Solving fully nonlinear fluid flow problems in channels having arbitrary bottom topography

by

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Abstract

- A class of fully nonlinear fluid flow problems is examined for their mathematical solutions by the aid of singular integral equations.
- The original nonlinear boundary value problems are formulated in terms of Dirichlet's problem for Laplace equation to be satisfied by the potential function governing the flow problems under consideration, which involves unknown boundaries to be determined along with the potential function.
- Finally these boundary value problems are shown to be amenable for their complete solution via integral equations of special types involving unknown curves of integration, which can also be determined fully.

What is Channel-flow problem ?

- Flow in a channel is called Channel-flow problem.
- Open-channel flows are those whose boundaries are not entirely a solid and rigid material; the other part of the boundary of such flows may be another fluid.
  •Example: Rivers, tidal currents, irrigation canals etc.
- Open-channel flow has a free surface, whereas pipe flow does not.

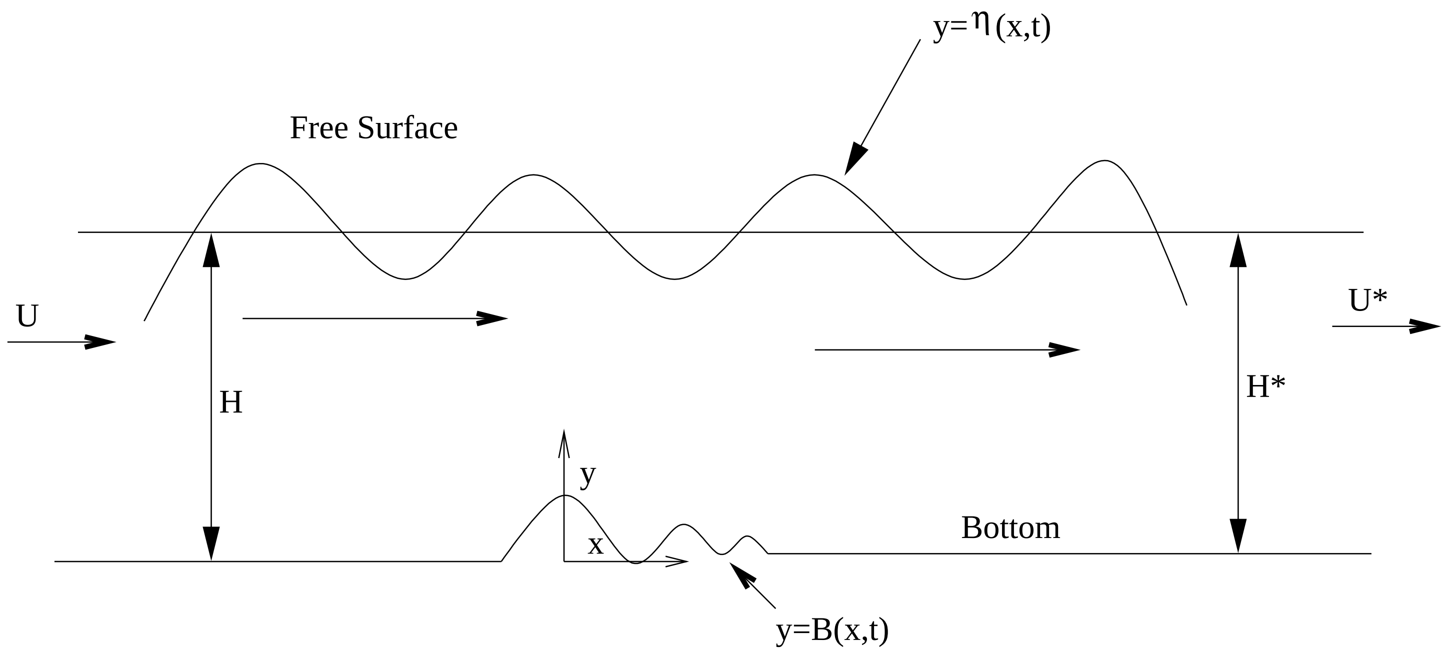
Figure 1: Open-channel flow

Applications

- The problems of free-surface fluid flow over submerged obstacles have created varieties of challenges to model the situations in engineering and in atmospheric and oceanographic sciences.

- In considering the problem of flow over obstacles in a meteorological situation.

