Assignment 4

Sairam V C Rebbapragada

***Abstract*—This document uses the concepts of Intercept theorem, Isosceles triangle in proving a statement.**

Download latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.tex

1 Problem

D is a point on side BC of a △ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that $\overline{AD}$ is the bisector of ∠BAC

2 Explanation

If two sides of a triangle are equal, then the corresponding angles are also equal.

$$\triangle ABC, \overline{AB} = \overline{AC} \implies \angle ABC = \angle ACB \tag{2.0.1}$$

From The Intercept or Thales' theorem, we can infer that - Lines cutting intercepts in equal ration on two intersecting lines are parallel.

3 Solution

Given that :

$$\frac{BD}{CD} = \frac{AB}{AC} \tag{3.0.1}$$

Extend $\overline{BA}$ to E such that $\overline{AE} = \overline{AC}$. From properties of Isosceles triangle we can say that,

$$\overline{AE} = \overline{AC} \implies \angle AEC = \angle ACE \tag{3.0.2}$$

Using equation 3.0.2, we can re-write equation 3.0.1 as :

$$\frac{BD}{CD} = \frac{AB}{AE} \tag{3.0.3}$$

From Thales' theorem, we can now say that :

$$\overline{AD} \parallel \overline{EC} \tag{3.0.4}$$

Consecutive Interior Angles :

$$\angle CEA = \angle DAB \tag{3.0.5}$$

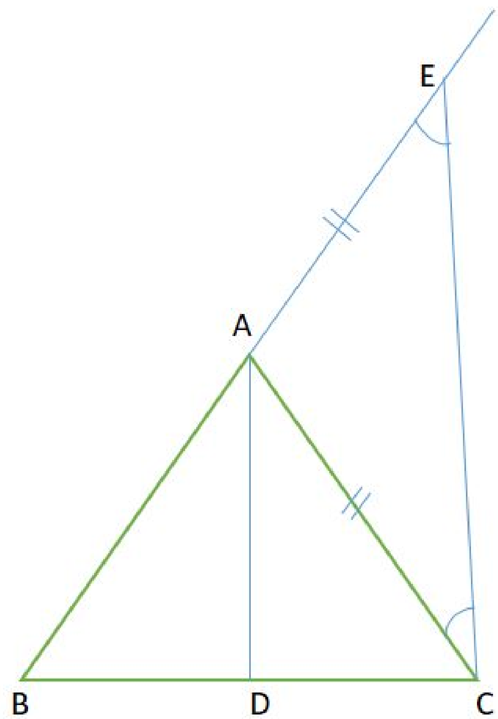

Fig. 0: Construction

Alternate Interior Angles :

$$\angle ECA = \angle CAD \tag{3.0.6}$$

But We already know that :

$$\angle CEA = \angle ECA \tag{3.0.7}$$

So,

$$\angle DAB = \angle DAC \tag{3.0.8}$$

Hence it is proved that AD is the bisector of ∠BAC