Predator-Prey Lab

The goal of this lab is to study the dynamics of a spotted owl and wood rat population, where the rats compose a significant portion of the owl population's diet, and likewise, the owls are a major predator of the rats. Let O_k denote the owl population size in month k, R_k the rat population size in month k. Suppose we can model the population dynamics with the following linear equations:

$$\begin{aligned} O_{k+1} &= .5O_k + .4R_k \\ R_{k+1} &= -pO_k + 1.1R_k \end{aligned}$$

where p is a parameter representing predation.

The model can be understood as follows:

pO_k is the number of rats consumed per month by the owls. 1.1 is the monthly growth rate of the rat population in the absence of owls (that is, the rat population increases by 10% monthly in the absence of owls). In the absence of wood rats, the owl population decreases by half each month (hence the term $.5O_k$ in the first equation). In the presence of wood rats, .4 is the growth rate of the owl population per wood rat (as a result of consumption of the rats).

Note that this system can be written in matrix notation:

$\mathbf{x}_{k+1} = A\mathbf{x}_k$, where $A = \begin{bmatrix} .5 & .4 \\ -p & 1.1 \end{bmatrix}$ and $\mathbf{x}_k = \begin{bmatrix} O_k \\ R_k \end{bmatrix}$. Suppose $\mathbf{x}_0 = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$.

1. Suppose λ_1, λ_2 are eigenvalues of A and $\mathbf{v}_1, \mathbf{v}_2$ eigenvectors corresponding to λ_1 and λ_2, respectively. Recall that there exist scalars c_1, c_2 such that the initial condition vector $\mathbf{x}_0$ can be written as $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$. Write an expression for $\mathbf{x}_k$ in terms of the eigenvalues, eigenvectors, and c_i's below:

2. Suppose $p = 0.2$. Calculate the eigenvalues of A and use the above expression for $\mathbf{x}_k$ to draw a conclusion about whether the populations grow, decline, or approach a steady-state.

3. Simulate the population using MATLAB for $p = .2$. Is the simulation consistent with your expectation?

4. Suppose instead that $p = 0.1$. Calculate the eigenvalues of A and again use the expression for $\mathbf{x}_k$ to draw a conclusion about whether the populations grow, decline, or approach a steady-state.

5. Simulate the population with $p = 0.1$. Is the simulation consistent with your expectation?

6. Imagine now that your goal is to protect the owl population, but you don't want the wood rat population to grow out of control either. What would p need to be in order to maintain constant owl and rat populations? [Hint: Think about the role that the eigenvalues play in the growth of the populations].

7. Set p to be the value you determined in the previous question. Suppose that a mysterious parasite is infecting the rats and as a result, the monthly rat population growth rate is reduced from 1.1 to 1.0. What happens to the owls under this scenario?

8. Consider the original matrix A with $p = 0.125$. Determine the eigenvalues of A and corresponding eigenvectors (determine eigenvectors by hand!). Calculate the scalars c_i described in question (1). Use these values to write an expression for $\mathbf{x}_k$ and simulate the population dynamics in MATLAB.

9. Describe aspects of this model that are unrealistic. What factors might you want to include to improve the model?