m15w06, Quiz 1	Name:	Section:

Answer ALL questions. Unless instructed otherwise, you should show ALL your work and simplify your final answer as much as possible. Please box your final answer to each part.

Problem 1: [8 pts] A metal sheet is bent into the shape described by rotating the curve $z = x^2$ with $0 \leq x \leq \sqrt{2}$ all the way around the z-axis. The metal has density $\rho(x, y, z) = 3\ kg/m^2$. Find the total mass of the sheet.

Problem 2: [8 pts] Find the flux of the vector field $\vec{F} = \begin{pmatrix} z^2 x \\ \frac{1}{3}y^3 + \tan z \\ x^2 z + y^2 \end{pmatrix}$ across the surface $x^2+y^2+z^2 = 1$, $z \geq 0$ oriented upwards.

Problem 3: [9 pts] An infinite cylinder filling the region $x^2 + z^2 \leq 9$ has charge density $q(x, y, z) = \sqrt{x^2 + z^2}$. Find the electric field generated by this cylinder.

Some useful identities in cylindrical coordinates (r, θ, y)

$$\nabla f = \frac{\partial f}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{e}_\theta + \frac{\partial f}{\partial y}\vec{j}$$

$$\text{div } \vec{F} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}F_r\right) + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_y}{\partial y}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial y^2}.$$