Chapter 4 Homework

Directions: Work on the problems in this order: yellow , green , blue . Do this for Sections 4.1-4.4 first, then Sections 4.5-4.9 (there are no problems for Section 4.9).

You will generally be given about two days' notice that a certain color grouping will be due in Top Hat. The expectation is that you are working on a few of these problems every day, so two days should be plenty of time to wrap up your work and submit your answers.

Some of the red problems (if any) may be discussed in class as time permits.

The uncolored problems can be done for additional practice.

Section 4.1

Problem 0. What is the definition of the CDF of a continuous random variable?

Solution: The definition is the same as it was for a discrete random variable in Chapter 3: $F(x) = P(X \leq x)$. What differentiates the CDF of a continuous random variable from the CDF of a discrete random variable is the former is a continuous function, whereas the latter is discontinuous. $\square$

Problem 1. Consider the CDF given by

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \dfrac{x^2}{10} & \text{if } 0 < x < 2 \\ \dfrac{x}{5} & \text{if } 2 \leq x \leq 5 \\ 1 & \text{if } 5 < x \end{cases}$$

(a) Determine $P(3.5 \leq X)$.

(b) Determine $P(1 \leq X \leq 4)$.

(c) Determine k so that $P(X \leq k) = 0.20$.

(d) Graph $F(x)$.

Solution:

(a) $P(3.5 \leq X) = 1 - F(3.5) = 1 - \frac{3.5}{5} = 0.30$

(b) $P(1 \leq X \leq 4) = F(4) - F(1) = \frac{4}{5} - \frac{1^2}{10} = 0.70$

(c) Since $F(2) = 0.40$, it should be clear that we need $0 < k < 2$. Solve $P(X \leq k) = F(k) = \frac{k^2}{10} = 0.20$ to get $k = \sqrt{2}$.

$\square$

Problem 2. Use the CDF from Problem 1 to determine the following probabilities:

(a) $P(3.5 < X)$

(b) $P(1.5 \leq X)$

(c) $P(2.5 \leq X \leq 4.25)$

(d) $P(1.5 \leq X \leq 1.7)$

(e) $P(3 \leq X \leq 6)$

(f) $P(-1 \leq X \leq 1)$

(g) $P(X \leq 1.2)$

(h) $P(X \leq 3.8)$

Problem 3. Use the CDF from Problem 1 to determine the following $\dfrac{\Delta F}{\Delta x}$ quotients:

(a) $\dfrac{F(1.7) - F(1.5)}{1.7 - 1.5}$

(b) $\dfrac{F(1.6) - F(1.5)}{1.6 - 1.5}$

(c) $\dfrac{F(1.55) - F(1.5)}{1.55 - 1.5}$

(d) $\dfrac{F(1.51) - F(1.5)}{1.51 - 1.5}$

(e) $\dfrac{F(1.501) - F(1.5)}{1.501 - 1.5}$

(f) $\dfrac{F(1.5001) - F(1.5)}{1.5001 - 1.5}$

(g) $\dfrac{F(1.50001) - F(1.5)}{1.50001 - 1.5}$

(h) Use the previous results to determine $F'(1.5)$.

Solution:

(a) $$\frac{F(1.7) - F(1.5)}{1.7 - 1.5} = \frac{\frac{1.7^2}{10} - \frac{1.5^2}{10}}{1.7 - 1.5} = 0.32$$

$\square$

Problem 4. If the random variable X has CDF $F(x) = \dfrac{e^x}{1+e^x}$ for $-\infty < x < \infty$, determine the following probabilities:

(a) $P(X \le 0)$

(b) $P(X \le -1)$

(c) $P(X \le -4)$

(d) $P(X \le -8)$

(e) $P(X \le 1)$

(f) $P(X \le 4)$

(g) $P(X \le 8)$

(h) $P(-1.5 \le X \le 1.5)$

Problem 5. If the random variable X has CDF

$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 - e^{-x} & \text{if } 0 < x, \end{cases}$$

determine the following probabilities:

