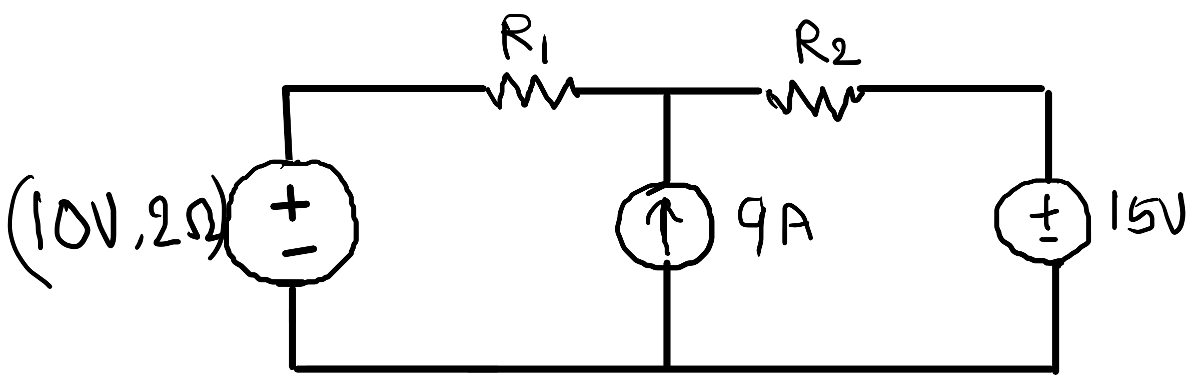
Superposition Theorem

Statement

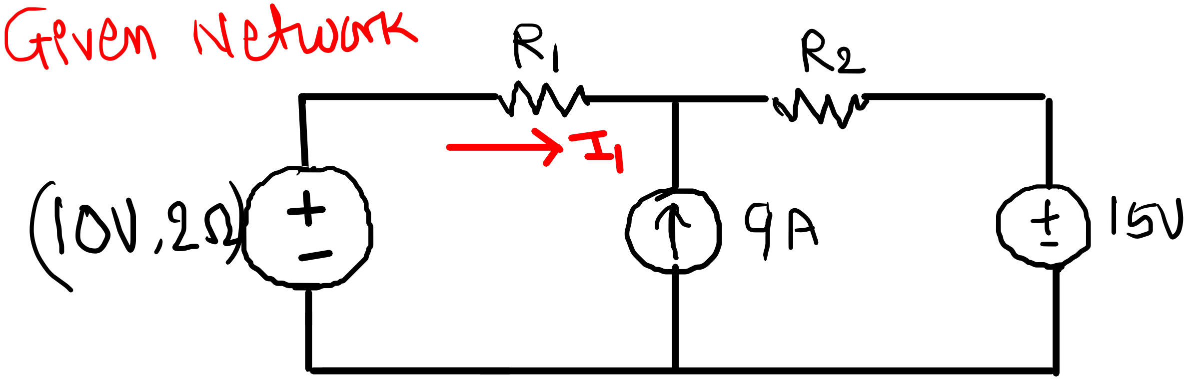
In a network of linear element containing more than one source(Current/ Voltage) of energy , the current flowing/Voltage across any element is the sum of all the currents/Voltages which would result if each source is considered separately and all the other sources replaced for the time being by their internal resistances.

e.g. If the current through R1 is to be found in the network below

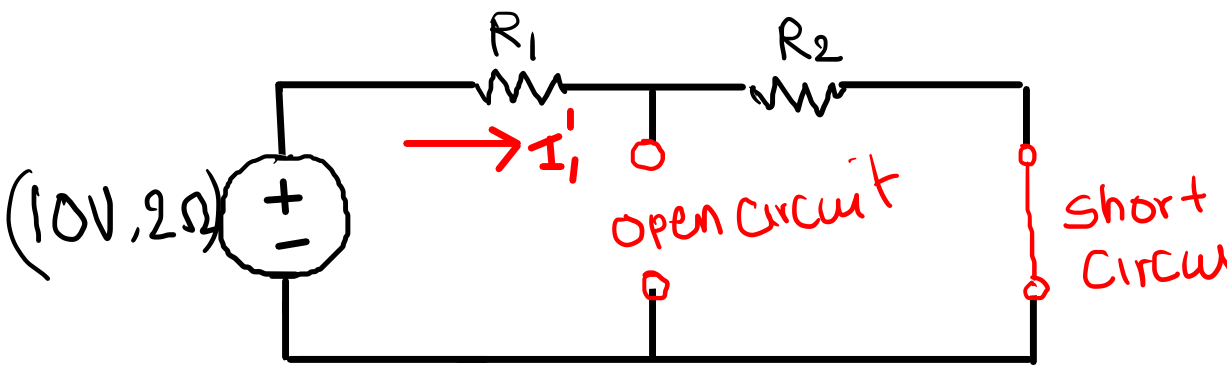


Superposition Theorem

Given Network



1. Consider 10V source in the circuit and replace 9A and 15V by their internal resistances
   Ideal current source has infinite internal resistance (Parallel to it) so It is replaced by open circuit and Ideal Voltage source has zero series resistance so it is replaced by short circuit.

