**School of Science and Technology, Singapore**
Mathematics Department
Secondary One

**Name:** ______________________ **Index No:** ________ **Class: Sec 1 -** ________

**Unit 4: Basic Algebra and Algebraic Manipulation** **Date:** ____________

**Topical EU**
- Algebra is the language upon which the world can be modelled mathematically.
- Algebra involves using symbols to represent relationships.

**Topical EQ**
- What is algebra and its role?
- How does algebra explain and predict relationships?

**At the end of the lesson, students should be able to**
- differentiate between like terms and unlike algebraic terms
- add or subtract algebraic expressions
- simplify linear algebraic expressions
- expand and simplify algebraic expressions
- factorise an expression by common factors
- factorise an expression by grouping terms

**Big Ideas: Notation and Equivalence**

**Key Points (Learning Outcomes)**
- Algebraic Manipulation Rules

**Difficult Point**
- Transition from Model Method to Algebraic Representation

**Critical Point**
- Recognising the same rules for manipulating algebraic expressions apply to numbers

## History of Algebra - Who invented Algebra? Why was Algebra invented?

Algebra was invented by Mohammed ibn-Musa al-Khwarizmi. (Around 800 A.D. to 847 A.D.). He was a mathematician and astronomer who wrote many books on arithmetic and algebra. His most significant work is a book entitled “al-Kitab al-mukhtasar fi hisab al-jabr wa'l-muqabala” or "The Compendious Book on Calculation By Completion and Balancing" in 830 B.C.. This was his first book that began the discussion on algebraic equations in the first and second degree. The word “al-jabr” meant “restore” or “complete”. The word “muqabalah” refers to "reduce" or "balance". Thus, it was through the title of this book “Al-jabr” that led to the origin of the word “algebra”.

Through the use of algebra, Al-Khwarizmi derived methods to solve quadratic equations in a simple, easy and systematic way. His methods became popular and his book was used in the universities in Europe until the 16th century.

**Teaching to the Big Idea.**

| Lesson sequence in the unit | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Student Learning Outcomes | Dimensions (Please tick the appropriate boxes) | | | | | | | |
| | FUNCTIONS<br>F | INVARIANCE<br>I | NOTATIONS<br>N | DIAGRAMS<br>D | MEASURES<br>M | EQUIVALENCE<br>E | PROPORTIONALITY<br>P | MODELS<br>M |
| Introducing the Algebraic variable | | | ✓ | | | ✓ | | |
| Writing Algebraic expressions | | | ✓ | | | ✓ | | |
| Like, Unlike terms, co-efficients and constant terms. | | | ✓ | | | ✓ | | |
| Addition and Subtraction of algebraic terms and expressions | | | ✓ | | | ✓ | | |
| Multiplication and Division of algebraic terms and expressions | | | ✓ | | | ✓ | | |
| Distributive Law and Algebraic Identities. | | | ✓ | | | ✓ | | |
| Factorisation of algebraic expressions. | | | ✓ | | | ✓ | | |

