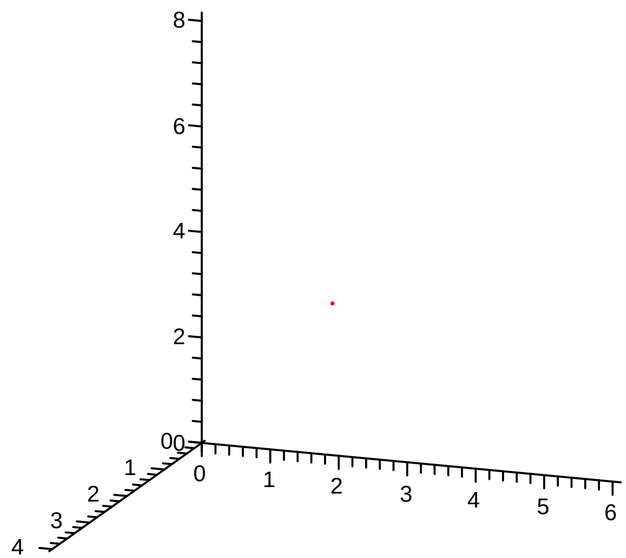
Coordinates in $\mathbb{R}^2$ and $\mathbb{R}^3$

October 23, 2006

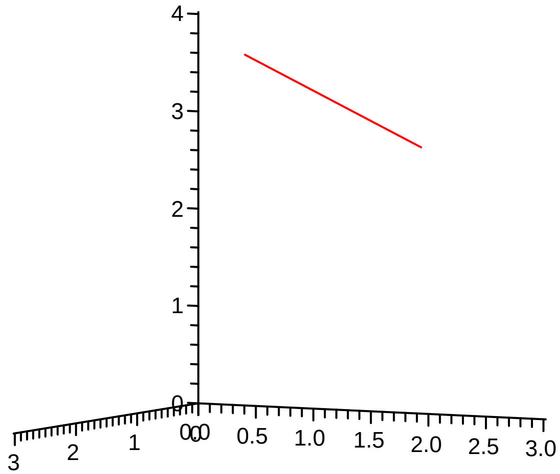
Lecture 15

Three-dimensional coordinate systems



- A point P in space is represented by a triple (a, b, c)
- a is the x-coordinate
- b is the y-coordinate
- c is the z-coordinate
- This correspondence between points and triples (a, b, c) in $\mathbb{R}^3$ is called a three dimensional rectangular coordinate system.

