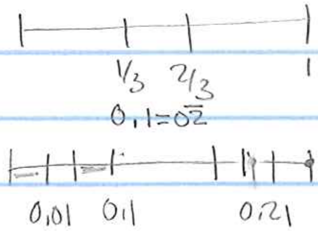
HW #5

T4.2



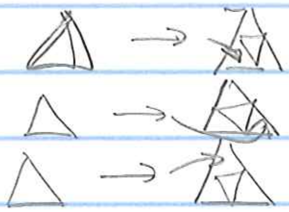
all left hand endpts of removed intervals end w/ $1 = 0\overline{2}$.

T4.3 (a) Goal: find f_1, f_2, f_3 to express this function.

f_1 maps

f_2 maps

f_3 maps



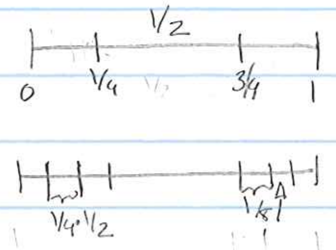
$$f_1(x,y) = \left(x/2,\ y/2\right)$$
$$f_2(x,y) = \left(1/2 + x/2,\ y/2\right)$$
$$f_3(x,y) = \left(1/4 + x/2,\ 1/2 + y/2\right)$$

b) 

$1/4\sqrt{3}$ so the exponential rate is $\left(1/4\sqrt{3}\right)\left(1/2\right)^k$.

4.2 middle 1/2 cantor set.

(a)



	removed	step
	$1/2$	1
	$\frac{1}{4}\cdot\frac{1}{2}\cdot 2 = 1/4$	2
	$(1/2)^n$	n

Total amount removed is $\sum_{n=1}^{\infty} (1/2)^n = \frac{1}{1-(1/2)} - 1$

$= 1/(1/2) - 1$

$= 2 - 1 = 1$

(b) any nonterminating sequence of 0's & 3's.

For example $0.33003\ldots$

(c) Goal: show that $1/5 \in K(4)$.

→ rewrite in base 4 & show it has a nonterminating sequence of 0 & 3 only.

multiply by 4 until you get something bigger than 1.

$1/5 \rightarrow 4/5 \rightarrow \frac{16}{5} \rightarrow \frac{1}{5} \rightarrow \frac{4}{5} \rightarrow \frac{16}{5}$

$3\cdot 4^{-2}$ $\quad$ $3\cdot 4^{-4}$

$\rightarrow 1/5 = 0.\overline{03}$ in base 4.

$17/21 \rightarrow 68/21 \rightarrow 5/21 \rightarrow 20/21 \rightarrow 80/21 \rightarrow 17/21$

$3\cdot 4^{-1}$ $\quad$ $0\cdot 4^{-2}$ $\quad \rightarrow 3\cdot 4^{-3}$

$\Rightarrow 17/21 = 0.\overline{303}$ in base 4

e) $a=4$

4.4 Goal: show that the rational #s has measuer zero.

let $\{Q_n\}$ denote a collection of intervals
st the n^{th} rational # is contained in Q_n.
& the length of each of the intervals is $\varepsilon/2^n$.

So the length of the intervals containing the rationals is

$$\varepsilon \sum_{n=1}^{\infty} 1/2^n = \varepsilon\left(\frac{1}{1-1/2} - 1\right) = \varepsilon(2-1) = \varepsilon$$

we take the limit as $\varepsilon \to 0$ to get the "length" of the rationals. so the rationals have measure 0.

Goal: Show that if $|z| > 2$ & $|c| < 2$ then the iteration on $P_c(z)$ diverges.

We can prove this by showing the iterates are increasing and unbounded in magnitude.

let l be st $|z_l| > 2$.

now $z_{l+1} = z_l^2 + c$

$\Rightarrow |z_l|^2 = |z_{l+1} - c| \leq |z_{l+1}| + |c|$ by triangle inequality.

So $|z_l|^2 - |c| \leq |z_{l+1}|$

$2|z_l| - |c| < |z_{l+1}|$ since $|z_l| > 2$

$|z_l| < |z_{l+1}|$ since $|c| < 2$

So $|z_l| < |z_{l+1}|$ $\Rightarrow$ increasing sequence.

Furthermore

$2(2|z_l| - |c|) - |c| < 2|z_{l+1}| - |c| < |z_{l+2}|$

$2^2|z_l| - 2|c| - |c| < |z_{l+2}|$

NOTE: $2|z_{l+2}| - |c| < |z_{l+3}|$

$2(2^2|z_l| - 2|c|) - |c| <$

$2^3|z_l| - 2^2|c| - |c| <$

Thus. $2^k|z_l| - 2^{k-1}|c| - |c| < |z_{l+k}|$

$2^{k-1}(\underbrace{2|z_l| - |c|}_{>2}) - |c| <$

$2^k - |c| <$

$\to \infty$ as $k \to \infty$.