# Math 46: Applied Math: Homework 8

due Wed May 25

**p.345-346**: #6.

#2. a. [Hint: get the general solution with $y$ held const]

d. [If you're ever unsure you have the right solution, substitute back into the PDE to check it works!]

e.

#3. You'll need to think how to satisfy the BC and IC, check it does. [Hint: subtract something].

#1. Note this is 1D equivalent of the heat spreading function you studied in 3D in the early dimensional analysis worksheet.

A) Write the integral solution of the 1D heat equation with the IC $f(x) = \sin \mu x$ for a constant 'wavenumber' $\mu$, and change variable to show that $u(x,t) = T(t) \sin \mu x$, where $T(t)$ is some nasty integral (make sure it's independent of $x$). Thus $f(x)$ is an eigenfunction of the integral operator; this reminds you of blurring (convolving) an image, which is exactly what the heat equation does! Finally compute this integral by a trick: stick the above form for $u$ into the PDE to solve for $T(t)$, as in separation of variables from Math 23.

**p.365-367**: #3.

#5. Here you derive that the radial part of the laplace operator in 3D cylindrical (or 2D polar) coordinates is $\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}\cdot)$

#11. Note that $z$ is the only dimensionless parameter you can make from $x$, $k$ and $t$. The situation is sticking an initially uniform-temperature rod against a hot oven at constant temperature; also it gives the probability of having hit the left wall in a random walk (see 6.2.4 for random walk connection).

#13. Cute that energy method can work for some non-linear PDEs too.

**p.371-374**: #5. easy if you look up the radial part of the 3D Laplacian operator

#6. Adapt the method from 1D. In fact $-\Delta$ is a 'positive operator'. Note the $\lambda$ values would be eigenvalues of the Laplacian.

**p.381-382**: #3. b. Use the result from a, which states that the $L$ given is *self-adjoint* when certain BCs are imposed. [Hint: see proof we did for Fredholm operators (or, even, symmetric matrices), and it should not be hard].