Tutorial 4

(1) $T(n) = 3T(n/2) + n^2$

Ans $a = 3$, $b = 2$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (Case 3)

$\therefore$ according to masters Theorem

$T(n) = \theta(n^2)$

(2) $T(n) = 4T(n/2) + n^2$

$a = 4$, $b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (Case 2)

$\therefore$ according to masters theorem $T(n) = \theta(n^2 \log n)$

(3) $T(n) = T(n/2) + 2^n$

$a = 1$, $b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (Case 3)

$\therefore$ According to masters theorem $T(n) = \theta(2^n)$

(4) $T(n) = 2^n T(n/2) + n^n$

$\therefore$ Master's theorem is not applicable as a is function of n.

(5) $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$, $f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (Case 1)

$\therefore T(n) = \theta(n^2)$

⑥ $T(n) = 2T(n/2) + n\log n$

$a=2,\ b=2,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

$\therefore$ According to masters $T(n) = \Theta(n\log n)$

⑦ $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a=2,\ b=2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ According to master theorem $T(n) = \Theta(n)$

⑧ $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a=2,\ b=4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ According to master Theorem $T(n) = \Theta(n^{0.51})$

⑨ $T(n) = 0.5T(n/2) + \frac{1}{n}$

$\therefore$ Master's Not applicable as $a < 1$

⑩ $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ According to master, $T(n) = \Theta(n!)$

(11) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

Ans $a = 4,\ b = 2$, $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to master's, $T(n) = \Theta(n^2)$.

(12) $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

$\therefore$ Master's Not applicable as a is not Constant.

(13) $T(n) = 3T(n/2) + n$

Ans $a = 3,\ b = 2$, $f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to master's theorem, $T(n) = O(n^{\log_2 3})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

Ans $a = 3,\ b = 3$, $f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ According to master theorem, $T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

Ans $a = 4,\ b = 2$, $f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c * n$

$\therefore$ According to master's theorem, $T(n) = \Theta(n^2)$

(16) $T(n) = 3T(n/4) + n\log n$

Ans $a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ According to master's theorem, $T(n) = \theta(n\log n)$

(17) $T(n) = 3T(n/3) + n/2$

Ans $a = 3,\ b = 3,\ f(n) = \frac{n}{2}$

$n^{\log_b a} = n^{\log_3 3} = n$

$\theta(n) = \theta(\frac{n}{2})$

$\therefore$ According to master's theorem
$T(n) = \theta(n\log n)$.

(18) $T(n) = 6T(n/3) + n^2\log n$

Ans $a = 6,\ b = 3,\ f(n) = n^2\log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2\log n$

$\therefore$ According to master's theorem $T(n) = \theta(n^2\log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

Ans $a = 4,\ b = 2,\ f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$\therefore$ According to master's theorem $T(n) = \theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

Ans: Master's theorem is not applicable as $f(n)$ is not increasing function.

(21) $T(n) = 7T(n/3) + n^2$

Ans: $a = 7$, $b = 3$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore$ According to master's, $T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Ans: Master's theorem isn't applicable since regularity condition is violated in Case 3.