Lagrange Multipliers

Lecture 28

February 7, 2007

Example

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Find the extreme values of $f(x, y) = x^2 + 2y^2$ on the set

$$D = \{(x, y) \ : \ x^2 + y^2 \leq 1\}.$$

Method of Lagrange Multipliers

Fact

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$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

and

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2. *Evaluate f at all the points (x, y, z) that result from step 1. The largest of these values is the maximum value of f; the smallest is the minimum value of f.*

Example (cont'd)

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- Let's use the Lagrange multiplier method to find the extreme values of $f(x, y) = x^2 + 2y^2$ on the unit circle.

Example (cont'd)

Example

- Let's use the Lagrange multiplier method to find the extreme values of $f(x, y) = x^2 + 2y^2$ on the unit circle.
- Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closed to and farthest from the point $(3, 1, -1)$.

Thank you and good luck!

The End!