Bayesian estimation of optical properties of the human head via 3D structural MRI

June 23, 2003 at ECBO 2003

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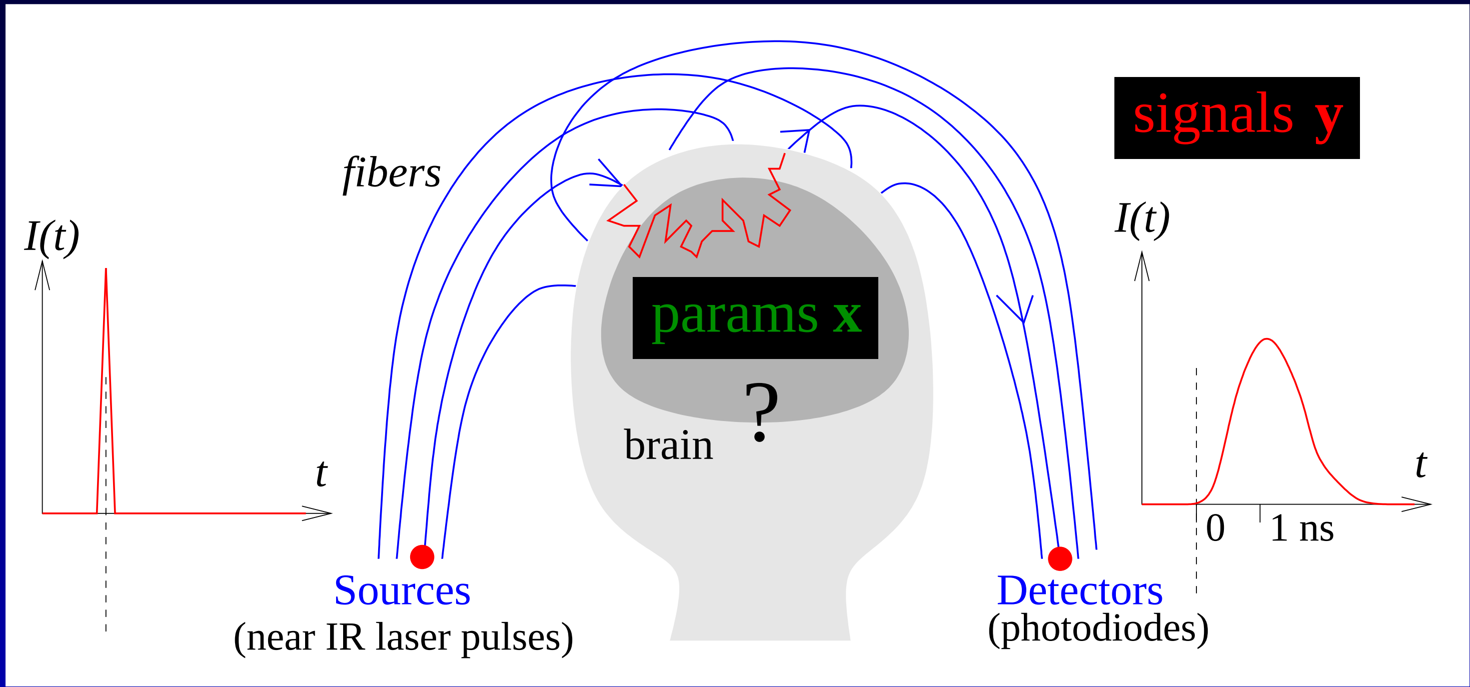
Collaborators (*NMR Center, Mass. Gen. Hosp., Boston*)

David Boas	Joe Culver	Anna Custo (MIT)
Gregory Sorensen	Anders Dale	

Funding: CIMS, NIH, CIMIT

The big picture

Diffuse Optical Tomography (DOT):



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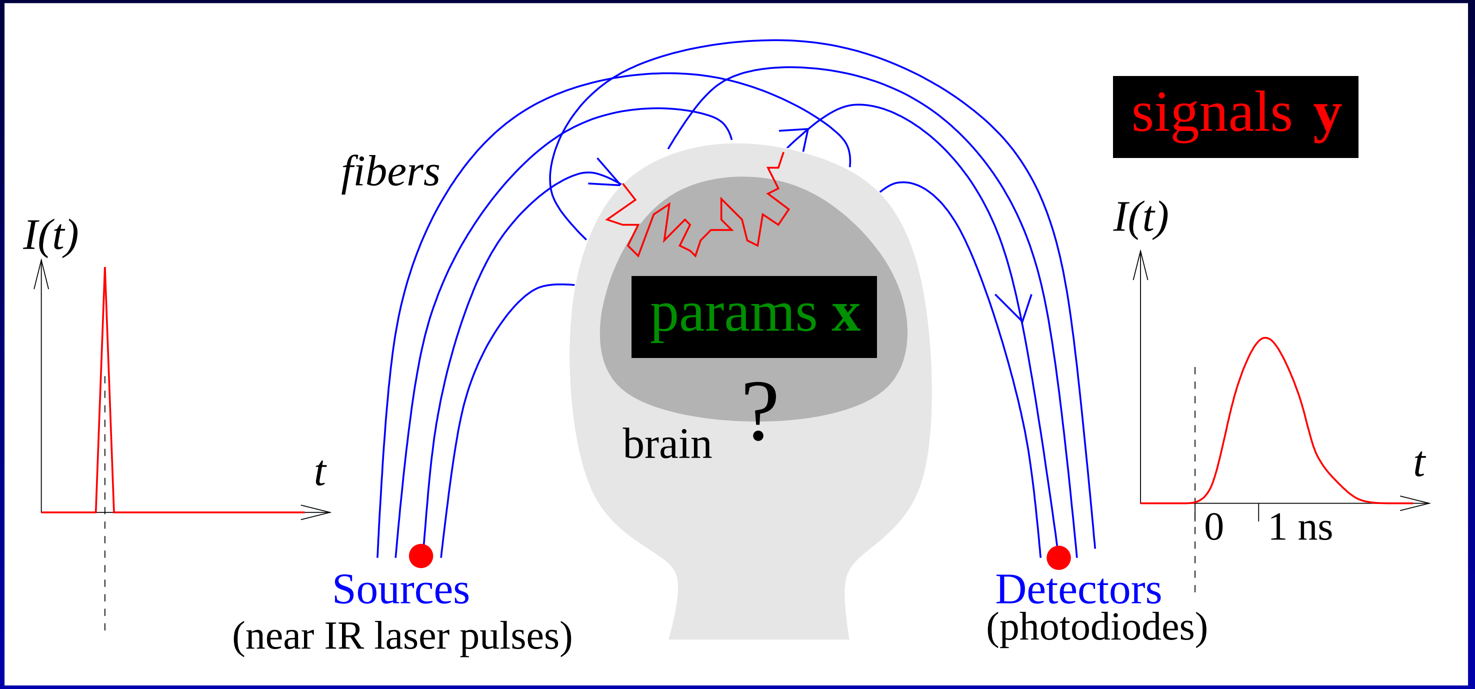

vector **x** = spatial absorption & scattering info

vector **y** = components of measured signals

Inverse problem: given **y** find **x**.