**Unit Checklist:**

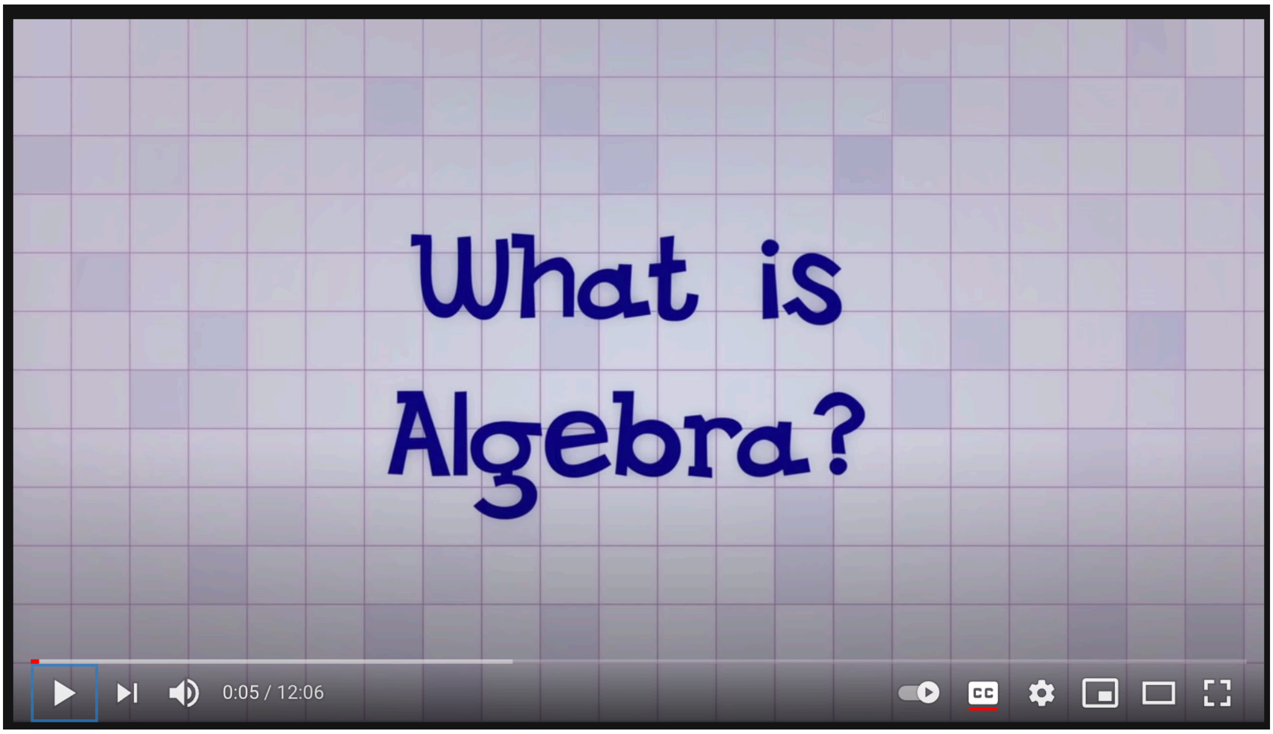
| Cognitive Level | Know, Understand, Demonstrate | Checklist |
|---|---|---|
| Level 0: Memorisation | Explain what is a variable, co-efficient and identify algebraic and constant terms. | |
| Level 1: Procedural tasks without connections | Apply the four operations for algebraic expressions | |
| Level 2: Procedural tasks with connections | Apply the distributive law of expansion, algebraic identities into algebraic expansion and factorization. | |
| Level 3: Problem Solving | Form algebraic expressions/equations* and resolve them to solve unknowns. | |

**Introduction to Algebra.**

To give us an introduction to how Algebra is an extension of what we are already doing in terms of numerical expressions and equations, let us start with this video.

Watch the Youtube video "What is Algebra?" linked here:

https://www.youtube.com/watch?v=NybHckSEQBI




After watching the video, are you able to link the usage of variables in Algebra to one of our Big Ideas in Mathematics?

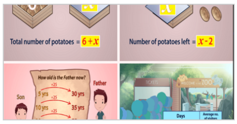
Big Idea:

## 4.1 Introducing Algebra and the Variable

**Definition:**

**Algebra** is a branch of mathematics in which general properties of numbers are studied by using symbols, usually letters, to represent variables and unknown quantities.

Activity 1: Watch the video in ‘4.1 Part I - Algebra and Letters’ in SLS.




After watching the video, what can you share about the use of Algebra:

***Discussion:***

(a) ♣ + 5 = 12

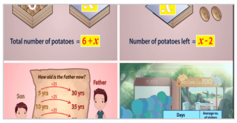
(b) 5 × ♠ + 3 × ♠ = 4

(c) ♦ ÷ ♥ = 10

What numbers could ♣, ♦, ♥ and ♠ be?

