

Math 11, Fall 2007

Lecture 1

Scott Pauls

¹Department of Mathematics
Dartmouth College

9/26/07

Outline

- 1 Introduction to Math 11
- 2 Today's material
 - Describing objects in \mathbb{R}^3
 - Vectors
- 3 Further discussion
 - Examples
 - Group Work
- 4 Summary
- 5 Next class

Instructor information

Scott Pauls

Office: 303 Kemeny Hall

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Course Information

- Meeting time: MWF 12, 006 Kemeny Hall
- Book: Stewart, Calculus

Exams:

- 1 Midterm I: 10/22/07 4-6 pm Kemeny 007/008
- 2 Midterm II: 11/12/07 4-6 pm Kemeny 007/008
- 3 Final Exam: 12/07/07 11:30-2:30

Homework

- Daily reading assignments and homework problems
- Regular homework (daily) via webwork
- Written homework assigned daily and due weekly.
- Tutorial sessions: Sun, Tues, Thurs 7-9pm, location 008 Kemeny

Grades:

- Homework: 25 points
- Quizzes: 25 points
- Midterms: 100 each
- Final: 150

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Coordinates in three space

Subtitles are optional.

- Use (x, y, z) to describe points in space
- Coordinate planes: $x = \text{const}$, $y = \text{const}$, $z = \text{const}$
- Other planes: a linear relation between variables
- Spheres: constant distance from a central point

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Surfaces in \mathbb{R}^3

planes

- A line in the plane, \mathbb{R}^2 , is a linear relation between variables:

$$t = ms + b, \text{ or } t - ms - b = 0 \text{ or } \alpha t + \beta s + \gamma = 0$$

- A plane is a linear relation between variables in \mathbb{R}^3 :

$$\alpha x + \beta y + \gamma z + \delta = 0$$

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Surfaces in \mathbb{R}^3

Spheres

- A sphere is the collection of all points located a fixed distance, r , from a given point $P_0 = (x_0, y_0, z_0)$.
- Distance from $P_1 = (x_1, y_1, z_1)$ to P_0 is

$$\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

- Sphere:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

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Points vs. vectors

- Vectors are quantities that have both *magnitude* and *direction*
- Points specify location while vectors specify direction
- We often confuse points and vectors (intentionally) but, for clarity, we use different notation:

$$(x, y, z) = \text{point}, \quad \langle x, y, z \rangle = \text{vector}$$

- *Magnitude of a vector*: distance from the tip to the origin:

$$|\langle x, y, z \rangle| = \sqrt{x^2 + y^2 + z^2}$$

- *Direction of a vector*: if $\vec{u} = \langle x, y, z \rangle$, the direction of \vec{u} is

$$\frac{\vec{u}}{|\vec{u}|}$$

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Vector operations

- $\vec{u} = \langle a, b, c \rangle$, $\vec{v} = \langle d, e, f \rangle$ and α, β are real numbers:

$$\alpha \vec{u} + \beta \vec{v} = \langle \alpha a + \beta d, \alpha b + \beta e, \alpha c + \beta f \rangle$$

- This can also be seen geometrically.
- Basis vectors:

$$\vec{i} = \langle 1, 0, 0 \rangle, \vec{j} = \langle 0, 1, 0 \rangle, \vec{k} = \langle 0, 0, 1 \rangle$$

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Example

Suppose two points $(1, 2, 3)$ and $(0, 2, -2)$ are antipodal points on a sphere. Find an equation for the sphere.

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Problems to work on

Projections:

- $P_{xy}((x, y, z)) = (x, y, 0)$
- $P_{yz}((x, y, z)) = (0, y, z)$
- $P_{xz}((x, y, z)) = (x, 0, z)$

Question:

Let $S = \{(x, y, z) | x^2 + y^2 + z^2 = 1\}$. What is $P_{xy}(S)$?

Summary

- Coordinates in \mathbb{R}^3 and describing geometric objects
- Vectors: numeric and geometric
- Operations

Work for next class

- Reading: review 13.1-13.2, read 13.3,13.4
- Homework set: f07hw1, f07hw2