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Adaptive neural dynamic surface control of output constrained non-linear systems with unknown control direction

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Abstract: This study investigates the adaptive neural dynamic surface control (DSC) of output constrained non-linear systems, subject to unknown system dynamics and uncertain control direction. A Nussbaum-type dynamic gain algorithm is used to handle the effect of unknown control direction. Integral barrier Lyapunov functions (iBLFs) are directly utilised to tackle the effect of output constraint. The prominent feature of iBLFs is that the feasible initial output signals are relaxed to the whole constraint range compared with pure tracking errors-based barrier Lyapunov function. Also, the DSC technology is developed to avoid the explosion of complexity in traditional control design, and adaptive neural networks are adopted to estimate the uncertainty that comprises the unknown system dynamics and parametric uncertainties. By utilising Lyapunov synthesis, it is proven that the proposed control is able to guarantee semi-global uniformly ultimately bounded of all signals in the closed-loop system. Simulation results are provided to verify the effectiveness of the proposed approach.

1 Introduction

During recent decades, adaptive neural network (NN) control of uncertain non-linear systems has attracted a great deal of attention [1–4]. Adaptive NN control has been used to solved for non-linear systems with unknown parameters [5–9]. For the case with unknown functions, the adaptive control design is merged as the problem to model the unknown functions by using the adaptive systems in [10–14]. These works have received an active topic for the adaptive NN control of non-linear systems. Although adaptive neural tracking control of non-linear systems has been studied extensively, the systems with output constraints are rarely considered.

Barrier Lyapunov functions (BLFs) are effective tools for handling the constraint, because BLFs will grow to infinity when their values approach limitations. This property combining with the Lyapunov direct method is employed to ensure that the states are in predefined constrained sets. In [15], the BLFs are used to design control methods for single-input single-output (SISO) non-linear systems with output constraint. The boundedness of BLFs is constructed to ensure that the output constraint is not transgressed. In [16, 17], the tracking problem of time-varying output constraint is solved by employing asymmetric time-varying BLFs to ensure constraint satisfaction. Besides, BLFs also have been used to solve for tracking problems of strict-feedback systems [18, 19], pure-feedback systems [20–22], multiple-input multiple-output (MIMO) systems [23–25], as well as the application in robotics [26, 27] and flexible marine risers [28]. After that, compared with previous BLFs, the integral BLFs (iBLFs) are proposed for non-linear systems in [29], where the original state constraints are directly mixed with the error terms. This proposed iBLFs can reduce some of the conservatism associated with the use of purely error-based functions with transformed error constraints. Some research results have been developed based on the analysis in [29]. The iBLFs have been used to design control strategies for constrained systems in the strict-feedback systems [30, 31] and pure-feedback systems [32]. However, those works either need the information of control gain in [30, 31], or require the bounds of the derivatives of non-linear functions in [32], which may be very difficult to acquire in practical applications. Meanwhile, another restriction in these works is that the sign of control direction is compelled to be known. Therefore, the tracking problem of constrained non-linear systems needs to be further developed.

In this paper, Nussbaum-type functions are proposed for non-linear systems without control direction and it is first defined in [33]. Based on the analysis about Nussbaum-type functions in [33], a number of research results have been proposed. In [34], an observer-based fuzzy adaptive controller for non-linear systems with unknown control gain is investigated by using Nussbaum-type functions. In [35], a novel Nussbaum-type function is introduced for linearly parameterised multi-agent systems with unknown identical control direction. Based on Nussbaum-type functions and NN approximation, adaptive neural control is presented of strict-feedback non-linear systems, where Nussbaum-type functions are used to handle unknown virtual control coefficients in [36, 37]. Besides, a fuzzy adaptive control strategy employing Nussbaum-type functions is introduced for MIMO non-linear systems with unknown control direction in [38]. However, constraint is not taken into consideration in those control schemes. In [39], Nussbaum gain adaptive control design is studied for SISO non-linear systems with unknown control gain and the systems are enforced to subject to state constraints. Although the issue of constrained non-linear systems with unknown control direction by using Nussbaum-type functions is tackled in [39], the research result only focuses on the non-linear system with unknown parameters and cannot be directly utilised to the control of non-linear systems with unknown functions. What is more, the control scheme developed in [39] still employs the traditional BLFs to handle the effect of state constraint, which results in the conservative step of being transformed into new bounds on tracking errors [31]. This key point motivates us to deal with the control problem of non-linear systems with unknown functions and constraint.

Based on the above discussions, this paper will plan to stabilise of uncertain non-linear systems with the output constraint and unknown control direction. However, the complexity of controller will grow rapidly as the order n of the system increases, resulting in the fact that the control design suffers from the problem of 'explosion of the complexity' which is caused by the repeated differentiations of certain non-linear functions such as virtual control [40]. Thus, dynamic surface control (DSC) is used to overcome the problem of 'explosion of the complexity' by introducing first-order filters of the synthetic virtual control input at each step of traditional backstepping approach. The DSC technology has been widely used for the control design of non-linear systems [41–46]. In this paper, adaptive neural DSC is proposed of uncertain non-linear systems with unknown control direction and iBLFs are introduced to handle the effect of output constraint. Compared with the existing work, the contributions of this paper are summarised as follows:

i. Compared with the conventional Lyapunov functions [15, 18, 47], iBLFs are developed to deal with output constraint directly without carrying out an additional mapping to the error space. The initial states can be relaxed to whole constrained space.
ii. An adaptive controller incorporating of Nussbaum function is introduced of non-linear systems with completely uncertain time-varying control coefficients, where there is no need to get any priori knowledge of control coefficients.
iii. In contrast to the traditional backstepping methods, DSC employing the adaptive NN structure is introduced for the control of non-linear systems with the unknown control directions and output constraint. The explosion of complexity in the previous design methods is avoided.

The remaining of this paper is organised as follows. Section 2 elaborates control problem and introduces several basic assumptions. In Section 3, control design is elucidated which is developed by using adaptive neural DCS for a class of non-linear systems. Some simulation examples are given to show the performance and validity of the designed control in Section 4. Section 5 contains the conclusion of the paper.

2 Problem formulation and preliminaries

2.1 Problem formulation

Throughout this paper, we denote by $\mathbb{R}_+$ the set of non-negative real numbers, $\| \bullet \|$ the Euclidean vector norm in $\mathbb{R}^m$, and $\lambda_{\max}(\bullet)$ and $\lambda_{\min}(\bullet)$ the maximum and minimum eigenvalues of $\bullet$, $\bar{x}_i = [x_1, x_2, \dots, x_i]^T$, $\bar{z}_i = [z_1, z_2, \dots, z_i]^T$, respectively. From (1), in actual applications, $f_i(\bar{x}_i)$ and $g_i(\bar{x}_i)$ cannot be obtained forehand. The non-linear system under study in this paper is given as follows:

$$\begin{aligned} \dot{x}_i(t) &= f_i(\bar{x}_i(t)) + g_i(\bar{x}_i(t))x_{i+1}(t) + d_i(t), \quad i = 1, 2, \dots, n-1 \\ \dot{x}_n(t) &= f_n(\bar{x}_n(t)) + g_n(\bar{x}_n(t))u(t) + d_n(t) \\ y(t) &= x_1(t) \end{aligned} \tag{1}$$

where $f_1(\bar{x}_1), f_2(\bar{x}_2), \dots, f_n(\bar{x}_n)$ are unknown smooth functions; $g_1(\bar{x}_1), g_2(\bar{x}_2), \dots, g_n(\bar{x}_n)$ are the unknown differentiable control gains, the signs of $g_i(\bar{x}_i), i = 1, 2, \dots, n$, are also unknown; $d_1(t), d_2(t), \dots, d_n(t)$ are smooth and bounded external disturbance, and $| d_i(t) | \le D^*$, where $D^* > 0$ is an unknown constant. $y(t)$ and u denote the output signal and control input, respectively. In the control design process, the output $y(t)$ is asked to hold on the set $\{|y| \le k_c, \forall t \ge 0\}$, where k_c is a positive constant. Meanwhile, $y(t)$ is demanded to track the desired trajectory $y_d(t)$ which is continuously differentiable function and satisfy $| y_d(t) | < k_c$ as Assumption 1.

Assumption 1 [15]: The $y_d(t)$ and its time derivatives holds that $| y_d(t) | < k_c$, $| \dot{y}_d(t) | < Y$, $\forall t \ge 0$ is bounded, where $k_c > 0$, $Y > 0$ are constants.

