Name: ____________________________

Math 20 Midterm 1

November 16, 2011

Instructions: This is a closed book, closed notes, computer-free exam. You are not to give nor to receive help from any outside source during the exam. Remember that your instructor can clarify any questions that are not clear to you.

Please show all of your work and justify all of your answers.

Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own work.

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Signature

Question	Points	Score
1	18	
2	10	
3	10	
4	12	
5	12	
6	8	
7	5	
8	3	
Total:	78	

1. Short Answer:

   Answer each of the following questions. You do not need to show any work. (3 points each)

   (a) [3 points] We are playing a game of ping pong to the score of 21. I win each point with probability .7. Find an expression for the probability I win by exactly 6 points.

   (b) [3 points] The number of years a radio functions is an exponential random variable with expected value of 8 years (so $\lambda = 1/8$). Your radio is now 2 years old. What is the probability it is functioning 8 years from now?

   (c) [3 points] Let X be a non-negative random variable with $E(X) = 12$. What can you say about $P(X \geq 16)$?

(d) [3 points] There are 200 students in a dorm, each of whom will enroll in Math 20 next fall with probability $p = 1/50$. Let X be the number of such students who enroll. Estimate $P(X = 4)$.

(e) [3 points] For X a continuous random variable with density $f(x)$, find the density $g(y)$ of Y where $Y = X/3$.

(f) [3 points] Briefly explain why it is necessary to standardize S_n when applying the Central Limit Theorem.

2. One technique to estimate the charge of an electron works as follows: over time t, the average current at a given point is a random variable given by $I_t = eN/t$ where e is the charge of an electron and N is the number of electrons passing by that point. We can readily measure I and t. Here, N can be modeled as a Poisson random variable with parameter λt where λ is unknown.

   (a) [4 points] Find the standard deviation of the current at time t in terms of e, t and λ.

   (b) [6 points] Using (a) and $E(I_t)$, find the value of e. (We can estimate $E(I_t)$ and $SD(I_t)$ through experimentation)

3. A bank accepts rolls of pennies and gives 50 cents credit to a customer without counting the contents. Assume a roll contains 49 pennies 30% of the time, 50 pennies 60% of the time and 51 pennies the remaining 10%.

   (a) [4 points] Find the expected value and the variance of the amount the bank loses on a typical roll.

   (b) [6 points] Estimate the probability the bank loses 16 cents or less on 100 rolls.

4. In a chemistry class, we are extracting one compound from another. The amount we expect to extract is 12 grams. Our equipment is imprecise, with the error known to be normally distributed with $\sigma = .02$. However, the professor has never heard of significant figures, and rumor has it will fail you if your answer is off by more than .01. Rather than fudge the numbers, you realize you can perform the experiment multiple times and average to get a value in the correct range.

   (a) [5 points] Using Chebyshev's inequality, find the number of experiments you would need to perform to have a 95% or better chance of your average error falling within the desired range.

   (b) [5 points] Using the Central Limit Theorem, find the number of experiments you would need to perform to have a 95% or better chance of your average error falling within the desired range.

   (c) [2 points] Why is it appropriate to use the Central Limit Theorem, knowing the value in (b)?

5. Let S_n be the number of successes on n Bernoulli trials with probability .8 of success on each trial. Let $A_n = S_n/n$ be the average number of successes. Compute the following and justify your answer:

   (a) [4 points] $\lim_{n\to\infty} P(A_n = .8)$

   (b) [4 points] $\lim_{n\to\infty} P(.7n < S_n < .9n)$

   (c) [4 points] $\lim_{n\to\infty} P(S_n < .8n + .8\sqrt(n))$

6. [8 points] Let $X_1, X_2, \ldots, X_n$ be independent random variables (but not identically distributed) with $E(X_i) = m_i$ and $V(X_i) = \sigma_i^2$. Let $\mu = m_1 + \cdots + m_n$ and assume there exists an $R > 0$ such that $\sigma_i^2 \leq R$ for $i = 1, \ldots, n$. For $S_n = X_1 + \cdots + X_n$ and any $\epsilon > 0$, show as $n \to \infty$ that

$$P(|\frac{S_n}{n} - \frac{\mu}{n}| < \epsilon) \to 1.$$

7. [5 points] Let U be a uniformly distributed random variable on the interval $(0, 5)$. Find the probability that the polynomial $x^2 + 4Ux + 4U + 2$ has all real roots (recall the quadratic equation).

8. [3 points] BONUS: Show that if X is an integer valued random variable that exhibits the memoryless property, then X is a geometric random variable.