Three Phase AC Circuits

Syllabus

3.1 Star and Delta connected balanced circuits: Three phase voltages, current and power, delta/star equivalence and analysis for various loading conditions.

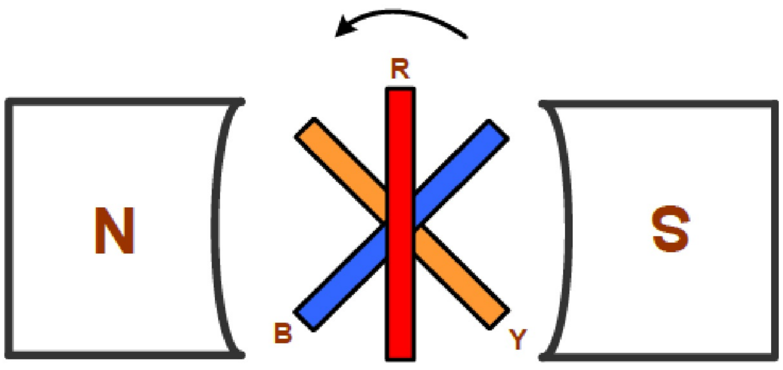
3.2 Measurement of power using two watt meter method.

Introduction

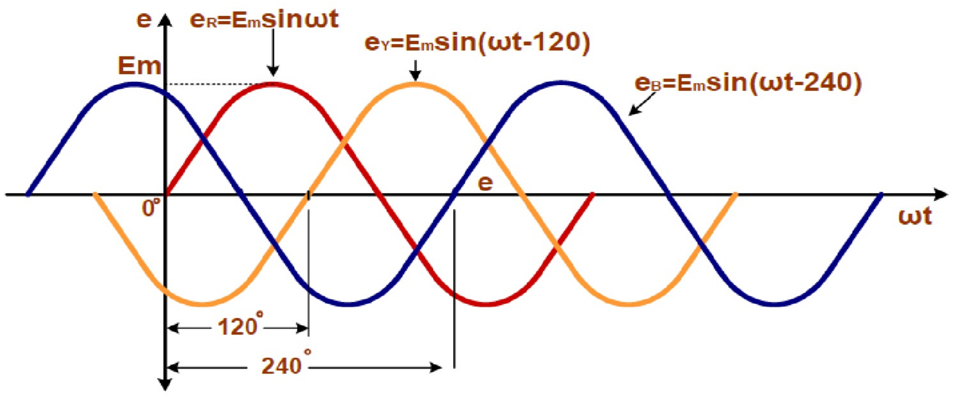
- For transmission of large amounts of electrical power, three-phase systems are invariably used.

- The reasons for the immense popularity of three-phase apparatus are that
  1. It is more efficient
  2. it uses less material for a given capacity
  3. it costs less than single-phase apparatus etc.

Generation of three phase EMF



Demonstrative diagram



Waveforms: Three phase emf

- According to Faraday's law of electromagnetic induction: whenever a coil is rotated in a magnetic field, there is a sinusoidal emf induced in that coil.
- Three coils C1(R-phase), C2(Y-phase) and C3(B-phase), which are displaced 120^0 from each other on the same axis.
- The coils are rotating in a uniform magnetic field produced by the N and S poles in the counter clockwise direction with constant angular velocity.
- The emf induced in these three coils will have phase difference of 120^0. i.e. if the induced emf of the coil C1 has phase of 0^0, then induced emf in the coil C2 lags that of C1 by 120^0 and C3 lags that of C2 120^0.

Three phase EMF

- The instantaneous values of the three e.m.fs.

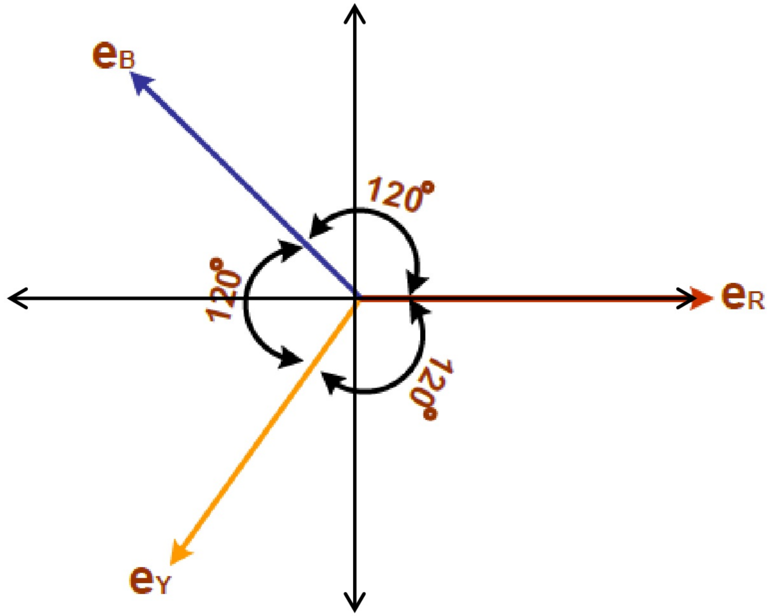
$$e_R = E_m \sin \omega t$$

$$e_Y = E_m \sin\left(\omega t - 120^0\right)$$

$$e_B = E_m \sin\left(\omega t - 240^0\right)$$

- Phasor representation:

  Three alternating voltages may be represented by revolving vectors which indicate their maximum values (or r.m.s. values if desired).



Three phase EMF…

- It can be shown that the sum of the three phase e.m.f.s. is zero in the following three ways :

  1. The sum of the above three equations

$$e_R = E_m \sin \omega t$$

$$e_Y = E_m \sin\left(\omega t - 120^0\right)$$

$$e_B = E_m \sin\left(\omega t - 240^0\right)$$

Resultant instantaneous e.m.f. = $e_R + e_Y + e_B$

$$= Em \sin \omega t + Em \sin(\omega t - 120) + Em \sin(\omega t - 240)$$

$$= Em\,[\sin \omega t + 2 \sin(\frac{\omega t - 120 + \omega t - 240}{2}).\cos(\frac{\omega t - 120 - \omega t + 240}{2})]$$

$$= Em\,[\sin \omega t + 2 \sin(\frac{2\omega t - 360}{2}).\cos(\frac{120}{2})]$$

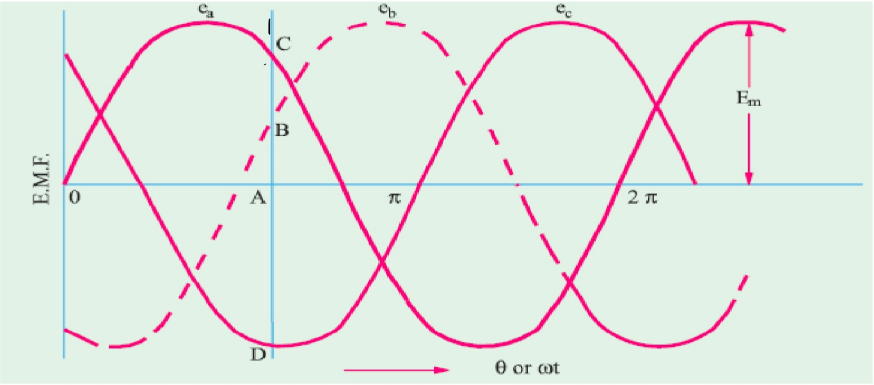
$$= Em\,[\sin \omega t + 2 \sin(\omega t - 180).\cos(60)]$$

$$= Em\,[\sin \omega t - 2 \sin(\omega t).1/2]$$

$$= 0$$

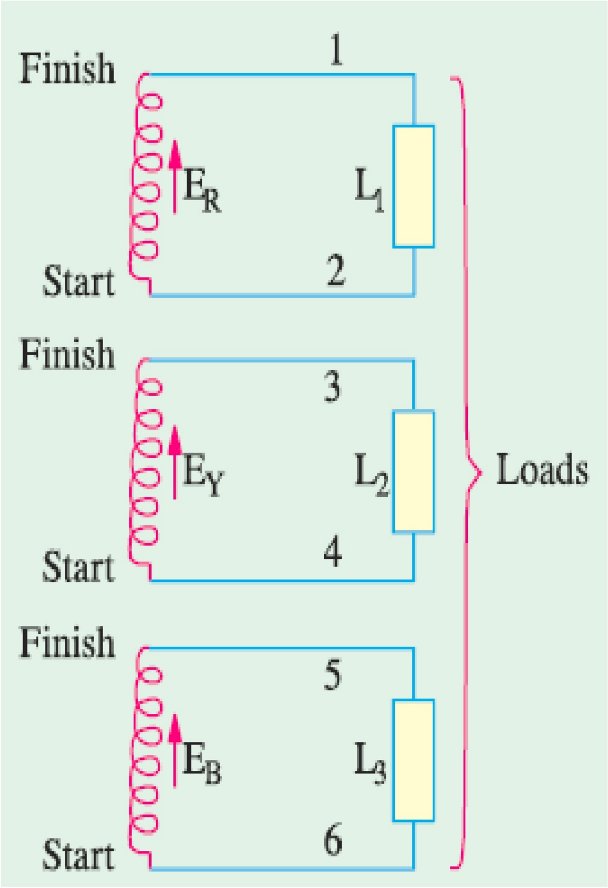
- The sum of ordinates of three e.m.f curves is zero. For example, taking ordinates *AB* and *AC* as positive and *AD* as negative, it can be shown by actual measurement that

  AB + *AC* + (– *AD*) = 0



Interconnection of Three Phases

- If the three armature coils of the 3-phase alternator are not interconnected but are kept separate, as shown below.



- Then each phase or circuit would need two conductors, the total number of conductors, in that case, being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive.

- Hence, the three phases are generally interconnected which results in substantial saving of copper. The general methods of interconnection are

(*a*) Star or Wye (Y) connection and

(*b*) Mesh or Delta (Δ) connection.

