Workshop Problems 2

Problem 1. Let A be an $m \times n$ matrix and let $\mathbf{b}$ and $\mathbf{c}$ be vectors in $\mathbb{R}^m$. Show that if $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{c}$ are both consistent then so is $A\mathbf{x} = \mathbf{b} + \mathbf{c}$.

Problem 2. Let A be an $m \times n$ matrix and let $\mathbf{u}$ be any vector in $\mathbb{R}^m$ that satisfies the equation $A\mathbf{x} = \mathbf{0}$. Show that if c is a scalar then $c\mathbf{u}$ also satisfies $A\mathbf{x} = \mathbf{0}$.

Problem 3. Let A be an $m \times n$ matrix and suppose that the vectors $\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_p}$ each satisfy the equation $A\mathbf{x} = \mathbf{0}$. Show that any linear combination of $\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_p}$ also satisfies $A\mathbf{x} = \mathbf{0}$.

Problem 4. Theorem 6 of our textbook states the following: *Suppose the system* $A\mathbf{x} = \mathbf{b}$ *is consistent for some given* $\mathbf{b}$*, and let* $\mathbf{p}$ *be a solution. Then the solution set of* $A\mathbf{x} = \mathbf{b}$ *is the set of all vectors of the form* $\mathbf{q} = \mathbf{p} + \mathbf{v}$*, where* $\mathbf{v}$ *is any solution of the homogeneous equation* $A\mathbf{x} = \mathbf{0}$. In this problem you will prove this theorem.

a. Let $\mathbf{p}$ be a solution to $A\mathbf{x} = \mathbf{b}$ and let $\mathbf{v}$ be any solution to $A\mathbf{x} = \mathbf{0}$. Show that $\mathbf{p} + \mathbf{v}$ is also a solution to $A\mathbf{x} = \mathbf{b}$.

b. Let $\mathbf{p}$ and $\mathbf{q}$ be solutions to $A\mathbf{x} = \mathbf{b}$. Show that $\mathbf{q} - \mathbf{p}$ is a solution to $A\mathbf{x} = \mathbf{0}$.

c. Use parts (a) and (b) together to give a proof of Theorem 6.