Math 9 Fall 2002, 2nd Midterm Exam, November 11

1. Calculate $\int_0^{0.05} \frac{\sin(2x)}{x}dx$ with error $< 10^{-5}$. (You may leave your answer as a sum of a number of terms.)

2. **a** Find the Taylor series for the function $f(x) = e^x$ centered around $a = \ln 2$.

   b Use the Taylor remainder formula and the series you have obtained in [**a**] to calculate $e^{0.1+\ln 2}$ with error less than 10^{-4}. You might want to use the fact that $e^{0.1} < 2$. (You may leave your answer as a sum of a number of terms.)

3. The acceleration of a particle moving in space is $\mathbf{a}(t) = \sin(t)\mathbf{i} + \cos(t)\mathbf{j}$. The initial velocity of the particle is $\mathbf{v}(0) = -\mathbf{i}$ and the initial position is $\mathbf{r}(0) = -\mathbf{j} + 3\mathbf{k}$.

   a Find the position of the particle as a function of time: $\mathbf{r}(t) =?$

   b Find the distance the particle traveled from $t_0 = 1$ to $t_1 = 5$.

4. Consider the depicted parallelogram determined by the vectors $\mathbf{A}$ and $\mathbf{B}$, and its diagonals, which are lightly drawn.

   a Assign directions to the diagonals, and express the resulting two vectors in terms of $\mathbf{A}$ and $\mathbf{B}$.

   b If $\mathbf{A}$ and $\mathbf{B}$ have the same length, show that the diagonals are orthogonal.

5. Consider the four points $A = (1,1,0)$, $B = (4,0,0)$, $C = (1,2,2)$ and $D = (2,6,4)$.

   a Find the distance from the point D to the plane containing A, B and C.

   b Find the volume of the parallelepiped determined by the vectors $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$.

6. Show that the following limit does not exist:

$$\lim_{(x,y)\to(1,1)} \frac{y\ \sin(x-1)}{x+y-2}.$$

7. The position of a particle moving subject to a force $\mathbf{F}$ is given by the vector-valued function $\mathbf{r}(t)$; the velocity vector is $\mathbf{v}(t) = \mathbf{r}'(t)$. The *momentum* vector is defined by $\mathbf{p} = m\mathbf{v}$, where m is the particle's mass; by Newton's Law, $\mathbf{F} = m\mathbf{v}' = \mathbf{p}'$. The *angular momentum vector* $\mathbf{L}$ is defined by $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. Suppose that the force vector is always in the direction of the position vector $\mathbf{r}$. Show that in this case angular momentum is conserved, i.e., that the angular momentum vector is constant. [Hint: Show that the derivative of $\mathbf{L}$ is zero.]

8. How should the constant c be chosen so that the line $\frac{x-1}{2} = \frac{y+1}{c} = \frac{z-5}{3}$ is contained in the plane $3x - 2y = 5$?