1 1.8.30)

If $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$ then

$T(\mathbf{x}+\mathbf{y}) = A(\mathbf{x}+\mathbf{y})+\mathbf{b} = A\mathbf{x}+A\mathbf{y}+\mathbf{b} \neq A\mathbf{x}++\mathbf{b}+A\mathbf{y}+\mathbf{b} = T(\mathbf{x})+T(\mathbf{y})$

unless $\mathbf{b} = \mathbf{0}$

2 1.8.32)

If $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$ then $T(-3(0,1)) = T(0,-3) = (6,9) \neq (6,-9) = -3T(0,1)$

3 1.9.20)

If $T(x_1, x_2, x_3, x_4) = 2x_1+3x_3-4x_4$ we have $T(\mathbf{x} = A\mathbf{x}$ where $A = \begin{bmatrix} 2 & 0 & 3 & -4 \end{bmatrix}$

Since $\begin{bmatrix} 2 & 0 & 3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 2x_1 + 3x_3 - 4x_4 = T(x_1, x_2, x_3, x_4)$

4 1.9.26)

If A is the standard matrix for T, then T is one-to-one if and only if the columns of A are linearly independent in $\mathbb{R}^2$. Now from problem 2 of this section we know that

$$A = \begin{bmatrix} 1 & 4 & -5 \\ 3 & -7 & 4 \end{bmatrix}$$

This gives us three vectors in $\mathbb{R}^2$ which implies by theorem 1.7.8 that the columns of A are linearly dependent. Hence T is not one-to-one.

T is onto if and only if the columns of A span $\mathbb{R}^2$ by theorem 1.9.12 if and only if A has a pivot position in every row by theorem 1.4.4. So it suffices to row reduce A

$R_2 : R_2 - 3R_1$

$$\begin{bmatrix} 1 & 4 & -5 \\ 0 & -19 & 19 \end{bmatrix}$$

$R_2 : -(1/19)R_2$

$$\begin{bmatrix} 1 & 4 & -5 \\ 0 & 1 & -1 \end{bmatrix}$$

Hence A has a pivot in every row which implies T is onto.

5 1.S.14)

Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} a \\ a+2 \end{bmatrix}$

First we will find all a such that $\mathbf{v_1}, \mathbf{v_2}$ are linearly dependent. If $\mathbf{v_1}, \mathbf{v_2}$ are linearly dependent, there exists $c \in \mathbb{R}$ such that

$$c \begin{bmatrix} 1 \\ a \end{bmatrix} = \begin{bmatrix} a \\ a+2 \end{bmatrix}$$

giving

$$\begin{bmatrix} c \\ ca \end{bmatrix} = \begin{bmatrix} a \\ a+2 \end{bmatrix}$$

hence we must have $c = a$ and $ca = a + 2$ hence $a^2 - a - 2 = 0$ or $(a-2)(a+1) = 0$. Thus we have $\mathbf{v_1}, \mathbf{v_2}$ are linearly dependent if and only if $a \in \{-1, 2\}$ which implies $\mathbf{v_1}, \mathbf{v_2}$ are linearly independent when $a \notin \{-1, 2\}$.

6 1.S.22

If $\mathbf{x} \mapsto A\mathbf{x}$ is onto then A has a pivot in each row. A is a 3×3 matrix hence A has 3 pivots. Hence A has a pivot in each column, which implies $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.