## 4.1 Basic Algebraic Concepts & Notations: Writing Algebraic Equations

Activity 2: Watch the video in ‘4.1 Part II – Writing Algebraic Expressions in SLS and complete the accompanying questions.



| Practice: Write an algebraic expression for each of the following: | | | |
|---|---|---|---|
| (a) | the sum of the square root of $x$ and the cube of $y$. | (b) | the quotient of the sum of $xy$ and 5 when divided by $z$ is $c$, where $z \neq 0$. |
| (c) | the product of eleven times the difference between $3x$ and the cube root of $y$. | (d) | Adam’s mother is thrice as old as him. Adam’s sister is 4 years older than him. What is the sum of their ages if $y$ represents Adam’s age? |

## Evaluating Algebraic Expressions

| Complete the following by showing your workings clearly. | |
|---|---|
| 1. If $p = 3$, $q = 2$ and $r = -4$, find the value of<br><br>(a) $p^2 - q^2 + r^3$<br><br>(b) $\dfrac{2+p}{3} - \dfrac{5+q}{4} + \dfrac{r-1}{2}$ | 2. If $a = \dfrac{1}{12}$, $b = -\dfrac{3}{2}$ and $c = 25$, find the value of<br><br>(a) $3ab\sqrt{c}$<br><br>(b) $\dfrac{1}{a} - \dfrac{b}{2} + 2c$ |

## 4.1 Definitions – Like and Unlike Terms

1. A collection of **Algebraic Terms** that are connected by the signs '+', '–', '×' or '÷' makes up an **Algebraic Expression**.

   Example: The algebraic expression $2x+4y-5$ has **3** terms; $2x$, $4y$ and 5.

2. In the term $7y$, the numerical part 7 is called the **coefficient** of $y$.
   (This means that the number that is in front of the variable or a group of variables is called the coefficient.)

   Example: In the term $9p$, **9** is the coefficient of $p$.
   In the term $x$, **1** is the coefficient of $x$.
   In the term $-3w^2$, -3 is the coefficient of $w^2$.
   In the term $\frac{1}{2}xy$, $\frac{1}{2}$ is the coefficient of $xy$.

3. Any algebraic term that does not have a variable attached to it is called a **constant**.

   Example: In the algebraic expression, $11x+2$, the constant term is 2.

4. Algebraic terms that have the same variables where each variable has the same power are called **like terms**.

   Example: The algebraic terms $5x$ and $9x$ are like terms because they have <u>the same variable</u> and the <u>variable has the same power</u>.

5. If two terms are not like terms, then they are called **unlike terms**.
   Example: The algebraic terms $5x$ and $9x^2$ are unlike terms because even though they share the same variable, the powers are different.

6. **We collect all the like terms together to simplify an algebraic expression. *Only like terms can be added or subtracted.***

## 4.1 Coefficient, Constant & Terms

***Exercise*:**

Write down the coefficient of $x$, the constant term and the number of Algebraic Terms in each of the following.

| Algebraic Expression | Coefficient of $x^2$ | Coefficient of $x$ | Constant Term | Number of Algebraic Terms |
|---|---|---|---|---|
| (a) $7x^2+2x-3$ | 7 | 2 | -3 | 3 |
| (b) $5x-11x^2+2$ | | | | |
| (c) $9x+x$ | | | | |

**Activity:**
*Adapted from Think! Mathematics New Syllabus Mathematics 1A (8th Edition)*

Complete the table below on practice on expressing mathematical operations and simple real-world situations using algebraic expressions.

