Math 11, Fall 2007

Lecture 3

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Outline

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Dot Poduct

- dot product measures angle: $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos(\theta)$
- Projections, components
- Work: $W = \vec{F} \cdot \vec{D}$

Cross product

- measures volume/area
- torque: $\vec{\tau} = \vec{r} \times \vec{F}$
- cross product is perpendicular to components
- $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin(\theta)$

Outline

Concepts from reading

Lines

- Lines:

$$\vec{r} = \vec{r}_0 + t\vec{v}$$

- Lines are determined by a point and a direction
- Parametric form: $\vec{r} =< x, y, z >$, $\vec{r}_0 =< x_0, y_0, z_0 >$, $\vec{v} =< a, b, c >$

$$x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$$

- Symmetric form:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Concepts from reading
Planes

- A plane is determined by a point and a normal vector:

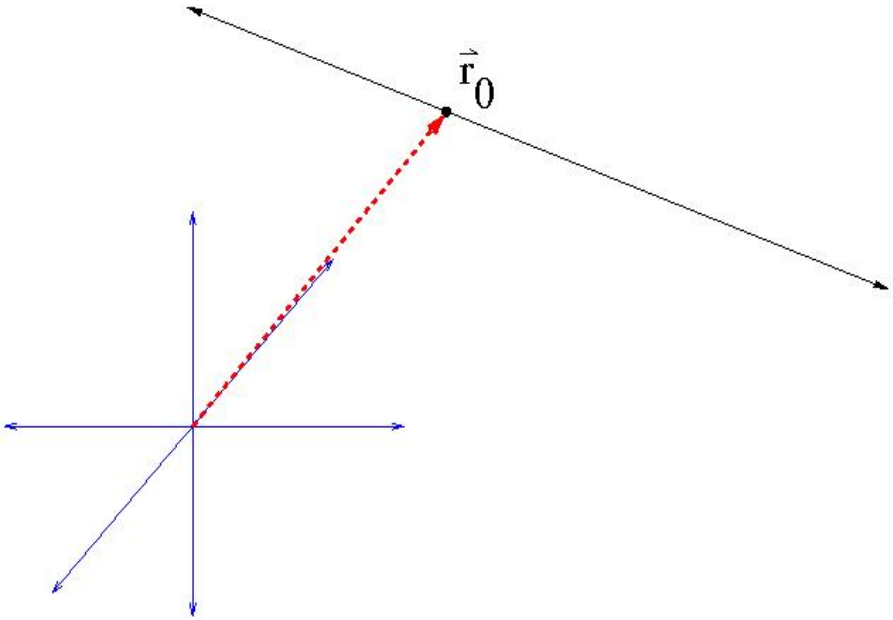
$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

- Letting $\vec{n} =< a, b, c >$, $\vec{r} =< x, y, z >$, $\vec{r}_0 =< a_0, y_0, z_0 >$, we can reduce this to the scalar equation of the plane:

