

Math 11, Fall 2007

Lecture 5

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Outline

- 1 Recap and overview
 - Last class
 - Additional examples
- 2 Today's material
 - Review of reading topics
 - Group Work
- 3 Summary
- 4 Next class

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Spacecurves

- A space curve is an example of a vector valued function of one variable.
- Curves generalize lines:

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

- Limits, derivatives and integrals are taken component-wise
- Sketching space curves.

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Examples

What do these curves look like? Where are they smooth? What are do their tangent vectors look like? Are the tangent vectors continuous?

- $\vec{r}(t) = \langle t, t^2, 0 \rangle$
- $\vec{r}(t) = \langle t^2, t^4, 0 \rangle$

Example

$$\vec{r}(t) = \langle t^2, t^3, t^4 \rangle$$

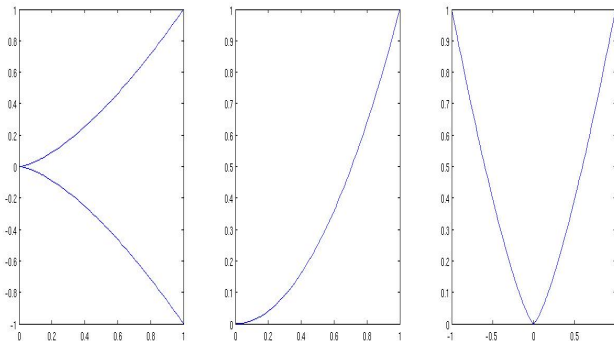


Figure: Coordinate plane projections

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Concepts from reading

Arclength

Given a spacecurve, $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ with $a \leq t \leq b$, its arclength is given by

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2} dt$$

or, equivalently

$$L = \int_a^b |\vec{r}'(t)| dt$$

- Parametrization and reparametrization by arclength

$$s(t) = \int_a^t |\vec{r}'(\tau)| d\tau$$

Concepts from reading

Position, velocity and acceleration

- $\vec{r}(t) = \vec{p}(t)$ can be interpreted as the position of an object traveling through space.
- $\vec{v}(t) = \vec{p}'(t)$ is velocity ($|\vec{v}(t)|$ is the speed)
- $\vec{a}(t) = \vec{v}'(t) = \vec{p}''(t)$ is the acceleration

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Problems to work on

- A projectile is fired with an angle of elevation α and initial velocity \vec{v}_0 . Find the position function $\vec{r}(t)$ and determine for which α is the range maximized.
- Derive the equations of projectile motion in three dimensional space: Assume that the only force acting on a particle is gravity: $\vec{F} = -g\vec{k}$ where g is the gravitational constant. If \vec{p}_0 is the initial position of the particle and \vec{v}_0 is the initial velocity, find an equation for $\vec{p}(t)$.

Summary

- Tangent vectors
- Arclength
- Motion of a particle: position, velocity and acceleration

Work for next class

- Reading: 15.1,15.2
- f07hw6