Matrix Assignment - Conic

T.Sai Raghavendra(FWC22087)

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

CONTENTS

I. PROBLEM

If the line x-1=0 is the directrix of the parabola to $y^2 - kx + 8 = 0$ then find one of the values of k
1) 4
2) $\frac{1}{4}$
3) 8
4) $\frac{1}{8}$

II. SOLUTION

we know that the vector equation of the line is

$$\mathbf{n}^\top x = c \tag{1}$$

By comparing the given line with (1) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ c = 1}$$

We know that the equation of a conic with directrix $\mathbf{n}^\top x = c$, eccentricity e and focus $\mathbf{F}$ is given by

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top x + f = 0 \tag{2}$$

Compare the given parabola with (2) we get,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{-k}{2} \\ 0 \end{pmatrix}, \text{ f = 8}$$

Finding the vector $\mathbf{u}$ we can obtain the k value, To find vector $\mathbf{u}$ we have,

$$\mathbf{u} = ce^2\mathbf{n} - ||\mathbf{n}||^2\mathbf{F} \tag{3}$$

To find Focus $\mathbf{F}$ in (3) we have,

$$f = ||\mathbf{n}||^2||\mathbf{F}||^2 - c^2e^2 \tag{4}$$

By substituting the f,c,e,**n** in (4) we get,

$$\mathbf{F} = 3\mathbf{e_1} \implies \mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \implies \mathbf{F} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

By substituting the $\mathbf{F}$,c,e,**n** in (3) we get,

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

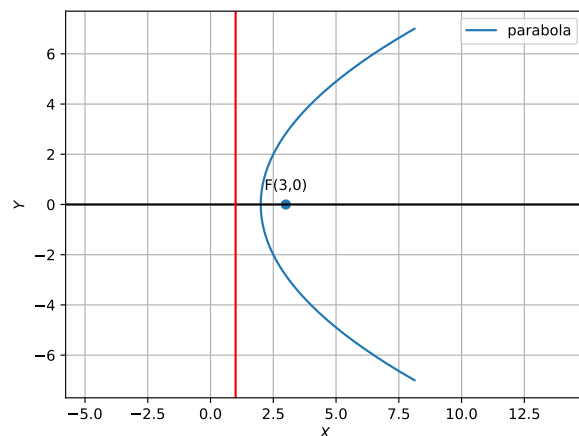Equating the vectors $\mathbf{u}$ we get,

$$k = 4$$

III. FIGURE

Fig. 1. To find the value of k and plotting the parabola

IV. CodeLink

https://github.com/Sairaghavendra36/Fwc−2022/ blob/main/Matrices/Code/Conic.py

Execute the code by using the command **python3 conic.py**