Distance between two points

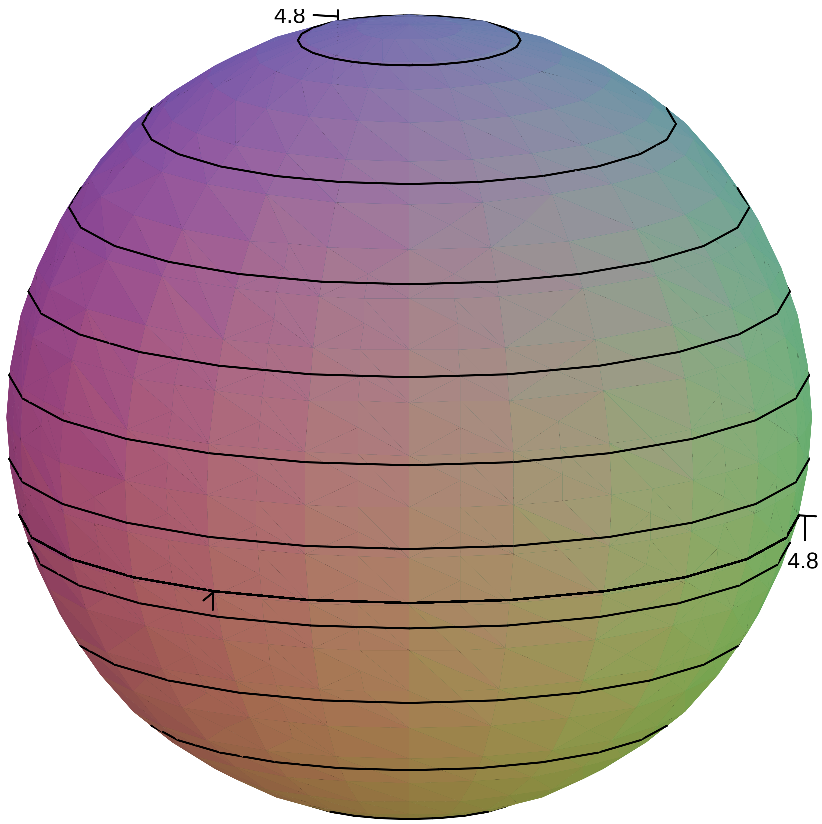


Distance formula

- The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a sphere



Equation of a Sphere

- An equation of a sphere with center $C(h, k, l)$ and radius r is

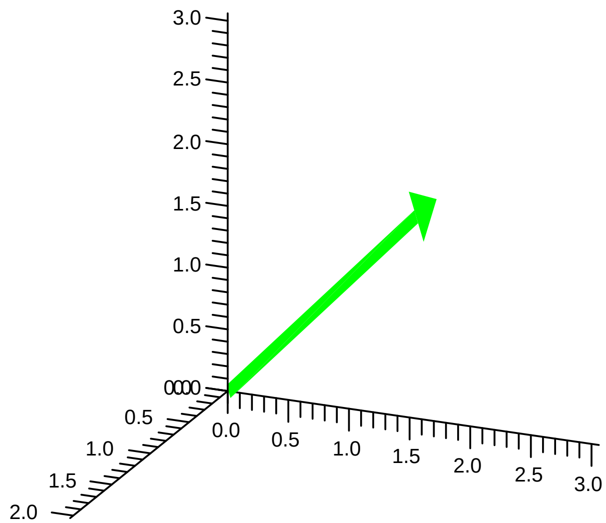
$$(x-h)^2 + (y-k)^2(z-l)^2 = r^2.$$

- If the center is the origin

$$x^2 + y^2 + z^2 = r^2.$$

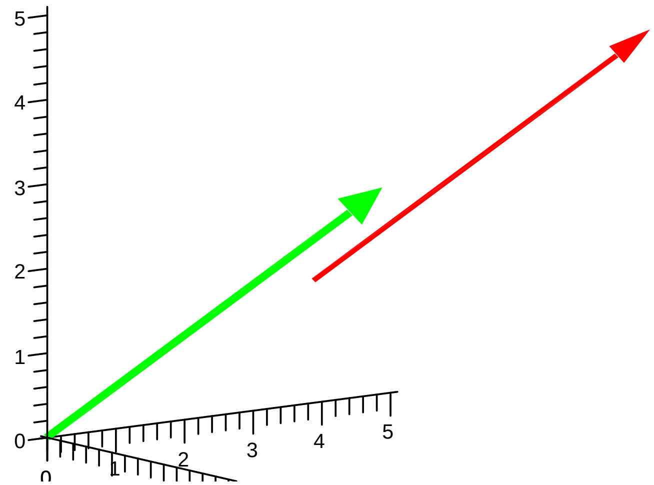
Vectors

- A **vector** has **initial point** A and **terminal point** B



- We write $\vec{AB}$

- Two vectors u and v are **equivalent** (or **equal**) and we write $u = v$ if the have the same length and the same direction



Vector Addition

- If u and $v$are vectors positioned so the initial point of v is at the terminal point of u, then the **sum** $u + v$ is the vector from the initial point of u to the terminal point of v.

Scalar multiplication

- If c is a scalar and v is a vector, then the **scalar multiple** cv is the vector whose length is $|c|$ times the length of v and whose direction is the same as v if $c > 0$ and is opposite to v if $c < 0$. If $c = 0$ or $v = 0$ then $cv = 0$.

- We call $-v$ the **negative** of v.