Introduction: Fully Nonlinear Flow Problem



Assumption:

- Fluid is incompressible, inviscid and homogenous
- Flow is two dimensional and irrotational

Equation of continuity $\Longrightarrow$ $\boxed{\frac{\partial^2 \Phi^*}{\partial x^2} + \frac{\partial^2 \Phi^*}{\partial y^2} = 0}$

Let

$\Phi^*(x, y, t) \equiv \Phi(x', y)$ where $x' = x - ct$ (c is a known positive constant)

Then Laplace's equation:

$$\frac{\partial^2 \Phi^*}{\partial x^2} + \frac{\partial^2 \Phi^*}{\partial y^2} = 0 \equiv \frac{\partial^2 \Phi}{\partial x'^2} + \frac{\partial^2 \Phi}{\partial y^2}$$

Condition on free surface: Free surface: $f \equiv y - \eta(x,t) \equiv y - \eta(x-ct)$

Kinematic Condition: $\boxed{\frac{Df}{Dt} = 0}$

$$\Rightarrow \quad f_t + uf_x + vf_y = 0 \quad [(u,v) \text{ are the velocity components}]$$

$$\Rightarrow \quad -\eta_t - u\eta_x + v = 0 \qquad [u = \frac{\partial \Phi^*}{\partial x},\ v = \frac{\partial \Phi^*}{\partial y}]$$

$$\Rightarrow \quad -\eta_{x'}\frac{\partial x'}{\partial t} - u\eta_x + v = 0$$

$$\Rightarrow \quad c\eta_{x'} - \frac{\partial \Phi^*}{\partial x}\eta_x + \frac{\partial \Phi^*}{\partial y} = 0$$

$$\Rightarrow \boxed{c\eta_{x'} - \frac{\partial \Phi}{\partial x'}\eta_{x'} + \frac{\partial \Phi}{\partial y} = 0} \quad \text{on} \quad y = \eta \qquad [\because \frac{\partial \Phi^*}{\partial x} = \frac{\partial \Phi}{\partial x'}].$$

Dynamic free surface condition:

$$\boxed{\frac{\partial \Phi *}{\partial t} + \frac{1}{2}|\overrightarrow{\nabla}\Phi^*|^2 = -\frac{p}{\rho} - gy + c_0(t)}$$

$$\Rightarrow \quad \frac{\partial \Phi}{\partial x'}\frac{\partial x'}{\partial t} + \frac{1}{2}\left[\left[\left(\frac{\partial \Phi}{\partial x'}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2\right]\right] = -g\eta + c_0(t), \quad \text{on } y = \eta,$$

$$(p = 0 = \text{atmospheric pressure})$$

$$\Rightarrow \boxed{-c\frac{\partial \Phi}{\partial x'} + \frac{1}{2}\left(\frac{\partial \Phi}{\partial x'}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 = -g\eta + c_0(t), \quad \text{on } y = \eta,}$$

Condition on bottom:

$$\boxed{cB_{x'} - \frac{\partial \Phi}{\partial x'}B_{x'} + \frac{\partial \Phi}{\partial y} = 0, \quad \text{on } \; y = B(x')}$$

Set: $\phi = \Phi - cx'$

We obtain:

$$\frac{\partial \phi}{\partial x'} = \frac{\partial \Phi}{\partial x'} - c$$

$$\Rightarrow \quad \frac{\partial \Phi}{\partial x'} = c + \frac{\partial \phi}{\partial x'}.$$

Also,

$$\frac{\partial \Phi}{\partial y} = \frac{\partial \phi}{\partial y}.$$

Then the BVP becomes

New BVP

$$\frac{\partial^2\phi}{\partial x'^2} + \frac{\partial^2\phi}{\partial y^2} = 0, \text{ in the fluid region,}$$

with boundary conditions:

$$\phi_x'^2 + \phi_y^2 + 2g\eta = U^2 + 2gH, \text{ on } y = \eta(x'), \text{ (free surface cond}$$

$$\phi_y = \phi_x'\eta_x', \text{ on } y = \eta(x'), \text{ (free surface condition)}$$

$$\phi_y = \phi_x'B_x', \text{ on } y = B(x'), \text{ (bottom condition)}$$

other boundary condition

$$\phi_x' \rightarrow U, \text{ as } x' \rightarrow -\infty, \text{ (upstream condition)}$$

$$\phi_x' \rightarrow U^*, \text{ as } x' \rightarrow +\infty, \text{ (downstream condition)}$$