2.2 Nussbaum-type gain

The sign of $g_i(\bar{x}_i)$ always be called the control direction, which represents the motion direction of the non-linear systems. Unknown control function direction will increase control variables as explosive speed. This paper utilises Nussbaun-type gain to overcome the mentioned difficulty. If a continuously differentiable function $N(s)$ has the following properties, it is called a Nussbaum-type function [37]

$$\lim_{s \to \infty} \sup \frac{1}{s} \int_0^s N(\zeta)\,d\zeta = +\infty \tag{2}$$

$$\lim_{s \to \infty} \inf \frac{1}{s} \int_0^s N(\zeta)\,d\zeta = -\infty \tag{3}$$

The continuous function usually adopts $\zeta^2\cos(\zeta)$, $\zeta^2\sin(\zeta)$, or $e^{\zeta^2}\cos((\pi/2)\zeta)$ and so on. For clarity, we choose $N(\zeta) = \zeta^2\cos(\zeta)$ in this paper.

Lemma 1 [37]: Let $V(\bullet)$ and $N(\bullet)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0, \forall t \in [0, t_f)$, and $N(\bullet)$ be an even smooth Nussbaum-type function. If there is

$$V(t) \le C_0 + \int_0^t (gN(\zeta) + 1)\dot{\zeta}\,dt, \quad \forall t \in [0, t_f) \tag{4}$$

where C_0 is a suitable constant, g denotes a non-zero constant, then $V(t)$, $N(\zeta)$ and $\int_0^t (gN(\zeta) + 1)\dot{\zeta} d\tau$ must be bounded on $[0, t_f)$.

Lemma 2 [48]: For the Gaussian radial basis function NN, let $\varrho \triangleq \min_{i \ne j} \| \mu_i - \mu_j \|$ and ψ is the width of Gaussian function, and q is the dimension of input Z. Then $\| S(Z) \|$ is upper bounded as

$$\| S(Z) \| \ge \sum_{k=0}^{\infty} 3q(k+1)^{q-1} e^{-2\varrho^2 k^2/\psi^2} \triangleq s^* \tag{5}$$

where $s^* > 0$ is a constant, l and Z are the node number and inputs of NNs, respectively.

Lemma 3: The adaptation law is designed as $\dot{\hat{W}}_i = \Gamma_i[S_i(Z_i)z_i - \delta_i z_i \hat{W}_i]$, where Γ_i is the constant gain matrix, $S_i(\cdot)$ is the basis function of NNs, Z_i is the input of NNs, and γ_i, $i = 1, 2, \dots, n$, is a positive number. For an adaptive law mentioned above, there exits a compact set

$$\Omega_{W_i} = \left\{ \hat{W}_i \,|\, \| \hat{W}_i \| \le \left(\frac{\lambda_{\max}(\Gamma_i)}{\lambda_{\min}(\Gamma_i)} \right)^{1/2} \frac{s_i^*}{\gamma_i} \right\} \tag{6}$$

where $s_i^*, i = 1, 2, \dots, n$, is an unknown positive constant such that $\hat{W}_i(t) \in \Omega_{W_i}, \forall t \ge 0$ provided that $\hat{W}_i(0) \in \Omega_{W_i}$.

Proof: Let the Lyapunov function $V_{W_i} = \frac{1}{2}\hat{W}_i^T \Gamma_i^{-1} \hat{W}_i$, the time derivative of V_{W_i} is

$$\begin{aligned} \dot{V}_{W_i} &= \hat{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i \\ &= \hat{W}_i^T [S_i(Z_i)z_i - \gamma_i z_i \hat{W}_i] \\ &\le \| W_i \| \, z_i \,| [s_i^* - \gamma_i \| \hat{W}_i \|] \end{aligned} \tag{7}$$

if $\| \hat{W}_i \| > s_i^*/\gamma_i$, $\dot{V}_{W_i} < 0$, which yields

$$\dot{V}_{W_i} < 0 \;\; \text{if} \;\; V_{W_i} > \frac{1}{2}[\lambda_{\min}(\Gamma_i)]^{-1} \left(\frac{s_i^*}{\gamma_i} \right)^2 = \bar{V}_{W_i} \tag{8}$$

where the range of V_{W_i} is $[0, \bar{V}_{W_i}]$. We have

$$\frac{1}{2}[\lambda_{\max}(\Gamma_i)]^{-1} \| \hat{W}_i \|^2 \le \frac{1}{2}[\lambda_{\min}(\Gamma_i)]^{-1} \left(\frac{s_i^*}{\gamma_i} \right)^2 \tag{9}$$

where

$$\| \hat{W}_i \| \leq \left(\frac{\lambda_{\max}(\Gamma_i)}{\lambda_{\min}(\Gamma_i)}\right)^{1/2} \frac{s_i^*}{\gamma_i} \tag{10}$$

□

Lemma 4: The adaptive law is designed as $\dot{\hat{p}}_i = z_i \tanh(z_i/\delta_i) - \eta_{p_i} z_i \hat{p}_i$, where δ_i and η_{p_i}, i=1, 2, …, n, are small positive constants. For an adaptive law mentioned above, there exits a compact set

$$\Omega_{p_i} = \left\{\hat{p}_i \,||\, \hat{p}_i \,| \leq \left(\frac{1}{\eta_{p_i}}\right)^{1/2}\right\} \tag{11}$$

such that $\hat{p}_i(t) \in \Omega_{p_i}, \forall \geq 0$ provided that $\hat{p}_i(0) \in \Omega_{p_i}$.

Proof: Let the Lyapunov function $V_{p_i} = (1/2)(\hat{p}_i)^2$, the time derivative of $\dot{V}_{p_i}$ is

$$\begin{aligned} \dot{V}_{p_i} &= \hat{p}_i \dot{\hat{p}}_i \\ &= \hat{p}_i \left[z_i \tanh\left(\frac{z_i}{\delta_i}\right) - \eta_{p_i} z_i \hat{p}_i \right] \\ &\leq | p_i || z_i | \left[\left| \tanh\left(\frac{z_i}{\delta_i}\right) \right| - \eta_{p_i} | \hat{p}_i \right| \Big] \end{aligned} \tag{12}$$

if $| \hat{p}_i | > \tanh(z_i/\delta_i) \leq 1/\eta_{p_i}$, $\dot{V}_{p_i} < 0$

$$\dot{V}_{p_i} < 0 \text{ if } V_{p_i} > \frac{1}{2}\frac{1}{\eta_{p_i}} = \bar{V}_{p_i} \tag{13}$$

where the range of V_{p_i} is $[0, \bar{V}_{p_i}]$. We have

$$\frac{1}{2}\hat{p}_i^2 \geq \frac{1}{2}\frac{1}{\eta_{p_i}} \tag{14}$$

which implies $| p_i | \leq \sqrt{1/\eta_{p_i}}$.

□

Lemma 5 [49]: The following inequality holds for $\delta > 0, \forall x \in \mathbb{R}$

$$0 \leq | x | - x \tanh\left(\frac{x}{\delta}\right) \leq 0.2785\delta \tag{15}$$

3 Control design

In this section, we design an adaptive neural dynamic surface controller to identify the uncertainty and handle the effect of output constraint. The tracking errors are defined as

$$z_1 = x_1 - y_{\rm d} = x_1 - \omega_1 \tag{16}$$

$$z_i = x_i - \omega_i = x_i - y_i - \alpha_i \quad i = 2, 3, \ldots, n \tag{17}$$

where z_1 and z_i are the system tracking errors, $y_{\rm d} = \omega_1$ is the desired trajectory function which is continuously differentiable, $y_i = \omega_i - \alpha_i$ is the virtual control errors, ω_i is the filtered virtual control, α_i is the virtual control that will be extended for the corresponding subsystem as far as $(n-1)$th. An overall control input u is reckoned at the last step. Comparing with the previous design methods [15, 18], it is obviously different that the quantity $\dot{\alpha}_{i-1}$ is replaced by ω_i at each step of recursion. It turned out that the arithmetic of differentiation is reduced to straight-forward algebraic operation. The system structure is plotted in Fig. 1.

