Math 13. Multivariable Calculus. Written Homework 3.

Due on Monday, 4/16/12.

You may leave this homework in the boxes outside of Kemeny 108 by 12:30 pm on Monday. Please write problems 1-3 on separate pages from problems 4-6 and turn them in in the corresponding columns.

1. (Ch 15.8, #24) Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.

2. (Ch 15.8, #28) Find the mass of a ball B given by $x^2 + y^2 + z^2 \leq a^2$ if the density at any point is proportional to its distance from the z-axis.
Hint: even though both cylindrical and spherical coordinates work for this problem, spherical coordinates give a simpler integral.

3. (Ch 15.9, #28) Find the average distance from a point in a ball of radius a to its center.

4. (Ch 12.4, #48) If $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$, show that

$$\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}.$$

5. (Ch 15.10, #18) Evaluate $\iint_R (x^2 - xy + y^2)\, dA$, where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$. Use the change of variables $x = \sqrt{2}\, u - \sqrt{2/3}\, v$, $y = \sqrt{2}\, u + \sqrt{2/3}\, v$.

6. (Ch 15.10, #14) Let R be the region bounded by hyperbolas $y = 1/x$, $y = 4/x$, and the lines $y = x$, $y = 4x$, in the first quadrant. Find equations for a transformation T that maps a rectangular region S in the uv-plane onto R, where the sides of S are parallel to the u- and v- axes.