# Chapter 2 Homework

**Directions:** Work on the problems in this order: yellow , blue , green , gray .

You will generally be given about two days' notice that a certain color grouping will be due in Top Hat. The expectation is that you are working on a few of these problems every day, so two days should be plenty of time to wrap up your work and submit your answers.

Some of the red problems may be discussed in class as time permits.

The uncolored problems can be done for additional practice.

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### Sample Spaces, Outcomes, and Events

**Problem 1.** A coin is tossed ($H$ or $T$), and then a die is rolled $(1, 2, 3, 4, 5, 6)$. Write out the sample space for this experiment, and draw a tree diagram that shows the 12 outcomes.

**Problem 2.** A die is rolled $(1, 2, 3, 4, 5, 6)$, and then a coin is tossed ($H$ or $T$). Write out the sample space for this experiment, and draw a tree diagram that shows the 12 outcomes.

**Problem 3.** Four cards labeled $A, B, C, D$ are lying face down on a desk. A card will be chosen, and then, without replacing the first card, a second card will be chosen. Write out the sample space, and draw a tree diagram that shows the outcomes.

**Problem 4.** Four cards labeled $A, B, C, D$ are lying face down on a desk. A card will be chosen and then replaced on the desk. A second card will be chosen. Write out the sample space, and draw a tree diagram that shows the outcomes.

**Problem 5.**

(a) A die is rolled. If the number on the die is odd, the experiment ends. If the number on the die is even, we toss a coin twice ($H$ or $T$). Write out the sample space, and draw the tree diagram for this experiment.

(b) A coin is tossed. If the coin lands on heads, the experiment ends. Otherwise, we roll a die. Write out the sample space, and draw the tree diagram for this experiment.

### Cardinality, Unions, Intersections, and Complements

**Problem 6.** Suppose that we have two events with $C(E_1) = 3$, $C(E_2) = 5$, and $C(\mathcal{S}) = 20$. If $E_1 \subset E_2$, determine the cardinality of each of the following:

(a) $E_1 \cup E_2$

(b) $E_1 \cap E_2$

(c) $E_1^C \cap E_2$

(d) $E_2^C \cap E_1$

(e) $E_1^C \cap E_2^C$

**Problem 7.** Suppose that we have two events with $C(E_1) = 3$, $C(E_2) = 5$, and $C(\mathcal{S}) = 20$. If $E_1 \cap E_2 = \emptyset$ (i.e., the events are disjoint), determine the cardinality of each of the following:

(a) $E_1 \cup E_2$

(b) $E_1 \cap E_2$

(c) $E_1^C \cap E_2$

(d) $E_2^C \cap E_1$

(e) $E_1^C \cap E_2^C$

**Problem 8.** Suppose that we have two events with $C(E_1) = 3$, $C(E_2) = 5$, and $C(\mathcal{S}) = 20$. If $E_1$ and $E_2$ are not disjoint, and $E_1$ is not a subset of $E_2$, determine the maximum value of the cardinality and the minimum value of the cardinality for each of the following:

(a) $E_1 \cup E_2$

(b) $E_1 \cap E_2$

(c) $E_1^C \cap E_2$

(d) $E_2^C \cap E_1$

(e) $E_1^C \cap E_2^C$

**Problem 9.** A single die is rolled. Consider the events $E_1 = \{1, 2, 3, 5\}$ and $E_2 = \{2, 3, 4, 6\}$. Determine $E_1 \cap E_2$, $(E_1 \cap E_2)^C$, $E_1 \cup E_2$, $(E_1 \cup E_2)^C$, $E_1^C \cap E_2^C$, and $E_1^C \cup E_2^C$.

**Problem 10.** A sample space for some experiment consists of 8 cards, each with a letter on it. The sample space is $\mathcal{S} = \{A, B, C, D, E, F, G, H\}$. Consider the events $E_1 = \{A, B, C, D, E\}$ and $E_2 = \{A, E, G\}$. Determine $E_1 \cap E_2$, $(E_1 \cap E_2)^C$, $E_1 \cup E_2$, $(E_1 \cup E_2)^C$, $E_1^C \cap E_2^C$, and $E_1^C \cup E_2^C$.