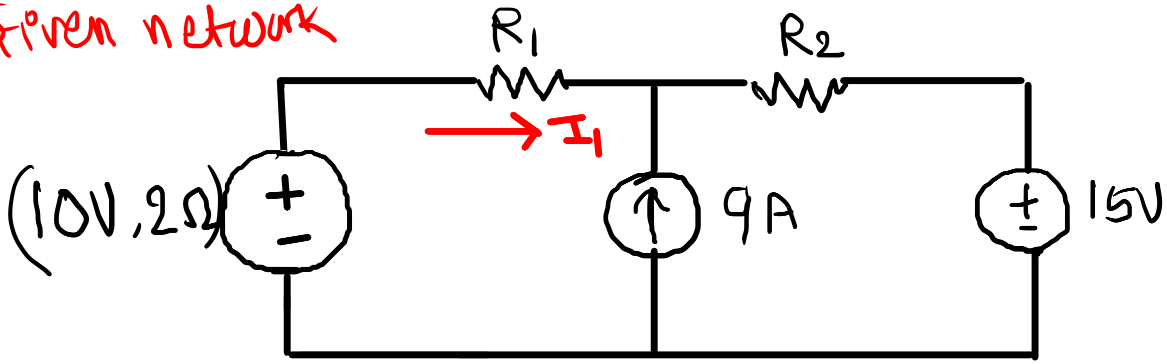


Find I_1' by any suitable analysis method.

e.g. $I_1' = \left(\dfrac{10}{2 + R_1 + R_2}\right)(\rightarrow)$

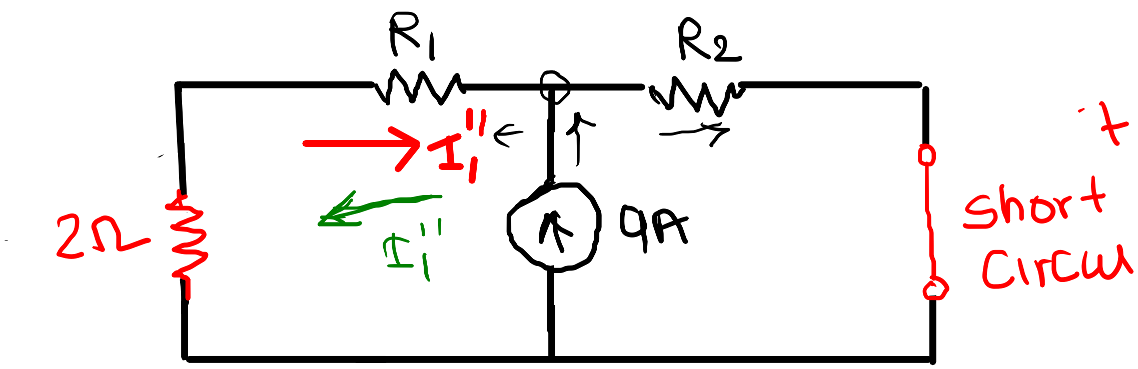
Superposition Theorem

Given network



2. Consider 9A source in the circuit and replace 10V and 15V by their internal resistances

10V source has internal resistance 2 Ohm so it is replaced by 2Ohm resistance.
and Ideal Voltage source has zero series resistance so it is replaced by short circuit.



Find I_1'' by any suitable analysis method.

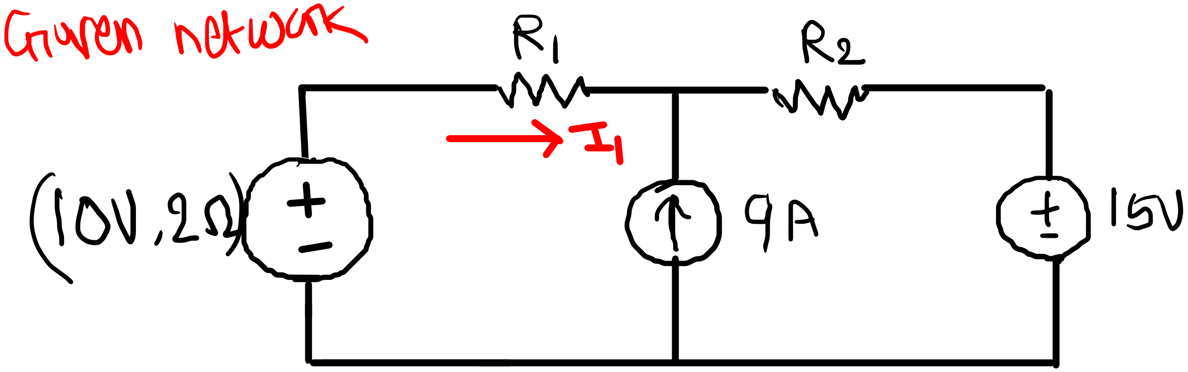
$$I_1'' = -\left(\frac{R_2 \times 9}{R_1+R_2+2}\right)(\rightarrow)$$

↑ Using Current division formula

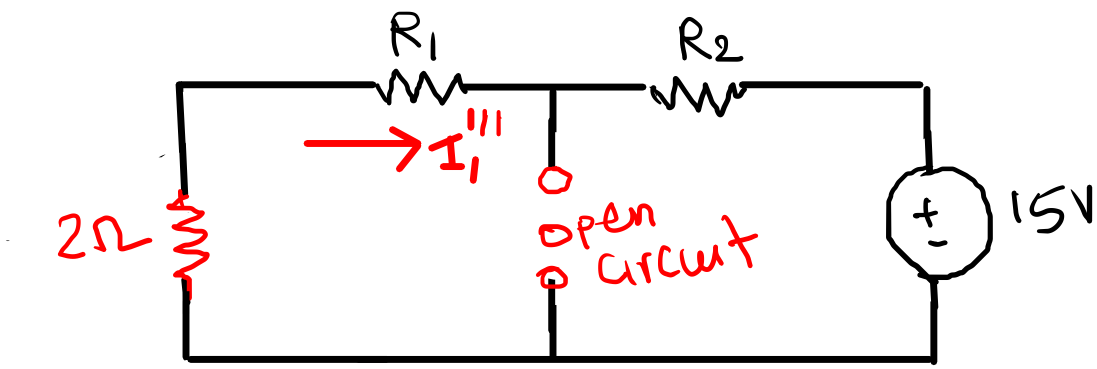
$$I_1'' = \frac{R_2 \times 9}{R_1+R_2+2}\,(\leftarrow)$$

