

Answer ALL questions. Unless instructed otherwise, you should show ALL your work and simplify your final answer as much as possible. Please box your final answer to each part.

**Problem 1:** [12 pts]

(a) Compute the 3rd order Taylor polynomial for  $x^{1/3}$  centered at  $x = 1$ .

(b) The Taylor series for  $x^{-1/2}$  centered at  $x = 1$  is given by

$$1 + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!} (x-1)^{2n+1}.$$

Use the Taylor series to compute  $\sqrt{\frac{10}{11}}$  as a fraction, accurate to within  $10^{-4}$ . (You must justify that your answer is within this accuracy).

**Problem 2:** [13 pts] Consider the three points  $A(1, 0, -1)$ ,  $B(2, -1, 2)$  and  $C(1, 3, 1)$ .

(a) What is the radius of the sphere centered at the point  $A$  that passes through the point  $B$ ?

(b) What is the angle  $\angle BAC$ ? (i.e. the angle at  $A$ .) You may leave your answer in the form of an arccos.

(c) Find the vector projection of  $\vec{AB}$  onto  $\vec{AC}$ .

(d) A point  $D$  is constructed on the straight line through  $A$  and  $C$  so that the triangle  $\triangle ABD$  has a right angle at  $D$ . Find the coordinates of the point  $D$ . (You may find it useful to use your answer to (c)).