Derivatives of the Trigonometric Functions

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The Derivative of \sin

$$\frac{d}{dx}\sin x = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \sin x \lim_{h \to 0} \frac{\cos h - 1}{h} + \cos x \lim_{h \to 0} \frac{\sin h}{h}$$

Theorem

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1,$$

 $\quad \text{and} \quad$

$$\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0.$$

Example

Evaluate

$$\lim_{x \to 0} \frac{\sin 3x}{x}$$

Theorem

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x.$$

Examples

• Differentiate

$$\sin(2x)$$

$$\sin\left(x^2 + \frac{1}{x}\right)$$

$$\cos(3x + \sqrt{x})$$

• Differentiate

$$y = \sin x \cos x$$
$$y = \sin^2(\cos(x^2 + 2))$$

Theorem

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Example

$$\frac{d}{dx}\sec^2 x$$

Example

• Compute the derivative of

$$y = (\sin^3(\tan^2(2x)))^4.$$