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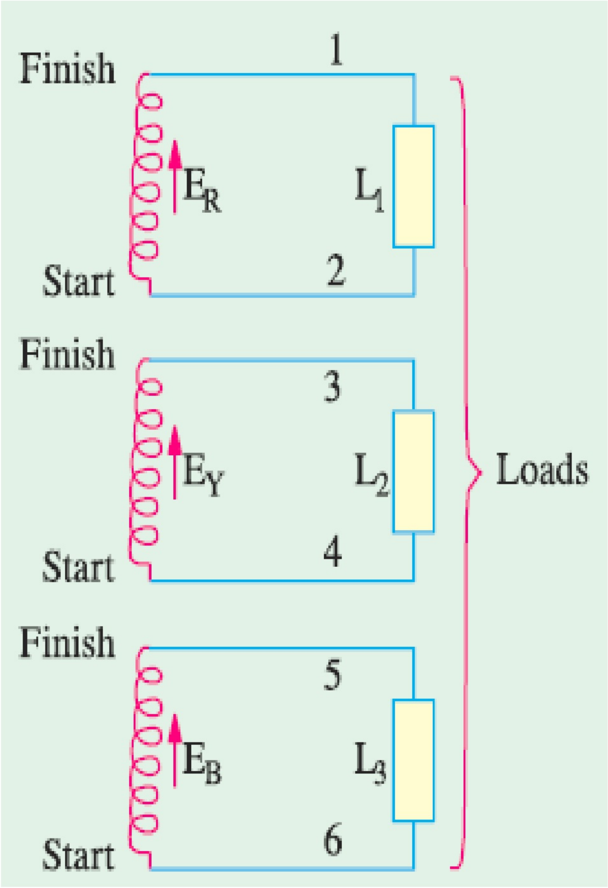

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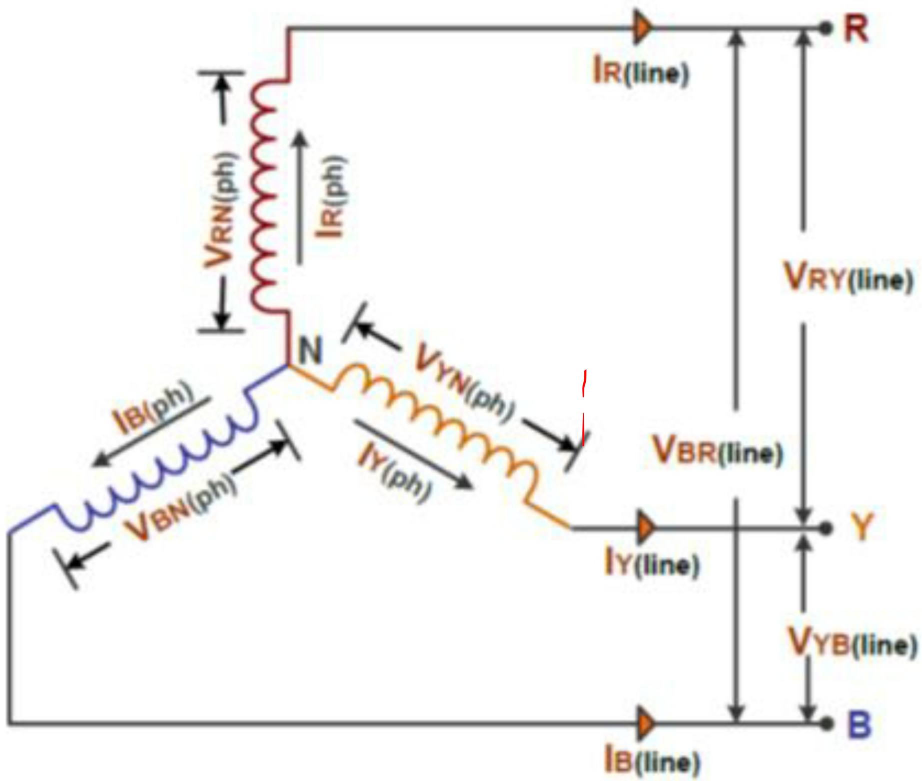
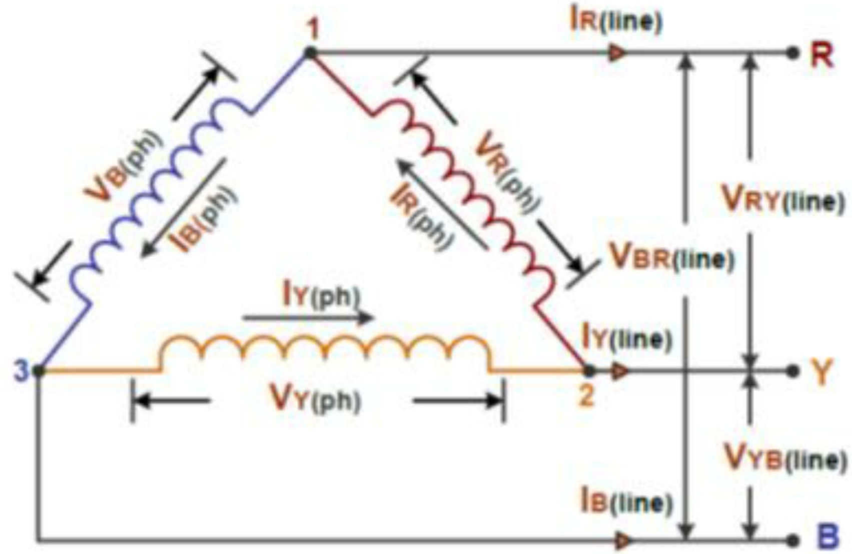
Interconnection of Three Phases..

Three phase star and Delta connected circuits





Interconnection of Three Phases..

Important definitions:

- **Phase Voltage**: It is defined as the voltage across either phase winding or load terminal. It is denoted by Vph. Phase voltage VRN, VYN and VBN are measured between R-N, Y-N, B-N for star connection and between R-Y, Y-B, B-R in delta connection.

- **Line voltage**: It is defined as the voltage across any two-line terminal. It is denoted by VL. Line voltage VRY, VYB, VBR measure between R-Y, Y-B, B-R terminal for star and delta connection both.

- **Phase current**: It is defined as the current flowing through each phase winding or load. It is denoted by Iph. Phase current IR(ph), IY(ph) and IB(Ph) measured in each phase of star and delta connection. respectively.

- **Line current**: It is defined as the current flowing through each line conductor. It denoted by IL. Line current IR(line), IY(line), and IB((line) are measured in each line of star and delta connection.

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Interconnection of Three Phases..

Important definitions:

➢ **Phase sequence**: The order in which three coil emf or currents attain their peak values is called the phase sequence. It is customary to denoted the 3 phases by the three colours. i.e. red (R), yellow (Y), blue (B).

➢ **Balance System:** A system is said to be balance if the voltages and currents in all phase are equal in magnitude and displaced from each other by equal angles.

➢ **Unbalance System**: A system is said to be unbalance if the voltages and currents in all phase are unequal in magnitude and displaced from each other by unequal angles.

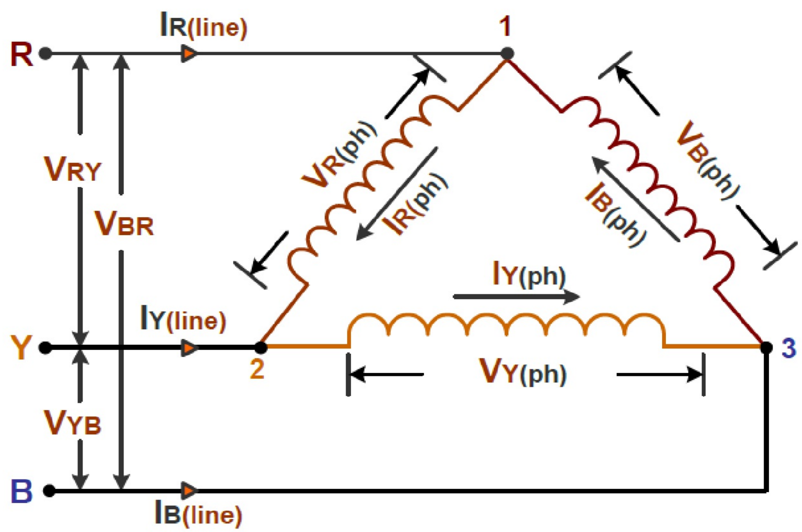
➢ **Balance load**: In this type the load in all phase are equal in magnitude. It means that the load will have the same power factor equal currents in them.

➢ **Unbalance load:** In this type the load in all phase have unequal power factor and currents.

Relation between line/phase values for voltage/current and power in case of balanced delta connection.

➢ Delta (Δ) or Mesh connection, starting end of one coil is connected to the finishing end of other phase coil and so on which gives a closed circuit.

Circuit Diagram



KCL at node 1

$\bar{I}_{R(L)} + \bar{I}_{B(ph)} = \bar{I}_{R(ph)}$

$\bar{I}_{R(L)} = \bar{I}_{R(ph)} - \bar{I}_{B(ph)}$

KCL at node

$I_{Y(line)} + I_{R(ph)} = I_{Y(ph)}$

$I_{Y(line)} = I_{Y(ph)} - I_{R(ph)}$

$\bar{I}_{Y(ph)} + I_{B(line)} = I_{B(ph)}$

$\bar{I}_{B(line)} = \bar{I}_{B(ph)} - \bar{I}_{Y(ph)}$

- Let,

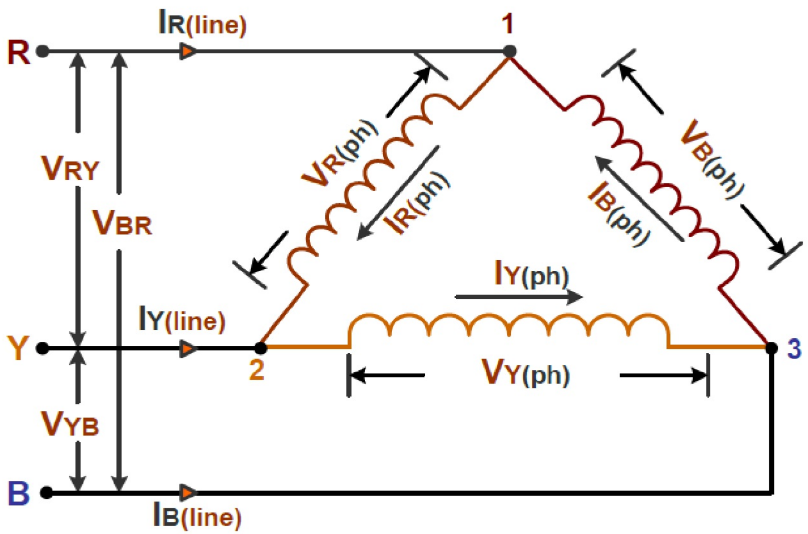
  Line voltage, $V_{RY} = V_{YB} = V_{BR} = V_L$

  Phase voltage, $V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$

  Line current, $I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$

  Phase current, $I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$

Relation between line/phase values for voltage/current and power in case of balanced delta connection………



Relation between line and phase voltage

- For delta connection line voltage V_L and phase voltage V_{ph} both are same.

$$V_{RY} = V_{R(ph)}$$

$$V_{YB} = V_{Y(ph)}$$

$$V_{BR} = V_{B(ph)}$$

$$\boxed{\therefore V_L = V_{ph}}$$

Line voltage = Phase Voltage

Relation between line/phase values for voltage/current and power in case of balanced delta connection………

Relation between line and phase current

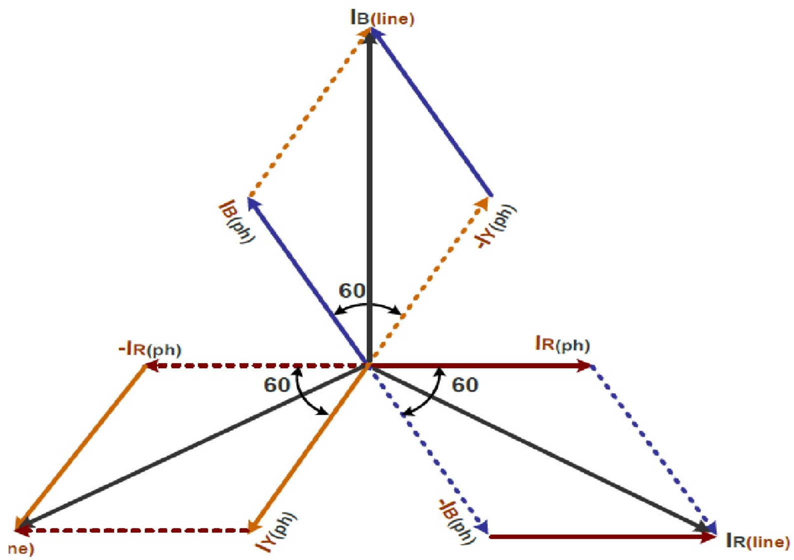
- For delta connection,

$$\bar{I}_{R(line)} = \bar{I}_{R(ph)} - \bar{I}_{B(ph)}$$

$$\bar{I}_{Y(line)} = \bar{I}_{Y(ph)} - \bar{I}_{R(ph)}$$

$$\bar{I}_{B(line)} = \bar{I}_{B(ph)} - \bar{I}_{Y(ph)}$$

Please note its vector addition i.e current in each line is vector difference of two phase currents.



