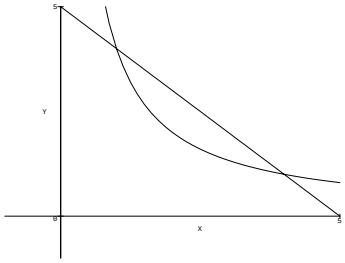
1. (20) Write your answer in the space provided. (*Show all work*).

   (a) (10) The region in the first quadrant bounded by the curve $xy = 4$ and the line $x + y = 5$ is rotated about the line $x = 1$. Write down (**but do not evaluate**) a definite integral for the volume of the solid of revolution.



   (b) (10) Suppose that a function $f(x)$ has a continuous second derivative on the interval $[a, b]$ and satisfies $-50 \leq f''(x) \leq 13$ for all x in that interval. Recall that the error formula for the Midpoint Rule says that if $|f''(x)|$ is bounded by a constant K on the interval $[a, b]$, then the error E_n involved in using n subintervals for the approximation of $\int_a^b f(x)\,dx$ satisfies

$$E_n \leq \frac{K(b-a)^3}{24n^2} = \frac{K(b-a)(\Delta x)^2}{24}, \quad \Delta x = (b-a)/n.$$

   Find an inequality which determines a number n of subintervals required to guarantee an error of less than 10^{-7}. **Note:** Your answer will be in terms of a and b.

2. (20) Determine whether the following integrals converge or diverge (*Show all work*).

   (a) (10) $\int_1^\infty e^{-x^2}\,dx$

   (b) (10) $\int_1^\infty \frac{dx}{2^x}$

3. (20) Write your answer in the space provided. (*Show all work*).

   (a) (10)Find all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{5^n}$ converges.

   (b) (10) Let $p > 1$ be a real number. Use the integral test to determine whether $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^p}$ converges or diverges.

4. (20) Short Answer (*No work need be shown, but no partial credit without some correct work*).

   (a) Discuss briefly (but more than true or false) the validity of the following argument:

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_{-1}^{1} = -1 - 1 = -2$$

   (b) If $P_n(x) = a_0 + a_1 x + \cdots + a_n x^n$ is the Taylor polynomial of degree n of a function $f(x)$ about $x = 0$, then for $1 \le k \le n$, $f^{(k)}(0) =$?

   (c) $\sum_{n=2}^{\infty} \frac{2^{2n}}{5^n} =$

   (d) Write the first three terms of a convergent series which equals the infinite repeating decimal: $0.081081081081\ldots$

   (e) Write down a sequence which is non-monotonic (i.e., one that is neither increasing nor decreasing).

5. $[-10, +20]$ Multiple Choice (No partial credit). Circle the appropriate answer. **Warning:** Each correct answer of Convergent or Divergent earns 4 points; each answer of "Don't Know" earns 0 points; each incorrect answer of Convergent or Divergent earns -2 points.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n-1}$ Convergent Divergent Don't Know

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2}$ Convergent Divergent Don't Know

(c) $\displaystyle\sum_{n=1}^{\infty} \ln(\frac{n+1}{n})$ Convergent Divergent Don't Know

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n+1}\right)$ Convergent Divergent Don't Know

(e) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$ Convergent Divergent Don't Know