Non-dimensionalize

$$
\begin{aligned}
\frac{1}{U}\left(\frac{\partial\phi}{\partial x'},\frac{\partial\phi}{\partial y}\right) &= \left(\frac{\partial\phi}{\partial x},\frac{\partial\phi}{\partial y}\right) \\
\frac{\eta(x')}{H} &= \eta(x), \\
\frac{B(x')}{H} &= B(x)
\end{aligned}
$$

and dropping dash's [x' to x], we have

Then the flow problem becomes to solve:

$$(i) \qquad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0, \quad -\infty < x < \infty,\ \eta(x) < y < B(x),$$

with

$$(ii) \qquad \phi_x^2 + \phi_y^2 + \frac{2y}{F^2} = 1 + \frac{2}{F^2},\ \text{on } y = \eta(x),$$

$$(iii) \qquad \phi_y = \phi_x \eta_x,\ \text{on } y = \eta(x),$$

$$(iv) \qquad \phi_y = \phi_x B_x,\ \text{on } y = B(x),$$

$$(v) \qquad \phi_x \to 1, \eta(x) \to 1,\ \text{as } x \to -\infty,$$

$$(vi) \qquad \phi_x \to \gamma, \eta(x) \to \frac{H^*}{H} = \eta_0 \text{(say)},\ \text{as } x \to +\infty,$$

where $\gamma = \frac{U^*}{U}$, $F = \frac{U}{\sqrt{g}H}$ is the Froude number

Using stream function $\psi(x, y)$ with:

$$\frac{\partial \phi}{\partial x} = +\frac{\partial \psi}{\partial y} \quad \text{and} \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x},$$

and using the fact that both the free surface as well as the bottom surface are stream-lines:

$$\begin{array}{ll} & \psi(x, y) = 1, \quad \text{on} \quad y = \eta(x) \\ \text{and} \qquad & \psi(x, 0) = 0, \quad \text{on} \quad y = B(x) \end{array} \Bigg\}$$

New BVP

Then the BVP becomes:

$$(i)' \qquad \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0, \quad -\infty < x < \infty, \ \eta(x) < y < B(x),$$

$$(ii)' \qquad \psi_y^2 + \psi_x^2 + \frac{2\eta(x)}{F^2} = 1 + \frac{2}{F^2}, \text{ on } y = \eta(x),$$

$$i.e, \text{where } \ \psi = 1,$$

$$(iii)' \qquad \psi_x + \psi_y \eta_x = 0, \text{ on } y = \eta(x), \text{ i.e., where } \ \psi = 1,$$

$$(iv)' \qquad \psi_x + \psi_y B_x = 0, \text{ on } y = B(x), \text{ i.e., where } \ \psi = 0,$$

$$(v)' \qquad \psi_y \to 1, \ \ \& \ \ \eta(x) \to 1, \text{ as } x \to -\infty,$$

$$(vi)' \qquad \psi_y \to \gamma, \ \ \& \ \ \eta(x) \to \eta_0 \text{(say)} \ \equiv \frac{H^*}{H}, \text{ as } x \to +\infty.$$

Considering a region bounded by $x = -L,\ x = +L,\ y = B(x)$ and $y = \eta(x)$, where L is very large (+ve).

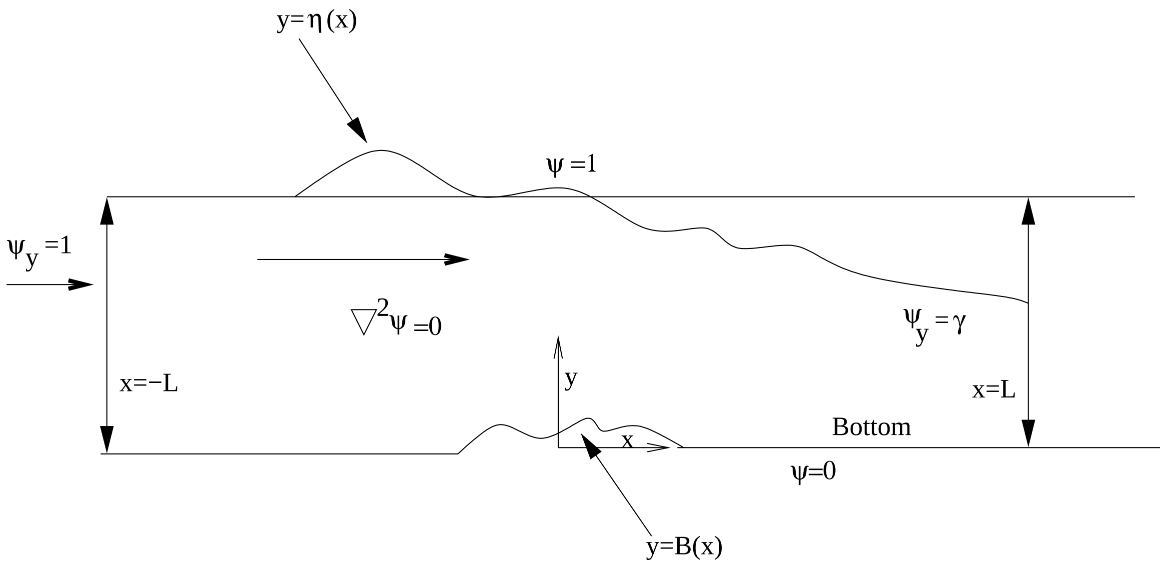


Figure 2

Then, using the fact that $\psi_y = 1$, on $x = -L$ and $\psi_y = \gamma$, on $x = L$, we obtain (on integration),

$$\left.\begin{array}{lllll} (a) & \psi(x,y) & = & y & \text{on} \quad x = -L, \\ (b) & \psi(x,y) & = & \gamma y, & \text{on} \quad x = +L \end{array}\right\}$$

