1. Let $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} -6 \\ 3 \end{bmatrix}$.

(a) Show that $S = \{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal basis for $\mathbb{R}^2$.

(b) Express $\mathbf{y}$ as a linear combination of the vectors in S.

(c) In part (b), you should have derived an expression for $\mathbf{y}$ of the form $\mathbf{y} = \hat{\mathbf{y}}_1 + \hat{\mathbf{y}}_2$. Determine $\hat{\mathbf{y}}_1$ and $\hat{\mathbf{y}}_2$.

(d) Draw the 1-dimensional subspaces $L_i = Span(\{\mathbf{u}_i\})$ for $i = 1, 2$ on the axes below. On the same graph, plot the vectors $\hat{\mathbf{y}}_1$, $\hat{\mathbf{y}}_2$, and $\mathbf{y}$.

(e) Describe what $\hat{\mathbf{y}}_1$ and $\hat{\mathbf{y}}_2$ represent using the terminology of projections developed today.