**Problem 11.** If possible, create a sample space $\mathcal{S}$ and two events $E_1$ and $E_2$ such that $E_1$ is non-empty, $E_2$ is non-empty, $E_1 \neq E_2$, and $C(E_1) = C(E_2) = C(E_1 \cap E_2) = C(E_1 \cup E_2) = C\left((E_1 \cup E_2)^C\right)$.

### Definition of a Probability Function

**Problem 12.** Suppose we have a probability space, i.e, a sample space $\mathcal{S}$ along with a probability function $P$. Determine whether each of the following statements is true or false:

(a) It is possible for $P(E) = 0$.

(b) It is possible for $P(E) = 1$.

(c) It is possible for $P(E) = 1.1$.

(d) It is possible for $P(E) = -0.2$.

(e) If $P(E_1) = 0.7$ and $P(E_2) = 0.5$, it is possible for $E_1$ to be a subset of $E_2$.

(f) If $P(E_1) = 0.7$ and $P(E_2) = 0.5$, it is possible for $E_2$ to be a subset of $E_1$.

(g) If $P(E_1) = 0.7$ and $P(E_2) = 0.5$, it is possible for $E_1$ and $E_2$ to be disjoint.

(h) If $P(E_1) = 0.5$ and $P(E_2) = 0.5$, it is possible for $E_1$ and $E_2$ to be disjoint.

**Problem 13.** For each function $P_i$ on the sample space $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$ given below, determine the value of $c$ that makes $P_i$ a valid probability function, or say that no such value exists.

| $s$ | $P_1(s)$ | $P_2(s)$ | $P_3(s)$ | $P_4(s)$ | $P_5(s)$ | $P_6(s)$ | $P_7(s)$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.2 | 0.1 | 0.2 | $c$ | $\frac{1}{8}$ | 0.1 | 0.15 |
| 2 | 0.2 | 0.1 | 0.1 | 0.1 | $\frac{1}{7}$ | 0.1 | 0.1 |
| 3 | $c$ | $c$ | -0.1 | 0.2 | $\frac{1}{6}$ | 0.2 | 0.1 |
| 4 | 0.2 | 0.1 | 0.2 | 0.1 | $\frac{1}{5}$ | 0.2 | 0.1 |
| 5 | 0.2 | 0.1 | $c$ | 0.2 | $\frac{1}{4}$ | 0.25 | $c$ |
| 6 | 0.2 | 0.1 | 0.3 | 0.5 | $c$ | $4c$ | $0.85 - 2c$ |

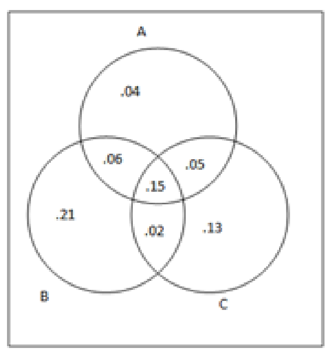
**Problem 14.** Suppose that the sample space for an experiment is $\mathcal{S} = \{1, 2, 3, \ldots\}$.

(a) Is $P(n) = \frac{1}{n+1}$ a valid probability function on the sample space?

(b) Determine the value of $c$ so that $P(n) = c^n$ is a valid probability function on the sample space, or say that no such value exists.

**Rules of Probability**

**Problem 15.** Use the figure below to determine the following probabilities:




(a) $P\left(B^C\right)$

(b) $P(A)$

(c) $P(A \cup B \cup C)$

(d) $P(A \cap C)$

(e) $P(A \cup B)$

(f) $P(C \cap A)$

(g) $P(C \cup B)$

(h) $P\left((A \cup B)^C\right)$

(i) $P(C \cap B \cap A)$

(j) $P\left(\left(B^C\right)^C\right)$

(k) $P\left((A \cup B \cup C)^C\right)$

(l) $P(A - B)$

(m) $P\left(A - (B \cup C)\right)$

(n) $P\left(A - (B \cap C)\right)$

**Problem 16.** Two events $A$ and $B$ are such that $P(A) = 0.45$, $P(B) = 0.65$, and $P(A \cap B) = 0.25$. Determine $P(A \cup B)$.

**Problem 17.** Two events $A$ and $B$ are such that $P(A) = 0.45$, $P(B) = 0.65$, and $P(A \cup B) = 0.7$. Determine $P(A \cap B)$.

**Problem 18.** Two events $A$ and $B$ are such that $P(A) = 0.45$, $P(B) = 0.65$, and $P(A \cap B) = 0.25$. Determine $P\left(B \cap A^C\right)$.