So, considering the parallelogram formed by I_R and I_B.

$$I_{R(line)} = \sqrt{I_{R(ph)}^2 + I_{B(ph)}^2 + 2I_{R(ph)}I_{B(ph)}\cos\theta}$$

$$\therefore I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\cos 60^\circ}$$

$$\therefore I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + \cancel{2}I_{ph}^2 \times \left(\frac{1}{\cancel{2}}\right)}$$

$$\therefore I_L = \sqrt{3I_{ph}^2}$$

$$\boxed{\therefore I_L = \sqrt{3}I_{ph}}$$

Similarly, $I_{Y(line)} = I_{B(line)} = \sqrt{3}\, I_{ph}$

Thus, in delta connection Line current = $\sqrt{3}$ Phase current

Relation between line/phase values for voltage/current and power in case of balanced delta connection………

Power

$$P = V_{ph} I_{ph} \cos\phi + V_{ph} I_{ph} \cos\phi + V_{ph} I_{ph} \cos\phi$$

$$\boxed{P = 3 V_{ph} I_{ph} \cos\phi}$$

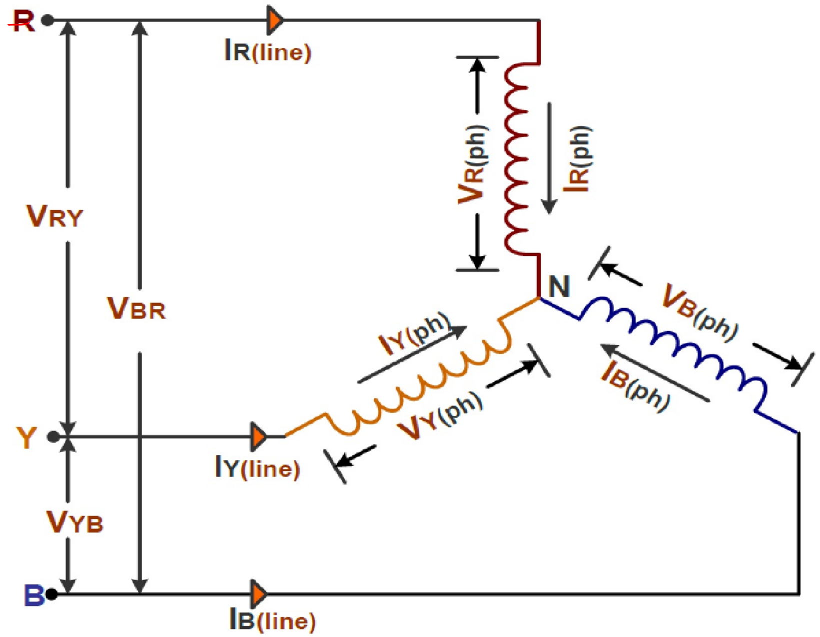
$$P = 3 V_L \left(\frac{I_L}{\sqrt{3}} \right) \cos\phi$$

$$\boxed{\therefore P = \sqrt{3} V_L I_L \cos\phi}$$

Relation between line/phase values for voltage/current and power in case of balanced star connection………

In the **Star Connection**, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point.

Circuit Diagram



- Let,

line voltage, $V_{RY} = V_{BY} = V_{BR} = V_L$

phase voltage, $V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$

line current, $I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$

phase current, $I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$

Relation between line/phase values for voltage/current and power in case of balanced star connection………

Relation between line and phase current

- For star connection, line current I_L and phase current I_{ph} both are same.

$$I_{R(line)} = I_{R(ph)}$$

$$I_{Y(line)} = I_{Y(ph)}$$

$$I_{B(line)} = I_{B(ph)}$$

$$\therefore \quad I_L = I_{ph}$$

Line Current = Phase Current

Relation between line/phase values for voltage/current and power in case of balanced star connection………

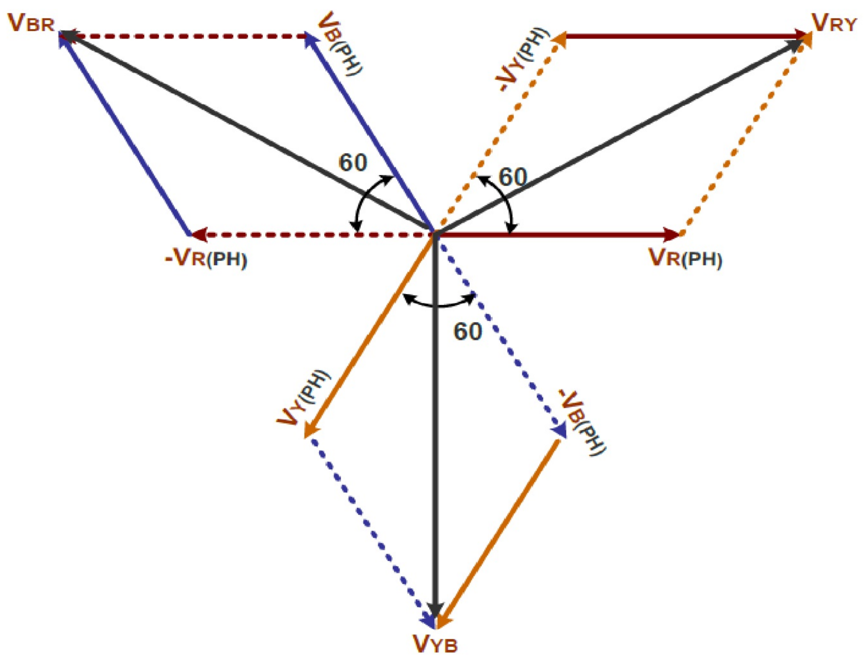
Relation between line and phase voltage

$$\overline{V}_{RY} = \overline{V}_{R(ph)} - \overline{V}_{Y(ph)}$$

$$V_{YB} = V_{Y(ph)} - V_{B(ph)}$$

$$V_{BR} = V_{B(ph)} - V_{R(ph)}$$

Please note its vector addition i.e line voltage is vector difference of two phase voltages.



From parallelogram,

$$V_{RY} = \sqrt{V_{R(ph)}^{2} + V_{Y(ph)}^{2} + 2V_{R(ph)}V_{Y(ph)}\cos\theta}$$

$$\therefore V_L = \sqrt{V_{ph}^{2} + V_{ph}^{2} + 2V_{ph}V_{ph}\cos 60^\circ}$$

$$\therefore V_L = \sqrt{V_{ph}^{2} + V_{ph}^{2} + 2V_{ph}^{2} \times \left(\frac{1}{2}\right)}$$

$$\therefore V_L = \sqrt{3V_{ph}^{2}}$$

$$\boxed{\therefore V_L = \sqrt{3}V_{ph}}$$

Similarly, $V_{YB} = V_{BR} = \sqrt{3}\ V_{ph}$

Thus, in star connection Line voltage = $\sqrt{3}$ Phase voltage

Relation between line/phase values for voltage/current and power in case of balanced star connection……..

Power

$$P = V_{ph} I_{ph} \cos\phi + V_{ph} I_{ph} \cos\phi + V_{ph} I_{ph} \cos\phi$$

$$\boxed{P = 3 V_{ph} I_{ph} \cos\phi}$$

$Q = 3 V_{ph} I_{ph} \sin\phi$

$S = 3 V_{ph} I_{p}$

$$P = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\phi$$

$$\boxed{\therefore P = \sqrt{3} V_L I_L \cos\phi}$$

$Q = \sqrt{3} V_L I_L \sin\phi$

$S = \sqrt{P^2 + Q^2} = \sqrt{3} V_L I_L$

Difference between star and delta connection

Star Connection	Delta Connection
In STAR Connection, the starting or finishing ends (similar ends) of three coils are connected together to form the neutral point. A common wire is taken out from the neutral point which is called Neutral.	In **DELTA Connection**, the opposite ends of three coils are connected together. In other words, the end of each coil is connected with the starting point of another coil, and three wires are taken out from the coil joints.
There is a Neutral or Star Point.	No Neutral Point in Delta Connection.
Three phase four wire system is derived from Star Connections (3-Phase, 4 Wires System). We may Also derive 3 Phase 3 Wire System from Star Connection.	Three phase three wire system is derived from Delta Connections (3-Phase, 3 Wires System). i.e. 3 Phase, Wire system is not possible in Delta Connection.
Line Current is Equal to the Phase Current. i.e. Line Current = Phase Current IL = IPH	Line Current is √3 times of Phase Current. i.e. IL = √3 IPH
Line Voltage is √3 times of Phase Voltage. i.e. VL = √3 VPH	Line Voltage is Equal to the Phase Voltage. i.e. Line Voltage = Phase Voltage VL = VPH
In Star Connection, the Total Power of three Phases P = √3 x VL x IL x CosΦ …. Or P = 3 x VPH x IPH x CosΦ	In delta Connection, the Total Power of three Phases P = √3 x VL x IL x CosΦ …. Or P = 3 x VPH x IPH x CosΦ
The speeds of Star connected motors are slow as they receive 1/√3 voltage.	The speeds of Delta connected motors are high because each phase gets the total of line voltage.
In Star Connection, the phase voltage is low as 1/√3 of the line voltage. Therefore, it needs a low number of turns, hence saving in copper and low insulation cost.	In Delta connection, The phase voltage is equal to the line voltage, hence it needs more number of turns which increase the total cost. Also high insulation cost.
Star Connection is a common and general system which is used in Power transmission.	Delta Connection is a typical system used in Distribution system and Industries.