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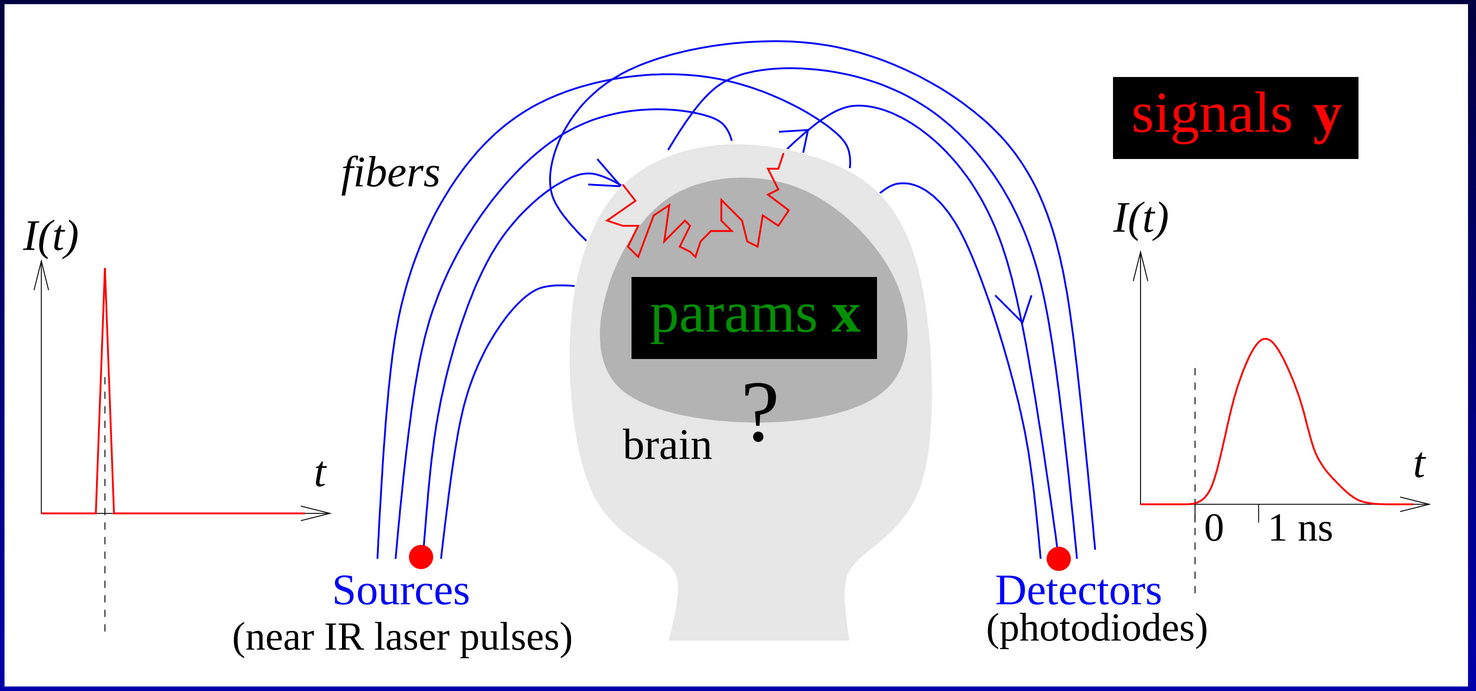

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Many wavelengths $\longrightarrow$ [HbO_2], [HbR], activation...

Baseline optical measurement

Assuming head tissues optically homogeneous:
How well could we measure their baseline properties?

- *absolute* cortical absorption μ_a
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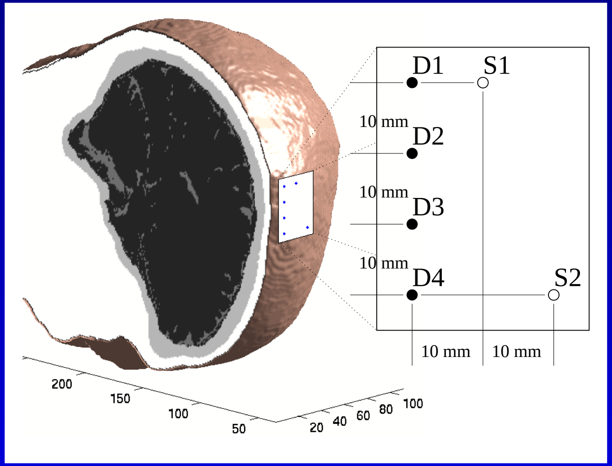
$\mathbf{x} \equiv \{\mu_a, \mu_s'\}$ scalp, skull, brain

- time-resolved DOT
- small system eg 2 Src, 4 Det



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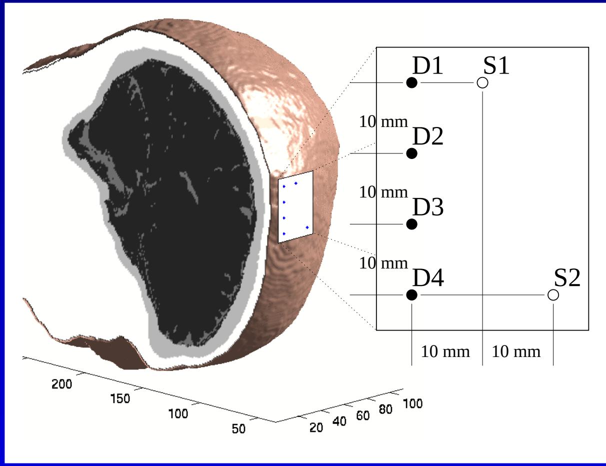
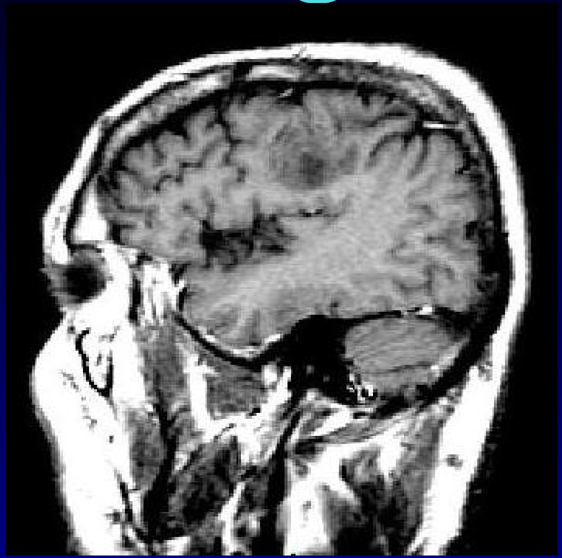
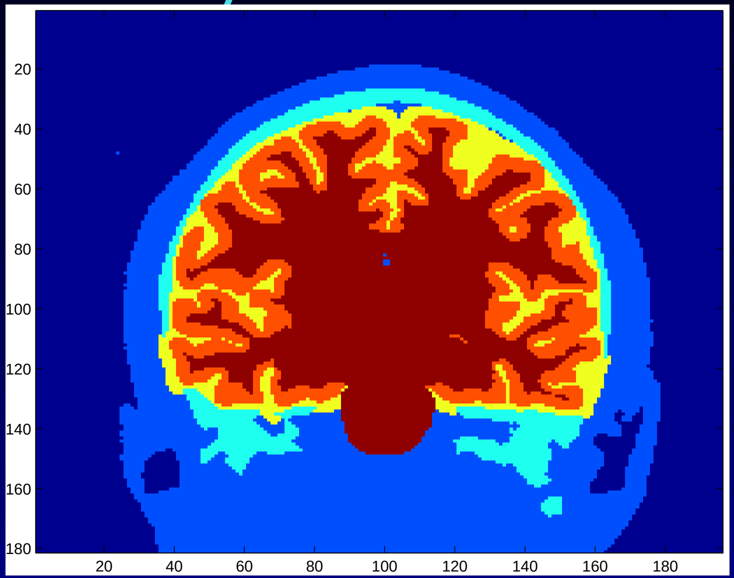


Numerical study: simulated noisy signals $\mathbf{y}$

MRI segmented geometry



segment
$\longrightarrow$



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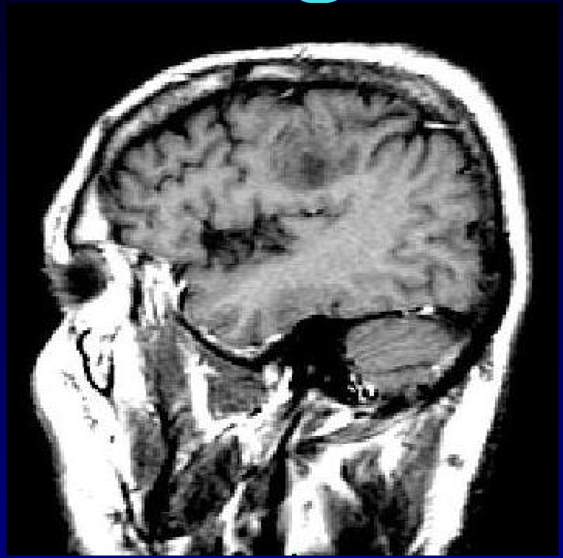

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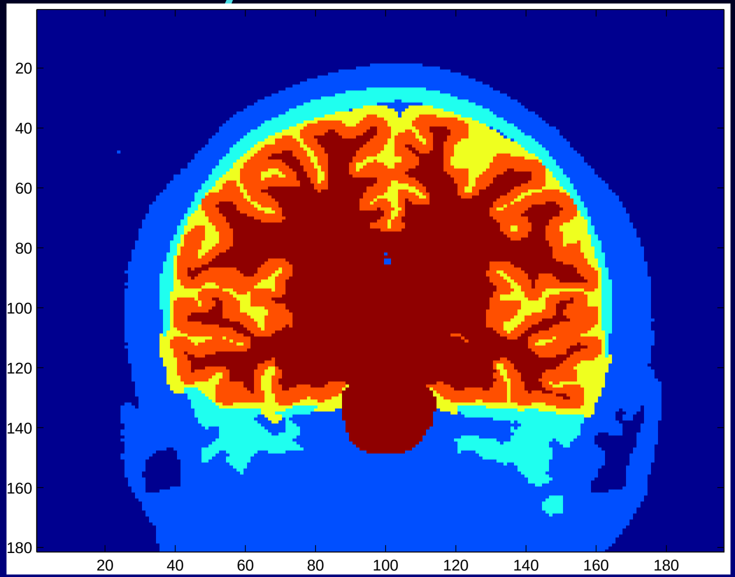


