A Diophantine equation with triangular numbers and Stirling numbers

Florian Luca

A theorem of Siegel asserts that the Diophantine equation

$$f(x) = y^2$$

has only finitely many integer solutions (x, y) when $f(x)$ is a polynomial with at least three roots of odd multiplicity. Let $S(n, k)$ be the Stirling number of the second kind which counts the number of ways of partitioning a set with n elements in k nonempty disjoint subsets. It turns out that if n is a fixed integer, then for varying integer $x > n$, the expression $S(x, x-n)$ is a polynomial of degree n is x. Thus, in order to decide whether for a fixed positive integer n, the equation

$$S(x, x-n) = \binom{y}{2}$$

has only finitely many integer solutions (x, y), it would be sufficient to prove that the degree n polynomial $8S(x, x-n) + 1$ has at least three roots of odd multiplicity. In my talk, I will give the main ideas of the proof of the fact that this is indeed so. The proof uses estimates from the theory of prime numbers. This is joint work with Clemens Fuchs (ETH, Zürich) and Akos Pínter (Univ. Debrecen).