Math 22 Workshop II
15 July 2010

1. For each of the following, decide if the assertion is true or false. Justify your answer with a proof or example, as appropriate.

(a) If $\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \}$ is linearly independent, then so is the set $\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \}$.

(b) If $\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \}$ and $\{ \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \}$ are both linearly independent, then so is $\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \}$.

(c) Suppose that $\mathbf{v}_1$ and $\mathbf{v}_2$ are nonzero vectors which are not multiplies of each other and that $\mathbf{u} \notin \operatorname{span}\{ \mathbf{v}_1, \mathbf{v}_2 \}$. Then $\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{u} \}$ is linearly independent.[1]

2. Suppose that V and W are vector spaces and that $T : V \to W$ is a linear transformation. If $\{ \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p \}$ are vectors in V and if $\{ T(\mathbf{v}_1), T(\mathbf{v}_2), \dots, T(\mathbf{v}_p) \}$ is linearly independent, then show that $\{ \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p \}$ is linearly independent.[2]

3. Suppose that V and W are vector spaces and that $T : V \to W$ is a linear transformation. Suppose that $\{ \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p \}$ is a linearly independent set of vectors in V. Must it be the case that $\{ T(\mathbf{v}_1), T(\mathbf{v}_2), \dots, T(\mathbf{v}_p) \}$ is linearly independent?

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[1] Note that in the version passed out in class, the $\mathbf{u}$ in the last set was incorrectly listed as $\mathbf{v}_3$.

[2] Start your argument with "Suppose that $c_1\mathbf{v}_1 + \dots c_p\mathbf{v}_p = \mathbf{0}$." You need to show that this forces $c_1 = \dots = c_p = 0$. To do this, think about what happens when you evaluate T at the vector $c_1\mathbf{v}_1 + \dots c_p\mathbf{v}_p$.