Your name:

Instructor (please circle): Zajj Daugherty Erik van Erp

Math 11 Fall 2011, Homework 6, due Wed Nov 2

Please show your work. No credit is given for solutions without justification.

(1) Choose the correct answer. Show relevant work (it will not be graded).

(a) What is the average value of the product xy for $0 \le x \le 10$, $0 \le y \le 10$.

(A) 20 (B) 25 (C) 30 (D) 40 (E) 45 (F) 50

(b) Let $\mathcal{D}$ be the region in $\mathbb{R}^2$ bounded by the two parabolas $y = 2x^2 - 1$ and $y = x^2 + 1$. Evaluate

$$\iint_{\mathcal{D}} \sin(xy)\, dA$$

(A) 0 (B) π (C) 2π (D) 3π (E) 4π (F) 5π

(c) Evaluate the triple integral

$$\iiint_B \frac{z}{x}\, dV$$

over the rectangular box $B = [1,3] \times [0,2] \times [0,4]$.

(A) $\ln 3$ (B) $2\ln 3$ (C) $4\ln 3$ (D) $8\ln 3$ (E) $16\ln 3$ (F) $32\ln 3$

1a. $R = [0,10] \times [0,10]$.

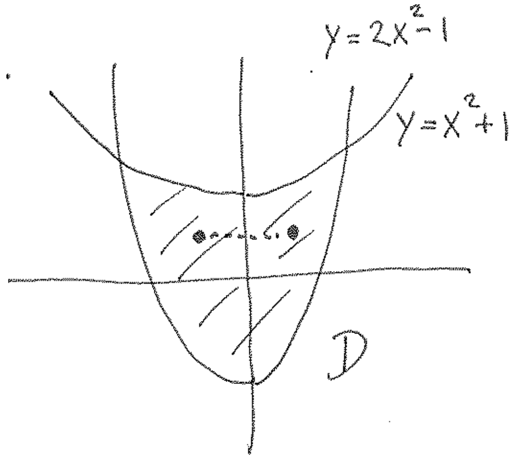
"Total" xy for $R = \iint_R xy\, dA = \int_0^{10}\int_0^{10} xy\, dx dy$

$= \int_0^{10} x dx \cdot \int_0^{10} y dy = \frac{1}{2}x^2\Big|_0^{10} \cdot \frac{1}{2}y^2\Big|_0^{10} = 50 \cdot 50 = 2500$

Area $R = 100$

Average of xy on $R = \frac{2500}{100} = \boxed{25}$

1b.



Region D is symmetric for $x \leftrightarrow -x$

and also

$\sin(-xy) = -\sin(xy)$.

Therefore $\iint_D \sin(xy)\, dA = 0$.

1c. The triple integral is a simple product of three integrals:

$$\iiint_B \frac{z}{x}\,dV = \int_{x=1}^{3}\int_{y=0}^{2}\int_{z=0}^{4} \frac{z}{x}\,dz\,dy\,dx$$

$$= \int_1^3 \frac{1}{x}\,dx \cdot \int_0^2 dy \cdot \int_0^4 z\,dz$$

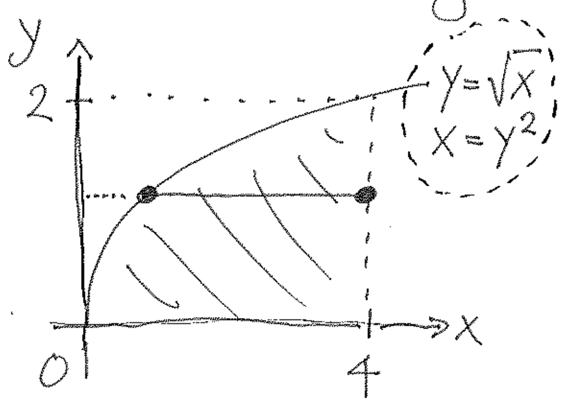
$$= \ln|x|\,\Big|_{x=1}^{3} \cdot y\,\Big|_{y=0}^{2} \cdot \frac{1}{2}z^2\,\Big|_{z=0}^{4}$$

$$= \ln 3 \quad \cdot \quad 2 \quad \cdot \quad 8 \quad = 16\ln 3.$$

(2) Evaluate the following iterated integral by reversing the order of integration.

$$\int_0^4 \int_0^{\sqrt{x}} \frac{e^y}{4-y^2}\, dy\, dx. \quad = \iint_D \frac{e^y}{4-y^2}\, dA$$

Recover the region D from the integrals:



$0 \le x \le 4$
$0 \le y \le \sqrt{x}$

Slice this horizontally. Then

$0 \le y \le 2$
$y^2 \le x \le 4$

New integral:

$$\int_0^2 \int_{y^2}^4 \frac{e^y}{4-y^2}\, dx\, dy$$

Evaluate: $\displaystyle\int \frac{e^y}{4-y^2}\, dx = \frac{xe^y}{4-y^2}\Bigg|_{x=y^2}^{x=4} = \frac{(4-y^2)e^y}{4-y^2} = e^y$

$$\int_0^2 e^y\, dy = e^y \Big|_{y=0}^{2} = e^2 - e^0 = \boxed{e^2 - 1.}$$

(3) Assume that $\mathcal{W}$ is the solid region above the xy-plane (i.e., $z \geq 0$) bounded by the cylinder $x^2 + y^2 = 1$ and the plane $z = y$. Set up the iterated integral that corresponds to the triple integral $\iiint_W f(x, y, z)\, dV$ in the following order of integration,

$$\int_{\dots}^{\dots} \int_{\dots}^{\dots} \int_{\dots}^{\dots} f(x, y, z)\, dz\, dy\, dx$$

Then set up the **same** triple integral as an iterated integral in the order $\iiint \dots dx\, dz\, dy$. Finally, set it up in the order $\iiint \dots dy\, dz\, dx$.

