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10/20/11.

SOLUTIONS

Math 5: Music and Sound, 2011. Midterm

2 hours, 6 questions, 60 points total

Please show working. Points per question is shown to help judge time. Useful info on last page. Good luck!

1. [12 points] The famous shower scene in the film *Psycho* has a soundtrack where violins repeat a note whose fundamental frequency is 2500 Hz. [1]

   [3] (a) Compute the note name (and octave number) of this note. [You may use the closest note.]

   # semitones from A4 is $n = 12 \frac{\log \frac{2500}{440}}{\log 2}$

   $= 30.\overset{\text{ignore}}{08}$ $\qquad n = 30 = (2\text{ octaves}) + 6$

   6 semi's above A6 = E♭7 (since wraps past C).

   [2] (b) Very shortly they are joined by more violins playing another note at fundamental of 1325 Hz. Compute the (closest) musical interval formed by these two notes, and give its name.

   Interval semitones $n = 12 \frac{\log \frac{2500}{1325}}{\log 2} = 10.99$

   v. close to 11 semi's = major seventh (see table of intervals)

   [2] (c) How many cents sharp or flat of the equal-tempered version of this interval are the violins playing?

   Easiest is convert the above n to cents

   or via $n = 1200 \frac{\log \frac{2500}{1325}}{\log 2} = 1099.1 \ldots$ ¢

   Equal-tempered is 1100 ¢ so the psycho violins are playing only 0.9¢ flat (narrow) of equal tempered.

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[1] This idea inspired by one of your Aural Postings!

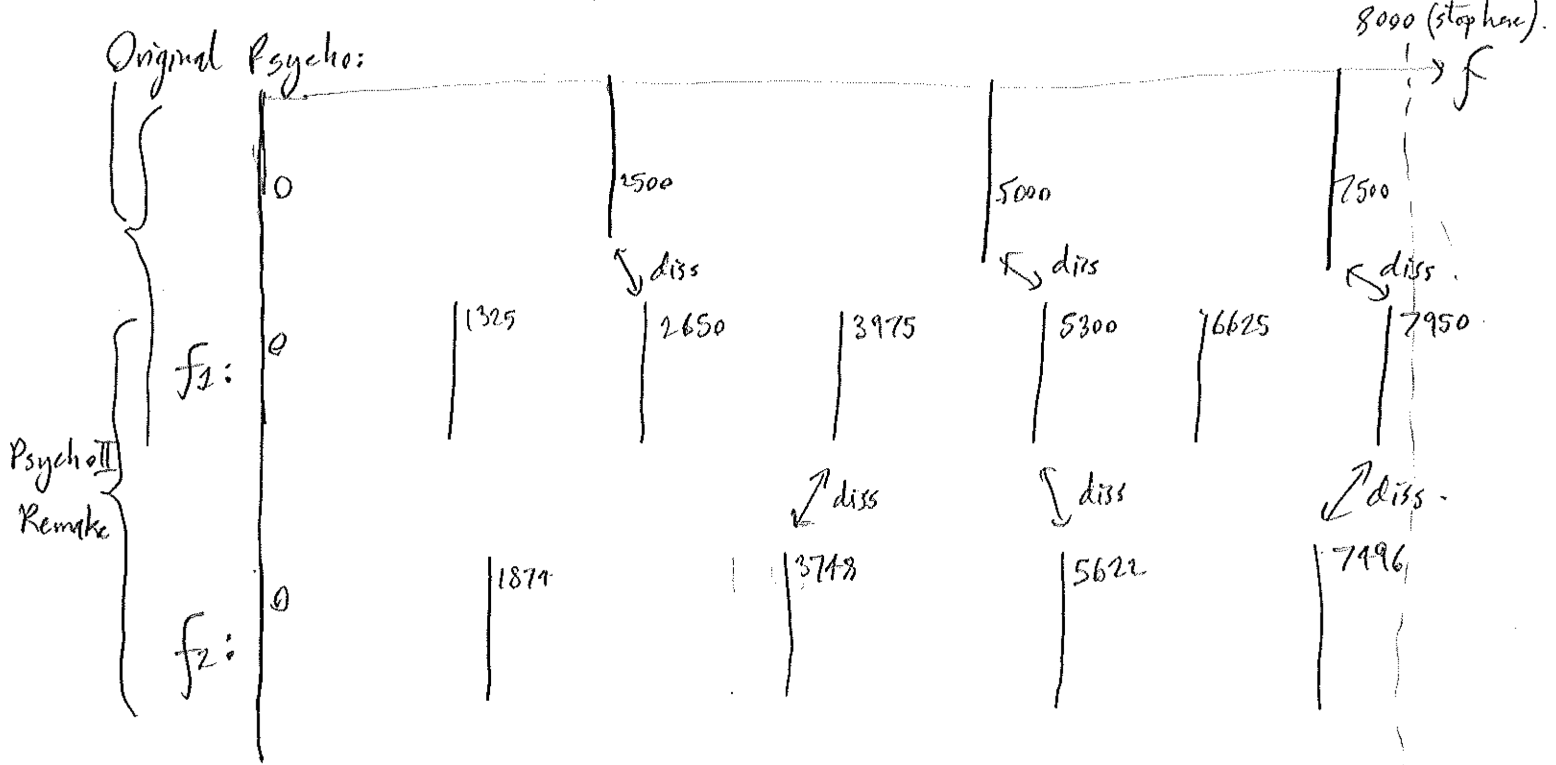
ie above 1325 Hz.

