Workshop Problems 5

Problem 1. Let V be a vector space. Use the axioms of a vector space to show that the zero vector is unique. That is, show that if a vector $\mathbf{w}$ in V has the property that $\mathbf{w} + \mathbf{v} = \mathbf{v}$ for all $\mathbf{v}$ in V, then $\mathbf{w} = \mathbf{0}$. *Hint:* What happens if you choose $\mathbf{v} = \mathbf{0}$?

Problem 2. Let V be a vector space. Use the axioms of a vector space to show that $0\mathbf{v} = \mathbf{0}$ for all vectors $\mathbf{v}$ in V. *Hint:* As in class, use the fact that $0 = 0 + 0$.

Problem 3. Let V be a vector space and let $H \subset V$ be a subspace. Show that if $\mathbf{u}$ and $\mathbf{v}$ are two vectors in H, then $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is contained in H.

Problem 4. Let V be a vector space and let $H, K \subset V$ be subspaces. The *intersection* of H and K, denoted $H \cap K$, is the collection of all vectors that belong to both H and K simultaneously. In set notation

$$H \cap K = \{\mathbf{v} : \mathbf{v} \text{ is in both } H \text{ and } K\}.$$

Show that $H \cap K$ is a subspace of V.

Problem 5. Let $V = \mathbb{R}^+$ be the set of positive real numbers. We define *addition* in V as follows: if x and y are in V then

$$x \oplus y = xy$$

where the right-hand side is ordinary multiplication of real numbers. If c is a scalar (real number) and x is in V then we define *scalar multiplication* by

$$c \odot x = x^c$$

where the right-hand side is ordinary exponentiation of a real number. Show that V together with the operations $\oplus$ and $\odot$ is a vector space.