HW5 solutions.

12.10 #8

$f(x) = \cos(3x)$

$$\cos 3x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\cos 3x$	1
1	$-3\sin 3x$	0
2	$-3^2\cos 3x$	-3^2
3	$3^3 \sin 3x$	0
4	$3^4 \cos 3x$	3^4
⋮	⋮	⋮

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{2n}$$

For Radius of convergence

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{3^{2n+2}}{(2n+2)!} \frac{x^{2n+2}\,(2n)!}{3^{2n}\, x^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^2\, 3^2}{(2n+2)(2n+1)} \right|$$

$$= \lim_{n\to\infty} \left| \frac{9x^2}{(2n+2)(2n+1)} \right|$$

$$= 0 \quad \text{for all } x$$

Hence $\boxed{R = \infty.}$

#14 $f(x) = x - x^3, \quad a = -2.$

n	$f^{(n)}(x)$	$f^n(a) = f^n(-2)$
0	$x - x^3$	6
1	$1 - 3x^2$	-11
2	$-6x$	12
3	-6	-6
4	0	0
⋮	⋮	⋮

Hence $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(-2)}{n!} (x+2)^n$$

$$= \frac{6(x+2)^0}{0!} + \frac{-11}{1!}(x+2) + \frac{12}{2!}(x+2)^2 + \frac{-6}{3!}(x+2)^3$$

$$= 6 - 11(x+2) + 6(x+2)^2 - (x+2)^3$$

\# 32 $f(x) = e^{x} + 2e^{-x}$

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Hence $$2e^{-x} = \sum_{n=0}^{\infty} \frac{2(-x)^n}{n!} = \sum_{n=0}^{\infty} \frac{2(-1)^n x^n}{n!}$$

& $$e^{x} + 2e^{-x} = \sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{2(-1)^n x^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(1+2(-1)^n)}{n!} x^n$$

\# 64

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!} \quad \left(= \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{6}\right)^{2n}}{2n!} \right)$$

$$\parallel$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

12.10

10

(a) dist from $(3,7,-5)$ to xy plane $= 5$

(b) dist from $(3,7,-5)$ to yz plane $= 3$

(c) " ——— " ... xz plane $= 7$

(d) " ——— " to the x-axis $= \sqrt{74}$

(because $\text{dist}\big((3,7,-5),\ (3,0,0)\big) = \sqrt{74}$)

(e) " ——— " to the y-axis $= \sqrt{34}$

(f) " ——— " to the z-axis $= \sqrt{58}$.

32

$x^2+y^2+z^2 > 2z$ is equivalent to

$x^2+y^2+z^2-2z+1-1 > 0$

which is $x^2+y^2+(z-1)^2 > 1$

This region consists of all points (x,y,z) which are <u>outside</u> the <u>sphere</u> <u>with radius</u> <u>1</u> & center $(0,0,+1)$.

13.2

#24. length of $\langle -2,4,2\rangle = \sqrt{4+16+4} = \sqrt{24}$

A unit vector in the dir^n of $\langle -2,4,2\rangle$

is $\vec{u} = \frac{1}{\sqrt{24}}\langle -2,4,2\rangle$.

Hence a vector that has the same dir^n as

$\langle -2,4,2\rangle$ with length 6

$$= 6\vec{u} = \frac{6}{\sqrt{24}}\langle -2,4,2\rangle$$

$$= \left\langle \frac{-12}{\sqrt{24}}, \frac{24}{\sqrt{24}}, \frac{12}{\sqrt{24}} \right\rangle$$

13.3 #18 $\vec{a} = \langle 4,0,2\rangle$, $\vec{b} = \langle 2,-1,0\rangle$

~~$\|a\|$~~

$\vec{a}\cdot\vec{b} = 8+0+0 = 8$

$\|\vec{a}\| = \sqrt{16+4} = \sqrt{20}$

$\|\vec{b}\| = \sqrt{4+1} = \sqrt{5}$

Hence $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \dfrac{8}{\sqrt{20}\sqrt{5}}$ $\left(= \dfrac{8}{\sqrt{100}} = \dfrac{4}{5}\right)$

$\theta = \cos^{-1}\left(\dfrac{8}{\sqrt{20}\sqrt{5}}\right)\left(= \cos^{-1}\left(\dfrac{4}{5}\right)\right)$

#24

(a) $\vec{u} = \langle -3, 9, 6 \rangle$, $\vec{v} = \langle 4, -12, -8 \rangle$

$\vec{u} \cdot \vec{v} = -12 - 108 - 48$

$\neq 0$ & hence not orthogonal.

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$

$\Rightarrow -168 = \sqrt{9+81+36}\ \sqrt{16+144+64}\ \cos\theta$

$= \sqrt{126}\ \sqrt{224}\ \cos\theta$

$= (3\sqrt{14})\ (4\sqrt{14})\ \cos\theta$

$= (12 \times 14)\ \cos\theta = 168 \cos\theta$

$\Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$

Hence $\vec{u}$ & $\vec{v}$ are parallel.

[Alternatively (simpler):

$\vec{u} = -\frac{3}{4}\vec{v}$

$\vec{u}$ & $\vec{v}$ are scalar multiples of each other

& hence $\vec{u}$ & $\vec{v}$ are parallel.]

(b) $\vec{u} = \vec{i} - \vec{j} + 2\vec{k}$, $\vec{v} = 2\vec{i} - \vec{j} + \vec{k}$

$\vec{u} \cdot \vec{v} = 2 + 1 + 2 = 5 \neq 0$ so $\vec{u}$ & $\vec{v}$ are not orthogonal

Also, $\vec{u}$ ~~is~~ <u>not</u> a scalar multiple of $\vec{v}$

& hence they are not parallel.

(c) $\vec{u} = \langle a, b, c \rangle$, $\vec{v} = \langle -b, a, 0 \rangle$

$\vec{u} \cdot \vec{v} = a(-b) + b(a) + 0 = 0$

So $\vec{u}$ & $\vec{v}$ are orthogonal. (not parallel)

[Note: If $a = b = c = 0$, then $\vec{u}$ & $\vec{v}$ are parallel.
You don't need to write it on your HW though]