Superposition Theorem

Given network



3. Consider 15 V source in the circuit and replace 10V and 9A by their internal resistances
 10V source has internal resistance 2 Ohm so it is replaced by 2Ohm resistance.
 and Ideal 9A current source is replaced by open circuit.



$$-2I_1''' - R_1 I_1''' - R_2 I_1''' - 15 = 0$$

Find I_1''' by any suitable analysis method.

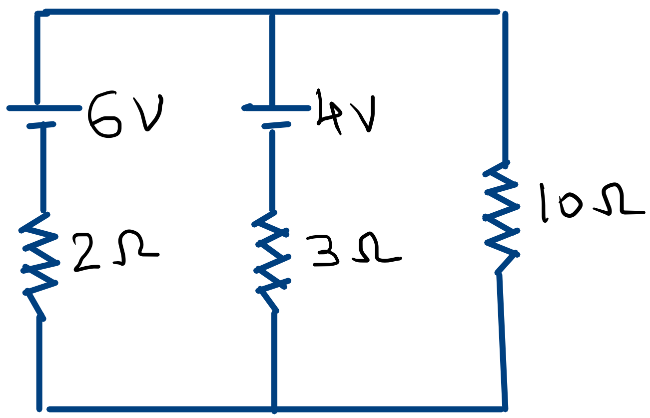
$$I_1''' = -\left(\frac{15}{R_1+R_2+2}\right)(\rightarrow)$$

4. The current flowing through R1 is $I_1 = I_1' + I_1'' + I_1'''$

Superposition Theorem

Q.1 Find current flowing through 10 Ohm resistance using superposition principles.

0.464A



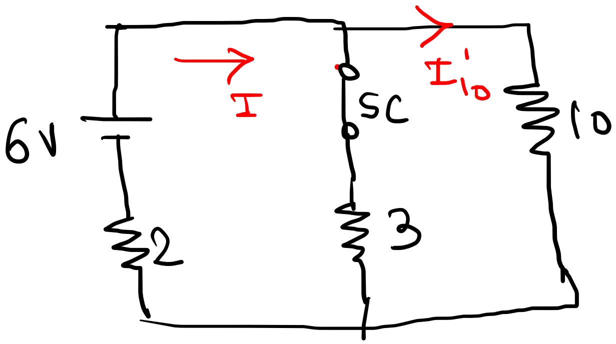
$R_T = (10 \| 3) + 2$

$R_T = \frac{30}{13} + 2 = 4.3\,\Omega$

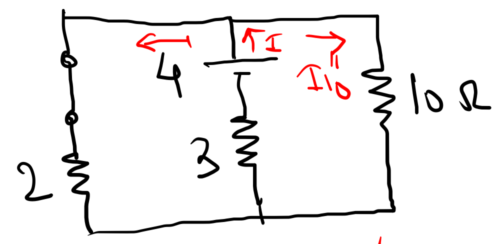
$I = \frac{6}{R_T} = \frac{6}{4.3\Omega} = 1.39$ | $I_{10}' = \frac{I \times 3}{3+10}$

$I_{10}' = \frac{1.39 \times 3}{13} = 0.32\,A\,(\downarrow)$

① Consider 6V source & SC 4V



→ ② Consider 4V source & SC 6V



$R_T = (10 \| 2) + 3$

$R_T = \frac{20}{12} + 3$

$R_T = 4.67\,\Omega$

$I = \frac{4}{4.67} = 0.86\,A$

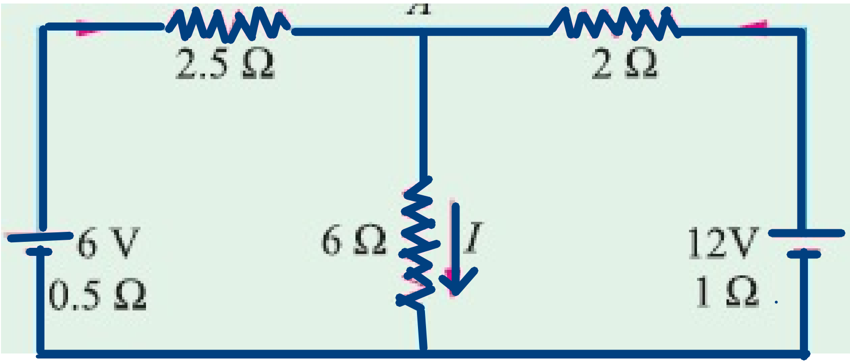
$I_{10}'' = \frac{0.86 \times 2}{10+2} = 0.14\,A\,(\downarrow)$

③ $I_{10} = I_{10}' + I_{10}'' = 0.32 + 0.14 = 0.46\,A\,(\downarrow)$

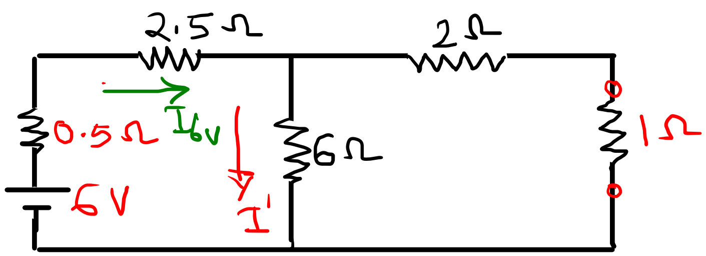
Superposition Theorem

Q.2 Find current flowing through 6 Ohm resistance using superposition principles.

