Review and overview
Today's material
Next class

Math 12, Fall 2007
Lecture 11

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Review and overview
Today's material
Next class

Outline

Review and overview
Today's material
Next class

Last class

Outline

Review and overview
Today's material
Next class

Last class

Differentiation
The gradient vector field

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$$\nabla f = \langle f_x, f_y \rangle$$

- The gradient encodes all the derivative information:
    1. $D_{\vec{u}} f = \nabla f \cdot \vec{u}$
    2. The gradient points in the direction of maximal ascent
    3. Extremal values must occur when $|\nabla f| = 0$
    4. For $f : \mathbb{R}^3 \to \mathbb{R}$, ∇f gives a normal vector to the level surface, $f(x, y, z) = 0$.

Review and overview
Today's material
Next class

Extremal values
Canonical examples
More examples

Outline

Review and overview
Today's material
Next class
Extremal values
Canonical examples
More examples

Locating Max/Min values

If $z = f(x, y)$ has a maximum or minimum value at (x_0, y_0) then $\nabla f(x_0, y_0) = \vec{0}$.

Example: $f(x, y) = x^3 y + 12x^2 - 8y$

$$f_x = 3x^2 y + 24x, \ f_y = x^3 - 8$$

Review and overview
Today's material
Next class
Extremal values
Canonical examples
More examples

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Review and overview
Today's material
Next class

Extremal values
Canonical examples
More examples

Second derivative test

Suppose the second partial derivatives of f are continuous on a disk with center (x_0, y_0) and that $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$. Let

$$D = f_{xx}(x_0, y_0)f_{yy}(x_0, y_0) - (f_{xy}(x_0, y_0))^2$$

- If $D > 0$ and $f_{xx}(x_0, y_0) > 0$ then $f(x_0, y_0)$ is a local minimum.
- If $D > 0$ and $f_{xx}(x_0, y_0) < 0$ then $f(x_0, y_0)$ is a local maximum.
- If $D < 0$ then $f(x_0, y_0)$ is a not a local minimum or maximum.
- If $D = 0$ then the test is inconclusive

Review and overview
Today's material
Next class

Extremal values
Canonical examples
More examples

Outline

Review and overview
Today's material
Next class

Extremal values
Canonical examples
More examples

Simple examples

- $f(x, y) = x^2 + y^2$
- $f(x, y) = xy$
- $f(x, y) = -x^2 - y^2$

Review and overview
Today's material
Next class

Extremal values
Canonical examples
More examples

Outline

Review and overview
Today's material
Next class

Extremal values
Canonical examples
More examples

Examples

- $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$
- $f(x, y) = \sin(x^2 + y^2)$

Review and overview
Today's material
Next class

Work for next class

- Reading: 15.7
- Exam 1!
- f07hw12