(HW3 solns) Barnett

T2.7a key:

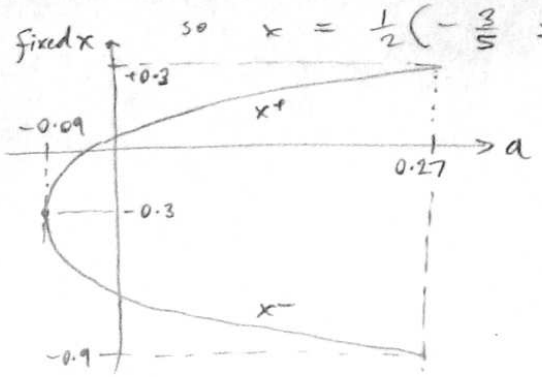
fixed pt ($b=0.4$) $f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}a-x^2+\frac{2}{5}y\\ x\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$

sub $x=y$ into $a-x^2+\frac{2}{5}y = x$

$\Rightarrow x^2 + \frac{3}{5}x - a = 0$

so $x = \frac{1}{2}\left(-\frac{3}{5} \pm \sqrt{\frac{9}{25}+4a}\right) = -\frac{3}{10} \pm \sqrt{\frac{9}{100}+a}$

for $a=0.27$ gives 0.6



graph it:

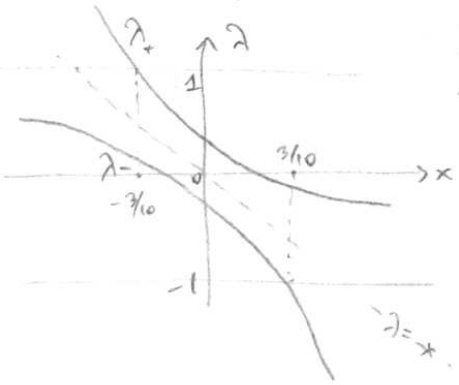
x^+ and x^- more monotonically w/ a.

for $-0.09 < a < 0.27$, $|x^+| < 0.3$

Then $D\vec{f} = \begin{pmatrix}-2x & \frac{2}{5}\\ 1 & 0\end{pmatrix}$ so its eigvals are via $(-2x-\lambda)(-\lambda) - \frac{2}{5} = 0$

$\lambda^2 + 2x\lambda - \frac{2}{5} = 0$

$\lambda = -x \pm \sqrt{x^2+\frac{2}{5}}$



also monotonic wrt x.

For $|x| < 0.3$ both λ's have $|\lambda| < 1 \Rightarrow$ sink.

solve b): check $x = 3/10$ $\Rightarrow \lambda = -3/10 \pm \sqrt{\frac{9}{100}+\frac{40}{100}} = -1, +0.4$. ← critical case.

For $-0.9 < x < -0.3$, one $|\lambda|<1$ & one $|\lambda|>1$ $\Rightarrow$ saddle.

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for A) see Appendix A of book, p. 558.