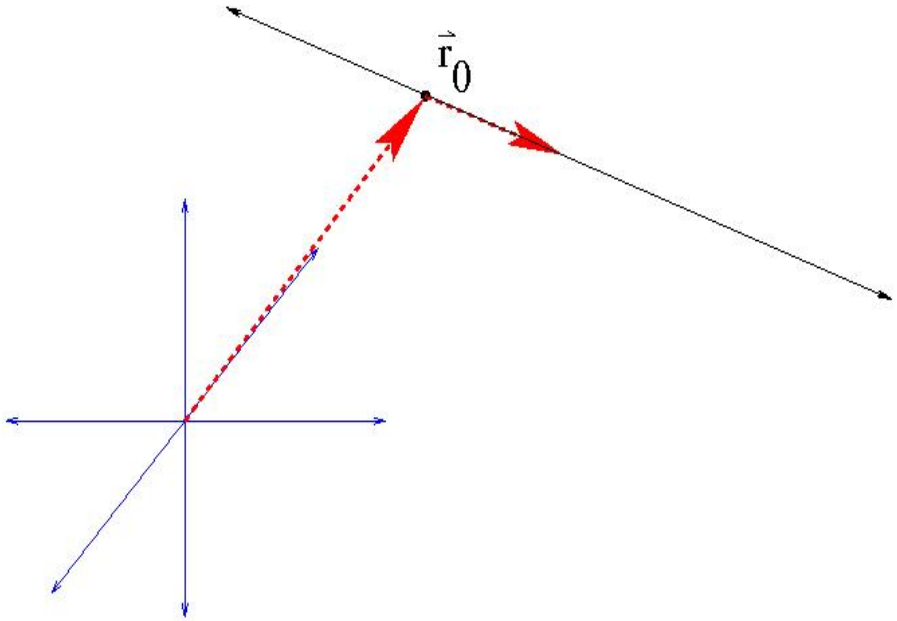
$$ax + by + cz + d = 0$$

where $d = -(ax_0 + by_0 + cz_0)$

Geometric derivation of the line



Geometric derivation of the line



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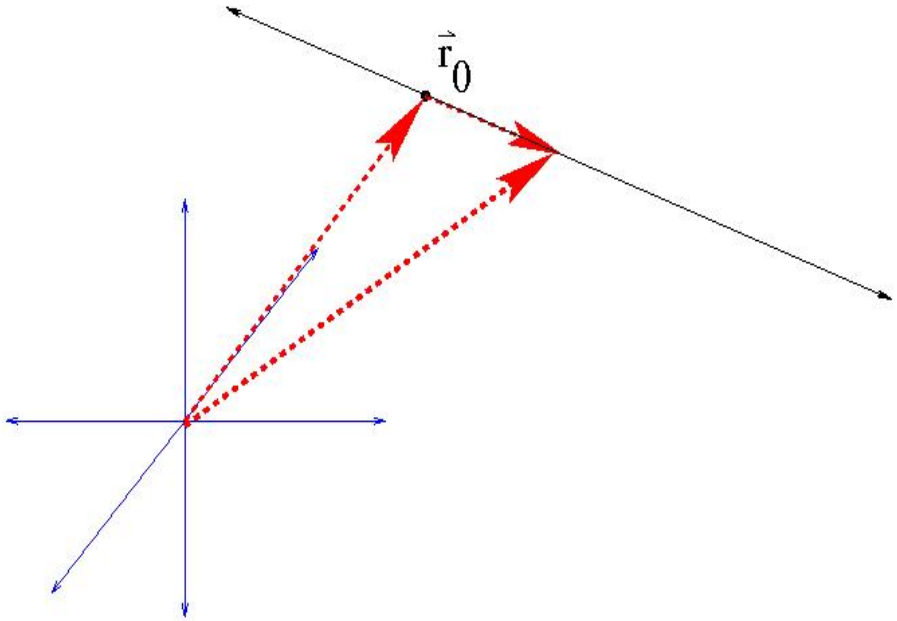
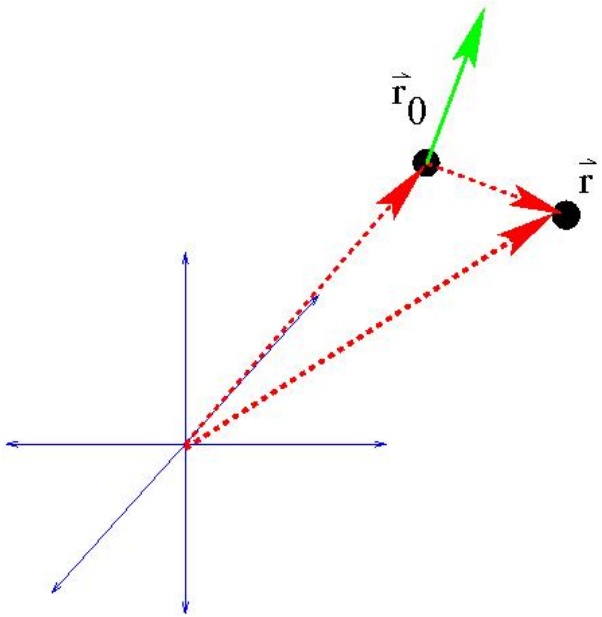


Geometric derivation of a plane



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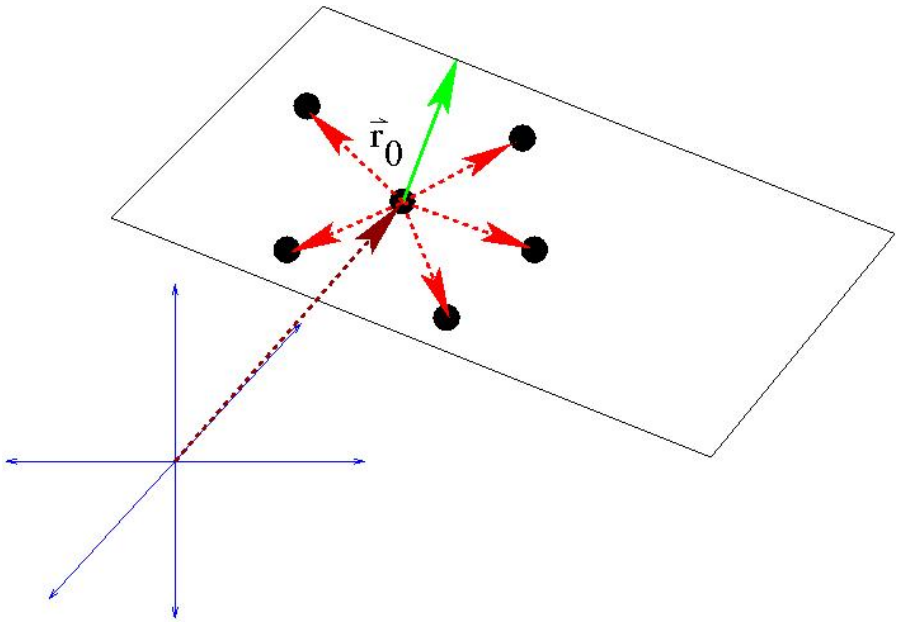

Outline

Problems to work on

From elementary geometry, there are many ways to characterize lines and planes. Generate an algorithm for each case to determine the vector equation of the line/plane using the given data.

- Lines
  1. Two points determine a line
  2. Two intersecting planes determine a line
- Planes
  1. Three non-colinear points determine a plane
  2. Two parallel lines determine a plane
  3. Two intersecting lines determine a plane
  4. A point and a line determine a plane

Summary

Lines and Planes!

Work for next class

- Reading: 14.1,14.2
- f07hw4