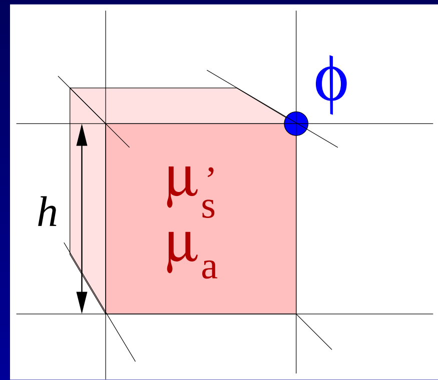
tissue	μ_a (mm^{-1})	μ'_s (mm^{-1})	shape
scalp	0.015	0.8	$\sim$ 7 mm layer
skull	0.01	1.0	$\sim$ 7 mm layer
CSF	0.0004	0.01*	folded 1–3 mm sheet
brain	0.018	1.3	$\sim$ 1 cm folds (sulci)

Much uncertainty. * Diffusion, we use $\mu'_{s,\mathrm{eff}} \sim 0.4$ mm^{-1}.

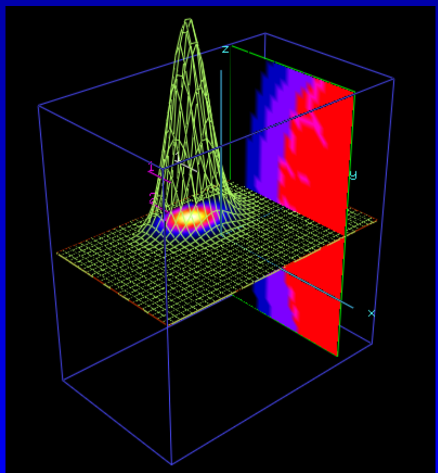
Diffusion forward model

$$\frac{1}{c}\frac{\partial \phi}{\partial t} = \nabla \cdot \left(\frac{1}{3\mu_s'(\mathbf{r})} \nabla \phi \right) - \mu_a(\mathbf{r})\phi + q(\mathbf{r}, t)$$

- Finite difference, lattice size h
- Forward Euler, timestep $\Delta t \sim$ ps
- accuracy $O(h^2)$, typ few % error
- $h = 2$ mm: 4×10^4 cells, 10s CPU



Signals: time-dep fluence at detectors = vector $\mathbf{f}(\mathbf{x})$



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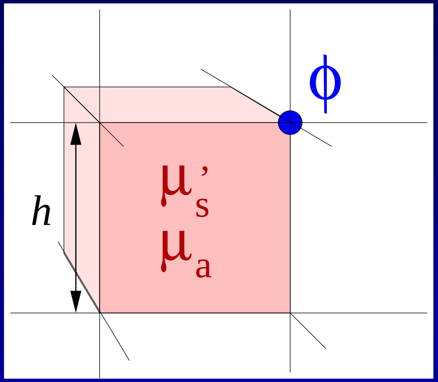

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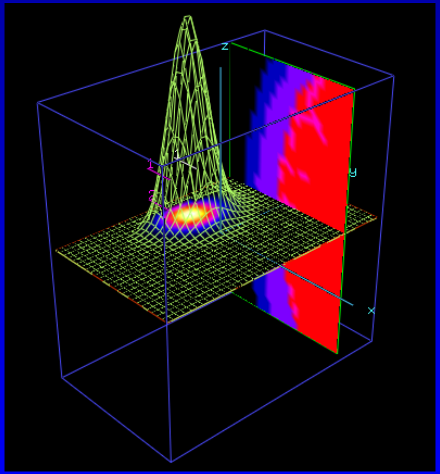
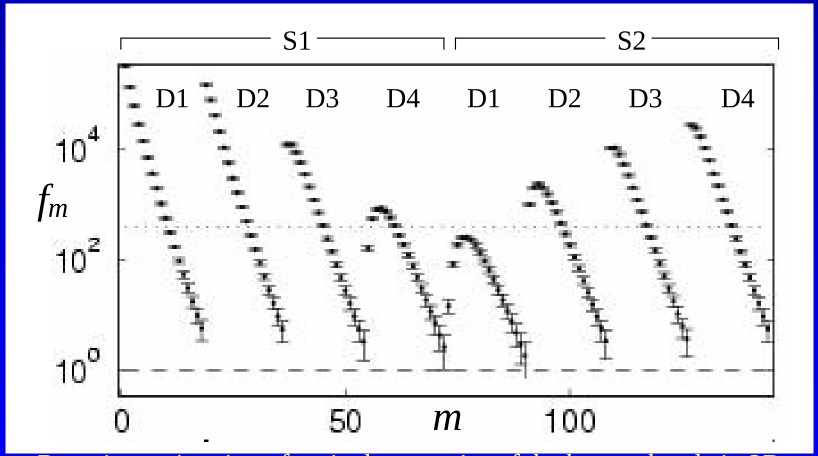


detect $\longrightarrow$



Bayesian inverse problem

$\frac{\partial f_m}{\partial x_n}$ sing. vals. $\longrightarrow 0$: ‘ill-posed’ (many $\mathbf{x}$ equally valid)

Incomplete info on $\mathbf{x} \longrightarrow$ *probability density function*

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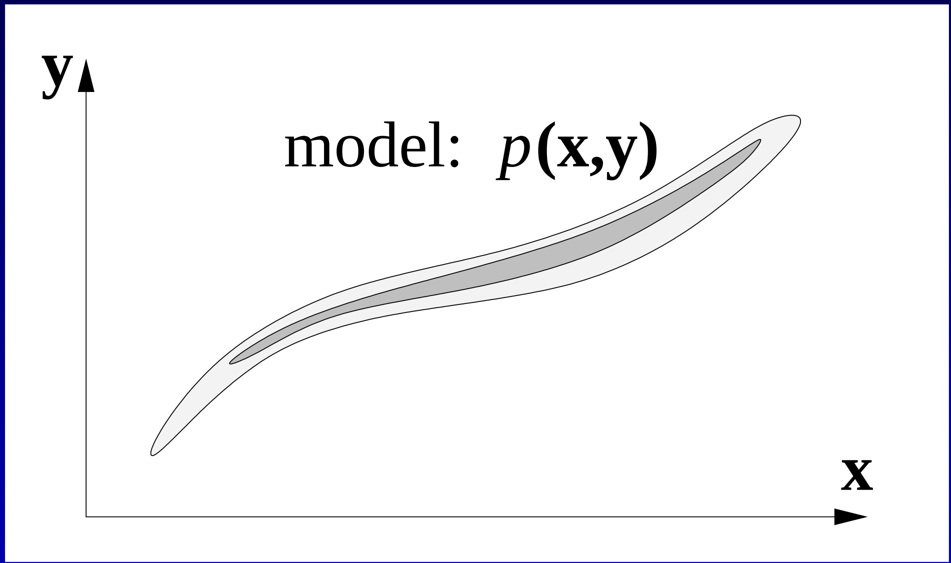

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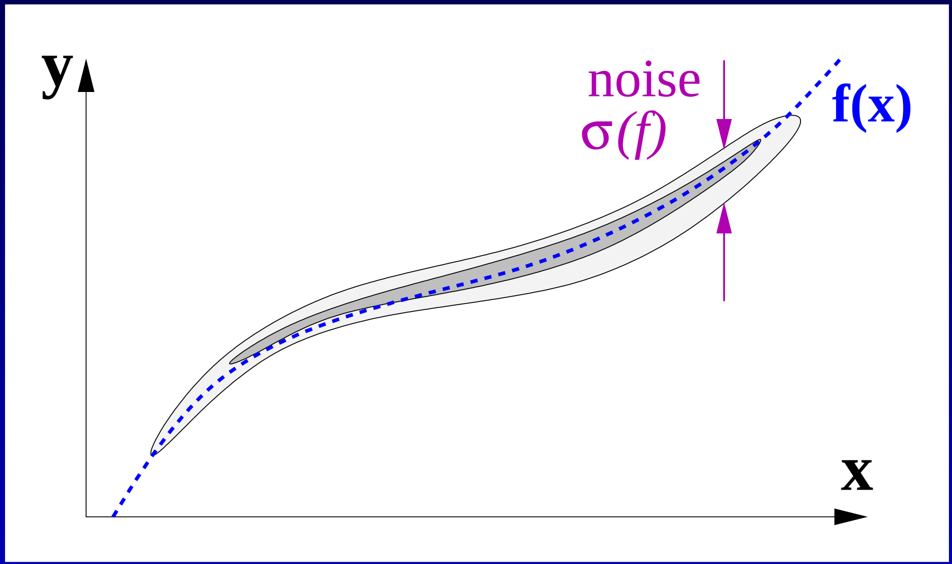

