Math 13. Multivariable Calculus. Practice Homework 9.

This homework set is not to be turned in. It is for you to practice and prepare for the exam. Solutions will be posted on the assignments section of the website.

1. (Chapter 16.7, #29) Let $\mathbf{F} = \langle x, 2y, 3z \rangle$, and let S be the cube with vertices $(\pm 1, \pm 1, \pm 1)$ with positive orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

2. (Chapter 16.7, #49) Let $\mathbf{F}$ be an inverse square vector field (that is, $\mathbf{F}(\mathbf{r}) = c\mathbf{r}/|\mathbf{r}|^3$ for some constant c, where $\mathbf{r} = \langle x, y, z \rangle$). Show that the flux of $\mathbf{F}$ across a sphere S centered at the origin is independent of the radius of S.

3. (Chapter 16.9, #18) Let $\mathbf{F} = \langle z \tan^{-1}(y^2), z^3 \ln(x^2+1), z \rangle$. Find the flux of $\mathbf{F}$ across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$ and is oriented upwards.

4. (Chapter 16.9, #24) Use the Divergence Theorem to evaluate

$$\iint_S (2x + 2y + z^2)\, dS,$$

where S is the sphere $x^2 + y^2 + z^2 = 1$.

5. (Chapter 16.8, #17) A particle moves along line segments from the origin to the points $(1, 0, 0)$, $(1, 2, 1)$, $(0, 2, 1)$, and then back to the origin under the influence of the force field $\mathbf{F} = \langle z^2, 2xy, 4y^2 \rangle$. Find the work done in two separate ways: (a) by directly calculating this line integral, and (b) by using Stokes' Theorem with a suitable choice of surface S.

6. (Chapter 16.8, #18) Evaluate

$$\int_C (y + \sin x)\, dx + (z^2 + \cos y)\, dy + x^3\, dz,$$

where C is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$, $0 \le t \le 2\pi$.
(Hint: Observe that C lies on the surface $z = 2xy$.)