Math 12, Fall 2007

Lecture 1

Scott Pauls

[1] Department of Mathematics
Dartmouth College

9/26/07

Outline

Instructor information

Scott Pauls
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Course Information

- Meeting time: MWF 2, 006 Kemeny Hall
- Book: Stewart, Calculus

Exams:

1. Midterm I: 10/23/07 6-8 pm Carpenter 13
2. Midterm II: 11/13/07 6-8 pm Carpenter 13
3. Final Exam: 12/07/07 11:30-2:30

Homework

- Daily reading assignments and homework problems
- Regular homework (daily) via webwork
- Written homework assigned daily and due weekly.
- Tutorial sessions: Sun, Tues, Thurs 7-9pm, location 006 Kemeny

Grades:

- Homework: 20% (10% WebWork, 10% written)
- Midterms: 25% each
- Final: 30%

Outline

Coordinates in three space

Subtitles are optional.

- Use (x, y, z) to describe points in space
- Coordinate planes: $x = const$, $y = const$, $z = const$
- Other planes: a linear relation between variables
- Spheres: constant distance from a central point

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Surfaces in $\mathbb{R}^3$

planes

- A line in the plane, $\mathbb{R}^2$, is a linear relation between variables:

$$t = ms + b, \quad \text{or } t - ms - b = 0 \quad \text{or } \alpha t + \beta s + \gamma = 0$$

- A plane is a linear relation between variables in $\mathbb{R}^3$:

$$\alpha x + \beta y + \gamma z + \delta = 0$$

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Surfaces in $\mathbb{R}^3$

Spheres

- A sphere is the collection of all points located a fixed distance, r, from a given point $P_0 = (x_0, y_0, z_0)$.
- Distance from $P_1 = (x_1, y_1, z_1)$ to P_0 is

$$\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

- Sphere:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

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1. Introduction to Math 12
2. Today's material
    - Describing objects in $\mathbb{R}^3$
    - Vectors
3. Further discussion
    - Examples
    - Group Work
4. Summary
5. Next class

Points vs. vectors

- Vectors are quantities that have both *magnitude* and *direction*
- Points specify location while vectors specify direction
- We often confuse points and vectors (intentially) but, for clarity, we use different notation:

$$(x, y, z) = \text{point}, \quad < x, y, z >= \text{vector}$$

- *Magnitude of a vector:* distance from the tip to the origin:

$$| < x, y, z > | = \sqrt{x^2 + y^2 + z^2}$$

- *Direction of a vector:* if $\vec{u} =< x, y, z >$, the direction of $\vec{u}$ is $\frac{\vec{u}}{|\vec{u}|}$

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Vector operations

- $\vec{u} =< a, b, c >, \vec{v} =< d, e, f >$ and α, β are real numbers:

$$\alpha\vec{u} + \beta\vec{v} =< \alpha a + \beta d, \alpha b + \beta e, \alpha c + \beta f >$$

- This can also be seen geometrically.
- Basis vectors:

$$\vec{i} =< 1, 0, 0 >, \vec{j} =< 0, 1, 0 >, \vec{k} =< 0, 0, 1 >$$

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Example

Suppose two points $(1, 2, 3)$ and $(0, 2, -2)$ are antipodal points on a sphere. Find an equation for the sphere.

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Problems to work on

Projections:

- $P_{xy}((x, y, z)) = (x, y, 0)$
- $P_{yz}((x, y, z)) = (0, y, z)$
- $P_{xz}((x, y, z)) = (x, 0, z)$

Question:

$$\boxed{\text{Let } S = \{(x, y, z) | x^2 + y^2 + z^2 = 1\}. \text{ What is } P_{xy}(S)?}$$

Summary

- Coordinates in $\mathbb{R}^3$ and describing geometric objects
- Vectors: numeric and geometric
- Operations

Work for next class

- Reading: review 13.1-13.2, read 13.3,13.4
- Practice problems: (13.1 # 1-4,6; 13.2 #2,4,5,17,19), 13.3 #1,3,5,29,35; 13.4 #1,3
- Homework set: f07hw1, f07hw2