

$$A = \begin{bmatrix} 0 & \sin\theta & 0 & -\cos\theta \\ 0 & 0 & 1 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 1 & 0 & 0 & 0 \end{bmatrix}_{4 \times 4}$$

For any matrix A^{-1} to be equal to the transpose of A , which is A^T . The matrix should follow below condition:

$$A \cdot A^T = A^T \cdot A = I$$

$A \rightarrow$ Original matrix
 $A^T \rightarrow$ Transpose of A
 $A^{-1} \rightarrow$ Inverse of A

$\cdot \rightarrow$ Multiplication of matrices
 $I \rightarrow$ Identity matrix

$$A^T = \begin{bmatrix} 0 & 0 & 0 & 1 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\cos\theta & 0 & \sin\theta & 0 \end{bmatrix}_{4 \times 4}$$

Below is the multiplication of A and A^T

$$A \cdot A^T = \begin{bmatrix} 0 & \sin\theta & 0 & -\cos\theta \\ 0 & 0 & 1 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 1 & 0 & 0 & 0 \end{bmatrix}_{4 \times 4} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\cos\theta & 0 & \sin\theta & 0 \end{bmatrix}_{4 \times 4}$$

$$A \cdot A^T = \begin{bmatrix} 0 \cdot 0 + \sin^2\theta + 0 + \cos^2\theta & 0 + 0 + 0 + 0 & 0 + \sin\theta\cos\theta + 0 - \sin\theta\cos\theta & 0 + 0 + 0 + 0 \\ 0 + 0 + 0 + 0 & 0 + 0 + 1 + 0 & 0 + 0 + 0 + 0 & 0 + 0 + 0 + 0 \\ 0 + \sin\theta\cos\theta + 0 - \sin\theta\cos\theta & 0 + 0 + 0 + 0 & 0 + \cos^2\theta + 0 + \sin^2\theta & 0 + 0 + 0 + 0 \\ 0 + 0 + 0 + 0 & 0 + 0 + 0 + 0 & 0 + 0 + 0 + 0 & 1 + 0 + 0 + 0 \end{bmatrix}_{4 \times 4}$$

$$A \cdot A^T = \begin{bmatrix} \sin^2 \theta + \cos^2 \theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sin^2 \theta + \cos^2 \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$$

Since, $\boxed{\sin^2 \theta + \cos^2 \theta = 1}$

$$A \cdot A^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4} = I_{4 \times 4}$$

$\Rightarrow \boxed{A \cdot A^T = I}$

Below is the multiplication of A^T and A

$$A^T \cdot A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ \sin \theta & 0 & \cos \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\cos \theta & 0 & \sin \theta & 0 \end{bmatrix}_{4 \times 4} \cdot \begin{bmatrix} 0 & \sin \theta & 0 & -\cos \theta \\ 0 & 0 & 1 & 0 \\ 0 & \cos \theta & 0 & \sin \theta \\ 1 & 0 & 0 & 0 \end{bmatrix}_{4 \times 4}$$

$$A^T \cdot A = \begin{bmatrix} 0+0+0+1 & 0+0+0+0 & 0+0+0+0 & 0+0+0+0 \\ 0+0+0+0 & \sin^2 \theta + 0 + \cos^2 \theta + 0 & 0+0+0+0 & -\sin \theta \cos \theta + 0 + \sin \theta \cos \theta + 0 \\ 0+0+0+0 & 0+0+0+0 & 0+1+0+0 & 0+0+0+0 \\ 0+0+0+0 & -\sin \theta \cos \theta + 0 + \sin \theta \cos \theta + 0 & 0+0+0+0 & \cos^2 \theta + 0 + \sin^2 \theta + 0 \end{bmatrix}_{4 \times 4}$$

$$A^T \cdot A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4} = I_{4 \times 4}$$

$$\boxed{A^T \cdot A = I}$$

Hence, matrix A follows below condition:

$$\boxed{A \cdot A^T = A^T A = I}$$

Hence, for any value of θ , Inverse of A is equal to the Transpose of A.