15.1

#8

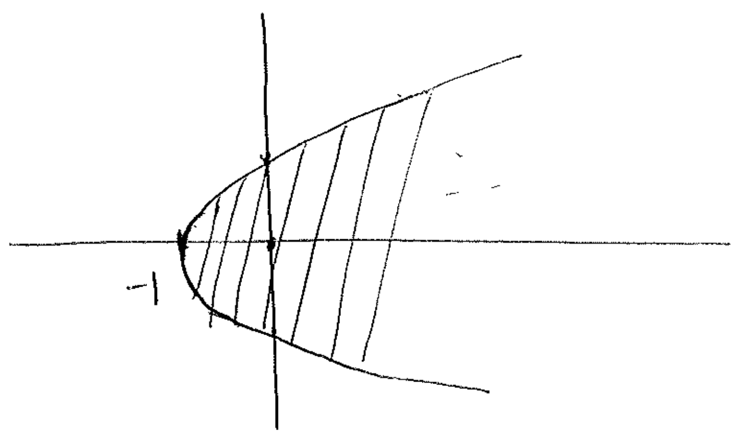
$f(x,y) = \sqrt{1+x-y^2}$

$f(x,y)$ is defined when $1+x-y^2 \geq 0$

$\Rightarrow \quad x \geq y^2-1$

So domain of $f = \{(x,y) \mid x \geq y^2-1\}$.

Sketch



optional: (the domain is the set of all pts which are on or to the right of the parabola $x = y^2-1$)

The range of f is $[0,\infty)$.

15.2

#10

$$f(x,y) = \frac{x^2+\sin^2 x}{2x^2+y^2}$$

Along x-axis i.e. $y=0$

$$f(x,0) = \frac{x^2}{2x^2} = \frac{1}{2} \quad \text{if } x \neq 0$$

Hence $\displaystyle\lim_{\substack{(x,y)\to(0,0) \\ \text{along } x\text{-axis}}} \frac{x^2+\sin^2 y}{2x^2+y^2} = 1/2$

Along y axis i.e. $x=0$

$$f(0,y) = \frac{\sin^2 y}{y^2}$$

hence $\displaystyle\lim_{\substack{(x,y)\to(0,0) \\ \text{along } y\text{-axis}}} f(x,y) = \lim_{y\to 0}\left(\frac{\sin y}{y}\right)^2$

$$= 1^2 = 1$$

Since f has two different limits along two different paths, the limit does not exist.

#38. $$f(x,y) = \frac{xy}{x^2+xy+y^2} \quad \text{if } (x,y) \neq (0,0)$$

Since f is a rational fun on $\{(x,y) / (x,y) \neq (0,0)\}$ it is continuous on $\{(x,y) / (x,y) \neq (0,0)\}$.

We have to check the continuity at $(0,0)$

First see if $\lim_{(x,y)\to(0,0)} f(x,y)$ exists or not.

Along x-axis: $f(x,0) = 0$ if $x \neq 0$

& $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along x axis

Along y-axis $f(0,y) = 0$ $y \neq 0$

So $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along y-axis

along $y = x$: $f(x,x) = \dfrac{x^2}{x^2+x^2+x^2} = \dfrac{1}{3}$ if $x \neq 0$

Hence $f(x,y) \to \frac{1}{3}$ as $(x,y) \to (0,0)$ along $y = x$.

Since along different paths, we get different limits the $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

& hence f is not continuous at $(0,0)$.

So the set of the pts at which the fun^n is continuous is $\{(x,y) \mid (x,y) \neq (0,0)\}$.

15.3

#20

$$z = \tan xy$$

$$\frac{\partial z}{\partial x} = y \sec^2(xy)$$

$$\frac{\partial z}{\partial y} = x \sec^2(xy)$$

22

$$f(x,y) = x^y$$

$$f_x = y x^{y-1}$$

$$f_y = x^y \ln x$$

15.4

6

$$z = e^{x^2 - y^2}$$

$$f_x(x,y) = 2x\, e^{x^2-y^2}$$

$$f_y = -2y\, e^{x^2-y^2}$$

$$f_x(1,-1) = 2e^0 = 2$$

$$f_y(1,-1) = 2$$

Eqn of the tgt plane is

$$z - 1 = 2(x-1) + 2(y+1)$$

$$\left(\text{i.e. } z = 2x + 2y + 1\right)$$

#16

$$f(x,y) = \sin(2x+3y)$$

$$f_x = 2\cos(2x+3y)$$

$$f_y = 3\cos(2x+3y)$$

$$f_x(-3,2) = 2\cos(0) = 2$$

$$f_y(-3,2) = 3$$

linearizaⁿ of f at $(-3,2)$ is

$$L(x,y) = f(-3,2) + f_x(-3,2)(x+3) + f_y(-3,2)(y-2)$$

$$= 0 + 2(x+3) + 3(y-2)$$

$$L(x,y) = 2x+3y$$

15.5

#4

$z = \tan^{-1}(y/x) \qquad x = e^t \qquad y = 1-e^{-t}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= \frac{-(y x^{-2})}{1+(y/x)^2}e^t + \frac{(1/x)}{1+(y/x)^2}(e^{-t})$$

#12

$z = \tan(u/v) \qquad u = 2s+3t, \qquad v = 3s-2t$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial s} = \sec^2\left(\frac{u}{v}\right)\left(\frac{1}{v}\right)(2) + \sec^2\left(\frac{u}{v}\right)(-uv^{-2})(3)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial t}$$

$$= \sec^2\left(\frac{u}{v}\right)\left(\frac{1}{v}\right)(3) + \sec^2\left(\frac{u}{v}\right)(-uv^{-2}) \times (-2)$$