Math 5: Music and Sound. Midterm 1

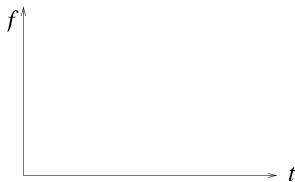
2 hours, 7 questions, 55 points total

Please show your working and pay attention to the indicated number of points available per question. The last page has useful information.

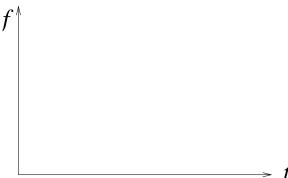
1.	[8 points]
	(a) What is the frequency of the pitch Eb6 in the equal-tempered system?
	(b) What musical pitch is the frequency 350 Hz nearest, and what is the error from this pitch in cents?
	(c) Construct the frequency ratio between E and C in the Pythagorean C major (diatonic) scale. (Briefly show working)

2.	[7 pc	pints + bonus] An ice cream truck produces a constant pure tone at pitch A5 (880 Hz).
	(a)	Assuming you and the truck are at rest, write a mathematical formula for the pressure signal heard as a function of time t in seconds.
	(b)	If you stand still and the truck comes towards you at 41 m/s, what frequency do you hear?
	()	
	(c)	Assume the truck is now stationary. What speed and direction (towards or away from the truck) would you have to be traveling at to lower the pitch to D5? (a perfect fifth; you may use 3:2).
	(4)	DONIE. You and the touch are stationary but a name strong wind of 24 m/s is blowing in the
	(a)	BONUS: You and the truck are stationary but a very strong wind of 34 m/s is blowing in the direction from the truck to you. What frequency do you hear?

- 3. [7 points] Sketch spectrograms on the axes provided which could realistically match the following sounds. Feel free to explain any features in words too:
 - (a) A pure tone rising in pitch but getting quieter, then followed by a hissing sound (no apparent pitch)



(b) A musical note with harsh timbre which is a periodic signal, followed by a bell-like note with no definite pitch.



4. [5 points] A bell produces the following partials all at roughly equal amplitudes: 302, 781, 1168, 1560, 2964. What 'strike tone' frequency is perceived, and why?

5.		points] A tuning fork is struck and produces a pure sinusoid at 300 Hz. A listener is a distance 2 ers from the tuning fork.
		Initially the tuning fork radiates 0.005 W acoustic power in all directions. What intensity in dB does the listener hear?
	(b)	The Q-factor of the tuning fork is 1000. What is the decay time?
		How long since it was struck with the above initial strength does it take until the intensity at the listener reaches the lower threshold of human hearing which is about 10^{-10} W/m ² at 300 Hz? (careful, not 10^{-12} W/m ²)
		If mass is added to the prongs of the tuning fork so that their <i>effective</i> mass doubles (viewing the fork as a mass-spring oscillator), what frequency does the fork sound now?

6.	[7 pc	pints + bonus]
	(a)	Draw a space-time diagram showing why a flutter echo is heard by a listener standing beside a wall a distance L from another wall, when they produce a short sound such as a clap. Label the walls and any sound pulses.
	(b)	What period of signal is heard if the spacing between the walls is $L=10$ meters?
	(a)	BONUS: If the listener instead stands <i>half way</i> between the walls, what periodicity is heard?
	(c)	(draw a space-time diagram)

7.	[10 p	points + bonus] Random short-answer questions.
	(a)	Explain briefly what is a Fourier series (you may use an equation). What kinds of functions does it apply to?
	(b)	State as clearly as you can what you will hear when pure tones of frequencies 500 and 507 Hz are played together, giving any relevant new frequencies.
	(c)	What frequency has a wavelength about equal to the size of the human ear (4 cm) ?
	(d)	According to the Helmholtz theory, state briefly why an octave is less dissonant than the major seventh interval.
	(e)	BONUS: When sounds are heard together their signals add. We learned that doubling the amplitude of a signal caused intensity to be multiplied by 4. However when two violins (for example) play the same music together the intensity is merely doubled relative to one violin. Resolve the paradox!

Useful information

$$\omega = 2\pi f$$

$$c = f\lambda$$

$$dB = 10 \log_{10} \frac{I}{10^{-12} W/m^2}$$

$$Q = \pi \frac{\tau}{T}$$

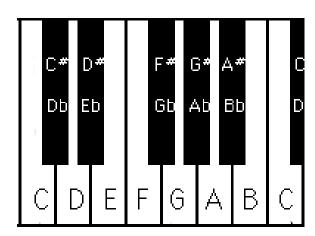
$$\frac{f_{obs}}{f} = \frac{1}{1 - v/c} \quad \text{or} \quad 1 + v/c$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin a + \sin b = 2\cos(\frac{a-b}{2})\sin(\frac{a+b}{2})$$

Intervals by number of semitones:

- 1. minor second
- 2. whole tone (major second)
- 3. minor third
- 4. major third
- 5. perfect fourth
- 6. tritone (augmented fourth)
- 7. perfect fifth
- 8. minor sixth
- 9. major sixth
- 10. minor seventh
- 11. major seventh
- 12. octave



The standard musical pitch A4 is 440 Hz

You can use the speed of sound as 340 m/s.