भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Matrix Assignment - Conic

T.Sai Raghavendra(FWC22087)

CONTENTS

I. PROBLEM

If the line x-1=0 is the directrix of the parabola to $y^2 - kx + 8 = 0$ then find one of the values of k is

$$a)\frac{1}{8} \qquad b)4 \qquad c)\frac{1}{4} \qquad d)8$$

II. SOLUTION

Given line, x-1 = 0
we know that the vector equation of the line is

$$\mathbf{n}^\top x = c$$

Compare the given line with the vector equation

$$\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 1$$

By comparing we get,

$$n = \begin{pmatrix}1\\0\end{pmatrix}, \text{ c = 1}$$

From $y^2 - kx + 8 = 0$

We know that the equation of a conic with directrix $\mathbf{n}^\top x = c$, eccentricity e and focus F is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top x + f = 0$$

Compare the given parabola with equation of conic we get,

$$\mathbf{V} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \quad \mathbf{u} = \begin{pmatrix}\frac{-k}{2}\\0\end{pmatrix} \quad \text{f = 8}$$

If we can find vector $'\mathbf{u}'$ just by comparing the $'\mathbf{u}'$ vector we can obtain the $\mathbf{k}$ value

To find $\mathbf{u}$ we have,

$$\mathbf{u} = ce^2\mathbf{n} - ||\mathbf{n}||^2\mathbf{F} \tag{1}$$

To find Focus $\mathbf{F}$ in equation(1) we have,

$$f = ||\mathbf{n}||^2||\mathbf{F}||^2 - c^2e^2 \tag{2}$$

By substituting the values of f,c,e,$\mathbf{n}$ we get,

$$||\mathbf{F}||^2 = 9$$

$$||\mathbf{F}||^2 = \mathbf{F}^\top\mathbf{F}$$

After comparing the conic equation with parabola we know that the y co-ordinate of $\mathbf{u}$ is zero(0) such that,

$||\mathbf{F}||^2$ can be written in two cases as y co-ordinate is zero..,

$$\text{case 1: } \begin{pmatrix}-3 & 0\end{pmatrix}\begin{pmatrix}-3\\0\end{pmatrix}$$

$$\text{case 2: } \begin{pmatrix}3 & 0\end{pmatrix}\begin{pmatrix}3\\0\end{pmatrix}$$

From case 1: The $\mathbf{F}$ is $\begin{pmatrix}-3\\0\end{pmatrix}$

By substituing all the values of c,e,$\mathbf{n}$,$\mathbf{F}$ in equation(1) we get,

$$\mathbf{u} = \begin{pmatrix}4\\0\end{pmatrix}$$

we got, k = -8

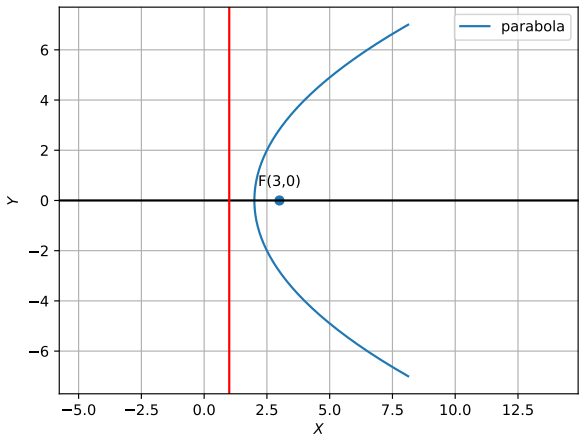
From case 2: The $\mathbf{F}$ is $\begin{pmatrix}3\\0\end{pmatrix}$

By substituing all the values of c,e,$\mathbf{n}$,$\mathbf{F}$ in equation(1) we get,

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

we got, k = 4

III. FIGURE



IV. CodeLink

https://github.com/Sairaghavendra36/Fwc−2022/blob/main/Matrix/Conic/Conic.py

Execute the code by using the command
python3 line.py