(a) $P(X \le 0)$

(b) $P(X \le -1)$

(c) $P(X \le -4)$

(d) $P(X \le -8)$

(e) $P(X \le 1)$

(f) $P(X \le 4)$

(g) $P(X \le 8)$

(h) $P(-1.5 \le X \le 1.5)$

Problem 6. If the random variable X has CDF

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - e^{-x} & \text{if } 0 < x, \end{cases}$$

determine the following $\dfrac{\Delta F}{\Delta x}$ quotients:

(a) $\dfrac{F(1.7) - F(1.5)}{1.7 - 1.5}$

(b) $\dfrac{F(1.6) - F(1.5)}{1.6 - 1.5}$

(c) $\dfrac{F(1.55) - F(1.5)}{1.55 - 1.5}$

(d) $\dfrac{F(1.51) - F(1.5)}{1.51 - 1.5}$

(e) $\dfrac{F(1.501) - F(1.5)}{1.501 - 1.5}$

(f) $\dfrac{F(1.5001) - F(1.5)}{1.5001 - 1.5}$

(g) $\dfrac{F(1.50001) - F(1.5)}{1.50001 - 1.5}$

(h) Use the previous results to determine $F'(1.5)$.

Problem 7. Determine the value of a so that the function

$$F(x) = \begin{cases} 0 & \text{if } x \leq a \\ \dfrac{x^3 - 5}{x^3 + 7} & \text{if } a < x \end{cases}$$

is the CDF of a continuous random variable X.

Solution: In order for $F(x)$ to be the CDF of a continuous random variable X, we need four conditions to be met: (i) $\lim_{x \to -\infty} F(x) = 0$, (ii) $\lim_{x \to \infty} F(x) = 1$, (iii) $F(x)$ must be non-decreasing, and (iv) $F(x)$ must be a continuous function. Conditions (i) and (ii) are obviously true. Condition (iii) can be verified by noting that $F'(x) > 0$ wherever it is defined. In order for condition (iv) to be satisfied, notice that since $F(a) = 0$, we need $\lim_{x \to a^+} F(x) = \dfrac{a^3 - 5}{a^3 + 7} = 0$. This can only happen when $a^3 - 5 = 0$ so that $a = \sqrt[3]{5}$. $\square$

Problem 8. Determine the values of a and b so that the function

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ x^3 - 1 & \text{if } a \leq x \leq b \\ 1 & \text{if } b < x \end{cases}$$

is the CDF of a continuous random variable X.

Problem 9. Determine the values of a and b so that the function

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ e^x - 1 & \text{if } a \leq x \leq b \\ 1 & \text{if } b < x \end{cases}$$

is the CDF of a continuous random variable X.

Problem 10. Determine the values of a and b so that the function

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \ln x - 2 & \text{if } a \le x \le b \\ 1 & \text{if } b < x \end{cases}$$

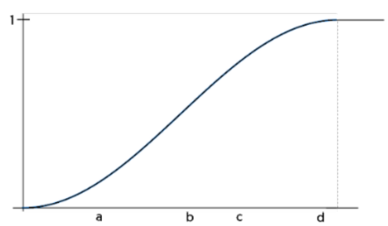
is the CDF of a continuous random variable X.

Problem 11. Determine the values of a and b so that the function

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x^2 - 1}{x} & \text{if } a \le x \le b \\ 1 & \text{if } b < x \end{cases}$$

is the CDF of a continuous random variable X.

Problem 12. Use the graph of the CDF below to estimate the median of X. Note that the median of a distribution, denoted by $\widetilde{x}$, is defined as the 50^{th} percentile. In other words, the median $\widetilde{x}$ satisfies $F(\widetilde{x}) = P(X \le \widetilde{x}) = 0.50$. Is the best answer a, b, c, or d?



Problem 13. Let X be a continuous random variable with CDF $F(x) = \dfrac{4x^2}{4x^2 + 10}$ for $x \ge 0$. Determine the median of the distribution (as defined in Problem 12).

