HW #6

T4.9

$b_1 = 1/3$

$N(b_1) = 3 \cdot 2$

$b_2 = 1/9$

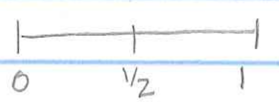
$N(b_n) = 9 \cdot 4$

$$\Rightarrow b_n = 1/3^n \quad N(b_n) = 3^n 2^n$$

$$\text{box dim} = \lim_{n\to\infty} \frac{\ln(N(b_n))}{\ln(1/b_n)} = \lim_{n\to\infty} \frac{n\ln(3\cdot 2)}{n\ln 3}$$

$$= \frac{\ln 3}{\ln 3} + \frac{\ln 2}{\ln 3} = 1 + \frac{\ln 2}{\ln 3}$$

4.11 b only Find box dimension of rational # on [0,1].



$b_1 = 1/2 \quad N(b_1) = 2$

$b_2 = 1/4 \quad N(b_2) = 4$

$\vdots$

$b_n = 1/2^n \quad N(b_n) = 2^n$

$$\text{boxdim} = \lim_{n\to\infty} \frac{\ln(N(b_n))}{\ln(1/b_n)} = \lim_{n\to\infty} \frac{n\ln 2}{n\ln 2} = 1$$

4.7 (a) middle 1/5 cantor set.

$b_1 = 2/5 \quad N(b_1) = 2$

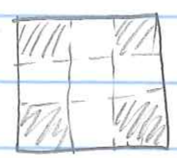
$b_2 = (2/5)^2 \quad N(b_2) = 4$

$b_n = (2/5)^n \quad N(b_n) = 2^n$

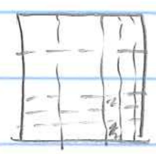
$$\text{boxdim} = \lim_{n\to\infty} \frac{\ln(N(b_n))}{\ln(1/b_n)} = \lim_{n\to\infty} \frac{n\ln(2/5)}{n\ln(2)}$$

$$= \frac{\ln 2}{\ln 2} - \frac{\ln 5}{\ln 2} = 1 - \ln 5$$

(b)



$b_1 = 1/3$
$N(b_1) = 4$

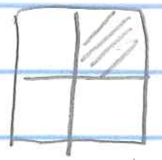


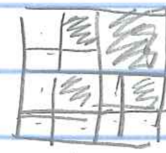
$b_2 = 1/9$
$N(b_2) = 4^2$

$b_n = (1/3)^n$
$N(b_n) = 4^n$

$$\text{boxdim} = \lim_{n\to\infty} \frac{\ln(N(b_n))}{\ln(1/b_n)} = \lim_{n\to\infty} \frac{n\ln 4}{n\ln 3} = \frac{\ln 4}{\ln 3}$$

4.9 a)





$b_1 = 1/2$

$N(b_1) = 3$

$b_n = (1/2)^n$

$N(b_n) = 3^n$

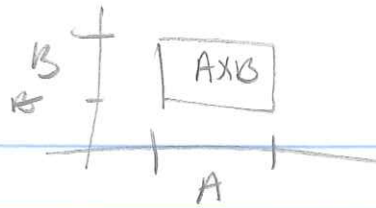
$b_2 = 1/4$

$N(b_2) = 3^2$

$$\text{boxdim} = \lim_{n\to\infty} \frac{\ln(N(b_n))}{\ln(1/b_n)}$$

$$= \lim_{n\to\infty} \frac{n\ln 3}{n \ln 2} = \frac{\ln 3}{\ln 2}$$

b)



◁— example of A×B. (Not all will look like this)

<u>4.10</u>

$$\text{boxdim}(A) = \lim_{\varepsilon \to 0} \frac{\ln(N_A(\varepsilon))}{\ln(1/\varepsilon)}$$

$N_A(\varepsilon) = \#$ of A boxes.

likewise, $$\text{boxdim}(B) = \lim_{\varepsilon \to 0} \frac{\ln(N_B(\varepsilon))}{\ln(1/\varepsilon)}$$

$$\text{Box dim}(A \times B) = \lim_{\varepsilon \to 0} \frac{\ln(N_A(\varepsilon) \cdot N_B(\varepsilon))}{\ln(1/\varepsilon)}$$

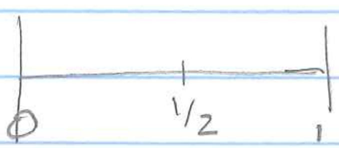
$$= \lim_{\varepsilon \to 0} \frac{\ln(N_A(\varepsilon))}{\ln(1/\varepsilon)} + \frac{\ln(N_B(\varepsilon))}{\ln(1/\varepsilon)}$$

$$= \text{boxdim}(A) + \text{boxdim}(B).$$

<u>4.12</u> (a) Note dist $1/n$ & $1/n+1$ is

$$1/n - 1/n+1 = \frac{n+1-n}{n(n+1)} = 1/n(n+1)$$

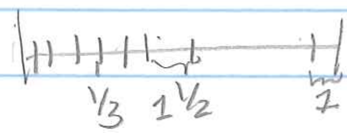
if we take $b_n = 1/n(n+1)$ what happens?