(ignoring arbitrary constants which do not contribute to the velocity)

Transforming to the new dependent variable $\Psi(x,y)$:

$$\psi(x,y) = \Psi(x,y) + y + \left(\frac{x+L}{2L}\right)(\gamma-1)y$$

Then, the problem for Ψ becomes the NEW problem, to solve:

$$(i)'' \qquad \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} = 0, \quad -L < x < L, \;\; \eta(x) < y < B(x),$$

$$(ii)'' \qquad \left[\Psi_y + 1 + \left(\frac{x+L}{2L}\right)(\gamma-1)\right]^2 + \left[\Psi_x + \frac{(\gamma-1)}{2L}y\right]^2 + \frac{2\eta(x)}{F^2} = 1 + \frac{2}{F^2},$$

on $y = \eta(x)$,

$$(iii)'' \qquad \left[\Psi_x + \frac{(\gamma-1)}{2L}\eta(x)\right] + \left[\Psi_y + \left(\frac{x+L}{2L}\right)(\gamma-1) + 1\right]\eta_x = 0, \;\; \text{on } y = \eta(x),$$

$$(iv)'' \qquad \left[\Psi_x + \frac{(\gamma-1)}{2L}B(x)\right] + \left[\Psi_y + \left(\frac{x+L}{2L}\right)(\gamma-1) + 1\right]B_x = 0, \;\; \text{on } \; y = B(x),$$

$$(v)'' \qquad \Psi \to 0, \;\; \text{on } x = -L,$$

$$(vi)'' \qquad \Psi \to 0, \;\; \text{on } x = +L.$$

and, finally, $(vii)''$ $\quad \eta(x) \to 1,$ $\qquad$ on $x = -L \; (-\infty)$,

$\quad \& \;\; \eta(x) \to \eta_0,$ (a known constant), $\quad$ on $x = +L \; (+\infty)$.

<u>NOTE</u> that, the conditions $(iii)''$ and $(iv)''$ are representatives of the fact that $y=\eta(x)$ and $y=B(x)$ are strem-lines, which can be replaced by the relations:

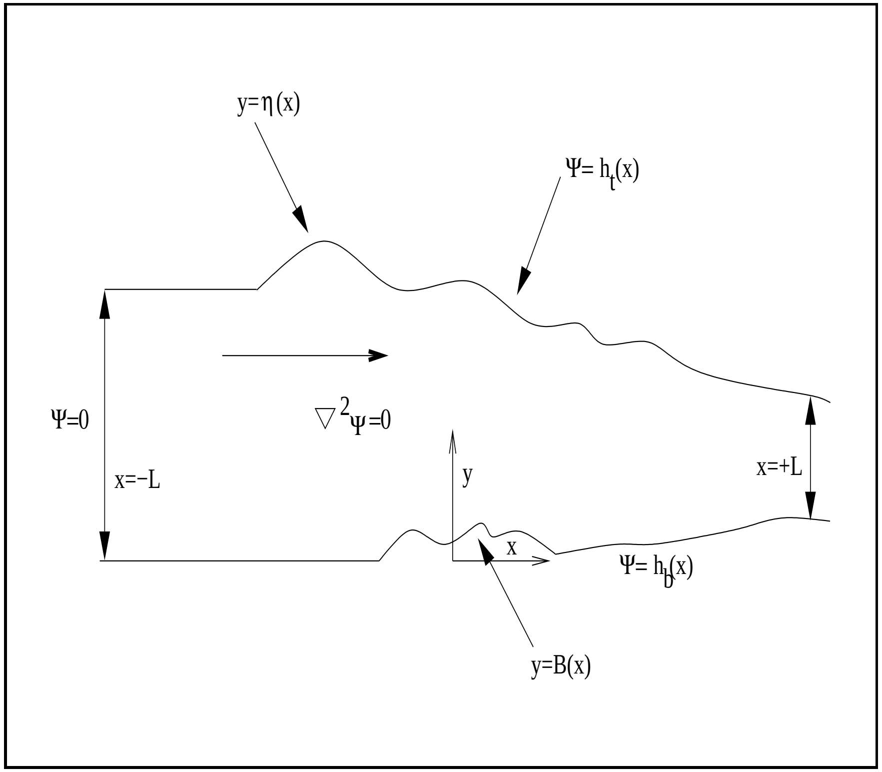
$$\left.\begin{array}{llll} & (iii)''' & \psi(x,y)=1, & \text{on} \quad y=\eta(x), \\ \text{and} & (iv)''' & \psi(x,y)=0, & \text{on} \quad y=B(x) \end{array}\right\}$$

