Equations of Planes

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- If $\mathbf{r}_0$ is t he position vector of P_0 and $\mathbf{r}$ then the **vector equation of the plane**

$$\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$$

The Scalar equation of a plane

- The scalar equation of the plane trough $P_0(x_0, y_0, z_0)$ with normal vector $n = \langle a, b, c \rangle$ is

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- Example: Find an equation of the plane trough the point $(2, 1, -3)$ with normal vector $\mathbf{n} = \langle 3, 1, 1 \rangle$

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- Example: Find an equation of a plane trough the point $(-2, -1, 2)$ which is parallel to the plane $-3x + 2y + z = 7$.

The angle between two planes

- Two planes are parallel if their normal vectors are parallel.
- If two planes are not parallel, then they intersect in a straight line and the angle between them is the (acute) angle between their normal vectors.

Examples

1. Find the angle between the planes $x-y+z=1$ and $2x+y-3=1$.

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2. Find a equation for the line of intersection L of these two planes

- Find a equation of a plane containing the three points $(-2, 2, 0)$, $(-1, 3, 1)$ and $(-3, -3, 2)$

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- Find an equation of a plane trough the point $(0, 4, 1)$ which is orthogonal to the line $x = 1 + t$, $y = 2 - 3t$, $z = 5 + 2t$ in which the coefficient of x is 5.

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- Find an equation of a plane containing the line $\mathbf{r} = \langle -2, -2, 1\rangle + t\langle -4, 0, 1\rangle$ which is parallel to the plane $-2x + 2y + z = 5$