Solution: We need to determine the number $\widetilde{x}$ so that $F(\widetilde{x}) = 0.50$:

$$\begin{aligned} F(\widetilde{x}) &= 0.50 \\ \frac{4\widetilde{x}^2}{4\widetilde{x}^2 + 10} &= 0.50 \\ 4\widetilde{x}^2 &= 2\widetilde{x}^2 + 5 \\ 2\widetilde{x}^2 &= 5 \\ \widetilde{x}^2 &= 2.5 \\ \widetilde{x} &= \sqrt{2.5} \end{aligned}$$

□

Problem 14. Let X be a continuous random variable with CDF $F(x) = \dfrac{e^x}{1+e^x}$ for $-\infty < x < \infty$. Determine the following:

(a) the median of X (as defined in Problem 12)

(b) the value of k so that $P(X \le k) = 0.90$

(c) the value of k so that $P(X \le k) = 0.10$

(d) the value of k so that $P(k \le X) = 0.05$

Problem 15. Let X be a continuous random variable with CDF

$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 - e^{-x} & \text{if } 0 < x \end{cases}$$

Determine the following:

(a) the median of X (as defined in Problem 12)

(b) the value of k so that $P(X \le k) = 0.90$

(c) the value of k so that $P(X \le k) = 0.10$

(d) the value of k so that $P(k \le X) = 0.05$

Problem 16. Let X be a continuous random variable with CDF

$$F(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x^3 - 5}{x^3 + 7} & \text{if } a < x \end{cases}$$

Determine the following:

(a) the median of X (as defined in Problem 12)

(b) the value of k so that $P(X \le k) = 0.90$

(c) the value of k so that $P(X \le k) = 0.10$

(d) the value of k so that $P(k \le X) = 0.05$

Section 4.2

Problem 17. Let X be a continuous random variable with CDF

$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \dfrac{x^2}{10} & \text{if } 0 < x < 2 \\ \dfrac{x}{5} & \text{if } 2 \le x \le 5 \\ 1 & \text{if } 5 < x \end{cases}$$

Determine the pdf $f(x)$.

Solution: $f(x) = F'(x) = \begin{cases} \dfrac{x}{5} & \text{if } 0 < x < 2 \\ \dfrac{1}{5} & \text{if } 2 < x < 5 \\ 0 & \text{otherwise} \end{cases}$ $\square$

Problem 18. Let X be a continuous random variable with the given CDF. Determine the pdf $f(x)$.

(a) $F(x) = \dfrac{e^x}{1+e^x}$ for $-\infty < x < \infty$

(b) $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } 0 \leq x \end{cases}$

Problem 19. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \dfrac{1}{4} & \text{if } 1 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

Determine the CDF $F(x)$.

Solution: It should be clear that $F(x) = 0$ for $x \leq 1$ and $F(x) = 1$ for $x \geq 5$. For $1 < x < 5$, we have

$$\begin{aligned} F(x) &= \int_1^x \frac{1}{4}\,dt \\ &= \frac{t}{4}\Big|_1^x \\ &= \frac{x-1}{4} \end{aligned}$$

□

Problem 20. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ e^{-x} & \text{if } 0 < x \end{cases}$$

Determine the CDF $F(x)$.

Problem 21. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the CDF $F(x)$.

Problem 22. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0.5 & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Determine the CDF $F(x)$.

Problem 23. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 4x^3 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $P(X \leq 0.5)$.

(b) Determine $P(0.75 \leq X)$.

Problem 24. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0.25 & \text{if } 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Determine the CDF $F(x)$.

Problem 25. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} kx^2 & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k that makes $f(x)$ a valid pdf.

Solution: Recall that the definite integral of the pdf over the support must equal 1. This gives

$$\begin{aligned} \int_{-\infty}^{\infty} f(x)\,dx &= \int_0^2 kx^2\,dx \\ &= \frac{kx^3}{3}\bigg|_0^2 \\ &= \frac{8k}{3} \end{aligned}$$

so that $k = \frac{3}{8}$. □

Problem 26. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x^2 & \text{if } 0 < x < k \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k that makes $f(x)$ a valid pdf.

Problem 27. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0.3 & \text{if } -1 < x < 0 \\ 0.2 + kx & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k that makes $f(x)$ a valid pdf.

Problem 28. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0.3 & \text{if } -0.5 < x < 0 \\ 0.2 + kx & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k that makes $f(x)$ a valid pdf.

