

A few more problems.

1. The recommended homework assignment.
2. Classify the critical point at $(0,0)$ for the system

$$\begin{aligned}x' &= -\sin(x+y) \\ y' &= -y(x+1)^2\end{aligned}$$

by describing its stability properties and giving all possibilities for its type.

3. Find the radius of convergence any series solution about $x_0 = 1$ of

$$(1+x^2)y'' + y = 0.$$

Find the fourth order polynomial approximation (about $x_0 = 1$ again) to the initial value problem with $y(0) = 1$, $y'(0) = -1$ and give the recurrence relation that describes the full solution.

Answers on next page.

1. check back of book
2. $a = 1$: asymptotically stable, node or spiral sink
3. $\sqrt{2}$,

$$y = 1 - (x - 1) - \frac{1}{4}(x - 1)^2 + \frac{1}{6}(x - 1)^3 - \frac{5}{96}(x - 1)^4 + \dots$$

$$2(n + 2)(n + 1)a_{n+2} + 2n(n + 1)a_{n+1} + (n(n - 1) + 1)a_n = 0.$$