call f_1

[5] (d) A studio wishes to remake the film, and hires a composer who wants to create an even more dissonant interval than in the original! They keep the lower note at 1325 Hz, and now choose a higher one a *tritone* above (you may round this new frequency to the nearest Hz). Use the Helmholtz theory of dissonance to predict: do they succeed in their goal? [Please show working—a frequency axis will help—and only consider partials up to 8000 Hz.]

tritone = 6 semi's (from table of intervals)

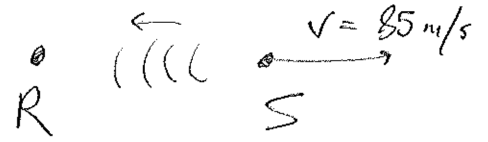
$$f_2 = 2^{6/12} \cdot f_1 = \sqrt{2} f_1 = 1873.8 \approx 1874 \text{ Hz}.$$



Both Original & Remake have 3 dissonant pairs (≤10%) ⇒ not more dissonant.

2. [9 points]

[3] (a) You are stationary. A fast train emits a pure tone at frequency 1000 Hz while rushing away from you at 85 m/s. What frequency do you hear?



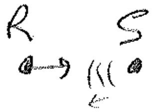
moving source formula Doppler:

$$f' = \frac{f}{1+\frac{v}{c}}$$

← away

$$f' = \frac{1000}{1+\frac{85}{340}} = \frac{4}{5}1000 = 800 \text{Hz}$$

[3] (b) You drive towards an outdoor concert with the windows down, and you hear the guitars tuning to an $\underbrace{\text{F\#4 instead of to the E4}}$ to which they actually are tuning. How fast are you traveling?



