Introduction to Prim's algorithm:

We have discussed Kruskal's algorithm for Minimum Spanning Tree. Like Kruskal's algorithm, Prim's algorithm is also a Greedy algorithm. This algorithm always starts with a single node and moves through several adjacent nodes, in order to explore all of the connected edges along the way.

The algorithm starts with an empty spanning tree. The idea is to maintain two sets of vertices. The first set contains the vertices already included in the MST, and the other set contains the vertices not yet included. At every step, it considers all the edges that connect the two sets and picks the minimum weight edge from these edges. After picking the edge, it moves the other endpoint of the edge to the set containing MST.

A group of edges that connects two sets of vertices in a graph is called cut in graph theory. So, at every step of Prim's algorithm, find a cut, pick the minimum weight edge from the cut, and include this vertex in MST Set (the set that contains already included vertices).

How does Prim's Algorithm Work?

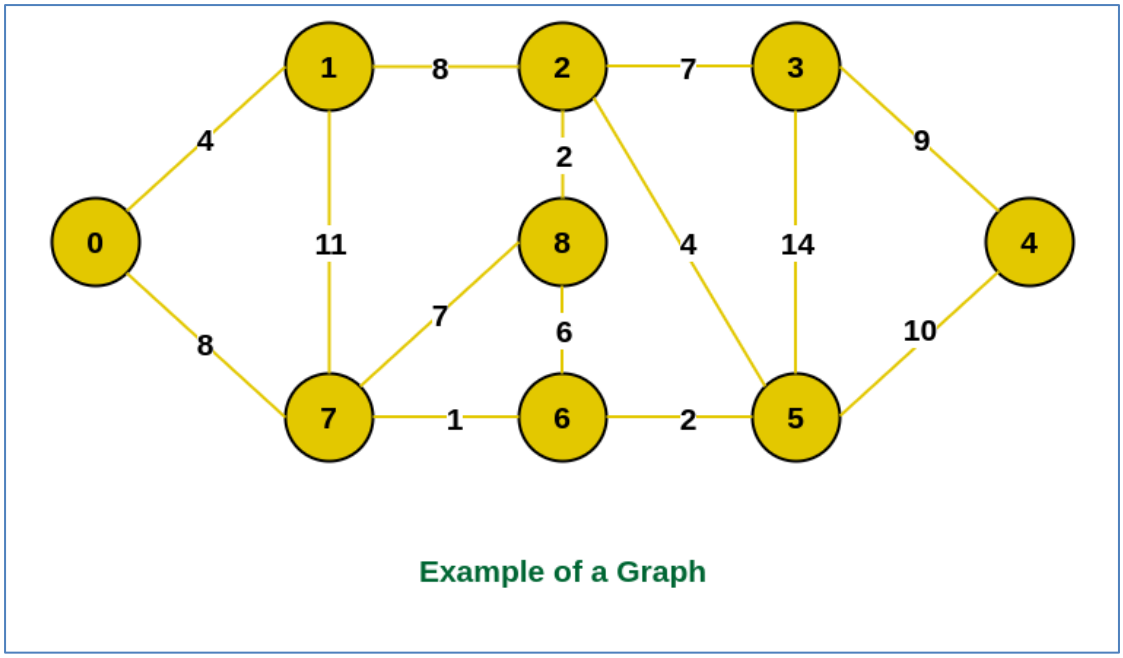
The working of Prim's algorithm can be described by using the following steps:

Step 1: *Determine an arbitrary vertex as the starting vertex of the MST.*
Step 2: *Follow steps 3 to 5 till there are vertices that are not included in the MST (known as fringe vertex).*
Step 3: *Find edges connecting any tree vertex with the fringe vertices.*
Step 4: *Find the minimum among these edges.*
Step 5: *Add the chosen edge to the MST if it does not form any cycle.*
Step 6: *Return the MST and exit*

Note: For determining a cycle, we can divide the vertices into two sets [one set contains the vertices included in MST and the other contains the fringe vertices.]

