Math 14 Fall 2005

Multivariable Calculus–Honors

Second Midterm Exam

Monday February 21, 6-8 PM
Bradley 102

Your name (please print): ________________________________

Instructor Vladimir Chernov.

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You must justify all of your answers to receive credit, unless instructed otherwise in a given problem.

You have two hours to work on all **11** problems. The total score is the sum of your **10** best scores. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only

1. ____________ /10
2. ____________ /10
3. ____________ /10
4. ____________ /10
5. ____________ /10
6. ____________ /10
7. ____________ /10
8. ____________ /10
9. ____________ /10
10. ____________ /10
11. ____________ /10

Total: ________ /100

(1) **Prove** that $\operatorname{div}(\nabla \times F) = 0$ for every C^2 vector field $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$, with $F(x, y, z) = \big(F_1(x, y, z), F_2(x, y, z), F_3(x, y, z)\big)$.

(2) Let $c : [a, b] \to \mathbb{R}^4$ be a differentiable path, and let $f : \mathbb{R}^4 \to \mathbb{R}$ be a differentiable function. **Prove** that $\int_c \nabla f \cdot d\mathbf{s} = f(c(b)) - f(c(a))$.

(3) Let $\alpha : [a, b] \to \mathbb{R}^4$ be a differentiable path, and let $f : \mathbb{R}^4 \to \mathbb{R}$ be a differentiable function. Let $h : [c, d] \to [a, b]$ be a differentiable decreasing bijection. Let $\beta = \alpha \circ h$ be an orientation reversing parameterization of α. Express $\int_\beta f ds$ through $\int_\alpha f ds$ and **prove** your answer.

(4) Compute the following integral $\int_0^1 \int_{\sqrt{y}}^1 \frac{\sin(x^2)}{x} dx dy$

(5) Find the center of mass of a cylinder $\left\{(x,y,z)\middle|x^2+y^2\leq 1, 0\leq z\leq 1\right\}$ with density $\delta(x,y,z)=z$.

(6) Find the surface area of the part of the sphere of radius **three** centered at the origin that is located outside of the cone $z^2 = x^2 + y^2$. (That is you have to find the area of the part of the sphere formed by points $\left\{(x, y, z)\middle| -\sqrt{x^2+y^2} \leq z \leq \sqrt{x^2+y^2} \text{ and } x^2+y^2+z^2 = 3^2\right\}$.)

(7) Rewrite the triple integral

$$\int_{\frac{1}{2}}^{1} \int_{0}^{\sqrt{1-z^2}} \int_{0}^{\sqrt{1-x^2-z^2}} f(x,y,z)dydxdz$$

as

$$\int \int \int f(x,y,z)dzdydx$$

with the appropriately chosen limits of integration.

(8) Compute the integral $\int_{\mathbf{c}} y \sin z dx + x \sin z dy + xy \cos z dz$ for the path $\mathbf{c}(t) = (\sqrt{t}, \frac{t}{2}, e^t)$, $0 \leq t \leq 4$. (Hint: think before integrating.)

(9) Find the volume of the shape enclosed between the paraboloids $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$.

(10) Compute the **improper** integral $\displaystyle\int\int_D \frac{1}{y\sqrt{1-x}}dA$, where $D = \{(x,y)|0 \le x \le 1; 1 \le y \le 2\}$. **Be careful** and write all the appropriate limits.

(11) Compute the following integral $\int\int_D \sqrt{\frac{x-1}{x+3y}}\,dA$, where D consists of all (x,y) such that $x+3y-2 \leq 0$, $-x-3y+1 \leq 0$, $x \leq 5$, and $x \geq 1$.