UDE-based Robust Control for a Class of Nonaffine Nonlinear Systems with Uncertainties and Output Constraint

Tian Chen¹, Sainan Zhang², Shuzhi Sam Ge³

- 1. School of Computer Science and Engineering, Center for Robotics, University of Electronic Science and Technology of China.
 - 2. Department of Mechanical Engineering, Biomechatronics and Intelligent Robotics, the City College of New York.
- 3. Department of Electrical and Computer Engineering, National University of Singapore, and also with the Institute for Center for Robotics, University of Electronic Science and Technology of China.

 E-mail: samge@nus.edu.sg

Abstract: In this paper, we study the robust control problem of a class of nonaffine nonlinear systems based on uncertainty and disturbance estimator (UDE) under unknown system dynamics and output constraint. A proposed UDE control method is used to estimate uncertainties that contain the uncertainty of parameters. The prominent feature of UDE is to avoid the construction of inverse operators. The control design requires only the bandwidth information of uncertainty. Moreover, the barrier Lyapunov function (BLF) is used directly to address the problem of output constraint. The remarkable feature of the BLF compared to the Lyapunov function based on pure tracking error is to relax the feasible initial output signal to the whole constraint range. Based on the control of UDE, by using Lyapunov's stability theory, the consistent boundary of the closed-loop system is obtained. Through the simulation studies of model uncertainty and external disturbance, the validity of the proposed UDE-based control method is proved and compared with the traditional UDE control method.

Key Words: Nonaffine Nonlinear Systems, Uncertainty and Disturbance Estimator, Barrier Lyapunov Function

1 Introduction

A variety of control methods for nonlinear systems are proposed, for example, feedback linearization control [1], invariance control [2], non-overshooting control [3]. In recent years, nonlinear systems have been extensively used in various engineering projects, such as the power industry in [4], physics in [5], and have also been used in science and technology research, such as artificial intelligence in [6]. They have also been extensively utilized in academic research, including aperiodic sampled-data control in [7], reliable output feedback control in [8], adaptive output trajectory tracking control in [9], and a multi-observe in [10].

However, many of the actual nonlinear systems are nonaffine, for example, chemical reactions are essentially nonlinear, and the input variables may not be in the form of affine. In fact, the control of nonaffine nonlinear systems is academically challenging due to the lack of mathematical tools. It is impossible to directly deal with the control problem of nonaffine nonlinear systems, because it is difficult to construct it by parsing even if the inverse is known to exist. For the control of nonaffine nonlinear systems, some researchers recommend using neural networks (NNs) as simulators of the discrete-time systems [11, 12]. The primary point is that for a system with limited relativity, the mapping between the input and output of the system is one-to-one so that the left inverse of the nonlinear system can be constructed by NNs. Some scholars have studied the state feedback control of nonaffine singleinput single-output (SISO) systems for the first time by neural network approximation [13]. For a class of non-affine nonlinear systems, an adaptive output feedback control scheme is proposed, in which the output signal can track the reference signal [14]. It is also proposed to use adaptive fuzzy control to control nonaffine nonlinear systems [15]. Although the control problem of nonaffine nonlinear systems has been solved, its output constraints are rarely considered. This paper aims to solve the output constraint problem of the nonaffine nonlinear systems.

To solve the control problem of nonlinear constrained systems, a variety of control methods are proposed, including extremum seeking control [16] and error transformation [17]. Based on the spirit of remodeling control Lyapunov function with potential barrier function, a barrier Lyapunov function (BLF) is proposed to guarantee constraint satisfaction. Under certain limit conditions, the design method of BLF is to ensure the boundary of BLF in the closed-loop system to ensure that the stability and constraint satisfaction of the closed-loop system. The BLF is a finite method to solve this limitation. The basic idea is that when the value of the independent variable tends to the boundary of some areas, the purpose of limiting the system state can be achieved by ensuring the boundary of the BLF. Some research has been carried out, for example, full state constraint based on BLF is developed in [18]. In [19], the state constraint control problem of a class of output constraint flexible manipulator system with different payload is studied. Besides, BLF is also used to resolve strict feedback systems in [20], pure-feedback systems in [21]. The slight defect in BLF leads to the conservatism of the BLF-based approach because it is always caused by tracking errors rather than the original state [22]. In this paper, BLF is used to solve the output constraint problem of the nonaffine nonlinear

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systems.

