# The Comparison Tests (cont'd)

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# The Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

1. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all $n$, then $\sum a_n$ is also convergent.

2. If $\sum b_n$ is divergent and $a_n \geq b_n$ for all $n$, then $\sum a_n$ is also divergent.

# Examples

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- $\sum_{n=1}^{\infty} \frac{1}{n!}$

# The Limit Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where $c$ id a finite number and $c > 0$, then either both series converge or both diverge.

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- $\sum_{n=1}^{\infty} \frac{1+n \ln n}{n^2+5}$
- $\sum_{n=1}^{\infty} \frac{\sin n \sqrt{n}}{4n+1}$

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# The Alternating Series Test

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$$a_n = (-1)^{n-1} b_n \text{ or } a_n = (-1)^n b_n,$$

where each $b_n$ is a positive number.

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where each $b_n$ is a positive number.

- **The Alternating Series Test:** If the alternating series

$$\sum_{n=1}^{\infty}(-1)^n b_n$$

satisfies

$$\begin{aligned} b_{n+1} &\leq b_n \text{ for all } n \\ \lim_{n\to\infty} b_n &= 0 \end{aligned}$$

then the series is convergent.

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