Gaussian Quadrature

Do not use uniform spacing.

Goal: Define

Ex.) Construct a rule which integrates all polynomials up to exactly over [-1, 1]

$\int_{-1}^{1} f(x)\,dx =$ let be unknown

let $f(x) = 1$: $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} \, dx = \quad =$

$f(x) = x$: $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} \, dx = \quad =$

$f(x) = x^2$: $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} \, dx = \quad =$

$f(x) = x^3$: $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} \, dx = \quad =$

From $f(x) = x$, $w_2 =$

Use in $f(x) = x^3$ $\Rightarrow$ $=$

$\Rightarrow$ $=$

Since , $=$

From $w_2 =$ $=$ $=$ $\Rightarrow$ $=$

Then

from $f(x)=1$: $=$ $=$ $\Rightarrow$ $=$ $=$

from $f(x)=x^2$: $=$ $=$ $=$

$\Rightarrow x_1 =$ & $x_2 =$ $=$

Thus, if $\int_{-1}^{1} f(x)\,dx \approx$ $+$

then all polynomials up to order 3 are integrated ,

Called a rule as $n=2$ points used

ex.) $f(x)=$ w/ $\int_{-1}^{1} f(x) =$

$f(-1/\sqrt{3}) =$

$f(+1/\sqrt{3}) =$

In general an will integrate polynomials up to exactly.

n	x_i	w_i	Max exact order
2			
3			
4			
⋮			

What if $x \notin [-1, 1]$

Do a change of variables:

$$\int_a^b f(x)\,dx = \int \qquad dt$$

w/ $g(t) =$

We need $g(-1) = \quad = \quad \alpha =$

$g(1) = \quad = \quad \beta =$

$\Rightarrow g(t) = \qquad g'(t) =$

$$\Rightarrow \int_a^b f(x)\,dx = \int_{-1}^{1} f\left[\qquad \right] dt$$

Other non-uniform quadratures:

① Gauss - Kronrod:

② Chebyshev - Gauss: Quadrature for

③ Gauss - Hermite: Quadrature for

Others, ..