Relation between power in star and delta connected load

For Star Connection

$V_L = \sqrt{3} V_{ph}$, $I_L = I_{ph}$

$P_Y = 3 V_{ph} I_{ph} \cos\phi$

$= 3 V_{ph} \cdot I_{ph} \cdot \frac{R}{Z}$

$= 3 I_{ph} \times \cancel{Z} \cdot I_{ph} \times \frac{R}{\cancel{Z}}$

$P_Y = 3 I_{ph}^2 \cdot R$

$= 3 \left(\frac{V_{ph}}{R}\right)^2 \times R$

$= 3 \times \left(\frac{V_L}{\sqrt{3} R}\right)^2 \times R$

$= \cancel{3} \times \frac{V_L^2}{\cancel{3} R^{\cancel{2}}} \times \cancel{R}$

$P_Y = \frac{V_L^2}{R}$ ---- (1)

For Delta Connection

$V_L = V_{ph}$, $I_L = \sqrt{3} I_{ph}$

$P_\Delta = 3 \times I_{ph}^2 \times R$

$= 3 \times \left(\frac{V_{ph}}{R}\right)^2 \times R$

$= 3 \times \left(\frac{V_L}{R}\right)^2 \times R$

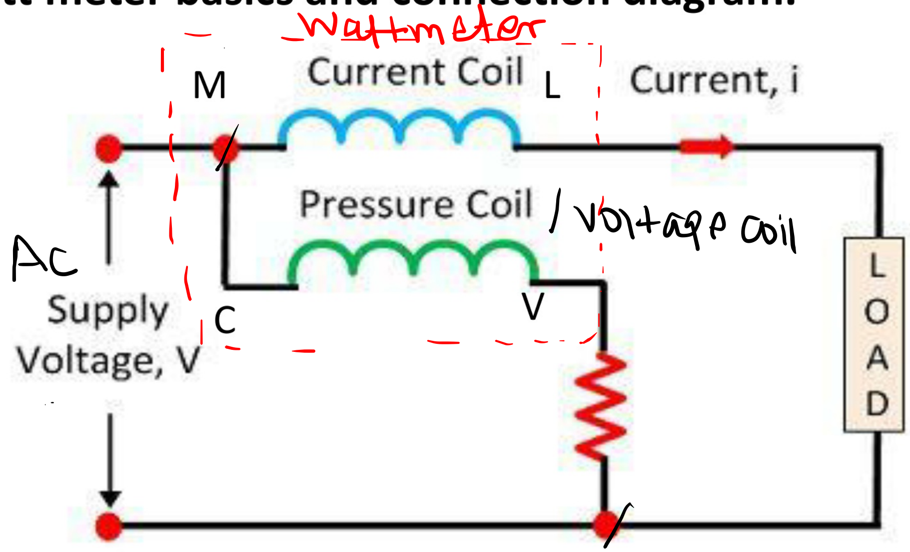
$= 3 \times \frac{V_L^2}{R^2} \times R$

$P_\Delta = 3 \frac{V_L^2}{R}$ From (1)

$$\boxed{P_\Delta = 3 P_Y}$$

Measurement of power in balanced three phase circuit

- **The watt meter basics and connection diagram.**



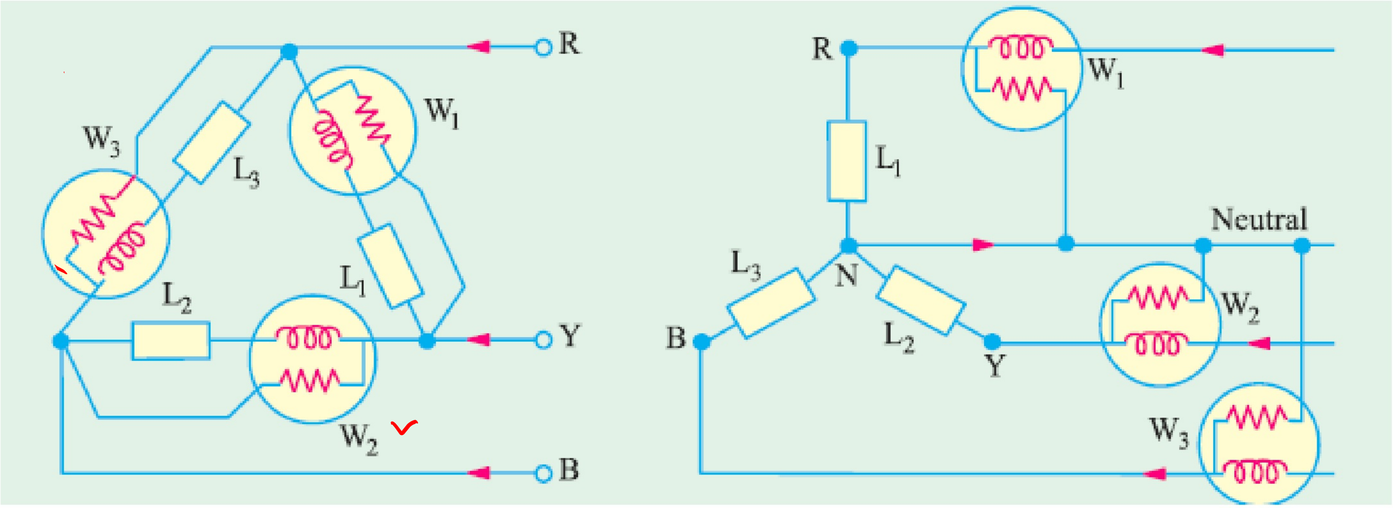
- Watt meter terminal M(main), L(Line), C(common), V(Voltage)
- To measure the current the measuring instrument should be placed in series (current Coil) with the load.
- In case of voltage, the instrument should be connected in parallel (pressure Coil) to the load . therefore M from the current coil and C from the pressure coil are can be short-circuited to measure the power of the given circuit
- Wattmeter reading is product of current in current coil, voltage across pressure coil and cosine of angle between voltage and current.

Measurement of power in balanced three phase circuit..

- The power in three phase circuits can be measured by using following methods.

1. **One wattmeter method** In this method, a single wattmeter is used to obtain the two readings which are obtained by two wattmeters by the two-wattmeter method. This method can, however, be used only when the load is balanced.

2. **Three wattmeter method** In this method, three wattmeters are inserted one in each phase and the algebraic sum of their readings gives the total power consumed by the 3-phase load.



3. **Two wattmeter method** in which sum of the readings of the two wattmeter gives total power in three phase system. This method is suitable to measure power in balanced as well as un balanced load

Measurement of power using Two wattmeter method..

Two wattmeter method

Star Connected Load **Delta Connected Load**

1. **Resistive Load**
2. **Inductive Load**
3. **Capacitive Load**

- There are total six possibilities to prove that two watt meter method is suitable to measure total power in three phase systems. i.e. three possible loads for delta and three for star.

- To prove that sum of wattmeter readings = $W_1+W_2 = P$ i.e.

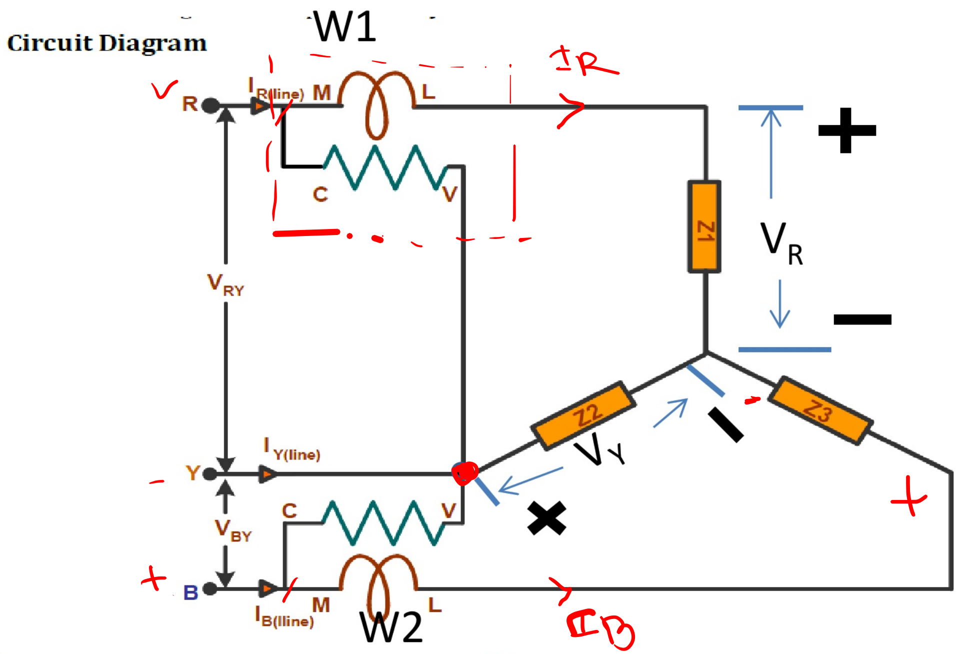
  $$P=\sqrt{3}\, V_L\, I_L \cos\Phi$$

- In general

wattmeter Reading= [Current through current coil] X [Voltage across Pressure coil] X [cos(angle between Voltage and current)]

Measurement of power using Two wattmeter method..

- **Star Connected Load wattmeter connection**



- **Two wattmeter readings**

$W_1 = I_R \times V_{RY} \times \cos$ (angle between I_R and V_{RY}) where $\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$

$W_2 = I_B \times V_{BY} \times \cos$ (angle between I_B and V_{BY}) where $\bar{V}_{BY} = \bar{V}_B - \bar{V}_Y$

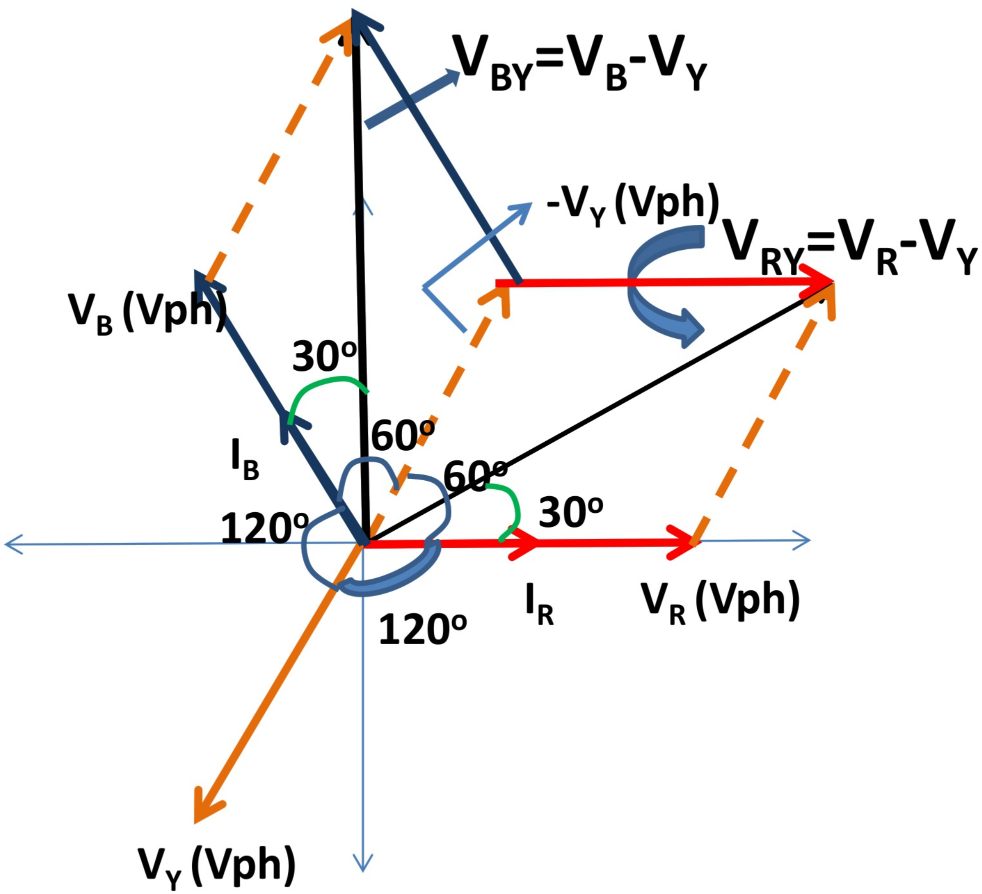
- To find angle between voltage and current , draw phasor diagram for given load

  Here I_R , I_Y and I_B are Line Current (IL) = Phase current (Iph)

  Line voltage V_{RY}, V_{BY} and V_{BR} are vector addition/ subtraction of Phase Voltage (Vph) namely V_R, V_Y and V_B

Measurement of power using Two wattmeter method..

1. Star Connected Resistive Load phasor diagram.



- **Steps to draw phasor diagram**

1. Draw Phase voltages V_R, V_Y and V_B with 120° phase shift between them.
2. Draw phase currents I_R, I_Y and I_B with respect to corresponding phase voltages. Considering nature of the load. (here resistive load so phase voltages and currents are in phase)
3. Find the line voltages from phase voltages. Here V_{RY} and V_{BY} using law of parallelogram.

