

1. (20) Evaluate the integral  $\int (\sin x + x)^2 dx$ . (*Show all work*).
2. (20) Find the general solution to the differential equation  $\frac{dy}{dx} = \frac{y^2}{(x-1)(x-2)}$ . (*Show all work*).
3. (15) Write your answer in the space indicated. (*Show all work*).
  - (a) (7) A 5000 liter container is filled with 1000 liters of a brine solution containing 20 kg of dissolved salt. Two solutions are added to the container. The first solution contains .03 kg/l of salt and is added at 10 l/min, while the second is pure water and is added at 20 l/min. The solution is kept thoroughly mixed and drains from the container at a rate of 20 l/min. Find (but do not solve) a differential equation which describes this system.

Answer: \_\_\_\_\_

- (b) (8) Consider the homogeneous differential equation  $\frac{d^2y}{dt^2} + 9y = 0$ . Find the general solution as well as the particular solution which satisfies the initial conditions  $y(0) = 3$  and  $y'(0) = 6$ .

General: \_\_\_\_\_ Particular: \_\_\_\_\_

4. (15) Write your answer in the space indicated. (*Show all work*).

- (a) (7) Consider the differential equation  $\frac{dy}{dx} = \frac{\ln x + 1}{y}$ , and a solution  $y = f(x)$  which satisfies  $f(1) = 2\sqrt{2}$ . Is this solution increasing or decreasing at the point  $(1, 2\sqrt{2})$ ? Justify your answer. Hint: Don't try to solve the differential equation; rather think about the field of slopes associated to this differential equation.

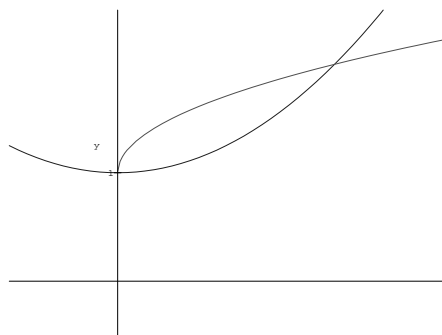
Answer: \_\_\_\_\_

- (b) (8) Consider the differential equation  $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ , where  $b$  and  $c$  are constants and  $b = 2\sqrt{c}$ . Determine the general solution of this differential equation.

Answer: \_\_\_\_\_

5. (15) Write your answer in the space indicated. (*Show all work*).

- (a) (8) Write a definite integral which whose value is the volume of the solid of revolution obtained by revolving the region (pictured) between the curves  $y = x^2 + 1$  and  $y = \sqrt{x} + 1$  about the  $x$ -axis. Do NOT evaluate this integral.



Answer: \_\_\_\_\_

- (b) (7) Find a complex number which satisfies the equation  $z^4 = 1 + i$ .

Answer: \_\_\_\_\_

6. (15) Write your answer in the space indicated. (*Show all work*).

- (a) (7) Evaluate the integral  $\int \tan^3 \theta \sec^2 \theta d\theta$ .

Answer: \_\_\_\_\_

- (b) (8) Determine the correct integrating factor to solve the differential equation:  $(1 + x^2)y' - 3y = \sin x$

Answer: \_\_\_\_\_