Math 105
Homework Problems 2

January 19, 2006

Just to keep you honest (to yourself, of course!), the first two exercises here ask you to verify a couple of the statements I've made in lecture. The third exercise has an asterisk next to it simply because I don't (at the time of assignment) have a complete solution, and so I don't know how hard it actually might be.

Exercise 1. Let K be a field with equivalent absolute values $|\cdot|_1$ and $|\cdot|_2$. We know that there is an $s > 0$ so that $|x|_1 = |x|_2^s$ for all $x \in K$. Let $(K_1, |\cdot|_1)$ and $(K_2, |\cdot|_2)$ be completions of K relative to $|\cdot|_1$ and $|\cdot|_2$, respectively. Show that there is a K-isomorphism $\sigma : K_1 \to K_2$ so that $|x|_1 = |\sigma x|_2^s$ for all $x \in K_1$. Is σ unique?

Exercise 2. Let $F_1, \ldots, F_n$ and $L_1, \ldots L_m$ be fields. Suppose there is an isomorphism of rings

$$F_1 \times \cdots \times F_n \cong L_1 \times \cdots \times L_m.$$

Show that $n = m$ and that there is a permutation $\tau \in S_n$ so that $F_i \cong L_{\tau(i)}$ for $i = 1, \ldots, n$.

Exercise 3.* Exercise 2 on page 107.

Exercise 4. Exercise 1 on page 118.

Exercise 5. Exercise 2 on page 118.