Assignment 2

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***Abstract*—This document explains the concepts of Matrix transpose, Matrix Equality by solving a problem.**

Download the python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_2/assignment_2.py

and latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_2/assignment_2.tex

1 Problem

If $\mathbf{A} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, and $\mathbf{A} + \mathbf{A}^\top = \mathbf{I}$, then find the value of angle α.

2 Explanation

* The Complex number equivalent to the matrix $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is $\begin{pmatrix} a \\ b \end{pmatrix}$
* So, If $\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix}$, then transpose of $\mathbf{A} = \begin{pmatrix} a \\ -b \end{pmatrix}$
* And addition of $\mathbf{A}$ with $\mathbf{A}^\top$ results in :

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} a \\ -b \end{pmatrix} = \begin{pmatrix} 2a \\ 0 \end{pmatrix} \tag{2.0.1}$$

3 Solution

So, According to the given question $\mathbf{A} + \mathbf{A}^\top$ is :

$$\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} + \begin{pmatrix} \cos\alpha \\ -\sin\alpha \end{pmatrix} = \begin{pmatrix} 2\cos\alpha \\ 0 \end{pmatrix} \tag{3.0.1}$$

Given that $\mathbf{A} + \mathbf{A}^\top = \mathbf{I}$:

$$\begin{pmatrix} 2\cos\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.2}$$

That Implies,

$$2\cos\alpha = 1 \implies \cos\alpha = \frac{1}{2} \tag{3.0.3}$$

As per the cosine values, the angle α is :

$$\alpha = \frac{\pi}{3} = 1.047 \tag{3.0.4}$$

The cosine function is plotted along with the point (x, cos(x)) = (1.047, 0.5) as shown in Fig. 0:

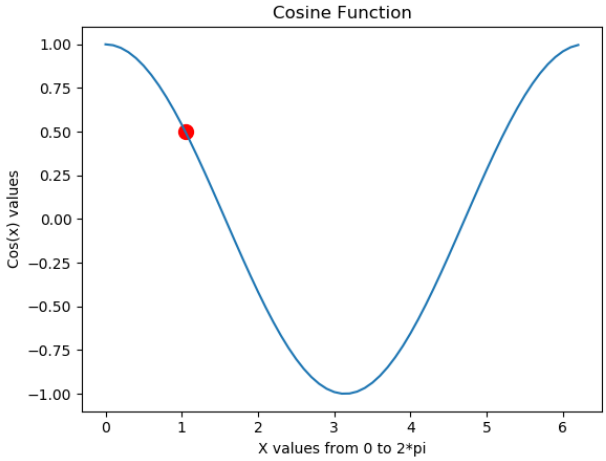

Fig. 0: Cosine Function