

Math 11, Fall 2007

Lecture 20

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Outline

- 1 Recap and overview
 - Last classes
- 2 Today's material
 - Vector fields
- 3 Group Work
- 4 Summary
- 5 Next class

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Integration of functions of more than one variable

- Double and triple integrals
- Fubini's theorem, iterated integrals, non-rectangular domains
- Change of coordinates: polar/cylindrical, spherical

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Multivariable functions

Functions from \mathbb{R}^m to \mathbb{R}^n

- spacecurves: one input, many outputs e.g $\vec{r} : \mathbb{R} \rightarrow \mathbb{R}^3$
- Functions of more than one variable: many inputs, one output eg $f : \mathbb{R}^3 \rightarrow \mathbb{R}$
- many inputs/many outputs: $W : \mathbb{R}^m \rightarrow \mathbb{R}^n$
- Vector fields: $V : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $W : \mathbb{R}^2 \rightarrow \mathbb{R}^2$

Visualizing vector fields

$$V(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$$

The gradient vector field

A vector field \vec{F} is called a conservative vector field if it is the gradient of some scalar function, f (i.e. $\nabla f = \vec{F}$). f is called the potential function of \vec{F} .

Integration along curves

- Integrate a function of more than one variable along a curve

- 1 Given a curve C parametrized by $\vec{r}(t) = \langle x(t), y(t) \rangle$, $a \leq t \leq b$
- 2 Given $f(x, y)$, a function of two variables
- 3 Integrate along the curve with respect to arclength, x or y :

$$\int_C f \, ds$$

- 4 Substitute and evaluate

$$\int_C f \, ds = \int_a^b f(x(t), y(t)) \sqrt{x'(t)^2 + y'(t)^2} \, dt$$

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Integration along curves

Variants:



$$\int_C f \, dx, \quad \int_C f \, dy$$



$$\int_C f \, dx = \int_a^b f(x(t), y(t)) x'(t) \, dt$$

$$\int_C f \, dy = \int_a^b f(x(t), y(t)) y'(t) \, dt$$

Examples



$$\int_C ye^x ds$$

where C is the line segment joining $(1, 2)$ to $(4, 7)$



$$\int_C z dx + x dy + y dz$$

where C is given by $\vec{r}(t) = \langle t^2, t^3, t^2 \rangle$, $0 \leq t \leq 1$.

Group work



$$\int_C xz \, ds$$

where C is given by $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$, $0 \leq t \leq 2\pi$.

Summary

- Vector fields and general functions
- Integration along curves

Work for next class

- Reading: 17.3
- Exam II on monday
- f07hw21 (due Wednesday)