Assignment 4

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***Abstract*—This document uses the concepts of Median and Right angled triangles in proving a statement.**

Download Python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.py

Download latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.tex

1 Problem

AD is a median of a $\triangle ABC$ and $AM \perp BC$. Prove that

$$AC^2 = AD^2 + (BC)(DM) + \left(\frac{BC}{2}\right)^2 \tag{1.0.1}$$

2 Explanation

In a right angled triangle, square of hypotenuse is equal to sum of squares of the other two sides. Also, a median divides a side into two equal halves.

3 Solution

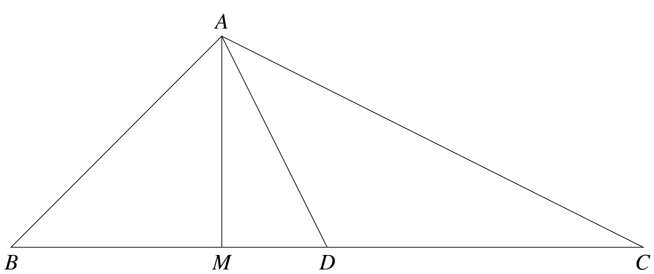

Fig. 0: $\triangle ABC$ with AD as median and $AM \perp BC$

See Fig.0, It is given that :

$$\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{D}-\mathbf{C}\| \tag{3.0.1}$$

$$AM \perp BC \tag{3.0.2}$$

We have to prove that :

$$\|\mathbf{A}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{B}-\mathbf{C}\|\,\|\mathbf{D}-\mathbf{M}\| + \left\|\frac{\mathbf{B}-\mathbf{C}}{2}\right\|^2 \tag{3.0.3}$$

From $\triangle AMC$, we know that :

$$\|\mathbf{A}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{M}\|^2 + \|\mathbf{M}-\mathbf{C}\|^2 \tag{3.0.4}$$

Using Pythagoras theorem we can write it as:

$$= \|\mathbf{A}-\mathbf{D}\|^2 - \|\mathbf{M}-\mathbf{D}\|^2 + \|\mathbf{M}-\mathbf{C}\|^2 \tag{3.0.5}$$

$$= \|\mathbf{A}-\mathbf{D}\|^2 - \|\mathbf{M}-\mathbf{D}\|^2 + (\|\mathbf{M}-\mathbf{D}\| + \|\mathbf{D}-\mathbf{C}\|)^2 \tag{3.0.6}$$

Expanding $(a+b)^2$ and solving, we get :

$$= \|\mathbf{A}-\mathbf{D}\|^2 + 2\,\|\mathbf{M}-\mathbf{D}\|\,\|\mathbf{D}-\mathbf{C}\| + \|\mathbf{D}-\mathbf{C}\|^2 \tag{3.0.7}$$

As AD is a median, we can write :

$$\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{D}-\mathbf{C}\| = \frac{\|\mathbf{B}-\mathbf{C}\|}{2} \tag{3.0.8}$$

Using (3.0.8), we can re-write (3.0.7) as

$$= \|\mathbf{A}-\mathbf{D}\|^2 + 2\,\|\mathbf{M}-\mathbf{D}\|\,\frac{\|\mathbf{B}-\mathbf{C}\|}{2} + \left\|\frac{\mathbf{B}-\mathbf{C}}{2}\right\|^2 \tag{3.0.9}$$

$$= \|\mathbf{A}-\mathbf{D}\|^2 + \|\mathbf{M}-\mathbf{D}\|\,\|\mathbf{B}-\mathbf{C}\| + \left\|\frac{\mathbf{B}-\mathbf{C}}{2}\right\|^2 \tag{3.0.10}$$

Hence it is proved.