Math 11, Fall 2007

Lecture 18

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Outline

1. Review and overview
   - Last class
2. Today's material
   - Triple Integrals: coordinate transformations
3. Next class

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Triple integrals

- Triple integrals allow us to integrate over solid regions in $\mathbb{R}^3$
- Fubini's theorem and the theory of iterated integrals still apply.
- The main new difficulty is visualizing and parametrizing three dimensional solids.

Outline

Integrals of functions of three variables

- Polar coordinates: Change any two variables via polar coordinates:

$$u = r\cos(\theta),\ v = r\sin(\theta),\ dudv = rdrd\theta$$

This use of polar coordinates is also called cylindrical coordinates

- Spherical coordinates:

$$x = \rho\sin(\phi)\cos(\theta), y = \rho\sin(\phi)\sin(\theta), z = \rho\cos(\phi)$$

$$dV = \rho^2\sin(\phi)d\rho d\theta d\phi$$

Corrdinate transformations

1. Coordinate transformations help describe complicated regions in a simpler way
2. Circles, Cylinders $\rightarrow$ polar/cylidrical coordinates
3. Spheres$\rightarrow$ spherical coordinates

Examples

1. Find the volume enclosed by the sphere of radius r.
2. Find
$$\iiint_R z\, dV$$
where R is the region the lies between $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=9$ in the first octant.
3. Find the volume of the solid that lies above the xy -plane, inside the sphere of radius two centered at the origin and below the cone $z=\sqrt{x^2+y^2}$.
4. A torus is a surface generated by taking a circle of radius r centered at $(x_0, 0)$ (with $x_0 > r$) and rotating it (in three space), about a line. How can we describe a torus most efficiently? What is its volume?

Work for next class

- Read 16.8,17.1-17.2
- f07hw19