



# Matrix Assignment - Conic

T.Sai Raghavendra(FWC22087)

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### I. PROBLEM

If the line  $x-1=0$  is the directrix of the parabola to  $y^2 - kx + 8 = 0$  then find one of the values of  $k$

### II. SOLUTION

we know that the vector equation of the line is

$$\mathbf{n}^\top \mathbf{x} = c \quad (1)$$

By comparing the given line with (1) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = 1$$

$$y^2 - kx + 8 = 0 \quad (2)$$

We know that the equation of a conic with directrix  $\mathbf{n}^\top \mathbf{x} = c$ , eccentricity  $e$  and focus  $F$  is given by

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (3)$$

Compare the given parabola (2) with (3) we get,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{u} = \begin{pmatrix} \frac{-k}{2} \\ 0 \end{pmatrix}, f = 8$$

Finding the vector  $\mathbf{u}$  we can obtain the  $k$  value,  
To find vector  $\mathbf{u}$  we have,

$$\mathbf{u} = ce_1^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \quad (4)$$

To find Focus  $F$  in (4) we have,

$$\mathbf{F} = \frac{\frac{\eta}{4\sqrt{\lambda_2}} \sqrt{\lambda_2} e_1 - \frac{\eta}{2} e_1}{\lambda_2}$$

From the given parabola and line we have,

$$\lambda_2 = 1, c = 1, e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$c = \frac{\eta}{4\sqrt{\lambda_2}} \implies \eta = 4.c.\lambda_2 = 4$$

On substituting  $\eta, \lambda_2, e_1$  in (5) we get,

$$\mathbf{F} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

By substituting the  $\mathbf{F}, c, e_1, \mathbf{n}$  in (4) we get,

$$\mathbf{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Equating the vectors  $\mathbf{u}$  we get,

$$k = 4$$

### III. FIGURE

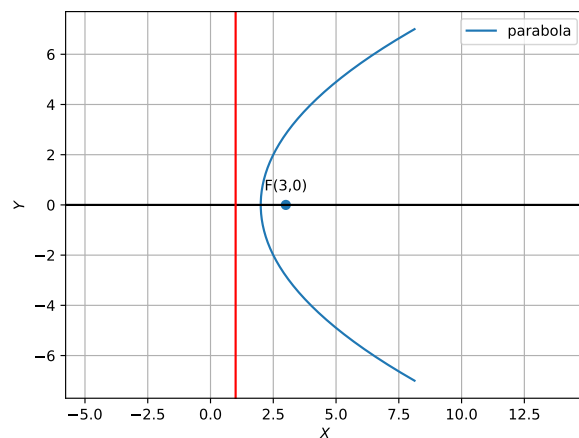


Fig. 1. To find the value of  $k$  and plotting the parabola

### IV. CodeLink

<https://github.com/Sairaghavendra36/Fwc-2022/blob/main/Matrices/Code/Conic.py>

(5) Execute the code by using the command **python3 line.py**