**Problem 19.** Two events $A$ and $B$ are such that $P(A) = 0.45$, $P(A \cap B) = 0.35$, and $P(A \cup B) = 0.6$. Determine $P(B)$.

### Probability as Relative Frequency

**Problem 20.** The following data records blood type (O, A, B, AB) by ethnic group ($G_1$, $G_2$, $G_3$) for a group of 2,000 people and should be used to compute the probabilities below:

| | **O** | **A** | **B** | **AB** | **Total** |
|---|---|---|---|---|---|
| $G_1$ | 300 | 150 | 175 | 25 | 650 |
| $G_2$ | 125 | 650 | 15 | 10 | 800 |
| $G_3$ | 165 | 145 | 175 | 65 | 550 |
| **Total** | 590 | 945 | 365 | 100 | 2,000 |

(a) $P(G_1)$

(b) $P(\mathrm{A})$

(c) $P(\mathrm{A} \cup \mathrm{B})$

(d) $P(\mathrm{A} \cap G_2)$

(e) $P(G_1 \cup G_2)$

(f) $P(G_3 \cap \mathrm{O})$

(g) $P(G_3 \cup \mathrm{O})$

(h) $P(G_1 \cup G_2 \cup \mathrm{AB})$

(i) $P((G_1 \cup G_2) \cap \mathrm{AB})$

(j) $P\left(\mathrm{B}^C\right)$

(k) $P\left((\mathrm{A} \cup G_3)^C\right)$

(l) $P\left((G_2 \cap \mathrm{AB})^C\right)$

**Problem 21.** A deck of cards is well-shuffled, and a card is drawn at random. Determine the probability of each of the following events:

(a) a face card is selected

(b) the card is not a 7

(c) the card is a club

(d) a red face card is selected

(e) the card is red or a king

(f) the card is red or a club

**Problem 22.** Two dice (one red and one green) are to be rolled. The sample space consists of the 36 outcomes shown in the figure below:

$$S = \begin{Bmatrix} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3, & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{Bmatrix}$$

Suppose the first number listed is the result of the red die, and the second number listed is the result of the green die. Determine the following probabilities:

(a) $P$(at least one of the dice is a 5)

(b) $P$(sum of the dice is equal to 7)

(c) $P$(sum of the dice is 11 or more)

(d) $P$(both results are less than 3)

(e) $P$(red result is larger than green result)

(f) $P$(sum is greater than 9)

(g) $P$(red result = 6)

(h) $P$(largest number is a 5)

(i) $P$(smallest number is a 5)

### Counting Techniques

**Problem 23.** Five coins are to be tossed. Determine the cardinality of the sample space.

**Problem 24.** A coin is to be tossed, followed by rolling two dice. Determine the cardinality of the sample space.

**Problem 25.** Two dice are to be rolled, and the total number of spots observed. Determine the cardinality of the sample space. Are these outcomes equally likely?

**Problem 26.** Three dice are to be rolled. Determine the cardinality of the sample space.

**Problem 27.** In how many ways can we arrange 7 red chips and 4 blue chips in 11 slots?

**Problem 28.** In how many ways can we arrange 5 white chips, 4 blue chips, and 3 red chips in 12 slots?

**Problem 29.** Determine the value of each of the following: $\binom{6}{3}$, $\binom{8}{3}$, $\binom{8}{5}$, $\binom{9}{3,2,4}$, $\binom{9}{2,3,4}$, and $\binom{9}{4,2,3}$.

**Problem 30.** Suppose we have 9 distinct objects to place in 9 slots. The 9 slots are partitioned into 3 groups. The first group has 3 slots, the second group has 2 slots, and the third group has 4 slots. In how many ways can we place the objects in the slots if the order of the objects within each group does not matter?

**Problem 31.** Three cards are selected from a deck of 52 cards. Determine the probability that all of the cards are face cards.

**Problem 32.** A bin contains 10 red chips, 8 white chips, and 2 blue chips. Three chips are selected without replacement. Determine the probability of each of the following:

(a) all three chips are white

(b) all three chips are red

(c) all three chips are blue

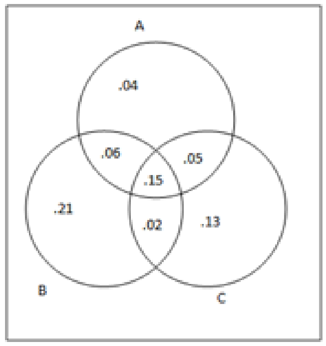
**Problem 33.** Four cards are selected from a deck of cards. Determine the probability that all four of the cards are hearts.