1.2A



(I) Considering 6V source & replace 12V by its internal resistance

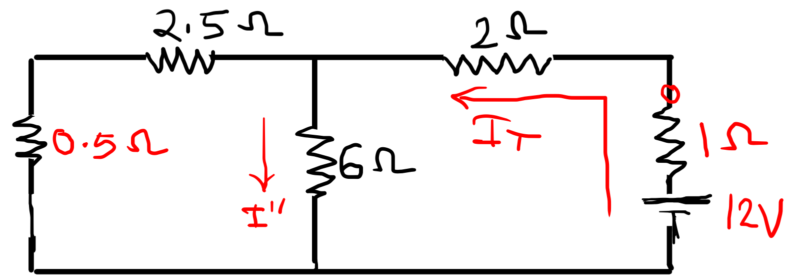


$R_T = (6 \| (2+1)) + (2.5+0.5)$

$R_T = (6 \| 3) + 3 = 5\Omega$

$I_{6V} = \frac{6}{5} = 1.2A$ | $I' = \frac{1.2 \times 3}{6+3} = 0.4A \downarrow$

(II) Considering 12V source & replace 6V by its internal resistance



$R_T = ((2.5+0.5) \| 6) + (2+1)$

$R_T = (3 \| 6) + 3 = 5\Omega$

$I_T = \frac{12}{5} = 2.4A$

$I'' = \frac{2.4 \times 3}{3+6} = 0.8A \downarrow$

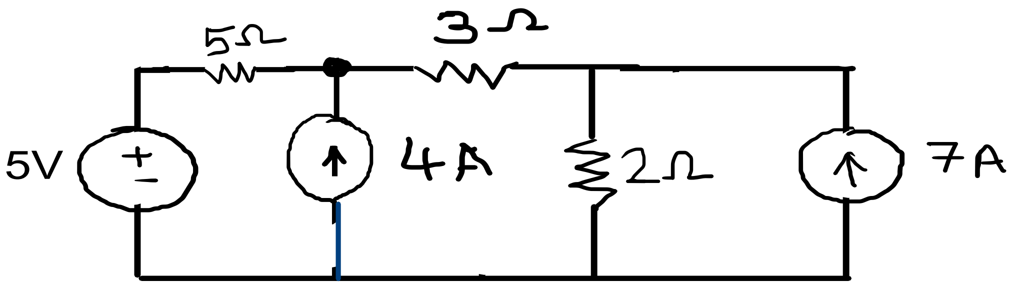
(III) $I = I' + I'' = (0.4 + 0.8)$

$\boxed{I = 1.2A \downarrow}$

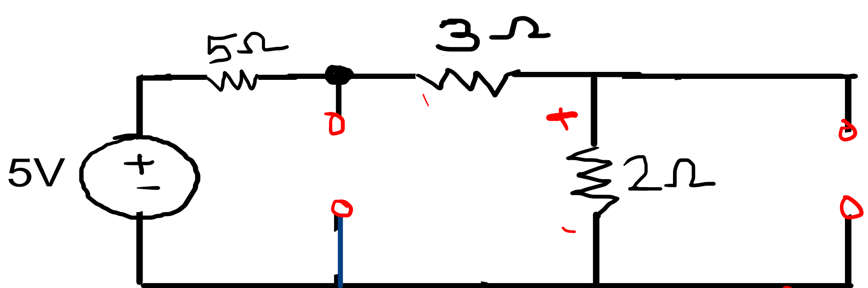
Superposition Theorem

Example:- 3. Find voltage across 2 Ohm resistor in the following network.

16.2V



① Consider 5V source & replace 4A & 7A by open circuit.

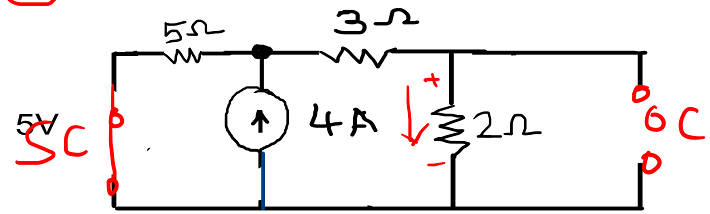


using voltage division formula

$$V'_{2\Omega} = \frac{5\times 2}{5+3+2} = \frac{10}{10} = 1V$$

$$\boxed{V_{2\Omega} = V'_{2\Omega} + V''_{2\Omega} + V'''_{2\Omega} = 1+4+11.2 = 16.2V}$$

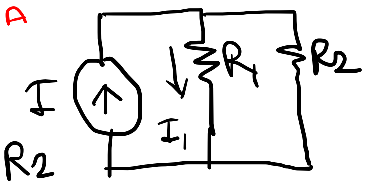
② Consider 4A, SC 5V & OC 7A



using current division formula

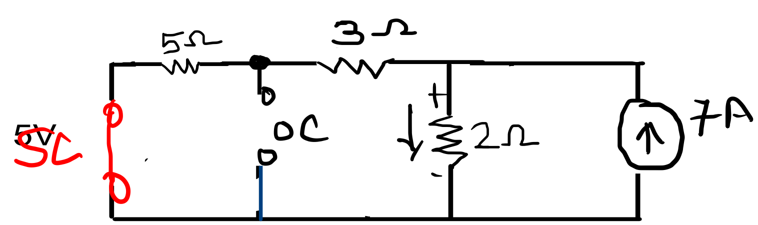
$$I_{2\Omega} = \frac{4\times 5}{5+3+2} = 2A$$

$$V''_{2\Omega} = 2\times 2 = 4V$$



$$I_1 = \frac{I\times R_2}{R_1+R_2}$$

③ Consider 7A & replace 5V by SC & 4A OC.