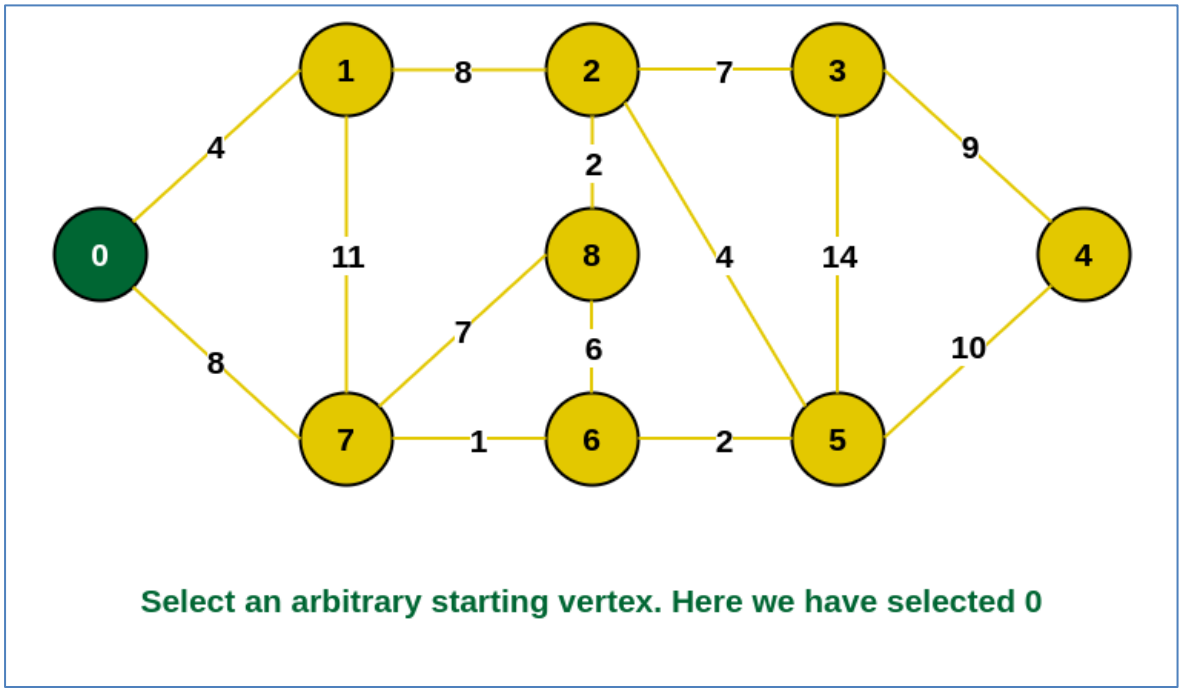
Illustration of Prim's Algorithm:

Consider the following graph as an example for which we need to find the Minimum Spanning Tree (MST).



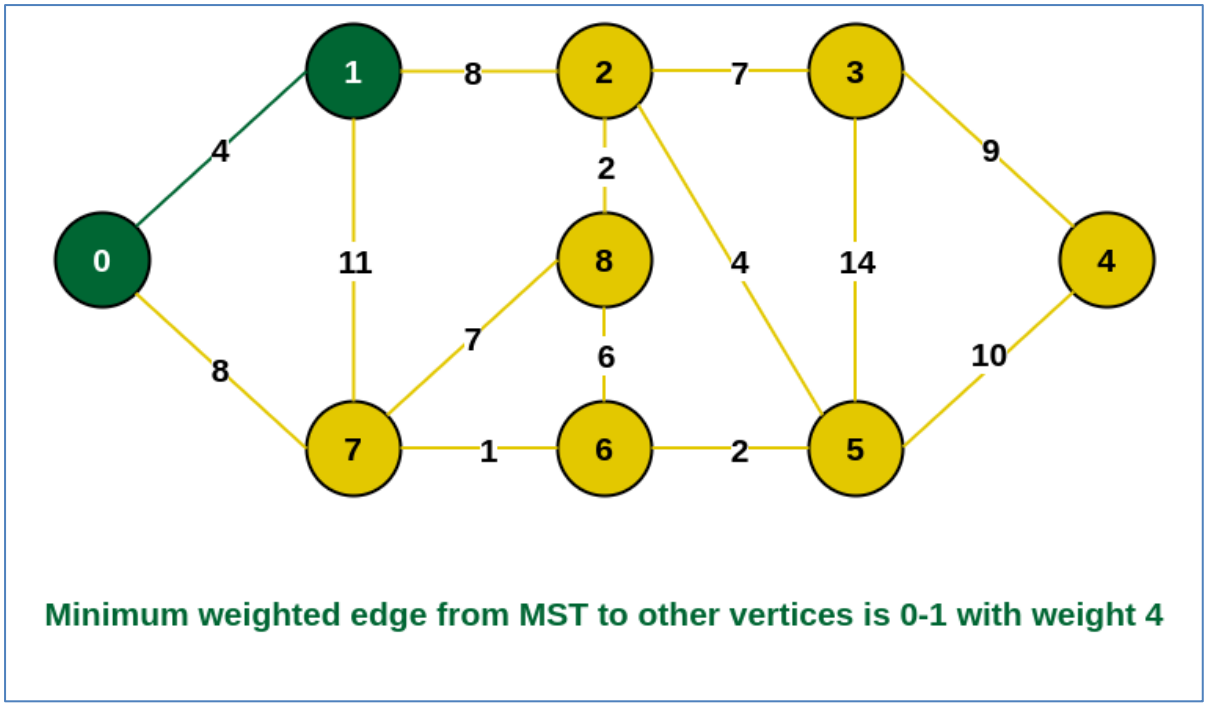
Example of a graph

Step 1: *Firstly, we select an arbitrary vertex that acts as the starting vertex of the Minimum Spanning Tree. Here we have selected vertex* ***0*** *as the starting vertex.*



