

Assignment 8

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Abstract—This document uses the concepts of vector lines and planes in solving a problem.

Download Python code from

https://github.com/Sairam13001/AI5106/blob/main/Assignment_8/assignment_8.py

Download latex-tikz codes from

https://github.com/Sairam13001/AI5106/blob/main/Assignment_8/assignment_8.tex

1 PROBLEM

Write the equation of line through $\begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$ and perpendicular to the plane

$$2x - y + 2z - 5 = 0 \quad (1.0.1)$$

Determine the co-ordinates of the point in which the plane is met by this line.

2 EXPLANATION

The general equation of a plane is given by

$$px + qy + rz = c \quad (2.0.1)$$

and can be expressed as

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.2)$$

where

$$\mathbf{n} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (2.0.3)$$

The equation of a line passing through the point \mathbf{a} and having direction vector \mathbf{b} is given by

$$\mathbf{x} = \mathbf{a} + \lambda \mathbf{b} \quad (2.0.4)$$

3 SOLUTION

Comparing (1.0.1) with (2.0.1), we get

$$\mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad (3.0.1)$$

The required line is perpendicular to (1.0.1). So, its direction vector is parallel to the normal vector of (1.0.1). Thus, from (2.0.4) :

$$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad (3.0.2)$$

So, the equation of required line is

$$\mathbf{x} = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad (3.0.3)$$

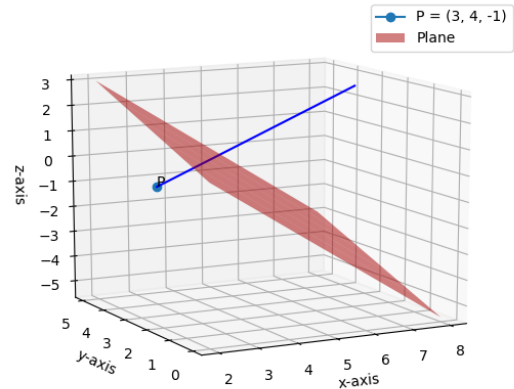


Fig. 0: Line passing through $\begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$ and perpendicular to the plane $2x - y + 2z - 5 = 0$.

Substituting (3.0.3) in (2.0.4), we get

$$\begin{pmatrix} 2 & -1 & 2 \end{pmatrix} \left(\begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \right) = 5 \quad (3.0.4)$$

$$\begin{aligned} \Rightarrow 6 + 4\lambda - 4 + \lambda - 2 + 4\lambda &= 5 \\ \Rightarrow \lambda &= \frac{5}{9} \end{aligned} \quad (3.0.5)$$

substituting (3.0.5) in (3.0.3), we get:

$$\mathbf{x} = \frac{1}{9} \begin{pmatrix} 37 \\ 31 \\ 1 \end{pmatrix} \quad (3.0.6)$$