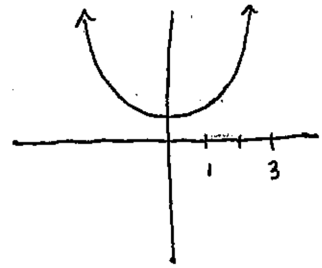

Math 2 - 1/18/06

Example:

Find the area under the graph $f(x) = x^2 + 1$ from 1 to 3.

$$A_{CP} = \sum_{k=1}^{n} f(v_k)\Delta x$$

Max occurs @ right endpoint of interval, so $V_k = X_k$

$$= \sum_{k=1}^{n} f(x_k)\Delta x$$

$$= \sum_{k=1}^{n} \left((x_k)^2 + 1\right)\Delta x$$

$\Delta x = \frac{2}{n}$ since $b-a = 3-1 = 2$

$$= \sum_{k=1}^{n} \left(\left(1+\frac{2k}{n}\right)^2 + 1\right)\left(\frac{2}{n}\right)$$

$x_k = 1 + \frac{2k}{n}$ since interval starts @ 1 and $x_{k+1} = x_k + \frac{2}{n}$ for each k

$$= \left(\frac{2}{n}\right)\sum_{k=1}^{n}\left(\left(1+\frac{4k}{n}+\frac{4k^2}{n^2}\right)+1\right)$$

lots of algebra and properties of Σ !

$$= \left(\frac{2}{n}\right)\left(\sum_{k=1}^{n} 2 + \frac{4}{n}\sum_{k=1}^{n} k + \frac{4}{n^2}\sum_{k=1}^{n} k^2\right)$$

$$= \left(\frac{2}{n}\right)\left(2n + \left(\frac{4}{n}\right)\frac{n(n+1)}{2} + \left(\frac{4}{n^2}\right)\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 4 + 4 + \frac{4}{n} + \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}$$

$$= \frac{32}{3} + \frac{8}{n} + \frac{4}{3n^2}$$

$$A = \lim_{\Delta x \to 0} A_{CP}$$

$$= \lim_{n\to\infty}\left(\frac{32}{3} + \frac{8}{n} + \frac{4}{3n^2}\right)$$

$$= \frac{32}{3}$$