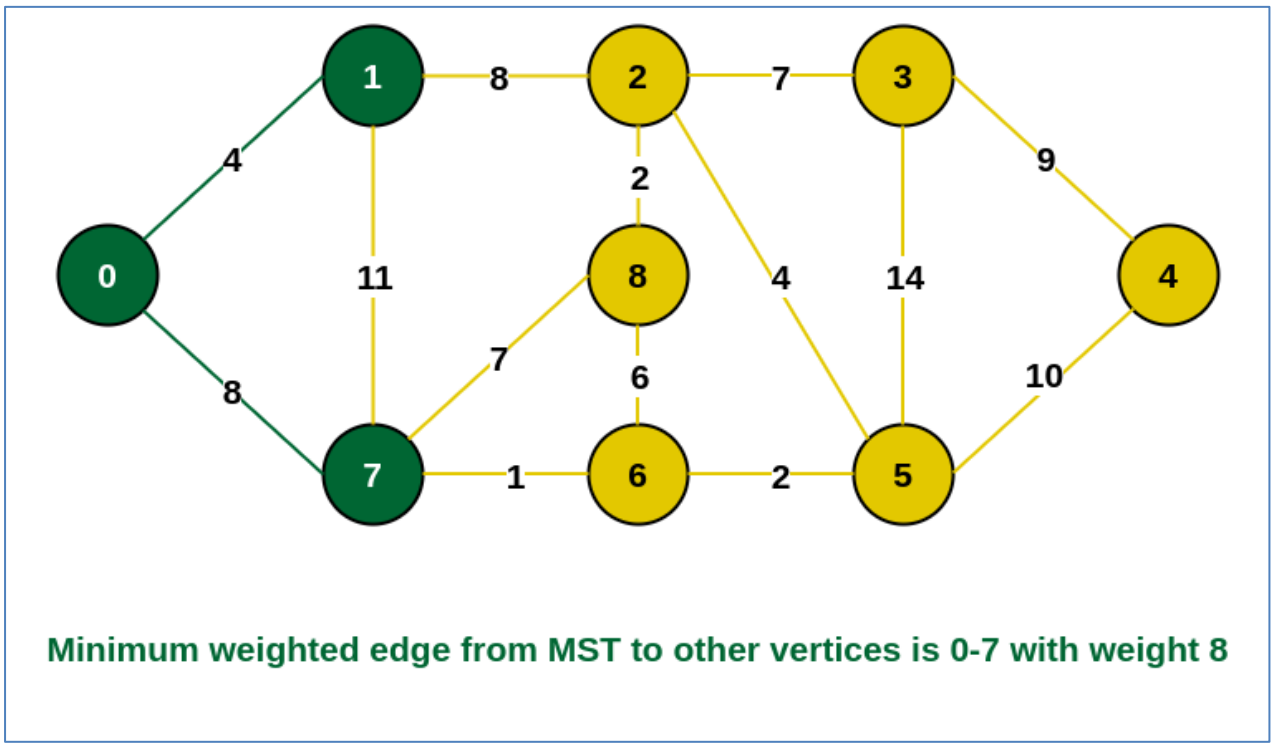
0 is selected as starting vertex

Step 2: *All the edges connecting the incomplete MST and other vertices are the edges {0, 1} and {0, 7}. Between these two the edge with minimum weight is {0, 1}. So include the edge and vertex 1 in the MST.*



