Math 118. **Combinatorics.** Spring 2013

Problem Set 4. Due on Wednesday, 5/29/2013.

1. Consider a hexagon with equal angles and integer side lengths given counter-clockwise by r, s, t, r, s, t. How many equilateral rhombi (consisting of two equilateral triangles of side 1) are needed to tile the hexagon?

2. Find a formula for the number of non-intersecting k-tuples $(P_1, P_2, \dots, P_k)$ of paths, where P_i is a path from $(-2i, 0)$ to $(2i, 0)$ with steps $(1, 1)$ and $(1, -1)$ that does not go below the x-axis.

3. Find a formula for the number of non-intersecting k-tuples $(P_1, P_2, \dots, P_k)$ of paths, where P_i is a path from $(-2i+1, 0)$ to $(2i-1, 0)$ with steps $(1, 1)$, $(1, -1)$ and $(2, 0)$ that does not go below the x-axis.

4. One can think of labeled trees on n vertices as spanning trees of the complete graph K_n. Recall Cayley's formula $\sum_T x_1^{\rho_T(1)} \cdots x_n^{\rho_T(n)} = (x_1 + \cdots + x_n)^{n-2}$, where T ranges over all spanning trees of K_n, and $\rho_T(i)$ denotes the degree of vertex i in T minus one.
   Find the number of spanning trees in a complete bipartite graph $K_{m,n}$, and give an analogue of Cayley's formula in this case.

5. Let Γ be a region on a square grid, and let τ be a tiling of Γ by dominoes.
   (a) Prove that the parity of the number of vertical dominoes in τ only depends on Γ, not on the particular tiling τ.
   (b) Prove that if Γ is simply connected (it has no "holes"), then every two tilings τ and τ' are connected by a series of 2×2 flips (one such flip switches two adjacent vertical dominoes with two horizontal ones).