

Assignment 3

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Abstract—This document explains the concepts of Matrix multiplication, Matrix Addition and Matrix Inverse by solving a problem.

Download the python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_3/assignment_3.py

and latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_3/assignment_3.tex

1 PROBLEM

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \quad (1.0.1)$$

Show that

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = 0. \quad (1.0.2)$$

Hence find \mathbf{A}^{-1} .

2 EXPLANATION

Square of a matrix is the product of matrix with itself :

$$\mathbf{A}^2 = \mathbf{A}\mathbf{A} \quad (2.0.1)$$

Product of a scalar with a matrix is the product of that scalar with every element of the matrix :

$$k \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix} \quad (2.0.2)$$

Inverse of a matrix \mathbf{A} is defined as :

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \quad (2.0.3)$$

3 SOLUTION

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} \quad (3.0.1)$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - 5 \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (3.0.2)$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \quad (3.0.3)$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (3.0.4)$$

Thus, It is proved that

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = 0. \quad (3.0.5)$$

Multiplying above equation with \mathbf{A}^{-1} on both sides, We get :

$$\Rightarrow \mathbf{A}^2\mathbf{A}^{-1} - 5\mathbf{A}\mathbf{A}^{-1} + 7\mathbf{I}\mathbf{A}^{-1} = 0 \quad (3.0.6)$$

$$\Rightarrow \mathbf{A}\mathbf{A}\mathbf{A}^{-1} - 5\mathbf{I} + 7\mathbf{A}^{-1} = 0 \quad (3.0.7)$$

$$\Rightarrow \mathbf{A}\mathbf{I} - 5\mathbf{I} + 7\mathbf{A}^{-1} = 0 \quad (3.0.8)$$

$$\Rightarrow \mathbf{A}^{-1} = \frac{1}{7}(5\mathbf{I} - \mathbf{A}) \quad (3.0.9)$$

Solving for \mathbf{A}^{-1} , we get :

$$\mathbf{A}^{-1} = \frac{1}{7} \left(5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \right) \quad (3.0.10)$$

$$\Rightarrow \mathbf{A}^{-1} = \frac{1}{7} \left(\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \right) \quad (3.0.11)$$