# Math 46: Applied Math: Homework 2

due Wed Apr 13 . . . but best if do relevant questions after each lecture

From p.100 #2 onwards, which is the meat of the problem set, always check how many terms the question asks for, *e.g.* $y_0 + \varepsilon y_1$ is 2-term. You'll also need to allow time to get Matlab to produce the right plots.

**p.40-44**: #5. A warm-up question (no pun intended). Write your answer to b in the following way: move both exponential terms into the integral to simplify to a single exponential. Be sure to match ICs in both a and b (asnwer to b will involve a *definite* integral). Please interpret as a weighted average of $\theta(t)$, i.e. explain for which times $\theta$ has a large effect on $T(t)$. This convolution result is called *Duhamel's principle*.

**p.52-54**: #6. You will see in c why this is called a 'pitchfork bifurcation'—please show the pitchfork on your plot.

#10. (quick). This can be a sketch, but label clearly where stable and unstable lie.

**p.67-68**: #2. For this you'll need to look up your phase plane linear stability from Math 23. The point is to see that stability can suddenly change with a parameter. Try to visualize how the two eigenvalues move in the complex plane as $b$ varies. Note you don't need a full solution for each case of $b$, just discussion of behavior (type of critical point), including the equal-roots case.

**p.100-104**: #1. This is a quick and easy review of Lecture 2 (see the Errata in the formula).

#2. This is a lovely example. Please leave enough time to get it right and produce the plots—you will love it when it works. Please print out your code and state who else you worked with. First ask yourself, is the unperturbed ODE oscillatory or decaying/growing? You will find the ICs given cause the unperturbed solution to be special (how?), and the perturbation messes this up in a dramatic way. Please don't bother finding, or plotting, the Taylor series. Instead produce the following two plots at $\varepsilon = 0.04$:

- compare $u(t)$, $u_0(t)$, $\varepsilon u_1(t)$, and $u_a(t)$ on the same axes in the domain $t \in [0, 5]$
- show error $E(\varepsilon, t) := u_a(t) - u(t)$ in the domain $t \in [0, 3]$, making sure your axes illustrate its size

You should find the error is very small, staying much smaller than $10^{-3}$ in most of the latter domain. If you don't find this, you'll need to debug your algebra! [*e.g.* make sure $u_1(t)$ satisfies the correct ICs]

#3. Be careful: actually proving this isn't trivial.

#4 (easy algebra review; remember to substitute for $y$!)

#5 d, g (should be easy).

#8. a. This ODE could have come from a mass on a nonlinear spring that got weaker with speed squared.

#11. (connects to the planet-projectile ODE scaling problem from Lecture 3). Getting the 3rd term involves some high powers of $t$; do not be alarmed. However, only compute $t_m$ and $h_{max}$ to order $\varepsilon$ since order $\varepsilon^2$ is an algebra nightmare.