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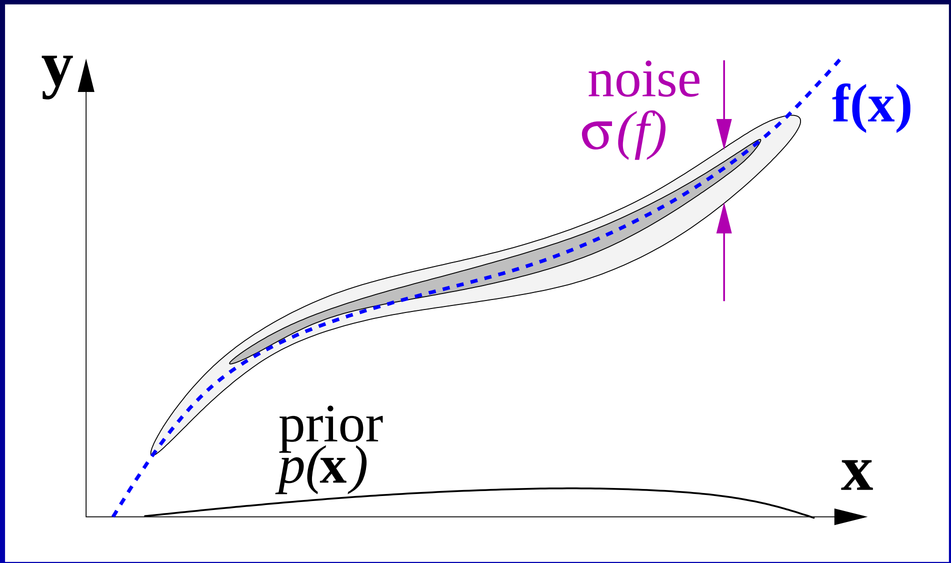

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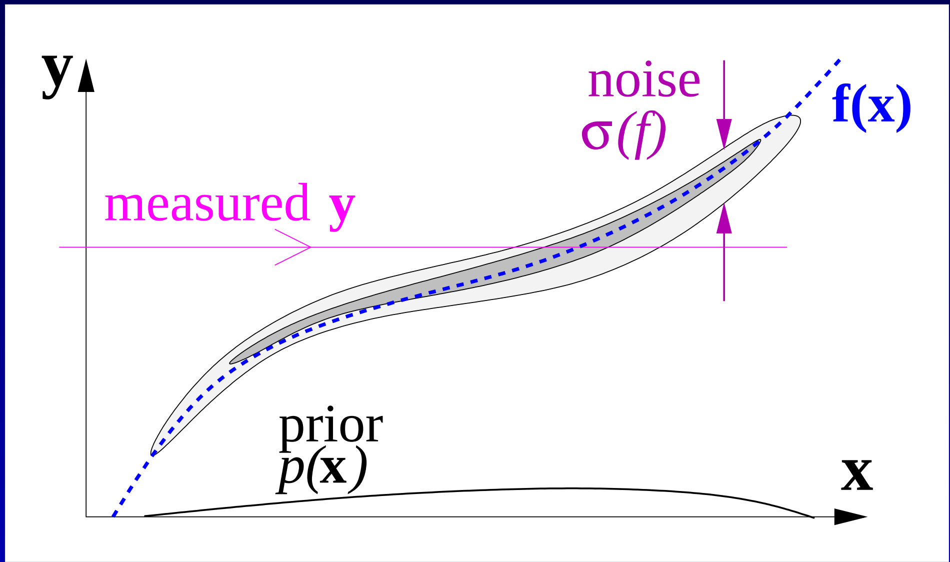

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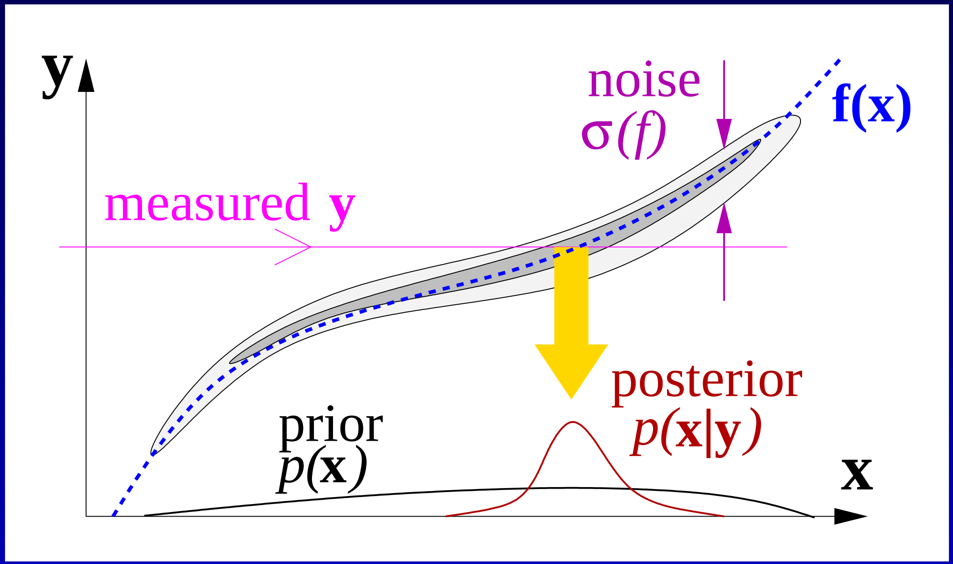

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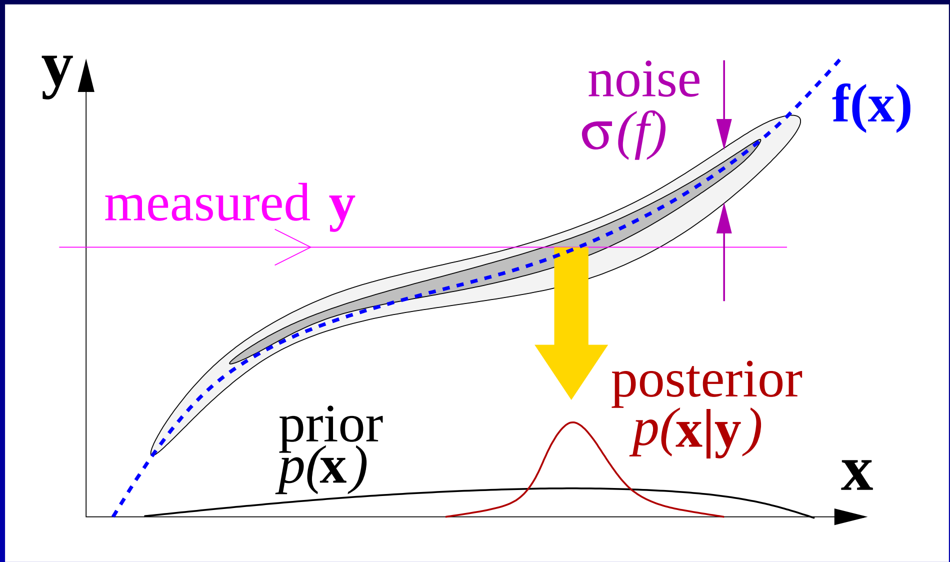

Bayesian inference

$$\begin{aligned} p(\mathbf{x}|\mathbf{y}) &\propto p(\mathbf{x}, \mathbf{y}) \\ &= p(\mathbf{y}|\mathbf{x}) \cdot p(\mathbf{x}) \end{aligned}$$

posterior likelihood prior

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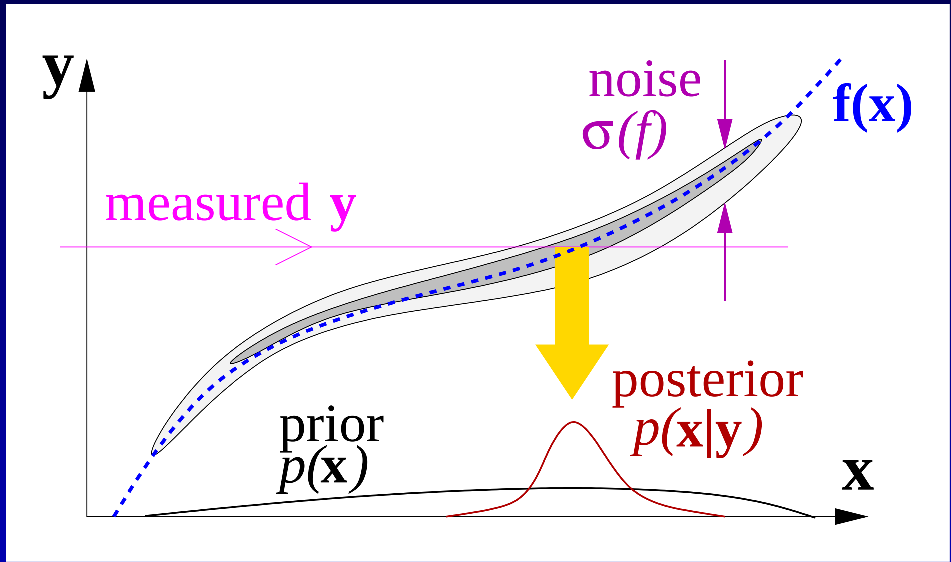

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posterior likelihood prior

- assumptions about noise $\rightarrow$ width of likelihood
- Embraces ill-posedness, statistically rigorous.
- Need to explore N-dim posterior: many $\mathbf{f}(\mathbf{x})$ evals required ($> 10^2$).