Note: $-V_Y$ is phasor 180° phase shifted compared to V_Y which is equal and opposite.

4. Find angle between corresponding line current and voltages required to estimate wattmeter readings .

- **Two wattmeter readings**

$W_1= I_R \times V_{RY}$ X cos (angle between I_R and V_{RY}) = $I_L V_L \cos(30)$

$W_2= I_B \times V_{BY}$ X cos (angle between I_B and V_{BY}) = $I_L V_L \cos(30)$

$P= W_1 + W_2 = I_L V_L \cos(30) + I_L V_L \cos(30)= 2 I_L V_L \cos(30) = \sqrt{3} I_L V_L$

Note: cos Φ= 1 in this case since Φ=0 for resistive load i.e. phase current and voltages are in phase.

$W_1 = I_R \times V_{RY} \cos(\text{angle betn } I_R \text{ \& } V_{RY})$

$I_R = I_B = I_L$

$W_2 = I_B \times V_{BY} \cos(\text{angle betn } I_B \text{ \& } V_{BY})$

$V_{RY} = V_{BY} = V_L$

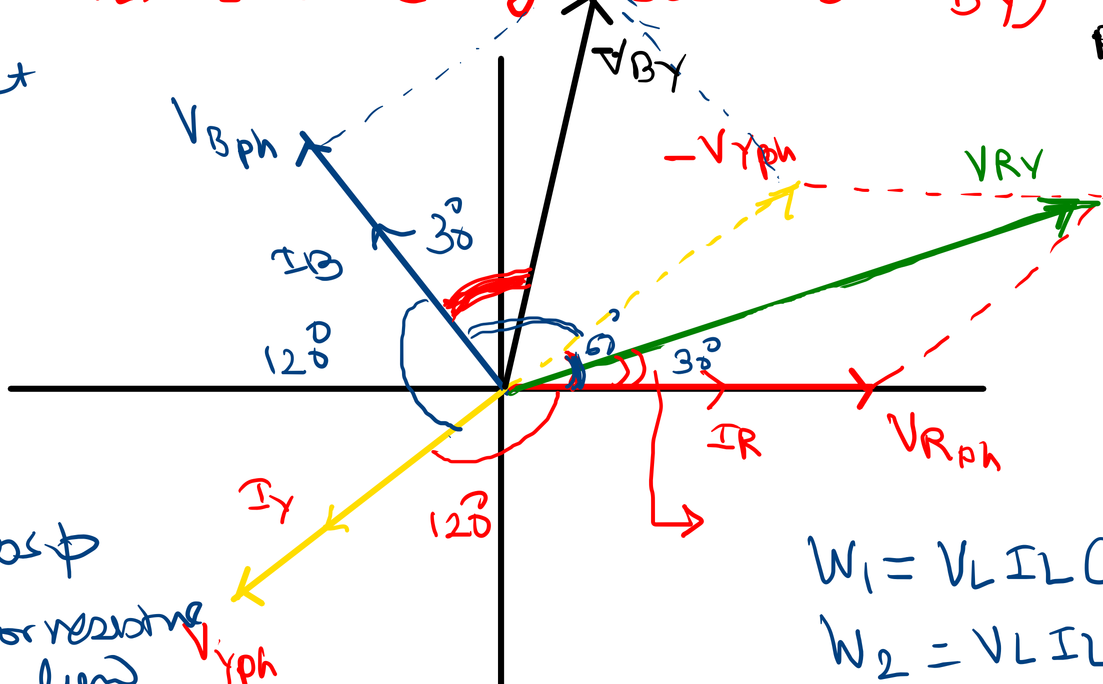
$W_1 = V_L \times I_L \cos(\text{angle betn } I_R \text{ \& } V_{RY})$

$W_2 = V_L \times I_L \cos(\text{angle betn } I_B \times V_{BY})$

Star Connect
Resistive

phase sequence
RYB

$I_{ph} = I_L$



$\bar{V}_{RY} = \bar{V}_{R_{ph}} - V_{Y_{ph}}$

$\bar{V}_{BY} = \bar{V}_{B_{ph}} - \bar{V}_{Y_{ph}}$

$P = \sqrt{3} V_L I_L \cos\phi$

$\cos\phi = 1$ for resistive load

$P = \sqrt{3} V_L I_L$

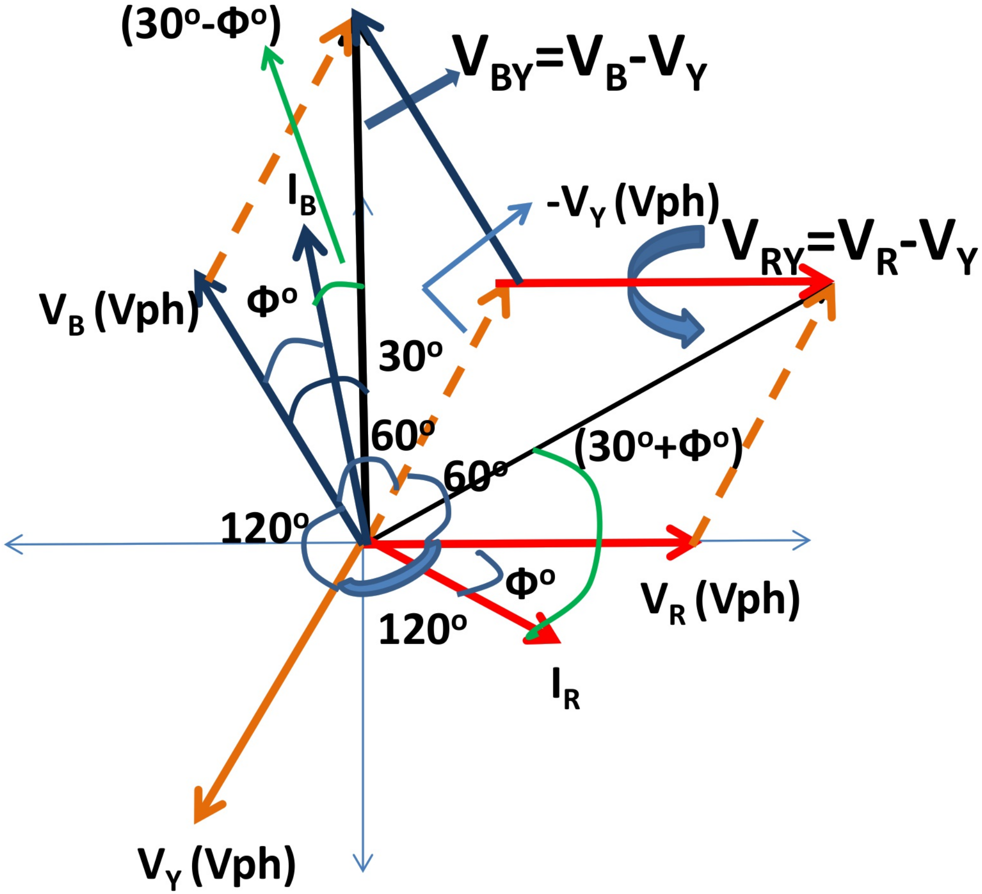
$W_1 = V_L I_L \cos(30°) = \frac{\sqrt{3}}{2} V_L I_L$

$W_2 = V_L I_L \cos(30°) = \frac{\sqrt{3}}{2} V_L I_L$

$P = W_1 + W_2 = \sqrt{3} V_L I_L$

Measurement of power using Two wattmeter method..

2. Star Connected inductive Load phasor diagram.



- **Steps to draw phasor diagram**

1. Draw Phase voltages V_R, V_Y and V_B with 120° phase shift between them.
2. Draw phase currents I_R, I_Y and I_B with respect to corresponding phase voltages. Considering nature of the load. (**here inductive load so phase currents lags phase voltages by Φ°**)
3. Find the line voltages from phase voltages. Here V_{RY} and V_{BY} using law of parallelogram.

Note: $-V_Y$ is phasor 180° phase shifted compared to V_Y which is equal and opposite.

4. Find angle between corresponding line current and voltages required to estimate wattmeter readings .

- **Two wattmeter readings**

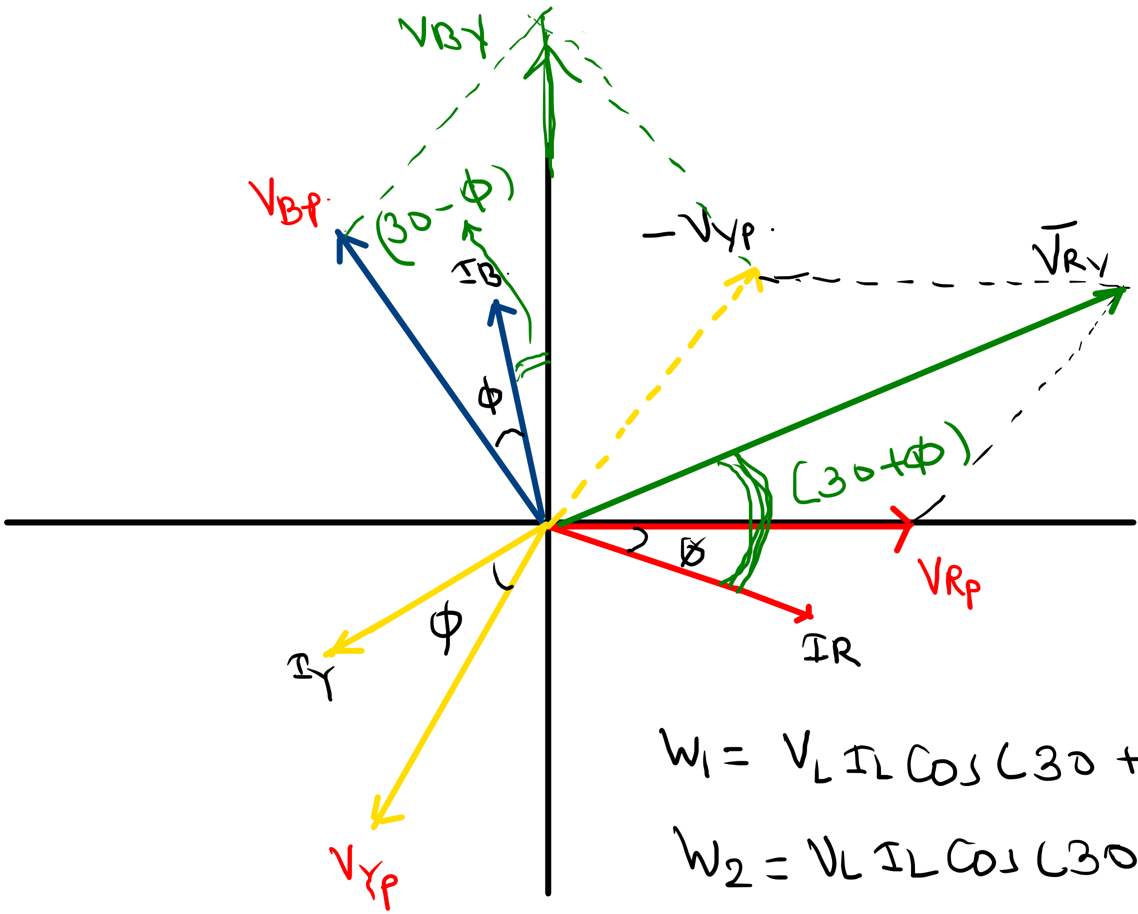
$W_1= I_R \times V_{RY}$ X cos (angle between I_R and V_{RY}) = $I_L V_L \cos(30+\Phi)$

