Math 74: Algebraic Topology

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Problem 1. Suppose that $f : X \to Y$ is a homotopy equivalence. Show that the map $\pi_0(f) : \pi_0(X) \to \pi_0(Y)$ from Homework 1, Problem 5 is a bijection.

Solution.

Problem 2. (0.11) Show that a continuous map $f : X \to Y$ is a homotopy equivalence if there exist continuous maps $g, h : Y \to X$ such that $f \circ g \simeq \mathrm{id}_Y$ and $h \circ f \simeq \mathrm{id}_X$. More generally, show that f is a homotopy equivalence if $f \circ g$ and $h \circ f$ are homotopy equivalences.

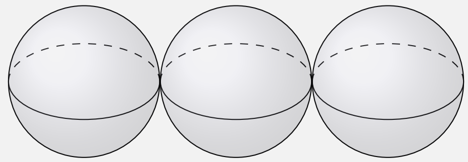
Solution.

Problem 3. Suppose that G is a group and $G_1, \ldots, G_n$ are subgroups that generate G such that $G_i \cap G_j = \{1\}$ for $i \neq j$. Show that G is the free product $G_1 * \ldots * G_n$ if and only if for every group H and collection of homomorphisms $h_j : G_j \to H$, there exists a unique homomorphism $h : G \to H$ such that $h_j = h \circ i_j$ where $i_j : G_j \to G$ is the inclusion.

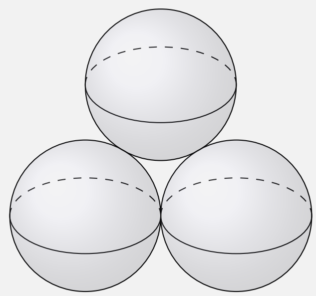
Solution.

Problem 4. Find the fundamental group of each arrangement of three spheres below.

1. A chain.



2. A triangle.



Solution.

Problem 5. (1.2.3) Let X be the union of n lines through the origin in $\mathbb{R}^3$. Compute the fundamental group of $\mathbb{R}^3 \setminus X$.

Solution.