m8s05	Final	Exam Time: Fri. June 3rd, 8:00 - 11:00

Name:	Student No.:

Instructions:

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT seperate the pages of your exam.

Problem	Points	Score
A1	15	
A2	16	
A3	16	
A4	16	
A5	15	
A6	10	
A7	12	
Total	100	

Name:

Section A: Answer ALL questions.

Problem A1: [15 pts]

(**a**) Find an equation for the plane that passes through the points $A(1,0,1)$, $B(2,-1,0)$ and $C(1,3,2)$.

(**b**) Does the line $\vec{r}(t) = \langle 1,2,1\rangle + t\langle 1,-1,-1\rangle$ intersect the plane $x+2y-z=-2$? (Justify your answer.)

(**c**) What is the distance from the point $P(1,2,1)$ to the plane $x-y+3z=-2$?

Name:

Problem A2: [16 pts]

(**a**) Find $\int \frac{1}{x}\sqrt{1+4x^2}\,dx$.

(**b**) Evaluate the arc-length of the curve $\vec{r}(t) = \langle t, 2e^t \rangle$ between the points $(0, 2)$ and $(1, 2e)$.

Name:

Problem A3: [16 pts] Consider the function $f(x, y) = ye^{\sin x}$.

(**a**) Find the gradient ∇f.

(**b**) Find the tangent plane to the surface $z = f(x, y)$ at the point $(0, 1, 1)$.

(**c**) Use the function $f(x, y)$ to approximate $(0.9)e^{\sin 0.2}$ as a fraction.

(**d**) Find a tangent vector (at $(\pi/2, 2)$) to the contour (level set) of $f(x, y)$ that passes through the point $(\pi/2, 2)$.

Name:

Problem A4: [16 pts] Consider the function $f(x, y) = (x - 1)(x^2 + y^2) - 8x$.

(**a**) Find and classify all the critical points of f.

(**b**) Find the absolute max and min of $f(x, y)$ on the region $x^2 + y^2 \leq 9$.

Name:

Problem A5: [15 pts]

(**a**) What is the radius of convergence of the power series $\sum_{n=2}^{\infty} (-1)^n \frac{4^n}{n+1}(x-2)^{2n}$?

(**b**) Expand $\dfrac{1}{(3+x)^2}$ as a power series. What is its radius of convergence?

Name:

Problem A6: [10 pts] Does the improper integral

$$\int_0^1 \frac{\ln x}{\sqrt{x}} dx$$

converge or diverge? If it converges, what does it converge to?

Name:

Problem A7: [12 pts] The probability of the bird seeing a worm depends upon its position in space according to the formula

$$P(x, y, z) = \frac{\cos^2(x + y)}{1 + z^2}.$$

(**a**) A birds flight path is given by the curve

$$\vec{r}(t) = \langle t, t^2 \cos t, e^{-t} \rangle.$$

At time $t = 0$, is the bird's chance of spotting a worm increasing or decreasing?

(**b**) If the bird starts at its location at $t = 0$, in which direction should it fly to make its chances of finding lunch increase most rapidly? (Your answer should be a unit vector.)