MAE 298 Aeroacoustics – Homework #3
Generalized Differentiation and Farassat's Formulation

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Nomenclature

L	Subscript for loading parameters
n	Surface normal direction
s	Surface coordinate variable
t	time
$\vec{x}$	Observer location vector
r	Distance between source and observer
$\hat{r}$	Unit vector between source and observer
θ	Angle between $\hat{r}$ and $\vec{x}$
M	Mach number
$W(r)$	Radial distribution of mean axial velocity
c	Speed of Sound
$\overline{\rho}$	Mean density
γ	Specific heat ratio
p	Pressure
$\widetilde{p}$	Pressure (Discontinuous across data surface)
$\overline{p}$	Mean pressure
p'	Perturbation pressure
ΔP	Pressure difference from CFD solution
k	Wavenumber
$\overline{\partial}$	Generalized derivative
δ	Delta function
ω	Wave oscillating frequency
i	Imaginary number $\sqrt{-1}$
exp	Exponential (e)
E	Exponential term: $kz + n\theta - \omega t$
λ	Constant term in Bessel equation
J	First-order Bessel function
Y	Second-order Bessel function
$H^{(n)}$	nth-order Hankel function
x	Placeholder variable for λr
A, B	Arbitrary Bessel function constants
C, D	Arbitrary Hankel constants
$\vec{V}$	General velocity vector
V_r	Velocity component in radial direction
ν	Constant velocity parameter
χ	Constant position parameter
ζ	Position of vortex sheet dividing inner/outer solution
$+/-$	Outer/Inner solution, respectively

Overview

Explore techniques of generalized differentiations and solve FW-H loading term with Farassat's Formulation 1A

$r = |\vec{x} - \vec{y}|$

I. Problem 1 – Generalized Differentiation of Wave Equation

The acoustic wave equation without considering the source is expressed as follows:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0 \tag{1}$$

We can define a new function $\widetilde{p}$ using the imbedding technique as follows:

$$\widetilde{p} = \begin{cases} p, & f > 0 \\ 0, & f < 0 \end{cases} \tag{2}$$

where $f = 0$ describes the arbitrary moving body. Show that the wave equation whose sound is generated by an arbitrary moving body (f=0) can be expressed as follows:

$$\frac{1}{c^2}\frac{\overline{\partial}^2\widetilde{p}}{\partial t^2} - \overline{\nabla}^2\widetilde{p} = -\left[\frac{M_n}{c}\frac{\partial p}{\partial t} + p_n\right]\delta(f) - \frac{1}{c}\frac{\partial}{\partial t}\left[M_n p\delta(f)\right] - \nabla\cdot\left[p\vec{n}\delta(f)\right] \tag{3}$$

where $\vec{n}$ is the unit normal vector on the surface and $p_n = \nabla p \cdot \vec{n}$. Now we can use the Greens function of the wave equation in the unbounded space, the so-called free-space Greens function, to find the unknown function $p(\vec{x}, t)$ everywhere in space. The result is the Kirchhoff formula for moving surfaces.

I.A. Term 1

II. Problem 2 – Farassat Formulation 1A for Loading Noise

Farassats formulation 1 for the loading noise is given as

$$4\pi p'_L = \frac{1}{c}\frac{\partial}{\partial t}\int_{f=0}\left[\frac{L_r}{r(1-M_r)}\right]_{ret} ds + \int_{f=0}\left[\frac{L_r}{r^2(1-M_r)}\right]_{ret} ds \tag{4}$$

where $L_r = \Delta P\vec{n}\cdot\hat{r} = \Delta P\cos\theta$. This formulation 1 is difficult to compute since the observer time differentiation is outside the integrals. A much more efficient and practical formulation can be derived by carrying the observer time derivate inside the integrals (formulation 1A). Show that formulation 1A for the loading noise becomes

$$4\pi p'_L = \frac{1}{c}\int_{f=0}\left[\frac{\dot{L}_r}{r(1-M_r)^2}\right]_{ret} ds + \int_{f=0}\left[\frac{L_r - L_M}{r^2(1-M_r)}\right]_{ret} ds + \frac{1}{c}\int_{f=0}\left[\frac{L_r[r\dot{M}_r + c(M_r - M^2)]}{r^2(1-M_r)^3}\right]_{ret} ds \tag{5}$$

where $L_M = \vec{L}\cdot\vec{M}$.

Conclusion

conclude