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Matrix Assignment - Conic

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CONTENTS

I. PROBLEM

If the line x-1=0 is the directrix of the parabola to $y^2 - kx + 8 = 0$ then find one of the values of k is
$a)\frac{1}{8}$
$b)4$
$c)8$
$d)\frac{1}{4}$

II. SOLUTION

we know that the vector equation of the line is

$$\mathbf{n}^\top x = c \tag{1}$$

By comparing the given line with (1) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ c = 1}$$

From $y^2 - kx + 8 = 0$

We know that the equation of a conic with directrix $\mathbf{n}^\top x = c$, eccentricity e and focus F is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top x + f = 0 \tag{2}$$

Compare the given parabola with (2) we get,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} \frac{-k}{2} \\ 0 \end{pmatrix} \quad \text{f = 8}$$

If we can find vector $'\mathbf{u}'$ just by comparing the $'\mathbf{u}'$ vector we can obtain the $\mathbf{k}$ value

To find $\mathbf{u}$ we have,

$$\mathbf{u} = ce^2\mathbf{n} - ||\mathbf{n}||^2\mathbf{F} \tag{3}$$

To find Focus $\mathbf{F}$ in (3) we have,

$$f = ||\mathbf{n}||^2||\mathbf{F}||^2 - c^2e^2 \tag{4}$$

By substituting the values of f,c,e,$\mathbf{n}$ in (4) we get,

$$||\mathbf{F}||^2 = 9$$

$$||\mathbf{F}||^2 = \mathbf{F}^\top\mathbf{F}$$

After comparing the conic equation with parabola we know that the y co-ordinate of $\mathbf{u}$ is zero(0) such that,

$||\mathbf{F}||^2$ can be written in two cases as y co-ordinate is zero..,

$$\text{case 1: } \begin{pmatrix} -3 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

$$\text{case 2: } \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

From case 1: The $\mathbf{F}$ is $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

By substituing all the values of c,e,$\mathbf{n}$,$\mathbf{F}$ in (3) we get,

$$\mathbf{u} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

we got, k = -8

From case 2: The $\mathbf{F}$ is $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

By substituing all the values of c,e,$\mathbf{n}$,$\mathbf{F}$ in (3) we get,

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

we got, k = 4

III. FIGURE

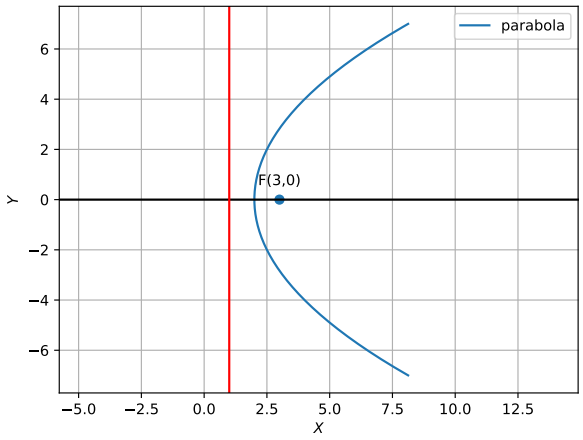

Fig. 1. To find the value of k and plotting the parabola

IV. CodeLink

https://github.com/Sairaghavendra36/Fwc−2022/blob/main/Matrices/Code/Conic.py

Execute the code by using the command
python3 line.py