

1. Let A, B , and C be sets. Prove that:
 - (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 - (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.These are known as De Morgan's Laws.
 2. Do Problem 1.9 in the text.
 3. The set of symmetry-preserving moves of a regular triangle, with the operation of function (move) composition, is the group called D_3 .
 - (a) Describe, with pictures or words (or both!) all elements of D_3 .
 - (b) Write down the Cayley table for D_3 .
 - (c) Is D_3 abelian? Why or why not?
 4. Write down the Cayley table for the group (\mathbb{Z}_6, \oplus) . Compare it with the Cayley table for D_3 . State two properties that these groups share (beyond the ones guaranteed by the definition of a group, like the existence of an identity), and two ways to tell them apart.
 5. Give two reasons why the set of odd integers does not form a group under addition.
 6. Let X be a set and let $P(X)$ be the set of subsets of X . Does $P(X)$ with the binary operation $A * B = A \cap B$ form a group? Why or why not?
 7. Let G be the set of all 2×2 matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Show that G forms a group under matrix multiplication.
 8. Let a and b be elements of a group G .
 - (a) Prove that $(ab)^{-1} = b^{-1}a^{-1}$.
 - (b) Give an example of a group G and elements a, b such that $(ab)^{-2} \neq b^{-2}a^{-2}$.
 - (c) Are there any groups G for which $(ab)^{-1} = a^{-1}b^{-1}$, for all $a, b \in G$?
- * Prove that for every $n \geq 1$, 3 divides $n^3 - n$.