**Problem 34.** Four cards are selected from a deck of cards. Determine the probability that all four of the cards are red.

**Problem 35.** Four cards are selected from a deck of cards. Determine the probability that all four of the cards are jacks.

**Conditional Probability**

**Problem 36.** Use the Venn diagram below to determine the following probabilities:




(a) $P(A \mid B)$

(b) $P(B \mid A)$

(c) $P(A \mid C)$

(d) $P(C \mid A)$

(e) $P(A \mid (B \cap C))$

(f) $P((B \cap C) \mid A)$

(g) $P(A \mid (B \cup C))$

(h) $P((B \cup C) \mid A)$

(i) $P\left(B \mid A^C\right)$

(j) $P\left(A^C \mid B\right)$

(k) $P((A \cap B) \mid (A \cap C))$

(l) $P((A \cap B) \mid (A \cup B))$

**Problem 37.** The following data records blood type (O, A, B, AB) by ethnic group ($G_1$, $G_2$, $G_3$) for a group of 2,000 people and should be used to compute the probabilities below:

| | **O** | **A** | **B** | **AB** | **Total** |
|---|---|---|---|---|---|
| $G_1$ | 300 | 150 | 175 | 25 | 650 |
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| $G_3$ | 165 | 145 | 175 | 65 | 550 |
| **Total** | 590 | 945 | 365 | 100 | 2,000 |

(a) $P(\text{A} \mid G_1)$

(b) $P(G_1 \mid \text{A})$

(c) $P(\text{A} \mid \text{B})$

(d) $P(G_1 \mid (\text{A} \cup \text{B}))$

(e) $P((G_1 \cup G_2) \,|\, (\mathrm{A} \cup \mathrm{B}))$

(f) $P((G_2 \cup G_3) \,|\, \mathrm{AB})$

**Problem 38.** Given $P(A \cap B) = 0.15$ and $P(B) = 0.55$, determine $P(A \,|\, B)$.

**Problem 39.** Given $P(A \cup B) = 0.6$, $P(A) = 0.22$, and $P(B) = 0.5$, determine $P(A \,|\, B)$.

**Problem 40.** An urn contains 6 white chips, 8 red chips, and 3 blue chips. A person selects four chips without replacement. Determine the following probabilities:

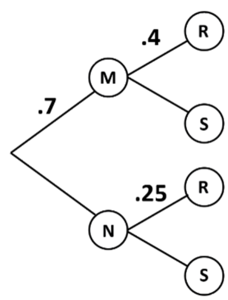
(a) $P$(exactly two chips are white)

(b) $P$(all of the chips are blue)

(c) $P$(the third chip is red)

(d) $P$(the fourth chip is blue | the first three were white)

**Problem 41.** A person has to make a sequence of two choices. The first choice is between $M$ and $N$, and the second choice is between $R$ and $S$, with probabilities as given in the tree diagram below. Determine the following probabilities:




(a) $P(S)$

(b) $P(R)$

(c) $P(N)$

(d) $P(M \cap R)$

(e) $P(N \cup R)$

(f) $P(S \,|\, N)$

(g) $P(M \,|\, S)$

**Problem 42.** Seven cards are selected from a standard 52-card deck. Determine the probability of each of the following:

(a) there are 5 hearts

(b) there are 3 hearts and 3 diamonds

(c) there are 3 hearts, 3 diamonds, and 1 spade

(d) there are 2 aces and 2 kings

(e) there are 2 aces and 3 kings

**Problem 43.** Five cards are selected from a standard 52-card deck. Determine the probability of each of the following (note that these are not easy):

(a) a full house

(b) two pair

(c) three of a kind

(d) a straight

(e) a flush

(f) all card values are distinct

### Law of Total Probability and Bayes' Theorem

**Problem 44.** An urn contains 6 white chips, 5 red chips, and 3 blue chips. A person selects 4 chips without replacement. Determine the following probabilities:

(a) $P(\text{the third chip is blue} \mid \text{the first two were white})$

(b) $P(\text{the third chip is blue} \mid \text{neither of the first two were white})$

(c) $P(\text{the fourth chip is blue} \mid \text{the first two were white})$

(d) $P(\text{the fourth chip is blue} \mid \text{neither of the first two were white})$

**Problem 45.** Four identical bowls are labeled 1, 2, 3, and 4. Bowl 1 contains 5 white chips. Bowl 2 contains 3 white chips and 2 black chips. Bowl 3 contains 1 white chip and 3 black chips. Bowl 4 contains 2 white chips and 3 black chips.

