Workshop 9

Eigenvalues

Instructions:

Get into groups and work on the following exercises. Each group is expected to turn in *one neatly written copy* of their solutions at the end of the class period.

Exercise 1. Let A be an $n \times n$ matrix with the property that the row sums all equal the same number s. Show that s is an eigenvalue of A. [*Hint:* Find an eigenvector. To see what's going on you may want to take $n = 2$ or 3 first.]

Exercise 2. Let A be an $n \times n$ matrix with the property that the column sums all equal the same number s. Show that s is an eigenvalue of A. [*Hint:* Use Exercise 1 and the fact, proven in homework, that A and A^T have the same eigenvalues.]

Exercise 3. Let A be an $n \times n$ matrix. Show that if $A^2 = 0$ then the only eigenvalue of A is 0. [*Hint:* Multiply both sides of the equation $A\mathbf{v} = \lambda\mathbf{v}$ by A.]

Exercise 4.* Let

$$A = \begin{pmatrix} -2 & 4 & -4 \\ 3 & -3 & 4 \\ 6 & -8 & 9 \end{pmatrix}, B = \begin{pmatrix} -6 & -2 & 3 \\ 10 & 4 & -4 \\ -11 & -3 & 6 \end{pmatrix}.$$

a. Show that A and B have the same characteristic polynomials (and hence the same eigenvalues with the same multiplicities).

b. Show that A is diagonalizable but that B is not. Conclude that A and B are *not* similar. [*Hint:* Show that if a matrix C is diagonalizable so is any matrix similar to C.]

c. Show that A and B are both roots of their characteristic polynomials. Show further that A is a root of a degree 2 divisor of its characteristic polynomial but that B does *not* have this property. It is far from obvious, but it is precisely this fact that prevents A from being similar to B.