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Matrix - Line Assignment

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I. Problem

The vertices of a triangle are $[at_1t_2, a(t_1+t_2)], [at_2t_3, a(t_2+t_3)], [at_3t_1, a(t_3+t_1)]$. Find the orthocentre of the triangle.

II. Solution

Orthocenter of a triangle is the point where perpendiculars drawn to the opposite side from each vertex of the triangle intersect.

To find the orthocenter first we find the equation of line AP which is given by

$$\mathbf{m_1}^\top(\mathbf{x}-\mathbf{A})=0 \tag{1}$$

where $\mathbf{m_1}=(\mathbf{B}-\mathbf{C})$

Similarly the equation of line AP is given by

$$\mathbf{m_2}^\top(\mathbf{x}-\mathbf{B})=0 \tag{2}$$

where $\mathbf{m_2}=(\mathbf{B}-\mathbf{C})$

By Solving eq1 and eq2 we get two line equations are represented in matrix form

$$\begin{pmatrix} \mathbf{at_3(t_2-t_1)} & \mathbf{a(t_2-t_1)} & \mathbf{-a^2(t_2-t_1)[t_1t_2t_3+(t_1+t_2)} \\ \mathbf{at_1(t_2-t_3)} & \mathbf{a(t_2-t_3)} & \mathbf{-a^2(t_2-t_3)[t_1t_2t_3+(t_2+t_3)} \end{pmatrix}$$

$$R_1 \rightarrow \frac{1}{a(t_2-t_1)} \qquad R_2 \rightarrow \frac{1}{a(t_2-t_3)}$$

$$\begin{pmatrix} \mathbf{t_3} & \mathbf{1} & \mathbf{-a[t_1t_2t_3+(t_1+t_2)} \\ \mathbf{t_1} & \mathbf{1} & \mathbf{-a[t_1t_2t_3+(t_2+t_3)} \end{pmatrix}$$

$$R_1 \rightarrow R_1-R_2$$

$$\begin{pmatrix} \mathbf{(t_3-t_1)} & \mathbf{0} & \mathbf{a(t_3-t_1)} \\ \mathbf{t_1} & \mathbf{1} & \mathbf{-a[t_1t_2t_3+(t_2+t_3)} \end{pmatrix}$$

$$R_1 \rightarrow \frac{1}{(t_3-t_1)}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{a} \\ \mathbf{t_1} & \mathbf{1} & \mathbf{-a[t_1t_2t_3+(t_2+t_3)} \end{pmatrix}$$

$$R_2 \rightarrow R_2-t_1R_1$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{a} \\ \mathbf{0} & \mathbf{1} & \mathbf{-a[t_1t_2t_3+(t_1+t_2+t_3)} \end{pmatrix}$$

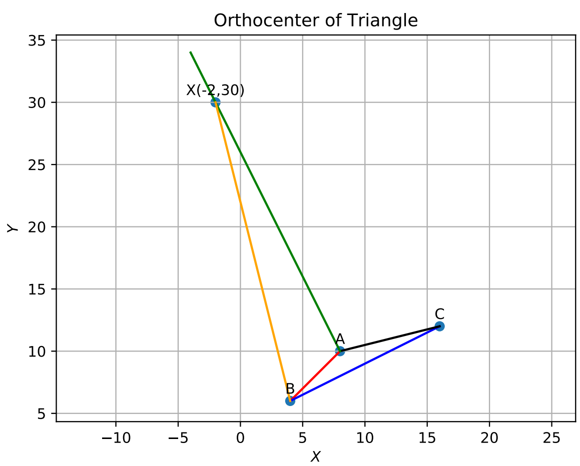
By making X and Y Coordinates of eq1 and eq2 as Identity Matrix there obtained Intersection point i.e.., **Orthocentre**

Therefore the Orthocentre of triangle is

$$\mathbf{X} = \begin{pmatrix} \mathbf{a} \\ \mathbf{-a[t_1t_2t_3+(t_1+t_2+t_3)} \end{pmatrix}$$

Symbol	Co-ordinates	Description
m1	$\begin{pmatrix} at_3(t_2-t_1) \\ a(t_2-t_1) \end{pmatrix}$	direction vector of m1
m2	$\begin{pmatrix} at_1(t_2-t_3) \\ a(t_2-t_3) \end{pmatrix}$	direction vector of m2
A	$\begin{pmatrix} at_1t_2 \\ a(t_1+t_2) \end{pmatrix}$	direction vector of m1
B	$\begin{pmatrix} at_2t_3 \\ a(t_2+t_3) \end{pmatrix}$	direction vector of m1
C	$\begin{pmatrix} at_3t_1 \\ a(t_3+t_1) \end{pmatrix}$	direction vector of m1

III. Figure



IV. CodeLink

https://github.com/Sairaghavendra36/Fwc−2022/blob/main/Matrix/Line/line.py

Execute the code by using the command
python3 line.py