1 5.4.12

solution: As in example 4 the matrix $P = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, an easy computation shows $P^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix}$. Thus the matrix for the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ with respect to the basis $\mathfrak{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ is

$$B = P^{-1}AP = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

2 5.4.20

solution: if A is similar to B then there exists an invertible matrix P such that $B = P^{-1}AP$. So $B^2 = P^{-1}APP^{-1}AP = P^{-1}AIAP = P^{-1}A^2P$, hence A^2 is similar to B^2

3 6.1.24

solution: $\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = \langle \mathbf{u}+\mathbf{v}, \mathbf{u}+\mathbf{v}\rangle + \langle \mathbf{u}-\mathbf{v}, \mathbf{u}-\mathbf{v}\rangle$

$$= \langle \mathbf{u}, \mathbf{u}+\mathbf{v}\rangle + \langle \mathbf{v}, \mathbf{u}+\mathbf{v}\rangle + \langle \mathbf{u}, \mathbf{u}-\mathbf{v}\rangle + \langle -\mathbf{v}, \mathbf{u}-\mathbf{v}\rangle$$

$$= \langle \mathbf{u}, \mathbf{u}\rangle + \langle \mathbf{u}, \mathbf{v}\rangle + \langle \mathbf{v}, \mathbf{u}\rangle + \langle \mathbf{v}, \mathbf{v}\rangle + \langle \mathbf{u}, \mathbf{u}\rangle + \langle \mathbf{u}, -\mathbf{v}\rangle - \langle \mathbf{v}, \mathbf{u}\rangle - \langle \mathbf{v}, -\mathbf{v}\rangle$$

$$= \langle \mathbf{u}, \mathbf{u}\rangle + \langle \mathbf{u}, \mathbf{v}\rangle + \langle \mathbf{v}, \mathbf{u}\rangle + \langle \mathbf{v}, \mathbf{v}\rangle + \langle \mathbf{u}, \mathbf{u}\rangle - \langle \mathbf{u}, \mathbf{v}\rangle - \langle \mathbf{v}, \mathbf{u}\rangle + \langle \mathbf{v}, \mathbf{v}\rangle$$

$$= \langle \mathbf{u}, \mathbf{u}\rangle + \langle \mathbf{v}, \mathbf{v}\rangle + \langle \mathbf{u}, \mathbf{u}\rangle + \langle \mathbf{v}, \mathbf{v}\rangle = 2\langle \mathbf{u}, \mathbf{u}\rangle + 2\langle \mathbf{v}, \mathbf{v}\rangle = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

4 6.1.30

solution:

a) Pick $z \in W^{\perp}$ and let c be any scalar then for $u \in W$ we have $\langle cz, u\rangle = c\langle z, u\rangle = c(0) = 0$. Since this holds for all $u \in W$ we have $cz \in W^{\perp}$

b) Pick $z_1, z_2 \in W^{\perp}$, the for $u \in W$ we have $\langle z_1+z_2, u\rangle = \langle z_1, u\rangle + \langle z_2, u\rangle = 0 + 0 = 0$. Since this holds for all $u \in W$ we have $z_1 + z_2 \in W^{\perp}$

c) $0 \in W^\perp$ since for $u \in W$ $\langle 0, u\rangle = \langle 0 \cdot 0, u\rangle = 0\langle 0, u\rangle = 0$ Hence $W^\perp$ is a subspace by definition, since (a) and (b) show that $W^\perp$ is closed under addition and scalar multiplication.

5 6.2.8

solution: We will first show that $u_1 \perp u_2$ now $\langle u_1, u_2\rangle = 3(-2) + 1(6) = -6 + 6 = 0$ thus $u_1 \perp u_2$. This implies that $\{u_1, u_2\}$ is linearly independent. Now since $\{u_1, u_2\}$ is a set of two linearly independent vectors in $\mathbb{R}^2$ we must have that $\{u_1, u_2\}$ forms a basis for $\mathbb{R}^2$. Now by theorem 5 we have $x = \frac{\langle x,u_1\rangle}{\langle u_1,u_1\rangle} u_1 + \frac{\langle x,u_2\rangle}{\langle u_2,u_2\rangle} u_2 = -\frac{3}{2} u_1 + \frac{3}{4} u_2$