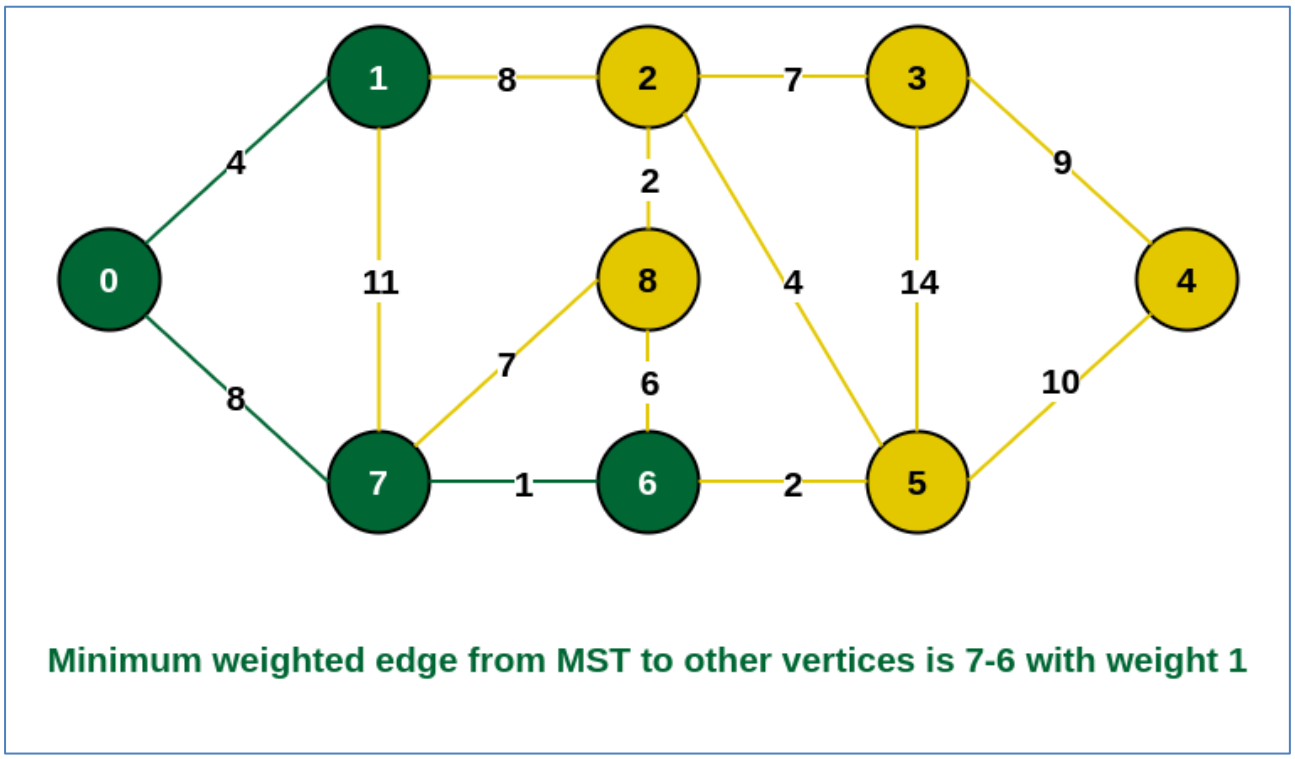
1 is added to the MST

Step 3: *The edges connecting the incomplete MST to other vertices are {0, 7}, {1, 7} and {1, 2}. Among these edges the minimum weight is 8 which is of the edges {0, 7} and {1, 2}. Let us here include the edge {0, 7} and the vertex 7 in the MST. [We could have also included edge {1, 2} and vertex 2 in the MST].*



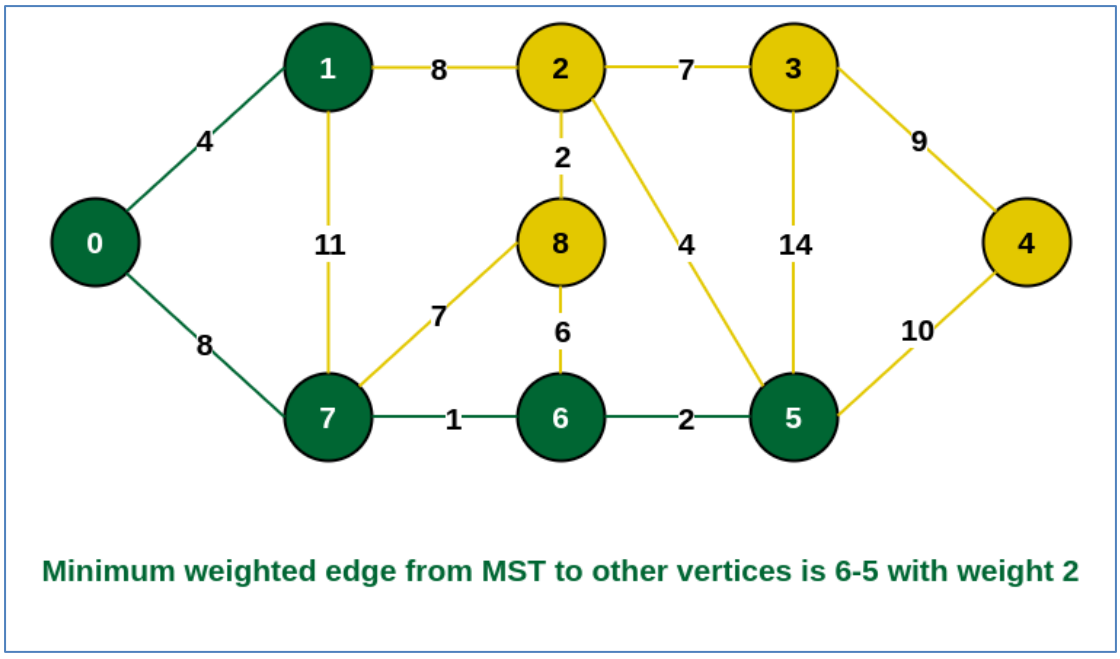
7 is added in the MST

Step 4: *The edges that connect the incomplete MST with the fringe vertices are {1, 2}, {7, 6} and {7, 8}. Add the edge {7, 6} and the vertex 6 in the MST as it has the least weight (i.e., 1).*



