Lecture Notes

Math 3 / Fall 2012

Week 10

Limits

Evaluate: $\lim_{x \to \infty} \dfrac{5x - 3x^2 + 7}{\sqrt{3x^2 - x + 1}}$

(a) $5/\sqrt{3}$

(b) $-\sqrt{3}$

(c) ∞

(d) $-\infty$

Continuity and differentiability

Consider the function

$$f(x) = \begin{cases} \sin x & \text{if } x \leq \pi/4, \\ \cos x & \text{if } x > \pi/4. \end{cases}$$

Which one of the following is true?

(a) $f(x)$ is continuous wherever it is defined

(b) $f(x)$ is discontinuous at $x = \pi/4$ but has a removable discontinuity there

(c) $f(x)$ is discontinuous at $x = \pi/4$ and the discontinuity there is not removable

(d) $f(x)$ is differentiable (i.e., has a derivative) at $x = \pi/4$

Tangent lines

The tangent line to $f(x) = x \ln x$ at $(1, 0)$ is:

(a) $y = (\ln x + 1)(x - 1)$

(b) $1 = x - y$

(c) $y = x$

(d) $y = 2x - 2$

Derivatives

Evaluate: $\lim_{h \to 0} \dfrac{\sec(3+h) - \sec(3)}{h}$

(a) $\sec(3)$

(b) $-\cos(3)$

(c) $\sec(3)\tan(3)$

(d) Does not exist

Differentiation rules

Evaluate: $\dfrac{d}{dx}\left(\dfrac{\arcsin(x)}{\tan(x)}\right)$

(a) $\dfrac{\cot(x)}{\sqrt{1-x^2}} - \arcsin(x)\cdot\csc^2 x$

(b) $\dfrac{\tan(x)}{\sqrt{1-x^2}} + \dfrac{\arcsin x}{\sec^2(x)}$

(c) $\left(\arcsin(x)\sec^2(x) + \dfrac{\tan(x)}{\sqrt{1-x^2}}\right) / \tan^2(x)$

(d) $\dfrac{\cos^2 x}{\sqrt{1-x^2}}$

Graph sketching

Where does the function $f(x) = \frac{x^5}{20} - \frac{x^4}{12} + \frac{x}{8} + \frac{1}{24}$ have inflection points?

(a) At $x = 1$ but not at $x = 0$

(b) At $x = 0$ but not at $x = 1$

(c) At $x = 0$ and $x = 1$

(d) Neither at $x = 0$ nor at $x = 1$

Implicit differentiation

The slope of the tangent line to the curve $(x^2 + y^2)^3 = 8x^2y^2$ at the point $(-1, 1)$ is:

(a) -1

(b) 0

(c) 1

(d) 2

Linear approximation

Use linearization centered at $1/2$ to approximate $\arcsin(3/5)$:

(a) $\arcsin(3/5) \approx \frac{\pi}{6} + \frac{1}{5\sqrt{3}}$

(b) $\arcsin(3/5) \approx \frac{\pi}{3} + \frac{1}{5\sqrt{3}}$

(c) $\arcsin(3/5) \approx \frac{\pi}{3} + \frac{\sqrt{3}}{5}$

(d) $\arcsin(3/5) \approx \frac{\pi}{6} + \frac{\sqrt{3}}{5}$

Newton's method

If you use Newton's method to approximate a root of $f(x) = x - \cos x$ starting with $x_0 = 1$, then:

(a) $x_1 = 1 - \dfrac{1 - \cos 1}{1 + \sin 1}$

(b) $x_1 = 1 - \dfrac{1 - \cos 1}{1 - \sin 1}$

(c) $x_1 = 1 + \dfrac{1 - \cos 1}{1 + \sin 1}$

(d) $x_1 = 1 + \dfrac{1 - \cos 1}{1 - \sin 1}$

Antiderivatives

$\int (x^2 - 3)^2 \, dx$ equals:

(a) $\frac{1}{3}(x^2 - 3)^3 + C$

(b) $\frac{1}{3}(x^2 - 3)^3(2x) + C$

(c) $\frac{1}{6}x^6 - \frac{3}{2}x^4 + 9x + C$

(d) $\frac{1}{5}x^5 - 2x^3 + 9x + C$

Velocity and acceleration

A baseball is popped straight up from sea level with an initial velocity of 40 meters per second. After 5 seconds, the ball is:

(a) 20.0 meters high and still rising

(b) 49.5 meters high and falling

(c) 88.0 meters high and still rising

(d) 77.5 meters high and falling

Related Rates

The area of a circular puddle is growing at a rate of $20\,\text{in}^2/\text{min}$. How fast is the circumference of the puddle growing at the instant when it equals $5\,\text{in}$?

(a) $\pi\,\text{in}/\text{min}$

(b) $2\pi\,\text{in}/\text{min}$

(c) $4\pi\,\text{in}/\text{min}$

(d) $8\pi\,\text{in}/\text{min}$

Separable differential equations

Solve: $y' = 2x \sec y,\ y(0) = 0$

(a) $y = \arcsin x$

(b) $y = -\arcsin x^2$

(c) $y = -\arcsin x$

(d) $y = \arcsin x^2$

Euler's method

If Euler's method with step size 1 is used to for the initial value problem

$$y' = x - y, \qquad y(1) = -3,$$

then:

(a) $y_0 = -3, y_1 = 1$

(b) $y_0 = 1, y_1 = 1$

(c) $y_0 = -3, y_1 = -5$

(d) $y_0 = 1, y_1 = -3$

Optimization

The absolute maximum of the function $f(x) = x^3 - 3x + 1$ on the interval $[-3, 3]$ occurs at:

(a) $x = -3$

(b) $x = -1$

(c) $x = 1$

(d) $x = 3$

Definite integral

Evaluate: $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)^3$

(a) $1/4$

(b) $15/4$

(c) 4

(d) $4/3$

The fundamental theorem of calculus

$\dfrac{d}{dx}\displaystyle\int_x^5 \sin(t^3)\,dt$ equals:

(a) $\sin(x^3)$

(b) $-\sin(x^3)$

(c) $\cos(x^3)$

(d) $x^2\sin(x^3)$

Substitution

$\displaystyle\int \frac{\cos x}{\sin^2 x}\,dx$ equals:

(a) $\csc x + C$

(b) $\cot x + C$

(c) $-\cot x + C$

(d) $-\csc x + C$

Areas between curves

The area of the figure bounded by the graphs of $y = x^3 - 2x$ and $y = x^2$ is:

(a) 0

(b) 27/12

(c) 37/12

(d) 3

Arc length

The length of the graph of $f(x) = 2x^{3/2}$ from $(1, 2)$ to $(4, 16)$ is:

(a) $\dfrac{2}{27}(37^{3/2} - 10^{3/2})$

(b) $37^{3/2} - 10^{3/2}$

(c) 7

(d) $\dfrac{2}{27}\sqrt{7}$