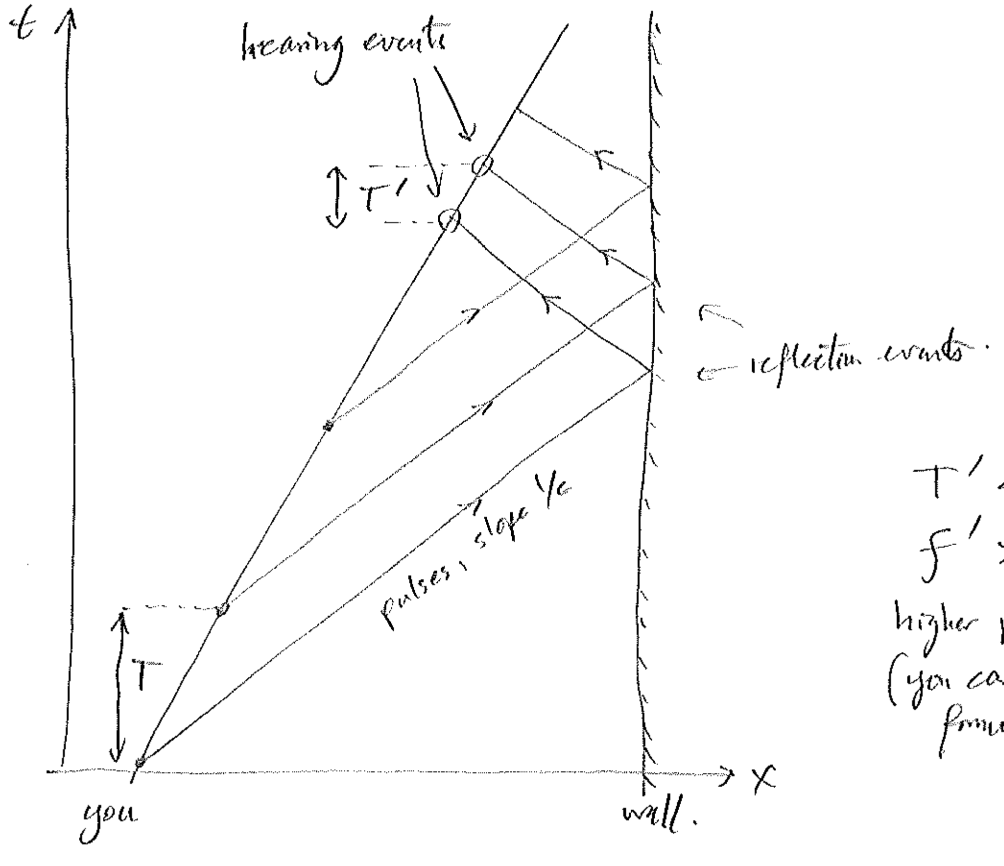
tells you f' is 2 semi's above f ie $f' = 2^{2/12} f$

Moving receiver formula: $f' = f\left(1 \overset{\uparrow}{+} \frac{v}{c}\right)$ since approaching

Divide the two equations (eliminates f' & f!): $1 = \frac{2^{2/12}}{1 + \frac{v}{c}}$

$\Rightarrow 1 + \frac{v}{c} = 2^{2/12} \Rightarrow v = c\left(2^{2/12} - 1\right) = 41.6$ m/s fast (93 mph!)

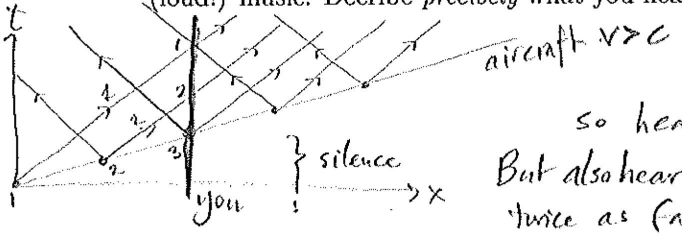
[3] (c) You run at constant speed towards a fixed wall while singing (i.e. emitting a periodic signal), and hear the echo of your own voice off the wall. Draw a spacetime diagram below (labeling your axes) which illustrates this—is the echo a higher or lower pitch?



$T' < T$ so $f' > f$, echo is higher pitch. (you can actually derive a formula for it — try it!)

BONUS: You are stationary. A supersonic aircraft passes right by you at speed 3c/2 while playing (loud!) music. Decribe *precisely* what you hear as it approaches, and then as it recedes.

here you hear silence.



