Fock spaces (what did that physicist just say?)

February 29, 2008

Fock spaces came from *spin chains*, which are made up of sites $b_1, b_2, \ldots, b_k$, with particles with spin at each site. $B = \{\text{spins}\} = \{v_1, \ldots, v_k\}$. Let $V = \mathbb{C}\text{-span}B$. So $\mathbb{C}\text{-span}\{\text{sites with spins}\} = V^{\otimes k}$. In other words, the spins are the basis for the vector space V, and the sites are the tensor factors in $V^{\otimes k}$. *Bosonic* stuff lives in $S^k(V)$, and *Fermionic* stuff lives in $\wedge^k(V)$.

The game in physics is to find *eigenstates* (eigenvectors) and *energies* (eigenvalues) of the *Hamiltonian* H (linear transformations of $V^{\otimes k} = \mathcal{H}$ of choice). But really, there are so many particles in any situation that $V^{\otimes\infty}$ (the *Fock space*), $S^\infty(V)$ (the *Bosonic Fock Space*), and $\wedge^\infty(V)$(the *Fermionic* Fock Space) are the primary objects. Usually H is somehow imbedded in a larger algebra of operators (linear combinations of *gauge symmetries* - for example, if my linear transformation of choice is the Casimir, then my gauge symmetries would either be the enveloping algebra, or the center-there-of. This is how stuff like Hecke algebras, BMW stuff, etc. comes into the picture).

Of special note are the cases where $\wedge^\infty$ is a module for the affine Lie algebra of the Virisoro algebra (this is *conformal field theory*). In many cases, $\wedge^\infty(V) = L(\wedge_0)$, where $\wedge_0$ is the fundamental weight of extra node in the affine Dynkin diagram.