Example 1:

$$\begin{array}{rcrcl} x_1 & + & 5x_2 & = & 7 \\ -2x_1 & - & 7x_2 & = & -5 \end{array} \qquad \left(\begin{array}{ccc} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right)$$

$$\begin{array}{rcrcl} x_1 & + & 5x_2 & = & 7 \\ & & 3x_2 & = & 9 \end{array} \qquad \left(\begin{array}{ccc} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right)$$

$$\begin{array}{rcrcl} x_1 & + & 5x_2 & = & 7 \\ & & x_2 & = & 3 \end{array} \qquad \left(\begin{array}{ccc} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right)$$

$$\begin{array}{rcrcl} x_1 & & & = & -8 \\ & & x_2 & = & 3 \end{array} \qquad \left(\begin{array}{ccc} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right)$$

Example 2:

$$\begin{array}{rcrcrcl} & & 2x_2 & + & x_3 & = & -8 \\ x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array}\right)$$

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{array}\right)$$

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ & - & x_2 & - & x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{array}\right)$$

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & 2x_2 & + & x_3 & = & -8 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 2 & 1 & -8 \end{array}\right)$$

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & & - & x_3 & = & -2 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & & & = & 6 \\ & - & x_2 & & & = & 5 \\ & & & - & x_3 & = & -2 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & 0 & 6 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$$\begin{array}{rcrcrcl} x_1 & & & & & = & -4 \\ & - & x_2 & & & = & 5 \\ & & & - & x_3 & = & -2 \end{array} \qquad \left(\begin{array}{cccc} 1 & 0 & 0 & -4 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$$\begin{array}{rcrcrcl} & x_1 & & & & = & -4 \\ & & x_2 & & & = & -5 \\ & & & & x_3 & = & 2 \end{array} \qquad \left(\begin{array}{cccc} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

Example 3:

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array}\right)$$

$$\left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{array}\right)$$

$$\left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{array}\right)$$

$$\left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

$$\left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{array}\right) \qquad \begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ & - & 2x_2 & - & 6x_3 & = & 10 \\ & & & & 0 & = & -1 \end{array}$$

Since the equation

$$0 = -1$$

is never true, this system has no solutions. That is, the system is inconsistent.