1. (14) Find the radius of convergence and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(x-8)^n}{10^n \ln n}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(x-8)\ln n}{10 \ln(n+1)}\right|$$

$$= \left|\frac{x-8}{10}\right| \lim_{n\to\infty} \left|\frac{\ln n}{\ln(n+1)}\right|$$

$$= \left|\frac{x-8}{10}\right|$$

$$\left(\lim_{n\to\infty} \frac{\ln n}{\ln n+1} = \lim_{x\to\infty} \frac{\ln x}{\ln(x+1)} = \lim_{x\to\infty} \frac{x+1}{x} \text{ (L'Hospital's Rule)} = 1 \right)$$

Hence by ratio test the power converges if $|x-8| < 10$ & diverges ~~otherwise~~ if $|x-8| > 10$

Hence $R = 10$

If $-2 < x < 18$, series diverges.

End pt: (i) $x = -2$, $\sum_{n=1}^{\infty} \frac{(-10)^n}{10^n \ln n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ is convergent by Alternating series test. Here seq^n $\left\{\frac{1}{\ln n}\right\}$ is decreasing & $\lim_{n\to\infty} \frac{1}{\ln n} = 0$.

(ii) $x = 18$, $\sum_{n=1}^{\infty} \frac{10^n}{10^n \ln n} = \sum \frac{1}{\ln n}$ is dgt by comparison test. $\frac{1}{\ln n} > \frac{1}{n}$ & $\sum \frac{1}{n}$ is dgt (Harmonic series)

Hence interval of convergence $= [-2, 18)$.

2. (14) Find a power series representation for the following function and find its interval of convergence:

$$f(x) = \frac{x^2}{x+2}.$$

(Write the representation in a form so that all the coefficients and powers of x are inside the Σ)

$$\frac{x^2}{x+2} = \frac{x^2}{2(1+x/2)} = \frac{x^2}{2(1-(-x/2))}$$

$$= \frac{x^2}{2}\left(\frac{1}{1-(-x/2)}\right)$$

$$= \frac{x^2}{2}\sum_{n=0}^{\infty}(-x/2)^n \quad \text{if } |x|<2$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+2}}{2^{n+1}}$$

Interval of convergence $= \underline{(-2, 2)}$

because it's a geometric series with common ratio $(x/2)$ & we know that geometric series converges if |common ratio| < 1

i.e. this series converges if $|x| < 2$

& diverges at the end pts.

3. (14) Let

$$f(x) = \ln(1 + x^2).$$

Find the first 3 nonzero terms in the Taylor series for $f(x)$ centered at $a = 2$. Write down the Taylor polynomial with 3 nonzero terms.

n	$f^{(n)}(x)$	$f^{(n)}(2)$
0	$\ln(1+x^2)$	$\ln(5)$
1	$\frac{1}{1+x^2} * 2x$	$\frac{4}{5}$
2	$\frac{2-2x^2}{(1+x^2)^2}$	$\frac{-6}{25}$

Taylor polynomial with 3 nonzero terms.

$$= \ln(5) + \frac{4}{5}\frac{(x-2)}{1!} + \left(\frac{-6}{25}\right)\frac{(x-2)^2}{2!}.$$

$$\left(= \ln(5) + \frac{4(x-2)}{5} - \frac{6}{50}(x-2)^2 \right)$$

4. (14) Find an equation of the plane which contains the x-axis as well as the line given by the parametric equations $x = t$, $y = 2t$, $z = 3t$.

~~Plane~~ The plane contains pt $(0,0,0)$

Contains x-axis so $\langle 1, 0, 0 \rangle$ is in the plane.

Also it contains $x=t$, $y=2t$, $z=3t$

& hence $\langle 1, 2, 3 \rangle$ is in the plane.

Normal vector to the plane $\langle 1,2,3 \rangle \times \langle 1,0,0 \rangle$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 1 & 0 & 0 \end{vmatrix} = \langle 0, 3, -2 \rangle$$

So the eq^n of the plane

$$0(x-0) + 3(y-0) - 2(z-0) = 0$$

i.e. $3y - 3z = 0$.

5. (14) Let $\vec{b} = \langle 2, 1, 1 \rangle$. Find all the values of x such that the scalar projection of the vector $\vec{b}$ onto $\vec{a}$ is 2 where $\vec{a} = \langle 3, x, 0 \rangle$ (i.e. $\text{comp}_{\vec{a}}\vec{b} = 2$).

$$\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|} = \frac{6+x}{\sqrt{9+x^2}} = 2 \text{ (given)}$$

$$6+x = 2\sqrt{9+x^2}$$

$$\Rightarrow x^2+12x+36 = 4(9+x^2)$$

$$\Rightarrow 3x^2-12x=0$$

$$\Rightarrow 3x(x-4)=0$$

$$\Rightarrow \boxed{x=0 \text{ or } x=4}$$

6. (14) Find parametric equations for the tangent line to the curve

$$\vec{r}(t) = \langle e^{-t}\cos t, e^{-t}\sin t, e^{-t} \rangle$$

at point $(1, 0, 1)$.

~~Sol~~ At pt $(1,0,1)$, $t=0$.

$\vec{r}\,'(t) = \langle -e^{-t}\cos t - e^{-t}\sin t,\ e^{-t}\cos t - e^{-t}\sin t,\ -e^{-t} \rangle$

so $\vec{r}\,'(0) = \langle -1, 1, -1 \rangle$

tgt line at $t=0$ is the line passing thru $(1,0,1)$ & $\parallel^{lel}$ to $\langle -1,1,-1\rangle$

so parametric eq^ns:

$$\left.\begin{aligned} x &= 1-t \\ y &= t \\ z &= 1-t \end{aligned}\right\} \quad t \text{ - scalar.}$$

7. (16) Multiple choice. Circle the correct response. You do not need to show your work and no partial credit will be given on this problem.

(a) Suppose for all n,

$$f^{(n)}(3) = \frac{(-1)^n n!}{4^n (n+2)}$$

Then the Taylor series of f centered at 3 is given by

$$(A) \sum_{n=1}^{\infty} \frac{(-1)^n n! x^n}{4^n (n+2)}, \quad (B) \sum_{n=1}^{\infty} \frac{(-1)^n n! (x-3)^n}{4^n (n+2)}, \quad (C) \sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{4^n (n+2)},$$

$$(D) \sum_{n=1}^{\infty} \frac{(-1)^n (x+3)^n}{4^n (n+2)}.$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(3)}{n!} (x-3)^n$$

(b) The series $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$ converges to

$$(A) \ln 2, \quad (B) 2, \quad (C) \frac{1}{2}, \quad (D) \text{None of these.}$$

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(\cancel{\ln 2})^n}{n!} = e^{-\ln 2}$$

$$= e^{\ln(1/2)}$$

$$= \cancel{0} \qquad = 1/2.$$

(c) The planes $x + 4y - 3z = 2$ and $-3x + 6y + 7z = -2$ are

(A) parallel, (B) perpendicular, (C) neither.

$\langle 1, 4, -3 \rangle \cdot \langle -3, 6, 7 \rangle = 0$

(d) Let $\vec{a}$ and $\vec{b}$ be two perpendicular vectors with $||\vec{a}|| = 3$ and $||\vec{b}|| = 7$.
Then $||\vec{a} \times \vec{b}|| = ?$.

$(A)\,10,\quad (B)\,0,\quad (C)\,21,\quad (D)$ the given information is not sufficient.

$$||\vec{a} \times \vec{b}|| = ||\vec{a}||\,||\vec{b}|| \sin \pi/2$$
$$= 21$$