| | In words | Algebraic expression |
|---|---|---|
| **(a)** | Sum of $2x$ and $3z$ | |
| **(b)** | Product of $x$ and $7y$ | |
| **(c)** | Divide $3ab$ by $2c$ | |
| **(d)** | Subtract $6q$ from $10z$ | |
| **(e)** | | $(p + q) - xy$ |
| **(f)** | | $\frac{3+y}{5}$ |
| **(g)** | | $\sqrt{b} - 2c$ |
| **(h)** | There are three times as many girls as boys in a school. Find an expression, in terms of $x$, for the total number of students in the school, where $x$ represents the number of boys in the school. | It is given that $x$ represents the number of boys.<br>$\therefore$ represents the number of girls.<br>Total number of students = |
| **(i)** | Nadia's father is three times as old as her. Nadia's brother is 5 years older than her. Find an expression, in terms of $y$, for the sum of their ages, where $y$ represents Nadia's age. | It is given that $y$ represents Nadia's age.<br>$\therefore$ Nadia's father is years old.<br>Nadia's brother is years old.<br>Sum of their ages = years |
| **(j)** | The is times as long as the of the rectangle. Find an expression, in terms of $b$, for the perimeter and the area of the rectangle, where $b$ represents the breadth of the rectangle. | It is given that $b$ represents the breadth of the rectangle in m.<br>$3b$ represents the length of the rectangle in m.<br>Perimeter of the rectangle = m<br>Area of the rectangle = $m^2$ |

***Extension:***
*(Adapted from MOE Syllabus Guide).*

Think of three different algebraic expressions and get your classmate to interpret the mathematical relationships in each of them.

Share some of them with the class?

## 4.2 Addition and Subtraction of Like and Unlike Algebraic Terms

***Discuss the difference between "Like Terms" and "Unlike Terms" (refer to Textbook 1A)***

**Like terms** ______________________________________________

Simplify the following by collecting like terms.

| | |
|---|---|
| (a) $2m+3p+4m-6p+8m$ | (b) $6k^2-k^2-6$ |
| (c) $abc+2ab+3abc-6ab+8a+b+7c$ | (d) $2m^3+p^3+3p^3+3m^3+m^3p^3$ |
| (e) $27xy+\frac{1}{2}xz-8yx-\frac{1}{2}zx$ | *(f) $(3ab)^2+3a^2b^2+ab+3$ |

## 4.2 Linear Algebraic Expressions. (Addition and Subtraction)

Note the two examples below to show how we can add and subtract algebraic expressions.

***Example 1***
Simplify $(3x+5y)+(5x-3y)$

***Example 2***
Simplify $(3x+5y)-(5x-3y)$

***Exercise 1*:**
Simplify each of the following.

| | |
|---|---|
| (a) $(3x-5y)-(5x+2y)$ | (b) $7x-(2x-3y+4)$ |
| (c) $(-11a-13b)-(-3a+4b)$ | (d) $\left(x-\frac{1}{3}+2y\right)+\left(2x-5y-\frac{1}{4}\right)$ |
| * (e) $x-[y-3(2x-y)]$ | |

***Optional higher ability questions.**

## 4.2 Four Operations of Like and Unlike Algebraic Terms involving Fractions

**ADDITION & SUBTRACTION**

Simplify the following into a single fraction.

| | |
|---|---|
| (a) $\frac{x}{10}+\frac{3x}{2}+2$ | (b) $\frac{5y}{2}+\frac{y}{3}-\frac{2y}{6}$ |
| (c) $\frac{3}{x}+\frac{4}{x}-\frac{2}{x}$ | * (d) $\frac{2}{7x}+\frac{3}{3x}+\frac{1}{21}$ |

**Optional higher ability questions.*

**MULTIPLICATION & DIVISION**

Simplify the following algebraic fractions.

| | |
|---|---|
| (a) $\dfrac{3a}{2} \times \dfrac{12}{9a}$ | (b) $\dfrac{3mn}{2a} \times \dfrac{4a^2}{2n} \times \dfrac{m}{n}$ |
| (c) $\dfrac{ab}{27} \div \dfrac{a^2b}{3}$ | (d) $\dfrac{2}{3}x^3 \times \dfrac{4x}{5y}$ |
| *(e) $\sqrt{49x^2y^4} \div \dfrac{7}{2xy}$ | *(f) $\dfrac{\sqrt{4a^2}}{3} \times \dfrac{3}{(2a)^2}$ |

**Optional higher ability questions.*

## Assignment 4A

**Think! Mathematics New Syllabus Mathematics 1A (8$^{th}$ Edition) Chapter 4**

Exercise 4A (page 104)

Basic Tier:
Question: 2a, 3b, 5.

