Math 11, Fall 2007

Lecture 5

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Outline

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1. Recap and overview
   - Last classes
   - Review of reading topics
   - Group Work
2. Summary
3. Next class

Lines and Planes

- Lines: $\vec{r} = \vec{r}_0 + t\vec{v}$
- Planes: $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$
- We use dot product and cross product heavily in determining defining data

Outline

Concepts from reading

Spacecruves

- Curves generalize lines:

$$\vec{r}(t) =< f(t), g(t), h(t) >$$

- Line: $f(t) = x_0 + at, g(t) = y_0 + bt, h(t) = z_0 + ct$
- Limits, derivatives and integrals are taken component-wise:

$$\lim_{t \to t_0} \vec{r}(t) =< \lim_{t \to t_0} f(t), \lim_{t \to t_0} g(t), \lim_{t \to t_0} h(t) >$$

$$\frac{d}{dt}\vec{r}(t) =< \frac{d}{dt}f(t), \frac{d}{dt}g(t), \frac{d}{dt}h(t) >$$

$$\int_a^b \vec{r}(t)\, dt =< \int_a^b f(t)\, dt, \int_a^b g(t)\, dt, \int_a^b h(t)\, dt >$$

- A space cruve is an example of a vector valued function of one variable.

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Concepts from reading
Sketching spacecurves

General tips:

- Plot projections to the coordinate planes first using one variable techniques
- Combine the projection plots to sketch the curve in 3-d
- Plotting a few test points is often helpful.
- Use one variable techniques to help. e.g. if $h'(t) > 0$ for all t, then the spacecurve moves up in the z direction for all t.
- Example: $\vec{r}(t) =< \sin(t), \sin(t), \sqrt{2}\cos(t) >$

Practice Problem

Find a vector equation of the curve which is the intersection of the top half of the sphere $x^2 + y^2 + z^2 = 1$ and the plane $y - x + 1 = 0$.
What is the domain of this curve?

Smoothness

Definitions:

1. The unit tangent vector to a spacecurve $\vec{r}(t)$ is

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

2. A space curve $\vec{r}(t)$ is continuous at $t = t_0$ if $\lim_{t \to t_0} \vec{r}(t) = \vec{r}(t_0)$.
3. A space curve $\vec{r}(t)$ is smooth if $\vec{r}'(t)$ is continuous and $\vec{r}'(t) \neq 0$ for any t.

Outline

Problems to work on

- Sketch the curve $< \cos(t), \sin(t), t >$. Is this curve continuous? Is it smooth?
- Sketch the curve $< t^2, t^3, t^4 >$ for $-1 \leq t \leq 1$. Is the curve smooth?

Summary

- Spacecurves are generalizations of lines
- A spacecurve is a vector valued function of one variable
- Differentiation, integration and limits of spacecurves are performed componentwise

Work for next class

- Reading: 14.3-14.4
- f07hw5