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Math 11 Fall 2011, Homework 3, due Wed Oct 12

Please show your work. No credit is given for solutions without justification.

(1) A projectile is fired from a position at ground level with an initial speed of 40 m/s at an unknown angle. A large vertical pole placed 40 meters away from the position where the projectile is fired has a target placed on it 30 meters above ground. At what angle should the projectile be fired in order to hit the target? (There are two possible angles.) For this problem assume that the gravitational constant is $g = 10$ m/s^2 and ignore air resistance.

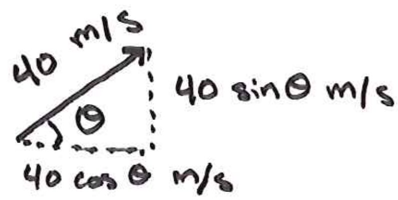
(*Hint*: To solve your final equation for t, use the equality $(t\cos\theta)^2 + (t\sin\theta)^2 = t^2$. You then should arrive at a quadratic equation in (t^2).)

The acceleration from gravity is $\vec{a} = \langle 0, -g \rangle$.

So $\vec{v} = \int \vec{a}\,dt = \vec{a}t + v_0$

and so $\vec{r} = \int \vec{v}\,dt = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{r}_0$.

since $\vec{v}(0) = \langle 40\cos\theta, 40\sin\theta \rangle$:



and $\vec{r}(0) = \langle 0, 0 \rangle$,

we get $\vec{r}(t) = \langle t40\cos\theta, -\frac{g}{2}t^2 + t \cdot 40\sin\theta \rangle$

$= \langle 40t\cos\theta, -5t^2 + 40t\sin\theta \rangle$.

On the other hand,

we know $\langle 40, 30 \rangle$ is a point on $\vec{r}(t)$.

Solve
$$\begin{cases} 40 = 40t\cos\theta \\ 30 = -5t^2 + 40t\sin\theta \end{cases}$$

from $40 = 40t\cos\theta$, we get $t = \sec\theta$.

So $30 = -5t^2 + 40t\sin\theta$

can be rewritten as

$$-6 = \sec^2\theta + (-8)\sec\theta\sin\theta$$

$$= 1 + \tan^2\theta - 8\tan\theta$$

so $\tan^2\theta - 8\tan\theta + 7 = 0$

$$\tan\theta = \frac{8 \pm \sqrt{64-28}}{2}$$

$$= 4 \pm 3 = 1, 7$$

so $\boxed{\theta = \frac{\pi}{4} \text{ or } \approx 1.43 \text{ radians.}}$

Check:

if $\theta = \frac{\pi}{4}$, then $v_0 = \langle 20\sqrt{2}, 20\sqrt{2} \rangle$

so $r(t) = \langle 20\sqrt{2}\,t, -5t^2 + 20\sqrt{2}\,t \rangle$

when $20\sqrt{2}\,t = 40$, $t = \sqrt{2}$,

so $-5(\sqrt{2})^2 + 20\sqrt{2}\cdot\sqrt{2} = -10 + 40 = 30$ ✓

(2) Evaluate each of the following limits. Prove or explain why the limit is what you say it is, or else prove that it does not exist.

(a)

$$\lim_{(x,y)\to(0,0)} \frac{3x^2-2y^2}{3x^2+3y^2}$$

(b)

$$\lim_{(x,y)\to(1,1)} \frac{3x^2-2y^2}{3x^2+3y^2}$$

(c)

$$\lim_{(x,y)\to(0,0)} \frac{6x^2y^2}{3x^2+3y^2}$$

(a) DNE:

Let $y = mx$. Then $z = \frac{3x^2-2(mx)^2}{3x^2+3(mx)^2} = \frac{x^2(3-2m^2)}{x^2(3)(1+m^2)}$

so along the path $y=mx$ to $(0,0)$,

the limit is $\frac{3-2m^2}{3(1+m^2)}$, which depends on m!

so the limit does not exist.

