Dynamical Systems from a Number Theorists Perspective

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(Tea 3:30 pm Math Lounge)

Abstract

A classical problem in the theory of dynamical systems is to describe the behavior of points under interation $\phi^n = \phi \circ \phi \circ \cdots \circ \phi$ of a rational map $\phi(z) = F(z)/G(z)$, i.e., where $F(z)$ and $G(z)$ are polynomials. The *orbit* of a point α under iteration of ϕ, denoted $O_\alpha(\phi)$, is the set of images of α under the iterates of ϕ, $O_\alpha(\phi) = \{\phi^n(\alpha) : n \geq 0\}$. The points with finite orbit, called *preperiodic points*, play a particularly important role in the dynamics of ϕ. For a number theorist, it is natural to take $F(z)$ and $G(z)$ to have integer coefficients and to study the orbits of rational numbers $\alpha \in \mathbb{Q}$. In this talk I will survey some of the known results and some of the outstanding conjectures related to this number-theoretic view of dynamics. Typical problems include: (1) How many preperiodic points can be rational numbers $\alpha \in \mathbb{Q}$? (2) For which rational maps ϕ can the orbit $O_\alpha(\phi)$ of a rational number α contain infinitely many integers?

This talk should be accessible to graduate students.

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