Assignment 4

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***Abstract*—This document uses the concepts of Intercept theorem, Isosceles triangle in proving a statement.**

Download latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.tex

1 Problem

D is a point on side BC of a $\triangle ABC$ such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠BAC

2 Explanation

If two sides of a triangle are equal, then the corresponding angles are also equal.

$$\|\mathbf{A}-\mathbf{B}\| = \left\|\mathbf{A}-\mathbf{C}\right\| \implies \angle ABC = \angle ACB \quad (2.0.1)$$

From The Intercept or Thales' theorem, we can infer that - Lines cutting intercepts in equal ration on two intersecting lines are parallel.

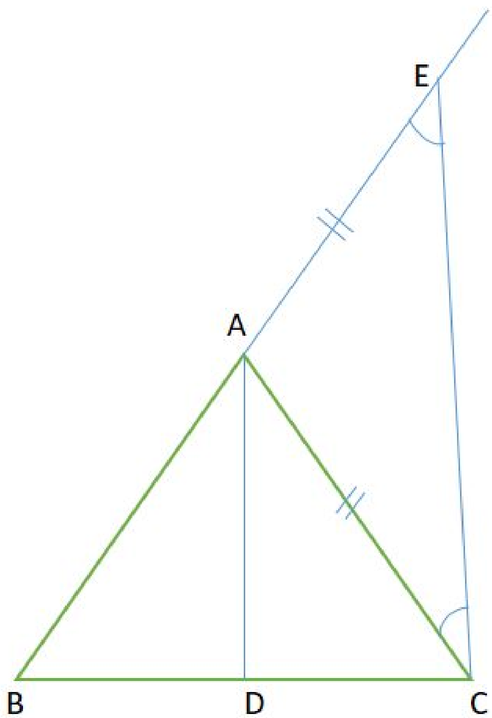

Fig. 0: Construction

3 Solution

Given that :

$$\frac{\|\mathbf{B}-\mathbf{D}\|}{\left\|\mathbf{C}-\mathbf{D}\right\|} = \frac{\|\mathbf{A}-\mathbf{B}\|}{\left\|\mathbf{A}-\mathbf{C}\right\|} \quad (3.0.1)$$

Extend BA to E such that $\|\mathbf{A}-\mathbf{E}\| = \left\|\mathbf{A}-\mathbf{C}\right\|$. From properties of Isosceles triangle we can say that,

$$\|\mathbf{A}-\mathbf{E}\| = \left\|\mathbf{A}-\mathbf{C}\right\| \implies \angle AEC = \angle ACE \quad (3.0.2)$$

Using equation 3.0.2, we can re-write equation 3.0.1 as :

$$\frac{\|\mathbf{B}-\mathbf{D}\|}{\left\|\mathbf{C}-\mathbf{D}\right\|} = \frac{\|\mathbf{A}-\mathbf{B}\|}{\|\mathbf{A}-\mathbf{E}\|} \quad (3.0.3)$$

From Converse of Thales' theorem, we can now say that :

$$AD \parallel EC \quad (3.0.4)$$

Consecutive Interior Angles :

$$\angle CEA = \angle DAB \quad (3.0.5)$$

Alternate Interior Angles :

$$\angle ECA = \angle CAD \quad (3.0.6)$$

But we already know that :

$$\angle CEA = \angle ECA \quad (3.0.7)$$

So,

$$\angle DAB = \angle DAC \quad (3.0.8)$$

Hence it is proved that AD is the bisector of ∠BAC