Why full PDF?

Common “Bayesian” method for DOT inversion: find single best-fit $\mathbf{x} = \mathbf{x}_{\mathrm{MAP}}$. (MAP = *maximum a-posteriori*)

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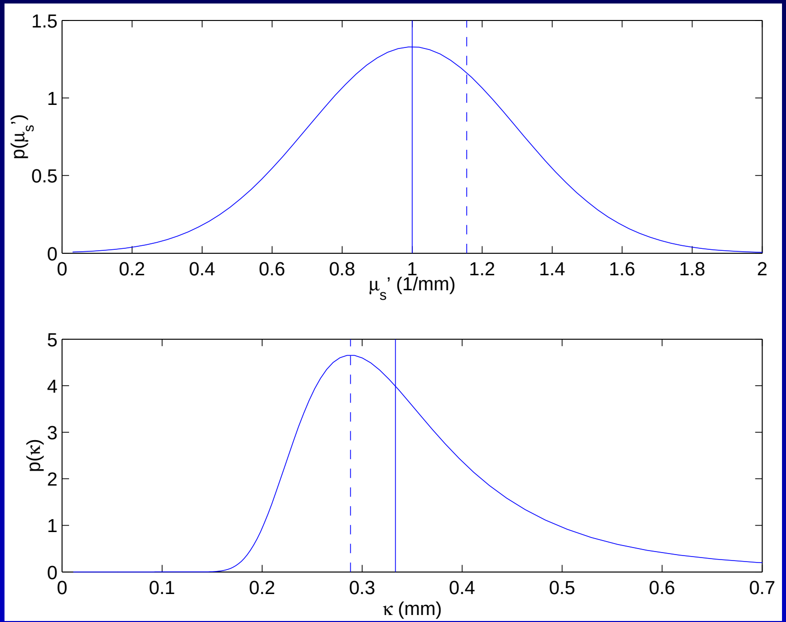

Example: broad PDF on μ_s' Normal, $\sigma = 30\%$ of mean.

16% shift in MAP between $p(\mu_s')$ and $p(\mu_s'^{-1})$

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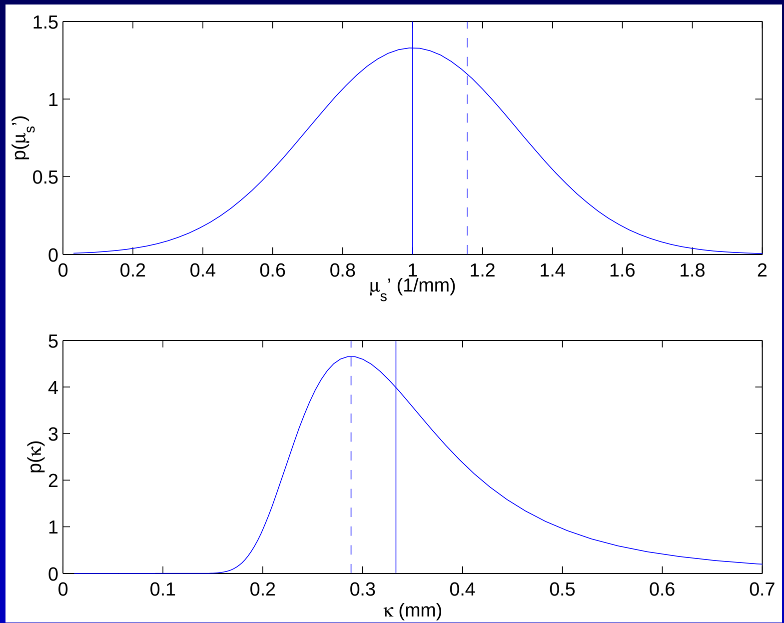

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- CPUs advance faster than DOT instrumentation $\longrightarrow$ best to make maximal use of data
- Statistical answers $\longrightarrow$ multimodal imaging.

Realistic new noise model

Each signal component $f_m(\mathbf{x})$ independent noise.

Photons Poissonian: gaussian approx $\sigma(f) = f^{1/2}$

E.g. 10^6 photons = 0.1% relative error

But: we do not trust forward model to 0.1% !

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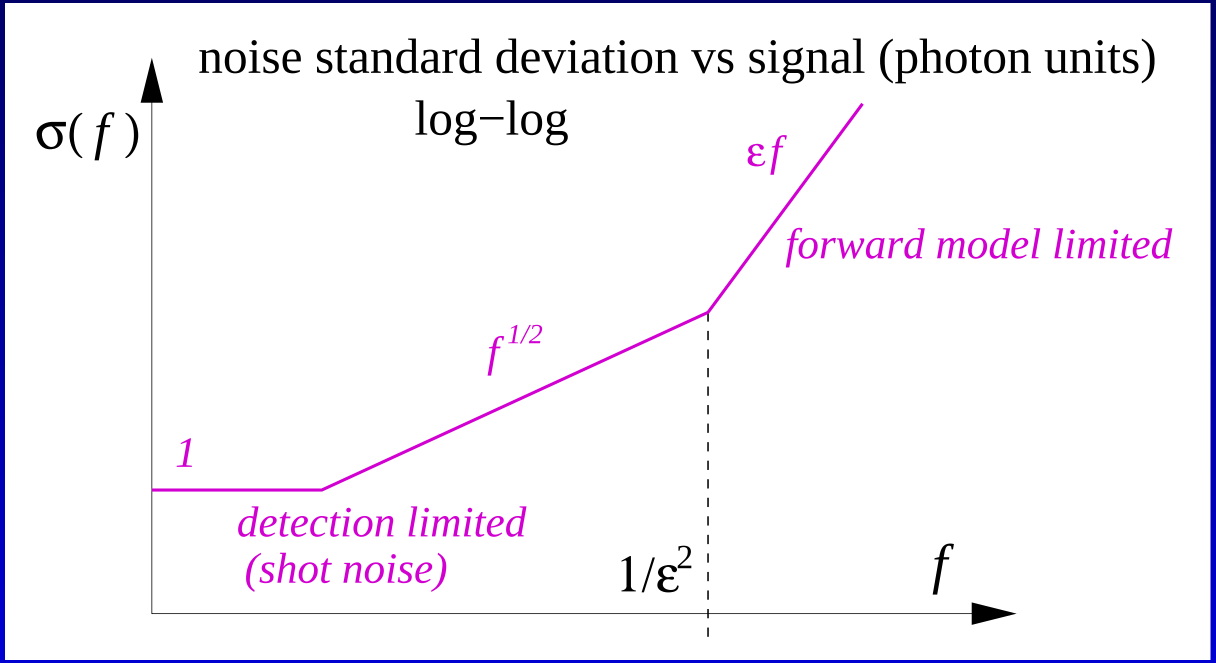
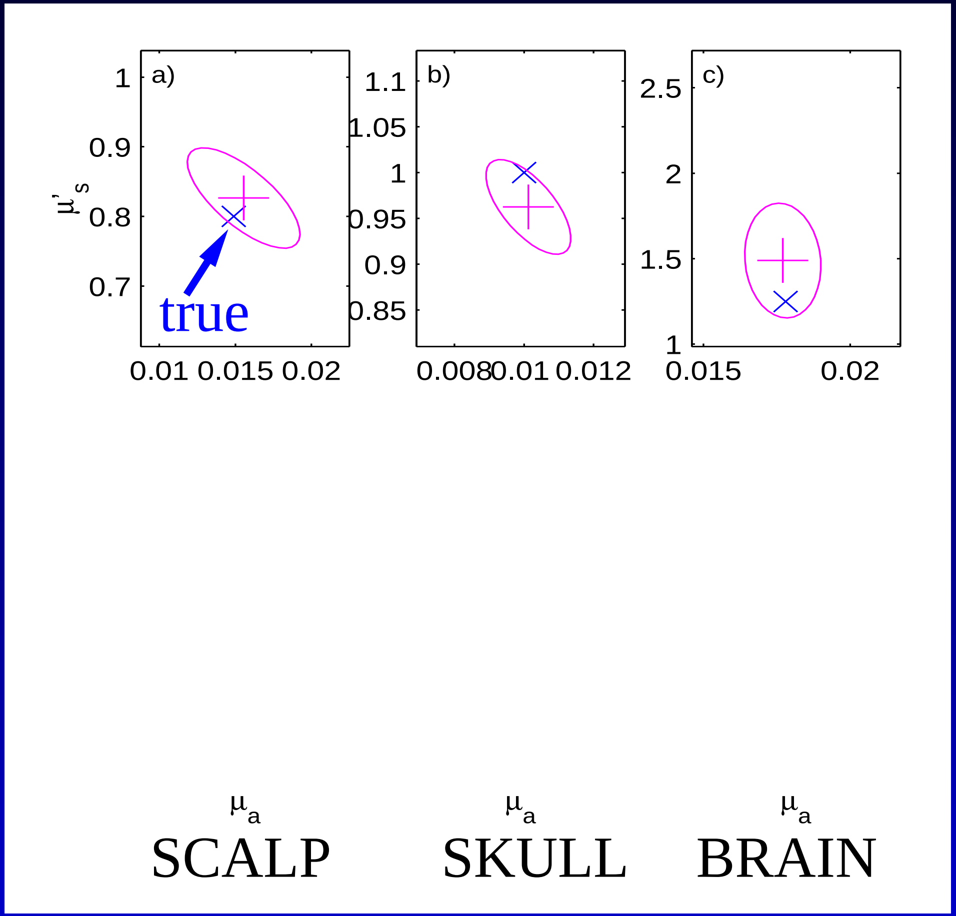