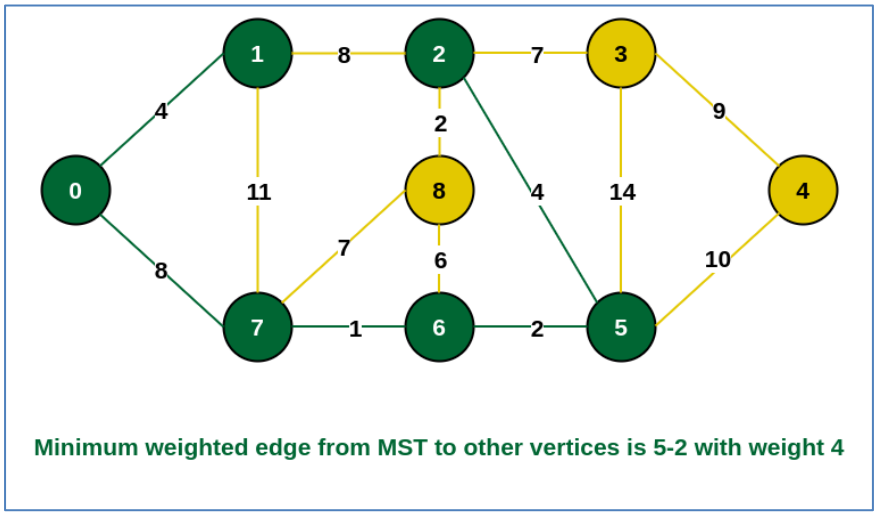
6 is added in the MST

Step 5: *The connecting edges now are {7, 8}, {1, 2}, {6, 8} and {6, 5}. Include edge {6, 5} and vertex 5 in the MST as the edge has the minimum weight (i.e., 2) among them.*



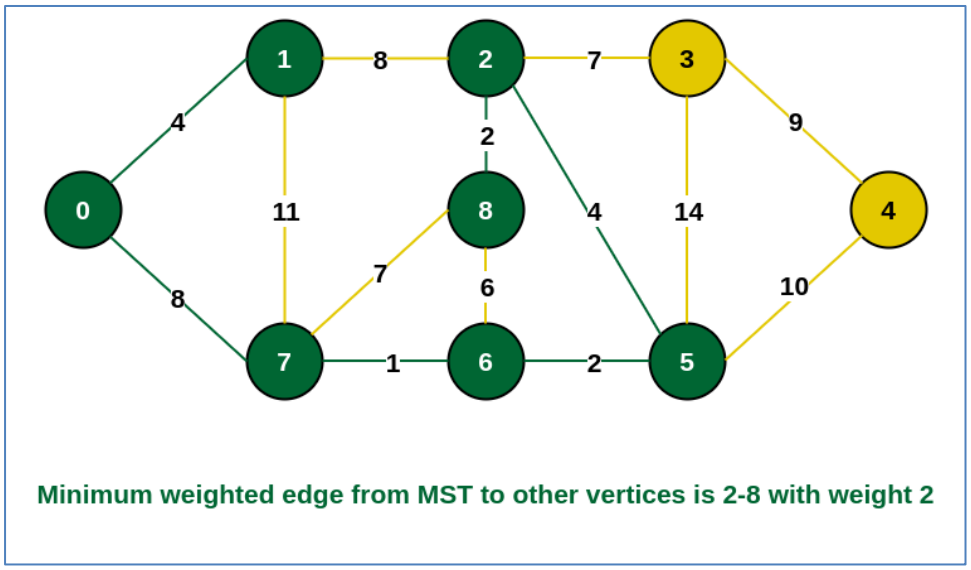
Include vertex 5 in the MST

Step 6: *Among the current connecting edges, the edge {5, 2} has the minimum weight. So include that edge and the vertex 2 in the MST.*



