

Assignment 2

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Abstract—This document explains the concepts of Matrix transpose, Matrix Equality by solving a problem.

Download the python code from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_2/assignment_2.py

and latex-tikz codes from

https://github.com/Sairam13001/AI5006/blob/master/Assignment_2/assignment_2.tex

1 PROBLEM

If $\mathbf{A} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$, and $\mathbf{A} + \mathbf{A}^T = \mathbf{I}$, then find the value of angle α .

2 EXPLANATION

- * If two matrices \mathbf{A} and \mathbf{B} are equal then the corresponding elements are equal.
- * Similarly, Addition of two matrices is addition of their corresponding elements.
- * If $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then transpose of \mathbf{A} is :

$$\mathbf{A}^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \quad (2.0.1)$$

3 SOLUTION

Given that,

$$\mathbf{A} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}. \quad (3.0.1)$$

So, the transpose of \mathbf{A} is :

$$\mathbf{A}^T = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}. \quad (3.0.2)$$

Sum of Matrices \mathbf{A} and its Transpose \mathbf{A}^T is :

$$\mathbf{A} + \mathbf{A}^T = \begin{pmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{pmatrix}. \quad (3.0.3)$$

Also given, $\mathbf{A} + \mathbf{A}^T = \mathbf{I}$, So :

$$\begin{pmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (3.0.4)$$

That Implies,

$$2\cos \alpha = 1 \implies \cos \alpha = \frac{1}{2} \quad (3.0.5)$$

As per the cosine values, the angle α is :

$$\alpha = \frac{\pi}{3} = 1.047 \quad (3.0.6)$$

The cosine function is plotted along with the point $(x, \cos(x)) = (1.047, 0.5)$ as shown below:

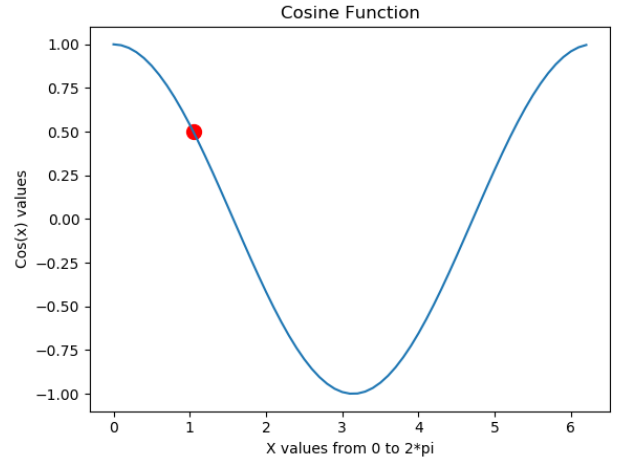


Fig. 0: Cosine Function