- The **difference** $u - v$ of two vectors is

$$u - v = u + (-v)$$

Components

- If we place the initial point of a vector a at the origin, then the terminal point of a has coordinates of the form (a_1, a_2) or (a_1, a_2, a_3).

- These coordinates are called **components** of a

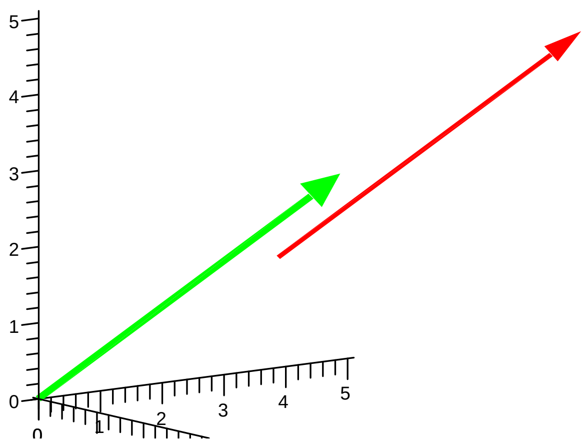
$$a = \langle a_1, a_2 \rangle \text{ or } a = \langle a_1, a_2, a_3 \rangle$$

- The particular representation from the origin is called the **position vector.**

- V_2 =the set of all two-dimensional vectors; V_3 =the set of all three dimensional vectors.

- Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector a with representation $\vec{AB}$ is

$$a = < x_2 - x_1, y_2 - y_1, z_2 - z_1 >$$



Magnitude or length

- The length of a vector v is the length of any representative and is denoted by $|v|$ or $\|v\|$.

- The length of a three-dimensional vector $a = \langle a_1, a_2, a_3 \rangle$ is

$$|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Addition, subtraction, multiplication by a scalar

- If $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$ then

$$
\begin{aligned}
\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle &= \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle \\
\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle &= \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle \\
c\langle a_1, a_2, a_3 \rangle &= \langle ca_1, ca_2, ca_3 \rangle
\end{aligned}
$$

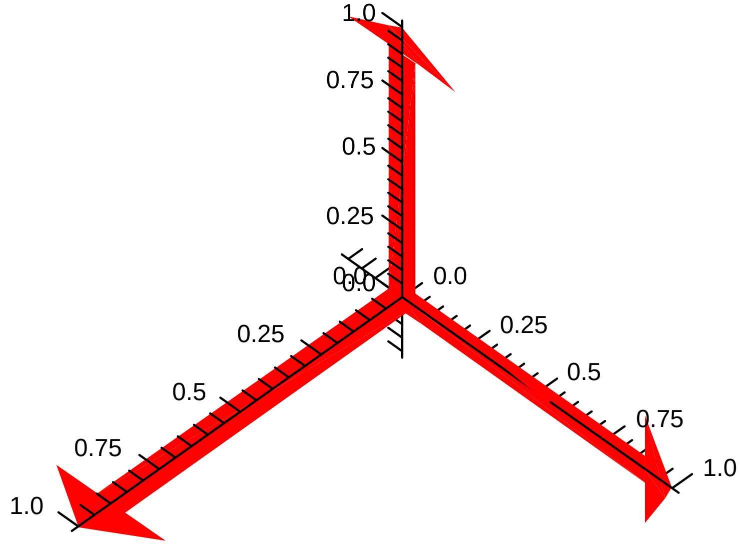
Standard vector basis

- Three vectors in V_3 play a special role

$$\mathbf{i} = \langle 1, 0, 0 \rangle \; \mathbf{j} = \langle 0, 1, 0 \rangle, \; \mathbf{k} =< 0, 0, 1 \rangle$$

- If $a = \langle a_1, a_2, a_3 \rangle$ then

$$a = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}.$$


1.0
0.75
0.5
0.25
0.0
0.0
0.0
0.25
0.5
0.75
1.0
0.25
0.5
0.75
1.0