Math 12, Fall 2007

Lecture 13

Scott Pauls [1]

[1] Department of Mathematics
Dartmouth College

10/26/07

Outline

Outline

1. Review and overview
   - Last class
2. Today's material
   - Integration in two variables
3. Group Work
4. Next class

Finding extrema

- First derivative test: $\nabla f = \vec{0}$
- Second derivative test: $D = f_{xx} f_{yy} - f_{xy}^2$
- Absolute max/min

Outline

If $f(x)$ is defined for $a \leq x \leq b$, we calculate the integral as follows:

1. Divide $[a, b]$ into n subintervals, $[x_{i-1}, x_i]$ of uniform width $\Delta x = (b-a)/n$.
2. Pick $x_i^* \in [x_{i-1}, x_i]$
3. Form the Riemann sum:

$$\sum_{i=1}^{n} f(x_i^*)\Delta x$$

   This is an approximation of the integral.
4. Take the limit as $\Delta x \to 0$, or equivalently $n \to \infty$.
5. This defines the definite integral:

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

The integral of a function of two variables

Given $f(x,y)$ defined on a rectangle $[a,b]\times[c,d]$. We follow similar reasoning:

1. Divide both $[a,b]$ and $[c,d]$ into equal portions to create a rectangle subdivision of $[a,b]\times[c,d]$. Precisely:
    1. $[a,b]$: sub intervals $[x_{i-1},x_i]$ of width $\Delta x=(b-a)/m$
    2. $[c,d]$: sub intervals $[y_{j-1},y_j]$ of width $\Delta y=(d-c)/n$
2. Pick a point (x^*_{ij},y^*_{ij}) in $R_{ij}=[x_{i-1},x_i]\times[y_{j-1},y_j]$
3. Form a Riemann sum:

$$\sum_{i=1}^{m}\sum_{i=1}^{n} f(x^*_{ij},y^*_{ij})\Delta x\,\Delta y$$

4. Take a limit to define the definite integral:

$$\int\int_R f(x,y)\,dxdy=\lim_{m,n\to\infty}\sum_{i=1}^{m}\sum_{i=1}^{n} f(x^*_{ij},y^*_{ij})\Delta x\,\Delta y$$

Estimation Techniques

The Endpoint, Midpoint, Trapezoid, and Simpson's Rules all have generalizations to the multivariable case, providing us with numerical estimations for integrals, i.e. volume under a surface.

Compute an integral!

Use the definition to compute the following integral: $f(x,y) = x^2 + y^2$, $a = 0, b = 1, c = 0, d = 1$. Find

$$\int_R (x^2 + y^2)\, dx\, dy$$

(Potentially) helpful formulae:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Work for next class

- Reading: 16.2
- f07hw14