Assignment 4

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***Abstract*—This document uses the concepts of Inner product and Cosines in proving a statement.**

Download Python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.py

Download latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.tex

1 Problem

D is a point on side BC of a $\triangle ABC$ such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of $\angle BAC$

2 Explanation

If cosines of two angles are equal and those angles are less than 180°, then we can say that the angles are also equal.

3 Solution

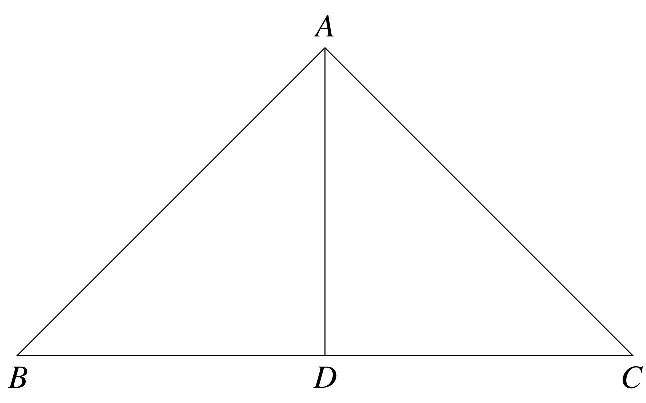

Fig. 0: $\triangle ABC$ with D as a point on side BC

See Fig. 0, It is given that :

$$\frac{\|\mathbf{B}-\mathbf{D}\|}{\|\mathbf{C}-\mathbf{D}\|} = \frac{\|\mathbf{A}-\mathbf{B}\|}{\|\mathbf{A}-\mathbf{C}\|} \tag{3.0.1}$$

Re-writing the above equation we get :

$$\frac{\|\mathbf{A}-\mathbf{C}\|}{\|\mathbf{A}-\mathbf{B}\|} = \frac{\|\mathbf{C}-\mathbf{D}\|}{\|\mathbf{B}-\mathbf{D}\|} \tag{3.0.2}$$

Taking inner product of the sides AB,AD we get :

$$\cos BAD = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{D}\|} \tag{3.0.3}$$

Taking inner product of the sides AC,AD we get :

$$\cos CAD = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{A}-\mathbf{D}\|} \tag{3.0.4}$$

Dividing equation (3.0.3) by (3.0.4), we get :

$$\frac{\cos BAD}{\cos CAD} = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D})\,\|\mathbf{A}-\mathbf{C}\|}{(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{D})\,\|\mathbf{A}-\mathbf{B}\|} \tag{3.0.5}$$

Let $A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then equation (3.0.5) becomes:

$$\frac{\cos BAD}{\cos CAD} = \frac{(\mathbf{B})^T(\mathbf{D})\,\|\mathbf{A}-\mathbf{C}\|}{(\mathbf{C})^T(\mathbf{D})\,\|\mathbf{A}-\mathbf{B}\|} \tag{3.0.6}$$

Using equation (3.0.2), (3.0.6) can be written as :

$$\frac{\cos BAD}{\cos CAD} = \frac{(\mathbf{B})^T(\mathbf{D})\,\|\mathbf{C}-\mathbf{D}\|}{(\mathbf{C})^T(\mathbf{D})\,\|\mathbf{B}-\mathbf{D}\|} \tag{3.0.7}$$

In a triangle that satisfies the equation (3.0.1), It can be seen that the equation (3.0.7) always equates to 1, which implies that :

$$\cos BAD = \cos CAD \implies \angle BAD = \angle CAD. \tag{3.0.8}$$

Hence it is proved that AD is the angle bisector of $\angle BAC$