Math 23 WORKSHEET :   Integrating factors

Fri 9/23/05
Alex Barnett

i) Find general solution to $ty' + 3y = 5t^2$

Hint : put into $\underline{standard\ form}$

so $p(t) =$

$g(t) =$

integrating factor $\mu(t) =$

ii) Find $y(t)$ if initial conditions are $y(1) = 2$

iii) Sketch $y(t)$:

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i) Find general solution to $ty' + 3y = 5t^2$

Hint: put into _standard form_

so $p(t) = \frac{3}{t}$

ie divide by t $\longrightarrow$ $y' + \frac{3}{t}y = 5t$

$g(t) = 5t$

integrating factor $N(t) = e^{\int p(t)dt} = e^{\int \frac{3}{t}dt} = e^{3\ln t}$

Now $3\ln t = \ln(t^3)$

$\Leftarrow$ check your algebra!

so $e^{3\ln t} = e^{\ln(t^3)} = t^3$

Note _not_ $e^3 + e^{\ln t}$
etc.

So $t^3 y' + \frac{3}{t}t^3 y = 5t^4$

$\underbrace{\qquad\qquad\qquad}$
$(t^3 y)'$

so $t^3 y = \int 5t^4 dt = t^5 + c$

$\Rightarrow y'(t) = \frac{t^5 + c}{t^3} = t^2 + \frac{c}{t^3}$

ii) Find $y(t)$ if initial conditions are $y(1) = 2$

$1 = 1^2 + \frac{c}{1^3}$ so $c = 1$

$\Rightarrow y(t) = t^2 + \frac{1}{t^3}$

iii) Sketch $g(t)$:

use $y \to \infty$ as $t \to 0$
$y \to \infty$ as $t \to \infty$.