The control of the unknown parts of the system has attracted extensive attention and several strategies for compensating uncertainty have been proposed [23, 24]. In [25], a stable trajectory tracking controller for a nonlinear system is designed by the backstepping. In [26], an adaptive neural dynamic surface controller is proposed to observe the uncertainty of the model by using a neural network. Although previous research has solved the problem of system uncertainty, these works require either linear parameterization as prior knowledge or boundary information of neural network approximation error [27]. This paper applies a control method based on uncertainty and disturbance estimator (UDE) to a class of non-affine nonlinear systems. A robot control method based on feedback linearization is proposed in [28], where the nonlinearity and uncertainty of the system are regarded as uncertainties and UDE technology is used to estimate uncertainty. Based on the proposed alternative control strategies, the control performance similar to time delay control (TDC) in the frequency domain. In [29], the UDE algorithm is first proposed based on the assumption that a filter with appropriate bandwidth is used to estimate the combined effects of interference and uncertainty. A lot of UDE studies have been presented since then, such as sliding mode control in [30, 31], and applications on [32, 33] vertical take-off and landing (VTOL) aircraft. The UDE algorithm performs well in dealing with uncertainty and interference in different systems and is used to linearize the robust input and output in [34, 35], SISO systems in [36]. In addition, it is more extended to uncertain systems with state delays, including nonlinear systems [37], especially a class of nonaffine nonlinear systems in [38].

The distinctive feature of UDE is that the use of UDE makes it possible to estimate uncertainties. It is the function of controlling input, state, disturbance, which relaxes the general assumptions about uncertainties and disturbance and only needs its bandwidth information in the control design. Also, this method is simple in structure, easy to implement, parameter tuning, and bring good robustness. Compared with the existing work, the main contributions of this paper include:

- (1) This paper proposes a class of nonaffine nonlinear systems based on UDE robust control, and the unknown terms in the system are estimated by UDE. Challenging problems using UDE design controllers are translated into the design of low-pass filters.
- (2) The BLF is designed to deal with the output constraints of the system.
- (3) The Lyapunov stability theory proves the stability of the system.

The effectiveness of the method is verified by simulation. The results show that the closed-loop system is consistent and bounded, and the output of the system can be arbitrarily kept near the desired trajectory.

2 Problem Formulation and Preliminaries

In this article, we use \mathbb{R} to represent a set of non-negative real numbers. $\| \bullet \|$ denotes the Euclidean vector norm in \mathbb{R}^n . To clarify the design procedure, consider the nonaffine feedback nonlinear system

$$\dot{x}(t) = f(x,u) + bu(t) + d(t) \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector, f(x, u) is an unknown smooth nonaffine nonlinear vector function of the state vector $x, b \in \mathbb{R}^{n \times r}$ is a know constant control matrix with full column rank $r, u = [u_1(t), u_2(t), \dots, u_n(t)]^T \in \mathbb{R}^r$ is the control input vector, d(t) is an unknown external disturbance vector.

The control goal is to design a controller u(t) to make the x(t) track the reference trajectory $x_d(t)$ and it also ensures that all closed-loop signals are restricted and don't violate the output constraints.

Assumption 1 [38] For all $(x, u) \in \mathbb{R}^n \times \mathbb{R}^r$, the nonlinear function $\frac{\partial [f(x, u) + bu]}{\partial u} \neq 0$.

Assumption 2 [39] The $x_d(t)$ holds that $|x_d(t)| < k_c$, where $k_c > 0$ is a constant vector.

Lemma 1 [40] For any restricted initial condition, if there is a continuous differentiable positive definite Lyapunov candidate function V(x), so that

$$\dot{V}(x) \le -\rho V(x) + C \tag{2}$$

where ρ and C are normal numbers, it can be concluded that the solution x(t) is consistent and bounded.