Step 1: As $z_1 = x_1 - w_1$, involving the transformed system (1), the time derivative of dynamic error z_1 is

$$\dot{z}_1 = f_1(x_1) + g_1(x_1)x_2 + d_1 - \dot{\omega}_1 \tag{18}$$

To derive the stabilising adaptive control law for the dynamic error system (18), we introduce a iBLF candidate

$$V_{z_1} = \int_0^{z_1} \frac{\sigma k_c^2}{k_c^2 - (\sigma + y_{\rm d})^2} {\rm d}\sigma \tag{19}$$

continuously differentiable satisfying $|x_1| < k_c$. Let $\sigma = \theta z_1$, we have

$$\frac{z_1^2}{2} \leq V_{z_1} \leq z_1^2 \int_0^1 \frac{\theta k_c^2}{k_c^2 - (\theta z_1 + y_{\rm d})^2} {\rm d}\theta \tag{20}$$

The time derivative of V_{z_1} is

$$\dot{V}_{z_1} = \frac{k_c^2 z_1}{k_c^2 - x_1^2}[f_1 + g_1(y_2 + \alpha_1 + z_2) + d_1 - \dot{y}_{\rm d}] + \frac{\partial V_{z_1}}{\partial y_{\rm d}}\dot{y}_{\rm d} \tag{21}$$

In (21), using the integration by parts and the substitution $\sigma = \theta z_1$, we have

$$\frac{\partial V_{z_1}}{\partial y_{\rm d}} = z_1\left(\frac{k_c^2}{k_c^2 - x_1^2} - \rho_1(z_1, y_{\rm d})\right) \tag{22}$$

where ρ_1 is

$$\rho_1(z_1, y_{\rm d}) = \frac{k_c}{2z_1}\ln\frac{(k_c + z_1 + y_{\rm d})(k_c - y_{\rm d})}{(k_c - z_1 - y_{\rm d})(k_c + y_{\rm d})} \tag{23}$$

Using L'Hôpital's rule, $\rho_1(z_1, y_{\rm d})$ becomes

$$\lim_{z_1 \to 0} \rho_1(z_1, y_{\rm d}) = \lim_{z_1 \to 0} \frac{k_c^2}{k_c^2(z_1 + y_{\rm d})^2} = \frac{k_c^2}{k_c^2 + y_{\rm d}^2} \tag{24}$$

Considering Assumption 1, since $| y_{\rm d} | < k_c$, we can find that $\rho_1(z_1, y_{\rm d})$ is well-defined in a neighbourhood of $z_1 = 0$. The desired control is usually unable to be achieved in practise since $f_1(x_1), g(x_1)$ are unknown. We define $h_1(Z_1) = f_1 - ((k_c^2 - x_1^2)/k_c^2)\rho_1\dot{y}_{\rm d} + (k_c^2 z_1 g_1^2/(k_c^2 - x_1^2))$, which is unknown continuous packaged function of x_1 and $\dot{\omega}_1$ and is approximated by $W_1^{*\rm T}S_1(Z_1)$ just like in [31]. The W_1^* is the ideal weight and satisfies $\hat{W}_1 = W_1^* - \tilde{W}_1$, $\hat{W}_1$ is the estimated weight, the $\tilde{W}_1$ is the error weight. The $S_1(Z_1)$ is the basis function. By utilising an $W_1^{*\rm T}S_1(Z_1)$ to approximate $h_1(Z_1)$, where $Z_1 = [x_1, \omega_1, \dot{\omega}_1]^{\rm T} \subset \mathbb{R}^3$, there is

$$h_1(Z_1) = \hat{W}_1^{\rm T}S_1(Z_1) - \tilde{W}_1^{\rm T}S_1(Z_1) + \epsilon_1 \tag{25}$$

where the NN $\hat{W}_1^{\rm T}S_1(Z_1)$ is the estimation of ideal NN $W^{*\rm T}S_1(z_1)$, ϵ_1 is the error of NNs and $| \epsilon_1 | < \epsilon_1^*$ with $\epsilon_1^* > 0$. The virtual control is set as

$$\alpha_1 = N(\zeta_1)\Big[K_1 z_1 + \hat{W}_1^{\rm T}S_1(Z_1) + \hat{p}_1\tanh\Big(\frac{(z_1 k_c^2/(k_c^2 - x_1^2))}{\delta_1}\Big)\Big] \tag{26}$$

where K_1 and σ_1 are positive constants. Design the adaptive laws as

$$\dot{\zeta}_1 = \frac{k_c^2 z_1}{k_c^2 - x_1^2}\Big[K_1 z_1 + \hat{W}_1^{\rm T}S_1(Z_1) + \hat{p}_1\tanh\Big(\frac{(z_1 k_c^2/(k_c^2 - x_1^2))}{\delta_1}\Big)\Big] \tag{27}$$

$$\dot{\hat{W}}_1 = \Gamma_1\Big[\frac{k_c^2 S_1(Z_1) z_1}{k_c^2 - x_1^2} - \gamma_1\hat{W}_1\Big] \tag{28}$$

$$\dot{\hat{p}}_1 = \frac{k_c^2 z_1}{k_c^2 - x_1^2}\tanh\Big(\frac{z_1 k_c^2}{\delta_1(k_c^2 - x_1^2)}\Big) - \eta_{p_1}\hat{p}_1 \tag{29}$$

where Γ_1 is an adaptation gain matrix and $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$; $\sigma_1 > 0$; γ_1, $\eta_{p_1} > 0$ are the γ_1-modification and η_{p_1}-modification to prevent the estimation from drifting to large values in the presence of estimation errors [50]. Furthermore, it is easy to see that $\hat{p}_1(t) > 0$ provided the updating law (29) and the non-negative initial value. Thus, the time derivative of V_{z_1} is

$$\begin{aligned}\dot{V}_{z_1} = & -\frac{K_1 k_c^2 z_1^2}{k_c^2 - x_1^2} + [g_1 N(\zeta_1) + 1]\dot{x}\zeta_1 + \frac{k_c^2 z_1 g_1 z_2}{k_c^2 - x_1^2} + \frac{k_c^2 z_1 d_1}{k_c^2 - x_1^2} \\ & -\frac{k_c^2 z_1}{k_c^2 - x_1^2}\hat{p}_1 \tanh\Big(\frac{z_1 k_c^2}{\delta_1 (k_c^2 - x_1^2)}\Big) + \frac{k_c^2 z_1 g_1 y_2}{k_c^2 - x_1^2} \\ & -\frac{k_c^2 z_1}{k_c^2 - x_1^2}\tilde{W}_1^{\mathrm{T}} S_1(Z_1) + \frac{k_c^2 z_1}{k_c^2 - x_1^2}\epsilon_1 - \frac{k_c^4 z_1^2 g_1^2}{(k_c^2 - x_1^2)^2}\end{aligned} \tag{30}$$

Using the Young's inequality, there are

$$\frac{k_c^2 z_1 g_1 z_2}{k_c^2 - x_1^2} \le \frac{1}{2}\Big(\frac{k_c^2 z_1 g_1}{k_c^2 - x_1^2}\Big)^2 + \frac{z_2^2}{2} \tag{31}$$

$$\frac{k_c^2 z_1 g_1 y_2}{k_c^2 - x_1^2} \le \frac{1}{2}\Big(\frac{k_c^2 z_1 g_1}{k_c^2 - x_1^2}\Big)^2 + \frac{y_2^2}{2} \tag{32}$$

Substituting (26)–(29) into (30), and considering $|d_1| \le D^*$ and $|\epsilon_1| \le \epsilon_1^*$, results in

$$\begin{aligned}\dot{V}_{z_1} \le & -\frac{K_1 k_c^2 z_1^2}{k_c^2 - x_1^2} + [g_1 N(\zeta_1) + 1]\dot{\zeta}_1 - \frac{k_c^2 z_1}{k_c^2 - x_1^2}\tilde{W}_1^{\mathrm{T}} S_1(Z_1) \\ & + \frac{k_c^2 z_1}{k_c^2 - x_1^2} p_1^* + \frac{z_2^2}{2} + \frac{y_2^2}{2} - \frac{k_c^2 z_1}{k_c^2 - x_1^2}\hat{p}_1 \tanh\Big(\frac{(z_1 k_c^2/(k_c^2 - x_1^2))}{\delta_1}\Big)\end{aligned} \tag{33}$$

where $p_1^* = D_1^* + \epsilon_1^*$. Introduce a first-order filter, and let α_1 pass through the first-order filter to obtain ω_2

$$\kappa_2 \dot{\omega}_2 + \omega_2 = \alpha_1, \quad \omega_2(0) = \alpha_1(0) \tag{34}$$

where κ_2 is a positive constant. Since $y_2 = \omega_2 - \alpha_1$, there is

$$\dot{\omega}_2 = \frac{\alpha_1 - \omega_2}{\kappa_2} = -\frac{y_2}{\kappa_2} \tag{35}$$

