Supplementary Homework for Math 43
Due Monday, May 6, 2002

S1: We showed in class that if the power series

$$\sum_{n=0}^{\infty} a_n (z - z_0)^n \tag{†}$$

converges when $z = z_1 \neq z_0$, then (†) converges absolutely for all z such that $|z - z_0| < R_1$ where $R_1 := |z_1 - z_0|$. Using this, prove the following assertion made in lecture: Let R be the least upper bound of the numbers

$$\{\, |z_2 - z_0| : (\dagger) \text{ converges when } z = z_2 \,\}.$$

Then prove that (†) converges absolutely for all z such that $|z - z_0| < R$, and that (†) diverges for all z such that $|z - z_0| > R$. (Recall that R is called the *radius of convergence* of (†).