Math 14 Fall 2005

Multivariable Calculus–Honors

First Midterm Exam

Monday January 31, 6-8 PM
Bradley 102

Your name (please print): ____

Instructor Vladimir Chernov.

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You must justify all of your answers to receive credit, unless instructed otherwise in a given problem.

You have two hours to work on all **11** problems. The total score is the sum of your **10** best scores. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only

1. ____________ /10
2. ____________ /10
3. ____________ /10
4. ____________ /10
5. ____________ /10
6. ____________ /10
7. ____________ /10
8. ____________ /10
9. ____________ /10
10. ____________ /10
11. ____________ /10

Total: ________ /100

(1) Find the second order Taylor polynomial $T(h_1, h_2)$ at the point $x = \frac{\pi}{2}, y = 0$ of the function $f(x, y) = \sin x e^y$. Use this second order Taylor polynomial to approximate the value of $\sin(\frac{\pi}{2} + 0.1)e^{-0.05}$.

(2) Let $f, g : \mathbb{R}^3 \to \mathbb{R}^3$ be defined as $f(x, y, z) = (x + y, xy, 3yz)$ and $g(u, v, w) = (3u + v, \cos u, -e^w)$. Put $h(u, v, w) = f(g(u, v, w))$. Find the derivative matrix of the composition h at the point $(0, 2, 0)$ and use it to compute $\dfrac{\partial h_3}{\partial w}$.

(3) Consider the system of equations $xy+z+e^{uv}+u=0$ and $x+y+\sin z+u-v=0$. Does the Implicit Function Theorem imply that the u and v coordinates of the solution set of this system of equations can be expressed as functions $u(x,y,z)$ and $v(x,y,z)$ close to the point $x=1, y=-1, z=0, u=0, v=0$? **Explain your answer.** If it is possible, find $\dfrac{\partial u}{\partial x}$ at $x=1, y=-1, z=0, u=0, v=0$.

(4) Use the ϵ, δ definition of the limit to **prove** that

$$\lim_{(x,y)\to(0,0)} \frac{3x^4}{x^2+y^2} = 0.$$

(5) Let $f(x, y, z) = x^2 + yz$. Find the directional derivative $D_{\mathbf{u}}f(1, 2, 3)$ in the direction of the vector $\mathbf{u} = (\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}})$. Find the **unit length** vector in the direction where f **decreases** fastest at the point $(1, 2, 3)$.

(6) Let $f(x_0, y_0, z_0)$ be a local minimum of the differentiable function $f(x, y, z)$. **Prove** that $\nabla f(x_0, y_0, z_0) = \vec{0}$.

(7) Let $f(x, y) = -e^{x^2+(y-1)^2}$. Find all the critical points of f and classify them as local maxima, local minima, and saddle points.

(8) A vector field $\mathbf{F}(x, y, z) = \nabla f(x, y, z)$ for a differentiable function f. Let $\mathbf{r}(t)$ be a curve which is everywhere orthogonal to the flow curves of $\mathbf{F}$. **Prove** that the composition function $h(t) = f(\mathbf{r}(t))$ is a constant function.

(9) Find the absolute maximum and the absolute minimum of the function $f(x, y) = e^{(x-0.5)^2+(y-0.5)^2}$ on the unit disk $D = \{(x, y)|x^2 + y^2 \leq 1\}$.

(10) Find the equation of the plane tangent to the sphere $x^2 + y^2 + z^2 = 1$ at the point $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

(11) A curve $\mathbf{r}(t) = (3t, 5t+3, 4-t)$. Starting with the point $\mathbf{r}(1) = (3, 8, 3)$ reparametrize the curve in terms of the arc length $\mathbf{r}(t(s)) =$