Math 22 Practice Problems

NOTE: In no sense should this collection of problems be construed as representative of the actual exam. These are simply some problems left over from the preparation of the exam or from previous exams which should serve to indicate the general level of expectation.

Problem 1. Solve the matrix equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 1 & 4 & 3 & -14 \\ 2 & 8 & 0 & -10 \\ 3 & 12 & 2 & -21 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -9 \\ -6 \\ -13 \end{pmatrix}.$$

Write your answer in parametric (vector) form.

Problem 2. Let

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} -2 \\ 3 \\ 1/2 \end{pmatrix}.$$

a. Write $\mathbf{v}$ as a linear combination of $\mathbf{e}_1, \mathbf{e}_2$ and $\mathbf{e}_3$

b. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation satisfying

$$T(\mathbf{e}_1) = \begin{pmatrix} -4 \\ 3 \end{pmatrix}, T(\mathbf{e}_2) = \begin{pmatrix} -2/3 \\ 5 \end{pmatrix}, T(\mathbf{v}) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

Find the standard matrix of T. [*Hint:* Use part (a) to help you find $T(\mathbf{e}_3)$.]

c. Is T one-to-one? Justify your answer.

Problem 3. Let

$$A = \begin{pmatrix} -4 & 1 & 0 \\ -2 & -1 & -2 \\ 4 & 1 & -5 \end{pmatrix}.$$

a. Are the columns of A linearly independent? Justify your answer.

b. Do the columns of A span $\mathbb{R}^3$? Justify your answer.

Problem 4. Fill in each blank below with a choice from the following list so that the resulting statement is always true.

(a) No solutions
(b) Exactly one solution
(c) At least one solution
(d) Infinitely many solutions

You *do not* need to justify your answers and no partial credit is possible.

i. If $T : \mathbb{R}^n \to \mathbb{R}^m$ is *not* onto, then there is a $\mathbf{b}$ in $\mathbb{R}^m$ so that $T(\mathbf{x}) = \mathbf{b}$ has ____________.

ii. If a matrix A has a column that is *not* a pivot column, then $A\mathbf{x} = \mathbf{0}$ has ____________.

iii. If $\mathbf{b}$ is a linear combination of the columns of the matrix A then $A\mathbf{x} = \mathbf{b}$ has ____________.

iv. The matrix equation $A\mathbf{x} = \mathbf{0}$ always has ____________.

v. If the matrix equation $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions then $A\mathbf{x} = \mathbf{b}$ *cannot* have ____________.

vi. If every row of a matrix A contains a pivot position then $A\mathbf{x} = \mathbf{b}$ has ____________.

vii. If the columns of a matrix A are linearly independent then $A\mathbf{x} = \mathbf{0}$ has ____________ with $\mathbf{x} \neq \mathbf{0}$.

viii. If T is a linear transformation then T is one-to-one if and only if $T\mathbf{x} = \mathbf{0}$ has ____________.

Problem 5. Let

$$A = \begin{pmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{pmatrix}.$$

a. Find A^{-1}.

b. Use the result from part (a) to solve the system $A\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

Problem 6. Determine h and k such that the system of equations

$$\begin{aligned} x_1 + 3x_2 &= k \\ 4x_1 + hx_2 &= 8 \end{aligned}$$

has

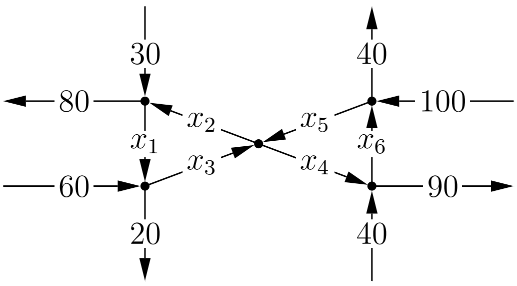
a. no solution;

b. exactly one solution;

c. infinitely many solutions.

In cases (b) and (c), solve the system and write the solution in parametric (vector) form.

Problem 7. Find the general flow pattern in the network shown below.



Problem 8. Let $S : \mathbb{R}^n \to \mathbb{R}^m$ and $T : \mathbb{R}^m \to \mathbb{R}^p$ be linear transformations. Show that the map $\mathbf{x} \mapsto T(S(\mathbf{x}))$ is a linear transformation (from $\mathbb{R}^n$ to $\mathbb{R}^p$).

(Work Page)