"1st & 2nd partial derivatives worksheet.
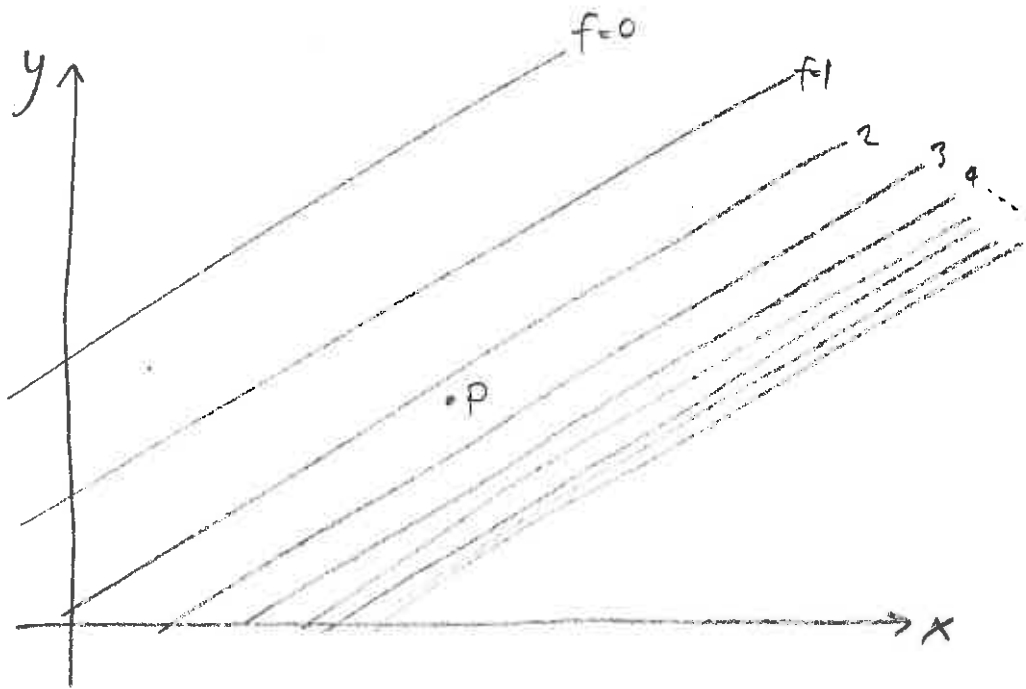
Contour lines of $f(x,y)$ shown. $P = (a,b)$

Are the following, evaluated at point P, <0, $=0$ or >0 ?

f_x
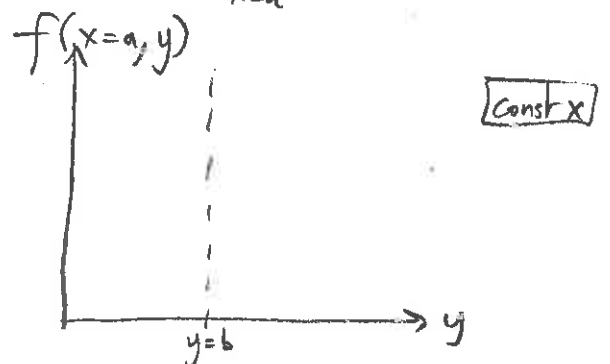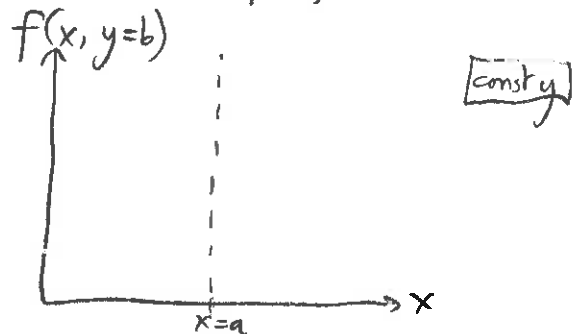
f_y

f_{xx}

f_{yy}

f_{xy}

Hint:
$\frac{\partial}{\partial y}(f_x)$

Hint: plot the 'traces' (slices)
of $z = f(x,y)$ below:

$f(x, y=b)$ const y

$f(x=a, y)$ const x

"1st & 2nd partial derivatives worksheet.

~~→ SOLUTIONS ←~~

shallow

$f=0$

$f=1$

2   3   4   steep

shape is a "half-pipe" at an angle.

f_b is smaller

P f_a

$y=b$ slice (const y)

x

Contour lines of $f(x,y)$ shown.

