Math 31 Lesson Plan

Day 3: Symmetry Groups

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Supplies needed:

- Quizzes!
- Labeled square and triangle (Patricia)
- 10 squares of tagboard.
- 30 copies of Gallian, p. 36-37
- Scratch paper?
- Sign in sheet

Goals for students: Students will:

- Begin to get an intuitive sense for the concept of a group.

[**Lecture Notes: Write everything in blue, and every equation, on the board. [Square brackets] indicate anticipated student responses. *Italics* are instructions to myself.**]

pass around sign-in sheet

Quiz! 5 mins

My Office hours this term will be: Weds 1:45-3:30; Thurs 11-12:30. You can also come meet with me after class any day of the week; and if none of these times works for you you can set up an appointment with me for some other time.

Please remember to meet with me in office hours this week! I'd like to get to know you, and I need to find out enough about your interests that I can start to separate you into groups for the presentations.

About the Homework:

- Did everyone find it on the website?
- I had to change Problem 8 this weekend, so please make sure you do the new Problem 8, not the old one.

I also posted my lecture notes for class on Friday on the course website. I'll try to do that in general, so that you have something to check your class notes against.

We're going to spend today introducing the idea of a group in a different way than your textbook does. I think that symmetry groups are the reason why group theory has so many applications to other arts and sciences, and they can give you an intuitive sense for what a group is, which will help when you're looking at the abstract definition and more mathematical examples. So we're going to talk about a particular example of a symmetry group today – the symmetry group of a square.

show square A square is very symmetric, right? What do we mean by that? *Think-Pair-Share?* [a square has lots of reflection lines] Basically, there are lots of ways to pick up the square and put it back in the same place, but in a different position. Does that make sense? Do I want to use a worksheet for this material?

The next question is, How many ways? That is, how many different positions are there for the square? *Trace around Patricia's square on board; label board corners a, b, c, d.* We can identify a position of the square via writing down where the corners are. For example, this position right now would be Position 1: 1a2b3c4d, and if I flip the square over this diagonal, I get Position 2: 1a4b3c2d. 5-10 mins

Ask students to work in groups of 3. Give each group an index-card square. Please label the corners of your square the way I labeled mine, and trace around it the way I did with mine, so that we're all using the same notation. That way it's easier to understand each other.

- Do the labels (1a2b3c4d etc) give us enough information to tell any two positions of the square apart?
- Does every pairing of the set $\{1, 2, 3, 4\}$ with the set $\{a, b, c, d\}$ correspond to a position of the square? Why or why not?

Please work on answering these questions in your group. 10 mins

Check answers to the questions as a class. List all possible positions on the board.

We can associate each position with the move we performed on the square to get it from Position 1, 1a2b3c4d, to the position in question. For example, what move does Position 2 correspond to? [Diagonal flip] *Write* No move *next to Position 1, and* Diagonal flip *next to Position 2.*

Next to each position, write down a (sequence of) moves to get to it from Position 1. Can you get to each position in 1 move? Can you get back to Position 1 from any other position in 1 move? 5-10 mins

So, each position has a "minimal" move that will get you there. What happens if you compose/combine two moves? Can you think of a binary operation on the set of "symmetry-preserving moves of the square"? Think about this in your groups for 2 minutes.

Agree on labels for each move. Write the Cayley table on the board; ask for input for completing it. This is called the Cayley table of a binary operation. *Emphasize that when we write $A * B$ we mean "Do B, then do A."* 8 mins

What's special about R_0? (two things) Think about this in your groups for a few minutes.

5 mins

For any move M, $R_0 * M = M * R_0 = M$. Also, for any move M, there is another move M^{-1} such that $M * M^{-1} = M^{-1} * M = R_0$.

The set of "symmetry preserving moves of the square," with this binary operation of move composition, is an example of a group. This group is called the dihedral group of order 8 or the symmetry group of the square and it's often abbreviated D_4. Any ideas why it's called D_4 and not D_8? *Think-pair-share* [This group is called D_4 because it's the symmetry group of a regular 4-sided polygon.] We can also have symmetry groups of regular n-sided polygons for any n – you'll get to explore this a little in the homework.

Any questions about D_4? *hand out copies from Gallian* This is optional reading, from our supplementary textbook Contemporary Abstract Algebra by J. Gallian. It talks a little more about applications of the dihedral groups to chemistry, and about other sorts of symmetry groups.

Now that we have D_4 in mind as an example, let's look at the formal definition of a group.

DEFINITION: Let $(G, *)$ be a set with an associative binary operation. We say that $(G, *)$ is a group if:

1. G has an element e that satisfies $e * x = x * e = x$ for all $x \in G$; and
2. For any element $x \in G$, there exists an element $y \in G$ such that $x * y = y * x = e$.

Note that in (1), the same element e has to work for all x; but in (2), the element y can be different for different x. We call e an identity element for the group, and we call y an inverse for x.

Who can tell me why D_4 is a group?

- We checked earlier that move composition is a binary operation on D_4.
- Move composition is like function composition, which we know is associative:

$$f \circ (g \circ h) = (f \circ g) \circ h.$$

- R_0 is the identity element.
- We checked earlier that every element has an inverse. For example, $R_{90} * R_{90} = R_0$, so R_{90} is its own inverse.

There are a lot more examples of groups in Section 2 of the textbook, which I'd like you to read for class on Wednesday. But for now, I'm going to jump ahead to Section 3 and prove a couple of important theorems about groups.

THEOREM 3.1: In a group $(G, *)$, there is only one identity element.

Ask students to prove it?

THEOREM 3.2: If x is an element of a group $(G, *)$, then there is only one inverse $y \in G$ for x.

The easiest way to prove either of these is proof by contradiction.

Read Sections 2 and 3. Post a comment by Tuesday at 10 PM. homework