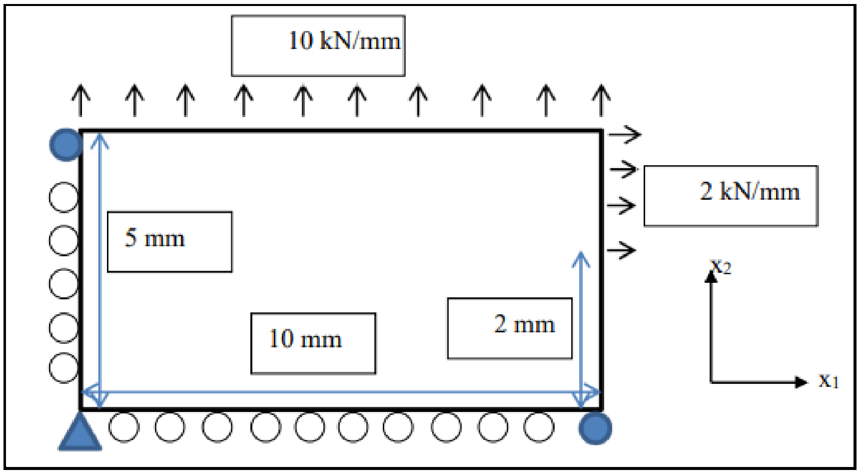
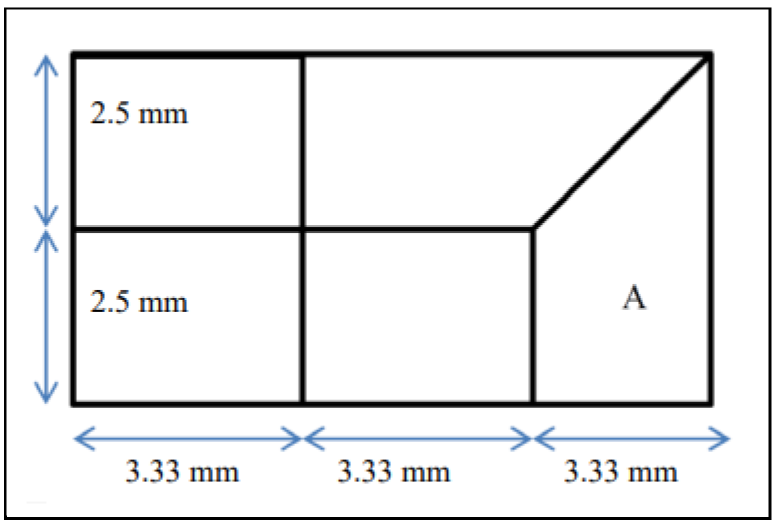
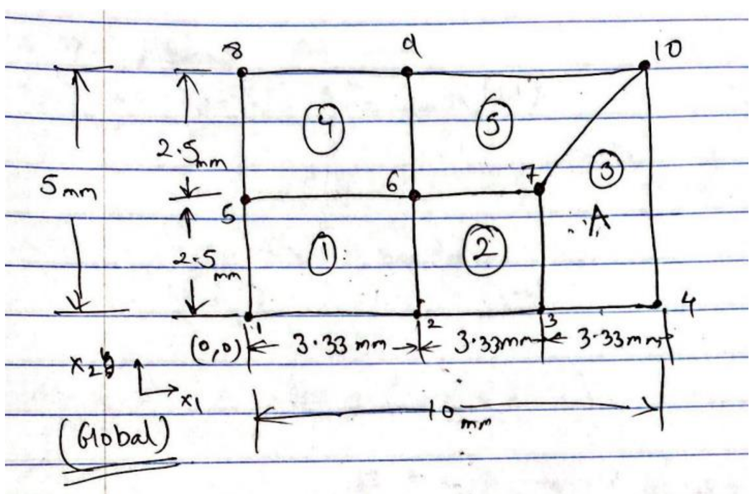
Problem statement-







Steps→

1. **Discretize the element into the given mesh condition.**
2. **Derive the Jacobians from their shape functions for each element.**
3. **Start the code by giving in the initial parameter like material properties, connectivity matrix, and coordinate matrix.**
4. **Find the shape function partial derivate with respect to zeta (ζ) and eta (η) to find the shape Function partial derivate with respect to x and y by inversing the Jacobian * shape function partial derivate with respect to zeta (ζ) and eta (η). This is will common through.**

$$\begin{bmatrix} \frac{\partial Ni}{\partial x} \\ \frac{\partial Ni}{\partial y} \end{bmatrix} = inv(Ji)_{2x2} * \begin{bmatrix} \frac{\partial Ni}{\partial \zeta} \\ \frac{\partial Ni}{\partial \eta} \end{bmatrix}_{2*m}$$

The Jacobian if a constant, it stays constant throughout the element but otherwise if it is a function of zeta (ζ) and eta (η), we find 4 Jacobians at the 4 Gauss Points.