$P = (a, b)$
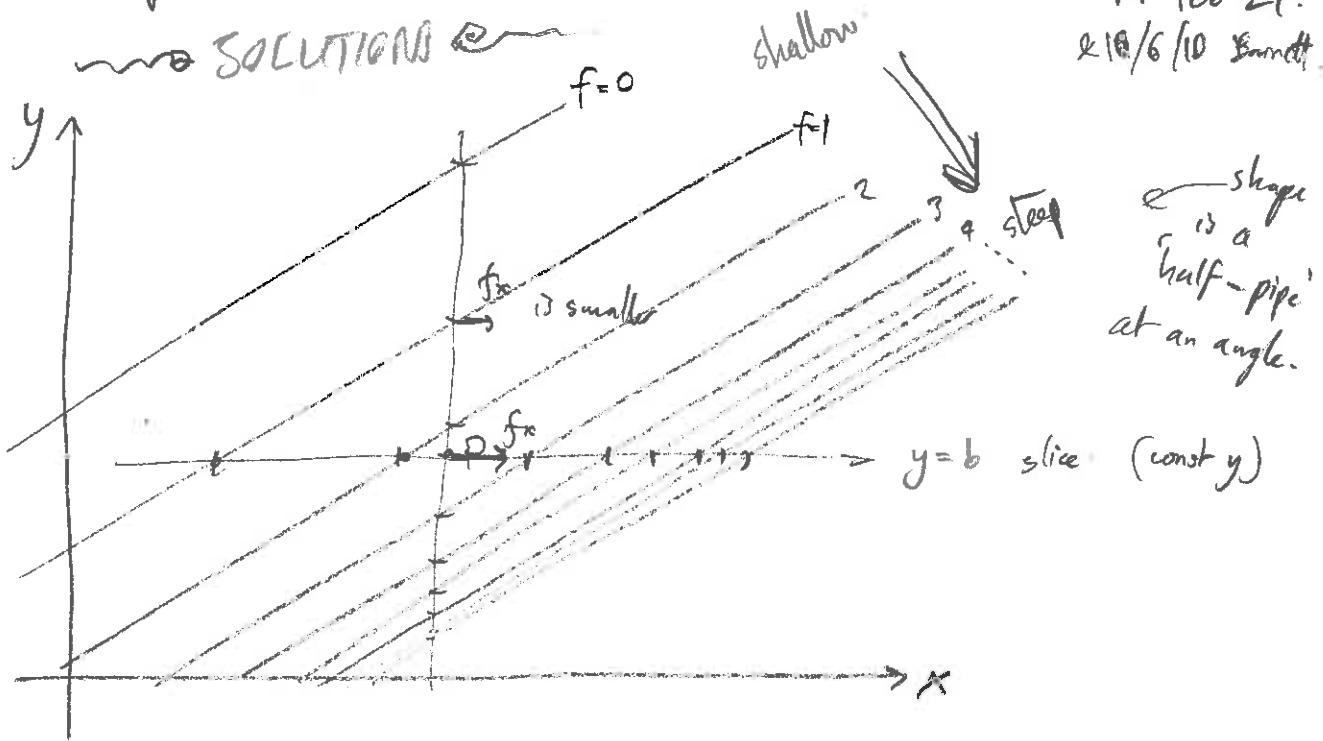
Are the following, evaluated at point P, <0, $=0$ or >0?

$f_x \quad > 0$

$f_y \quad < 0$

ask: "how does f change when go in x or y directions?"

$f_{xx} \quad > 0$

slice is concave up

$f_{yy} \quad > 0$

slice is also concave up!

$\boxed{f_{xy} \quad < 0}$

Hint: plot the 'traces' (slices) of $z = f(x,y)$ below:

$f(x, y=b)$

const y

$x=a$ x

note: positive 2nd deriv.

const x

$f(x=a, y)$

$y=b$ y
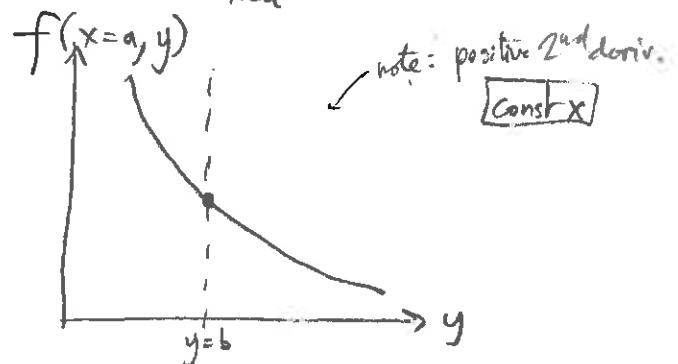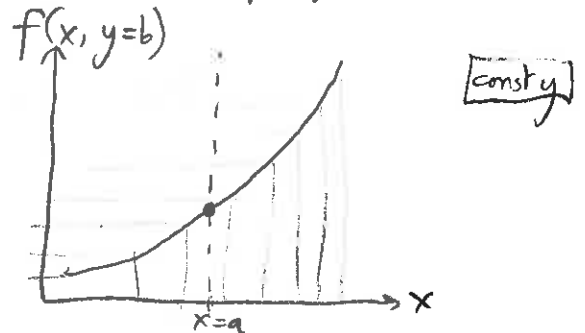
Hint:
$\frac{\partial}{\partial y}(f_x)$

harder:
so since f_x is smaller when increase $y=b$, its rate of change is <0, $f_{xy} < 0$