Additional Homework Problems

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Exercise 1.

a. Let A be an $n \times n$ matrix. Suppose that $A^k = 0$ for some integer $k \geq 1$.[1] Show that $I - A$ is invertible and that

$$(I - A)^{-1} = I + A + A^2 + \cdots + A^{k-1}.$$

b. Let

$$C = \begin{pmatrix} 8/5 & 1/5 & -6/5 \\ -1/5 & 8/5 & 2/5 \\ 3/5 & -4/5 & -1/5 \end{pmatrix}.$$

Use part (a) to show that C is invertible and find C^{-1} *without row reduction.* [*Hint:* Write $C = I - A$ and show that $A^3 = 0$. Then apply part (a).]

Exercise 2. Let $V = \mathbb{R}^+$ be the set of positive real numbers. We define *addition* in V as follows: if x and y are in V then

$$x \oplus y = xy$$

where the right-hand side is ordinary multiplication of real numbers. If c is a scalar (real number) and x is in V then we define *scalar multiplication* by

$$c \odot x = x^c$$

where the right-hand side is ordinary exponentiation of a real number. Show, by verifying the 10 axioms, that V together with the operations $\oplus$ and $\odot$ is a vector space.

[1] Such matrices are called *nilpotent*.