\tau^*) = |\vec{x} - \vec{x}_s(\tau^*)| = \{(x_1 - U\tau^*)^2 + \underbrace{(x_2 - l)^2 + x_3^2}_{ok}\}^{1/2}$$

$$= \{\underbrace{x_1^2 + x_2^2 + x_3^2}_{|\vec{x}|^2} - 2x_1 U\tau^* + U^2\tau^{*2} - 2x_2 l + l^2\}^{1/2}$$

$$= \{|\vec{x}|^2(1 - \frac{2x_1}{|\vec{x}|^2}U\tau^* + \cancel{\frac{U^2\tau^{*2}}{|\vec{x}|^2}}^{0} - \frac{2x_2 l}{|\vec{x}|^2} + \cancel{\frac{l^2}{|\vec{x}|^2}}^{0})\}^{1/2}$$

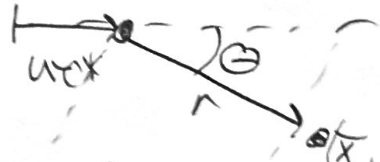
for $|\vec{x}| \gg 1$, $|\vec{x}| \gg |U\tau^*|$, $|\vec{x}| \gg l$

$$= \{|\vec{x}|^2(1 - \frac{2x_1 U\tau^* - 2x_2 l}{|\vec{x}|^2})\}^{1/2}$$

for small x: $(1-x)^{1/2} \simeq 1 - \frac{1}{2}x$

$$c(t-\tau^*) = |\vec{x}|\left(1 - \frac{2x_1 U\tau^* - 2x_2 l}{2|\vec{x}|^2}\right)$$

$$\boxed{\tau^* = \left(t - \frac{|\vec{x}|}{c} + \frac{x_2 l}{|\vec{x}|c}\right)\frac{1}{1 - M\cos\theta}}$$



$\hat{r} \cdot M = M_r = M\cos\theta$

ok.

for $\tau = \tau^*$

10/10

L. Halstrom

3. (b) [20 points] Show that the distant sound field produced by a source of strength

$$q(\vec{x},t) = \delta(x_1 - Ut)H(l^2 - x_2^2)\delta(x_3)Qe^{i\omega t}$$

is

$$p'(\vec{x},t) = \frac{cQ}{2\pi\omega x_2}\exp\left[\frac{i\omega\left(t - \frac{|\vec{x}|}{c}\right)}{1 - M\cos\theta}\right]\sin\left(\frac{\omega x_2 l}{|\vec{x}|c(1 - M\cos\theta)}\right)$$

when $|\vec{x}|$ is large in comparison with l and $|U\tau^*|$ and where $\cos\theta = x_1/|\vec{x}|$. Comment on the form when the source is compact, i.e. $\omega l \ll c(1 - M\cos\theta)$. Show the detailed derivations and do not jump on the final equation without justification. [Hint: H is the Heaviside function where $H(x) = 1$ when $x \geq 0$, $H(x) = 0$ when $x < 0$ and $H(l^2 - x_2^2) = H\{(l - x_2)(l + x_2)\}$.]

$$P'(\vec{x},t) = \frac{Q(\tau^*)}{4\pi r\,|1-M_r|} = \frac{\int \delta(x_1 - Ut)H(l^2 - x_2^2)\,\delta(x_3)\,Qe^{i\omega t}\,d^3y}{4\pi r\,|1-M_r|}$$

$|\vec{x} - \vec{x}_s(\tau^*)|$

$-x_2$ $\quad x_2$

$$= \int \delta(x_1 - Ut)\,H\{(l-x_2)(l+x_2)\}\,\delta(x_3)\,Qe^{i\omega t}\,d^3y$$

retarded time

$$\int_{-l}^{l} \frac{Qe^{i\omega\tau^*}}{4\pi r|1-M_r|}\,d\eta_2$$

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