Disclaimer: This set of problems is meant neither to indicate the length, difficulty, nor composition of the actual exam. However, it may give an indication of the type of problems which will appear on the exam.

1. (a) Show that the line
$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$$
does not intersect the plane $-x - y + z = 6$.

   (b) Find the distance between the line and the plane

2. Let $\mathbf{r}(t)$ be a vector-valued function of constant speed. Show that at any point t, the acceleration vector is always perpendicular to the velocity vector. **Hint:** Differentiate the square of the speed.

3. Let P be a point not on the line L determined by points Q and R. Let $\mathbf{v} = \overrightarrow{QR}$, and $\mathbf{w} = \overrightarrow{QP}$. Prove that the distance d from the point P to the line L is given by $d = \dfrac{|\mathbf{v} \times \mathbf{w}|}{|\mathbf{v}|}$.

4. Find the equation of the plane containing the point $(1, 0, -2)$ and the line given parametrically by $x = 2 - t$, $y = 2t$, and $z = -t$.

5. Let S be the surface in 3-space whose equation is $z = f(x, y)$. Consider the curve C obtained by intersecting the surface with the plane whose equation is $x = 2$. View C as a curve lying in the plane $x = 2$, with the y direction horizontal and the z direction vertical. Then the curve, viewed as an ordinary plane curve in this way, passes through the point $(2, 3, f(2, 3))$. Express the slope of the curve at this point as a partial derivative of f.

6. Let $f(x, y, z) = xy^2z^3 + e^{x^2+z^2}$. Find
$$\text{(i) } \frac{\partial f}{\partial x} \qquad \text{(ii) } \frac{\partial f}{\partial z} \qquad \text{(iii) } \frac{\partial f}{\partial y}(2, 1, 2)$$

7. Consider the curve given by the vector-valued function $\mathbf{r}(t) = \langle e^t \cos t, e^t \sin t, e^t \rangle$ for $1 \le t \le 2$.

   (a) Find the length L of the above curve.

   (b) Find the midpoint of the curve, that is the point P on the curve so that the length of the curve from $\mathbf{r}(1)$ to P is $L/2$.

8. A parallelogram has one vertex $P = (1, 0, 0)$, and two adjacent vertices $Q = (0, 1, 1)$ and $R = (2, 0, 1)$.

   (a) Find the area of the parallelogram

(b) Find the cosine of the angle between the side $\overrightarrow{PQ}$ and the diagonal from P to the opposite vertex.

9. Consider the planes given by $2x - y + 2z = 3$ and $2x - 2y = 4$.

   (a) What is the angle between the planes?

   (b) Find the equation of the plane which contains the point $(0, 0, 0)$ and is perpendicular to both planes.

10. A tennis player at practice hits the ball to a vertical wall 50 feet away. The ball is hit while it is 3 feet above ground level and its velocity immediately after being hit is 100 ft/sec at an angle of $\pi/3$ with the horizontal. When does the ball hit the wall, and how high off the ground is it when it does so? Note: The position of the ball is given by $\mathbf{r}(t) = -16t^2\mathbf{j} + t\mathbf{v}_0 + \mathbf{r}_0$ where $\mathbf{v}_0$ is the initial velocity vector, and $\mathbf{r}_0 = \mathbf{r}(0)$ is the initial position. Here we are thinking of this as a vector problem in the plane, not in 3-space.