$b_1 = 1/2 \quad N(b_1) = 2$

$b_2 = 1/6 \quad N(b_2) = 4$

$b_3 = 1/3.4 = 1/12 \quad N(b_3) = 6$



$b_n = 1/n(n+1) \quad N(b_n) = 2n$

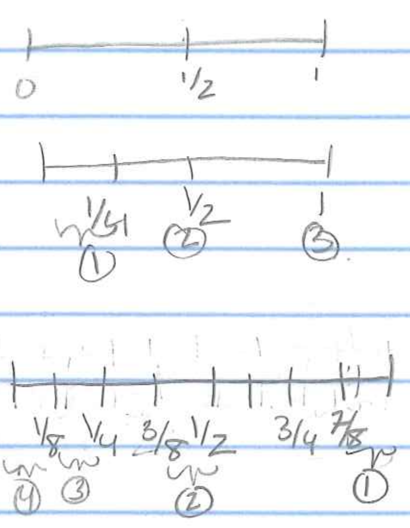
$$\text{boxdim}(S) = \lim_{n \to \infty} \frac{\ln(2n)}{\ln(n(n+1))}$$

$$\lim_{n\to\infty} \frac{\cancel{\ln 2}}{\ln(n(n+1))} + \frac{\ln(n)}{\ln(n(n+1))}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{1/n}{\frac{1}{n(n+1)}(2n+1)} = \lim_{n\to\infty} \frac{\cancel{n}(n+1)}{\cancel{n}(2n+1)}$$

$$= \quad = 1/2.$$

(b) Points are $1/2^{n-1}$ so distance between two consecutive pts is $1/2^{n-1} - 1/2^{n} = \frac{2-1}{2^n} = 1/2^n$



$b_1 = 1/2$ $N(b_1) = 2$

$b_2 = 1/4$ $N(b_2) = 3$

$b_3 = 1/8$ $N(b_3) = 4.$

$b_n = 1/2^n$ $N(b_n) = n+1$

$$\text{box dim}(S) = \lim_{n\to\infty} \frac{\ln(n+1)}{\ln(2^n)}$$

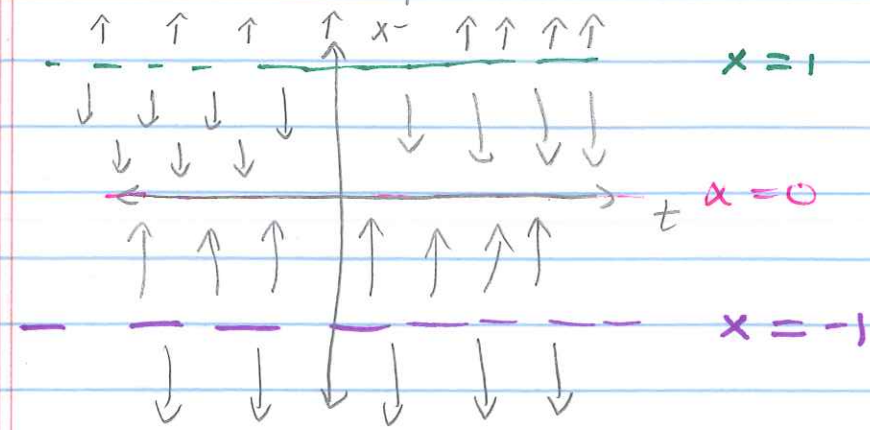
$$= \lim_{n\to\infty} \frac{\ln(n+1)}{n\ln 2} \overset{LH}{=} \lim_{n\to\infty} \frac{1}{(n+1)\ln 2} = 0 \checkmark$$

$$x' = x^3 - x$$

7.1 What are the equilibrium? ie for what x does

$$x(x^2-1) = 0 \ ?$$

$$x = 0, \pm 1,$$



if $|x_0| < 1$ then the solution is bounded.