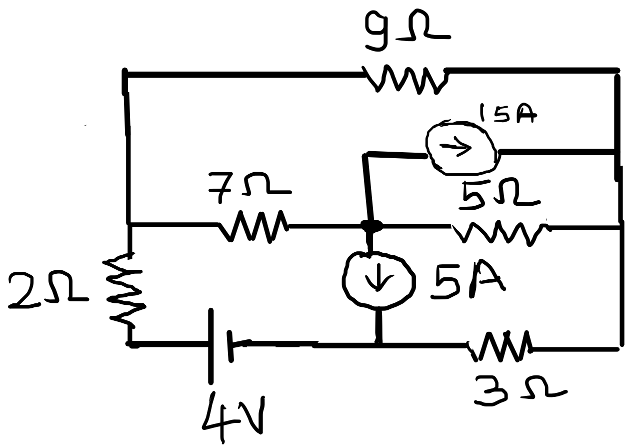
$$I_{2\Omega} = \frac{7\times 8}{10} = 5.6A$$

$$V'''_{2\Omega} = 5.6\times 2 = 11.2V$$

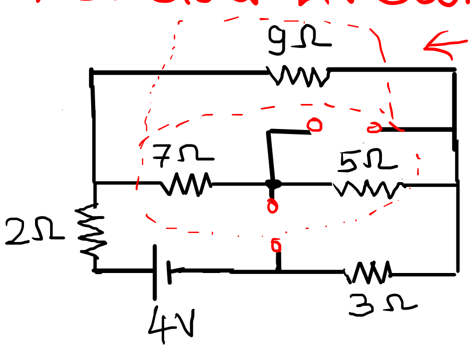
Superposition Theorem

Example:- 4. Find current through 3 Ohm resistor in the following network.

$I_3 = 1.09\,A$

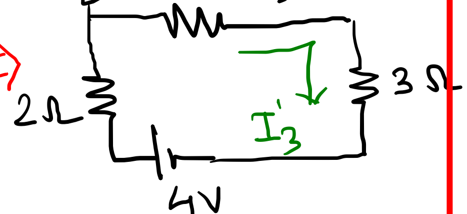


⇒ Consider 4V source & OC 15A & 5A.



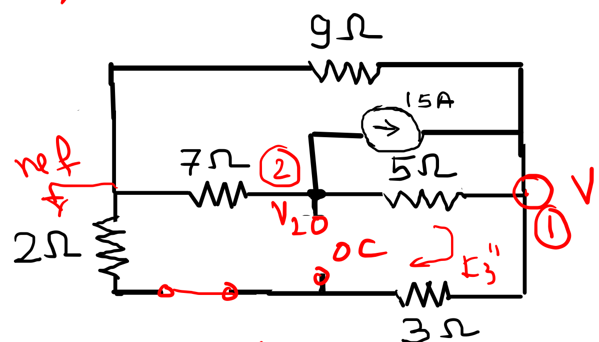
⇒

$[(7+5)||9] = 5.14\Omega$



$$I_3' = \frac{4}{2+5.14+3} = \frac{4}{10.14} = 0.39A(\downarrow)$$

⇒ Consider 15A source, SC 4V & OC 5A



Using Nodal Analysis.

KCL at node (1) $\frac{V_1}{9} + \frac{V_1}{5} + \frac{V_1 - V_2}{5} = 15$

$$5V_1 + 9V_1 + 9V_1 - 9V_2 = 675$$

$$23V_1 - 9V_2 = 675 \quad \text{---(1)}$$

KCL at node (2)

$$\frac{V_2}{7} + \frac{V_2 - V_1}{5} + 15 = 0$$

$$5V_2 + 7V_2 - 7V_1 + 525 = 0$$

$$7V_1 - 12V_2 = 525 \quad \text{---(2)}$$

$V_1 = 15.84V$, $V_2 = -34.5V$

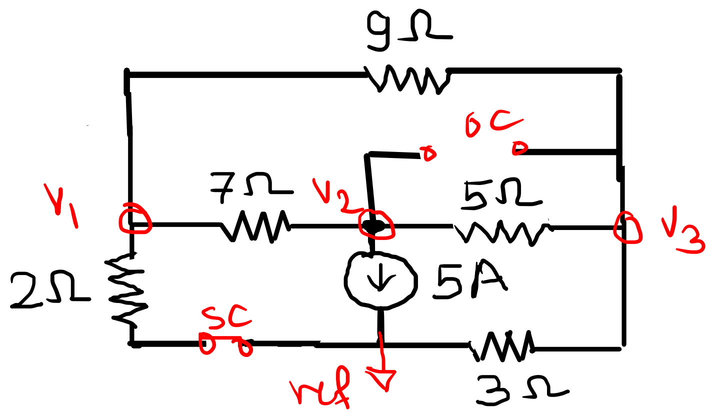
$$I_{3\Omega}'' = \frac{V_1}{5} = \frac{15.84}{5} = 3.168A(\downarrow)$$

Superposition Theorem

Example:- 4. .....

