Math 11, Fall 2007

Lecture 16

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10/31/07

Outline

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Integration of a function of two variables

General domains

- Iterated integrals and Fubini's theorem

$$\int\int_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dx\,dy$$

- Non-rectangular domains: parameterize boundary and introduce variables into the bounds of integration

Outline

Polar coordinates

- $(x, y) \to (r, \theta)$
- $x = r\cos(\theta), y = r\sin(\theta)$
- $r^2 = x^2 + y^2, \theta = \tan^{-1}(y/x)$

Change of variables

If f is a continuous function defined on a polar rectangle $R = [a, b] \times [\alpha, \beta] = \{(r, \theta) | a \leq r \leq b, \alpha \leq \theta \leq \beta\}$ where $0 \leq \beta - \alpha \leq 2\pi$ then

$$\int \int_R f(x, y)\, dA = \int_\alpha^\beta \int_a^b f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$$

General change of variables

If we are trying to integrate f over a region D, we may always change to polar coordinates:

$$\int\int_D f\ dA = \int\int_{D^*} f\ r dr d\theta$$

where D^* is the same region of D, described with respect to the polar variables.

Examples

1. $$\int\int_D (1 - x^2 - y^2)\, dA$$
   where D is the circle of radius 2 centered at the origin.
2. Let $R = \{(x, y) | 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$ and find
   $$\int\int_R \tan^{-1}(y/x)\, dA$$
3. Find the volume of a sphere is radius a centered at the origin.
4. Find
   $$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$$

Examples

A cylindrical drill with radius r_1 is used to bore a hole through the center of a solid ball of radius r_2. Find the volume of the ring-shpaed solid that remains. Express the volume in terms of the height, h, of the ring.

Work for next class

- Reading: 16.6
- f07hw17