Trigonometric Substitution

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- *The inverse substitution:*

$$\int f(x)\mathsf{d}x = \int f(g(t))g'(t)\mathsf{d}t \quad \text{if } x = g(t)$$

- For $\sqrt{a^2 - x^2}$ use the substitution $x = a\sin\theta$, $-\pi/2 \leq \theta \leq \pi/2$ and the identity $1 - \sin^2\theta = \cos^2\theta$.

- **Example:** $\int x^3\sqrt{9 - x^2}\mathrm{d}x$.

- For $\sqrt{a^2 + x^2}$ use the substitution $x = a \tan \theta$, $-\pi/2 < \theta < \pi/2$ and the identity $1 + \tan^2 \theta = \sec^2 \theta$.

- **Example:**

$$\int \frac{\mathsf{d}x}{\sqrt{4 + x^2}}.$$

$$\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}\mathsf{d}x$$

- For $\sqrt{x^2 - a^2}$ use the substitution $x = a \sec \theta$, $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$ and the identity $\sec^2 \theta - 1 = \tan^2 \theta$.

- **Example:**

$$\int \frac{\mathsf{d}t}{\sqrt{t^2 - 6t + 5}}$$