Thus, the time derivative of y_2 will be

$$\begin{aligned}\dot{y}_2 &= \dot{\omega}_2 - \dot{\alpha}_1 = -\frac{y_2}{\kappa_2} - \dot{\alpha}_1 \\ &= -\frac{y_2}{\kappa_2} - \Big[K_1 \dot{z}_1 + \dot{\hat{W}}_1^{\mathrm{T}} S_1(Z_1) + \hat{W}_1^{\mathrm{T}} \dot{S}_1(Z_1) \\ &\quad + \dot{\hat{p}}_1 \tanh\Big(\frac{(z_1 k_c^2/(k_c^2 - x_1^2))}{\delta_1}\Big) + \hat{p}_1 \dot{\tanh}\Big(\frac{(z_1 k_c^2/(k_c^2 - x_1^2))}{\delta_1}\Big)\Big]\end{aligned} \tag{36}$$

Considering a positive continuous function ϕ_2, i.e.

$$\Big|\dot{y}_2 + \frac{y_2}{\kappa_2}\Big| \le \phi_2(\bar{z}_2, y_2, \tilde{W}_1, y_{\mathrm{d}}, \dot{y}_{\mathrm{d}}, \ddot{y}_{\mathrm{d}}) \tag{37}$$

Then, we have

$$\dot{y}_2 y_2 \le -\frac{y_2^2}{\kappa_2} + |y_2|\phi_2 \le -\frac{y_2^2}{\kappa_2} + y_2^2 + \frac{1}{4}\phi_2^2 \tag{38}$$

Further, to handle the unknown bounds introducing the following Lyapunov function candidate:

$$V_1 = V_{z_1} + \frac{1}{2}y_2^2 + \frac{1}{2}\tilde{W}_1^{\mathrm{T}}\Gamma_1^{-1}\tilde{W}_1 + \frac{1}{2}\tilde{p}_1^2 \tag{39}$$

The time derivative of V_1 is

$$\begin{aligned}\dot{V}_1 \le & -\frac{K_1 k_c^2 z_1^2}{k_c^2 - x_1^2} + [g_1 N(\zeta_1) + 1]\dot{\zeta}_1 + \frac{z_2^2}{2} - \Big[\frac{1}{\kappa_2} - \frac{3}{2}\Big]y_2^2 \\ & + \Big|\frac{k_c^2 z_1}{k_c^2 - x_1^2}\Big| p_1^* - p_1^* \frac{k_c^2 z_1}{k_c^2 - x_1^2}\tanh\Big(\frac{(z_1 k_c^2/(k_c^2 - x_1^2))}{\delta_1}\Big) \\ & - \tilde{p}_1 \eta_{p_1}\hat{p}_1 - \gamma_1 \tilde{W}_1^{\mathrm{T}}\hat{W}_1 + \frac{1}{4}\phi_2^2\end{aligned} \tag{40}$$

Consider completion of squares, we can get

$$\begin{aligned}-\gamma_1 \tilde{W}_1^{\mathrm{T}}\hat{W}_1 &= -\gamma_1 \tilde{W}_1^{\mathrm{T}}(\tilde{W}_1 + W_1^*) \\ &\le -\gamma_1 \|\tilde{W}_1\|^2 - \gamma_1 \|\tilde{W}_1\|\|W_1^*\| \\ &\le -\frac{\gamma_1 \|\tilde{W}_1\|^2}{2} + \frac{\gamma_1 \|W_1^*\|^2}{2}\end{aligned} \tag{41}$$

Similar to (41), we have

$$-\tilde{p}_1 \eta_{p_1}\hat{p}_1 \le -\frac{1}{2}\eta_{p_1}|\tilde{p}_{11}|^2 + \frac{1}{2}\eta_p |p_1^*|^2 \tag{42}$$

Noting Lemma 5, we get that

$$\Big|\frac{k_c^2 z_1}{k_c^2 - x_1^2}\Big| p_1^* - p_1^* \frac{k_c^2 z_1}{k_c^2 - x_1^2}\tanh\Big(\frac{(z_1 k_c^2/(k_c^2 - x_1^2))}{\delta_1}\Big) \le 0.2785 p_1^* \delta_1 \tag{43}$$

Substituting (41)–(43) into (40), we can get the following inequality:

$$\begin{aligned}\dot{V}_1 \le & -\frac{K_1 k_c^2 z_1^2}{k_c^2 - x_1^2} + [g_1 N(\zeta_1) + 1]\dot{\zeta}_1 - \Big(\frac{1}{\kappa_2} - \frac{3}{2}\Big)y_2^2 + \frac{1}{2}z_2^2 \\ & + \frac{\gamma_1 \|W_1^*\|^2}{2} - \frac{\gamma_1 \|\tilde{W}_1\|^2}{2} + 0.2785 p_1^* \delta_1 + \frac{1}{4}\phi_2^2 \\ & - \frac{1}{2}\eta_{p_1}|\tilde{p}_1|^2 + \frac{1}{2}\eta_p |p_1^*|^2\end{aligned} \tag{44}$$

Considering the proposed Lyapunov function candidate (26), we have

$$\dot{V}_1 \le -\beta_1 V_1 + C_1 + [g_1 N(\zeta_1) + 1]\dot{\zeta}_1 + \frac{z_2^2}{2} \tag{45}$$

where

$$\beta_1 = \min\Big\{2K_1, \frac{\gamma_1}{\lambda_{\max}(\Gamma_1^{-1})}, 2\Big(\frac{1}{\kappa_2} - \frac{3}{2}\Big), \eta_{p_1}\Big\} \tag{46}$$

$$C_1 = \frac{\sigma_1 \|W_1^*\|^2}{2} + 0.2785 q_1^* \delta_1 + \frac{1}{2}\eta_{p_1} p_1^{*2} + \frac{1}{4}\phi_2^{*2} \tag{47}$$

where $\phi_2^{*2} > \phi_2^2$ is a positive number. Multiplying both sides of the in equation by $e^{\beta_1 t}$ and integrating it over $[0, t_f)$, there is

$$\begin{aligned}0 \le V_1(t) \le V_1(0)e^{-\beta_1 t} + \frac{C_1}{\beta_1} + \int_0^t \frac{z_2^2}{2} e^{-(t-\tau)\beta_1}\,\mathrm{d}\tau \\ + e^{-\beta_1 t}\int_0^t [g_1 N(\zeta_1) + 1]\dot{\zeta}_1 e^{\beta_1 \tau}\,\mathrm{d}\tau\end{aligned} \tag{48}$$

Remark 1: In (48), we can conclude that ζ_1, $V_1(t)$, and $e^{-\beta_1 t}\int_0^t [g_1 N(\zeta_1) + 1]\dot{\zeta}_1 e^{\beta_1 \tau}\,\mathrm{d}\tau$ are bounded on $[0, t_f)$ if there are no the coupling term $\int_0^t (z_2^2/2)e^{-(\tau - t)\beta_1}\,\mathrm{d}\tau$ within in the inequality. Because the presence of term $\int_0^t (z_2^2/2)e^{-(\tau - t)\beta_1}\,\mathrm{d}\tau$ in (48), Lemma 1 cannot be applied directly. However, if $z_2^2/2$ is bounded, this term can be reduced to C_1. Now assuming z_2 has upper bound, we have

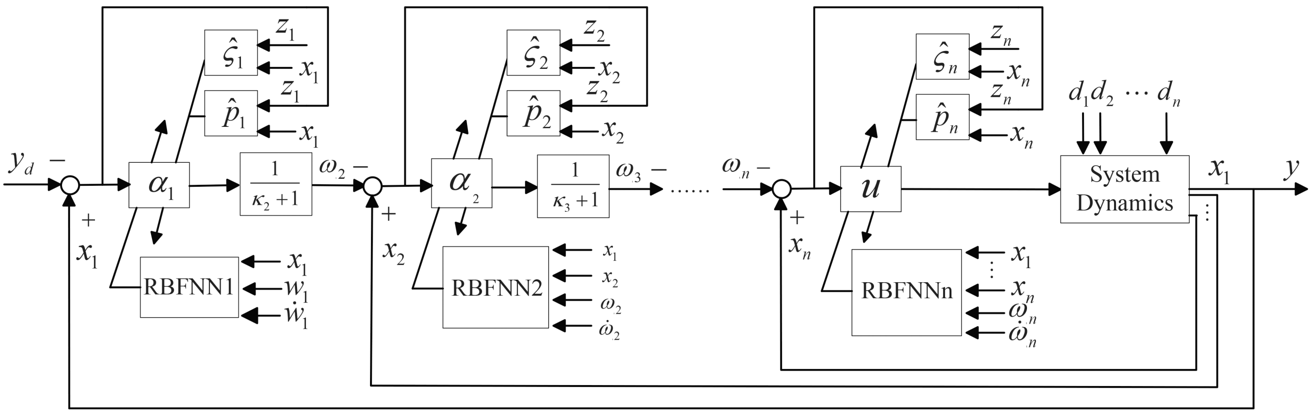

Fig. 1 *Block diagram of the proposed adaptive neural DSC scheme*

$$\int_0^t \frac{z_2^2}{2} e^{-(\tau - t)\beta_1} \mathrm{d}\tau \le \frac{1}{2} e^{-\beta_1 t} \sup_{\tau \in [0,t]} [z_2^2(\tau)] \int_0^t e^{\beta_1 t} \mathrm{d}\tau \le \frac{1}{\beta_1} \sup_{\tau \in [0,t]} [z_2^2(\tau)] \tag{49}$$

