



Matrix Assignment - Conic

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I. PROBLEM

If the line $x-1=0$ is the directrix of the parabola to $y^2 - kx + 8 = 0$ then find one of the values of k

- 1) 4
- 2) $\frac{1}{4}$
- 3) 8
- 4) $\frac{1}{8}$

II. SOLUTION

we know that the vector equation of the line is

$$\mathbf{n}^T \mathbf{x} = c \quad (1)$$

By comparing the given line with (1) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = 1$$

Given, equation of parabola is,

$$y^2 - kx + 8 = 0 \quad (2)$$

We know that the equation of a conic with directrix $\mathbf{n}^T \mathbf{x} = c$, eccentricity e and focus F is given by

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (3)$$

Compare the given parabola (2) with (3) we get,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{u} = \begin{pmatrix} \frac{-k}{2} \\ 0 \end{pmatrix}, f = 8$$

Finding the vector \mathbf{u} we can obtain the k value,
To find vector \mathbf{u} we have,

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \quad (4)$$

To find Focus F in (4) we have,

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \quad (5)$$

By substituting the f, c, e, n in (5) we get,

$$\mathbf{F} = 3\mathbf{e}_1 \implies \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \implies \mathbf{F} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

By substituting the F, c, e, n in (4) we get,

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

Equating the vectors \mathbf{u} we get,

$$k = 4$$

The equation of parabola obtained is

$$y^2 - 4x + 8 = 0 \quad (6)$$

III. FIGURE

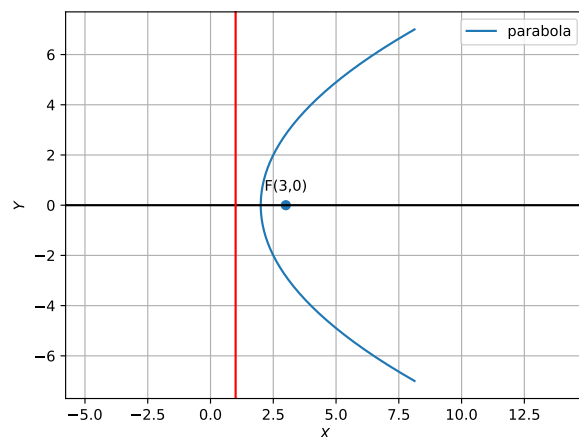


Fig. 1. To find the value of k and plotting the parabola

IV. CodeLink

<https://github.com/Sairaghavendra36/Fwc-2022/blob/main/Matrices/Code/Conic.py>

Execute the code by using the command **python3 conic.py**