8.1

#26 Evaluate the integral:

$$\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$$

Let $u = \arctan\left(\frac{1}{x}\right)$ $\qquad dv = dx$

$du = \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right)dx$ $\qquad v = x$

Integration by parts formula

$$\Rightarrow \int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx = x \arctan\left(\frac{1}{x}\right)\Big]_1^{\sqrt{3}} - \int_1^{\sqrt{3}} -\frac{x}{1+x^2}\,dx$$

$$= x \arctan\left(\frac{1}{x}\right)\Big|_1^{\sqrt{3}} + \frac{1}{2}\ln(1+x^2)\Big]_1^{\sqrt{3}}$$

$$= \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}\right) - \arctan(1) + \frac{1}{2}\left[\ln 4 - \ln 2\right]$$

$$= \boxed{\sqrt{3}\left(\frac{\pi}{6}\right) - \frac{\pi}{4} + \frac{1}{2}(\ln 4 - \ln 2)}$$

8.1

#34 First make a substitution & then use integration by parts to evaluate the integral

$$\int t^3 e^{-t^2} dt$$

→ Substitute $x = t^2$

$dx = 2t\,dt$

$t^3 = t^2 t$

Hence $\int t^3 e^{-t^2} dt = \int t^2 e^{-t^2} \underbrace{t\,dt}_{= dx/2}$

$= \frac{1}{2}\int x e^{-x} dx$

$= \frac{1}{2}\left[\int x e^{-x} dx\right]$ —— (*)

~~we~~ let $u = x$ & $dv = e^{-x} dx$

$\Rightarrow du = dx$ $\quad v = -e^{-x}$

By parts ~~on~~ in (*) ⇒

$\int t^3 e^{-t^2} dt = \frac{1}{2}\left[-xe^{-x} + \int e^{-x} dx\right]$

$= \frac{1}{2}\left[-xe^{-x} - e^{-x}\right] + C$

$$= \boxed{\frac{1}{2}\left[-t^2 e^{-t^2} - e^{-t^2}\right] + C}$$

8.2

#6 $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx$

Substitute $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

So the integral becomes $2\displaystyle\int \frac{\sin^3 u}{1}\,du$

Evaluate $2\displaystyle\int \sin^3 u\,du$

$= 2\displaystyle\int \sin^2 u \sin u\,du$

$= 2\displaystyle\int (1-\cos^2 u)\sin u\,du$

substitute $t = \cos u$ $\quad dt = -\sin u\,du$

$= -2\displaystyle\int (1-t^2)\,dt$

$$= -2\int (1-t^2)\,dt$$

$$= -2\left[t - t^3/3\right] + C$$

$$= -2\left[\cos u - \frac{\cos^3 u}{3}\right] + C$$

$$= \boxed{-2\left[\cos\sqrt{x} - \frac{\cos^3\sqrt{x}}{3}\right] + C}$$

8.2

#30 $$\int_0^{\pi/3} \tan^5 x \sec^6 x \, dx$$

$$= \int_0^{\pi/3} \tan^5 x \sec^4 x \sec^2 x \, dx$$

$$= \int_0^{\pi/3} \tan^5 x \,(1+\tan^2 x)^2 \sec^2 x \, dx$$

Substitute $u = \tan x$ $\quad du = \sec^2 x \, dx$

if $x = 0$, $u = \tan 0 = 0$

if $x = \pi/3$, $u = \tan \pi/3 = \sqrt{3}$

$$= \int_0^{\sqrt{3}} u^5 (1+u^2)^2 \, du$$

$$= \int_0^{\sqrt{3}} u^5 (1 + 2u^2 + u^4)\, du$$

$$= \frac{u^6}{6} + 2\frac{u^8}{8} + \frac{u^{10}}{10} \Big]_0^{\sqrt{3}}$$

$$= \frac{(\sqrt{3})^6}{6} + \frac{(\sqrt{3})^8}{4} + \frac{(\sqrt{3})^{10}}{10}$$

$$= \boxed{\frac{27}{6} + \frac{81}{4} + \frac{243}{10}}$$