1. (20) (**Show all work**). Find the solution to the boundary value problem:
$\frac{dy}{dx} - 3x^2y = x^2$, $y(0) = 1/3$.

2. (20) (**Show all work**). Find all points on the surface given by $3z = x^2 + xy$ where the normal line to the surface is parallel to the line given by: $x = 2t + 1$, $y = 4t - 1$, $z = -3t + 7$.

3. (20) (**Show all work**). The temperature at a point (x, y, z) is given by the function $T(x, y, z) = 200e^{x^2-4y^2-9z^2}$, measured in degrees Celsius. Find the rate of change of the temperature at the point $(2, 1, 0)$ in the direction from the point $(2, 1, 0)$ toward the point $(3, 2, 5)$.

4. (20) (**Show all work**). Find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

5. (70) **Multiple Choice** Circle the correct response. (No partial credit will be given)

   (a) $\boxed{\frac{1}{3+4i} =}$

   A. $\frac{1}{3} + \frac{1}{4}i$ **B.** $\frac{1}{3} - \frac{1}{4}i$ **C.** $\frac{3}{25} - \frac{4}{25}i$ **D.** $\frac{3}{5} - \frac{4}{5}i$

   E. none of the above

   (b) What is the arclength of the piece of the parabola $y = x^2$ from $(0, 0)$ to $(2, 4)$?

   A. $\int_0^4 (1 + 2t)\, dt$ **B.** $\int_0^4 \sqrt{1 + 4t^2}\, dt$ **C.** $\int_0^2 \sqrt{t^2 + t^4}\, dt$

   D. $\int_0^2 \sqrt{1 + 4t^2}\, dt$ **E.** none of the above

   (c) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-5)^{2n}}{n^3 3^n}$ is

   A. 1 **B.** 3 **C.** 5 **D.** 3/5 **E.** none of the above

(d) It is well-known by Math 8 students that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$. If we truncate this series after some point and use the partial sum as an approximation for $\pi/4$, what is the least number of terms needed in the partial sum so that the error is less than .01?

A. 49 **B**. 50 **C**. 98 **D**. 99 **E**. none of the above

(e) $\left(\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}\right)\left(\sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!}\right)$ is a solution to which differential equation?

A. $y'' + 5y' + 6y = 0$ **B**. $y'' - 5y' + 6y = 0$

C. $y'' + 4y' + 13y = 0$ **D**. $y'' - 4y' + 13y = 0$

E. none of the above

(f) Let $f(x, y) = |\langle x, y\rangle|$. What is the largest set on which f is continuous?

A. all of $\mathbb{R}^2$ **B**. all of $\mathbb{R}^2$ except the axes

C. all of $\mathbb{R}^2$ except the origin **D**. all of $\mathbb{R}^2$ except the x-axis

E. none of the above

(g) If $f(x, y) = \int_y^x \cos(t^3)\, dt$, then $\frac{\partial f}{\partial y} =$

A. $\cos(y^3)$ **B**. $3y^2 \cos(y^3)$ **C**. $\sin(y^3)$

D. $-3y^2 \sin(y^3)$ **E**. none of the above

(h) Suppose that you are given a function $f(x, y)$ and vectors $\mathbf{u} = \langle \frac{1}{2}, \frac{-\sqrt{3}}{2}\rangle$ and $\mathbf{v} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2}\rangle$. If $(D_{\mathbf{u}}f)(x_0, y_0) = 2$ and $(D_{\mathbf{v}}f)(x_0, y_0) = -1$, then $\frac{\partial f}{\partial x}(x_0, y_0) =$

A. $\frac{1}{2}$ **B.** 1 **C.** 2 **D.** $\sqrt{3}$ **E.** none of the above

(i) Suppose that f is a function of the variables x, y, and z, and that x is a function of the variables s and t, y is a function only of s and z is a function only of the variable t. What is the correct expression for $\dfrac{\partial f}{\partial t}$?

A. $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z}$ **B.** $\left(\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z}\right)\dfrac{\partial f}{\partial t}$

C. $\dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial s} + \dfrac{\partial f}{\partial z}\dfrac{\partial z}{\partial t}$ **D.** $\dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial z}\dfrac{\partial z}{\partial t}$

E. none of the above

(j) The altitude of a right circular cylinder is 8 inches and is increasing at a rate of .8 in/min. The base radius is 10 inches and is decreasing at a rate of .05 in/min. The volume ($V = \pi r^2 h$) is changing at the rate of

A. 72π **B.** -72π **C.** 88π **D.** -88π **E.** 800π

(k) Let $f(x, y, z) = xy + yz^2 + xz^3$ and $\mathbf{v} = \langle -2, 0, 1\rangle$. What can be said about the change in f at the point $(2, 0, 3)$ in the direction indicated by $\mathbf{v}$?

A. strictly increasing **B.** strictly decreasing

C. unchanging **D.** first increasing, then decreasing

E. impossible to be determined

(l) Suppose that $f(x, y)$ has continuous second partial derivatives and a critical point at $(2, 3)$. Suppose further that $f_{xx}(2, 3) = -3$, $f_{xy}(2, 3) = 2$, and $f_{yy}(2, 3) = -4$. Classify the critical point.

A. local minimum **B.** local maximum **C.** saddle point

D. local extremum, but can't determine which **E.** cannot be determined

(m) Suppose that there is a function $f(x, y)$ having continuous first partial derivatives with $f_x = (x^2 - 4)(y - 3)$ and $f_y = (y^2 - 9)(x - 2)$. How many critical points does f have in $\mathbb{R}^2$?

A. 2 **B**. 4 **C**. 5 **D**. 6 **E**. infinitely many

(n) Suppose that the graph of $z = f(x, y)$ represents the surface of a mountain, and you are standing at a point (x_0, y_0, z_0) on the surface. You are told that the gradient of f at (x_0, y_0) is $\nabla f(x_0, y_0) = \langle 1, 3\rangle$. If you move in the direction of the gradient, what is your initial angle of elevation?

A. $\tan^{-1} 3$ **B**. $\cos^{-1} 3$ **C**. $\tan^{-1} \sqrt{10}$ **D**. $\cos^{-1} \sqrt{10}$

E. none of the above