3 Control Design

The tracking error is defined as $z(t) = x(t) - x_d(t)$. According to the system (1), the time derivative of the dynamics error z(t) is designed as

$$\dot{z}(t) = \dot{x}(t) - \dot{x}_d(t)
= bu(t) - \dot{x}_d(t) + D$$
(3)

where D = d(t) + f(x, u) represents the unknown term in (1), including the external disturbance term d(t) and the nonaffine uncertainty term f(x, u).

The controller u(t) is designed as the feedback linearization part $u_l(t)$ and uncertainty compensation part $u_d(t)$, which is $u(t) = u_l(t) + u_d(t)$.

Design the stabilizing function as

$$u_l(t) = b^+ \left(-Kz(t) - \frac{z(t)}{k_c^2 - z^T(t)z(t)} + \dot{x}_d(t) \right)$$
(4)

where $b^+ = (b^T b)^{-1} b^T$ is the pseudoinverse of b, and K is a matrix.

Design the uncertainty part $u_d(t)$ as

$$u_d(t) = -b^+[f(x,u) + d(t)]$$

= $-b^+D$ (5)

Substituting (4) and (5) into (3), therefore, the closed-loop system becomes

$$\dot{z}(t) = -Kz(t) - \frac{z(t)}{k_c^2 - z^T(t)z(t)} + bu_d(t) + D$$
 (6)

To realize the output constraint, the following barrier Lyapunov function candidate is selected

$$V = \frac{1}{2} \log \frac{k_c^2}{k_c^2 - z^T(t)z(t)}$$
 (7)

We can see that V is positive definite in the set $|z(t)| < k_c$, continuously differentiable and bounded. The time derivative of V is

$$\dot{V} = \frac{z^{T}(t)}{k_{c}^{2} - z^{T}(t)z(t)} \dot{z}(t)
= \frac{z^{T}(t)}{k_{c}^{2} - z^{T}(t)z(t)} \left(-Kz(t) - \frac{z(t)}{k_{c}^{2} - z^{T}(t)z(t)} \right)
+bu_{d}(t) + D
= -\frac{Kz^{T}(t)z(t)}{k_{c}^{2} - z^{T}(t)z(t)} - \frac{z^{T}(t)z(t)}{(k_{c}^{2} - z^{T}(t)z(t))^{2}}
+ \frac{z^{T}(t)}{k_{c}^{2} - z^{T}(t)z(t)} [bu_{d}(t) + D]$$
(8)

Using Young's inequality, there is

$$\frac{z^{T}(t)}{k_{c}^{2} - z^{T}(t)z(t)} [bu_{d}(t) + D] - \frac{z^{T}(t)z(t)}{(k_{c}^{2} - z^{T}(t)z(t))^{2}} \le \frac{[bu_{d}(t) + D]^{2}}{2}$$
(9)

So, (8) becomes

$$\dot{V} \le -\frac{Kz^{T}(t)z(t)}{k_c^2 - z^{T}(t)z(t)} + \frac{[bu_d(t) + D]^2}{2}$$
(10)

Since the term D is unknown, we can not prove its stability directly. From (6), the uncertain and disturbance compensation D becomes

$$D = Kz(t) + \dot{z}(t) + \frac{z(t)}{k_a^2 - z^T(t)z(t)} - bu_d(t)$$
 (11)

It can be seen that the known signals $z(t), x(t), x_d(t)$ can be used to observe the uncertainty term. Since a filter with the appropriate bandwidth information of the uncertainty in [37] can be used to estimate the signal, we use an appropriate low-pass filter $G_f(s)$, which has zero phases and unity gain in the spectrum of D. Therefore, the estimator of D becomes

$$\hat{D} = G_f(s) * \left(Kz(t) + \dot{z}(t) + \frac{z(t)}{k_c^2 - z^T(t)z(t)} - bu_d(t) \right)
= \mathscr{L}^{-1} \left\{ \frac{G_f(s)}{1 - G_f(s)} \right\} * \left(Kz(t) + \frac{z(t)}{k_c^2 - z^T(t)z(t)} + \dot{z}(t) \right)$$
(12)

where $\mathcal{L}^{-1}\{\cdot\}$ is the operator of inverse Laplace transform.

