

Assignment 4

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Abstract—This document uses the concepts of Median and Right angled triangles in proving a statement.

Download Python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.py

Download latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_4/assignment_4.tex

1 PROBLEM

AD is a median of a $\triangle ABC$ and $AM \perp BC$. Prove that

$$AC^2 = AD^2 + (BC)(DM) + \left(\frac{BC}{2}\right)^2 \quad (1.0.1)$$

2 EXPLANATION

In a right angled triangle, square of hypotenuse is equal to sum of squares of the other two sides. Also, a median divides a side into two equal halves.

3 SOLUTION

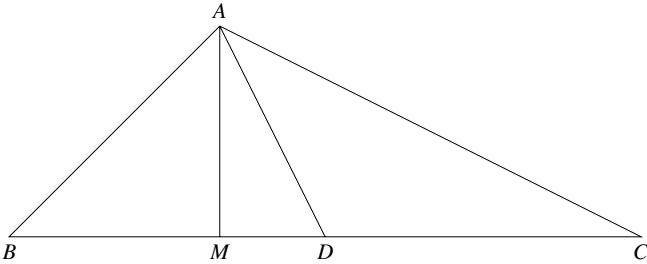


Fig. 0: $\triangle ABC$ with AD as median and $AM \perp BC$

See Fig.0, It is given that :

$$\|B - D\| = \|D - C\| \quad (3.0.1)$$

$$AM \perp BC \quad (3.0.2)$$

We have to prove that :

$$\|A - C\|^2 = \|A - D\|^2 + \|B - C\| \|D - M\| + \left\| \frac{B - C}{2} \right\|^2 \quad (3.0.3)$$

From $\triangle AMC$, we know that :

$$\|A - C\|^2 = \|A - M\|^2 + \|M - C\|^2 \quad (3.0.4)$$

Using Pythagoras theorem we can write it as:

$$= \|A - D\|^2 - \|M - D\|^2 + \|M - C\|^2 \quad (3.0.5)$$

$$= \|A - D\|^2 - \|M - D\|^2 + (\|M - D\| + \|D - C\|)^2 \quad (3.0.6)$$

Expanding $(a + b)^2$ and solving, we get :

$$= \|A - D\|^2 + 2 \|M - D\| \|D - C\| + \|D - C\|^2 \quad (3.0.7)$$

As AD is a median, we can write :

$$\|B - D\| = \|D - C\| = \frac{\|B - C\|}{2} \quad (3.0.8)$$

Using (3.0.8), we can re-write (3.0.7) as

$$\begin{aligned} &= \|A - D\|^2 + 2 \|M - D\| \frac{\|B - C\|}{2} + \left\| \frac{B - C}{2} \right\|^2 \\ &= \|A - D\|^2 + \|M - D\| \|B - C\| + \left\| \frac{B - C}{2} \right\|^2 \end{aligned} \quad (3.0.9)$$

Hence it is proved.