From (49), it can be concluded the coupling term $\int_0^t (z_2^2/2) e^{-(\tau - t)\beta_1} \mathrm{d}\tau$ is bounded if z_2 can be regulated as bounded. At the next steps, the effect of $\int_0^t (z_2^2/2) e^{-(\tau - t)\beta_1} \mathrm{d}\tau$ will be disposed.

Step i ($2 \le i \le n-1$): The derivative of $z_i = x_i - \omega_i$ is

$$\begin{aligned} \dot{z}_i &= f_i + g_i x_{i+1} + d_i - \dot{\omega}_i \\ &= f_i + g_i(z_{i+1} + y_{i+1} + \alpha_i) + d_i - \dot{\omega}_i \end{aligned} \tag{50}$$

Choosing the Lynapunov function candidate is $V_{z_i} = (1/2)z_i^2$, we have

$$V_{z_i} = z_i[f_i + g_i(z_{i+1} + y_{i+1} + \alpha_i) + d_i - \dot{\omega}_i] \tag{51}$$

Obviously since f_i, g_i, ω_i, d_i are unknown, we cannot directly design the controller. Defining $Z_i = [x_1, x_2, \ldots, x_i, \omega_i, \dot{\omega}_i]^{\mathrm{T}} \subset \mathbb{R}^{i+2}$, and utilising an $W_i^{*\mathrm{T}} S_i(Z_i)$ to approximate $h_i(Z_i) = f_i - \dot{\omega}_i - z_i g_i^2$, we can get

$$h_i(Z_i) = \hat{W}_i^{\mathrm{T}} S_i(Z_i) - \tilde{W}_i^{\mathrm{T}} S_i(Z_i) + \epsilon_i \tag{52}$$

where $\tilde{W}_i^{\mathrm{T}} = \hat{W}_i^{\mathrm{T}} - W_i^*$, $S_i(Z_i)$ are also the basis functions. ϵ_i is the error of NNs. Defining the error variable $z_{i+1} = x_{i+1} - y_{i+1} - \alpha_i$, and choosing the virtual control law is as follows:

$$\alpha_i = N(\zeta_i)\left[K_i z_i + \hat{W}_i^{\mathrm{T}} S_i(Z_i) + \hat{p}_i \tanh\left(\frac{z_i}{\delta_i}\right)\right] \tag{53}$$

where K_i is a positive number. The adaptive laws are as follows:

$$\dot{\zeta}_i = K_i z_i^2 + z_i \hat{W}_i^{\mathrm{T}} S_i(Z_i) + \hat{p}_i z_i \tanh\left(\frac{z_i}{\delta_i}\right) \tag{54}$$

$$\dot{\hat{W}}_i = \dot{\tilde{W}}_i = \Gamma_i[S_i(Z_i) z_i - \gamma_i \hat{W}_i] \tag{55}$$

$$\dot{p}_i = z_i \tanh\left(\frac{z_i}{\delta_i}\right) - \eta_{p_i} \hat{p}_i \tag{56}$$

Substituting (53)–(56) into (51), the time derivative of V_{z_i} becomes

$$\begin{aligned} \dot{V}_{z_i} = &- K_i z_i^2 + [g_i N(\zeta_i) + 1]\dot{\zeta}_i + \hat{p}_i z_i \tanh\left(\frac{z_i}{\delta_i}\right) + g_i z_i z_{i+1} \\ &+ z_i d_i + g_i z_i y_{i+1} - z_i \tilde{W}_i^{\mathrm{T}} S_i(Z_i) + z_i \epsilon_i - z_i^2 g_i^2 \end{aligned} \tag{57}$$

Using the Young's inequality, there is

$$g_i z_i z_{i+1} \le \frac{1}{2} z_i^2 g_i^2 + \frac{1}{2} z_{i+1}^2 \tag{58}$$

$$g_i z_i y_{i+1} \le \frac{1}{2} z_i^2 g_i^2 + \frac{1}{2} y_{i+1}^2 \tag{59}$$

Substituting (58) and (59) into (56), and considering $|d_i| \le D_i^*$ and $|\epsilon_i| \le \epsilon_i^*$, we have

$$\begin{aligned} \dot{V}_{z_i} \le &- K_i z_i^2 + [g_i N(\zeta_i) + 1]\dot{\zeta}_i + \hat{p}_i z_i \tanh\left(\frac{z_i}{\delta_i}\right) \\ &+ \frac{1}{2} z_{i+1}^2 + \frac{1}{2} y_{i+1}^2 - z_i \tilde{W}_i^{\mathrm{T}} S_i(Z_i) + z_i p_i^* \end{aligned} \tag{60}$$

where $p_i^* = D_i^* + \epsilon_i^*$. Defined κ_{i+1} is a positive constant, let α_i pass through a first-order filter with time constant κ_{i+1} to obtain ω_{i+1}

$$\kappa_{i+1} \dot{\omega}_{i+1} + \omega_{i+1} = \alpha_i, \quad \omega_{i+1}(0) = \alpha_i(0) \tag{61}$$

Due to $y_{i+1} = \omega_{i+1} - \alpha_i$, we can get

$$\begin{aligned} \dot{y}_{i+1} &= \dot{\omega}_{i+1} - \dot{\alpha}_i \\ &= -\frac{y_{i+1}}{\kappa_{i+1}} - \Big[K_i \dot{z}_i + \dot{\tilde{W}}_1^{\mathrm{T}} S_i(Z_i) + \hat{W}_i^{\mathrm{T}} \dot{S}_i(Z_i) \\ &\quad + \dot{\hat{p}}_i z_i \tanh\left(\frac{z_i}{\delta_i}\right) + \hat{p}_i z_i \dot{\tanh}\left(\frac{z_i}{\delta_i}\right)\Big] \end{aligned} \tag{62}$$

Considering positive continuous function ϕ_{i+1}, there is

$$\dot{y}_{i+1} y_{i+1} \le -\frac{y_{i+1}^2}{\kappa_{i+1}} + y_{i+1}^2 + \frac{1}{4}\phi_{i+1}^2 \tag{63}$$

Introducing the following Lyapunov function candidate:

$$V_i = V_{z_i} + \frac{1}{2} y_{i+1}^2 + \frac{1}{2} \tilde{W}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \frac{1}{2} \tilde{p}_i^2 \tag{64}$$

Then time derivative of V_i becomes

$$\begin{aligned} \dot{V}_i \le &- K_i z_i^2 + [g_i N(\zeta_i) + 1]\dot{\zeta}_i - \left(\frac{1}{\kappa_{i+1}} - \frac{3}{2}\right) y_{i+1}^2 \\ &+ \frac{1}{2} z_{i+1}^2 + \frac{1}{4}\phi_{i+1}^2 - \frac{1}{2}\gamma_i \| \tilde{W}_i \|^2 + \frac{1}{2}\gamma_i \| W_i^* \|^2 \\ &+ 0.2785 p_i^* \delta_i - \frac{1}{2}\eta_{p_i} | \tilde{p}_i |^2 + \frac{1}{2}\eta_{p_i} | p_i^* |^2 \end{aligned} \tag{65}$$

