Math 13. Multivariable Calculus. Written Homework 6.

Due on Wednesday, 5/8/13.

You may leave this homework in the boxes outside of Kemeny 108 by 1:45 pm on Wednesday. Please write problems 1-3 on separate pages from problems 4-6 and turn them in in the corresponding columns.

1. (Ch 16.3, #29) Show that if the vector field $\mathbf{F} = \langle P, Q, R\rangle$ is conservative and P, Q, R have continuous first-order partial derivatives, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \quad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$

2. (Ch 16.3, #14) Find a potential function $f(x, y)$ for $\mathbf{F} = \langle (1 + xy)e^{xy}, x^2e^{xy}\rangle$, and evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by $\mathbf{r}(t) = \langle \cos t, 2\sin t\rangle$, $0 \le t \le \pi/2$.

3. (Ch 16.3, #36a) Suppose that $\mathbf{F}$ is an inverse square field; that is,

$$\mathbf{F}(\mathbf{r}) = \frac{c\mathbf{r}}{|\mathbf{r}|^3}$$

for some constant c, where $\mathbf{r} = \langle x, y, z\rangle$. Find the work done by $\mathbf{F}$ in moving an object from a point P_1 along a path to a point P_2 in terms of the distances d_1, d_2 from these points to the origin.

4. (Ch 16.4, #2) Evaluate the line integral below by using two methods: direct evaluation and Green's Theorem, and check that the answers are identical.

$$\int_C xy\,dx + x^3\,dy,$$

where C is the rectangle (with positive orientation) with vertices $(0, 0), (3, 0), (3, 1), (0, 1)$.

5. Verify Green's Theorem for $P(x, y) = x$ and $Q(x, y) = xy$, where D is the unit disk $x^2+y^2 \le 1$.

6. Compute

$$\int_C (e^{x^2}\,dx + dy),$$

where C is the semicircle $x^2 + y^2 = 1$, $x \ge 0$ traced from $(0, -1)$ to $(0, 1)$.