Taylor and Maclaurin Series

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- Suppose that f is a function such that

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$$

  for $|x-a| < R$.

- Can we determine the coefficients?

The answer is: YES

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- The **Taylor series of the function f at a** is

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

The case $a = 0$

- The **Maclaurin series**

$$f(x) = f(0) + \frac{f^{'}(0)}{1!}x + \frac{f^{''}(0)}{2!}x^2 + \cdots$$

Examples

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- Find the first four nonzero terms in the Maclaurin series of $f(x) = \cos(3x)$.
- Find the first four nonzero terms of the Taylor series of $\sin x$ at $\pi/4$.
- Find the Taylor series for $f(x) = x^3$ at $a = -1$.

- Find the Maclaurin series for $\sin x$.

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$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$