There $-bu_d(t) = \hat{D}$, thus, the disturbance compensation part $u_d(t)$ is

$$u_{d}(t) = -b^{+} \left[\mathcal{L}^{-1} \left\{ \frac{G_{f}(s)}{1 - G_{f}(s)} \right\} \right.$$
$$\left. * \left(Kz(t) + \dot{z}(t) + \frac{z(t)}{k_{c}^{2} - z^{T}(t)z(t)} \right) \right] (13)$$

Control input is designed as follows

$$u(t) = b^{+} \left[-Kz(t) + \dot{x}_{d}(t) - \frac{z(t)}{k_{c}^{2} - z^{T}(t)z(t)} - \mathcal{L}^{-1} \left\{ \frac{G_{f}(s)}{1 - G_{f}(s)} \right\} \right]$$

$$* \left(Kz(t) + \dot{z}(t) + \frac{z(t)}{k_{c}^{2} - z^{T}(t)z(t)} \right)$$
(14)

According to the tracking error of the design, the appropriate control signal is generated on the basis of UDE robust control, which can make a class of nonaffine nonlinear systems achieves the desired purpose.

Substituting (3) and (12) into (14), the error dynamics can be obtained as

$$z(t) = -Kz(t) - \frac{z(t)}{k_c^2 - z^T(t)z(t)} - \tilde{D}$$
 (15)

where $\tilde{D} = \hat{D} - D$ is the estimation error of the uncertainty. The stability analysis of the system described in (15) is given by the following theorem.

Theorem 1 Considering the nonaffine nonlinear dynamics described in (1), the UDE-based controller (14) and the filter $G_f(s)$ are appropriately selected as the rigorously appropriate stable filters with unity gain and zero phase shift over the spectrum of the uncertainty D and zero gain, then, for any initial conditions in the constraint region, we can conclude that all the signals in the closed-loop system are uniformly bounded. Furthermore, the error signal z(t) converges uniformly to compact set $\Omega_{z(t)}$ as shown in (16), whose size can be reduced by appropriately choosing the control parameter K and designing the filter $G_f(s)$,

$$\Omega_{z(t)} := \left\{ z(t) \in \mathbb{R}^n \middle| \|z(t)\| \le \frac{\|\tilde{D}\|}{\min\{2K\}} \right\} \quad (16)$$

Proof: Consider the barrier Lyapunov function candidate

$$V = \frac{1}{2} \log \frac{k_c^2}{k_c^2 - z^T(t)z(t)}$$
 (17)

Substituting (15) into (17), the time derivative of V s

$$\dot{V} = \frac{z^{T}(t)}{k_{c}^{2} - z^{T}(t)z(t)} \left(-Kz(t) - \frac{z(t)}{k_{c}^{2} - z^{T}(t)z(t)} - \tilde{D} \right)
= -\frac{Kz^{T}(t)z(t)}{k_{c}^{2} - z^{T}(t)z(t)} - \frac{z^{T}(t)z(t)}{(k_{c}^{2} - z^{T}(t)z(t))^{2}}
- \frac{z^{T}(t)\tilde{D}}{k_{c}^{2} - z^{T}(t)z(t)}$$
(18)

Using Young's inequality

$$-\frac{z^{T}(t)\tilde{D}}{k_{o}^{2}-z^{T}(t)z(t)} \le \frac{1}{2} \frac{z^{T}(t)z(t)}{(k_{o}^{2}-z^{T}(t)z(t))^{2}} + \frac{\|\tilde{D}\|^{2}}{2}$$
(19)

