Homework Problems

November 18, 2005

Exercise 1. Consider the stochastic matrix

$$P = \begin{pmatrix} 4/5 & 1/2 \\ 1/5 & 1/2 \end{pmatrix}$$

and the Markov chain given by

$$\mathbf{x}_0 = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}$$

$$\mathbf{x}_{k+1} = P\mathbf{x}_k \text{ for } k \geq 0.$$

a. Diagonalize P. That is, find P's eigenvalues and a basis for $\mathbb{R}^2$ consisting of eigenvectors of P.

b. Express $\mathbf{x}_0$ in terms of the basis from part (a).

c. Use part (b) to find an expression for $\mathbf{x}_k$. Use this to determine what happens to $\mathbf{x}_k$ as $k \to \infty$.

Exercise 2. Refer to Exercise 2 on page 296.

a. Find the stochastic matrix P for the situation described.

b. Find the characteristic polynomial and the eigenvalues of the matrix P. It might be helpful to write the entries of P as fractions and not in decimal notation.

c. Show that the matrix P is diagonalizable. Find a basis for $\mathbb{R}^3$ consisting of eigenvectors of P. If you use a computer to help you with the row reduction here, be sure to use the suggestion from part (b).

d. Suppose that during the initial trial the animal described in the problem chooses the first food. The initial state describing this choice is

$$\mathbf{x}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

Write $\mathbf{x}_0$ in terms of the basis you found in part (c).

e. What food will the animal prefer after many trials? Does this surprise you?