(a) A bowl is randomly selected (equally likely), and two chips are simultaneously selected. Determine the probability that both chips are white.

(b) A bowl is randomly selected (equally likely), and two chips are simultaneously selected. Determine the probability that bowl 3 was chosen given that both chips were white.

(c) A bowl is randomly selected (equally likely), and two chips are simultaneously selected. Determine the probability that bowl 2 was chosen given that both chips were white.

**Problem 46.** Three cards are selected from a standard 52-card deck.

(a) Determine the probability that the first card is a heart, the second card is a diamond, and the third card is an ace.

(b) Determine the probability that the first card was a heart and the second card was a diamond given that the third card was an ace.

**Problem 47.** A card is selected from a standard 52-card deck. If the card is an ace, the experiment ends. If the card is not an ace, then a second card is selected from the deck. If the second card is an ace, the experiment ends. If the second card is not an ace, then a third card is selected, and the experiment ends no matter what card is selected. Determine the probability that the last card selected in the experiment is an ace.

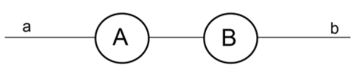
**Independent Events**

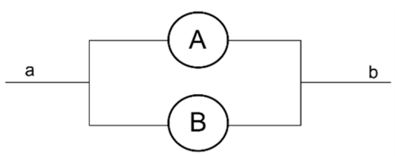
**Problem 48.** Suppose that $A$ and $B$ are independent events with $P(A) = 0.3$ and $P(B) = 0.4$. Determine $P(A \cap B)$.

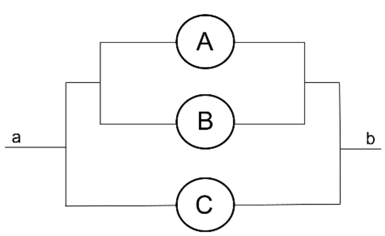
**Problem 49.** Suppose that $A$ and $B$ are independent events with $P(A) = 0.3$ and $P(A \cap B) = 0.06$. Determine $P(B)$.

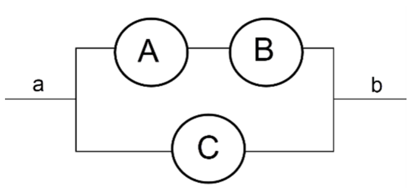
**Problem 50.** Each component in the circuits (or systems) shown below is functional (or "works") with probability 0.9, i.e., $P(A_W) = P(B_W) = P(C_W) = 0.9$. Let $S_W$ be the event that the circuit works.

(a) Determine $S_W$ for each of the circuits shown below.

(i)


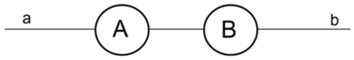

(ii)



(iii)


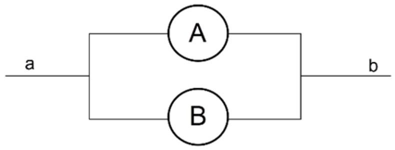

(iv)


(b) Determine $P(A_W \mid S_W)$ for the circuits shown below.

(i)




(ii)




**Problem 51.** We are going to be batch testing for a disease using batches of size 10. Suppose the current positivity rate is 2%.

(a) What is the probability that a batch of 10 independent individuals will produce a negative test result?

(b) You were in a batch that tested positive. Determine the probability that you tested positive.

**Problem 52.** We are going to be batch testing for a disease using batches of size 10. Suppose the current positivity rate is 5%.

(a) What is the probability that a batch of 10 independent individuals will produce a negative test result?

(b) You were in a batch that tested positive. Determine the probability that you tested positive.

**Problem 53.** We are going to be batch testing for a disease using batches of size 10. Suppose the current positivity rate is 10%.

(a) What is the probability that a batch of 10 independent individuals will produce a negative test result?

(b) You were in a batch that tested positive. Determine the probability that you tested positive.