Supplementary Homework for Math 43
Due Monday, April 29, 2002

S1: We showed in class that a connected open set D (a.k.a. a domain) in the plane could not be written as the disjoint union of two nonempty open sets. Prove the converse for an open set. More precisely, suppose that D is an open set in the plane with the property that whenever D is the union of two open sets U and V such that $U \cap V = \emptyset$ and $U \neq \emptyset$, then $V = \emptyset$. Then prove that D is connected.

I suggest you fix a point $z_0 \in D$, and consider the set U of points $z \in D$ for which there is a pologonal path in D from z_0 to z. Using the fact that an open disk is connected, you can show that U and its complement V in D are both open.