भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Optimisation

T. Sai Raghavendra

CONTENTS

I. PROBLEM STATEMENT

Let f(x) = $sin^3x + \lambda sin^2x, \frac{-\pi}{2} < x < \frac{\pi}{2}$. Find the intervals in which λ should lie in order that f(x) has exactly one minimum and exactly one maximum.

II. SOLUTION

Given function is,

$$f(x) = sin^3x + \lambda sin^2x \tag{1}$$

Theoriticalproof :

Let y=f(x)=$sin^3x + \lambda sin^2x, \frac{-\pi}{2} < x < \frac{\pi}{2}$

Let $sinx = t$

$$\frac{dy}{dt} = 3t^2 + 2t\lambda = t(3t + 2\lambda) \tag{2}$$

for exactly one minima and exactly one maxima $\frac{dy}{dx}$ must have two distinct roots $\in$ (-1,1)
$t = 0$ and $t = \frac{-2\lambda}{3} \in (-1, 1)$

$$-1 < \frac{-2\lambda}{3} < 1 \tag{3}$$

$$\frac{-3}{2} < \lambda < \frac{3}{2} \tag{4}$$

$$\lambda \in (\frac{-3}{2}, \frac{3}{2}) \tag{5}$$

Objectivefunction :

$$\min_x f(x) = sin^3x + \lambda sin^2x, \frac{-\pi}{2} < x < \frac{\pi}{2} \tag{6}$$

$$\max_x f(x) = sin^3x + \lambda sin^2x, \frac{-\pi}{2} < x < \frac{\pi}{2} \tag{7}$$

constraints :

$$x \in \{\frac{-\pi}{2}, \frac{\pi}{2}\} \tag{8}$$

A. Calculation of Minima using gradient descent algorithm:

Minima of the above (1), can be calculated from the following expression, Differentiating (10) yields,

$$\boxed{x_{n+1} = x_n - \alpha \nabla h(x_n)} \tag{9}$$

$$f(x) = sin^3x + \lambda sin^2x \tag{10}$$

$$\nabla f(x) = sinxcosx(3sinx + 2\lambda) \tag{11}$$

Taking $x_0 = \frac{-\pi}{2}, \alpha = 0.0001$ and precision = 0.000000001, values obtained using python are:

$$\boxed{\text{Minima} = -2.5} \tag{12}$$

$$\boxed{\text{Minima Point} = -1.5708} \tag{13}$$

B. Calculation of Maxima using gradient ascent algorithm:

Maxima of the above (1), can be calculated from the following expression, Differentiating (15) yields,

$$\boxed{x_{n+1} = x_n - \alpha \nabla h(x_n)} \tag{14}$$

$$f(x) = sin^3x + \lambda sin^2x \tag{15}$$

$$\nabla f(x) = sinxcosx(3sinx + 2\lambda) \tag{16}$$

Taking $x_0 = \frac{\pi}{2}, \alpha = 0.0001$ and precision = 0.000000001, values obtained using python are:

$$\boxed{\text{Maxima} = 2.5} \tag{17}$$

$$\boxed{\text{Maxima Point} = 1.5707} \tag{18}$$

III. PLOTS

Violation of λ:
λ violate the given condition beyond this range

$$x \in \{\frac{-3}{2}, \frac{3}{2}\}$$

Let us consider violated condition λ = 100

IV. SOFTWARE

https://github.com/Sairaghavendra36/Fwc−2022/blob/main/Matrix/Optimisation/opti.py

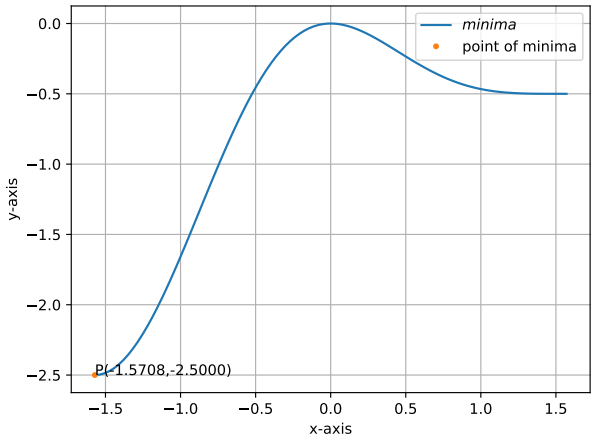

Fig. 1. Minima of $\lambda \in \{\frac{-3}{2}, \frac{3}{2}\}$

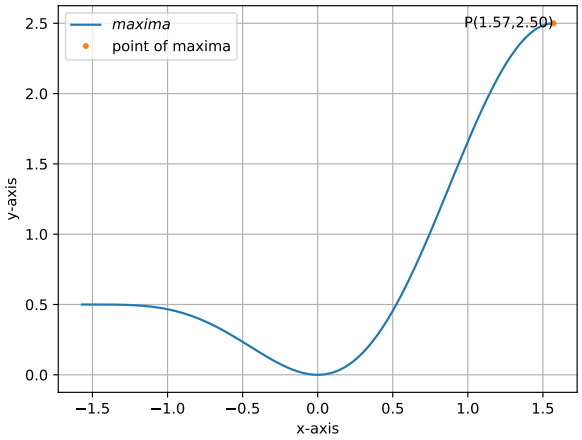

Fig. 2. Maxima of $\lambda \in \{\frac{-3}{2}, \frac{3}{2}\}$

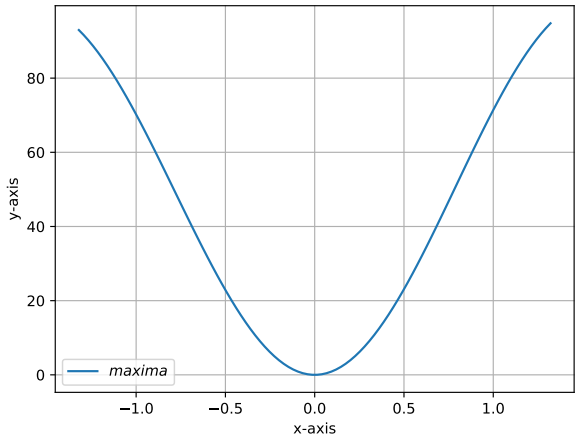

Fig. 3. violated condtion of lambda