The following are just basic answers without any supporting work; you will be expected to justify your answers on the exam. Also no guarantees that there are no typos ....

Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

   Absolutely convergent: Equal (in absolute value) to convergent p-series $\displaystyle\sum \frac{1}{n^{3/2}}$; $p = 3/2 > 1$

2. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\sqrt{n+1}+\sqrt{n-1})}$

   Absolutely convergent: Limit comparison to convergent p-series $\displaystyle\sum \frac{1}{n^{3/2}}$; $p = 3/2 > 1$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2-(n-1)^2}{n^3}$ (Hint: Which is it? $\sim \sum \frac{1}{n}$ or $\sim \sum \frac{1}{n^2}$)

   $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2-(n-1)^2}{n^{3/2}} = \sum_{n=1}^{\infty} \frac{4n}{n^3}$ which is absolutely convergent by (limit) comparison to $\sum 1/n^2$.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n-3^n}{4^n}$

   Divergent; LCT to $\sum (5/4)^n$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n+4^n}{5^n}$

   Convergent; LCT to $\sum (4/5)^n$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n^2}$

   Divergent: nth term does not go to zero.

7. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{4^n}\right)\left(\frac{1}{n}\right)$

   Convergent: direct comparison to $\sum 1/4^n$ (convergent geometric series)

8. Estimate $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ with an error less than 10^{-6}.

   $$\sum_{n=1}^{100}\frac{(-1)^n}{n^3}$$

9. $\sum_{n=1}^{\infty}\ln(1+1/n)^n$.

   Hmm, nice! Note $\lim_{n\to\infty}\ln(1+1/n)=\ln(1)=0$ so for n large enough $\ln(1+1/n)<1/2$, so $\ln(1+1/n)^n<(1/2)^n$. That is compare to convergent geometric series.

10. What is the value of $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\pi^n}{3^{2n}(2n)!}$

    $\cos(\sqrt{\pi}/3)$.

11. What is $\displaystyle\lim_{x\to 0}\frac{\sin(x^2)-x^2}{x^6}$?

    $-1/6$.

12. Let $J(x)=\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$. What is $J^{(6)}(0)$?

    $-6!/(2^6(3!)^2)$.

13. Consider the function $f(x)=\sin x\cos x$ (Hint: $\sin(2x)=2\sin x\cos x$)

    (a) Determine T_4 for f about $a=\pi$

    $(x-\pi)-(2/3)(x-\pi)^3$

    (b) Find the Maclaurin series for f.

    $$\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{(2n+1)!}(x-\pi)^{2n+1}.$$

14. Evaluate $\int_0^{0.1}\ln(x^3+1)dx$ with error less than $\frac{1}{100}$.

    0

15. Evaluate $e^{-\frac{1}{2}}$ with error less than $\frac{1}{10^3}$.

    0.606 $(n = 4)$

16. Find the radius and interval of convergence of the following power series:

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(ex-2)^n}{3^n e^n}$

    $R = 3$; Interval $(2/e - 3, 2/e + 3)$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(4-x)^n}{n!}$

    $R = \infty$; Interval $(-\infty, \infty)$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(23-6x)^{2n}}{\sqrt{n}}$

    $R = 1/6$; Interval $(22/6, 24/6)$

    (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(5-4x)^{2n+1}}{4^n}$

    $R = 1/2$; $(3/4, 7/4)$

    (e) $\displaystyle\sum_{n=1}^{\infty} \frac{2x^n}{n^n}$

    $R = \infty$; Interval $(-\infty, \infty)$