Math 11, Fall 2007

Lecture 15

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Outline

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Integration of a function of two variables

- Iterated integrals and Fubini's theorem

$$\int\int_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dx\,dy$$

Outline

1. Review and overview
   - Last class
2. Today's material
   - Integration in two variables
3. Group Work
4. Next class

Integration over complicated regions

- If the domain of integration is not a rectangle, then Fubini's Theorem cannot be directly applied.
- One idea: Any bounded region, D, is contained in a rectangle, R. Extending f to a function on R (by making it zero on $R \setminus D$), we have

$$\int_D f(x,y)dA = \int_R f(x,y)\, dA$$

Now we may use Fubini's Theorem

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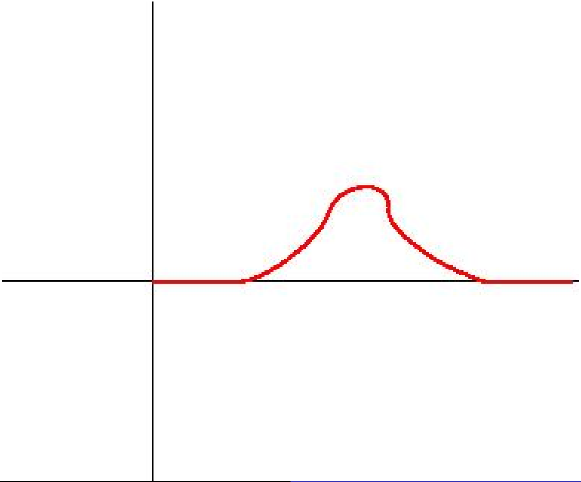
A new issue

New problem: What is the antiderivative of a function like this:



Parameterization of the boundary

Solution: Use a parameterization of the boundary to remove the portions where $f = 0$.

- This often introduces *variables* into the bounds of integration
- Switching order of integration is often helpful but requires a reparameterization of the boundary and a change in integration bounds.
- Example: $f(x, y) = xy$, D is the region bounded by $y = x^2, y = 0, y = 1$

Examples

For each domain, set up the integral $\int\int_D f(x,y)\, dA$

1. D is bounded is the triangular region with vertices $(0,2),(1,1),(3,2)$
2. $D = \{(x,y)|0 \leq x \leq 2, -x \leq y \leq x\}$
3. D is bounded by $y = \sqrt{x}, y = x$
4. D is a circle of radius 2 centered at the origin

Last, find the volume of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 4$.

Work for next class

- Reading: 16.4
- f07hw15