Math 12, Fall 2007

Lecture 5

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Outline

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1. Review and overview
   - Last class
2. Today's material
   - Review of reading topics
3. Group Work
4. Summary
5. Next class

More about spacecurves

- Tangent and normal vectors
- Arclength, curvature
- Motion of a particle: position, velocity and acceleration

Further investigation

Position, velocity and acceleration

- $\vec{r}(t)(=\vec{p}(t))$ can be interpreted as the position of an object traveling through space.
- $\vec{v}(t) = \vec{p}'(t)$ is velocity ($|\vec{v}(t)|$ is the speed)
- $\vec{a}(t) = \vec{v}'(t) = \vec{p}''(t)$ is the acceleration

Outline

Concepts from reading

Functions of more than one variable

- $f : \mathbb{R}^n \to \mathbb{R}$
- Common case: $n = 2$, $f(x, y)$
- Interpretation: a graph over the xy-plane

$$G = \{(x, y, z) | z = f(x, y)\}$$

or

$$z = f(x, y)$$

- Sketching graphs

Concepts from reading

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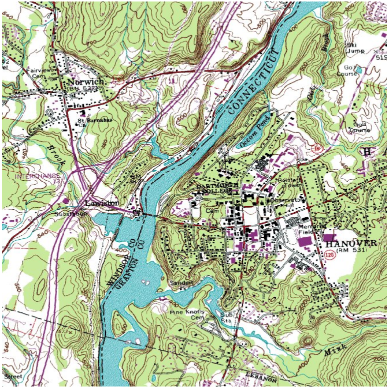
Concepts from reading
Contour plots

- Each line represented a path where the height remains constant or a "level" line.
- If we think of the height as a function, $h(x, y)$, this line is given by $h(x, y) = constant$.



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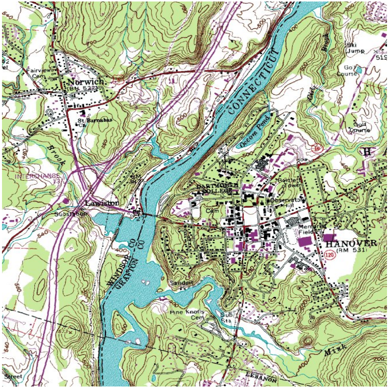

Concepts from reading
Contour plots

Method:

1. Sketch $f(x, y) = k$ for several values of k
2. Plot all of these in the plane (this is called a contour plot)
3. Lift each curve to height k in three dimensions to form a sketch of the surface.

Concepts from reading

Limits and continuity

Definition: Let f be a function of two variables whose domain D includes points arbitrarily close to (a, b). Then, we say that the limit of $f(x, y)$ as (x, y) approaches (a, b) is L and we write

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

if for every number $\epsilon > 0$, there is a corresponding number $\delta > 0$ so that $|f(x, y) - L| < \epsilon$ when $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$

Concepts from reading
Limits and continuity

A helpful test to determine if a limit does not exist:

If $f(x,y) \to L_1$ as $(x,y) \to (a,b)$ along a path C_1 and if $f(x,y) \to L_2$ as $(x,y) \to (a,b)$ along a path C_2 where $L_1 \neq L_2$ then the limit $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist.

e.g.

$$f(x,y) = \frac{xy^2}{x^2+y^4}$$

with $(a,b) = (0,0)$

Concepts from reading
Limits and continuity

A function f of two variables is continuous at (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$$

Group work

Find the limit, if it exists, or show that the limit does not exist

- $$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$$
- $$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

Summary

- Functions of more than one variable
- Limits
- Continuity

Work for next class

- Reading: 15.3
- f07hw6