- $\varepsilon =$ relative forward model error *e.g.* 1–5% (errors: physics, numerical. . .)

Result: marginal posterior PDF

Applied Optics, special issue on biomedical optics, June 2003.



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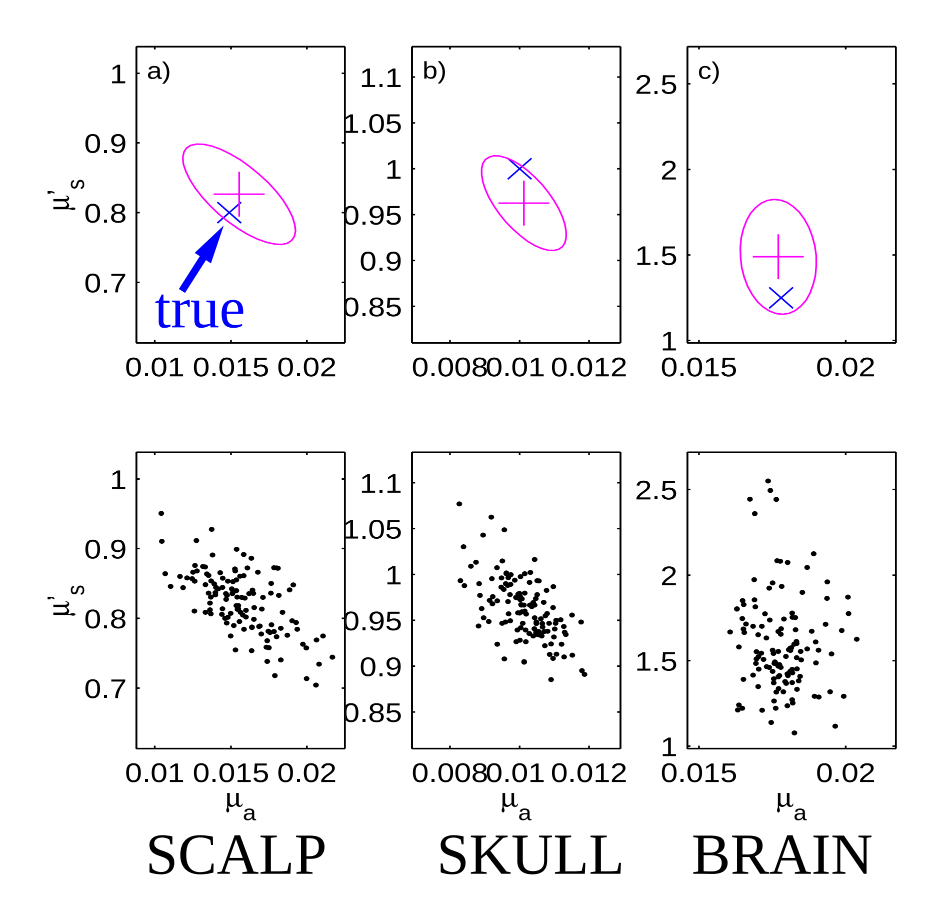

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Sampling exact PDF

Markov chain Monte Carlo $\sim 40\ \mathbf{f}(\mathbf{x})$ evals per sample (CPU intensive)

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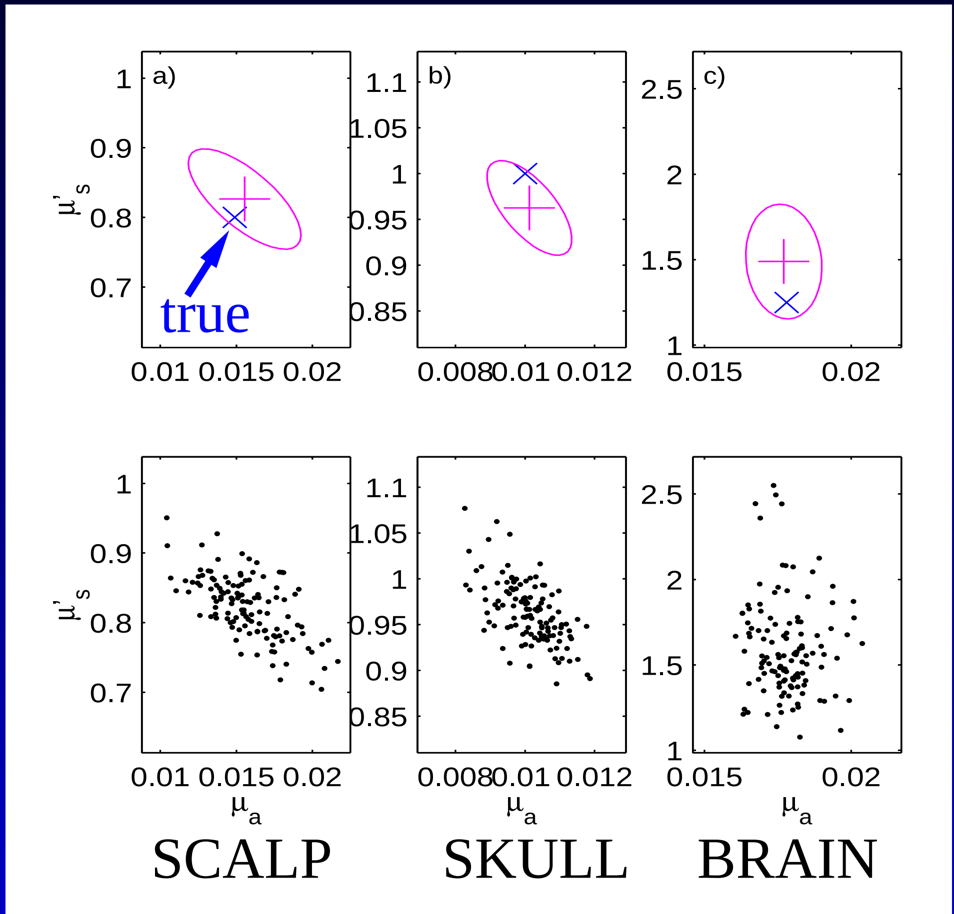