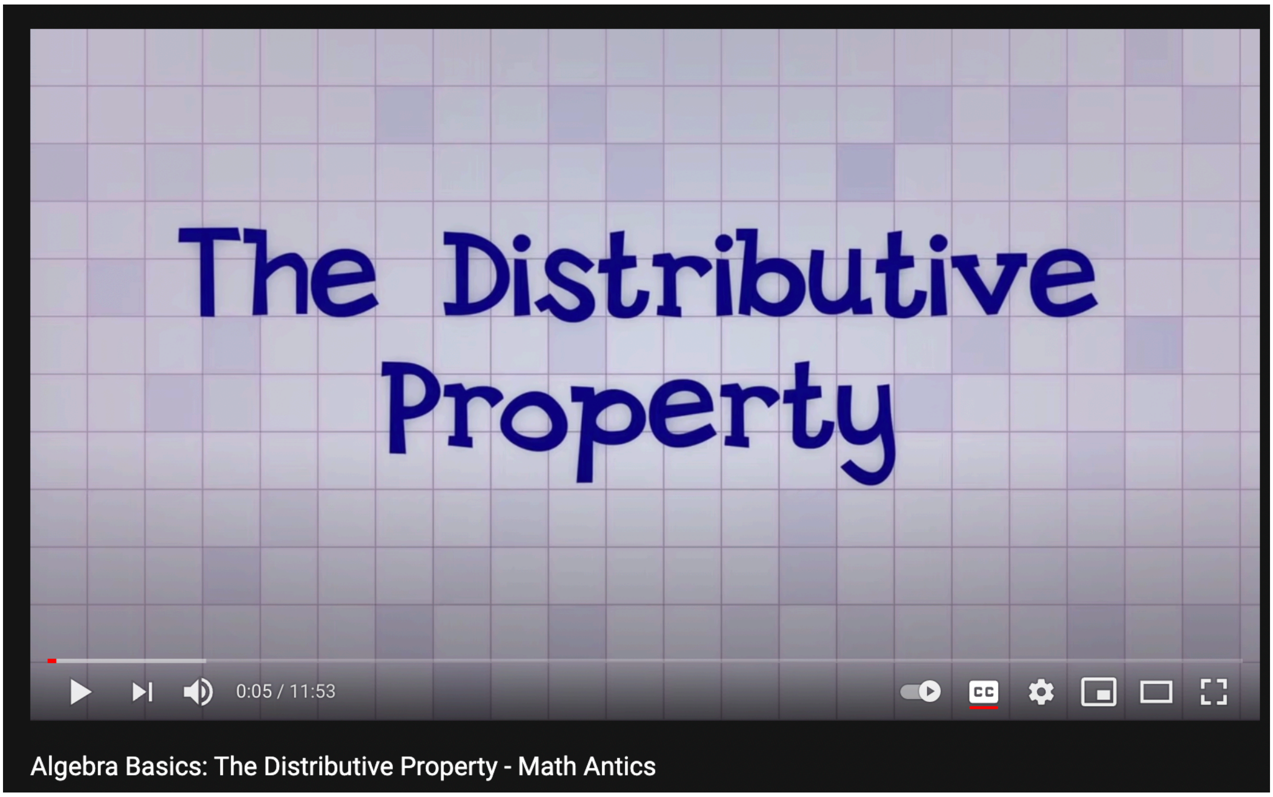
Intermediate Tier:
Question: 9a, 9g, 9d, 10c, 10e, 13.