Considering the proposed Lyapunov function candidate $V_{z_i} = (1/2)z_i^2$, we have

$$\dot{V}_i \leq -\beta_i V_i + C_i + [g_i N(\zeta_i) + 1]\dot{\zeta}_i + \frac{z_i^2}{2} \tag{66}$$

where β_i and C_i are given as follows:

$$\beta_i \leq \min\left\{2K_i, \frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}, 2\left(\frac{1}{\kappa_{i+1}} - \frac{3}{1}\right), \eta_{p_i}\right\} \tag{67}$$

$$C_i = \frac{\sigma_i \parallel W_1^* \parallel^2}{2} + \frac{1}{4}\phi_{i+1}^{*2} + 0.2785 p_i^* \delta_i + \frac{1}{2}\eta_{p_i} \mid p_i^* \mid^2 \tag{68}$$

where $\phi_{i+1}^{*2} > \phi_{i+1}^2$ is a positive number. Multiplying both sides of the inequation by $e^{\beta_i t}$ and integrating it over $[0, t_f)$, there is

$$\begin{aligned} 0 \leq V_i(t) \leq &\frac{C_i}{\beta_i} + V_i(0)e^{-\beta_i t} + \int_0^t \frac{z_2^2}{2} e^{-(\tau - t)\beta_i} \mathrm{d}\tau \\ &+ e^{-\beta_i t} \int_0^t [g_i N(\zeta_i) + 1]\dot{\zeta}_i e^{\beta_i \tau} \mathrm{d}\tau \end{aligned} \tag{69}$$

Remark 2: Similar to Step 1, if the boundedness of $z_{i+1}(t)$ can be guaranteed, we can conclude the boundedness of $z_i(t)$ and $V_i(t)$ can be ensured in Lemma 1. If z_{i+1} can be regulated in the subsequent steps, regulation of $z_i(t)$ can also be guaranteed.

Step n: We design the control input $u(t)$ in the final step. Due to $z_n = x_n - \omega_n$, its derivative becomes

$$\dot{z}_n = f_n + g_n u + d_n - \dot{\omega}_n \tag{70}$$

Considering the Lynapunov function candidate is $V_{z_n} = (1/2)z_n^2$, we have

$$V_{z_n} = z_n(f_n + g_n u + d_n - \dot{\omega}_n) \tag{71}$$

Utilising $W_n^{*\mathrm{T}} S_n(Z_n)$ to evaluate $h_n(Z_n) = f_n - \dot{\omega}_n$, where $Z_n = [\bar{z}_n, \omega_n, \dot{\omega}_n]^{\mathrm{T}} \subset \mathbb{R}^{n+2}$, we can get that

$$h_n(Z_n) = \hat{W}_n^{\mathrm{T}} S_n(Z_n) - \tilde{W}_n^{\mathrm{T}} S_n(Z_n) + \epsilon_n \tag{72}$$

Choosing the following control as:

$$u = N(\zeta_n)\left[K_n z_n + \hat{W}_n^{\mathrm{T}} S_n(Z_n) + \hat{p}_n \tanh\left(\frac{z_n}{\delta_n}\right)\right] \tag{73}$$

where K_n is a positive number. The adaptive rules are as follows:

$$\dot{\zeta}_n = K_n z_n^2 + z_n \hat{W}_n^{\mathrm{T}} S_n(z_n) + z_n \hat{p}_n \tanh\left(\frac{z_n}{\delta_n}\right) \tag{74}$$

$$\dot{\hat{W}}_n = \dot{\tilde{W}}_n = \Gamma_n[S_n(Z_n)z_n - \gamma_n \hat{W}_n] \tag{75}$$

$$\dot{p}_n = z_n \tanh\left(\frac{z_n}{\delta_n}\right) - \eta_{p_n} \hat{p}_n \tag{76}$$

Choosing the Lyapunov function candidate as $(1/2)z_n^2$, its time derivative is

$$\begin{aligned} \dot{V}_{z_n} = &-K_n z_n^2 + [g_n N(\zeta_n) + 1]\zeta_n + z_n d_n - z_n \hat{p}_n \tanh\left(\frac{z_n}{\delta_n}\right) \\ &+ z_n \epsilon_n - z_n \tilde{W}_n S_n(Z_n) \end{aligned} \tag{77}$$

Introducing the following Lyapunov function candidate:

$$V_n = V_{z_n} + \frac{1}{2}\tilde{W}_n^{\mathrm{T}} \Gamma_n^{-1} \tilde{W}_n + \frac{1}{2}\tilde{p}_n^2 \tag{78}$$

The time derivative of V_n is

$$\begin{aligned} \dot{V}_n \leq &-K_n z_n^2 + [g_n N(\zeta_n) + 1]\zeta_n - \frac{\gamma_n \parallel \tilde{W}_n \parallel^2}{2} \\ &+ \frac{\gamma_n \parallel W_n^* \parallel^2}{2} + 0.2785 p_n^* \delta_n - \frac{1}{2}\eta_{p_n} \mid \tilde{p}_n \mid^2 \\ &+ \frac{1}{2}\eta_{p_n} \mid p_n^* \mid^2 \end{aligned} \tag{79}$$

where $p_n^* = D_n^* + \epsilon_n^*$. Similar to Step 1 and Step i, we get

$$\dot{V}_n \leq -\beta_n V_n + C_i + [g_n N(\zeta_n) + 1]\dot{\zeta}_i \tag{80}$$

where β_n and C_n are given as follows:

$$\beta_n \leq \min\left\{2K_n, \frac{\gamma_n}{\lambda_{\max}(\Gamma_n^{-1})}, \eta_{p_n}\right\} \tag{81}$$

$$C_n = \frac{\gamma_n \parallel W_n^* \parallel^2}{2} + 0.2785 p_n^* \delta_n + \frac{1}{2}\eta_{p_n} \mid p_n^* \mid^2 \tag{82}$$

Multiplying both sides of the inequation by $e^{\beta_n t}$ and integrating it over $[0, t_f)$, we get

$$\begin{aligned} 0 \leq V_n(t) \leq &\frac{C_n}{\beta_n} + V_n(0)e^{-\beta_i t} \\ &+ e^{-\beta_n t} \int_0^t [g_n N(\zeta_n) + 1]\dot{\zeta}_n e^{\beta_n \tau} \mathrm{d}\tau \end{aligned} \tag{83}$$

It can be easily concluded by utilising Lemma 1 that $V_n(t)$, $\zeta_n(t)$ and $e^{-\beta_n t}\int_0^t [g_n N(\zeta_n) + 1]\dot{\zeta}_n e^{\beta_n \tau} \mathrm{d}\tau$ must be bounded on $[0, t_f)$. Because $V_n(t =)(1/2)z_n^2$ are bounded, z_n is bounded on $[0, t_f)$. Then the boundedness of $\int_0^t (g_n^2 z_2^2/4)e^{-(\tau - t)\beta_n} \mathrm{d}\tau$ at Step $n-1$ is guaranteed, and z_{n-1} is bounded. Similar to this way, we can conclude $V_{n-2}, \ldots, V_2, V_1$ and $z_{n-2}, \ldots, z_2, z_1$ is bounded. Therefore, $x_1(t)$ are uniformly ultimately bounded. The proof of stability and control performance will show in the following theorem.

Remark 3 [51]: Note that the large initial error $\tilde{W}_i(0)$ may lead to a large tracking error during the initial period of adaptation. Output tracking error may be reduced by increasing the controller gain K_i and adaptation gain Γ_i.

Theorem 1: Considering a class of non-linear systems with the output constraint and unknown control direction, under Assumption 1 and control (26), (53), (73), with the updating laws (27)–(29), (54)–(56), (74)–(76) supposed to the initial conditions $x_i(0)$, $y_{j+1}(0)$, $\hat{W}_i(0)$, $\hat{p}_i(0)$ are bounded, all signs in the closed-loop system are uniformly ultimately bounded, z_i, y_{j+1}, $\tilde{W}_i$, $\tilde{p}_1$ will remain within compact set Ω_{z_1}, Ω_{y_2}, $\Omega_{\tilde{W}_i}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n-1$, and Ω_{p_1}, respectively, which are defined by

$$\Omega_{z_j} := \left\{z_i \in \mathbb{R}^n \mid \ \parallel z_i \parallel \leq \sqrt{2(V_i(0) + \frac{C_i}{\beta_i} + c_i)}\right\} \tag{84}$$

$$\Omega_{y_{j+1}} := \left\{y \in \mathbb{R}^n \mid \ \parallel y_{j+1} \parallel \leq \sqrt{2(V_j(0) + \frac{C_j}{\beta_j} + c_j)}\right\} \tag{85}$$

$$\Omega_{\tilde{W}_i} := \left\{\tilde{W}_i \in \mathbb{R}^n \mid \ \parallel \tilde{W}_i \parallel \leq \sqrt{2(V_i(0) + \frac{C_i}{\beta_i} + c_i)/\lambda_{\min}(\Gamma_i)}\right\} \tag{86}$$

$$\Omega_{\tilde{p}_i} := \left\{\tilde{p}_i \in \mathbb{R}^n \mid \ \parallel \tilde{p}_i \parallel \leq \sqrt{2(V_i(0) + \frac{C_i}{\beta_i} + c_i)}\right\} \tag{87}$$

