m33s06	Final	Exam Time: Fri 6/2, 8:00 - 11:00
Name:		Student No.:

Instructions:

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT seperate the pages of your exam.

Problem	Points	Score
A1	10	
A2	10	
A3	10	
A4	10	
A5	10	
A6	10	
A7	10	
A8	10	
Total	80	

Name:

Section A: Answer ALL questions.

Problem A1: [10 pts]

(**a**) Find a fundamental solution for the operator $D^2 + 4D + 6$.

(**b**) Suppose $|f(t)| \leq 1$ for all t and $f(t) = 0$ for $t < 0$. Show that the solution to

$$y'' + 2y' + 5y = f(t), \quad y(0) = 0, y'(0) = 0$$

has the property that $|y(t)| \leq \frac{1}{2\sqrt{2}}$ for all $t > 0$.

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Problem A2: [10 pts] Solve the diffusion equation

$$\begin{cases} u_t = 2u_{xx}, \quad 0 < x < \pi, t > 0 \\ u_x(0,t) = u_x(\pi,t) = 1, \quad t > 0 \\ u(x,0) = x + \sin^2 x, \quad 0 < x < \pi \end{cases}$$

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Problem A3: [10 pts]

Movement of food through the intestine is modelled by the initial-boundary value problem for $x, t \geq 0$

$$u_t + \frac{1}{2+\cos x} u_x = 0.$$

(**a**) Find and sketch the characteristic curves. Which characteristic curve crosses the corner at $(0,0)$?

(**b**) A meal is modelled by the boundary condition $u(0,t) = \chi_{[1,2]}(t)$. Suppose the intestine to be initially empty. At what time does food first arrive at the point $x = \pi$.

(**c**) Solve the equation explicitly with the intial/boundary conditions $u(x,0) = 1$, $u(0,t) = e^{-t}$.

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Problem A4: [10 pts]

(**a**) Suppose $u(x,t)$ is a solution to the fixed-end wave equation

$$\begin{cases} u_{tt} = 4u_{xx}, \quad x > 0 \\ u(x,0) = (x-3)\chi_{[3,4]}(x), \quad x > 0 \\ u_t(x,0) = 0, \quad x > 0 \\ u(0,t) = 0 \end{cases}$$

On different axes, sketch $u(x,0)$, $u(x,1)$, $u(x,2)$ and $u(x,3)$.

(**b**) Repeat (a) with the boundary condition replaced by $u_x(0,t) = 0$.

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Problem A5: [10 pts] Solve the advection-diffusion equation

$$\begin{cases} u_t + a u_x - c u_{xx} = 0 \\ u(x,0) = \cos x \end{cases}$$

with $c > 0$. Sketch a few wave profiles for increasing values of t.

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Problem A6: [10 pts] Suppose Ω is a domain in the x,y-plane. Show that if u solves

$$\begin{cases} \triangle u = 0 \quad \text{in } \Omega \\ \vec{n} \cdot \nabla u \quad \text{on } \partial\Omega \end{cases}$$

where $\vec{n}$ is the outward point unit normal to Ω, then u must be a constant.

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Problem A7: [10 pts] Solve the following cooling problem on a spherical shell with fixed boundary temperatures

$$\begin{cases} u_t = k\nabla u \\ u(1,t) = 1 \\ u(2,t) = 1/2 \\ u(\rho,0) = 0 \end{cases}$$

Hint: for functions $u = u(\rho)$ that depend only on the radial variable, $\nabla u = u_{\rho\rho} + \frac{2}{\rho} u_\rho$.

Name:

Problem A8: [10 pts] For which values of c is there a positive valued wave-front solution $u = u(x - ct)$ to

$$u_t = ku_{xx} - \cos(u)u_x$$

with $k > 0$ such that $u(-\infty) = 0$ and $u(+\infty)$ is bounded? What range of values is possible for $u(+\infty)$?