

Example At 2:10 a 3-pound weight is hung from the end of the 3-foot long minute hand of a clock with a vertical clock face. Find the resulting torque (at the center of the clock).

First we need to set up a coordinate system. As it is traditional to consider the z -axis as pointing straight up, we will consider the clock face to be in the yz -plane, with the center at the origin, 12:00 on the positive z -axis, and 3:00 on the positive y -axis. We will label our axes in units of feet.

The minute hand is 3 feet long and at 2:10 it makes an angle of 30 degrees, or $\frac{\pi}{6}$, with the positive y -axis, so the coordinates of its tip are

$$\left(0, 3 \cos \frac{\pi}{6}, 3 \sin \frac{\pi}{6}\right) = \left(0, \frac{3\sqrt{3}}{2}, \frac{3}{2}\right).$$

These are also the coordinates of the vector \vec{r} from the center of the clock (the origin) to the end of the minute hand:

$$\vec{r} = \left\langle 0, \frac{3\sqrt{3}}{2}, \frac{3}{2} \right\rangle.$$

The force vector (the force in question being the gravitational force acting at the 3 pound weight) has magnitude 3 pounds and points straight downwards, so

$$\vec{F} = \langle 0, 0, -3 \rangle.$$

The torque is the cross product of \vec{r} and \vec{F} (in units of foot-pounds):

$$\vec{T} = \vec{r} \times \vec{F} = \left\langle 0, \frac{3\sqrt{3}}{2}, \frac{3}{2} \right\rangle \times \langle 0, 0, -3 \rangle =$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \frac{3\sqrt{3}}{2} & \frac{3}{2} \\ 0 & 0 & -3 \end{vmatrix} = \left\langle -\frac{9\sqrt{3}}{2}, 0, 0 \right\rangle.$$

Note that this has the correct direction, into the clock face, as the weight hung on the minute hand in that position tends to produce a clockwise rotation.