Matrix Assignment - Conic

T.Sai Raghavendra(FWC22087)

CONTENTS

I. PROBLEM

If the line x-1=0 is the directrix of the parabola to $y^2 - kx + 8 = 0$ then find one of the values of k

II. SOLUTION

we know that the vector equation of the line is

$$\mathbf{n}^\top x = c \tag{1}$$

By comparing the given line with (1) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ c = 1}$$

$$\mathbf{y^2} - k\mathbf{x} + 8 = 0 \tag{2}$$

We know that the equation of a conic with directrix $\mathbf{n}^\top x = c$, eccentricity e and focus F is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top x + f = 0 \tag{3}$$

Compare the given parabola (2) with (3) we get,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{u} = \begin{pmatrix} \frac{-k}{2} \\ 0 \end{pmatrix}, \text{ f = 8}$$

Finding the vector $\mathbf{u}$ we can obtain the k value, To find vector $\mathbf{u}$ we have,

$$\mathbf{u} = ce^2\mathbf{n} - ||\mathbf{n}||^2\mathbf{F} \tag{4}$$

To find Focus $\mathbf{F}$ in (4) we have,

$$\mathbf{F} = \frac{\frac{\eta}{4\sqrt{\lambda_2}}\sqrt{\lambda_2}e_1 - \frac{\eta}{2}e_1}{\lambda_2} \tag{5}$$

From the given parabola and line we have,

$$\lambda_2 = 1, \text{ c = 1}, \mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$c = \frac{\eta}{4\sqrt{\lambda_2}} \implies \eta = 4.c.\lambda_2 = 4$$

On substituting η,λ_2,$\mathbf{e_1}$ in (5) we get,

$$\mathbf{F} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

By substituting the $\mathbf{F}$,c,$\mathbf{e_1}$,$\mathbf{n}$ in (4) we get,

$$\mathbf{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Equating the vectors $\mathbf{u}$ we get,

k = 4

III. FIGURE

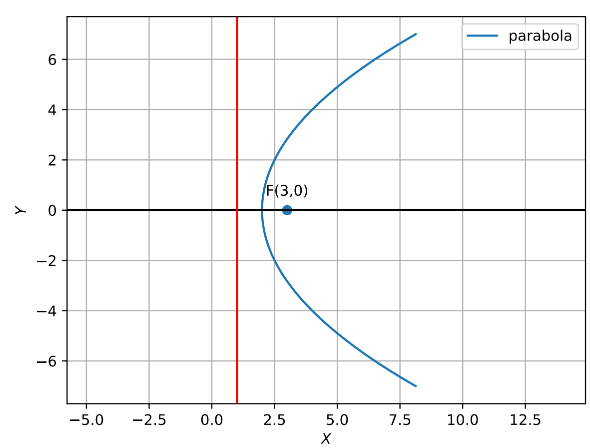

Fig. 1. To find the value of k and plotting the parabola

IV. CodeLink

https://github.com/Sairaghavendra36/Fwc−2022/blob/main/Matrices/Code/Conic.py

Execute the code by using the command **python3 line.py**