Problem 29. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0.1 & \text{if } -1 < x < 0 \\ 0.2 + kx & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k that makes $f(x)$ a valid pdf.

Problem 30. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } k < x < k+1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k that makes $f(x)$ a valid pdf.

Problem 31. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0.25 & \text{if } 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Determine the following probabilities:

(a) $P(X \le 0.5)$

(b) $P(1.5 \le X \le 2.5)$

(c) $P(0.5 \le X \le 2)$

(d) $P(2.5 \le X)$

(e) Determine the value of k so that $P(X \le k) = 0.05$.

Solution:

(a) $P(X \le 0.5) = \displaystyle\int_{-\infty}^{0.5} f(x)\,dx = \int_0^{0.5} x\,dx = \frac{x^2}{2}\Big|_0^{0.5} = \frac{1}{8}$

(b) $P(1.5 \le X \le 2.5) = \displaystyle\int_{1.5}^{2.5} f(x)\,dx = \int_{1.5}^{2.5} 0.25\,dx = 0.25x\Big|_{1.5}^{2.5} = 0.25$

(c) $P(0.5 \le X \le 2) = \displaystyle\int_{0.5}^{2} f(x)\,dx = \int_{0.5}^{1} x\,dx + \int_1^2 0.25\,dx = \frac{x^2}{2}\Big|_{0.5}^{1} + 0.25x\Big|_1^2 = \frac{1}{2} - \frac{1}{8} + 0.50 - 0.25 = \frac{5}{8}$

(d) $P(2.5 \le X) = \displaystyle\int_{2.5}^{\infty} f(x)\,dx = \int_{2.5}^{3} 0.25\,dx = 0.25x\Big|_{2.5}^{3} = 0.125$

(e) First, it should be clear that $k > 0$ since $f(x) = 0$ for $x \le 0$. Note that based on part (a), we know that $P(X \le 0.5) = 0.125$, so in order for $P(X \le k) = 0.05$, we need $k < 0.5$. Thus, we should solve $P(X \le k) = \displaystyle\int_0^k x\,dx = \frac{x^2}{2}\Big|_0^k = \frac{k^2}{2} = 0.05$ for k, which gives $k = \sqrt{0.10}$. Note that if we had determined that $k > 1$, then we would have had to use $P(X \le k) = \displaystyle\int_0^1 x\,dx + \int_1^k 0.25\,dx$ in order to solve a problem like this.

$\square$

Problem 32. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0.50 & \text{if } 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Determine the following probabilities:

(a) $P(X \leq 0.6)$

(b) $P(1.5 \leq X \leq 2.5)$

(c) $P(0.5 \leq X \leq 2)$

(d) $P(1 \leq X \leq 1.5)$

(e) Determine the value of k so that $P(X \leq k) = 0.20$.

(f) Determine the value of k so that $P(X \leq k) = 0.80$.

Problem 33. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0.25 & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Determine the following probabilities:

(a) $P(X \leq 4)$

(b) $P(-1 \leq X \leq 2)$

(c) $P(1 \leq X)$

Problem 34. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2} & \text{if } 0 \leq x \\ 0 & \text{otherwise} \end{cases}$$

Determine the following probabilities:

(a) $P(X \leq 4)$

(b) $P(-1 \leq X \leq 2)$

(c) $P(1 \leq X)$

(d) Determine $F(x)$.

Sections 4.3-4.4

Problem 35. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0.50 & \text{if } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Determine the mean of X.

Solution:

$$\begin{aligned} \mathrm{E}[X] &= \int_{-\infty}^{\infty} x f(x)\, dx \\ &= \int_0^1 x \cdot x \, dx + \int_1^2 x \cdot 0.50 \, dx \\ &= \frac{x^3}{3}\bigg|_0^1 + 0.25x^2\bigg|_1^2 \\ &= \frac{1}{3} - 0 + 1 - 0.25 \\ &= \frac{13}{12} \end{aligned}$$

□

Problem 36. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the expected value of X.

(b) Determine the variance of X.

(c) Determine $\mathrm{E}\left[\frac{1}{X}\right]$.

