Math 13. Multivariable Calculus. Written Homework 4.

Due on Wednesday, 4/24/13.

You may leave this homework in the boxes outside of Kemeny 108 by 12:30 pm on Monday. Please write problems 1-3 on separate pages from problems 4-6 and turn them in in the corresponding columns.

1. (Ch 15.10, #23) Use an appropriate change of variables to evaluate

$$\iint_R \frac{x-2y}{3x-y}\,dA,$$

   where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$ and $3x-y=8$.

2. (Ch 12.3, #54,) If $\mathbf{r} = \langle x, y, z\rangle$, $\mathbf{a} = \langle a_1, a_2, a_3\rangle$, $\mathbf{b} = \langle b_1, b_2, b_3\rangle$, show that the vector equation $(\mathbf{r}-\mathbf{a})\cdot(\mathbf{r}-\mathbf{b}) = 0$ represents a sphere, and find its center and radius.

3. (Ch 13.2, #34) At what point do the curves $\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2\rangle$ and $\mathbf{r}_2(s) = \langle 3-s, s-2, s^2\rangle$ intersect? Find their angle of intersection. (You can write your answer as the value of an inverse trigonometric function.)

4. Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t\rangle$ on the interval $0 \le t \le 1$.

5. (Ch 14.3, #72) If $g(x,y,z) = \sqrt{1+xz} + \sqrt{1-xy}$, find g_{xyz}. (Hint: use a different order of differentiation for each term if you want to keep calculations simple.)

6. (Ch 14.4, #42) Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2,1,3)$. You don't have an equation for S but you know that the curves

$$\mathbf{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2\rangle, \quad \mathbf{r}_2(u) = \langle 1+u^2, 2u^3-1, 2u+1\rangle$$

   both lie on S. Find an equation of the tangent plane at P.