Advanced Tier:
Question 14

## 4.3 Distributive Law

**Distributive Law of Algebra**

**Watch the Youtube Video at: https://www.youtube.com/watch?v=v-6MShC82ow**



**[OPTIONAL] Investigation** - *Adapted from Think! Mathematics New Syllabus Mathematics 1A (8th Edition)*

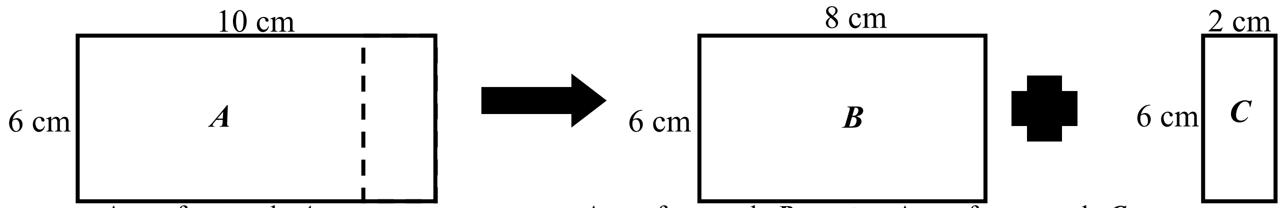
1. Evaluate the following:

   a. $5 \times (2+9)$
   $= 5\ (11)$
   $= 55$

   b. $5 \times 2 + 5 \times 9$
   $= 10 + 45$
   $= 55$

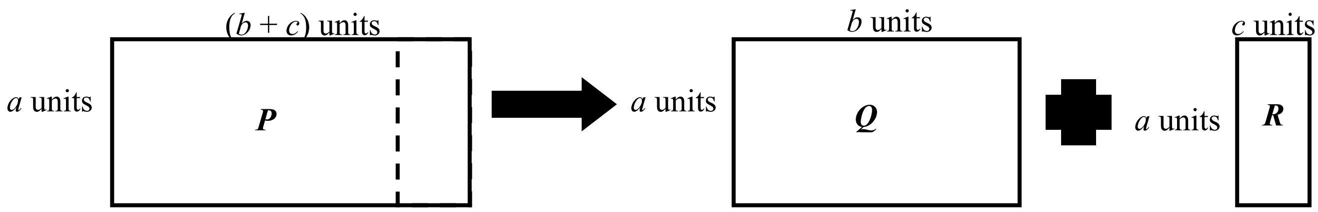
   Therefore $5 \times (2+9) = 5 \times 2 + 5 \times 9$

2. A rectangle ***A*** with sides of length 6 cm and 10 cm is shown below. The rectangle *A* is partitioned into 2 rectangles labelled ***B*** and ***C***.




Area of rectangle ***A*** = Area of rectangle ***B*** + Area of rectangle ***C***
= 6 (8) + 6 (2)
= 6 ( 8 + 2 ) = 60 square units

3. Here is a rectangle ***P*** with sides of length *a* units and (*b* + *c*) units. Similarly, it is partitioned into 2 rectangles ***Q*** and ***R*** as shown below.




Area of rectangle ***P*** = Area of rectangle ***Q*** + Area of rectangle ***R***
$= \boldsymbol{a(b)} + \boldsymbol{a(c)}$
$= \boldsymbol{a(\,b + c\,)}$

The reverse is also TRUE

**Distributive Law**

**1.** $a(x+y) = xa + ya$ **(Multiplication can be distributed over addition from the right.)**
$= ax + ay$

**2.** $a(x-y) = ax - ay$ **(Multiplication can be distributed over subtraction.)**

**3.** $a(x+y+z) = ax + ay + az$ **(Multiplication can be distributed over several terms.)**

***Exercise 2*:**

Simplify each of the following.

| | |
|---|---|
| 1. $11(3a-2p)$ | 2. $3(5a+\frac{1}{3}b)$ |
| 3. $2+x-\frac{x-3}{4}$ | 4. $\frac{x-4}{7}+\frac{x+3}{3}$ |

## 4.3 Expansion of Linear Algebraic Expression

*Think….*
How many terms would you expect to get when you multiply two terms together?