The time derivative V becomes

$$\dot{V} \leq -\frac{Kz^{T}(t)z(t)}{k_{c}^{2} - z^{T}(t)z(t)} + \frac{\|\tilde{D}\|^{2}}{2}
\leq -\rho V + C$$
(20)

where $\rho = \min\{2K\}, C = \frac{\|\tilde{D}\|^2}{2}$. Multiplying (20) by $e^{\rho t}$ yield $\frac{\mathrm{d}}{\mathrm{dt}}(Ve^{\rho t}) \leq Ce^{\rho t}$. Combining the above inequalities, it yields

$$0 \le V(t) \le e^{-\rho t} V(0) + \frac{C}{\rho} (1 - e^{-\rho t})$$
 (21)

For any initial conditions in the constraint region, when $t\to\infty$, V(t) is bounded by $\frac{C}{\rho}=\frac{0.5\|\tilde{D}\|^2}{\min\{2K\}}$. In other words, we can obtain $\|z(t)\|\leq\frac{\|\tilde{D}\|^2}{\min\{2K\}}$. Obviously, $\|z(t)\|$ can reduce the estimation error \tilde{D} by increasing the control parameter K and designing the filter $G_f(s)$. If the estimated error \tilde{D} converges to zero, then the error signal z(t) also converges to zero. According to the filter $G_f(s)$ designed as a strictly appropriate stable filter in [37], we can get $\tilde{D}\approx0$. Hence, according to Lemma 1,the error signal z(t) is uniformly bounded. This completes the proof.

4 Simulation

In this section, we consider two simulation cases to illustrate the effectiveness of the proposed method. The simulation aims to examine whether the UDE-based control (14) can make system output follow the required trajectory $x_d(t)$. Besides, the comparison with the traditional UDE control method shows the effectiveness of this method.

The nonaffine system is described as

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \end{bmatrix} u + \begin{bmatrix} 0 \\ f_2(\theta, \dot{\theta}, u) \end{bmatrix} + \begin{bmatrix} 0 \\ d_2(t) \end{bmatrix}$$
 (22)

where $f_2(\theta, \dot{\theta}, u)$ is the nonaffine uncertainty; θ and $\dot{\theta}$ are the system states; u is the control input and $d_2(t)$ is the external disturbance.

Compared to (1), we have
$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$
, $f(x,u) = \begin{bmatrix} 0 \\ f_2(\theta, \dot{\theta}, u) \end{bmatrix}$ and $d(t) = \begin{bmatrix} 0 \\ d_2(t) \end{bmatrix}$, where $f_2(\theta, \dot{\theta}, u) = \theta + \dot{\theta} + \arctan(u)$ and $d_2(t) = \cos(2\pi t)$. It is easy to prove that the formula (22) satisfies the Assumption 1 because $\frac{\partial [f(x,u)+bu]}{\partial u} = \begin{bmatrix} 0 \\ 80 + \frac{1}{1+u^2} \end{bmatrix} \neq 0$. The desired trajectory is described by $x_d(t) = \sin(0.1\pi t)$.

For the traditional UDE controller: $b = \begin{bmatrix} 0 \\ 80 \end{bmatrix}, K = \text{diag}[50].$

The UDE-based control (14) is activated by first-order low-pass filter $G_f(s) = \frac{1}{Ts+1}$. Theorem 1 guarantees the stability of the closed system. If filter $G_f(s)$ is selected properly as a strictly correct stable filter, it will have uniform gain and zero phase shift in the spectrum with uncertain u and zero gain. The first-order low-pass filter is chosen with T = 0.01s.

Then, for two case studies with external disturbance, the same parameters are used. We divide it into two parts: 1) there is no additional nonaffine uncertainties $f_2(\theta, \dot{\theta}, u) = 0$ and 2) there is additional nonaffine uncertainties $f_2(\theta, \dot{\theta}, u) = \theta + \dot{\theta} + \arctan(u)$.