i.e., for the NEW FUNCTION $\Psi(x,y)$, we have to replace the relations $(iii)''$ and $(iv)''$, by the relations:

$$\left.\begin{array}{rl} (iii)'''' \ \Psi(x,\eta(x)) & = 1-\eta(x)-\frac{(x+L)}{2L}(\gamma-1)\eta(x)=T(x)\text{(say)} \\ \text{and } (iv)'''' \ \Psi(x,B(x)) & = -B(x)-\frac{(x+L)}{2L}(\gamma-1)B(x)=G(x)\text{(say)} \end{array}\right\}$$

Final BVP: Dirichlet problem

Thus, the main problem becomes that of solving the Dirichlet problem involving $\Psi(x,y)$:



$$
\begin{array}{rll}
(I) & \Psi_{xx}+\Psi_{yy}=0 & \text{in fluid region} \\
(II) & \Psi=0, & \text{on } x=-L, \\
(III) & \Psi=0, & \text{on } x=+L, \\
(IV) & \Psi=\Psi(x,\eta(x))=T(x), & \\
 & & \text{on } y=\eta(x), \\
(IV) & \Psi=\Psi(x,B(x))=G(x), & \\
 & & \text{on } y=B(x).
\end{array}
$$

But, in our flow problem, $\eta(x)$ is not known beforehand, and that can be determined by utilizing the non-linear condition $(ii)''$.

Solution of Dirichlet problem

Let $\exists$ a function $\varphi(x, y)$ such that $\varphi_{xx} + \varphi_{yy} = 0$ giving an analytic function

$$w(u) = \Psi(x, y) + i\varphi(x, y), \text{ where } u = x + iy$$

Now $w(u)$ can be represented as Cauchy type integral:

$$w(u) = \frac{1}{2\pi i} \oint_{\partial D} \frac{\mu(t)dt}{t - u}, \text{ for } u \in D \text{ } (region),$$

where $\mu(t)$ is a suitably chosen Hölder continuous function and the point $t \in \partial D$.

Solution of Dirichlet problem

Applying Plemelj-Sokhotski formulae and taking the parametrization
$t = (x, \eta(x)) = (x(s), y(s))$ and $z = (x, \eta(x)) = (x(\sigma), y(\sigma))$

Solution of the Dirichlet problem:

$$\Psi(x,y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{x'(\sigma)(y(\sigma)-y) - y'(\sigma)(x(\sigma)-x)}{(x(\sigma)-x)^2 + (y(\sigma)-y)^2}\mu d\sigma + \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{(B(\sigma)-y) - B'(\sigma)(\sigma-x)}{(\sigma-x)^2 + (B(\sigma)-y)^2}\nu d\sigma, \qquad (1)$$

where $\mu = \mu(x(\sigma), y(\sigma))$, $\nu = \mu(\sigma, B(\sigma))$ can be obtained by solving the coupled integral

equations

Solution of Dirichlet problem

$$\Psi(x(s),y(s)) = \frac{\mu(x(s),y(s))}{2} + \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{x'(\sigma)(y(\sigma)-y(s))-y'(\sigma)(x(\sigma)-x(s))}{(x(\sigma)-x(s))^2+(y(\sigma)-y(s))^2}\mu d\sigma + \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{(B(\sigma)-y(s))-B'(\sigma)(\sigma-x(s))}{(\sigma-x(s))^2+(B(\sigma)-y(s))^2}\nu d\sigma, \quad (2)$$

and

$$\Psi(s,B(s)) = \frac{\mu(s,B(s))}{2} + \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{x'(\sigma)(y(\sigma)-B(s))-y'(\sigma)(x(\sigma)-s)}{(x(\sigma)-s)^2+(y(\sigma)-B(s))^2}\mu d\sigma + \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{(B(\sigma)-B(s))-B'(\sigma)(\sigma-s)}{(\sigma-s)^2+(B(\sigma)-B(s))^2}\nu d\sigma. \quad (3)$$

- Arclength satisfy:

