14.1

#28

$\vec{r}(t)$ intersects the sphere at pts where

$$\sin^2 t + \cos^2 t + t^2 = 5$$

i.e. when $1+t^2 = 5 \Rightarrow t = \pm 2$

Now ~~r(t) =~~ $\vec{r}(2) = \langle \sin 2, \cos 2, 2 \rangle$

& $\vec{r}(-2) = \langle \sin(-2), \cos(-2), -2 \rangle$

Hence pts of intersection are

$(\sin 2, \cos 2, 2)$ & $(\sin(-2), \cos(-2), -2)$.

14.2
p 865

24

$$\vec{r}(t) = \langle e^t, te^t, te^{t^2} \rangle$$

So $\vec{r}\,'(t) = \langle e^t, te^t + e^t, te^{t^2}(2t) + e^{t^2} \rangle$

pt $(1,0,0)$ corresponds to $t=0$

Now $\vec{r}\,'(0) = \langle 1, 1, 1 \rangle$

Hence the tgt line at $(1,0,0)$ is given by

$$x = 1+t$$
$$y = t$$
$$z = t$$

(where t is a scalar.)

#36

$$\int_1^2 \left(t^2\,\vec{i} + t\sqrt{t-1}\,\vec{j} + t\sin\pi t\,\vec{k}\right)dt$$

$$= \left(\int_1^2 t^2\,dt\,\vec{i} + \int_1^2 t\sqrt{t-1}\,dt\,\vec{j} + \int_1^2 t\sin\pi t\,dt\,\vec{k}\right)$$

Now

$$\int_1^2 t\sqrt{t-1}\,dt = \int_0^1 (u+1)\sqrt{u}\,du$$

$$(u = t-1\,,\quad t = u+1)$$

$$= \int_0^1 u^{3/2} + \sqrt{u}\,du$$

$$= \tfrac{2}{5}u^{5/2}\Big]_0^1 + \tfrac{2}{3}u^{3/2}\Big]_0^1$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$$

$$\int_1^2 t\sin\pi t\,dt =$$

$u = t \qquad dv = \sin\pi t\,dt$

$du = dt \qquad v = -\dfrac{\cos\pi t}{\pi}$

$$= -\frac{1}{\pi}\left[t\cos\pi t\right]_1^2 + \frac{1}{\pi}\int_1^2 \cos\pi t\,dt$$

$$= -\frac{1}{\pi}\left[2\cos 2\pi - \cos\pi\right] + \frac{1}{2}\ \frac{\sin\pi t}{\pi}\Big]_1^2$$

$$= -\frac{1}{\pi}[3] + 0$$

$$= -3/\pi$$

Hence

$$\int_1^2 t^2\vec{i} + t\sqrt{t-1}\ \vec{j} + t\sin\pi t\ \vec{k}\ dt$$

$$= \left(\frac{2^3}{3} - \frac{1}{3}\right)\vec{i} + \frac{16}{15}\vec{j} - \frac{3}{\pi}\vec{k}$$

$$= \frac{7}{3}\vec{i} + \frac{16}{15}\vec{j} - \frac{3}{\pi}\vec{k}$$

14.3

\# 6

$$\vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2\rangle$$

$$\vec{r}'(t) = \langle 12, 12\sqrt{t}, 6t\rangle$$

$$\|\vec{r}'(t)\| = \sqrt{144 + 144t + 36t^2} = \sqrt{36(t^2 + 4t + 4)}$$

$$= \sqrt{36(t+2)^2}$$

$$= \cancel{6(t+2} \quad 6(t+2)$$

Hence the length

$$= \int_0^1 6(t+2)\,dt$$

$$= 6\,\frac{t^2}{2} + 12t \Big]_0^1$$

$$= 3+12 = 15 .$$

14.4
#14

$$\vec{r}(t) = t\sin t\,\vec{i} + t\cos t\,\vec{j} + t^2\,\vec{k}$$

velocity $\vec{v}(t) = \vec{r}\,'(t)$

$$= (t\cos t + \sin t)\,\vec{i} + (-t\sin t + \cos t)\,\vec{j} + 2t\,\vec{k}.$$

acceleration $\vec{a}(t) = \vec{r}\,''(t)$

$$= (-t\sin t + \cos t + \cos t)\,\vec{i} + (-t\cos t - \sin t - \sin t)\,\vec{j} + 2\,\vec{k}$$

$$= (2\cos t - t\sin t)\,\vec{i} + (-2\sin t - t\cos t)\,\vec{j} + 2\vec{k}.$$

speed $= \|\vec{V}(t)\| = \sqrt{\cancel{t^2\sin} (t\cos t + \sin t)^2 + (-t\sin t + \cos t)^2 + 4t^2}$

$\left(= \sqrt{t^2\cos^2 t + 2t\sin t\cos t + \sin^2 t + t^2\sin^2 t - 2t\sin t\cos t + \cos t^2 + 4t^2} \right)$

$= \sqrt{5t^2 + 1}$

#16 $\vec{a}(t) = 2\vec{i} + 6t\vec{j} + 12t^2\vec{k}$

$\vec{V}(t) \left(= \int (2\vec{i} + 6t\vec{j} + 12t^2\vec{k})\,dt \right)$

$= 2t\vec{i} + 6\frac{t^2}{2}\vec{j} + 12\frac{t^3}{3}\vec{k} + \vec{C_1}$

$= 2t\vec{i} + 3t^2\vec{j} + 4t^3\vec{k} + \vec{C_1}$

Since $\vec{V}(0) = \vec{i}$, we have $\vec{C_1} = \vec{i}$

& $\vec{V}(t) = (2t+1)\vec{i} + 3t^2\vec{j} + 4t^3\vec{k}$

$\vec{r}(t) = \left(\frac{2t^2}{2} + t\right)\vec{i} + t^3\vec{j} + t^4\vec{k} + \vec{C_2}$

$\vec{r}(0) = j - k$

$\Rightarrow \quad c_2 = \vec{j} - \vec{k}$

$\therefore \vec{r}(t) = \cancel{t^2} \; (t^2 + t)\vec{i} + (t^3 + 1)\vec{j} + (t^4 - 1)\vec{k}$