(d) Determine $\mathrm{E}\left[\frac{1}{1+X^3}\right]$.

(e) Determine $\mathrm{E}\left[\frac{1}{1+X^2}\right]$.

Solution:

(c) $\mathrm{E}\left[\frac{1}{X}\right] = \int_0^1 \frac{1}{x} \cdot 3x^2 \, dx = \int_0^1 3x \, dx = \frac{3x^2}{2}\bigg|_0^1 = \frac{3}{2}$

□

Problem 37. Let X be a continuous random variable with CDF

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ \ln x & \text{if } 1 \le x \le e \\ 1 & \text{if } e < x \end{cases}$$

(a) Determine the mean of X.

(b) Determine the variance of X.

Problem 38. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \dfrac{2}{x^3} & \text{if } x \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the mean of X.

(b) Determine the variance of X.

Problem 39. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \dfrac{3}{x^4} & \text{if } x \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the mean of X.

(b) Determine the variance of X.

Problem 40. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \dfrac{1}{4} & \text{if } 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the mean of X.

(b) Determine the variance of X.

Problem 41. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ e^{-x} & \text{if } 0 < x \end{cases}$$

Determine the mean of X.

Problem 42. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0.25 & \text{if } 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Determine the mean of X.

Problem 43. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $\mathrm{E}[X]$.

(b) Determine $\mathrm{Var}[X]$.

Problem 44. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} kx^{k-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $\mathrm{E}[X]$.

(b) Determine $\mathrm{Var}[X]$.

Sections 4.5-4.6

Note: since we have formulas for the mean, variance, and CDF of the uniform and exponential distributions, no integration is necessary for the problems in this section.

Problem 45. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } 2 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $\mathrm{E}[X]$.

(b) Determine $\mathrm{Var}[X]$.

Problem 46. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } 5 < x < 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $\mathrm{E}[X]$.

(b) Determine $\mathrm{Var}[X]$.

Problem 47. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 2e^{-2x} & 0 < x \end{cases}$$

(a) Determine $\mathrm{E}[X]$.

(b) Determine $\mathrm{Var}[X]$.

Problem 48. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 5e^{-5x} & 0 < x \end{cases}$$

(a) Determine $\mathrm{E}[X]$.

(b) Determine $\mathrm{Var}[X]$.

Problem 49. Let X be a uniformly distributed random variable with support $2 < x < 8$.

(a) Determine $\mathrm{E}[X]$.

(b) Determine $\mathrm{Var}[X]$.

Problem 50. Let X be an exponentially distributed random variable with mean 3. Determine $\mathrm{Var}[X]$.

Problem 51. Let X be an exponentially distributed random variable with mean 3. Determine $P(2 < X < 5)$.

Problem 52. Let X be a uniformly distributed random variable with support $2 < x < 8$. Determine $P(3 < X < 5)$.

Sections 4.7-4.8

Problem 53. Let $Z \sim \mathrm{N}(0,1)$ be a standard normal random variable. Determine the following:

(a) $P(1.19 < Z)$

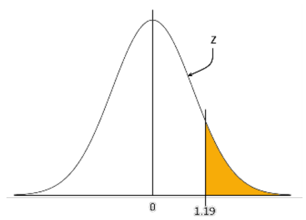
(b) $P(-1.36 \leq Z < -0.28)$

(c) the number k so that $P(Z < k) = 0.209$

(d) $z_{0.015}$

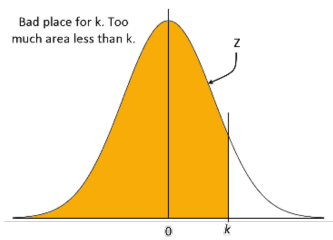
Solution:

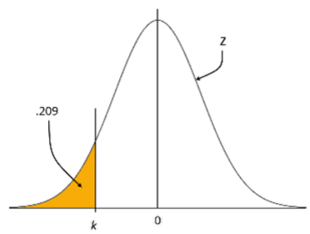
(a) $P(1.19 < Z) = 1 - \Phi(1.19) = 0.1170$



