Newton - Rhapson Method

To improve the convergence, need to start including derivative data.

Let x^* be the root: $f(x^*) = 0$.

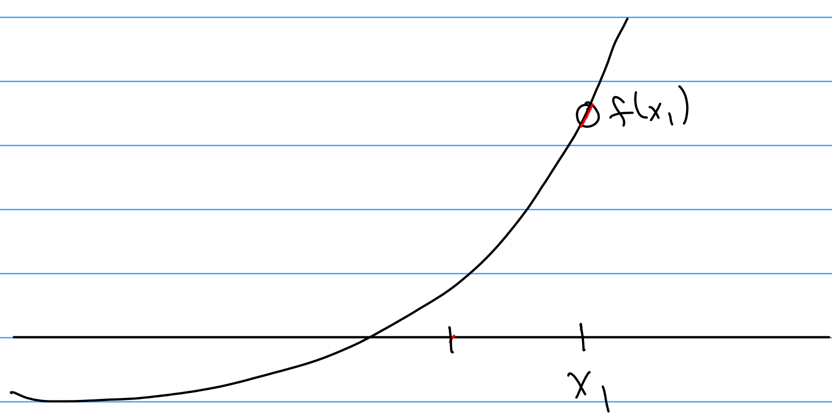
Let be a point near .

Taylor Series about x_1:

$f(x) =$

Create an iteration by finding x_2 such that

$= 0 \Rightarrow x_2 =$



Iteration is then $x_{i+1} =$

Look at convergence:

Let x^* be the true root, x_n the estimate at iteration n, and x_{n+1} the estimate at iteration $n+1$.

Assume that $|x^* - x_n| = \delta << 1$ (   )

Define errors as $e_n =$ & $e_{n+1} =$

Given e_n, how much smaller is e_{n+1}?

It can be shown that

$0 = f(x^*) =$

for some   such that

$\frac{f''(\xi)(x^* - x_n)^2}{2} =$

Newton - Rhapson: $x_{n+1} =$

$\Rightarrow f(x_n) =$

Then

① =

② =

③ =

$e_{n+1} =$

Or $e_{n+1} \propto$ ← convergence.

If $e_n \sim$

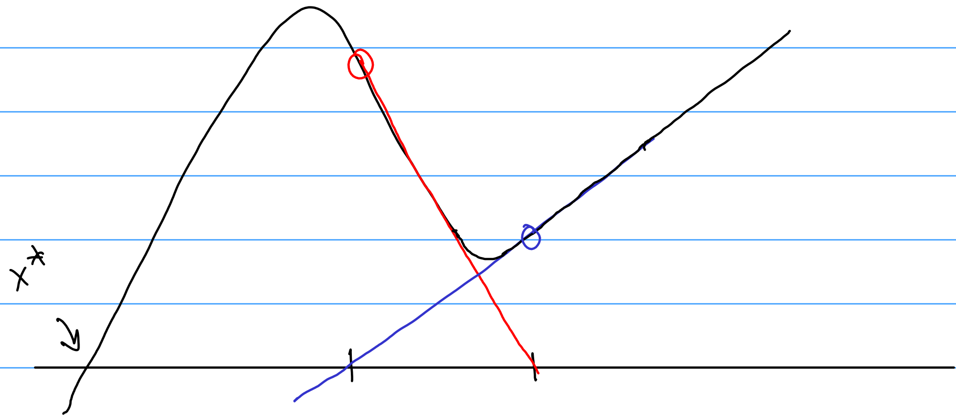
$e_{n+1} \sim$

$e_{n+2} \sim$

State the iteration has converged if

or if

Issues: (1)



⇒ Need to check for divergence!

(2)

(3)