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 = 1 \tag{4}$$

- Bernoulli's equation on the free surface:

$$\begin{aligned}\frac{1}{2}F^2\Bigg[\Psi_s^2 + 2\Psi_s\frac{dy}{ds}\left(1+\frac{(x(s)+L)}{2L}(\gamma-1)\right) \\ + 2\Psi_s\frac{dx}{ds}\frac{(\gamma-1)}{2L}y(s) + \left(\frac{x(s)+L}{L}\right)(\gamma-1)+1+ \\ \frac{(\gamma-1)^2}{4L^2}\left((y(s))^2+((x(s))^2+L)^2\right)\Bigg] + y(s) = 1+\frac{F^2}{2},\end{aligned} \tag{5}$$

- Upstream condition:

$$\Psi \to 0,\ \eta(x) = y(s) \to 1,\ \text{as } x(s) \to -L. \tag{6}$$

$\boxed{\text{Unknowns } x(s), y(s) \text{ and } \Phi(s) \text{ at the free surface are obtained by solving the equations (1)-(6)}}$

Numerical method

On the free surface:

We took N points $s_1, s_2, \ldots, s_N$ with $h = s_j - s_{j-1}$, $j = 2, 3, \ldots, N$ (here s_1 and s_N represent $-\infty$ and ∞, respectively)

On the bottom:

we took N points $\sigma_1, \sigma_2, \ldots, \sigma_N$ with $h = \sigma_j - \sigma_{j-1}$, $j = 2, 3, \ldots, N$ (σ_1 and σ_N represents $-\infty$ and ∞, respectively)

The domain of the first integration in the equations (2) and (3) is truncated upstream at s_1 and downstream at s_N and the second integration is truncated upstream at σ_1 and downstream at σ_N.

The equations are then evaluated at the midpoints $s_{k-\frac{1}{2}}$, and $\sigma_{k-\frac{1}{2}}$, ($k = 2, 3, \ldots, N$).

Because the singularity in one of the integrands in each equation now occurs symmetrically between the grid points, the integrals may be approximated by the trapezoidal rule, ignoring the singularity.

$\boxed{\text{Newton's method is used to solve the nonlinear equations and the algorithm is as follows:}}$

Numerical method: Algorithm

Step 1: Satisfy the upstream condition by specifying

$$y(s_1) \equiv y_1 = 1, \quad y_1' = 0,$$
$$x_1' = 1, \quad x(s_1) \equiv x_1 = s_1,$$
$$\Psi(s_1) \equiv \Psi_1 = 0, \quad \Psi_1' = 0.$$

Let the initial guess at the unknowns $y_2' = \ldots y_N' = 0$.

Step 2: To get x_j', for $j = 2, 3, \ldots, N$ we use arclength relation

$$x_k' = \sqrt{1 - y_k'^2}, \quad k = 2, 3, \ldots, N.$$

Step 3: Integrate y_j', x_j' (numerically) using trapezoidal rule to get y_j, x_j for $j = 2, 3, \ldots, N$:

$$\begin{aligned} x_k &= x_{k-1} + \frac{1}{2}h\left(x_k' + x_{k-1}'\right), \\ y_k &= y_{k-1} + \frac{1}{2}h\left(y_k' + y_{k-1}'\right), \; k = 2, 3, \ldots, N. \end{aligned}$$

Numerical method: Algorithm

Step 4: To get Ψ'_j, for $j = 2, 3, \ldots, N$ we use Bernoulli's relation

$$\Psi'_k = -A_k + \sqrt{A_k^2 - C_k}, \text{ with}$$

$$\begin{aligned} A_k &= y'_k\Big(1 + \frac{(x_k + L)}{2L}(\gamma - 1)\Big) + x'_k\frac{(\gamma - 1)}{2L}y_k, \\ C_k &= \Big(\frac{x_k + L}{L}\Big)(\gamma - 1) + 1 + \frac{2y_k}{F^2} - 1 - \frac{2}{F^2} \\ &\quad + \frac{(\gamma - 1)^2}{4L^2}\Big(y_k^2 + (x_k^2 + L)^2\Big), \; k = 2, 3, \ldots, N, \end{aligned}$$

and integrate Ψ'_j numerically using trapezoidal rule to get Ψ_j:

$$\Psi_k = \Psi_{k-1} + \frac{1}{2}h(\Psi'_k + \Psi'_{k-1}), \; k = 2, 3, \ldots, N.$$

Numerical method: Algorithm

Step 5: Obtain function values at the half-grid points:

$$x_{k-\frac{1}{2}} = \frac{1}{2}(x_{k-1} + x_k), \quad y_{k-\frac{1}{2}} = \frac{1}{2}(y_{k-1} + y_k),$$
$$x'_{k-\frac{1}{2}} = \frac{1}{2}(x'_{k-1} + x'_k), \quad \Psi'_{k-\frac{1}{2}} = \frac{1}{2}(\Psi'_{k-1} + \Psi'_k)$$
$$k = 2, 3, \ldots, N.$$