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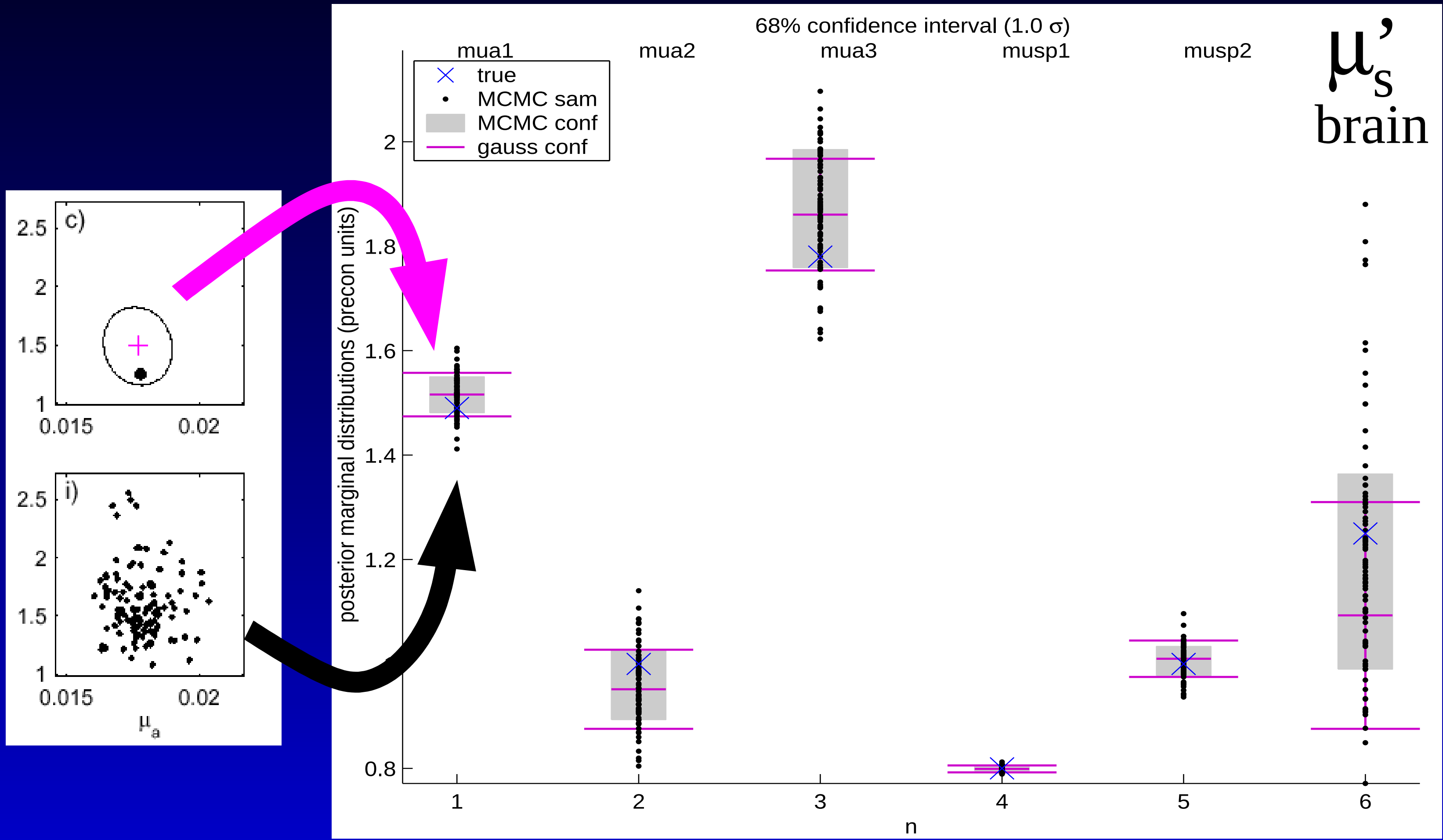
Pancake-like PDF: major-to-minor axis ratio $\sim 10^2$