5. **Compute the Finite Element B matrix at each gauss points for each element that means 4 B matrix per element.**

$$[Bi] = \begin{bmatrix} \frac{\partial Ni}{\partial x} & 0 \\ 0 & \frac{\partial Ni}{\partial y} \\ \frac{\partial Ni}{\partial y} & \frac{\partial Ni}{\partial x} \end{bmatrix}_{3x2*m}$$

6. **Computing Local Stiffness at each gauss point of an element.**

$$[Ki]_{2mx2m} = transpose(Bi)_{2mx3} * [E]_{3x3} * [Bi]_{3x2m} * det(Ji)$$

Where, $[E]_{3x3} = \frac{E}{(1+\mu)(1-2\mu)} * \begin{bmatrix} 1-\mu & \mu & 0 \\ \mu & 1-\mu & 0 \\ 0 & 0 & \frac{1-2*\mu}{2} \end{bmatrix}_{3x3} \dashrightarrow for\ plane\ strain\ isotropic$

Material Properties→ $\mu - Poissions\ Ratio$ **and** $E - \ Youngs\ Modulus$

$$[Ki]_{2mx2m} = [Ki\_1]_{2mx2m} + [Ki\_2]_{2mx2m} + [Ki\_3]_{2mx2m} + [Ki\_4]_{2mx2m}$$

7. **Populate/Combining Global Stiffness Matrix as per the nodal indexing**

$$[\boldsymbol{K}]_{5mx5m}$$

8. **Creating Global Force Vector $\{\boldsymbol{R\_trunc}\}_{13x1}$**

   As our case doesn't have any body forces, we only apply the given applied forces which are resolved in the following manner

   Add a pic here

9. **Imposing restricted degrees of Freedom by truncating the rows and column global stiffness matrix**

$$[\boldsymbol{K\_trunc}]_{13x13}\ \boldsymbol{The\ size\ of\ which\ will\ depend\ on\ the\ applied\ BC}$$

10. **Finding Displacements by solving the linear systems**

$$[\boldsymbol{D}]_{5mx1} = \boldsymbol{inv}[\boldsymbol{K\_trunc}]_{13x13} * \{\boldsymbol{R\_trunc}\}_{13x1}$$

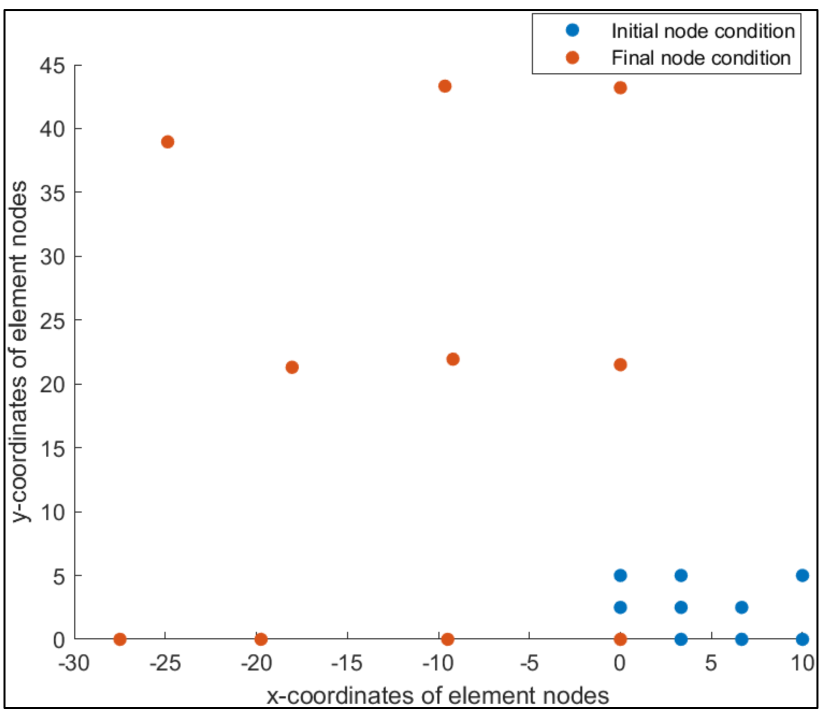
11. **Computing Strains and stresses for Element A (in my case element 3)**

$$\{\boldsymbol{\varepsilon}\}_{3x1} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}_{3x3} * \{\boldsymbol{u}\}_{dof=2x1}$$

