Chapter 5 Homework

Directions: Work on the problems in this order: <mark>yellow</mark> , <mark>green</mark> , <mark>blue</mark> . Do this for Sections 5.1-5.3 first, then Sections 5.4-5.8.

You will generally be given about two days' notice that a certain color grouping will be due in Top Hat. The expectation is that you are working on a few of these problems every day, so two days should be plenty of time to wrap up your work and submit your answers.

Some of the <mark>red</mark> problems (if any) may be discussed in class as time permits.

The uncolored problems can be done for additional practice.

Sections 5.1-5.3

Problem 1. Determine the value of k so that $f_{X,Y}(x, y) = k(2 + x + y^2)$ with support $x = 1, 2, 3$ and $y = 1, 2, 3$ is a valid joint pmf.

The joint pmf of the random variables X and Y is given in the table below. Use this distribution when answering Problems 2-3.

X \ Y	0	1	2
0	0.01	0.06	0.18
1	0.05	0.02	0.03
2	0.05	0.11	0.04
3	0.14	0.07	0.05
4	0.07	0.09	0.03

Problem 2. Use the pmf provided above to determine the following:

(a) $P(Y = 2)$

(b) $f_Y(y)$

(c) $f_{X \mid Y=1}(2)$

(d) $f_{X \mid Y=2}(x)$

Problem 3. Use the pmf provided above to determine the following:

(a) $P(X = 3)$

(b) $f_X(x)$

(c) $f_{Y \mid X=3}(1)$

(d) $f_{Y \mid X=2}(y)$

The joint pmf of the random variables X, Y, and Z is given in the tables below. Use this distribution when answering Problems 4-9.

$Z = 1$

X \ Y	1	2	3
1	0.001	0.006	0.007
2	0.002	0.005	0.008
3	0.003	0.004	0.009

$Z = 2$

X \ Y	1	2	3
1	0.07	0.08	0.09
2	0.06	0.05	0.04
3	0.01	0.02	0.03

$Z = 3$

X \ Y	1	2	3
1	0.03	0.02	0.07
2	0.01	0.145	0.06
3	0.05	0.04	0.08

Problem 4. Use the pmf provided above to determine the following:

(a) $P(Y = 2)$

(b) $P(Z = 2)$

(c) $P(X = 2)$

(d) $P(Y = X)$

(e) $P(X + Y = 3)$

(f) $P(X + Y = 4)$

Problem 5. Use the pmf provided above to determine the following:

(a) $P(Y = 1)$

(b) $P(Z = 1)$

(c) $P(X = 1)$

Problem 6. Use the pmf provided above to determine the following:

(a) $f_{X,Y}(1,2)$

(b) $f_{X,Y \mid Z=1}(1,2)$

(c) $f_{X,Y \mid Z=2}(1,2)$

(d) $f_{Z \mid X=Y}(z)$

(e) $f_{Z \mid X+Y=3}(z)$

(f) $f_{Z \mid X+Y=4}(z)$

Problem 7. Use the pmf provided above to determine the following:

(a) $f_{X,Y}(2,2)$

(b) $f_{X,Y \mid Z=1}(2,2)$

(c) $f_{X,Y \mid Z=2}(2,2)$

Problem 8. Use the pmf provided above to determine the following:

(a) $f_{X,Z}(x,z)$

(b) $f_{Y \mid X=1,Z=1}(y)$

(c) $f_{Y \mid X=1}(y)$

(d) $f_{Y \mid Z=1}(y)$

Problem 9. Use the pmf provided above to determine the following:

(a) $f_{X,Y}(x,y)$

(b) $f_{Z \mid X=1,Y=1}(z)$

(c) $f_{Z \mid X=1}(z)$

(d) $f_{Z \mid Y=1}(z)$

Use the joint pmf $f_{X,Y,Z}(x,y,z) = \dfrac{1}{81}(x+xy+z)$ **for** $x = 0,1,2$, $y = 0,1,2$, **and** $z = 0,1,2$ **when answering Problems 10-13.**

Problem 10. Use the pmf provided above to determine the following:

