4.9 #1.1 $\left(\frac{4}{3} x^{\frac{3}{2}}\right)'$

$$= \frac{\cancel{4}^2 \cancel{3}}{\cancel{3}} \cdot \frac{}{\cancel{2}} \cdot x^{\frac{1}{2}}$$

$$= 2\sqrt{x}$$

$$L = \int_0^1 \sqrt{1 + 4x} \; dx$$

$$= \cancel{\text{scribbled out}}$$

$$= \frac{1}{4} \cdot \frac{2}{3} (1+4x)^{\frac{3}{2}} \Big|_0^1$$

$$= \frac{1}{6} \times (5^{\frac{3}{2}} - 1)$$

#2 $y' = \frac{5}{20} \cdot x^4 + \frac{1}{3} \cdot (-3) \cdot x^{-4}$

$$= \frac{x^4}{4} - \frac{1}{x^4}$$

$$\int_3^4 \sqrt{1 + \left(\frac{x^4}{4} - \frac{1}{x^4}\right)^2} \; dx$$

$$= \int_3^4 \sqrt{\left(\frac{x^4}{4}\right)^2 + \left(\frac{1}{x^4}\right)^2 + \frac{1}{2}} \; dx$$

$$= \int_{3}^{4} \left(\frac{x^4}{4} + \frac{1}{x^4} \right) dx$$

$$= \left[\frac{1}{20} \cdot x^5 + \left(-\frac{1}{3}\right) x^{-3} \right]_{3}^{4}$$

$$= \left(\frac{1}{20} \cdot 4^5 - \frac{1}{3} \cdot \frac{1}{4^3} \right) - \left(\frac{1}{20} \cdot 3^5 - \frac{1}{3} \cdot \frac{1}{3^3} \right)$$

$$= \frac{4^5 - 3^5}{20} + \frac{1}{3^4} - \frac{1}{3 \cdot 4^3}$$

$$= 39.05714$$

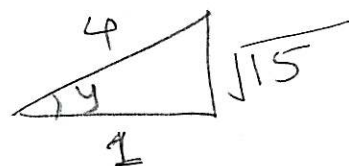4.10 #1 $8 \arcsin\left(-\frac{1}{2}\right)$

$$= 8 \times \left(-\frac{\pi}{6}\right)$$

$$= -\frac{4}{3}\pi$$

#3 $\sin\left(\text{arc sec}(4) \right)$

$$= \sin\left(\text{arc cos}\left(\frac{1}{4}\right) \right)$$

$$= \cancel{\frac{\sqrt{15}}{4}} = \frac{\sqrt{15}}{4}$$

#4　$\arccos(\cos(6\pi))$

$= \arccos(1)$

$= 0$

#5　$f'(x) = \dfrac{1}{\sqrt{1-\left(\frac{8x-2}{9}\right)^2}} \cdot \dfrac{8}{9}$

$= \dfrac{9}{\sqrt{9^2-(8x-2)^2}} \cdot \dfrac{8}{9}$

$= \dfrac{8}{\sqrt{81-(8x-2)^2}}$

$= \dfrac{8}{\sqrt{-64x^2+32x+77}}$

#6　(1) $y' = -\dfrac{1}{\sqrt{1-\left(\frac{x-8}{8}\right)^2}} \cdot \dfrac{1}{8}$

$= -\dfrac{8}{\sqrt{8^2-(x-8)^2}} \cdot \dfrac{1}{8} = -\dfrac{1}{\sqrt{8^2-(x-8)^2}}$

(2) $-1 \le \dfrac{x-8}{8} \le 1 \Rightarrow -8 \le 8x - 8 \le 8$

$\Downarrow$

$y \quad \boxed{0 \le x \le 16} \quad$ (arc cos).

$8^2 - (x-8)^2 > 0 \Rightarrow (x-8)^2 < 8^2 \Rightarrow \cancel{-8} < x - 8 < 8$

$\Downarrow$

$y' \quad \boxed{0 < x < 16}$

domain: $(0, 16)$

y	y'
L.L: 0	0
U.L. 16	16
A	D

(3) graph

decreasing funct'n: exclude B

domain: $(0, 16)$; exclude C

A or D (identical graphs).

#12 $\displaystyle \int_{-3}^{3} \dfrac{1}{\sqrt{81 - x^2}}\, dx = \int_{-3}^{3} \dfrac{1}{\sqrt{9^2 - x^2}}\, dx = \sin^{-1}\left(\dfrac{x}{9}\right)\Big]_{-3}^{3}$

$= \dfrac{1}{9}\int_{-3}^{3} \dfrac{1}{\sqrt{1 - \left(\frac{x}{9}\right)^2}} \cdot 9\, dx = \int_{-3}^{3} \dfrac{1}{\sqrt{1 - \left(\frac{x}{9}\right)^2}} \cdot d\left(\dfrac{x}{9}\right)$

$$= \quad \arcsin\left(\frac{x}{9}\right)\Big|_{-3}^{3}$$

$$= \quad \arcsin\frac{1}{3} - \arcsin\left(-\frac{1}{3}\right)$$

$$= 2\arcsin\left(\frac{1}{3}\right)$$

$$0.6797$$