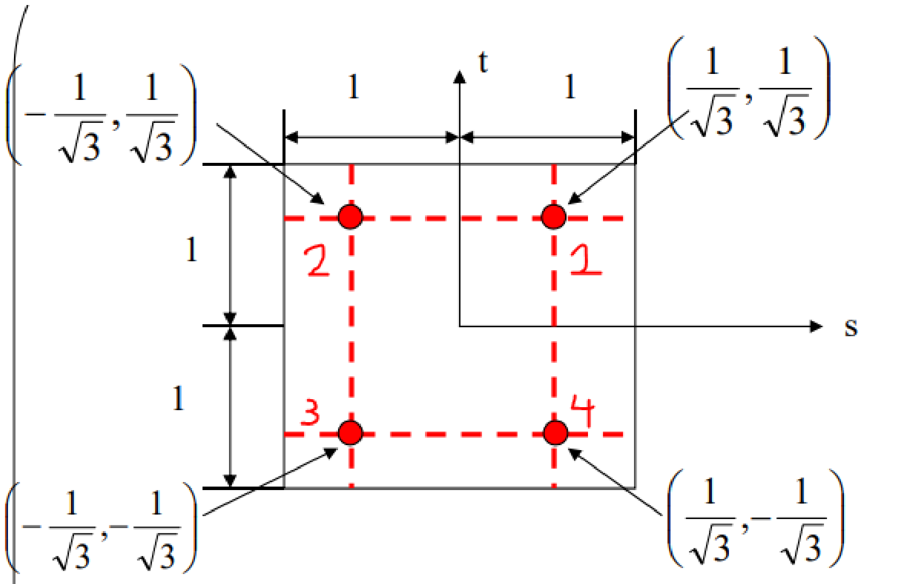
$$\{\boldsymbol{\sigma}\}_{3x1} = [\boldsymbol{E}]_{3x3} * \{\boldsymbol{\varepsilon}\}_{3x1}$$

Displacements→

Node with dof	Displacement (mm)_Matlab	Displacement (mm)_Abaqus
U2	**-9.48760407452011**	**-11.5691**
U3	**-19.7393745774389**	**-3.61331**
U4	**-27.4888247003979**	**-37.5530**
V5	**21.5058470071542**	**28.4539**
U6	**-9.19679826284809**	**1.12164**
V6	**21.9385927654481**	**19.4119**
U7	**-18.0319709230779**	**-24.0266**
V7	**21.3037129224173**	**32.6058**
V8	**43.1995053517166**	**34.8413**
U9	**-9.63701303310965**	**-14.3452**
V9	**43.3230911030585**	**60.7361**
U10	**-24.8663333235884**	**1.61301**
V10	**38.9593190893009**	**35.9252**



Deformation Scatter Plot



$$I \approx \sum_{i=1}^{2}\sum_{j=1}^{2} W_{ij} f(s_i, t_j)$$

$Stress\ (N/mm2)$_Matlab	Gauss Point 1	Gauss Point 2	Gauss Point 3	Gauss Point 4
σ_11	**1280.89549532283**	**306.935744089175**	**1590.32127222576**	**3028.49664439981**
σ_22	**9206.17024921344**	**6283.75082610577**	**9766.08180377926**	**14081.4055490910**
$Stress\ (N/mm2)$_Abaqus	Gauss Point 1	Gauss Point 2	Gauss Point 3	Gauss Point 4
σ_11	**1.40140E+03**	**1.13287E+03**	**604.822**	**613.842**
σ_22	**9.84075E+03**	**10.0879E+03**	**9.95194E+03**	**9.32161E+03**

$Strain$_Matlab	Gauss Point 1	Gauss Point 2	Gauss Point 3	Gauss Point 4
ε_11	-2.42479149644947	-2.17135129506010	-2.36157954574847	-2.73581621774168
ε_22	7.87806568360833	5.59850831156148	8.26690914527108	11.6329653583569
$Strain$_Abaqus	Gauss Point 1	Gauss Point 2	Gauss Point 3	Gauss Point 4
ε_11	-1.55083	-1.89351	-2.38076	-2.18264
ε_22	9.42033	9.74806	9.77050	9.13746

Reaction Force Table-

Coordinate Location	Reaction Forces (N)_MATLAB	Reaction Forces (N)_Abaqus
X1	-1587.23840342068	-2.54677E+03
Y1	-16764.0656813191	-17.2427E+03
X2	2.64396848605909e-12	0.
Y2	-33683.5322862437	-33.9445E+03
X3	-9.73878330477871e-12	0
Y3	-30908.1262357253	-32.7683E+03
X4	1800.00000000001	0
Y4	-18544.2757967119	-15.9445E+03
X5	-3011.03103529511	-1.88700E+03
Y5	7.16534590093373e-12	0
X6	4.30961822195189e-12	0
Y6	2.65138299758077e-11	0
X7	-8.94612434164227e-12	0
Y7	7.19325362961999e-13	0
X8	-1401.73056128420	-1.56623E+03
Y8	16650.0000000000	0
X9	-1.74493396492297e-12	0
Y9	49950.0000000000	0
X10	4200.00000000000	0
Y10	33300.0000000000	0