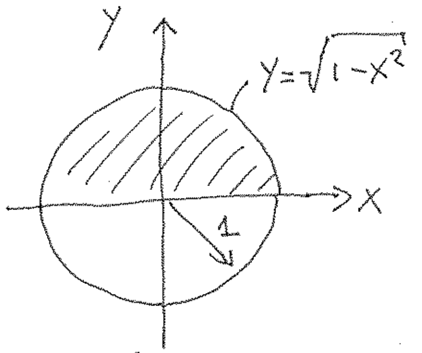


Fig. 1

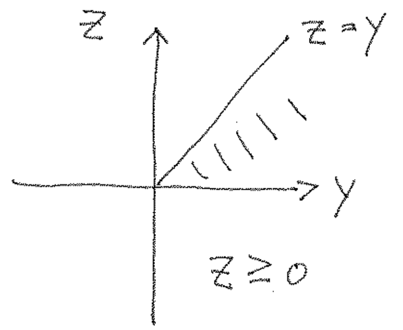


Fig. 2

Because $z \geq 0$
also $y \geq 0$
(in Fig. 2)

Combine into a 3D-sketch.

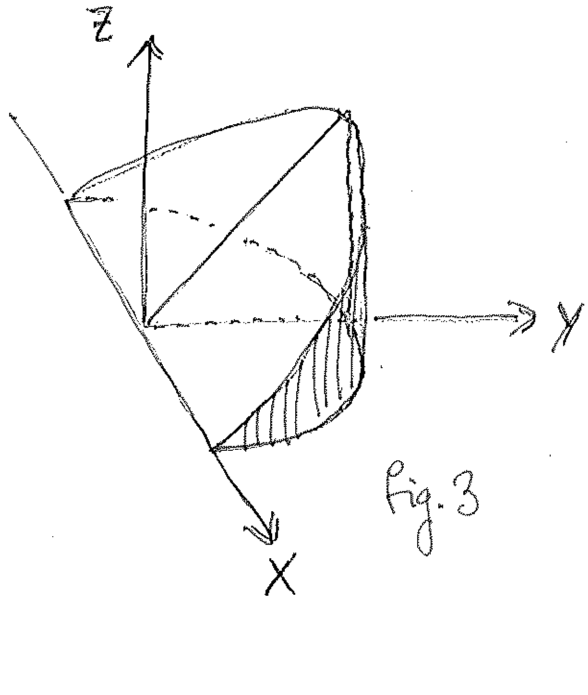


Fig. 3

(i) $\iiint \dots dz\, dy\, dx$

Need xy-shadow D_{xy}
See Fig. 1 above.

$$\left.\begin{array}{l} -1 \leq x \leq 1 \\ 0 \leq y \leq \sqrt{1-x^2} \end{array}\right\} D_{xy}$$

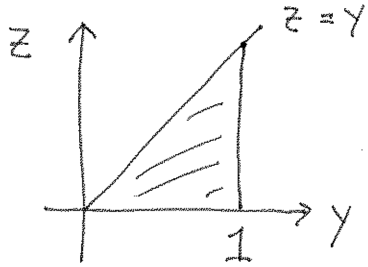
Then in Fig. 3:
"floor" $\Rightarrow z = 0$
"roof" $\Rightarrow z = y$

Then. $0 \leq z \leq y$

$$\int_{x=-1}^{1} \int_{y=0}^{\sqrt{1-x^2}} \int_{z=0}^{y} f(x, y, z)\, dz\, dy\, dx$$

(ii) $\iiint \ldots \, dx\,dz\,dy$.

Now need yz-shadow D_{yz}.



To get $dz\,dy$ integral:

$$0 \le y \le 1$$
$$0 \le z \le y$$

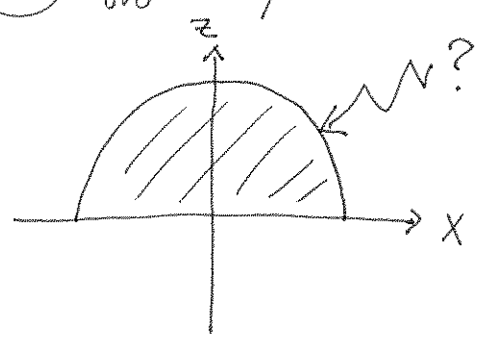
Now go back to fig. 3 and take "floor" and "roof" in direction of the x-axis

floor : $x = -\sqrt{1-y^2}$

roof : $x = \sqrt{1-y^2}$ ← from fig. 1

$$\int_{y=0}^{1} \int_{z=0}^{y} \int_{x=-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y,z)\, dx\,dz\,dy$$

(iii) $\iiint \ldots \, dy\,dz\,dx$. Need xz-shadow D_{xz}.



? intersection of $x^2+y^2=1$ cylinder

$y = z$ plane

Eliminate y to get $x^2+z^2=1$.

Therefore:

$$-1 \le x \le 1$$
$$0 \le z \le \sqrt{1-x^2}$$

Inspect fig. 3 to find roof + floor in y-direction

floor: $y = z$

roof: $y = \sqrt{1-x^2}$

$$\int_{x=-1}^{1} \int_{z=0}^{\sqrt{1-x^2}} \int_{y=z}^{\sqrt{1-x^2}} f(x,y,z)\, dy\,dz\,dx$$