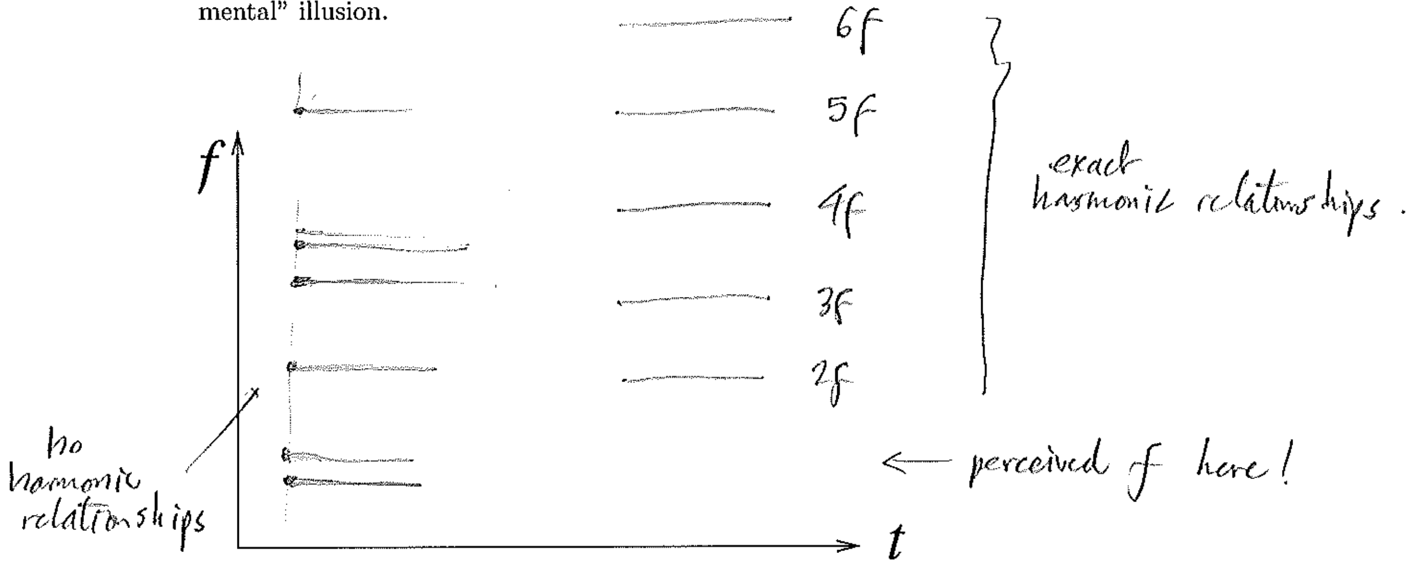
As recedes, $f' = f/(1 + v/c) = \frac{2}{5} f$

so hear slowed down & pitch-shifted music (to 40%)