$W_2= I_B \times V_{BY}$ X cos (angle between I_B and V_{BY}) = $I_L V_L \cos(30-\Phi)$

$P= W_1 + W_2 = I_L V_L \cos(30+\Phi) + I_L V_L \cos(30-\Phi)= I_L V_L [\cos(30+\Phi) + \cos(30-\Phi)]$

$= I_L V_L\ 2 \cos[(30+\Phi+30-\Phi)/2] \cos[(30+\Phi-30+\Phi)/2]= I_L V_L\ 2 \cos(30) \cos(\Phi)$

$= \sqrt{3} I_L V_L \cos \Phi$



$$\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$$

$$\bar{V}_{BY} = \bar{V}_B - \bar{V}_Y$$

$$W_1 = V_L I_L \cos(30+\phi)$$

$$W_2 = V_L I_L \cos(30-\phi)$$

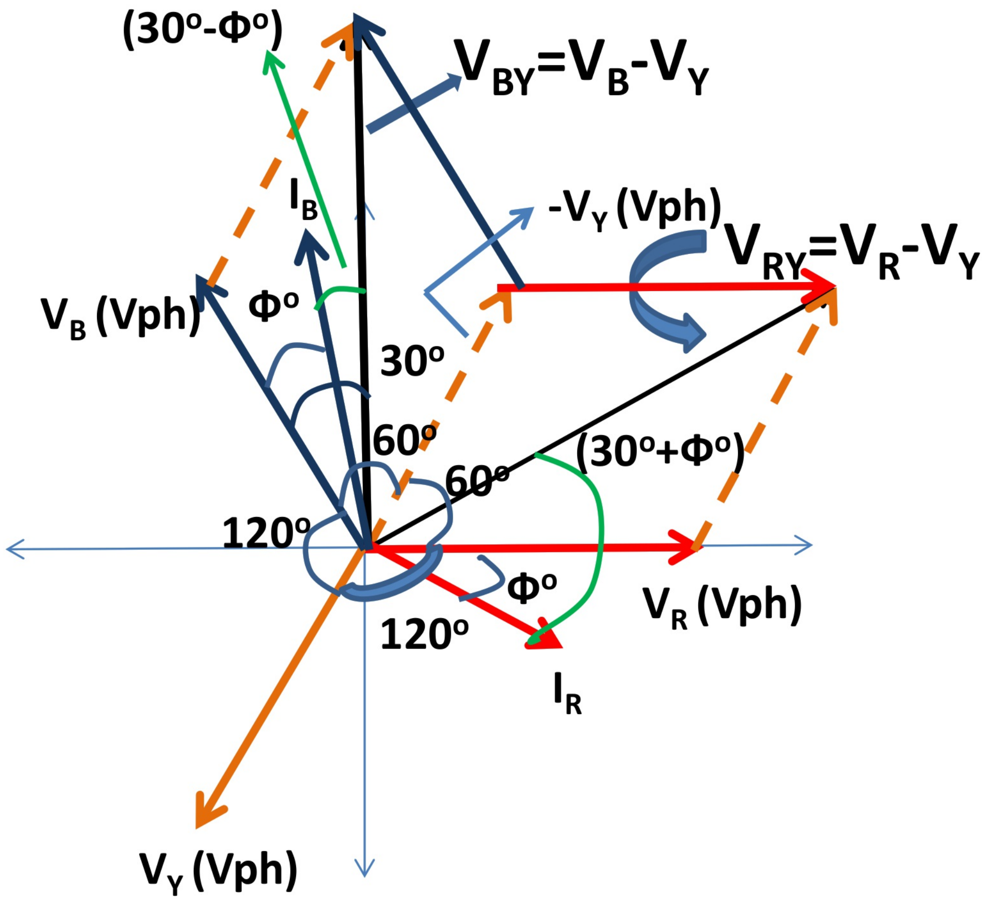
$$P = W_1 + W_2 = V_L I_L [\cos(30+\phi) + \cos(30-\phi)]$$

$$= V_L I_L 2 \left[\cos\left(\frac{30+\cancel{\phi}+30-\cancel{\phi}}{2}\right) \cos\left(\frac{\cancel{30}+\phi-\cancel{30}+\phi}{2}\right) \right]$$

$$= 2 V_L I_L [\cos 30 \cdot \cos\phi] = \sqrt{3} V_L I_L \cos\phi$$

Measurement of power using Two wattmeter method..

2. Star Connected inductive Load phasor diagram.



➢ **Steps to draw phasor diagram**

1. Draw Phase voltages V_R, V_Y and V_B with 120º phase shift between them.
2. Draw phase currents I_R, I_Y and I_B with respect to corresponding phase voltages. Considering nature of the load. (**here inductive load so phase currents lags phase voltages by Φº**)
3. Find the line voltages from phase voltages. Here V_{RY} and V_{BY} using law of parallelogram.

Note: $-V_Y$ is phasor 180º phase shifted compared to V_Y which is equal and opposite.

4. Find angle between corresponding line current and voltages required to estimate wattmeter readings .

➢ **Two wattmeter readings**

W_1= I_R x V_{RY} X cos (angle between I_R and V_{RY}) = I_L V_L cos(30+Φ)

W_2= I_Bx V_{BY} X cos (angle between I_B and V_{BY}) = I_L V_L cos(30-Φ)

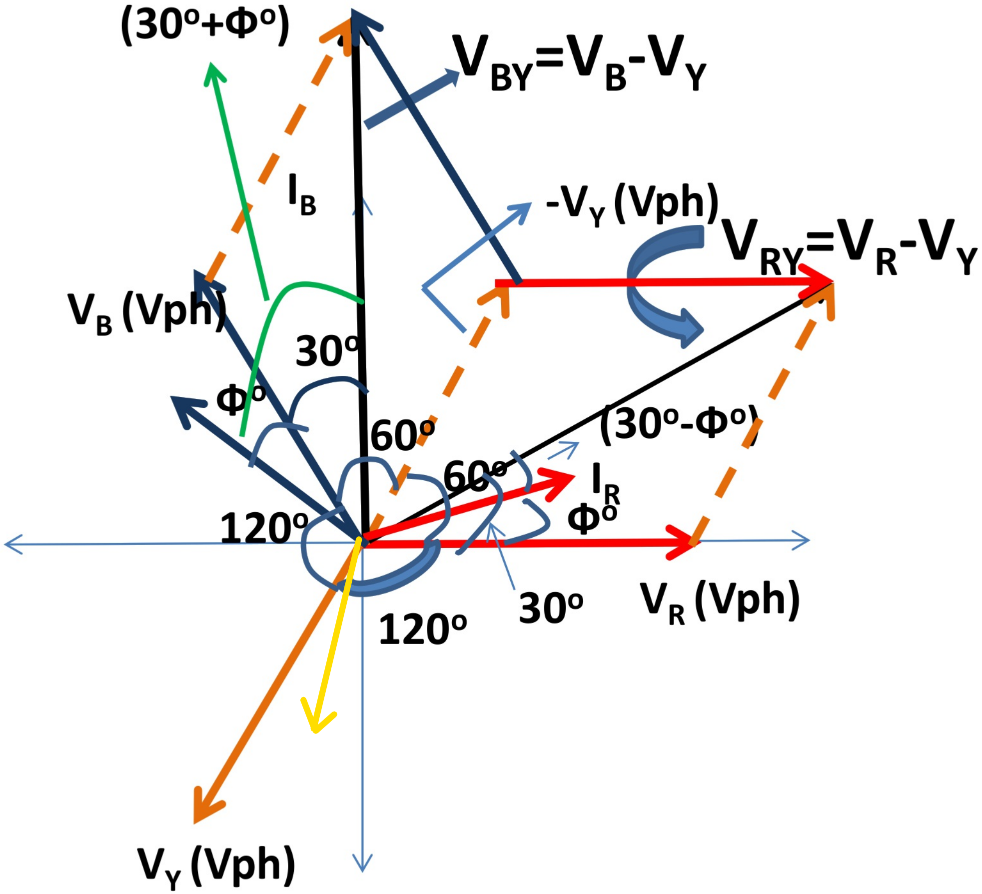
$P = W_1 + W_2 = I_L V_L \cos(30+\Phi) + I_L V_L \cos(30-\Phi) = I_L V_L [\cos(30+\Phi) + \cos(30-\Phi)]$

$= I_L V_L \ 2\cos[(30+\Phi+30-\Phi)/2] \cos[(30+\Phi-30+\Phi)/2] = I_L V_L \ 2\cos(30)\cos(\Phi)$

$= \sqrt{3}\, I_L V_L \cos \Phi$

Measurement of power using Two wattmeter method..

3. Star Connected capacitive Load phasor diagram.



- **Steps to draw phasor diagram**

1. Draw Phase voltages V_R, V_Y and V_B with 120º phase shift between them.
2. Draw phase currents I_R, I_Y and I_B with respect to corresponding phase voltages. Considering nature of the load. (**here capacitive load so phase currents leads phase voltages by Φº**)
3. Find the line voltages from phase voltages. Here V_{RY} and V_{BY} using law of parallelogram.

Note: -V_Y is phasor 180º phase shifted compared to V_Y which is equal and opposite.

4. Find angle between corresponding line current and voltages required to estimate wattmeter readings .

- **Two wattmeter readings**

W_1= I_R x V_{RY} X cos (angle between I_R and V_{RY}) = I_L V_L cos(30-Φ)

W_2= I_Bx V_{BY} X cos (angle between I_B and V_{BY}) = I_L V_L cos(30+Φ)

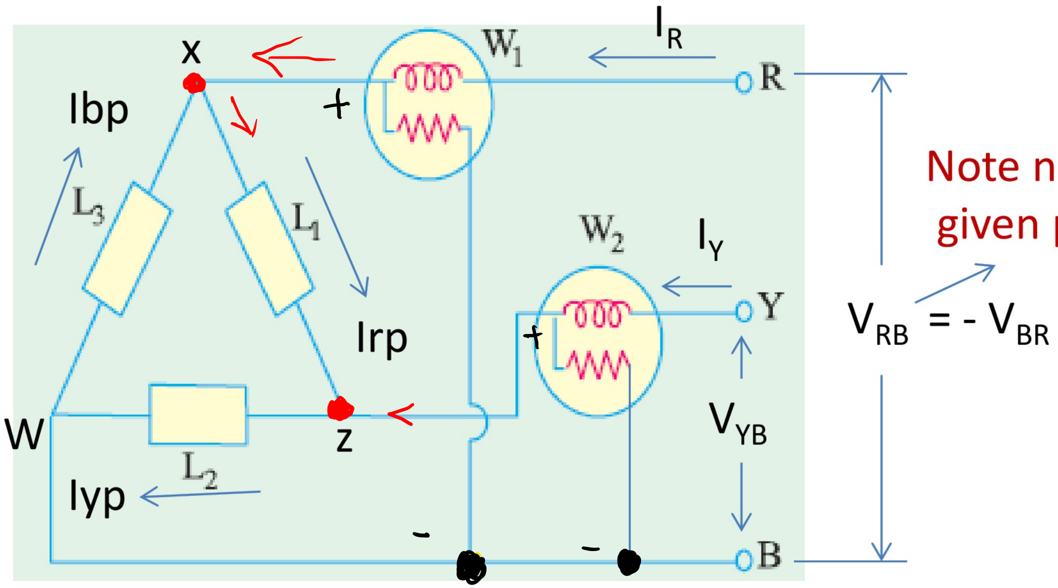
P= W_1 + W_2 = I_L V_L cos(30-Φ) + I_L V_L cos(30+Φ)= I_L V_L [cos(30-Φ) + cos(30+Φ)]

= I_L V_L 2 cos[(30-Φ+30+Φ)/2] cos[(30-Φ-30-Φ) /2]= I_L V_L 2 cos(30) cos(Φ)

= √3 I_L V_L cos Φ

Measurement of power using Two wattmeter method..

