**Math 42**
**Differential Geometry**
**Winter 2002**
**Assignment 2**
**Due Friday, January 18, 2001**

1. **Chapter 4:** 4.1, 4.3, 4.9, 4.12

2. Recall or observe for the first time the following definition.

   **Definition 1.** *A* **group** *is a set $G$ along with a binary operation $\cdot : G \times G \to G$ such that*

   *(a) There exists a unique element $e \in G$ such that $e \cdot g = g \cdot e = g$ for any $g \in G$. The element $e$ is known as the* **identity element**.

   *(b) For any $g \in G$ there exists an element $g^{-1}$ such that $g \cdot g^{-1} = g^{-1} \cdot g = e$. The element $g^{-1}$ is known as the* **inverse of** *$g$.*

   *(c) For any $a$, $b$, $c \in G$ we have $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.*

   *We sometimes denote the group along wth its binary operation as $(G, \cdot)$.*

   Examples of groups include $(\mathbb{R}, +)$ and $(\mathbb{R}_+, \cdot)$, where $\mathbb{R}_+ = \{x \in \mathbb{R} : x > 0\}$ and $\cdot$ denotes multiplication.

   Now let $\mathrm{SL}_2(\mathbb{R}) = \{A = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} : x_1, \dots, x_4 \in \mathbb{R}, \ \det(A) = 1\}$. First, show that $\mathrm{SL}_2(\mathbb{R})$ is a group under matrix multiplication, then do problem 4.15. The space $\mathrm{SL}_2(\mathbb{R})$ is an example of what's called a **Lie group**.

3. 4.16 (Note: The tangent space you calculated in this problem is an example of a Lie algebra. For $\mathrm{SL}_2(\mathbb{R})$ the Lie algebra is denoted by $\mathfrak{sl}_2(\mathbb{R})$ .)

4. **Chapter 5:** 5.1, 5.2, 5.9 & 5.10