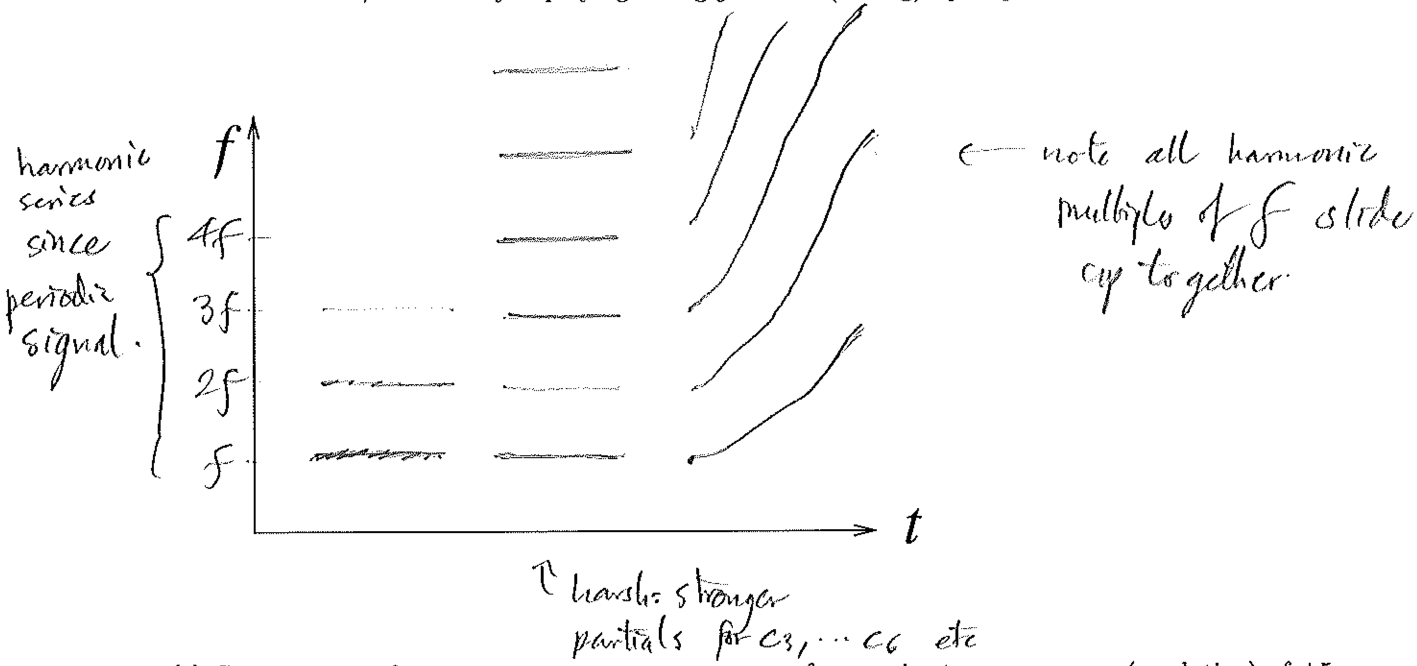
But also hear $f' = f/(1 - v/c) = -2f$ music played backwards twice as fast & one octave up. Weird!

3. [9 points] Sketch spectrograms on the axes provided which could realistically match the following sounds.

[4] (a) A struck bell of no definite pitch, followed by a constant sound that produces a "missing fundamental" illusion.



[4] (b) A violin playing a single note with mellow timbre, followed by it playing the same note with a harsh timbre, followed by it playing a long *glissando* (sliding) up in pitch.



[1] (c) Say you wanted to use a spectrogram to measure frequencies to an accuracy (resolution) of ±5 Hz, but preserve as much time detail as possible. What spectrogram time window should you choose?

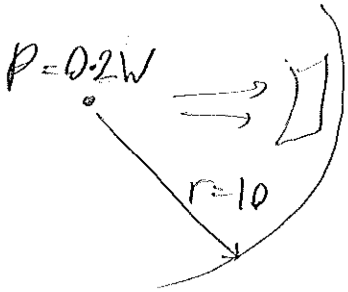
resolution $\Delta f \approx \frac{1}{T_w}$ ← time window.

so $T_w = \frac{1}{\Delta f} = \frac{1}{5}$ second

(don't want any longer since lose time detail then)

4. [11 points]

[3] (a) A trumpeter produces 0.2 W of acoustic power. Assuming it radiates equally in all directions, compute the resulting intensity when you are at a distance of 10 meters.



$$I = \frac{P}{4\pi r^2} = \frac{0.2}{4\pi(10^2)} = 1.59\times10^{-4} \text{ W/m}^2.$$

[3] (b) To what distance away would you have to move to so that the same trumpeter's intensity is 6 dB less than the situation in (a)?

Crude way to solve is convert (a) to dB: $dB = 10\log_{10}\frac{1.59\times10^{-4}}{10^{-12}}$
$= 82.0$ dB.

