Can a violently nonisotropic space be useful, interesting and beautiful?

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Abstract

Throughout this century the model of space provided by a riemannian manifold has dominated a large part of the literature on geometrical analysis. One distinguishing characteristic of such spaces is that they are, qualitatively speaking, isotropic. That is to say, if there are local coordinates $x_1, x_2, ... x_n$ then the distance is locally approximated by a square root of the sum of squares, with suitable weights. Models for spaces that are not isotropic in this sense have received much much less attention. However, questions arising in control theory, the study of diffusion processes and, in less specific way, small scale physics, have suggested that there are compelling reasons to move beyond qualitatively isotropic models toward the investigation of spaces in which different directions have qualitatively different metric properties. Perhaps the best developed line of work in this direction goes under the name of subriemannian geometry. Existing work already shows this subject to be both tractable and mathematically rich. In this talk I will introduce some of the main ideas and then will go on to discuss a set of questions that take the subject in a new direction, related to problems of current interest in control theory and classical mechanics.

This talk should be accessible to undergraduates.