(b) $P(-1.36 \leq Z < -0.28) = \Phi(-0.28) - \Phi(-1.36) = 0.3897 - 0.0869 = 0.3028$

(c) $k = -0.81$





	.00	.01	.0
0.0	.5000	.4960	.49
-0.1	.4602	.4562	.45
-0.2	.4207	.4168	.41
-0.3	.3821	.3783	.37
-0.4	.3446	.3409	.33
-0.5	.3085	.3050	.30
-0.6	.2743	.2709	.26
-0.7	.2420	.2389	.23
-0.8	.2119	.2090	.20
-0.9	.1841	.1814	.17

(d) Recall that $z_{0.015}$ is the z-value with area 0.015 to the right, which means $z_{0.015}$ is the z-value with area 0.985 to the left. Searching for 0.985 in the standard normal CDF chart, we find that $z_{0.015} = 2.17$.

	.00	.06	.07
0.0	.5000	.5239	.5279
0.1	.5398	.5636	.5675
0.2	.5793	.6026	.6064
1.7	.9554	.9608	.9616
1.8	.9641	.9686	.9693
1.9	.9713	.9750	.9756
2.0	.9773	.9803	.9808
2.1	.9821	.9846	.9850

$\square$

Problem 54. Let $Z \sim \mathrm{N}(0,1)$ be a standard normal random variable. Determine the following:

(a) $P(0.68 \leq Z < 0.78)$

(b) $P(-1.65 < Z)$

(c) $P(Z \leq 2.18)$

(d) $P(-0.68 \leq Z < 1.84)$

(e) $P(2.15 < Z)$

(f) $P(Z \leq -1.32)$

(g) the number k so that $P(Z < k) = 0.05$

(h) the number k so that $P(k < Z) = 0.04$

(i) $z_{0.003}$

(j) $z_{0.03}$

(k) $z_{0.006}$

Problem 55. Let X be a normally distributed random variable with mean 0.4 and standard deviation 0.2. Determine the following:

(a) $P(0.1 < X)$

(b) $P(-0.08 < X \leq 0.468)$

(c) the number k so that $P(k < X) = 0.025$

Solution:

(a) $P(0.1 < X) = P\left(\frac{0.1-0.4}{0.2} < \frac{X-0.4}{0.2}\right) = P(-1.5 < Z) = 1 - \Phi(-1.5) = 0.9332$

(b) $P(-0.08 < X \leq 0.468) = P\left(\frac{-0.08-0.4}{0.2} < \frac{X-0.4}{0.2} \leq \frac{0.468-0.4}{0.2}\right) = \Phi(0.34) - \Phi(-2.4) = 0.6249$

(c) Since $z_{0.025} = 1.96$, we solve $\frac{k-0.4}{0.2} = 1.96$ for k, so that $k = 0.4 + 0.2(1.96) = 0.792$.

$\square$

Problem 56. Let X be a normally distributed random variable with mean 1,000 and standard deviation 100. Determine the following:

(a) $P(1{,}226 \leq X)$

(b) $P(915 < X \leq 1{,}168)$

(c) $P(X \leq 860)$

(d) $P(795 \leq X < 905)$

(e) $P(745 < X \leq 1{,}212)$

(f) $P(X \leq 1{,}188)$

(g) the number k so that $P(k < X) = 0.025$

(h) the number k so that $P(X \leq k) = 0.025$

(i) the number k so that $P(k < X) = 0.0505$

(j) the number k so that $P(X < k) = 0.04$

Problem 57. Let X be a normally distributed random variable with mean 650 and standard deviation 75. Determine the following:

(a) $P(726 \leq X)$

(b) $P(700 < X \leq 865)$

(c) $P(X \leq 555)$

(d) $P(620 \leq X < 630)$

(e) $P(745 < X \leq 826)$

(f) $P(X \leq 847)$

(g) the number k so that $P(k < X) = 0.025$

(h) the number k so that $P(X \leq k) = 0.025$

(i) the number k so that $P(k < X) = 0.0505$

(j) the number k so that $P(X < k) = 0.04$

Problem 58. Let X be a normally distributed random variable with mean 75 and standard deviation 1.6. Determine the following:

