Alternating Series

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The Alternating Series Test

- The nth term is of the form

$$a_n = (-1)^{n-1} b_n \text{ or } a_n = (-1)^n b_n,$$

  where each b_n is a positive number.

- **The Alternating Series Test:** If the alternating series

$$\sum_{n=1}^{\infty} (-1)^n b_n$$

satisfies

$$\begin{aligned} b_{n+1} &\le b_n \text{ for all } n \\ \lim_{n\to\infty} b_n &= 0 \end{aligned}$$

then the series is convergent.

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- $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^2}{n^3+1}$

Absolutely Convergent and Conditional Convergent Series

- A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.

- A series $\sum a_n$ is called **conditionally convergent** if it is convergent but not absolutely convergent.

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- **Theorem:** If a series $\sum a_n$ is absolutely convergent, then it is convergent.

The Ratio Test

1. If
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1,$$
then the series $\sum a_n$ is absolutely convergent.

2. If
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1 \quad \text{or} \quad \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty,$$
then the series $\sum a_n$ is divergent.

3. If
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1,$$
the Ratio Test is inconclusive.

Examples

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- $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
- $\sum \frac{(n+3)!}{3!n!3^n}$