Step 6: To get μ_j and ν_j we will solve the discretized version of the coupled integral equations:

$$\Psi(x(s_k), y(s_k)) = \frac{\mu(x(s_k), y(s_k))}{2} + \frac{h}{2\pi}\sum_{j=1}^{N} w_j \frac{x'_j[y_j - y_{k-\frac{1}{2}}] - y'_j[x_j - x_{k-\frac{1}{2}}]}{[x_j - x_{k-\frac{1}{2}}]^2 + [y_j - y_{k-\frac{1}{2}}]^2}\mu_j$$
$$+ \frac{h}{2\pi}\sum_{j=1}^{N} w_j \frac{[B_j - y_{k-\frac{1}{2}}] - B'_j[\sigma_j - x_{k-\frac{1}{2}}]}{[\sigma_j - x_{k-\frac{1}{2}}]^2 + [B_j - y_{k-\frac{1}{2}}]^2}\nu_j,$$
$$k = 2, 3, \ldots, N, \qquad (7)$$

Numerical method: Algorithm

$$
\begin{aligned}
\Psi(s_l, B(s_l)) = \frac{\mu(s_l, B(s_l))}{2} \\
&+ \frac{h}{2\pi}\sum_{j=1}^{N} w_j \frac{x'_j[y_j - B_{l-\frac{1}{2}}] - y'_j[x_j - s_{l-\frac{1}{2}}]}{[x_j - s_{l-\frac{1}{2}}]^2 + [y_j - B_{l-\frac{1}{2}}]^2}\mu_j \\
&+ \frac{h}{2\pi}\sum_{j=1}^{N} w_j \frac{[B_j - B_{l-\frac{1}{2}}] - B'_j[\sigma_j - s_{l-\frac{1}{2}}]}{[\sigma_j - s_{l-\frac{1}{2}}]^2 + [B_j - B_{l-\frac{1}{2}}]^2}\nu_j, \\
&\qquad l = 2, 3, \ldots, N. \qquad (8)
\end{aligned}
$$

Here, w_j denotes the trapezoidal-rule weights, $w_1 = w_N = 1/2$, and $w_j = 1, j = 2, 3, \ldots, N-1$.

Step 7: We will use the solution of the Dirichlet problem as a cost function to update the initial values of y'_j.

Numerical method: Algorithm

The vector of residual errors:

$$E_k = \Psi_k - \frac{h}{2\pi}\sum_{j=1}^{N} w_j \frac{x'_j[y_j - y_{k-\frac{1}{2}}] - y'_j[x_j - x_{k-\frac{1}{2}}]}{[x_j - x_{k-\frac{1}{2}}]^2 + [y_j - y_{k-\frac{1}{2}}]^2}\mu_j$$

$$- \frac{h}{2\pi}\sum_{j=1}^{N} w_j \frac{[B_j - y_{k-\frac{1}{2}}] - B'_j[\sigma_j - x_{k-\frac{1}{2}}]}{[\sigma_j - x_{k-\frac{1}{2}}]^2 + [B_j - y_{k-\frac{1}{2}}]^2}\nu_j,$$

