Workshop 5
Subspaces

Instructions:

Get into groups and work on the following exercises. Each group is expected to turn in *one neatly written copy* of their solutions at the end of the class period.

Exercise 1. Let V be a vector space and let $H \subset V$ be a subspace. Show that if $\mathbf{u}$ and $\mathbf{v}$ are two vectors in H, then $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is contained in H. Can you generalize this statement?

Exercise 2. Let V be a vector space and let $H, K \subset V$ be subspaces. The *intersection* of H and K, denoted $H \cap K$, is the collection of all vectors that belong to both H and K simultaneously. In set notation

$$H \cap K = \{\mathbf{v} : \mathbf{v} \text{ is in both } H \text{ and } K\}.$$

Show that $H \cap K$ is a subspace of V.

Exercise 3. Let V be a vector space and let $H, K \subset V$ be subspaces. The *sum* of H and K, denoted $H + K$, is the collection of all vectors of the form $\mathbf{u} + \mathbf{v}$ where $\mathbf{u} \in H$ and $\mathbf{v} \in K$. In set notation

$$H + K = \{\mathbf{w} : \mathbf{w} = \mathbf{u} + \mathbf{v},\ \mathbf{u} \in H,\ \mathbf{v} \in K\}.$$

Show that $H + K$ is a subspace of V.

Exercise 4.* Let V and W be vector spaces and let $T : V \to W$ be a linear transformation. Let H be a subspace of W and let $T^{-1}(H)$ denote the set of all vectors $\mathbf{v} \in V$ so that $T(\mathbf{v}) \in H$. In set notation

$$T^{-1}(H) = \{\mathbf{v} \in V : T(\mathbf{v}) \in H\}.$$

Show that $T^{-1}(H)$ is a subspace of V.[1]

[1]The notation $T^{-1}(H)$ is *purely symbolic*. It *does not* mean that T is in invertible.