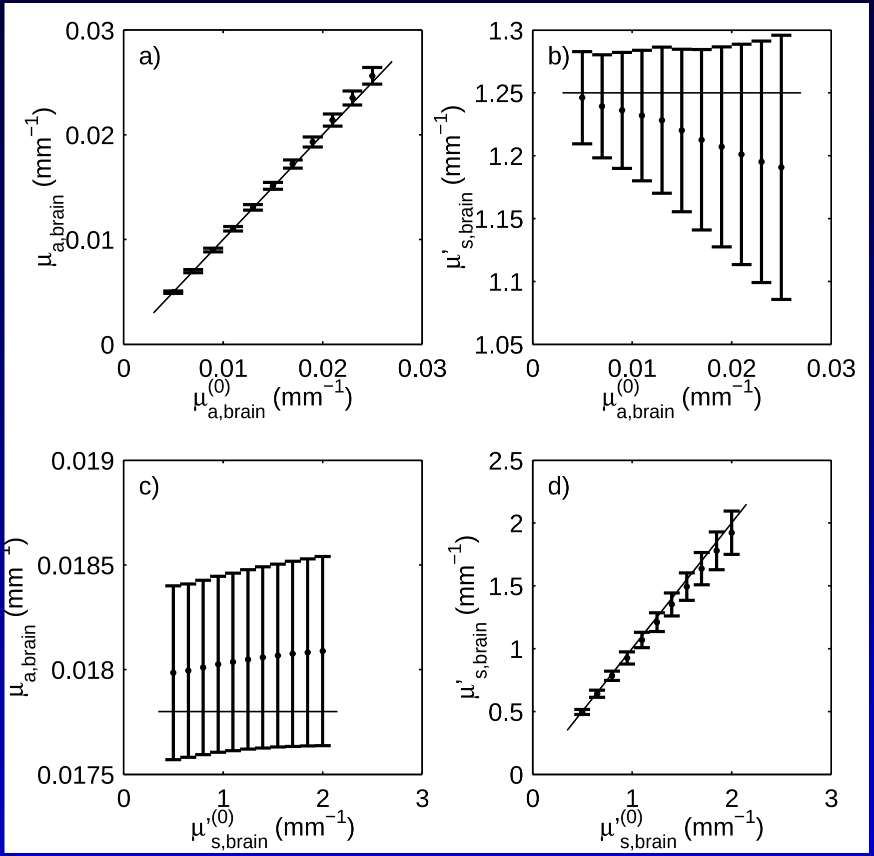
Results: confidence intervals



$\mu'_{s,\mathrm{brain}}$: Gaussian approx bad, need MCMC sampling

Tracking brain parameters

measured tracks true optical params, little crosstalk



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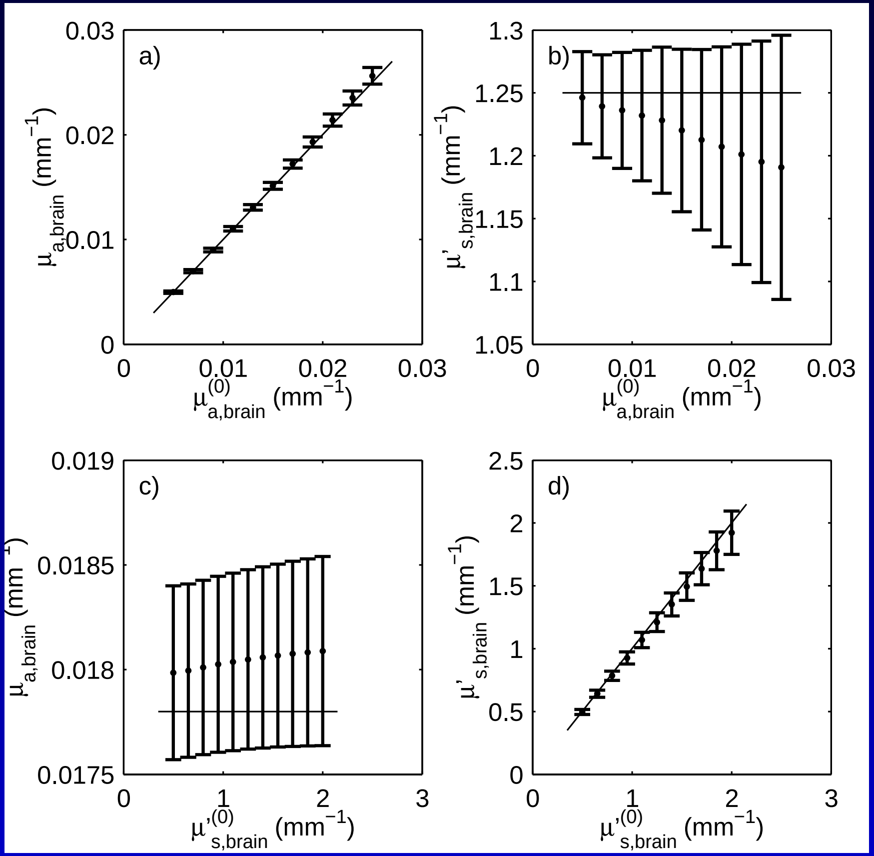
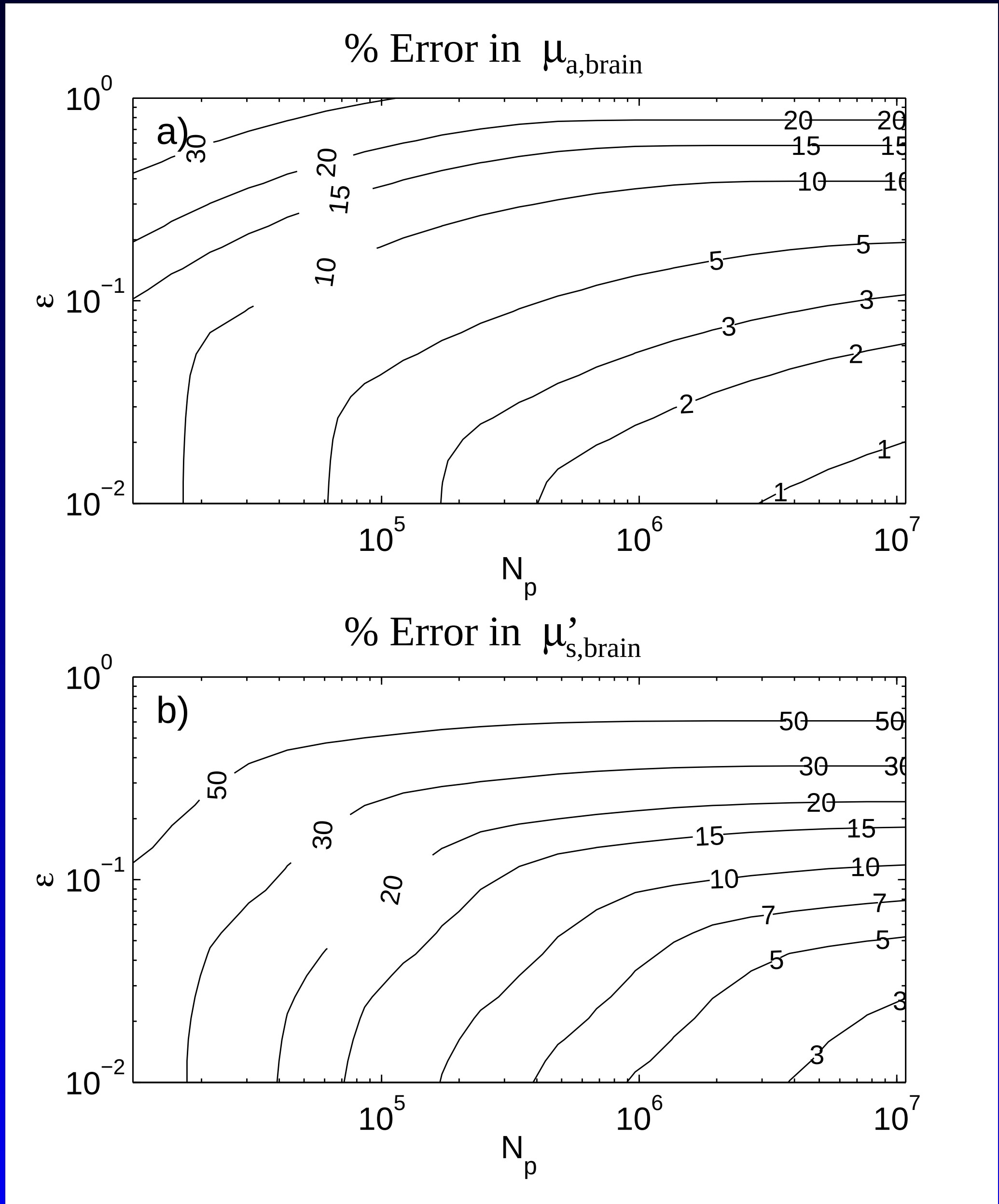


2×10^6 det photons: errorbars 3% $\mu_{a,\text{brain}}$, 10% $\mu'_{s,\text{brain}}$

for $\varepsilon = 3\%$, flat prior

How many photons needed?



Baseline brain errorbars vs…

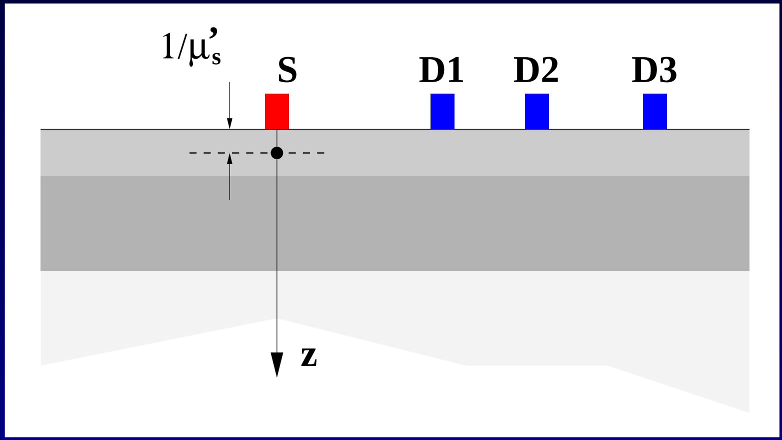
N_p = total det photons

ϵ = fwd model accuracy

Can optimize design of DOT apparatus

Partial geometric info

multilayer slab diffusion forward model:



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novel logarithmic timesteps

<1% error

Fast! typ. 0.2 sec CPU

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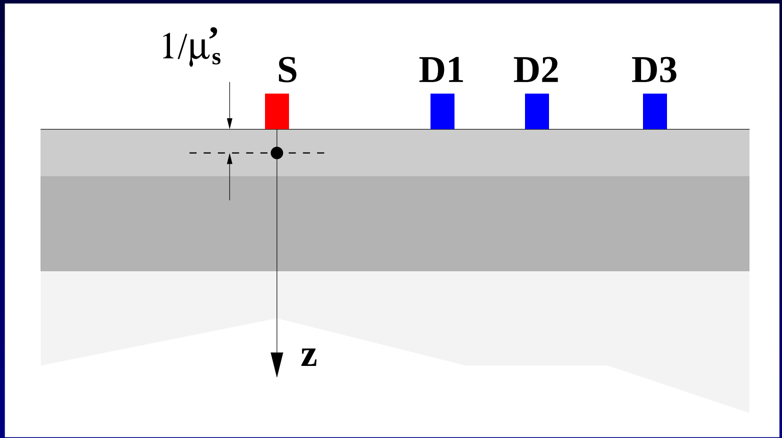

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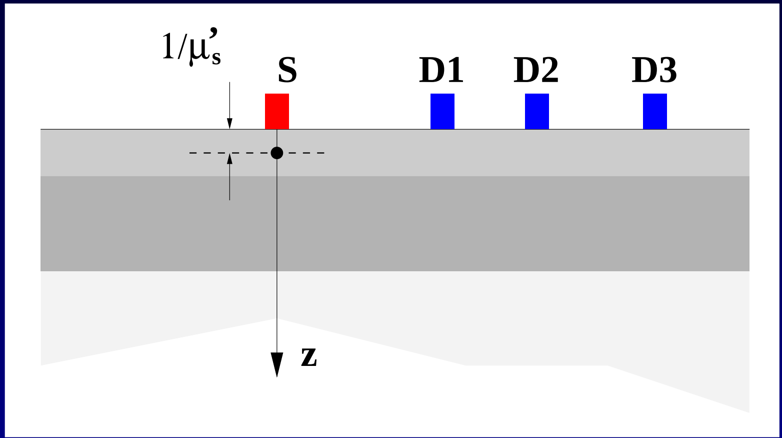

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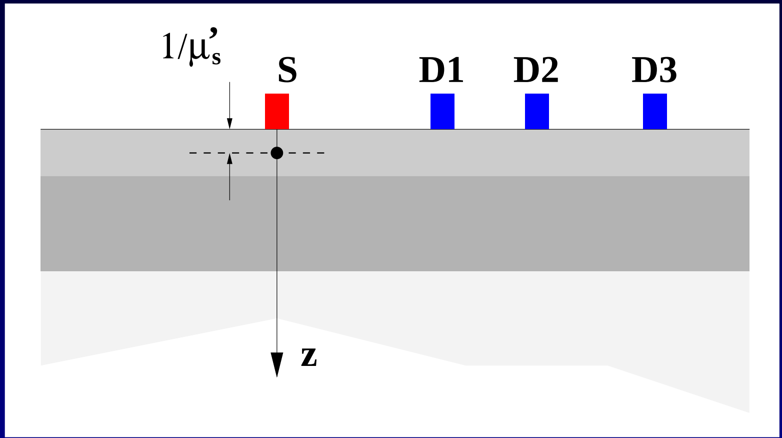

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Optode amplitude & time-offset calibrations work
(Bayes $\longrightarrow$ marginalize over the ‘nuisance’ params)

Conclusions

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- Bayes: understand full PDF on unknowns
  - · predict all errorbars, correlations
  - · CPU intensive but optimal use of data
  - · handle calibration (nuisance) params