(a) the number k so that $P(k < X) = 0.025$

(b) the number k so that $P(X \leq k) = 0.025$

(c) the number k so that $P(k < X) = 0.0505$

(d) the number k so that $P(X < k) = 0.04$

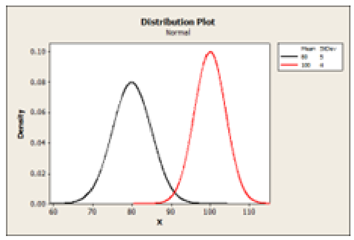
Problem 59. The tread life a certain brand of tire is known to be normally distributed with a mean of 40,000 miles and a standard deviation of 2,500 miles.

(a) Determine the probability that a randomly selected tire will last longer than 46,000 miles.

(b) Let X represent the lifetime of a randomly selected tire. Determine $P(36{,}000 < X \leq 46{,}000)$.

(c) The company wishes to set the warranty level so that only 2% of the tires will need to be replaced. Determine the mileage at which the warranty level should be set in order to achieve this (i.e., if the warranty level is set at k miles, we need $P(X < k) = 0.02$).

Problem 60. For patients with a certain disease, the amount of a certain protein in the blood is normally distributed with mean 100 and standard deviation 4. For patients without the disease, the amount of that protein is normally distributed with mean 80 and standard deviation 5. These distributions are displayed in the figure below. Use your answer to part (a) to answer parts (b)-(d).



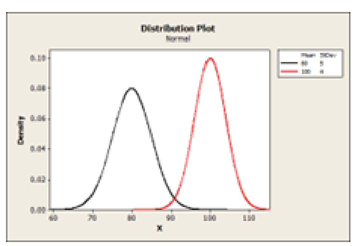
(a) Determine the amount of protein at which "tested positive" should start so that the probability of a person with the disease not testing positive is 0.004.

(b) Determine the probability of a false positive (i.e., a patient without the disease testing positive).

(c) A patient with the disease has a protein level of 94. Will we properly diagnose this patient?

(d) A healthy patient has a protein level of 94. Will we properly diagnose this patient?

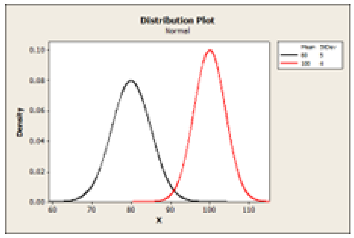
Problem 61. For patients with a certain disease, the amount of a certain protein in the blood is normally distributed with mean 100 and standard deviation 4. For patients without the disease, the amount of that protein is normally distributed with mean 80 and standard deviation 5. These distributions are displayed in the figure below. Use your answer to part (a) to answer part (b).



(a) Determine the amount of protein at which "tested positive" should start so that the probability of a person with the disease not testing positive is 0.003.

(b) Determine the probability of a false positive (i.e., a patient without the disease testing positive).

Problem 62. For patients with a certain disease, the amount of a certain protein in the blood is normally distributed with mean 100 and standard deviation 4. For patients without the disease, the amount of that protein is normally distributed with mean 80 and standard deviation 5. These distributions are displayed in the figure below. Use your answer to part (a) to answer parts (b)-(d).



(a) Determine the amount of protein at which "tested positive" should start so that the probability of a person with the disease not testing positive is 0.002.

(b) Determine the probability of a false positive (i.e., a patient without the disease testing positive).

(c) A patient with the disease has a protein level of 83. Will we properly diagnose this patient?

(d) A healthy patient has a protein level of 83. Will we properly diagnose this patient?

Problem 63. The distribution of systolic blood pressure values in a population is approximately normally distributed with mean 129 mm Hg and standard deviation 19.8 mm Hg. A person is chosen at random from the population, and their systolic blood pressure is measured.

(a) Determine the probability that the person's systolic blood pressure is greater than 141 mm Hg.

(b) Determine the probability that the person's systolic blood pressure is less than 115 mm Hg.

(c) Determine the probability that the person's systolic blood pressure is between 110 mm Hg and 160 mm Hg.

(d) Determine the systolic blood pressure level that is exceeded by only 5% of the population.