New intensity $I_2 = (10^{-12})10^{\frac{dB-6}{10}} = 10^{-12}10^{76/10} \approx 4.0\times10^{-5}$ W/m²

Finally $r = \sqrt{\frac{P}{4\pi I_2}}$ by rearranging

$= \sqrt{\frac{0.2}{4\pi(4\times10^{-5})}} = 19.95$ m

OR:
Neat way is to say $\frac{I_2}{I_1} = 10^{\frac{-6}{10}}$ and $I \propto \frac{1}{r^2}$ so $\frac{r_2}{r_1} = \sqrt{\frac{I_1}{I_2}} = 10^{+3/10} = 1.995$

[3] (c) A sound engineer compares the recorded amplitudes at the two distances in (a) and (b) above. What amplitude ratio will she find? [Hint: best solved independently of the solution of part (b)]

Intensity ratio $\frac{I_2}{I_1} = 10^{\frac{\text{change in dB}}{10}} = 10^{-6/10}$

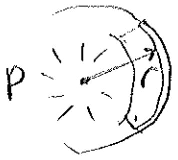
amplitude $A \propto \sqrt{I}$ so $\frac{A_2}{A_1} = \sqrt{\frac{I_2}{I_1}} = 10^{-3/10} = 0.501\ldots$

roughly half the original amplitude

(note ampl $\propto \frac{1}{r}$)

[2] (d) How much power would a trumpeter need to produce to hit the pain threshold of human hearing (130 dB) at the close distance of 1 meter away?

↰ so $I = (10^{-12})10^{\frac{dB}{10}} = 10^{-12} \cdot 10^{13} = 10 \text{ W/m}^2$

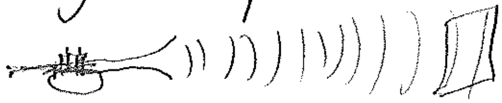


power P = intensity × (area of sphere rad 1m)

$= 10 \times 4\pi 1^2 = 125.6 \text{ W}$

yikes!

BONUS: Your power in the last question is certainly a lot *larger* than what is actually needed to cause pain/damage in real life. Explain why (there could be more than one reason).

↰ the above assumed radiation equally in all directions ('isotropic') but really a trumpet beams almost all its power in a narrow beam  , so much less power needed.

Second reason is that a lot of trumpet strong partials fall into 2–5 kHz where ear is more sensitive than at 1 kHz reference, → Damage!

5. [10 points] A distant tuning fork producing a *pure tone* at 425 Hz is used to test a two-microphone stereo recording set-up.

[2] (a) The amplitude (assume it's constant) at the first microphone is measured to be 1. Write a function of time t that could describe the recorded signal.

Pure tone amplitude 1

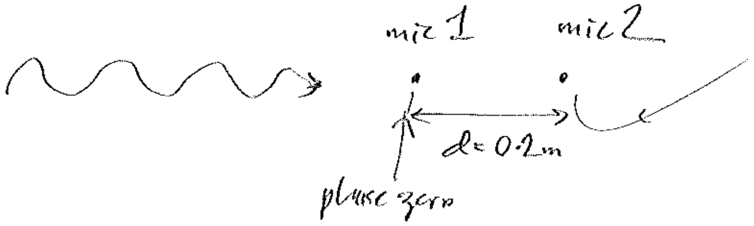
$$g(t) = \sin \omega t = \sin 2\pi f t = \sin(850\pi t)$$

↰ we left phase = 0;
$\sin(850\pi t + \phi)$ is more general.

[2] (b) What is the wavelength of this traveling pure tone?

$$\lambda = \frac{c}{f} = \frac{340}{425} = 0.8 \text{ m}.$$

[3] (c) In order to approximate the human ear separation, it is decided to place the second microphone 20 cm further away (i.e. 'downstream') from the first. Compute the phase ϕ (including correct sign) of the second microphone's signal, if the first microphone has phase zero.



$g_1(t) = \sin(850\pi t)$

$$\phi = -\frac{\omega d}{c}$$ derived in lecture.

$$= -\frac{2\pi f d}{c} = -2\pi\frac{d}{\lambda} = -2\pi\frac{0.2}{0.8}$$

$$= -\pi/2$$ or, negative 90°.

negative since mic 2 is downstream.

