1. (a) Determine a linear transformation T (using the idea of standard matrices) that maps the unit square with vertices $\mathbf{0}, \mathbf{e_1}, \mathbf{e_2}, \mathbf{e_1} + \mathbf{e_2}$ to the parallelogram with vertices $\mathbf{0}, \mathbf{e_1}, 2\mathbf{e_1} + \mathbf{e_2}, 3\mathbf{e_1} + \mathbf{e_2}$. In particular, choose T such that $\mathbf{e_1}$ is fixed under T and $2\mathbf{e_1} + \mathbf{e_2}$ is the image of $\mathbf{e_2}$ under T.

   (b) Apply the transformation found in part (a) to the 2×2 square with vertices $\mathbf{0}, 2\mathbf{e_1}, 2\mathbf{e_2}, 2(\mathbf{e_1} + \mathbf{e_2}$ and sketch the image of the 2×2 square under this transformation.

2. Recall problem from last class: "Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that contracts the unit square vertically by half, and results in a horizontal shear of the resulting rectangle to the the right by 3 units as shown on the board". Last time we showed that the standard matrix for T is $A = \begin{bmatrix} 1 & 3 \\ 0 & \frac{1}{2} \end{bmatrix}$. We also briefly discussed that T can be thought of as the composition of two transformations T_1 and T_2.

   (a) Find the standard matrices A_1 and A_2 for the intermediate transformations T_1 and T_2, respectively. [Hint: Idea of standard matrices may be used to find A_1. To find A_2, let $A_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and set up an appropriate system of equations to solve for a, b, c, d].

(b) Use matrix multiplication to show how A, A_1, A_2 are related.

(c) How does T_2 act on the **unit** square (either sketch the image or list the vertices of the image)?

3. Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Consider the augmented matrix $[A|I_2]$, where I_2 is the 2×2 identity matrix.

(a) Find the **reduced** echelon form of $[A|I_2]$. The resulting row equivalent matrix should be of the form $[I_2|B]$ for some matrix B.

(b) For the matrix B found in part (a), show that $AB = I_2 = BA$.

(c) The matrix B satisfying $AB = I_2 = BA$ is called the **inverse of** A and is denoted by A^{-1}. Using the method in parts (a) and (b), you can derive a general formula for A^{-1}, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$: $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. Under what condition is A^{-1} defined?

(d) Show that B can be written in the general form described in part (c).