Workshop 1
Induction Exercises

Exercise 1. Prove, by induction, that for any real number $r \neq 1$ and any integer $n \geq 1$,

$$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

Exercise 2. Use induction to prove that for every integer $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Exercise 3. Let $x > -1$. Prove, by induction, that for every integer $n \geq 1$,

$$(1 + x)^n \geq 1 + nx.$$

Exercise 4.* Use induction to prove the following.

a. For all integers $n \geq 1$, $n(n + 1)$ is even.

b. For all integers $n \geq 1$, $n^3 - n$ is divisible by 6.

Exercise 5.* Let a_n denote the number of subsets of $\{1, 2, 3, \ldots, n\}$ (including the empty set and the set itself).

a. Show that $a_n = 2a_{n-1}$. (No need for induction here.)

b. Guess a formula for the value of a_n and use induction to prove you are right.