Math 25 First Exam - Part b

October 20, 2009

Instructions: Show all work. You may feel free to consult your textbook or your notes, **but you may not consult any other source, animate or inanimate**.

You must show general procedures at all stages of a solution. Solving an equation or congruence by inspection or exhaustive search will get you no credit.

Your solutions are due at the beginning of class on Wednesday, October 21.

1. (20) Let $a, b, c \in \mathbb{Z}$ with $(a, b) = 1 = (b, c)$. Show that if $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} \in \mathbb{Z}$, then $b = \pm 1$, and $a = \pm c$. Hint: Express the fraction with a common denominator.

2. (20) Find all incongruent solutions to $784x \equiv 80 \pmod{832}$. Note that $784 = 2^4 \cdot 7^2$, $832 = 2^6 \cdot 13$, and $80 = 2^4 \cdot 5$.