$I_3 = 1.09A$

⇒ Consider 5A Source



Using Nodal Analysis

KCL at node (1)

$$\frac{V_1}{2} + \frac{V_1 - V_2}{7} + \frac{V_1 - V_3}{9} = 0$$

$$\frac{31.5V_1 + 9V_1 - 9V_2 + 7V_1 - 7V_3}{63} = 0$$

$$47.5V_1 - 9V_2 - 7V_3 = 0 \quad \text{--- (1)}$$

KCL at node (2)

$$\frac{V_2 - V_1}{7} + \frac{V_2 - V_3}{5} = -5$$

$$5V_2 - 5V_1 + 7V_2 - 7V_3 = -175$$

$$-5V_1 + 12V_2 - 7V_3 = -175 \quad \text{--- (2)}$$

KCL at node (3)

$$\frac{V_3}{3} + \frac{V_3 - V_2}{5} + \frac{V_3 - V_1}{9} = 0$$

$$15V_3 + 9V_3 - 9V_2 + 5V_3 - 5V_1 = 0$$

$$-5V_1 - 9V_2 + 29V_3 = 0 \quad \text{--- (3)}$$

Solving (1) (2) & (3) $V_1 = -5.07V$, $V_2 = -21$

$V_3 = -7.39V$ $\therefore I_3'' = \frac{V_3}{3} = -2.46A(\downarrow)$

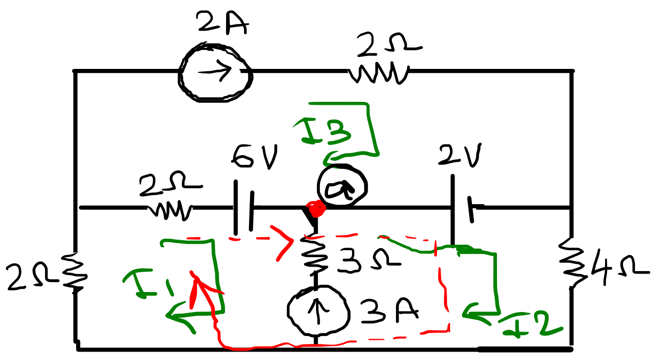
$$\boxed{I_{3\Omega} = 3.17 + 0.39 - 2.46 = 1.09A}$$

Practice Numerical solved using all methods

Example:- Determine current through 4 Ohm resistor

$I_3 = 1.09\,A$

1 Using Mesh Analysis



→ since 2A current source is residing on uncommon branch of mesh III

So $I_3 = 2A$

→ since 3A resides on common branch of mesh I & II so its supermesh

$I_1 + 3 = I_2 \quad \therefore I_1 - I_2 = -3 \quad \text{---} \; (1)$

KVL to supermesh

$-2(I_1 - I_3) - 6 - 2 - 4I_2 - 2I_1 = 0$

$-2I_1 + 2I_3 - 8 - 4I_2 - 2I_1 = 0$

$-4I_1 - 4I_2 = 8 - 2\times 2$

$4I_1 + 4I_2 = -4 \quad \text{---} \; (2)$

Solving (1) & (2)

$I_1 = -2A, \; I_2 = 1A$

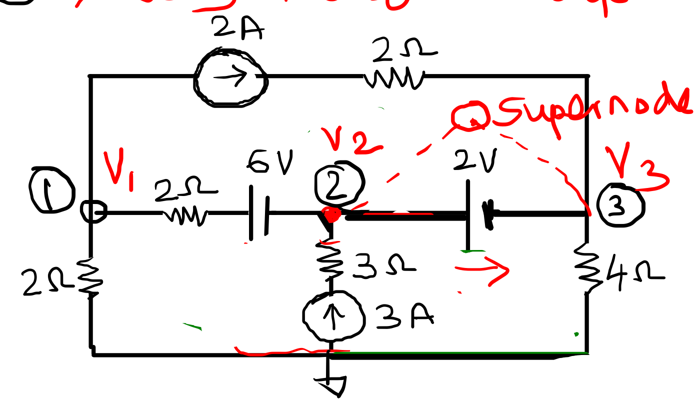
$\boxed{I_{4\Omega} = I_2 = 1A.}$

Practice Numerical solved using all methods

Example:- Determine current through 4 Ohm resistor mesh, Nodal, SPT, TH, NT

$I_3 = 1.09A$

(2) ⇒ Using Nodal Analysis



→ 2v source without series resistance resides betn node (2) & (3) so its supernode case

So $V_2 - 2 - V_3 = 0$

$V_2 - V_3 = 2$ ---- (1)

→ KCL to supernode

$$\frac{V_2+6-V_1}{2} - 3 + \frac{V_3}{4} - 2 = 0$$

$$\frac{2V_2+12-2V_1+V_3}{4} = 5$$

$-2V_1 + 2V_2 + V_3 = 20 - 12$

$-2V_1 + 2V_2 + V_3 = 8$ ---- (2)

→ KCL at node (1)

$$\frac{V_1-6-V_2}{2} + 2 + \frac{V_1}{2} = 0$$

$V_1 - 6 - V_2 + 4 + V_1 = 0$

$2V_1 - V_2 = 2$ ---- (3)

