

1. Consider the symmetric matrix  $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ . Perform the following steps to orthogonally diagonalize  $A$ .
- (a) Find the eigenvalues of  $A$ .
  - (b) Find a basis for each eigenspace.
  - (c) You should have found 2 bases in the previous step. Show that the basis with 2 vectors is not an orthogonal set.
  - (d) Use the Gram-Schmidt process to find an orthogonal basis for the 2-dimensional eigenspace of  $A$ .
  - (e) Using your orthogonal basis, construct an orthonormal basis for the 2-dimensional eigenspace.
  - (f) Find an orthogonal matrix  $P$  and diagonal matrix  $D$  such that  $A = PDP^{-1} = PDP^T$ .