Coordinate transformation mini-lab

1. Consider the equation

$$2x^2 - 4xy + 5y^2 = 1 \tag{1}$$

and the set $\mathcal{B} = \left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$.

(a) Explain why $\mathcal{B}$ is a basis for $\mathbb{R}^2$. How could you answer a question like this using MATLAB if $\mathcal{B}$ were a basis for $\mathbb{R}^{10}$? Apply this method to the given basis.

(b) What is the change of coordinate matrix $P_{\mathcal{B}}$ from $\mathcal{B}$ to the standard basis $\mathcal{E}$ in $\mathbb{R}^2$?

(c) Let (x, y) denote a point in the standard coordinate system defined by $\mathcal{E}$, and let (x', y') denote the same point in the basis $\mathcal{B}$. Express x and y in terms of x' and y'.

(d) Using your answer from (c), write equation (1) in terms of x' and y'. Describe the solution set geometrically, **without** plotting it.

(e) Create two figures with MATLAB (using the 'ezplot' function): one plotting the solution set to equation (1); the other plotting the solution set to the equation derived in part (d). Save your figures to a file.

(f) What do the vectors in $\mathcal{B}$ represent in terms of the solution set to equation (1)?

(g) Find the standard matrix of the linear transformation that transforms the standard coordinates of a vector $\mathbf{x} \in \mathbb{R}^2$ to the $\mathcal{B}$-coordinates of $\mathbf{x}$. That is, find $P_{\mathcal{B}}^{-1}$.

(h) Verify, using MATLAB that your expression for $P_{\mathcal{B}}^{-1}$ is correct by checking that $P_{\mathcal{B}} P_{\mathcal{B}}^{-1} = I = P_{\mathcal{B}}^{-1} P_{\mathcal{B}}$.

(i) Using $P_{\mathcal{B}}^{-1}$, determine (by hand) whether $\mathbf{x} = \frac{1}{\sqrt{5}} \begin{bmatrix} \sqrt{3} - \frac{1}{2\sqrt{6}} \\ \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{6}} \end{bmatrix}$ is a solution to equation (1).

(j) Recreate the plot of the solution set to the equation you derived in part (d) using the 'plot' function in MATLAB. Save your figure to a file.

(k) Write up the work you did above neatly and provide a print-out of the figures you created in parts (e) and (j). In this write-up, discuss in your own words the utility of the basis $\mathcal{B}$. (You may complete this portion of the assignment outside of class and turn in one copy per group on Tuesday 7/30/13. Make sure to put the name of each group member (maximum of 3 people per group) on the assignment. Every group member should contribute to the final product!)