MATH 53 — HW4, 3.7, Solution : $\boxed{\text{Writing Proofs!}}$ Daniel 10/21/09

A Theorem is usually, "Let X hold, then Y follows." To prove the theorem 'constructively' (ie not by contradiction) you need to show how Y follows given <u>any</u> instance of X. In our case X was "$x \in I$, $\varepsilon > 0$", so your method must take x, ε as <u>given to you</u>.

$\boxed{\text{HERE'S A NICE PROOF BY KYLE:}}$ (I've annotated too...)

3.7 Choose any x in I, choose any $\varepsilon > 0$

5 subintervals of k symbols have length 2^{-k} $\Rightarrow$

what Kyle didn't say here was:
since 2^{-k} has 0 as a limit as $k \to \infty$,
there is a k such that $2^{-k} < \varepsilon$, for any given $\varepsilon > 0$.
(constructively: choose any k larger than $\frac{\ln 1/\varepsilon}{\ln 2}$)

Kyle Konrad

$\exists k$ s.t. $N_\varepsilon(x) \overset{\text{contains}}{\supseteq} \underbrace{S_1 S_2 \ldots S_k}_{\text{the level-}k\text{ subinterval from itinerary of } x\text{: say this!}} \overset{\text{contains}}{\supseteq} S_1 S_2 \ S_k S_1$

the subinterval $S_1 S_2 \ldots S_k S_1$ exists because T has a complete transition graph and by corollary 3.18 contains a fixed point

this is essential, otherwise you might not be able to add S_1 to the end!

$\Rightarrow$ $N_\varepsilon(x)$ contains a fixed point
by Corollary 3.18 (follows from fixed pt. thm).

since ε and x were arbitrary we have

V. elegant.

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$\forall x \in I, \varepsilon > 0$ $N_\varepsilon(x)$ contains a fixed point of T

$\Rightarrow$ fixed points of T are dense in I $\square$

Proving a theorem is a <u>transaction</u>: I throw you <u>any</u> $x \in I$ & <u>any</u> $\varepsilon > 0$, and you have to construct a periodic orbit in $N_\varepsilon(x)$. I don't mind how you do it as long as you explain your steps. If the construction always works, the theorem is proved.