Math 46 Solutions Sor homework Day 18 problems Exercise 4 page 267 TECO(a,b) is a test function (a) Is it true that if $\psi_n = \frac{1}{n} \varphi(x)$ then $\frac{1}{n} = \frac{1}{n} \Phi^{(m)}(x)$ Since PECO(R) Ym 3 Km s.t. 19(m)(x)'-0/< km 4xeR since qual is continuous and because it has bounded support | P(m)(x) | attains the maximal value and we can take kun to be ON B twice thevalue 140m)(x) 0/< 1/2 Km 4xclR Thus goes to zero unformly => (4m)-0) also go to zero uniformly =) 4, ~ o indeed

~ C ~ (LP)

(P) A~ (X)= 7 4(X)

(page2

if the support of q is contained in [-R, R] then the support of yours contained in [-nR, nR] thus there is no bounded segment that contains the support of all yn simultaneously

=> 4, +0

(c) 4~(x)= t + (nx)

Let us look at

4, (0) = + n q'(n0) = q'(0)

Thus there is no reason

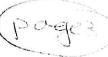
and thus why 4, (0) -30

it is not true that 4'n(x) -to

=> it is not true that

4~ → 0 in 0° (IR)

Exercise 6 page 268



Show that u(x, 2) = = [1x-31 is a fundamental solution for r= 9/5 Lu= dzy in the distributional sense. Thus we have to check that in the distributional sense. Lin= S(x-3) i.e $\left(\left(\frac{1}{2}\left(x-3\right)\right), \varphi(x)\right) \stackrel{?}{=} \left(S(x-3), \varphi(x)\right)$ of \frac{1}{2} \fr derivative $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3-x) \varphi''(x) dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x-3) \varphi''(x) dx$ $=\lim_{3\to -\infty} \frac{1}{2} (3-x) \varphi'(x)$ $= \lim_{3\to -\infty} \frac{1}{2} (3-x) \varphi'(x)$ $= \lim_{3\to -\infty} \frac{1}{2} (3-x) \varphi'(x)$ a test function - 5 = (-1) + (x) dx + - 1 im = (x-3) + (x)] - 5 = 4 (x) dx 1 - 300 = (x-3) + (x) dx = 3 = 4 (x) dx

serosuce de coo(B)

=
$$\frac{1}{2} \int_{-\infty}^{\infty} f'(x) dx = \frac{1}{2} \int_{-\infty}^{\infty} f'(x) dx$$

= $\lim_{x \to -\infty} \frac{1}{2} \int_{-\infty}^{\infty} f'(x) dx$
 $\lim_{x \to -\infty} f'(x) dx$
 \lim_{x

Exercise 3 page 3.45



Find a sormula for the solution to the equation

 $u_{xt} = f(x,t) \qquad x,t > 0$ that satisfies $u(x,0) = g(x) \times 0$ u(0,t) = h(t) + 0

where f, y, h are given nice functions satisfying g(0)=h(0)g'(0)=h'(0)

so that the conditions match

n(x,f) = 2 2 2 (x,7) dyd3+ A(x)+B(+)

 $u(x,0) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} f(x,j) dy dj + A(x) + B(0)$ g(x)

 $V(t) = \sum_{t=0}^{t} \sum_{t=0}^{\infty} f(x, \pm 1) dy d \pm (0) + B(4)$

=

$$g(x) = A(x) + B(0)$$
 $h(t) = A(0) + B(0)$
 $g(c) = A(0) + B(0)$
 $h(0) = A(0) + B(0)$
 $g'(0) = A'(0)$
 $h'(0) = B'(0)$
 $h'(0) = B'(0)$

So for example one can take $A(x) = g(x) - g(0)$
 $B(t) = h(t)$.

Then $g(x) = (g(x) - g(0)) + h(0)$
 $A(x) = (g(0) - g(0)) + h(1)$
 $A(c)$

Thus the solution is

 $u(x,t) = \sum_{i=1}^{n} f(x_i) dy dx + (g(x) - g(a)) + h(t)$

(page 7 Exercise 2 a page 345 u=u(x,y) bind the general Solution of Uxx+U= Gy sulution to the A particular equation is (x,y) = 0the general solution Let us find to the homogeneous equation Fix tet then we get U *x + U = 0 3 ~ (x,x) + (x,x) = 0 Thus we get $g_{\chi}^{\prime}(x) + g_{\chi}(x) = 0$ As we know brown gy (x) = Accesx + Besinx we can any hunchons Thus H(A) COSX + B (A) SINT n(x, 4)= to be the general solution of the homogeneous problem The total solution $u(x,y) = A(y)\cos(x) + B(y)\sin t + Gy$

Exercise 2 parth u(x,+) tuxx - 4 ux = 0 u=u(x,t) Put V=Wx tvx - 40 = 0 tix t=t 7 3 V(x, t) = c put 97(x)=V(x,7) => it gr (x) - 4 gr(x) => + 97(x)- 92(x)=E Soon ODE we unow that gz(x) = Ae. depends on T 4 x Thus v(x,t) = A(t) e V= Ux and a similar trick tells us that $U(x,t) = \frac{t}{4}A(t)e^{\frac{t}{4}x} + B(t)$

for arbitrary vice bune work