EAE 298 Aeroacoustics, Fall Quarter 2016

Homework #3: Generalized differentiation and Farassat's formulation
(Due Date: 11/17/2016)

Name : ______________________________

1. [50 points] The acoustic wave equation without considering the source is expressed as follows:

$$\frac{1}{\mathrm{c}^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0$$

We can define a new function $\tilde{p}$ using the imbedding technique as follows:

$$\begin{aligned} \tilde{p} &= p, \qquad f > 0 \\ \tilde{p} &= 0, \qquad f < 0 \end{aligned}$$

where $\mathrm{f} = 0$ describes the arbitrary moving body. Show that the wave equation whose sound is generated by an arbitrary moving body (f=0) can be expressed as follows:

$$\frac{1}{\mathrm{c}^2}\frac{\bar{\partial}^2 \tilde{p}}{\partial t^2} - \bar{\nabla}^2 \tilde{p} = -\left[\frac{M_n}{c}\frac{\partial p}{\partial t} + p_n\right]\delta(f) - \frac{1}{c}\frac{\partial}{\partial t}[M_n p \delta(f)] - \nabla \cdot [p\,\vec{n}\delta(f)]$$

where $\vec{n}$ is the unit normal vector on the surface and $\mathrm{p_n} = \nabla p \cdot \vec{n}$. Now we can use the Green's function of the wave equation in the unbounded space, the so-called free-space Green's function, to find the unknown function $\mathrm{p}(\vec{x}, t)$ everywhere in space. The result is the Kirchhoff formula for moving surfaces.

2. [50 points] Farassat's formulation 1 for the loading noise is given as

$$4\pi p_L'(\vec{x},t) = \frac{1}{c}\frac{\partial}{\partial t}\int_{f=0}\left[\frac{L_r}{r(1-M_r)}\right]_{ret} dS + \int_{f=0}\left[\frac{L_r}{r^2(1-M_r)}\right]_{ret} dS$$

where $L_r = \Delta P\ \vec{n}\cdot\hat{\vec{r}} = \Delta P\cos\theta$. This formulation 1 is difficult to compute since the observer time differentiation is outside the integrals. A much more efficient and practical formulation can be derived by carrying the observer time derivate inside the integrals (formulation 1A). Show that formulation 1A for the loading noise becomes

$$4\pi p_L'(\vec{x},t) = \frac{1}{c}\int_{f=0}\left[\frac{\dot{L_r}}{r(1-M_r)^2}\right]_{ret} dS + \int_{f=0}\left[\frac{L_r - L_M}{r^2(1-M_r)^2}\right]_{ret} dS + \frac{1}{c}\int_{f=0}\left[\frac{L_r(r\dot{M}_r + c(M_r - M^2))}{r^2(1-M_r)^3}\right]_{ret} dS$$

where $L_M = \vec{L}\cdot\vec{M}$.