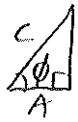
[3] (d) Later a mono recording is produced, so that these two signals from part (c) (assume they both have amplitude 1) are added together. Compute the resulting amplitude and phase of the new signal.

2 pure tones @ same freq

$$g_{mono}(t) = g_1(t) + g_2(t) = \sin 850\pi t + \underbrace{\sin(850\pi t - \pi/2)}$$

by trig this is $-\cos 850\pi t$. (cos is $\pi/2$ ahead, so $-\cos$ is $\pi/2$ behind).

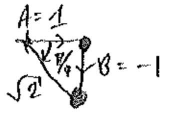
In form of



with $A=1$, $B=-1$

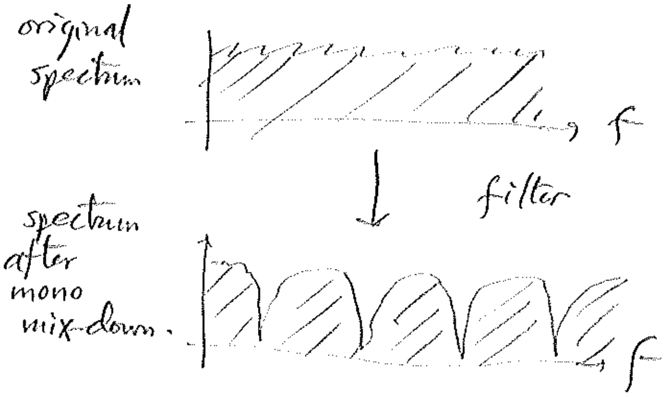
$$\Rightarrow C = \sqrt{A^2+B^2} = \sqrt{2}$$

$$\phi = \tan^{-1} B/A = \tan^{-1}(-1) = -\pi/4$$



BONUS: This mono recording produced will cause some probably undesirable changes in the *spectrum* compared to the true signal. Describe these as quantitatively as you can.

Well, for all freqs $f = (2n+1)\,850$, $n = 0, 1, 2, \ldots$ the phase difference as in (c) will be π, so the signals will cancel completely in part (d). This will kill off parts of the spectrum, a 'filter'.



yuk!

6. [9 points] Random short questions. Read the True/False ones carefully!

[4] (a) Place a check mark beside whichever of the following are true (could be all, some, or none):

- false • Every function $g(t)$ either has even or odd symmetry. no, but every $g(t)$ = odd part + even part.
- ✓ (But I had to allow false here too!) • If two pure tones at the same frequency are added (heard together), they always must give a single pure tone at that frequency. — in general it's true. Problem: if exactly cancel (π phase diff) you get zero ampl. which is not morally a pure tone.
- false • If two pure tones at the same frequency are added (heard together), you hear beats. — no, only if $f_1 \neq f_2$.
- false • Any periodic signal with period $1/f$ can include partials at f, $2f$, $3f$, ... but may also contain other partials. → no it can't

[2] (b) Compute the frequency ratio between D and F in the Pythagorean tuned scale. ie, the one in C major as in lectures.

'diatonic' scale in C: want.

C	D	E	F	G	A	B	C
1	9/8		4/3	3/2			2

3/2 down

$$\text{ratio} = \frac{4/3}{9/8} = \frac{32}{27}$$

[3] (c) A bell has strongest partials measured at 574, 637, 958, and 1273 Hz. What perceived frequency ('strike note') will you probably hear? [The note name is not needed.]

574 637 958 1273

compute ratios: (of adjacent pairs)

1.10 1.504 $\approx 3/2$ within <1% 1.329 $\approx 4/3$ within 1%

hypothesis: unrelated $2f$ $3f$ $4f$

so $f \approx \frac{637}{2} \approx \frac{958}{3} \approx \frac{1273}{4} \approx$ 318 Hz ($\pm$1 Hz or so)

Note: $\frac{958}{574} = 1.669 \approx 5/3$ to high accuracy but this doesn't fit into above hypothesis & is weaker since only involves one pair (hypothesis involves 3 partials).