Math 12, Fall 2007

Lecture 5

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Outline

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1. Recap and overview
    - Last class
    - Additional examples
2. Today's material
    - Review of reading topics
    - Group Work
3. Summary
4. Next class

Spacecurves

- A space curve is an example of a vector valued function of one variable.
- Curves generalize lines:

$$\vec{r}(t) = < f(t), g(t), h(t) >$$

- Limits, derivatives and integrals are taken component-wise
- Sketching space curves.

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Examples

What do these curves look like? Where are they smooth? What are do their tangent vectors look like? Are the tangent vectors continuous?

- $\vec{r}(t) =< t, t^2, 0 >$
- $\vec{r}(t) =< t^2, t^4, 0 >$

Example

$\vec{r}(t) = < t^2, t^3, t^4 >$

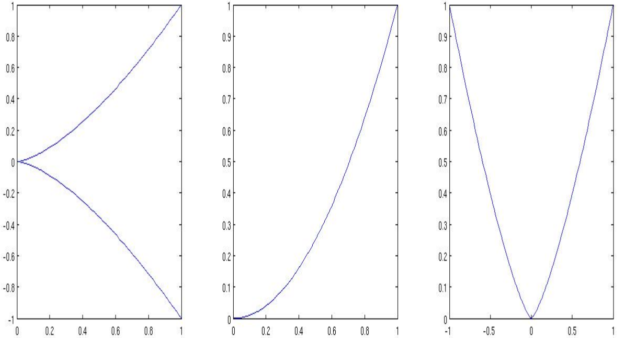

Figure: Coordinate plane projections

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Concepts from reading
Arclength

Given a spacecurve, $\vec{r}(t) =< f(t), g(t), h(t) >$ with $a \leq t \leq b$, its arclength is given by

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2}\, dt$$

or, equivalently

$$L = \int_a^b |\vec{r}'(t)|\, dt$$

- Parametrization and reparametrization by arclength

$$s(t) = \int_a^t |\vec{r}'(\tau)|\, d\tau$$

Concepts from reading
Curvature

- Recall: $\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$
- The curvature of a curve is

$$\kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

- Example: circles have constant curvature
- Normal vector:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

- Claim: $\vec{T}$ and $\vec{N}$ are perpendicular.

Concepts from reading

Position, velocity and acceleration

- $\vec{r}(t) = \vec{p}(t))$ can be interpreted as the position of an object traveling through space.
- $\vec{v}(t) = \vec{p}'(t)$ is velocity ($|\vec{v}(t)|$ is the speed)
- $\vec{a}(t) = \vec{v}'(t) = \vec{p}''(t)$ is the acceleration

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Problems to work on

- A projectile is fired with an angle of elevation α and initial velocity $\vec{v}_0$. Find the position function $\vec{r}(t)$ and determine for which α is the range maximized.
- Derive the equations of projectile motion in three dimensional space: Assume that the only force acting on a particle is gravity: $\vec{F} = -g\vec{k}$ where g is the graviational constant. If $\vec{p}_o$ is the initial position of the particle and $\vec{v}_0$ is the initial veclocity, find an equation for $\vec{p}(t)$.

Summary

- Tangent and normal vectors
- Arclength, curvature
- Motion of a particle: position, velocity and acceleration

Work for next class

- Reading: 15.1,15.2
- f07hw6