m15w06	Sample Final	Exam Time: , 8:00 - 11:00
Name:		Student No.:

Instructions:

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT seperate the pages of your exam.

Problem	Points	Score
A1	12	
A2	14	
A3	12	
A4	12	
A5	12	
A6	12	
A7	14	
A8	12	
Total	100	

Name:

Section A: Answer ALL questions.

Problem A1: [12 pts] Suppose $F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y^2 \\ x-y^2 \end{pmatrix}$ and $G\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 3u+4v \\ u-2v \end{pmatrix}$.

(**a**) If $\begin{pmatrix} z \\ w \end{pmatrix} = F\begin{pmatrix} x \\ y \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix} = G\begin{pmatrix} u \\ v \end{pmatrix}$, find $\frac{\partial z}{\partial v}$ when $u = 0$ and $v = -1$.

(**b**) Find the linear approximation to F at $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Name:

Problem A2: [14 pts] A solid metal shape is constructed by rotating the portion of the curve $x = \sqrt{1 - z^2}$ about the z-axis.

(**a**) The metal has density $\rho = y^2\ kg/m^3$. What is the total mass of the solid?

(**b**) The surface of the solid carries a uniform charge density of $q = +1\ coulombs/m^2$, with the rest of the solid uncharged. Find the total charge of the solid.

Name:

Problem A3: [12 pts] Find the maximum and minimum values the function $f(x, y, z) = xyz$ takes on the ellipsoid $x^2 + 2y^2 + 3z^2 = 6$.

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Problem A4: [12 pts] Compute the flux of $\vec{F} = \begin{pmatrix} xz + y^4 \\ x - \tan z \\ z + y^2 \end{pmatrix}$ over the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$ oriented upwards.

Name:

Problem A5: [12 pts] Let $\vec{F} = \begin{pmatrix} y + \sin x \\ z^2 + \cos y \\ x^3 \end{pmatrix}$ and C be the oriented curve parametrized by $\vec{r}(t) = \begin{pmatrix} \sin t \\ \cos t \\ \sin(2t) \end{pmatrix}$. Find $\int_C \vec{F} \cdot d\vec{r}$. (Recall: $\sin(2t) = 2 \sin t \cos t$.)

Name:

Problem A6: [12 pts] A thick spherical shell fills the region $1 \leq r \leq 2$ and carries a charge density of $q = \frac{1}{r}$ $coulombs/m^3$. Assuming everywhere outside the shell is uncharged, find the induced electric field.

Name:

Problem A7: [14 pts] A wire occupies the infinite cylinder $x^2 + y^2 = 1$. The wire is covered by a very thin layer of a different metal with a very thin layer of insulator between this and the interior. In cylindrical coordinates the magnetic field is given by

$$\vec{B} = \mu_0 \begin{cases} r\hat{e}_\theta + 4\vec{k}, & r < 1 \\ \frac{1}{r}\hat{e}_\theta, & r > 1 \end{cases}.$$

What is the current density in the wire? Is there a current density in the thin surface layer? If so, what is it?

Name:

Problem A8: [12 pts] An electric field is given by

$$\vec{E} = \begin{pmatrix} 2y - 4x - z - 2 \\ 2x - 2y \\ -x \end{pmatrix}$$

Consider the electric potential restricted to the xy-plane.
(**a**) Find and classify the critical points for this restricted potential

(**b**) If the potential vanishes at $(0, 0, 0)$. Find the max/min of the potential over the region $-2 \leq x, y \leq 2$. (in the xy-plane again)