- **Delta Connected Load wattmeter connections**



Note node W is at higher potential than X for given phase sequence R-Y-B. so VBR= Vw-Vx .

- **Two wattmeter readings**

W_1= I_R x V_{RB} X cos (angle between I_R and V_{RB}) where $\bar{I}_R$ = $\overline{Irp}$ -$\overline{Ibp}$ using KCL at node x

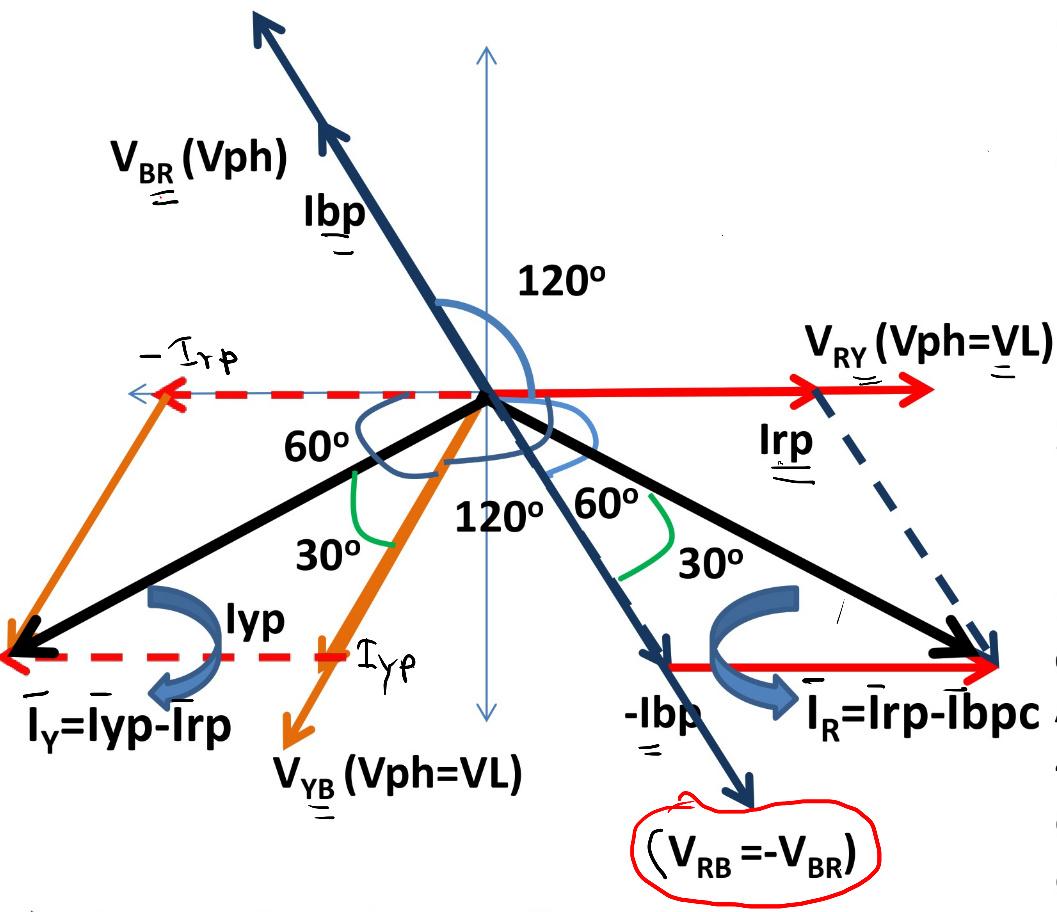
W_2= I_Yx V_{YB} X cos (angle between I_Y and V_{YB}) where $\bar{I}_Y$ = $\overline{Iyb}$- $\overline{Irp}$ using KCL at node Z

- To find angle between voltage and current , draw phasor diagram for given load
  Here V_{RB} , V_{YB} and V_{RY} are Line voltages (VL) = Phase voltages (Vph)

  Line currents I_{RY}, I_Y and I_B are vector addition/ subtraction of Phase currents.

Measurement of power using Two wattmeter method

1. Delta Connected Resistive Load phasor diagram.



- **Steps to draw phasor diagram**

1. Draw Phase voltages I_{RP}, I_{YP} and I_{BP} with 120° phase shift between them.
2. Draw phase voltages V_{RY}, V_{YB} and v_{BR} with respect to corresponding phase current considering nature of the load. (here resistive load so phase voltages and currents are in phase)
3. Find the line currents from phase currents. Here I_R and I_Y using law of parallelogram.

Note: -Ibp is phasor 180° phase shifted compared to Ibp which is equal and opposite. Also VRB is equal and opposite of VBR

4. Find angle between corresponding line current and voltages required to estimate wattmeter readings .

- **Two wattmeter readings**

W_1= I_R x V_{RB} X cos (angle between I_R and V_{RB}) = I_L V_L cos(30)

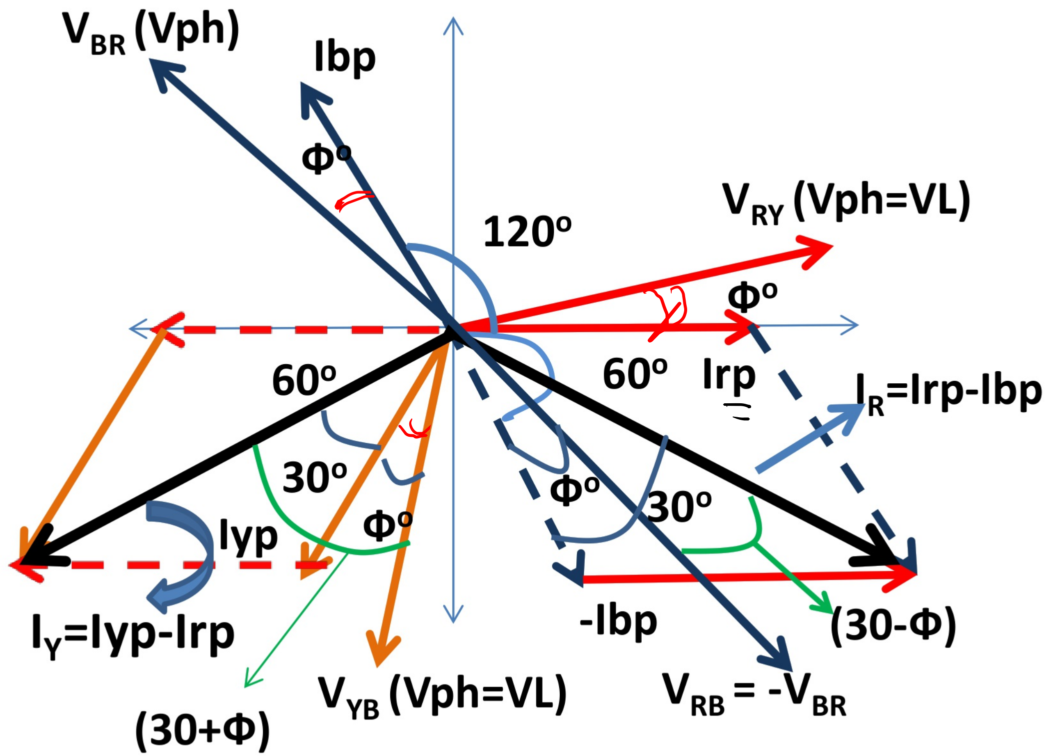
W_2= I_yx V_{YB} X cos (angle between I_Y and V_{YB}) = I_L V_L cos(30)

$P= W_1 + W_2 = I_L V_L \cos(30) + I_L V_L \cos(30)= 2 I_L V_L \cos(30) = \sqrt{3} I_L V_L$

Note: cos Φ= 1 in this case since Φ=0 for resistive load i.e. phase current and voltages are in phase.

Measurement of power using Two wattmeter method

2. Delta Connected inductive Load phasor diagram.



➢ **Steps to draw phasor diagram**

1. Draw Phase voltages I_{RP}, I_{YP} and I_{BP} with 120° phase shift between them.
2. Draw phase voltages V_{RY}, V_{YB} and v_{BR} with respect to corresponding phase current considering nature of the load. (here inductive load so phase voltages leads phase currents by Φ^o .
3. Find the line currents from phase currents. Here I_R and I_Y using law of parallelogram.

Note: -Ibp is phasor 180° phase shifted compared to Ibp which is equal and opposite.
Also VRB is equal and opposite of VBR.

4. Find angle between corresponding line current and voltages required to estimate wattmeter readings .

➢ Two wattmeter readings

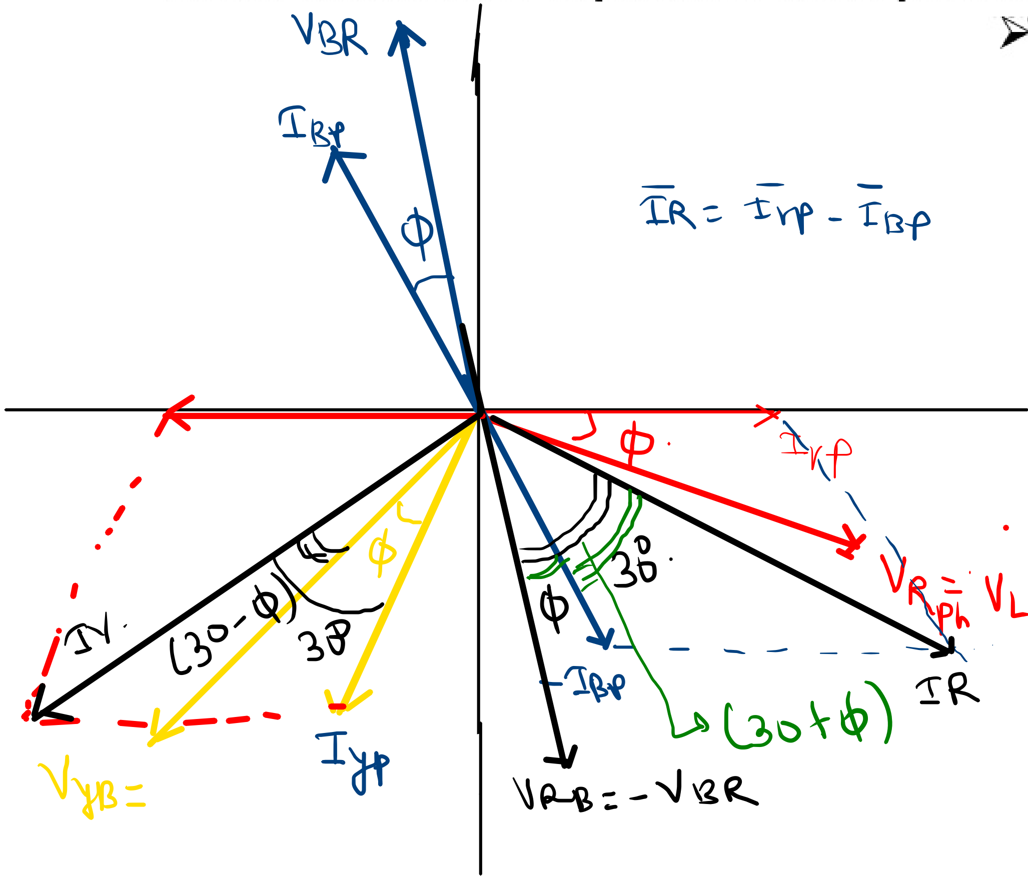
$W_1 = I_R$ x V_{RB} X cos (angle between I_R and V_{RB}) = $I_L V_L \cos(30-\Phi)$

$W_2 = I_y$x V_{YB} X cos (angle between I_Y and V_{YB}) = $I_L V_L \cos(30+\Phi)$

$P = W_1 + W_2 = I_L V_L \cos(30-\Phi) + I_L V_L \cos(30+\Phi) = 2 I_L V_L \cos(30) \cos\Phi = \sqrt{3} I_L V_L \cos\Phi$.