Solving (1), (2) & (3) $V_1 = 4V$ $V_2 = 6V$

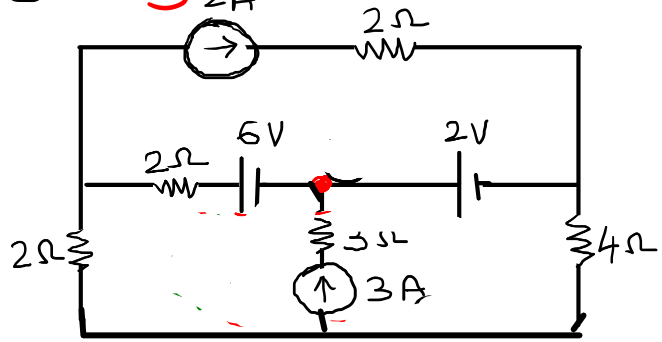
$V_3 = 4V$ $I_{4\Omega} = \frac{V_3}{4} = \frac{4}{4} = 1A \downarrow$

Practice Numerical solved using all methods

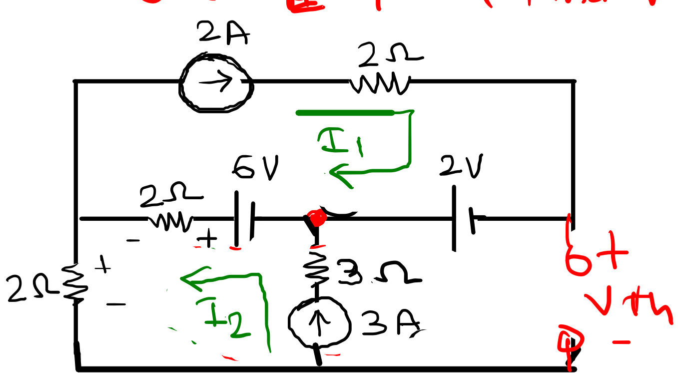
Example:- Determine current through 4 Ohm resistor

$I_3 = 1.09\,A$

③ Using Thevenin's Theorem.



→ Remove $R_L = 4\Omega$ & find V_{th}



$$V_{th} + 2 + 6 - V_{2\Omega} - V_{2\Omega} = 0$$

Using mesh Analysis

$I_1 = 2A$, $I_2 = 3A$

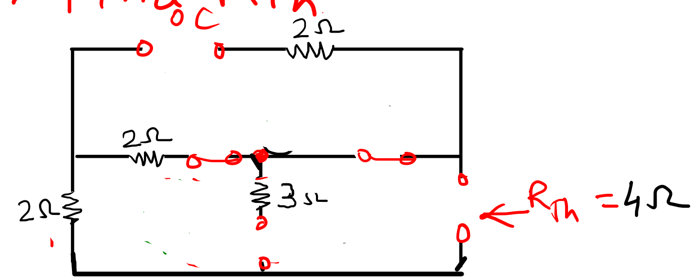
$$V_{th} + 8 - 2(I_1 + I_2) - 2I_2 = 0$$

$$V_{th} + 8 - 2(5) - 2 \times 3 = 0$$

$$V_{th} + 8 - 10 - 6 = 0$$

$$V_{th} = 8V$$

→ Find R_{Th}



→ Draw thevenin's Eq. circuit & connect load

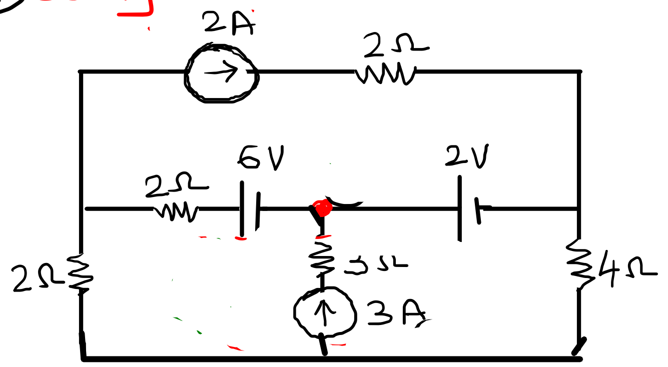
$$I_{4\Omega} = \frac{8}{4+4} = 1A$$

Practice Numerical solved using all methods

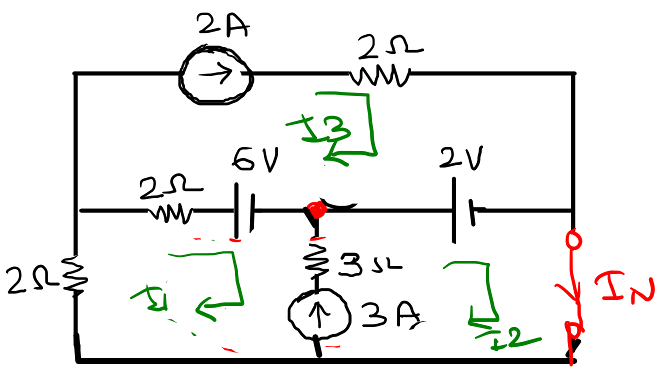
Example:- Determine current through 4 Ohm resistor

$I_3 = 1.09A$

4 Using Norton's Theorem.



→ Remove load, short circuit terminals & find I_N.



Using mesh Analysis.

