Series (cont'd)

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Series Laws

If $\sum a_n$ and $\sum b_n$ are convergent then so are the following series

- $\sum(a_n \pm b_n) = \sum a_n \pm \sum b_n$;
- $\sum c a_n = c \sum a_n$.

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- Example: Find the sum of the following series

$$\sum_{n=1}^{\infty} \frac{4}{n(n+1)} + \frac{2^n}{3^n}.$$

The Integral Test

Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$.

1. If $\int_1^\infty f(x)\mathrm{d}x$ is convergent, then $\sum_{n=1}^\infty a_n$ is convergent.

2. If $\int_1^\infty f(x)\mathrm{d}x$ is divergent, then $\sum_{n=1}^\infty a_n$ is divergent.

Examples

- Determine whether

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- Determine whether

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  is a convergent series.

- For what values of p is the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

  convergent?

- Determine whether

$$\sum_{n=1}^{\infty} ne^{-2n}$$

is a convergent series.

The Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

1. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

2. If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

Examples

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- $\sum_{n=1}^{\infty} \frac{\sin^2(n)\sqrt{n}}{n^2}$
- $\sum_{n=1}^{\infty} \frac{n}{n^3+2n+1}$
- $\sum_{n=1}^{\infty} \frac{1}{n!}$