Measurement of power using Two wattmeter method

3. Delta Connected capacitive Load phasor diagram.



➢ **Steps to draw phasor diagram**

1. Draw Phase voltages I_{RP}, I_{YP} and I_{BP} with 120° phase shift between them.
2. Draw phase voltages V_{RY}, V_{YB} and v_{BR} with respect to corresponding phase current considering nature of the load. (Here for capacitive load phase voltages will lag phase current by angle)
3. Find the line currents from phase currents. Here I_R and I_Y using law of parallelogram.

Note: -Ibp is phasor 180° phase shifted compared to Ibp which is equal and opposite.
Also VRB is equal and opposite of VBR.

4. Find angle between corresponding line current and voltages required to estimate wattmeter readings .

$$W_1 = I_R \times V_{RB} \times \cos(30+\phi)$$

$$W_2 = I_Y \times V_{YB} \times \cos(30-\phi)$$

$$P = W_1 + W_2 = V_L I_L [\cos(30+\phi) + \cos(30-\phi)] = \sqrt{3}\, V_L I_L \cos\phi$$

Measurement of power factor using Two wattmeter method

Let for star connected (lagging power factor) Inductive load

- As we know that

$$P = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \quad - - - - - (1)$$

Now,

$$W_1 - W_2 = V_L I_L \cos(30+\phi) - V_L I_L \cos(30-\phi)$$

$$= V_L I_L [\cancel{\cos 30 \cos\phi} + \underline{\sin 30 \sin\phi} - \cancel{\cos 30 \cos\phi} + \underline{\sin 30 \sin\phi}]$$

$$= V_L I_L [2 \sin 30 \sin\phi]$$

$$= V_L I_L \left[\cancel{2} \left(\frac{1}{\cancel{2}} \right) \sin\phi \right] = V_L I_L \sin\phi \quad - - - - (2)$$

Let

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\cancel{V_L I_L} \sin\phi}{\sqrt{3} \cancel{V_L I_L} \cos\phi}$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{1}{\sqrt{3}} \tan\phi$$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \right]$$

$$PF = \cos\phi$$

$$\therefore \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \tan\phi$$

$$\therefore \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

- Power factor of load given as,

$$\therefore \cos\phi = \cos\left(\tan^{-1} \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \right)$$

Measurement of power factor using Two wattmeter method

Let for star connected (lagging power factor) Inductive load

- As we know that

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

Now,

$$W_1 - W_2 = V_L I_L \cos(30+\phi) - V_L I_L \cos(30-\phi)$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi]$$

$$= V_L I_L [2 \sin 30 \sin\phi]$$

$$= V_L I_L \left[2\left(\frac{1}{2}\right)\sin\phi\right] = V_L I_L \sin\phi$$

$$\therefore \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \tan\phi$$

$$\therefore \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

- Power factor of load given as,

$$\therefore \cos\phi = \cos\left(\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right)$$

Effect power factor on watt meter reading in Two wattmeter method

Let for star connected (lagging power factor) Inductive load

$$W_1 = V_L I_L \cos(30+\phi)$$

$$W_2 = V_L I_L \cos(30-\phi)$$

- Thus, readings W_1 and W_2 will very depending upon the power factor angle ϕ.

p.f	ϕ	$W_1 = V_L I_L \cos(30+\phi)$	$W_2 = V_L I_L \cos(30-\phi)$	Remark
$\cos\phi = 1$	0^0	$\frac{\sqrt{3}}{2} V_L I_L$	$\frac{\sqrt{3}}{2} V_L I_L$	Both equal and +ve
$\cos\phi = 0.5$	60^0	0	$\frac{\sqrt{3}}{2} V_L I_L$	One zero and second total power
$\cos\phi = 0$	90^0	$-\frac{1}{2} V_L I_L$	$\frac{1}{2} V_L I_L$	Both equal but opposite

Example:- 1 The input to a 3-phase a.c. motor is measured as 5kW. If the voltage and current are 400 V and 8.6 A, respectively, determine the power factor of the system.

$$\Rightarrow \quad P = \sqrt{3} V_L I_L \cos\phi \quad --- \quad (1)$$

$$P = 5kW \qquad V_L = 400V \qquad I_L = 8.6A$$

From eqn (1)

$$PF = \cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{5000}{\sqrt{3} \times 400 \times 8.6} = 0.839 \text{ (lagging)}$$

Example:-2 Three identical coils, each of resistance 10 Ohm and inductance 42 mH are connected (a) in star and (b) in delta to a 415 V , 50 Hz , 3 phase supply. Determine the total power dissipated in each case.

$R = 10\,\Omega$, $L = 42\,mH$, $f = 50\,Hz$, $V_L = 415\,V$

$\Rightarrow X_L = 2\pi f L = 2 \times \pi \times 50 \times 42 \times 10^{-3} = 13.19\,\Omega$

$\tan\phi = \frac{X_L}{R}$

$\phi = \tan^{-1}\left(\frac{13.19}{10}\right) = 52.8^{\circ}$

$Z_{ph} = (R + jX_L)$

$|Z_{ph}| = \sqrt{(R^2 + X_L^2)} = \sqrt{(10)^2 + (13.19)^2} = 16.55\,\Omega$

$\rightarrow$ Star Connection

$V_L = \sqrt{3}\, V_{ph}$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\,V$

$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{16.55} = 14.46\,A$

$P = \sqrt{3}\, V_L I_L \cos\phi$

$P = \sqrt{3} \times 415 \times 14.46 \times \cos(52.8^{\circ})$

$\boxed{P_Y = 6279.78\ \text{Watts}}$

Delta Connection

$V_L = V_{ph} = 415\,V$

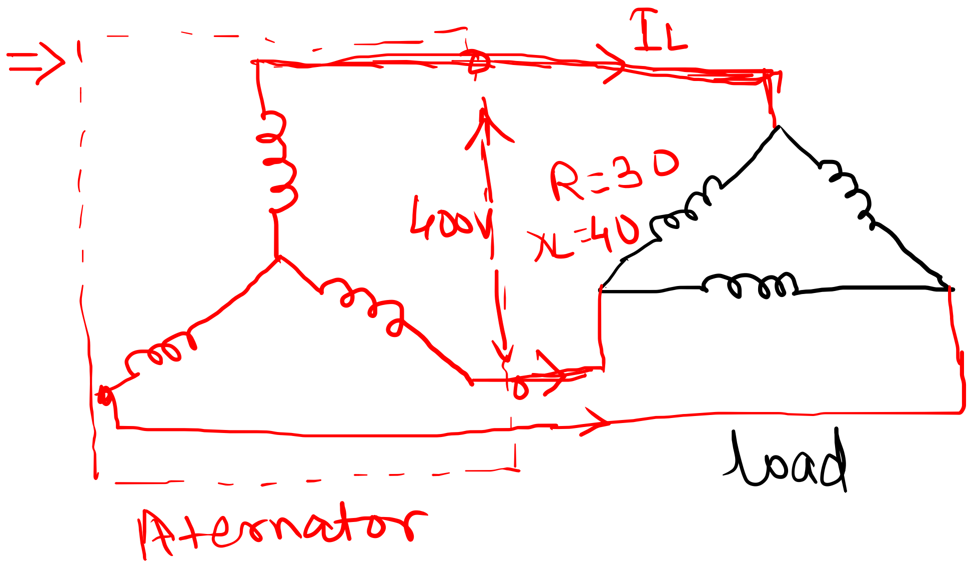
$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{16.55} = 25.07\,A$

$I_L = \sqrt{3}\, I_{ph} = 43.44\,A$

$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 415 \times 43.44 \times \cos(52.8^{\circ})$

$\boxed{P_{\Delta} = 18865.4\ \text{Watts}}$

Example:3 A 400 V , 3-phase star connected alternator supplies a delta-connected load with coil having resistance of 30 Ohm and inductive reactance 40 Ohm. Calculate (a) the current supplied by alternator (b)the output power and the kVA of the alternator, neglecting losses in the line between the alternator and load.



$V_L = 400V$

$Z_{ph} = 30 + j40$

Delta Connected

$V_{ph} = V_L = 400V$

$|Z_{ph}| = \sqrt{30^2 + 40^2} = 50\Omega$

$I_{ph\Delta} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{50}$

$I_{ph\Delta} = 8A$

$I_L = \sqrt{3} \times I_{ph\Delta}$

$= \sqrt{3} \times 8$

(a) $I_L = \sqrt{3} \times 8 = 13.86A$

(b) $P_{out} = \sqrt{3} \times V_L \times I_L \times \cos\phi$

$\cos\phi = \frac{R}{Z} = \frac{30}{50} = 0.6$

$P_{out} = \sqrt{3} \times 400 \times 13.86 \times 0.6$

$P_{out} = 5761.5$ watts

(c) KVA of alternator

$S = \sqrt{3} \times V_L I_L = \sqrt{3} \times 13.86 \times 400$

$S = 9602.5\ VA = 9.6\ KVA$

Example:-5 Two wattmeters are connected to measure the input power to a balanced 3-phase load by two wattmeter method. If the wattmeter readnings are 8kW and 4kW. Determine (a) the total input power and (b) the load power factor.

$\Rightarrow$ $W_1 = 8\,kW$ $W_2 = 4\,kW$

(a) $P = (8+4) = 12\,kW$

(b) Load power factor $= \cos\phi = \cos\left[\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right]$

$= \cos\left[\tan^{-1}\left(\frac{\sqrt{3}(8-4)}{8+4}\right)\right]$

$= \cos\left[\tan^{-1}\left(\frac{\sqrt{3} \times 4}{12}\right)\right]$

Power factor $= 0.866$ (lagging)

Example 6: Two wattmeter connected to a three phase motor indicate the total power input to be 12 kW. The power facctor is 0.6. Determine the readings of each wattmeter.

$\Rightarrow$ $P = 12\,kW$ $\quad pf = \cos\phi = 0.6$

Two wattmeter method

$P = W_1 + W_2$

$W_1 + W_2 = 12\,kW$ ---- (1)

$\cos\phi = 0.6$

$\phi = \cos^{-1}(0.6) = 53.13^{\circ}$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$W_1 - W_2 = (W_1 + W_2)\frac{\tan(53.13)}{\sqrt{3}}$$

$W_1 - W_2 = 9.23$ ---- (2)

Solving (1) & (2)

$W_1 = 10.61$ kWatts

$W_2 = 1.38$ kWatts