Coordinates in $\mathbb{R}^2$ and $\mathbb{R}^3$

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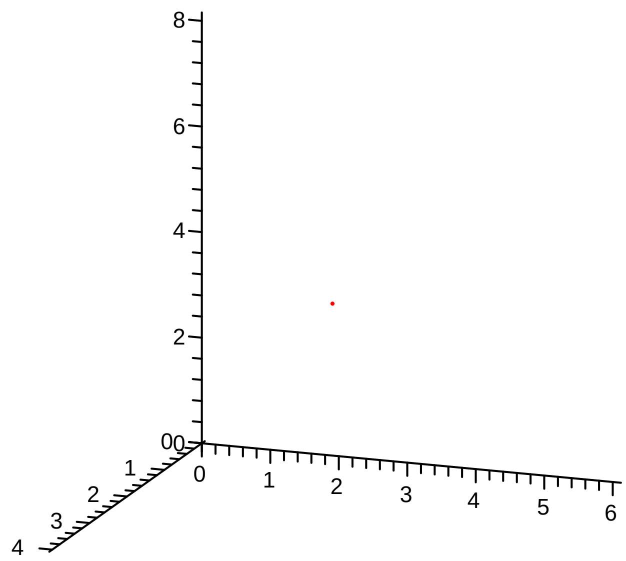
Three-dimensional coordinate systems



- A point P in space is represented by a triple (a, b, c)
- a is the x-coordintate
- b is the y-coordinate
- c is the z-coordinate
- This correspondence between points and triples (a, b, c) in $\mathbb{R}^3$ is called a three dimensional rectangular coordinate system.

Distance between two points

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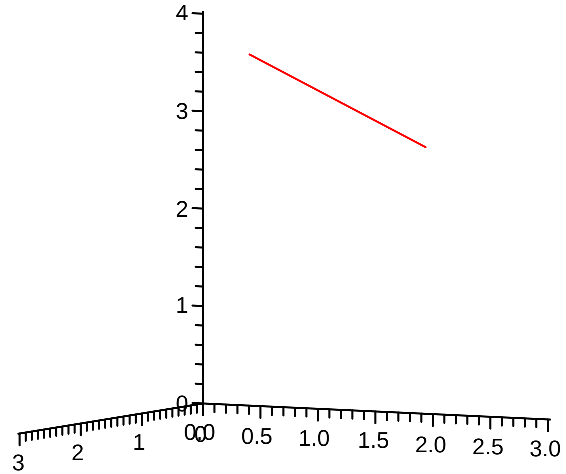
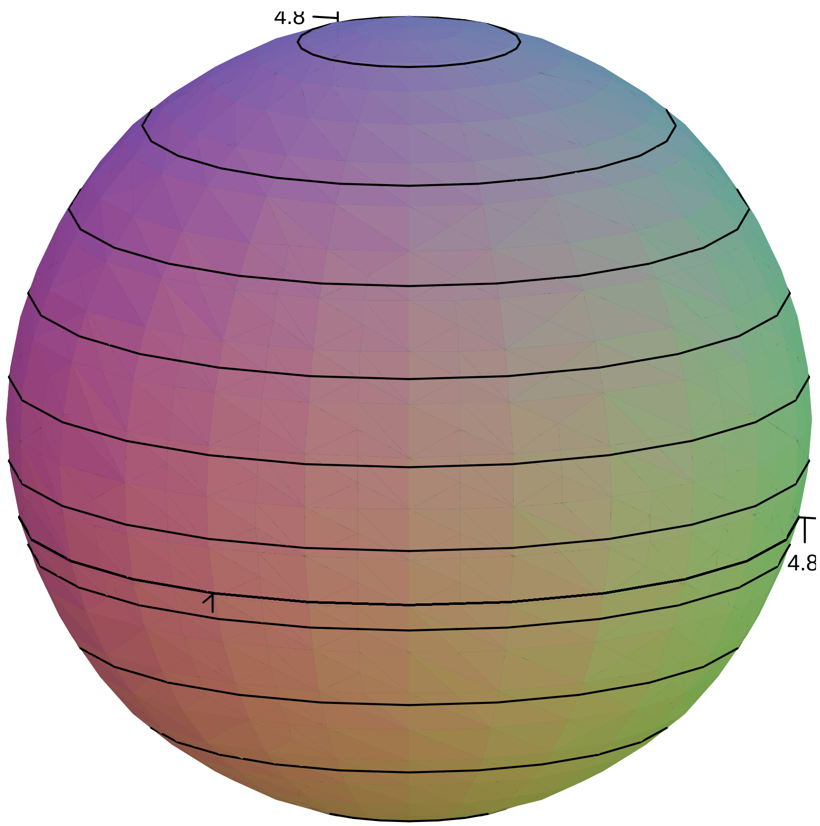


Distance formula

- The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a sphere



Equation of a Sphere

- An equation of a sphere with center $C(h, k, l)$ and radius r is

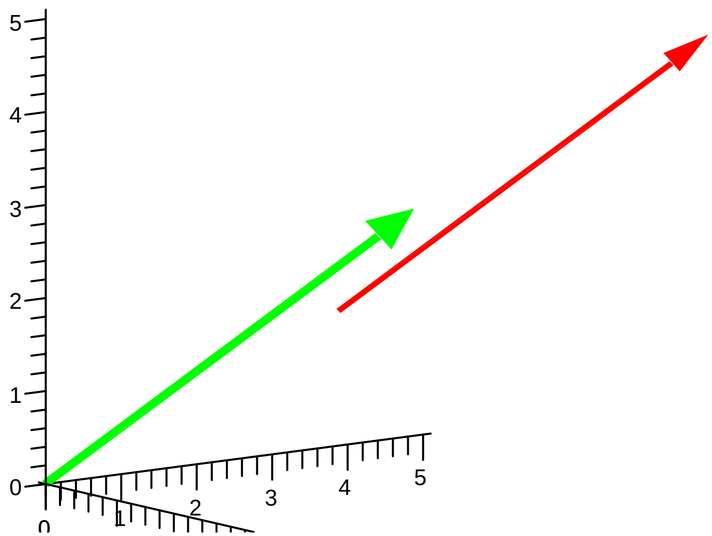
$$(x-h)^2 + (y-k)^2(z-l)^2 = r^2.$$

- If the center is the origin

$$x^2 + y^2 + z^2 = r^2.$$

Vectors

- A **vector** has **initial point**A and **terminal point** B
- We write $\vec{AB}$
- Two vectors u and v are **equivalent** (or **equal**) and we write $u = v$ if the have the same length and the same direction



0
1
2
3
4
5
0
1
2
3
4
5
0
1
2

Vector Addition

- If u and v are vectors positioned so the initial point of v is at the terminal point of u, then the **sum** $u + v$ is the vector from the initial point of u to the terminal point of v.

Scalar multiplication

- If c is a scalar and v is a vector, then the **scalar multiple** cv is the vector whose length is $|c|$ times the length of v and whose direction is the same as v if $c > 0$ and is opposite to v if $c < 0$. If $c = 0$ or $v = 0$ then $cv = 0$.

- We call $-v$ the **negative** of v.

- The **difference** $u - v$ of two vectors is

$$u - v = u + (-v)$$

Components

- If we place the initial point of a vector a at the origin, then the terminal point of a has coordinates of the form (a_1, a_2) or (a_1, a_2, a_3).

- These coordinates are called **components** of a

$$a = \langle a_1, a_2 \rangle \text{ or } a = \langle a_1, a_2, a_3 \rangle.$$