$(x+2)(x+3)=$

***Example***
Expand and simplify the following:

| | |
|---|---|
| 1. $(x-2)(x+4)$ | 2. $(2a-9)(3a-5)$ |
| *3. $(2x-1)(1+3x)(x+1)$ | 4. $(x+3)^2$ |
| 5. $(x-3)^2$ | 6. $(x+3)(x-3)$ |

## 4.3 Special Products of Algebraic Expressions (involving binomials)

| *Perfect Squares* | | *"Difference of Squares"* |
|---|---|---|
| *"Sum Squared"* | *"Difference Squared"* | |
| $(a+b)^2$<br>$= (a+b)(a+b)$<br>$= a(a+b)+b(a+b)$<br>$= a^2+ab+ba+b^2$<br>$= a^2+2ab+b^2$ | $(a-b)^2$<br>$= (a-b)(a-b)$<br>$= a(a-b)+b(a-b)$<br>$= a^2-ab-ba+b^2$<br>$= a^2-2ab+b^2$ | $(a+b)(a-b)$<br>$= (a+b)(a-b)$<br>$= a(a-b)+b(a-b)$<br>$= a^2-ab+ba-b^2$<br>$= a^2-b^2$ |

**What do you notice about the 3 expressions?*
*What are the similarities and differences?*

*Think…*
Why are squares numbers called "perfect squares"?

***Examples***

Using the above relationships write down the expansions of the following?

| | |
|---|---|
| 1. $(2+x)^2$ | 2. $(x-3)^2$ |
| 3. $(2+x)(2-x)$ | 4. $(3x+y)^2$ |
| 5. $(5a-2b)^2$ | 6. $(3x-2y)(3x+2y)$ |
| *7. $\left(\frac{1}{6}x+2\right)^2$ | *8. $\left(\frac{1}{4}x-\frac{y}{3}\right)^2$ |

**Optional higher ability questions.*

## 4.3 What is factorisation?

**<u>Factorisation</u>** is the process of writing an algebraic expression as a product of two or more other algebraic expressions.

There are many ways to factorise an algebraic expression.
(i) Grouping common factors
(ii) Special Algebraic Identities
(iii) Grouping terms
(iv) Cross multiplication

> **Important….**
> Always try to identify and take out common factors first. Sometimes you may need to apply more than one method to factorise an expression.

## 4.3 Factorisation by grouping common factors (method 1)

From **5.3**, we know that the <u>Distributive Law of Algebra</u> can be used to <u>expand</u> algebraic expressions.

$$a(b+c) = ab + ac$$

$$a(b-c) = ab - ac$$

When we <u>factorise</u> the algebraic expressions $ab + ac$ and $ab - ac$, we write them as a <u>product of its factors</u>.

$$ab + ac = a(b+c)$$

$$ab - ac = a(b-c)$$


Use this pictorial representation to aid your understanding.

> Imagine….
> ■★ + ■⊙ = ■(★ + ⊙)

***Example*** **:** Identification of common factor

| Algebraic Expression | Common Factors | Factorisation - Writing the algebraic expression as a product of its factors |
|---|---|---|
| (i) $ab + ac$ | $a$ | $ab + ac = a(b+c)$ |
| (ii) $ab - ac$ | | $ab - ac =$ |
| (iii) $3x + 12$ | | $3x + 12 =$ |
| (iv) $4x^2y + 16x^2$ | | $4x^2y + 16x^2 =$ |
| (v) $7cd - 21d + 14$ | | $7cd - 21d + 14 =$ |
| (vi) $2a^2bc - 4ab^2c + 10abc^2$ | | $2a^2bc - 4ab^2c + 10abc^2 =$ |
| (v) $2(x+3) + y(x+3)$ | | $2(x+3) + y(x+3) =$ |

## 4.3 Special Algebraic Identities **(method 2)**

Since factorization is the reverse of expansion, we have:

- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^2 - b^2 = (a+b)(a-b)$

***Examples***

Factorise the following:

| | |
|---|---|
| 1. $x^2 - 12x + 36$ | 2. $9b^2 + 12b + 4$ |
| 3. $4c^2 + 4cd + d^2$ | 4. $a^2 - 18a + 81$ |
| 5. $a^2 - 16$ | 6. $9c^2 - 6cd + d^2$ |
| 7. $(t^2 - 1)^2 - 9$ | 8. $4c^3 - 49c$ |
| *9. $9p^2 - 4(p - 2q)^2$ | *10. $\frac{1}{4}g^2 - \frac{1}{4}gh + \frac{1}{16}h^2$ |

**Optional higher ability questions.*

## 4.3 Factorisation by grouping terms (method 3)

**Question:** Factorise $ax + ay + bx + by$

**Step 1:** Factorise $ax + ay = a(x+y)$

**Step 2:** Factorise $bx + by = b(x+y)$

**Step 3:** Hence, factorise $ax + ay + bx + by$

$$= a(x+y) + b(x+y)$$

$$= (x+y)(a+b)$$

***Example*:**

Factorise the following:

| | |
|---|---|
| 1. $ab - ac + 6b - 6c$ | 2. $ax + bx - ay - by$ |
| 3. $8gx + 21hy + 6hx + 28gy$ | 4. $3a - 6b + 36bp - 18ap$ |

*Recap: to see this factorization method in action, visit:*

http://www.youtube.com/watch?v=KFWXSjPn87I

## 4.3 Factorisation of $ax^2 + bx + c$ (method 4)

An expression in the form $ax^2 + bx + c$ where the highest power of the variable $x$ is 2; $a$, $b$ and $c$ are real numbers and $a \neq 0$, is called a ***quadratic expression*** in $x$.

Which of the following are quadratic expressions? (Put a tick in the bracket next to it.)

| | | | | | |
|---|---|---|---|---|---|
| (a) | $5 + x^2 + 4x$ | (   ) | (d) | $3x - 8$ | (   ) |
| (b) | $2x + 3x^3 + 7$ | (   ) | (e) | $6x^2 - 2 - 7x$ | (   ) |
| (c) | $4x^2 - 7x$ | (   ) | (f) | $x^{0.5} + 1$ | (   ) |

**<u>Factorisation Using Cross Method:</u>**

The "Cross" method, commonly used in the factorisation of quadratic expressions (in the form $ax^2 + bx + c$), is a "trial and error" method.

**Step 1:** Find the factors of the term in $x^2$.
**Step 2:** Find the factors of the constant term.
**Step 3:** Place the term in $x^2$ and the constant term into their respective positions.
**Step 4:** Through trial and error, place the factors in various positions within their respective columns and cross-multiply to see which combination's result is equal to the term in $x$.
**Step 5:** Check the answer.

| | | |
|---|---|---|
| $x$ | $-2$ | $-2x$ |
| $x$ | $+3$ | $+3x$ |
| $x^2$ | $-6$ | $+x$ |

$x^2 + x - 6 = (x - 2)(x + 3)$

***<u>Examples</u>***

| | |
|---|---|
| 1. $x^2 - 5x + 6$ | 2. $15x^2 + 2x - 1$ |
| 3. $px^2 - 4xp - 21p$ | *4. $2x^2y^2 + 5xy - 12$ |

**Optional higher ability questions.*

## Assignment 4B

**Think! Mathematics New Syllabus Mathematics 1A (8th Edition) Chapter 4**

Exercise 4B (page 111)

Basic Tier:
Question: 3b, 3d, 5c, 5d.

Intermediate Tier:
Question: 8b, 8c, 8f, 8g, 8h

Advanced Tier:
Question 10a, 10c, 11a, 11c, 11f

Exercise 4C (page 116)

Basic Tier:
Question: 1b, 1d

Intermediate Tier:
Question: 3a, 3g, 3h, 5b, 5g, 5i,

Advanced Tier:
Question 6