(a) $f_{X,Y}(1,2)$

(b) $f_{X,Y \mid Z=1}(1,2)$

(c) $f_{X,Y \mid Z=2}(1,2)$

(d) $f_{Z \mid X=Y}(z)$

(e) $f_{Z \mid X+Y=3}(z)$

(f) $f_{Z \mid X+Y=4}(z)$

Problem 11. Use the pmf provided above to determine the following:

(a) $f_{X,Y}(2,2)$

(b) $f_{X,Y \mid Z=1}(2,2)$

(c) $f_{X,Y \mid Z=2}(2,2)$

Problem 12. Use the pmf provided above to determine the following:

(a) $f_{X,Z}(x,z)$

(b) $f_{Y \mid X=1,Z=1}(y)$

(c) $f_{Y \mid X=1}(y)$

(d) $f_{Y \mid Z=1}(y)$

(e) $f_{Y \mid X,Z}(y)$

(f) $f_{Y \mid X}(y)$

Problem 13. Use the pmf provided above to determine the following:

(a) $f_{X,Y}(x, y)$

(b) $f_{Z \mid X=1, Y=1}(z)$

(c) $f_{Z \mid X=1}(z)$

(d) $f_{Z \mid Y=1}(z)$

(e) $f_{Z \mid X,Y}(z)$

(f) $f_{Z \mid X}(z)$

Sections 5.4-5.8

Problem 14. Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x, y) = \begin{cases} k(x + y) & \text{if } 0 < x < 1, \ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of k so that $f_{X,Y}(x, y)$ is a valid joint pdf.

(b) Determine $f_X(x)$.

(c) Determine $f_Y(y)$.

(d) Determine $f_{X \mid Y}(x)$.

(e) Determine $f_{X \mid Y=0.5}(x)$.

(f) Determine $f_{Y \mid X}(y)$.

(g) Determine $f_{Y \mid X=0.25}(y)$.

(h) Determine $\mathrm{E}[X]$.

(i) Determine $\mathrm{Var}[X]$.

(j) Determine $\mathrm{E}[Y]$.

Problem 15. Determine the value of k so that

$$f_{X,Y}(x, y) = \begin{cases} kxy & \text{if } 0 < x < 1, \ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a valid joint pdf.

Problem 16. Determine the value of k so that

$$f_{X,Y}(x, y) = \begin{cases} kxy^2 & \text{if } 0 < x < 1, \ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a valid joint pdf.

Problem 17. Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} kx & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of k so that $f_{X,Y}(x,y)$ is a valid joint pdf.

(b) Determine $f_X(x)$.

(c) Determine $f_Y(y)$.

(d) Determine $f_{X\mid Y}(x)$.

(e) Determine $f_{X\mid Y=0.5}(x)$.

(f) Determine $f_{Y\mid X}(y)$.

(g) Determine $f_{Y\mid X=0.25}(y)$.

(h) Determine $E[X]$.

(i) Determine $\text{Var}[X]$.

(j) Determine $E[Y]$.

(k) Determine $\text{Var}[Y]$.

(l) Determine $\text{cov}(X,Y)$.

Problem 18. Determine the value of k so that

$$f_{X,Y}(x,y) = \begin{cases} k(x+y) & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a valid joint pdf.

Problem 19. Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} kxy & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of k so that $f_{X,Y}(x,y)$ is a valid joint pdf.

(b) Determine $f_X(x)$.

(c) Determine $f_Y(y)$.

(d) Determine $f_{X\,|\,Y}(x)$.

(e) Determine $f_{X\,|\,Y=0.5}(x)$.

(f) Determine $f_{Y\,|\,X}(y)$.

(g) Determine $f_{Y\,|\,X=0.25}(y)$.

(h) Determine $E[X]$.

(i) Determine $\text{Var}[X]$.

(j) Determine $E[Y]$.

(k) Determine $\text{Var}[Y]$.

(l) Determine $\text{cov}(X,Y)$.

Problem 20. Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} x+y & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $f_X(x)$.

(b) Determine $f_Y(y)$.

(c) Determine $f_{X\,|\,Y}(x)$.

(d) Determine $f_{X\,|\,Y=0.5}(x)$.

(e) Determine $f_{Y\,|\,X}(y)$.

(f) Determine $f_{Y\,|\,X=0.25}(y)$.

(g) Determine $E[X]$.

(h) Determine $\text{Var}[X]$.

(i) Determine $E[Y]$.

(j) Determine $\text{Var}[Y]$.