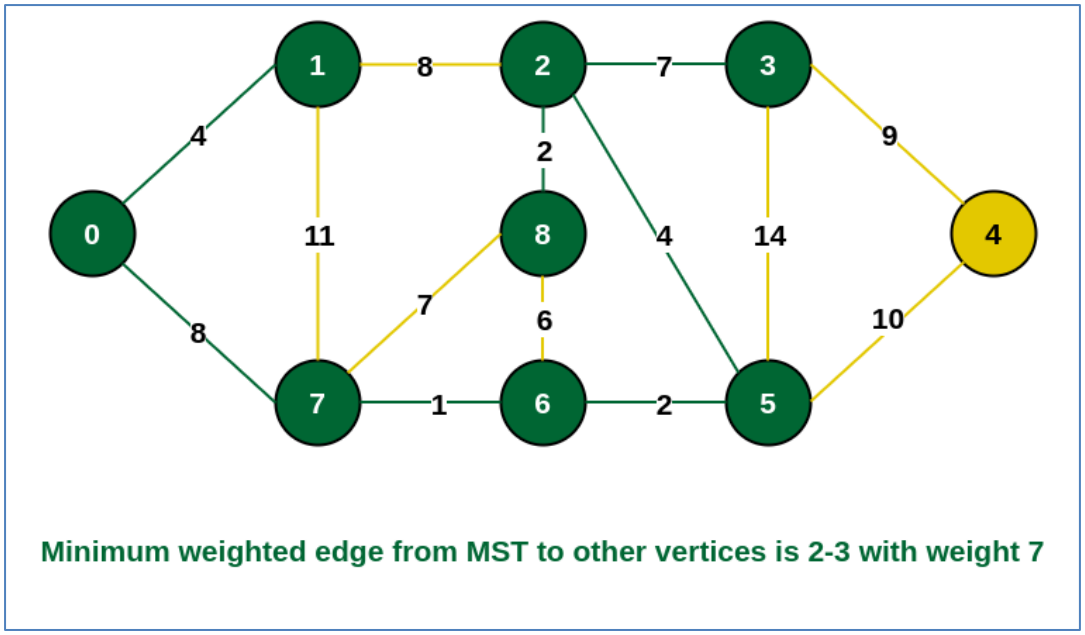
Include vertex 2 in the MST

Step 7: *The connecting edges between the incomplete MST and the other edges are {2, 8}, {2, 3}, {5, 3} and {5, 4}. The edge with minimum weight is edge {2, 8} which has weight 2. So include this edge and the vertex 8 in the MST.*



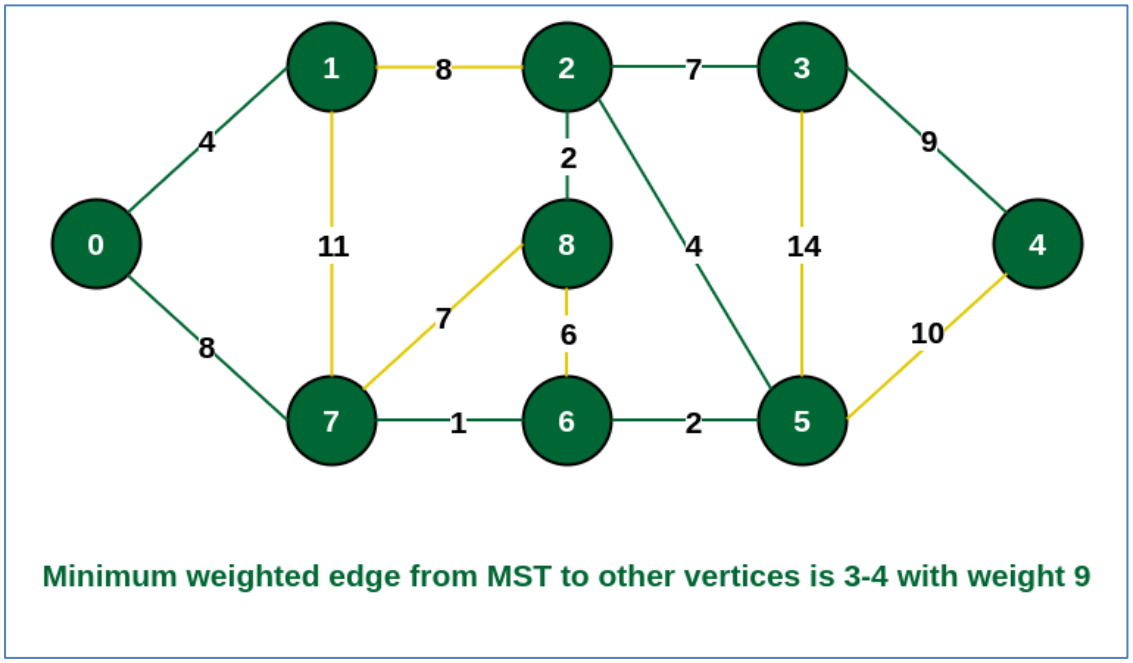
Add vertex 8 in the MST

Step 8: *See here that the edges {7, 8} and {2, 3} both have same weight which are minimum. But 7 is already part of MST. So we will consider the edge {2, 3} and include that edge and vertex 3 in the MST.*