$I_3 = 2A$,

Supermesh case

So $I_1 - I_2 = -3$ ---- (1)

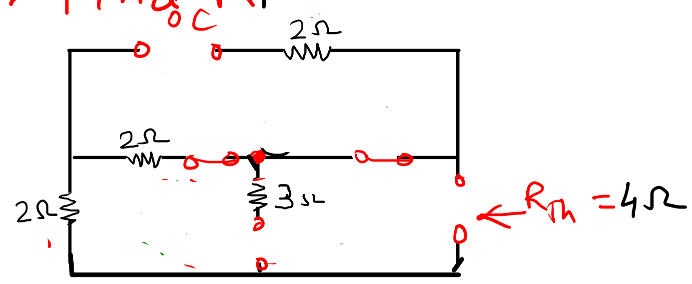
KVL to supermesh

$-2I_1 - 2(I_1 - I_3) - 6 - 2 = 0$

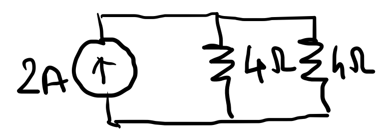
$\therefore I_1 = -1A$, $\therefore I_2 = -1 + 3 = 2A$

$I_N = I_2 = 2A$

→ Find R_N



→ Draw Norton's Eq. circuit & Connect load



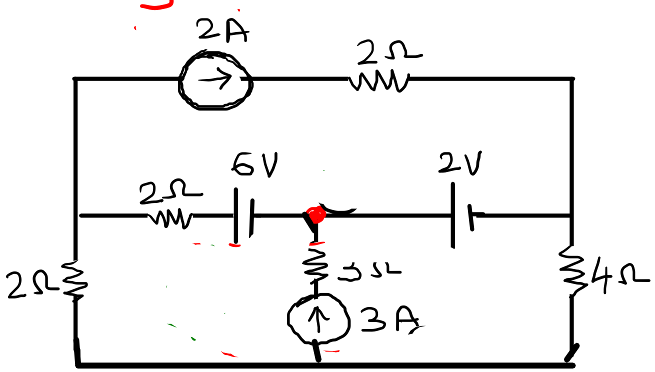
$I_{4\Omega} = \frac{2 \times 4}{4+4} = 1A$

Practice Numerical solved using all methods

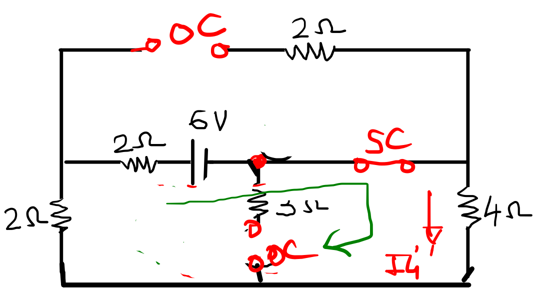
Example:- Determine current through 4 Ohm resistor

$I_3 = 1.09\,A$

(5) ⇒ Using Superposition Theorem



⇒ Consider 6V source & OC 2A & 3A, SC 2V

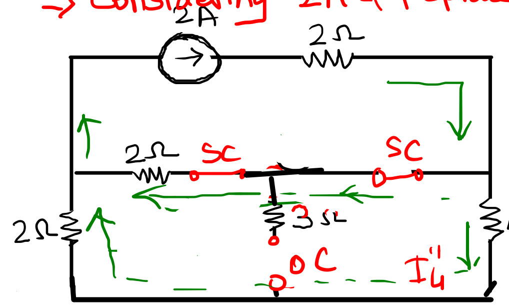


KVL to loop

$$-2I_4' - 2I_4' - 6 - 4I_4' = 0$$

$$I_4' = -\frac{6}{8} = -\frac{3}{4}\,A\,(\downarrow) \;---\; (1)$$

⇒ Considering 2A & replace other sources

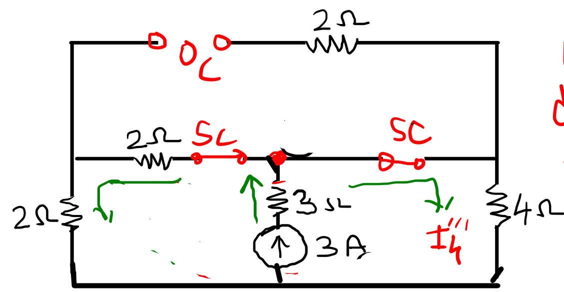


Using Current division formula

$$I''_{4\Omega} = \frac{2\times(2)}{4+2+2}$$

$$I''_{4\Omega} = \frac{4}{8} = \frac{2}{4}\,A\,(\downarrow) \;--\; (2)$$

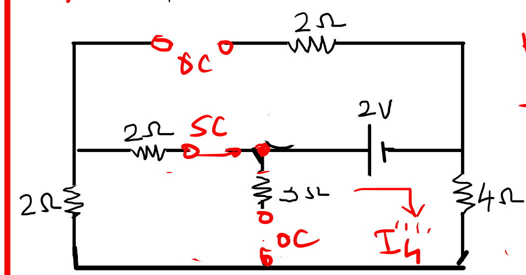
⇒ Consider 3A & replace other sources



Using Current division formula

$$I'''_{4\Omega} = \frac{3\times(2+2)}{4+2+2} = \frac{12}{8} = \frac{6}{4}\,A\,(\downarrow) \;--\; (3)$$

⇒ Consider 2V source & replace other sources.



KVL to loop

$$-2I''''_{4\Omega} - 2I''''_4 - 2 - 4I''''_4 = 0$$

$$I''''_4 = -\frac{2}{8} = -\frac{1}{4}\,(\downarrow) \;-\; (4)$$

Adding (1) (2) (3) & (4)

$$I_4 = -\frac{3}{4} + \frac{2}{4} + \frac{6}{4} - \frac{1}{4} = 1\,A\,(\downarrow)$$