Math 12, Fall 2007
Lecture 7

Scott Pauls [1]

[1] Department of Mathematics
Dartmouth College

10/11/07

Outline

Outline

Functions of more than one variable

- Functions of more than one variable: e.g. $x = f(x, y)$
- Contour plots
- Sketching graphs
- Limits
- Continuity

Example from last class

Find the limit, if it exists, or show that there is no limit:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{\sqrt{x^2+y^2}}$$

Outline

Concepts from reading

Differentiation

- Derivatives of functions of one variable: rate of change

$$f(x) = x^2$$

- Derivatives of spacecurves: rate of change plus direction

$$< x, x^2, 0 >$$

- Rates of change on a surface

Concepts from reading

Differentiation

- Derivatives of functions of one variable: rate of change

$$f(x) = x^2$$

- Derivatives of spacecurves: rate of change plus direction

$$< x, x^2, 0 >$$

- Rates of change on a surface

Concepts from reading

Differentiation

- Derivatives of functions of one variable: rate of change

$$f(x) = x^2$$

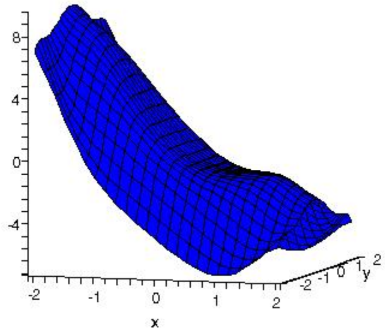
- Derivatives of spacecurves: rate of change plus direction

$$< x, x^2, 0 >$$

- Rates of change on a surface

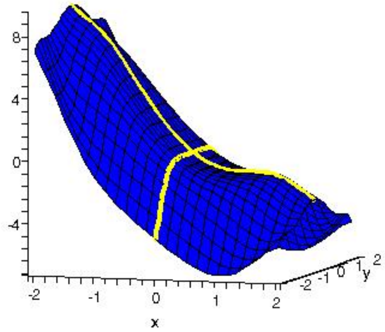
Concepts from reading

Derivatives of $f(x, y)$



Concepts from reading

Derivatives of $f(x, y)$



Concepts from reading

Derivatives of $f(x, y)$

Since there seem to be multiple derivatives (one for each direction), calculate them separately.
Method:

1. Pick a direction in $\mathbb{R}^2$, $\vec{v}$ and a base point $P = (x_0, y_0)$
2. Construct a line in the plane throught P in the direction of $\vec{v}$:
$$P + t\vec{v} = < x_0 + tv_1, y_0 + tv_2 >$$
3. Lift the line to a curve on the surface using $f(x, y)$:
$$< x_0 + tv_1, y_0 + tv_2, f(x_0 + tv_1, y_0 + tv_2) >$$
4. The derivative of this curve is
$$\left\langle v_1, v_2, \frac{d}{dt} f(x_0 + tv_1, y_0 + tv_2) \right\rangle$$

Concepts from reading
Directional derivative

Definition: the directional derivative of $f(x, y)$ at $P = (x_0, y_0)$ in the direction $\vec{v}$ is

$$D_v f(x_0, y_0) = \frac{d}{dt}\Big|_{t=0} f(x_0 + tv_1, y_0 + tv_2)$$

Concepts from reading
Partial derivatives

Special directions:

$$\frac{\partial f}{\partial x} := f_x := D_1 f := D_{<1,0>} f$$

$$\frac{\partial f}{\partial y} := f_y := D_2 f := D_{<0,1>} f$$

Some computation

Find $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$ for

- $f(x, y) = x^2 + y^2$
- $f(x, y) = x^2 - y^2$
- $f(x, y) = \sin(xy)$

Group work

Questions:

- Compute $D_v f$ in terms of f_x, f_y? Is there a general rule?
- What is the geometric meaning of f_{xx}? f_{yy}? f_{xy}?

Summary

- Directional derivatives
- Partial derivatives

Work for next class

- Reading: 15.4
- 15.4 # 1-3, 11-13
- f07hw8