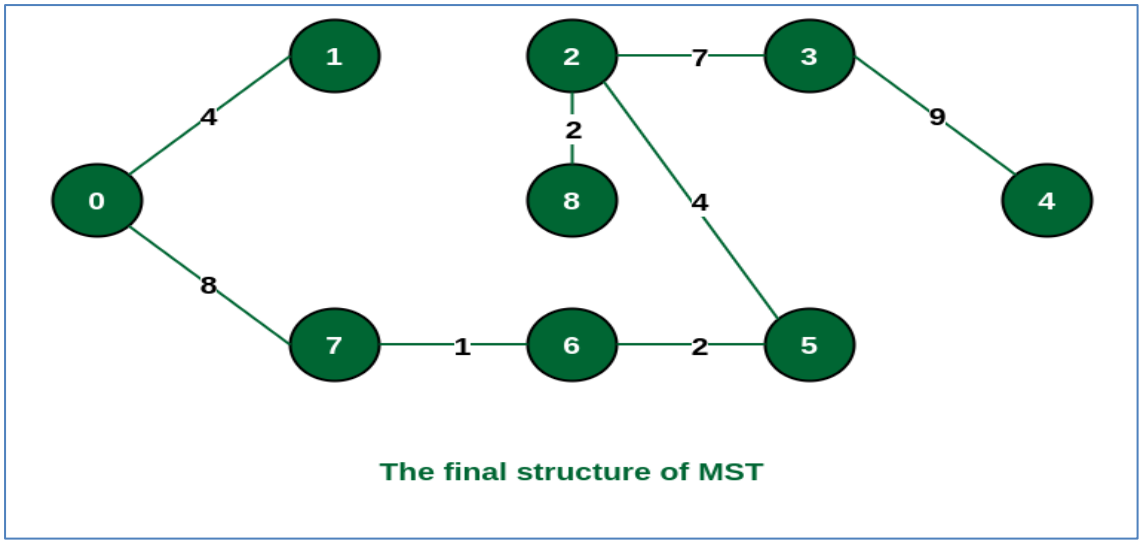
Include vertex 3 in MST

Step 9: *Only the vertex 4 remains to be included. The minimum weighted edge from the incomplete MST to 4 is {3, 4}.*



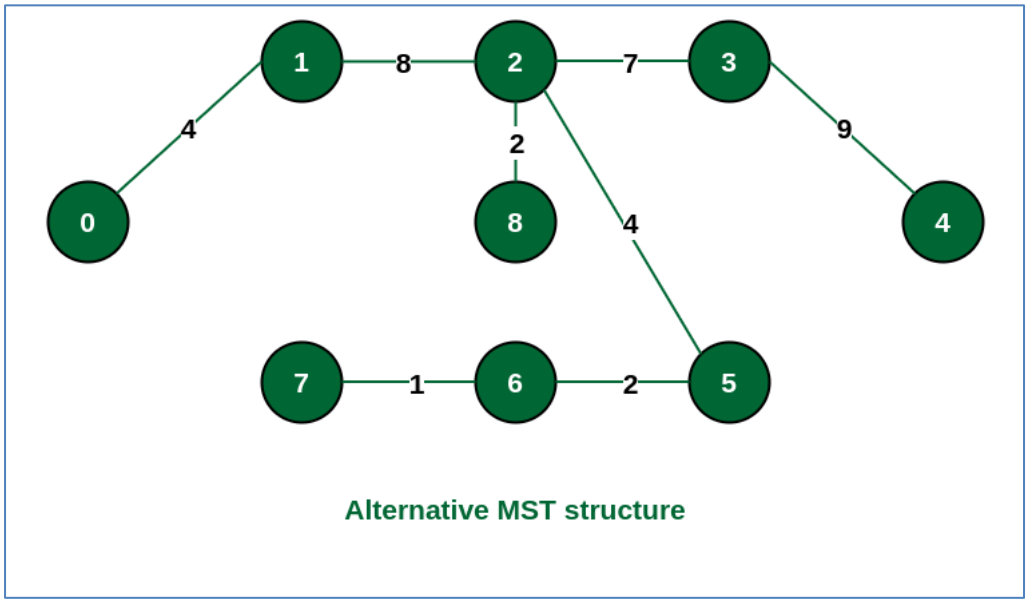
Include vertex 4 in the MST

The final structure of the MST is as follows and the weight of the edges of the MST is (4 + 8 + 1 + 2 + 4 + 2 + 7 + 9) = ***37****.*



The structure of the MST formed using the above method

Note: *If we had selected the edge {1, 2} in the third step then the MST would look like the following.*



Structure of the alternate MST if we had selected edge {1, 2} in the MST

How to implement Prim's Algorithm?

Follow the given steps to utilize the **Prim's Algorithm** mentioned above for finding MST of a graph:

- Create a set **mstSet** that keeps track of vertices already included in MST.
- Assign a key value to all vertices in the input graph. Initialize all key values as INFINITE. Assign the key value as 0 for the first vertex so that it is picked first.