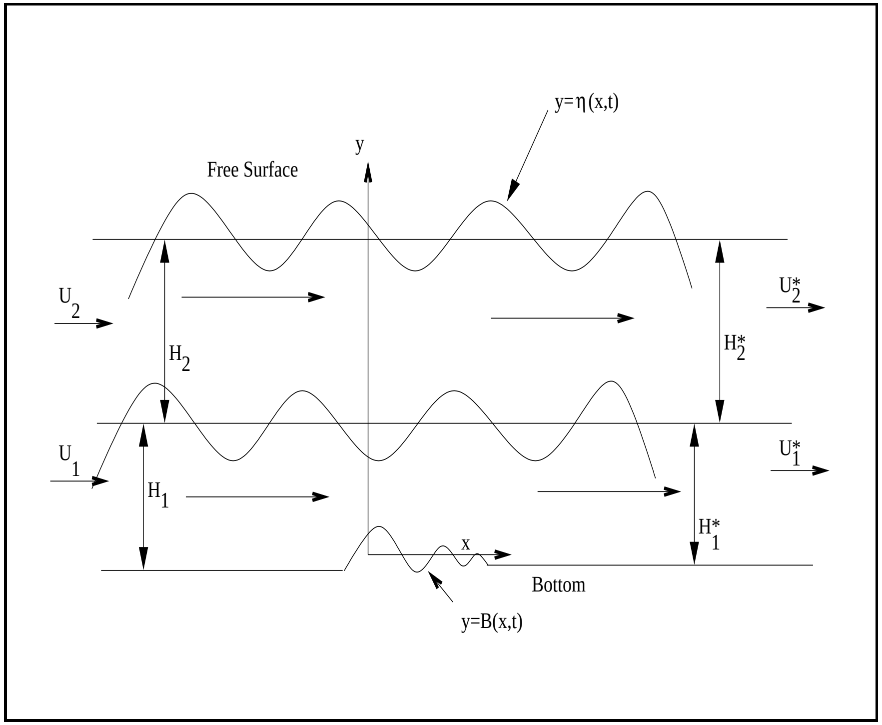
$$k = 2, 3, \ldots, N. \qquad (9)$$

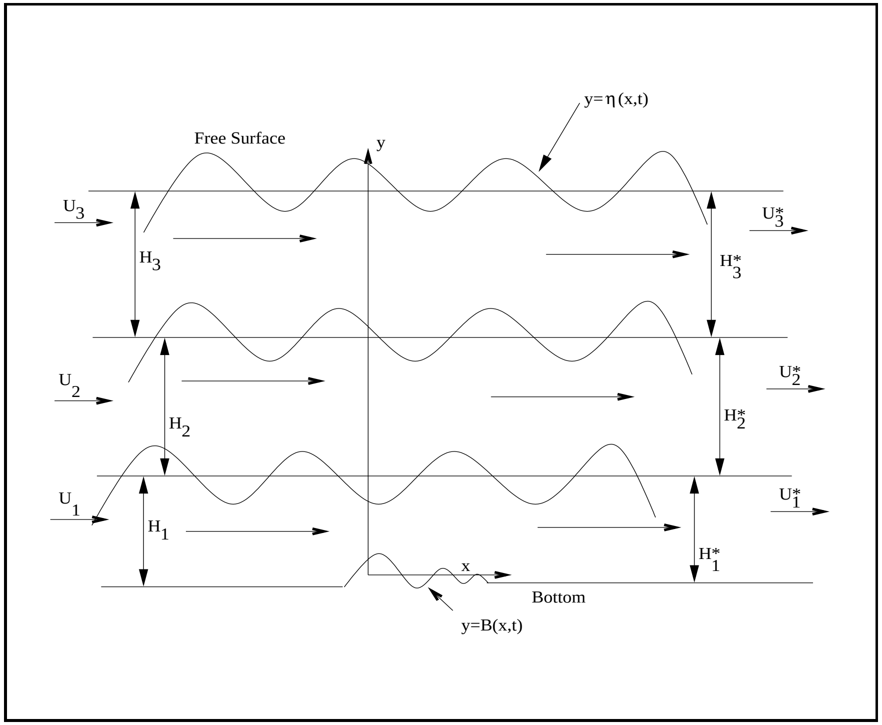
Step 8: The new approximation of the unknowns y'_j can be found out by adding the correction vector Δ_j to the vector y'_j, where Δ_j can be computed by solving the system of equations

$$\sum_{j=2}^{N}\left[\frac{\partial E_k}{\partial y'_j}\right]\Delta_j = -E_k, \quad k = 2, 3, \ldots, N$$

and the programme is returned to step 2, unless the Euclidean norm $||E||$ of the vector of residual errors is sufficiently small, where the programme is stopped.

Further work: Fully non-linear flow problem





Fully non-linear free-surface flow over arbitrary bottom topography in multi layers

References

[1] Forbes L.K., Schwartz L.W. (1982), Free-surface flow over a semicircular obstruction, J. Fluid Mech. 114: 299-314.

[2] Vanden-Broeck J.-M. (1987), Free-surface flow over a semi-circular obstruction in a channel, Phys. Fluids. 30: 2315-2317.

[3] Forbes L.K. (1988), Critical free-surface flow over a semi-circular obstruction, J. Eng. Math. 22: 3-13.

[4] Dias F., Vanden-Broeck J.-M. (1989), Open channel flows with submerged obstructions, J. Fluid Mech. 206: 155-170.

[5] Shen S.P., Shen M.C., Sun S.M. (1989), A model equation for steady surface waves over a bump, J. Engng. Math. 23: 315-323.

References

[6] Dias F., Vanden-Broeck J.-M. (2002), Generalised critical free-surface flows, J. Eng. Math. 42: 291-301.

[7] Chakrabarti A., Martha S.C. (2011), A review on the mathematical aspects of fluid flow problems in an infinite channel with arbitrary bottom topography, J. Appl. Math. & Informatics. 29: 1583-1602.

[8] Monacella V.J. (1967), On ignoring the singularity in the numerical evaluation of Cauchy Principal Value integral, Hydromechanics laboratory research and development report no. 2356, Washington D. C.

THANK YOU