(b) The function is continuous at $(1,1)$,

so the limit is

$$\frac{3-2}{3+3} = \boxed{\frac{1}{6}}$$

(c) Since

$$0 \leq \frac{6x^2y^2}{3x^2+3y^2} \leq \frac{6x^2y^2}{3x^2} = 2y^2 \quad \text{if } x \neq 0$$

and $0 \to 0$ and $2y^2 \to 0$ as $(x,y)\to(0,0)$,

by the squeeze theorem,

$$\frac{6x^2y^2}{3x^2+3y^2} \to \boxed{0} \quad \text{as } (x,y)\to(0,0).$$

(3) Consider the function

$$f(x,y) = \sqrt{\frac{x^4 y}{x^8 + y^2}}$$

(a) What is the domain of the function f?

(b) Derive an equation for the level curves of f by solving the equation $f(x,y) = c$ for y, and sketch a contour map of the function f. (*Hint:* The equation of the level curves can be brought in the form $y = Ax^n$. You must find a formula for A and determine the value of n.)

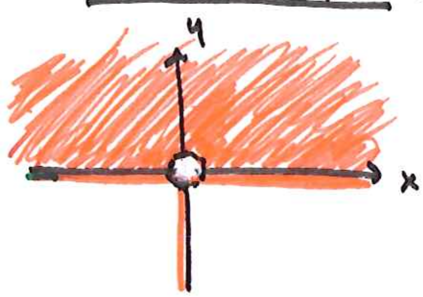
(c) Evaluate the limit of the function value $f(x,y)$ as (x,y) approaches $(0,0)$ along the straight lines $y = mx$.

(d) Do you believe $\lim_{(x,y)\to(0,0)} f(x,y)$ exists? Explain your answer using the contour map or the result of item (c).

(a) you need $\frac{x^4y}{x^8+y^2} \geq 0$ and to be defined.

This happens when either

$y>0$, $x = 0$ and $y \neq 0$, or $y=0$ and $x \neq 0$. :



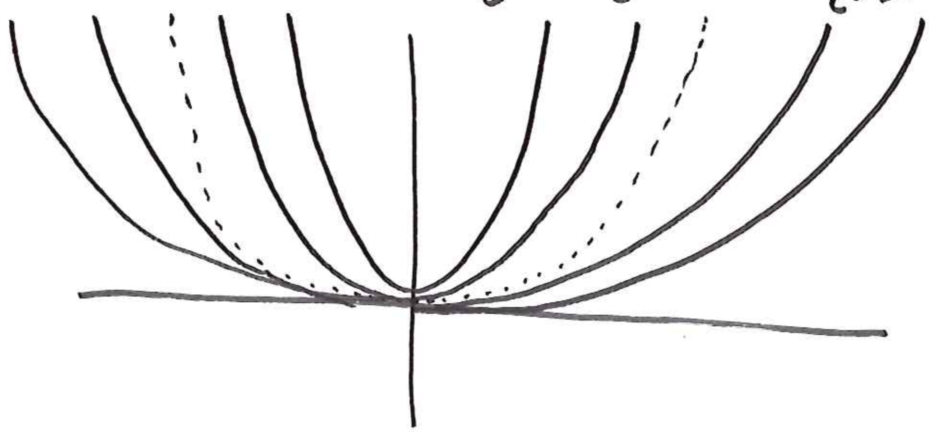
(b) if $c = \sqrt{\frac{x^4y}{x^8+y^2}}$, then $(x^8+y^2)c^2 = x^4y$.

So $c^2y^2 - x^4y + c^2x^8 = 0$

So

$$y = \frac{x^4 \pm \sqrt{x^8 - 4c^4x^8}}{2c^2} = x^4 \underbrace{\left(\frac{1 \pm \sqrt{1-4c^4}}{2c^2}\right)}_{A}$$

notice c can't be 0 in this formula (though z can be, when $x=0$ or $y=0$), and $c \leq \sqrt[4]{1/4} \approx .7$



(c) if $y=mx$,

$$z=\sqrt{\frac{mx^5}{x^8+m^2x^2}}=\sqrt{\frac{mx^3}{x^6+m^2}} \quad \text{if } x\neq 0$$

So along $y=mx$, as $x\to 0$,

$$\boxed{z\to 0},$$

which is inconclusive.

(d) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does <u>not</u> exist.

Choose paths Ax^4. These paths follow contours, so will have different limits depending on A.