The Ratio and Root Test

October 11, 2006

The Ratio Test

1. If
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1,$$
then the series $\sum a_n$ is absolutely convergent.

2. If
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1 \quad \text{or} \quad \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty,$$
then the series $\sum a_n$ is divergent.

3. If
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1,$$
the Ratio Test is inconclusive.

Examples

Determine whether the series is AC, CC, or D.

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- $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
- $\sum \frac{(n+3)!}{3!n!3^n}$

The Root Test

1. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum a_n$ is absolutely convergent.

2. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$, then the series $\sum a_n$ is divergent.

3. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, the Root Test is inconclusive.

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- $\sum \frac{(-1)^n}{(\arctan n)^n}$