1) There is no additional nonaffine uncertainties

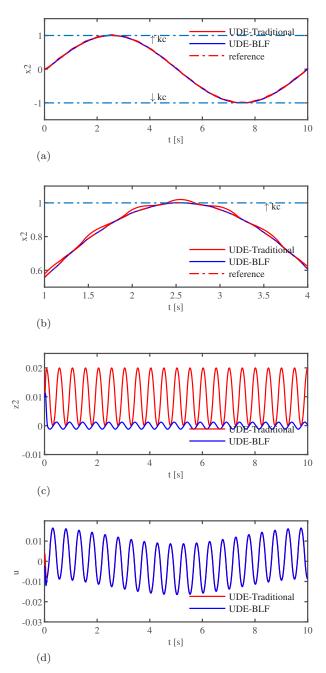


Fig. 1: In the case of the nonaffine term $f_2(\theta,\dot{\theta},u)=0$, the tracking experiment is carried out with the UDE-based and the traditional UDE controllers. (a) System output x_2 . (b) Magnify the system output x_2 . (c) Tracking error z_2 . (d) System input u.

 $f_2(\theta, \dot{\theta}, u) = 0$: The simulation results are shown in Fig. 1. Fig. 1(a) shows the tracking performance of x_2 . We can see from Fig. 1(a) that the proposed UDE control method (14) can perfectly track the desired trajectory, while the traditional UDE control method has some oscillations. Fig. 1(b) magnifies the tracking performance of x_2 . As can be seen from Fig. 1(b), the proposed UDE control method can ensure that the output is limited, while the traditional UDE control method exceeds the boundary, which means that the proposed control can guarantee output constraint. According to the tracking error shown in Fig. 1(c), compared with

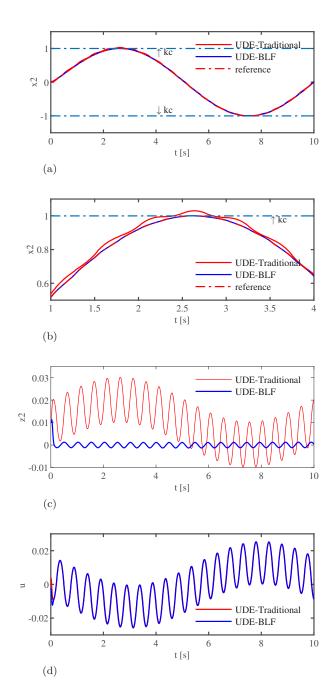


Fig. 2: In the case of the nonaffine term $f_2(\theta, \dot{\theta}, u) = \theta + \dot{\theta} + \arctan(u)$, the tracking experiment is carried out with the UDE-based and the traditional UDE controllers. (a) System output x_2 . (b) Magnify the system output x_2 . (c) Tracking error z_2 . (d) System input u.

the traditional UDE control, the UDE-based control can still maintain good tracking performance. Therefore, compared with the traditional UDE controller, the UDE-based controller has stronger robustness in dealing with external disturbance. Fig. 1(d) plots the control input of the system.

2) There is additional nonaffine uncertainties

 $f_2(\theta, \dot{\theta}, u) = \theta + \dot{\theta} + \arctan(u)$: The simulation results are shown in Fig. 2. Fig. 2(a) shows the tracking performance of x_2 . We can see from Fig. 1(a) that the proposed UDE control method (14) can perfectly track the desired trajectory, while the traditional UDE control

method has some oscillations. Fig. 2(b) magnifies the tracking performance of x_2 . We can see from Fig. 2(b) that the proposed UDE control method can ensure that the output is limited, while the traditional UDE control method exceeds the boundary, which means that the proposed control can guarantee output constraint. In Fig. 2(c), it is obvious that the UDE-based control has a better performance to handle the external disturbance and additional nonaffine uncertainties than the traditional UDE control. The control input of the system is shown in Fig. 2(d).

5 Conclusion

In this paper, for a class of nonaffine nonlinear systems, the UDE-based robust control strategy has been investigated. UDE has been used to estimate uncertainty, which is a function of the control input, state, and boundary conditions. This method has avoided the singular problem of the controller caused by the construction of the inverse operator. In the design process, in addition to bandwidth information, no uncertainty knowledge is required. Moreover, the BLF has been used to handle the impact of output constraint. The Lyapunov stability theory has ensured that the error signal converges to the predetermined set. The closed-loop system is asymptotically stable. The simulation results have shown that the UDE based robust control system has good performance and is easy to implement.

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