

Math 12, Fall 2007

Lecture 14

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10/29/07

Outline

- 1 Review and overview
 - Last class
- 2 Today's material
 - Integration in two variables
- 3 Group Work
- 4 Next class

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Integration of a function of two variables

- Integration measure the volume under a portion of surface $z = f(x, y)$.
- The definite integral is defined via Riemann sums analogously to the one variable case
- Evaluation of an integral via the definition can be difficult.

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Methods of integration

- In one variable calculus, the difficulty of evaluating integrals is overcome using the Fundamental Theorem of Calculus
- We can use the FTC in one variable by breaking up the two variable integral into two one variable integrals via Fubini's Theorem.

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Fubini's Theorem

Given $f(x, y)$, continuous on a rectangle $R = [a, b] \times [c, d]$,

$$\int \int_R f(x, y) \, dA = \int_a^b \int_c^d f(x, y) \, dy \, dx = \int_c^d \int_a^b f(x, y) \, dx \, dy$$

This same theorem holds more generally: if f is bounded on R , f is discontinuous only on a finite number of smooth curves, and the iterated integrals converge.

Example

$$f(x, y) = x^2 + y^2, R = [0, 1] \times [0, 1]$$

$$\begin{aligned} \int \int_R f(x, y) \, dA &= \int_0^1 \int_0^1 (x^2 + y^2) \, dx \, dy \\ &= \int_0^1 \left(\frac{x^3}{3} + y^2 x \right) \Big|_{x=0}^{x=1} dy \\ &= \int_0^1 \left(\frac{1}{3} + y^2 \right) dy \\ &= \left(\frac{y}{3} + \frac{y^3}{3} \right) \Big|_0^1 \\ &= \frac{2}{3} \end{aligned}$$

Examples

- 1 $f(x, y) = x + \sqrt{y}$, $R = [1, 4] \times [0, 2]$
- 2 $f(x, y) = (x + y)^{-2}$, $R = [1, 2] \times [0, 1]$
- 3 Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin(y)$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$, $z = 0$.

Work for next class

- Reading: 16.3
- f07hw14