Assignment 12

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***Abstract*—This document uses the concept of Linear Independence in solving a problem.**

Download latex-tikz codes from

https://github.com/Sairam13001/AI5106/blob/main/Assignment_12/assignment_12.tex

1 Problem

Let V denote the vector space of real valued continuous functions on the closed interval [0,1]. let W be the subspace of V spanned by {sin(x), cos(x), tan(x)}. Then the dimension of W over $\mathbb{R}$ is

1) 1
2) 2
3) 3
4) infinite

2 Explanation

Linearly Dependent vectors: The vectors in a subset $S = \{v_1, v_2, v_3, ..., v_k\}$ of a vector space V are said to be linearly dependent if $\exists$ scalars $a_1, a_2, ..., a_k$ not all zero such that

$$a_1v_1 + a_2v_2 + ... + a_kv_k = 0 \tag{2.0.1}$$

3 Solution

It is given that

$$W = \langle sin(x), cos(x), tan(x) \rangle \tag{3.0.1}$$

As W is spanned by three vectors, we can see that dimension$(W) \leq 3$.

Let us assume that the spanning set is linearly dependent $\implies \exists\ a, b, c \in \mathbb{R}$ not all zero, such that

$$asinx + bcosx + ctanx = 0, \forall x \in [0, 1]. \tag{3.0.2}$$

1) let $x = 0$:

$$a \times 0 + b \times 1 + c \times 0 = 0 \tag{3.0.3}$$
$$\implies b = 0. \tag{3.0.4}$$

2) $x = \frac{\pi}{6}$:

$$\frac{a}{2} + \frac{c}{\sqrt{3}} = 0 \tag{3.0.5}$$

3) $x = \frac{\pi}{4}$:

$$\frac{a}{\sqrt{2}} + c = 0 \tag{3.0.6}$$

From (3.0.5) and (3.0.6), we can observe that a = 0 and c = 0, which contradicts our assumption that the spanning set is linearly dependent. So, W is linearly independent which implies that the dimensions of W over $\mathbb{R}$ = 3.

Hence option 3 is correct.