Nodal lines of random eigenfunctions of the Laplacian

Igor Wigman CRM, University of Montreal

Thursday, November 6, 2008 007 Kemeny Hall, 4:00 pm (Tea 3:30 pm 300 Kemeny Hall)

Abstract

In this talk I will introduce the audience to nodal lines. The nodal lines were first discovered by the German physicist, musician and philosopher Ernst Chladni as the mysterious shapes appearing on musical instruments or highly excited membrane, varying with the membrane and the frequency.

Given a surface, one considers the zeros of the eigenfunctions of the Laplacian associated to large eigenvalues. In this situation it is widely accepted to model the nodal lines using Berry's random wave model. For surfaces like the torus or the sphere, one can use the multiplicities in the spectrum to endow the eigenspaces with Gaussian probability measure, and study the statistical properties of the corresponding nodal lines.

I plan to explain the background, our random model compared with Berry's random wave model, as well as the results we obtained. The one dimensional analogue of the nodal line is the zeros of random trigonometric polynomials, studied recently. I will also explain the analogue for smooth billiards - the number of the intersections of the nodal line with the boundary of the billiard.

(joint with Zeev Rudnick, Andrew Granville and John Toth)

This talk should be accessible to graduate students.