

## Homework 5

### trigonometric integrals

**Instructions:** You are encouraged to work out solutions to these problems in groups! Discuss problems with your classmates, the tutors and/or the instructors. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper, showing all of your work (this part should be done on your own). When you are asked to give explanations, be sure to use complete sentences. You are welcome (and sometimes encouraged) to use calculators or computing devices.

In this homework you will compute the integral

$$\int_0^{\pi} \sin^2(x) \cos^2(x) dx$$

in different ways, using different trigonometric identities.

We know the following trigonometric identities.

$$\sin^2(x) + \cos^2(x) = 1 \tag{1}$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x)) \tag{2}$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x)) \tag{3}$$

$$\sin(x)\cos(x) = \frac{1}{2}\sin(2x) \tag{4}$$

1. (3 Point) Use identity (2) and (3) to simplify the integral and then use (3) again to solve it.
2. (3 Points) Use identity (4) to simplify the integral and then use (2) to solve it.
3. (4 Points) Use identity (1) to express everything in terms of  $\cos(x)$  and then twice identity (3).
4. (2 Point) Bonus: Use identity (4) and the fact  $\int_0^{\pi} \sin^2(x) dx = \int_0^{\pi} \cos^2(x) dx$  to solve the integral.