Assignment 3

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***Abstract*—This document explains the concepts of Matrix multiplication, Matrix Addition and Matrix Inverse by solving a problem.**

Download the python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_3/assignment_3.py

and latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_3/assignment_3.tex

1 Problem

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

Show that $\mathbf{A}^2$ - 5$\mathbf{A}$ + 7$\mathbf{I}$ = 0. Hence find $\mathbf{A}^{-1}$.

2 Explanation

Square of a matrix is the product of matrix with itself :

$$\mathbf{A}^2 = \mathbf{A}.\mathbf{A} \tag{2.0.1}$$

Product of a scalar with a matrix is the product of that scalar with every element of the matrix :

$$k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix} \tag{2.0.2}$$

Inverse of a matrix $\mathbf{A}$ is defined as :

$$\mathbf{A}.\mathbf{A}^{-1} = \mathbf{I} \tag{2.0.3}$$

3 Solution

Square of the given matrix A is :

$$\mathbf{A}^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} \tag{3.0.1}$$

$\mathbf{A}^2$ - 5$\mathbf{A}$ + 7$\mathbf{I}$:

$$\begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.0.2}$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \tag{3.0.3}$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{3.0.4}$$

Thus, It is proved that

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = 0. \tag{3.0.5}$$

Multiplying equation (3.0.5) with $\mathbf{A}^{-1}$ on both sides, We get :

$$\mathbf{A}^{-1}\left(\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I}\right) = 0.\mathbf{A}^{-1} \tag{3.0.6}$$

$$\implies \mathbf{A}^2.\mathbf{A}^{-1} - 5\mathbf{A}.\mathbf{A}^{-1} + 7\mathbf{I}.\mathbf{A}^{-1} = 0 \tag{3.0.7}$$

$$\implies \mathbf{A}.\mathbf{A}.\mathbf{A}^{-1} - 5\mathbf{I} + 7\mathbf{A}^{-1} = 0 \tag{3.0.8}$$

$$\implies \mathbf{A}.\mathbf{I} - 5\mathbf{I} + 7\mathbf{A}^{-1} = 0 \tag{3.0.9}$$

$$\implies \mathbf{A}^{-1} = \frac{1}{7}\left(5\mathbf{I} - \mathbf{A}\right) \tag{3.0.10}$$

Solving for $\mathbf{A}^{-1}$, we get :

$$\mathbf{A}^{-1} = \frac{1}{7}\left(5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\right) \tag{3.0.11}$$

$$\implies \mathbf{A}^{-1} = \frac{1}{7}\left(\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\right) \tag{3.0.12}$$