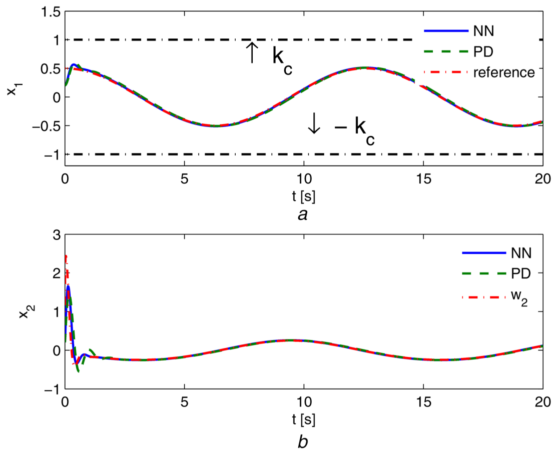

Fig. 2 *Case 1: Response of states*
(a)*, *(b)

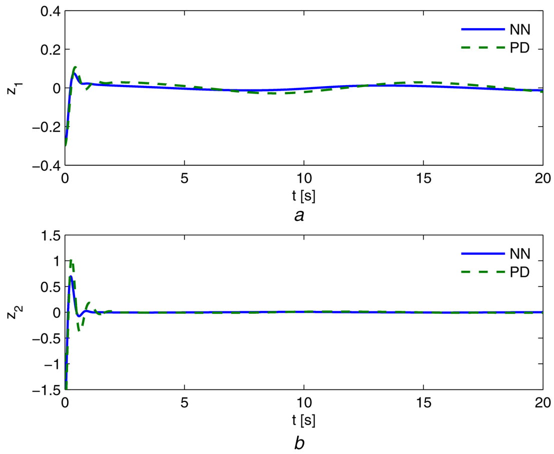

Fig. 3 *Case 1: Tracking errors*
(a)*, *(b)

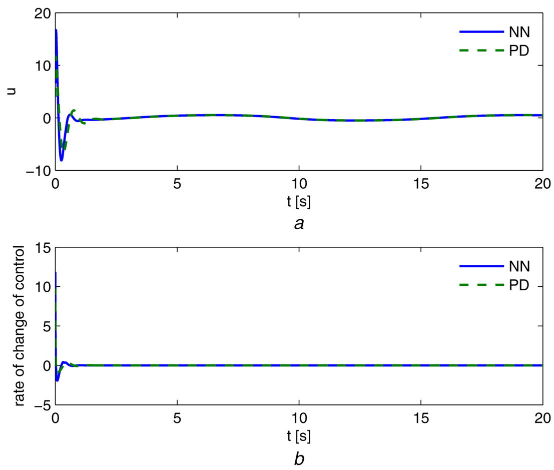

Fig. 4 *Case 1: Control signal*
(a)*, *(b)

where C_i is the upper bound of $e^{-\beta_i t}\int_0^t [g_i N(\zeta_i)+1]\dot{\zeta}_i e^{\beta_i \tau}\,\mathrm{d}\tau + \int_0^t (z_i^2/2)e^{-(\tau - t)\beta_i}\,\mathrm{d}\tau, \quad i = 1, 2, \ldots, n-1$, and $C_n = e^{-\rho_n t}\int_0^t [g_n N(\zeta_n)+1]\dot{\zeta}_n e^{\beta_n \tau}\,\mathrm{d}\tau$.

Remark 4: One issue may arise on the rate of change of control signal, i.e. the value of $\dot{u}$ may be too large to be realistic for some important applications, such as electric actuator, which means the actuator may respond slowly to the control object. In particular, the range of $\dot{u}$ actually depends on the careful selection of control parameters. In the practical applications, the control parameters should be carefully selected to ensure the satisfaction of $\dot{u}$.

Proof: In this paper, the proof is comprised of the above stages of design from Step 1 to Step n. From (46), we get

$$\frac{1}{2}z_1^2 \le V_1(t) \le V_1(0) + \frac{C_1}{\beta_1} + c_1 \tag{88}$$

which implies

$$\| z_1 \| \le \sqrt{2\left(V_1(0) + \frac{C_1}{\beta_1} + c_1\right)} \tag{89}$$

Similarly, we can obtain Ω_{z_i}, $\Omega_{y_{j+1}}$, $\tilde{W}_i$, $\tilde{p}_i$. □

4 Simulation

In this case, we provide two simulation cases to validate the proposed method. The first case is a second-order system that is modified based on [15] and the second case is a third-order system that is modified based on [40]. Both simulation results indicate that the proposed adaptive control strategy guarantee the output constraint satisfaction in non-linear systems subject to unknown control direction and uncertainty. Moreover, comparison to the PD control is also carried out to illustrate the effectiveness of the proposed method.

4.1 Case 1: Second-order non-linear systems

We consider the following second-order non-linear system:

$$\begin{cases} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 + d_1(t) \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)u + d_2(t) \\ y &= x_1 \end{cases} \tag{90}$$

where $f_1(x_1) = 0.1x_1^2$, $f_2(x_1, x_2) = 0.2x_1x_2 + x_1$, $g_1(x_1) = 1$, $g_2(x_1, x_2) = 1 + x_1^2$, $d_1(t) = 0.2\sin(t)$ and $d_2(t) = 0.2\cos(t)$. The output constraint is specified as $|x_1| < k_c$ with $k_c = 1$. The control objective is to drive the system output y to track the desired trajectory $y_\mathrm{d} = 0.5\cos(0.5t)$ as closely as possible subject to the output constraint, unknown control direction and uncertainty.

The control parameters are chosen as $K_1 = \mathrm{diag}[5, 5]$, $K_2 = \mathrm{diag}[5, 5]$. $N(\zeta_i) = \zeta_i^2\cos(\zeta_i)$ are chosen as Nussbaum-type functions with the initial value of ζ_i is $\zeta_1 = 0.3$ and $\zeta_2 = 0.5$. Besides, we have 8 nodes evenly spaced in $[-1, 1] \times [-1, 1] \times [-1, 1]$ and widths 10 for $\hat{W}_1^\mathrm{T} S_1(Z_1)$ and 64 nodes evenly spaced in $[-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1]$ and widths 10 for $\hat{W}_2^\mathrm{T} S_2(Z_2)$. The initial conditions are selected as $[x_1, x_2]^\mathrm{T} = [0.2, 0.2]^\mathrm{T}$, which lie in the predefined constrained set. Other parameters are selected as $\sigma_1 = \sigma_2 = 1$, $\eta_1 = \eta_2 = 0.1$, $\kappa_2 = 0.01$, $\Gamma_1 = \Gamma_2 = 1$, $\gamma_1 = 0.1$ and $\gamma_2 = 0.01$. The PD control is $\alpha_1 = -K_1 z_1$ and $u = -K_2 z_2$.

Figs. 2–5 show the simulation result of the second-order system (90). The tracking performance of x_1 is given in Fig. 2*a*, which implies that the output constraint can be guaranteed by using the proposed control. Besides, the response of x_2 is shown in Fig. 2*b*. The tracking errors z_1 and z_2 are given in Fig. 3a and Fig. 3b, respectively. It can be observed that the tracking errors of PD control are bigger than that of NN control, and the tracking errors of the proposed control can converge to a small value near zero. The control signal is given in Fig. 4*a*. The rate of change of control input is shown in Fig. 4*b*. Clearly, with the selected the parameters K_1 and K_2, the rate of change the control input is from −3 to 12, indicating the rate of change of realistic if applied to a practical system. Besides, the weights of NN are shown in Fig. 5.