- While **mstSet** doesn't include all vertices
    - Pick a vertex **u** that is not there in **mstSet** and has a minimum key value.
    - Include **u** in the **mstSet**.
    - Update the key value of all adjacent vertices of **u**. To update the key values, iterate through all adjacent vertices.
        - For every adjacent vertex **v**, if the weight of edge **u-v** is less than the previous key value of **v**, update the key value as the weight of **u-v**.

The idea of using key values is to pick the minimum weight edge from the cut. The key values are used only for vertices that are not yet included in MST, the key value for these vertices indicates the minimum weight edges connecting them to the set of vertices included in MST.

Below is the implementation of the approach:

```
# A Python3 program for

# Prim's Minimum Spanning Tree (MST) algorithm.

# The program is for adjacency matrix

# representation of the graph

# Library for INT_MAX

import sys

class Graph():

    def __init__(self, vertices):

        self.V = vertices
```

```
    self.graph = [[0 for column in range(vertices)]
                  for row in range(vertices)]
# A utility function to print
# the constructed MST stored in parent[]
def printMST(self, parent):
    print("Edge \tWeight")
    for i in range(1, self.V):
        print(parent[i], "-", i, "\t", self.graph[i][parent[i]])
# A utility function to find the vertex with
# minimum distance value, from the set of vertices
# not yet included in shortest path tree
def minKey(self, key, mstSet):
    # Initialize min value
    min = sys.maxsize
    for v in range(self.V):
        if key[v] < min and mstSet[v] == False:
            min = key[v]
```

```
            min_index = v

    return min_index

# Function to construct and print MST for a graph

# represented using adjacency matrix representation

def primMST(self):

    # Key values used to pick minimum weight edge in cut

    key = [sys.maxsize] * self.V

    parent = [None] * self.V  # Array to store constructed MST

    # Make key 0 so that this vertex is picked as first vertex

    key[0] = 0

    mstSet = [False] * self.V

    parent[0] = -1  # First node is always the root of

    for cout in range(self.V):

        # Pick the minimum distance vertex from

        # the set of vertices not yet processed.

        # u is always equal to src in first iteration

        u = self.minKey(key, mstSet)
```

```
        # Put the minimum distance vertex in

        # the shortest path tree

        mstSet[u] = True

        # Update dist value of the adjacent vertices

        # of the picked vertex only if the current

        # distance is greater than new distance and

        # the vertex in not in the shortest path tree

        for v in range(self.V):

            # graph[u][v] is non zero only for adjacent vertices of m

            # mstSet[v] is false for vertices not yet included in MST

            # Update the key only if graph[u][v] is smaller than key[v]

            if self.graph[u][v] > 0 and mstSet[v] == False \

            and key[v] > self.graph[u][v]:

                key[v] = self.graph[u][v]

                parent[v] = u

    self.printMST(parent)
```

```
if __name__ == '__main__':

    g = Graph(5)

    g.graph = [[0, 2, 0, 6, 0],

               [2, 0, 3, 8, 5],

               [0, 3, 0, 0, 7],

               [6, 8, 0, 0, 9],

               [0, 5, 7, 9, 0]]

    g.primMST()
```

Output

Edge Weight

0 - 1 2

1 - 2 3

0 - 3 6

1 - 4 5

Time Complexity: $O(V^2)$, If the input graph is represented using an adjacency list, then the time complexity of Prim's algorithm can be reduced to O(E * logV) with the help of a binary heap. In this implementation, we are always considering the spanning tree to start from the root of the graph

Auxiliary Space: O(V)

Other Implementations of Prim's Algorithm:

Given below are some other implementations of Prim's Algorithm

- Prim's Algorithm for Adjacency Matrix Representation – In this article we have discussed the method of implementing Prim's Algorithm if the graph is represented by an adjacency matrix.
- Prim's Algorithm for Adjacency List Representation – In this article Prim's Algorithm implementation is described for graphs represented by an adjacency list.
- Prim's Algorithm using Priority Queue: In this article, we have discussed a time-efficient approach to implement Prim's algorithm.

Prim's algorithm for finding the minimum spanning tree (MST):

Advantages:

1. Prim's algorithm is guaranteed to find the MST in a connected, weighted graph.
2. It has a time complexity of O(E log V) using a binary heap or Fibonacci heap, where E is the number of edges and V is the number of vertices.
3. It is a relatively simple algorithm to understand and implement compared to some other MST algorithms.

Disadvantages:

1. Like Kruskal's algorithm, Prim's algorithm can be slow on dense graphs with many edges, as it requires iterating over all edges at least once.
2. Prim's algorithm relies on a priority queue, which can take up extra memory and slow down the algorithm on very large graphs.
3. The choice of starting node can affect the MST output, which may not be desirable in some applications.