(k) Determine $\text{cov}(X,Y)$.

Problem 21. Consider two random variables X and Y with joint pdf $f_{X,Y}(x, y) = 2$ on S, where S is the region bounded by $x = 0$, $x = 1$, $y = x$, and $y = 1 - x$ as shown in the figure below:
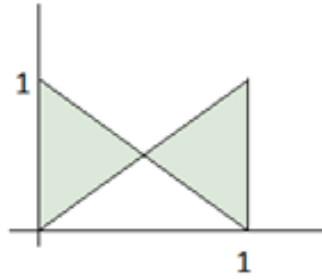
(a) Determine $f_X(x)$.

(b) Determine $f_Y(y)$.

(c) Determine $E[X]$.

(d) Determine $Var[X]$.

(e) Determine $E[Y]$.

(f) Determine $Var[Y]$.

Problem 22. Consider two continuous random variables X and Y with joint CDF (not pdf)

$$F_{X,Y}(x, y) = \begin{cases} \frac{1}{250}(20xy - x^2y - xy^2) & \text{if } 0 \le x \le 5, 0 \le y \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $P(3 \le X)$.

(b) Determine $P(2 \le X)$.

(c) Determine $P((2 \le X \le 3) \cap (1 \le Y \le 4))$.

(d) Determine $f_X(x)$.

(e) Determine $f_Y(y)$.

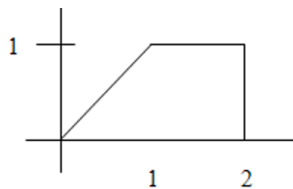**Problem 23.** Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x, y) = \begin{cases} \frac{x+y}{3} & \text{if } 0 < x < 2, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine $f_{Y|X}(y)$.

(b) Determine $f_{X|Y}(x)$.

(c) Determine $E[Y \mid X]$.

(d) Determine $E[X \mid Y]$.

Problem 24. Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} \frac{6}{11}x & \text{on the region } S \\ 0 & \text{otherwise} \end{cases},$$

where S is the region shown in the figure below:

Determine $f_Y(y)$.

Problem 25. Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 6x & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}.$$

Given that $\mathrm{E}[X] = \frac{1}{2}$ and $\mathrm{E}[Y] = \frac{3}{4}$, determine $\mathrm{cov}(X,Y)$.

Problem 26. Consider two random variables X and Y with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 12y^2 & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

Determine $\mathrm{E}[Y \mid X]$.

Problem 27. Determine $\mathrm{cov}(X,Y)$ for the random variables in Problem 23.

Problem 28. Determine $\mathrm{cov}(X,Y)$ for the random variables in Problem 24.

Problem 29. Given that $\mathrm{Var}[X] = 4.5$, $\mathrm{Var}[Y] = 2.8$, and $\mathrm{cov}(X,Y) = 3$, determine the following:

(a) $\mathrm{Var}[X+Y]$

(b) $\mathrm{Var}[X-Y]$

(c) $\mathrm{Var}[2X-4Y]$

(d) $\mathrm{Var}[Y+X]$

(e) $\mathrm{Var}[Y-X]$

(f) $\mathrm{Var}[4Y-2X]$

Problem 30. Given that Var $[X] = 1.2$, Var $[Y] = 2.8$, and cov$(X, Y) = -1$, determine the following:

(a) Var $[X + Y]$

(b) Var $[X - Y]$

(c) Var $[2X - 4Y]$

(d) Var $[Y + X]$

(e) Var $[Y - X]$

(f) Var $[4Y - 2X]$

Problem 31. Given that Var $[X] = 2.1$, Var $[Y] = 3.5$, and cov$(X, Y) = -2$, determine the following:

(a) Var $[X + Y]$

(b) Var $[X - Y]$

(c) Var $[2X - 4Y]$

(d) Var $[Y + X]$

(e) Var $[Y - X]$

(f) Var $[4Y - 2X]$

Problem 32. Given that Var $[X] = 2.1$, Var $[Y] = 1.5$, and cov$(X, Y) = 1.6$, determine the following:

(a) Var $[X + Y]$

(b) Var $[X - Y]$

(c) Var $[2X - 4Y]$

(d) Var $[Y + X]$

(e) Var $[Y - X]$

(f) Var $[4Y - 2X]$