

Assignment 2

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Abstract—This document explains the concepts of Matrix transpose, Matrix Equality by solving a problem.

Download the python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_2/assignment_2.py

and latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_2/assignment_2.tex

1 PROBLEM

If $\mathbf{A} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$, and $\mathbf{A} + \mathbf{A}^T = \mathbf{I}$, then find the value of angle α .

2 EXPLANATION

- * The Complex number equivalent to the matrix $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is $\begin{pmatrix} a \\ b \end{pmatrix}$ or $a + ib$
- * So, If $\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix}$, then transpose of $\mathbf{A} = \begin{pmatrix} a \\ -b \end{pmatrix}$
- * And addition of \mathbf{A} with \mathbf{A}^T results in :

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} a \\ -b \end{pmatrix} = \begin{pmatrix} 2a \\ 0 \end{pmatrix} \quad (2.0.1)$$

Or

$$(a + ib) + (a - ib) = 2a \quad (2.0.2)$$

3 SOLUTION

So, According to the given question $\mathbf{A} + \mathbf{A}^T$ is :

$$\begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} + \begin{pmatrix} \cos \alpha \\ -\sin \alpha \end{pmatrix} = \begin{pmatrix} 2\cos \alpha \\ 0 \end{pmatrix} \quad (3.0.1)$$

Given that $\mathbf{A} + \mathbf{A}^T = \mathbf{I}$:

$$\begin{pmatrix} 2\cos \alpha \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3.0.2)$$

That Implies,

$$2\cos \alpha = 1 \implies \cos \alpha = \frac{1}{2} \quad (3.0.3)$$

As per the cosine values, the angle α is :

$$\alpha = \frac{\pi}{3} = 1.047 \quad (3.0.4)$$

The cosine function is plotted along with the point $(x, \cos(x)) = (1.047, 0.5)$ as shown below:

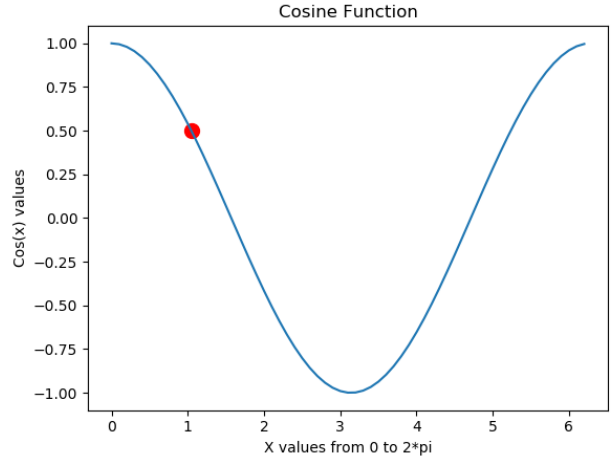


Fig. 0: Cosine Function