Rachel Margolese
Math 7
April 11, 2013
Professor Winkler

Don't Get the Goat

The contestant stands on a blindingly bright stage facing three garage-sized doors labeled A, B, and C in large black lettering. The host of the show explains the game to the contestant and the rapt audience. Each of the doors is concealing a prize—one door hides a car and the other two hide smelly goats. The contestant will pick a door. Then the host, who knows where all of the prizes are located, will open a second door revealing a goat. The contestant will then have the option to either switch her chosen door to the third, unopened door or to stick with her original door. After making her decision, her new door will be opened to reveal her prize, either a goat or a car.

The contestant wipes her sweaty palms against her legs and chooses door A. The host motions for door C to be opened and a goat is revealed. The contestant wants to win the car. Should she switch her choice to door B or stick with her original choice of door A?

This is the well known Monty Hall problem, first widely disputed in a 1990 column written by Marilyn vos Savant. I first heard it when I was eight and thought I was so smart for figuring out a probability problem (a word I could hardly pronounce) that a bunch of adults couldn't understand. Of course it doesn't matter if the contestant switches; there are two doors remaining closed, one has a car and one has a goat so the contestant has a fifty-fifty shot that the car is behind door A. Turns out I had a lot to learn about the subtleties of probability. The contestant should always switch doors to increase her chance of winning the car because the chance that the car is hidden behind door A is not 50% as I had thought. And it wasn't just an eight-year-old who made that elementary error, thousands of math professors and holders of Ph.D.'s wrote in to Marilyn's column adamantly arguing that switching from door A to door C would make no difference.

That initial apparent simplicity in a puzzle is why math puzzles so intrigue us. A well-done math puzzle states a problem that appears blatantly easy, so much so that the puzzle must be easily solvable in your head or in a few lines of paper during a quick coffee break. As you start to jot down the answer, you find that the puzzle is not so simple, that time spent thinking is needed to solve the puzzle. The good puzzles take what your intuition says is obvious and flips it on its head, stretching out a puzzle over pages and pages. A good puzzle, once you figure out the answer, makes you sit back and say, "Oh. Well duh,"

As such, the answer to the Monty Hall puzzle is more complicated than what my eight-year-old self believed. When the contestant first chooses a door, her chance of picking the door with the car is ⅓ (one car, three options) but the situation changes when the host opens door C. At that point the chance of door B containing the car becomes ⅔. Thus the contestant should always switch doors because the probability is greater that the new door contains the car. This logic is not flawed as I believed it to be when I was eight. The chance that the contestant chose the correct door the first try is ⅓ while the remaining ⅔ chance is split evenly between the remaining doors. If one of those closed door options is then removed, the remaining ⅔ chance is consolidated onto a single door. The probability does not reset to 50-50 between the two remaining doors because although new information has been learned about the doors, the scenario has not changed.

To better visualize the different probabilities linked to doors A and B, consider the extreme scenario that instead of only three doors there are fifty. The contestant chooses a door at random; lets call it door 1. The host then opens 49 of the doors including the door the contestant chose. Remember, the host knows which door contains the prize so there is no chance that he will open the door containing the car by mistake. Then the single door that is left closed must have a 100% chance of containing the car. Now consider the scenario that after the contestant chooses a door, the host opens 48 of the remaining 49 doors which leaves two closed doors: door 1 and lets call it door 45. Now door 1 was chosen completely randomly by the contestant while door 45 was carefully selected by the host, who knows where the car is, to be left closed. The contestant picked a door that had a $1/50^{th}$ chance of containing the car while the other $49/50^{th}$ of a chance of picking the correct door was split evenly among all the other doors. Now as there is only one door that the host has left closed, that $49/50^{th}$ of a chance of opening the correct door is concentrated all in the door that the host has left closed; the door that the host left open was carefully filtered by the host to be left closed while the contestant's chosen door is left closed simply because the contestant randomly chose it. Thus, the contestant should switch.

Another way to conceptualize the answer is to consider the six possible scenarios that the contestant could find herself.

Case 1: the contestant chooses the car, then switches. Loss.

Case 2: the contestant chooses one goat, then switches (she must switch to the car because the other goat has been reveled by the host). Win.

Case 3: the contestant chooses the other goat (as there are two distinct goats, we must consider two scenarios including goats. Imagine them as a white goat and a black goat. In case 1, the contestant chose the black goat, and in case 3 she chooses the white one.), then switches to the car. Win. Overall, when the contestant switched she won ⅔ and lost ⅓.

Now consider all the cases where she doesn't switch.

Case 4: the contestant chooses the black goat, stays. Loss.

Case 5: the contestant chooses the car, stays. Win.

Case 6: the contestant chooses the white goat, stays. Loss.

When the contestant stayed she won ⅓ and lost ⅔. This basic probability test shows that the contestant will win more often when she switches than when she stays with her original choice.

Logically the probabilities work out and if this problem were completed hundreds of times exactly as stated and the contestant switched every time she would indeed win about ⅔ of the time. The contestant back in the original problem knows this and opts to switch her choice to door B. The door is opened to reveal a car! The puzzle ends happily ever after for the contestant and everyone watching has learned an important lesson in puzzles. Logic knows no "gut instinct" or intuition. Riddles take time and pure logic and a good one won't be completed quickly in your head during a coffee break.

Citations

Tierney, John. "Behind Monty Hall's Doors: Puzzle, Debate and Answer?" *The New York Times*. The New York Times, 21 July 1991. Web. 04 Apr. 2013. <http://www.nytimes.com/1991/07/21/us/behind-monty-hall-s-doors-puzzle-debate-and-answer.html?pagewanted=all>.

"Understanding the Monty Hall Problem." *BetterExplained*. N.p., n.d. Web. 04 Apr. 2013. <http://betterexplained.com/articles/understanding-the-monty-hall-problem/>.