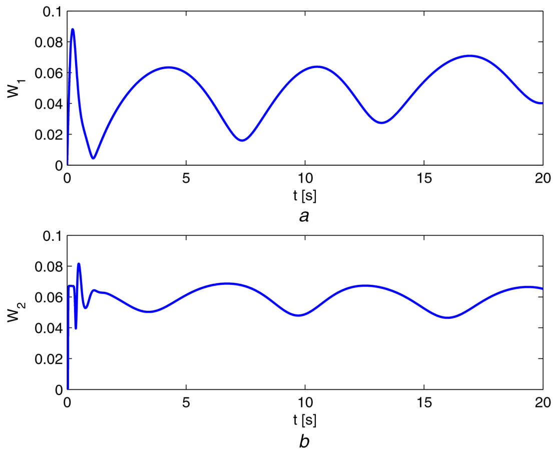

Fig. 5 *Case 1: NN weights*
(a)*, *(b)

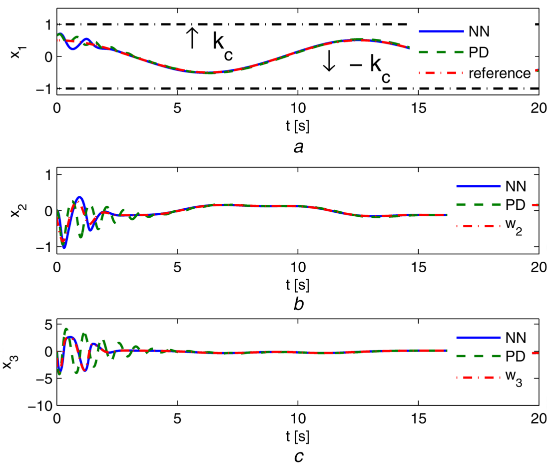

Fig. 6 *Case 2: Response of states*
(a)*, *(b)*, *(c)

4.2 Case 2: Third-order non-linear systems

To further elaborate the capability of the proposed control strategy, consider the following third-order non-linear systems:

$$\begin{cases} \dot{x}_1 &=& f_1(x_1) + g_1(x_1)x_2 + d_1(t) \\ \dot{x}_2 &=& f_2(x_1, x_2) + g_2(x_1, x_2)x_3 + d_2(t) \\ \dot{x}_3 &=& f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)u + d_3(t) \\ y &=& x_1 \end{cases} \tag{91}$$

where $f_1(x_1) = 2x_1^2\sin(x_1)$, $f_2(x_1, x_2) = x_1^2 + x_1x_2 + x_2\cos(x_1)$, $f_3(x_1, x_2, x_3) = x_1x_3 + x_2^2 + x_3\sin(x_2)$, $g_1(x_1) = 1 + \cos(x_1)$, $g_2(x_1, x_2) = 1 - \sin(x_2)$ and $g_3(x_1, x_2, x_3) = 1 + 0.1x_2^2$. The disturbance is set as $d_1(t) = 0.1\sin(t)$, $d_2(t) = 0.1\cos(t)$ and $d_3(t) = 0.1\cos(t)$. The output constraint is specified as $|x_1| < k_c$ with $k_c = 1$. The control objective is to drive the system output y to track the desired trajectory $y_d = 0.5\cos(0.5t)$ as closely as possible subject to the output constraint, unknown control direction and uncertainty.

The control parameters are chosen as $K_1 = \mathrm{diag}[5, 5, 5]$, $K_2 = \mathrm{diag}[6, 6, 6]$ and $K_3 = \mathrm{diag}[10, 10, 10]$. $N(\zeta_i) = \zeta_i^2\cos(\zeta_i)$ are chosen as Nussbaum-type functions with the initial value of ζ_i is $\zeta_1 = 2$, $\zeta_2 = 2$ and $\zeta_3 = 2$. Besides, we have 8 nodes evenly spaced in $[-1, 1] \times [-1, 1] \times [-1, 1]$ and widths 10 for $\hat{W}_1^{\mathrm{T}}S_1(Z_1)$, 64 nodes evenly spaced in $[-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1]$ and widths 10 for $\hat{W}_2^{\mathrm{T}}S_2(Z_2)$ and 256 nodes evenly spaced in $[-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1] \times [-1, 1]$
and widths 10 for $\hat{W}_3^{\mathrm{T}}S_3(Z_3)$.

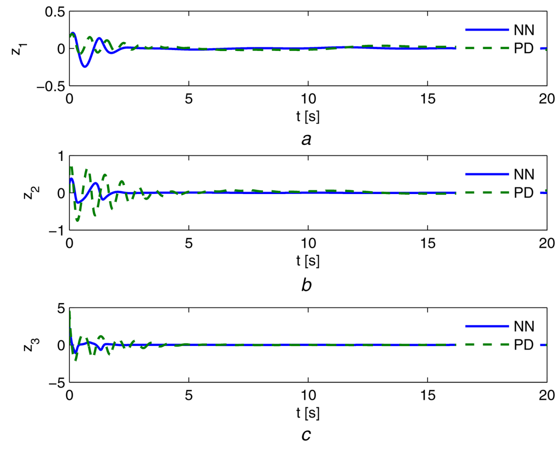

Fig. 7 *Case 2: Tracking errors*
(a)*, *(b)*, *(c)

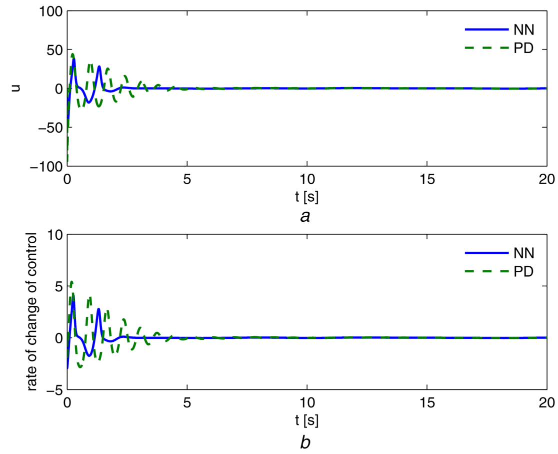

Fig. 8 *Case 2: Control signal*
(a)*, *(b)

The initial conditions are selected as $[x_1, x_2, x_3]^{\mathrm{T}} = [0.65, 0, 0]^{\mathrm{T}}$, which lie in the predefined constrained set. Other parameters are selected as $\sigma_1 = \sigma_2 = \sigma_3 = 10$, $\eta_1 = \eta_2 = \eta_3 = 0.01$, $\kappa_2 = \kappa_3 = 0.01$, $\Gamma_1 = \Gamma_2 = \Gamma_3 = 1$, $\gamma_1 = \gamma_2 = \gamma_3 = 0.01$. The PD control is $\alpha_1 = -K_1z_1$, $\alpha_2 = -K_2z_2$ and $u = -K_3z_3$.

The simulation results of the third-order system (91) are given in Figs. 6–9. The tracking performance of x_1 is given in Fig. 6*a*. Although the output of both control strategies (PD control and the proposed control) is in the constrained set, it can be observed that the output performance of PD control has some oscillations when $t \in [0, 3]$. The output of x_2 and x_3 is also given in Figs. 6*b* and *c*, respectively. It can be seen that there are some oscillations for PD control, whereas the tracking performance of the proposed control without any oscillation is better than that of PD control. The tracking errors z_1 and z_2 are given in Fig. 7*a* and Fig. 7*b*, respectively. It is clear that the tracking error of the proposed control is smaller than that of PD control. The control signal is given in Fig. 8*a*. The rate of change of control input is given in Fig. 8*b*. From Fig. 8*b*, it can be observed that the rate of change is from −3 to 6, which is realistic if applied to a practical platform. Moreover, the weights of NN are shown in Fig. 9.

5 Conclusion

In this paper, the adaptive neural DSC was developed of non-linear systems with output constraint and unknown control direction. In particular, the iBLFs were used to handle the effect of output constraint. This resulted in relaxing some of the conservatism associated with the use of purely error-based functions with transformed error constraints. Nussbaum-type functions were applied to address the control problem with unknown control direction. Furthermore, the adaptive NN was employed to

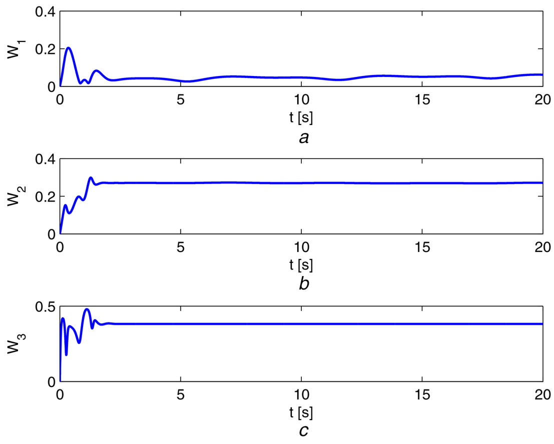

Fig. 9 *Case 2: NN weights*
(a)**, **(b)**, **(c)

compensate the model uncertainty and the DSC technology was used to deal with the problem of explosion of complexity in control design. The simulation studies showed the effectiveness of the proposed control strategy.

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