Hour Exam 1

Math 3

Oct. 19, 2005

Name:____

Instructor (circle): Lahr (8:45) Elizalde (10:00) Ionescu (11:15)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All your answers to the multiple choice questions must be marked on the Scantron form provided, and your responses to the remaining questions must be written in this exam booklet. Take a moment now to print your name and section clearly on your Scantron form, and on your exam booklet. With regard to the multiple choice questions, you may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 10 multiple choice problems each worth 6 points, and there are 3 additional problems totaling 40 points. Check to see that you have 8 pages of questions plus this cover page.

Non-multliple choice questions:

Problem	Points	Score
1	15	
2	10	
3	15	
Total	40	

1. The domain and the range of the function

$$f(x) = \frac{1}{\sqrt{x-2}}$$

are:

(a) the domain is $[2, \infty)$ and the range is $(0, \infty)$.

(b) the domain is $(-\infty, 2) \cup (2, \infty)$ and the range is the set of all real numbers.

(c) the domain is $[2, \infty)$ and the range is $[0, \infty)$.

(d) the domain is $(2, \infty)$ and the range is $(0, \infty)$.

(e) the domain is $(-2, \infty)$ and the range is $[0, \infty)$.

2. The limit

$$\lim_{x \to \infty} \frac{\sqrt{3x^2 + 2}}{x}$$

is equal to

(a) ∞

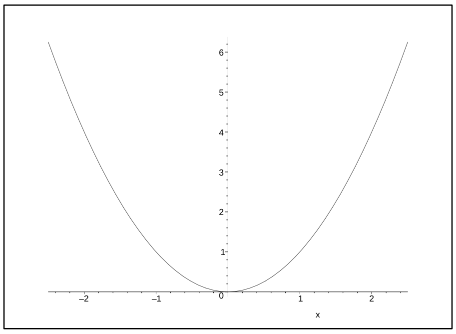
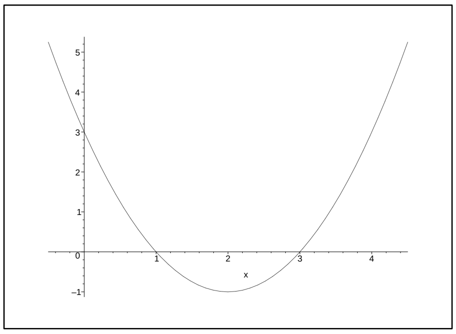
(b) 0

(c) 3

(d) $\sqrt{3}$

(e) the limit does not exist, but equals ∞

3. Considering that the figure on the left is the graph of $y = x^2$, the function whose graph is depicted in the figure on the right is:

(a) $(x-2)^2 - 1$

(b) $(x-1)^2 - 2$

(c) $(x+2)^2 - 1$

(d) $2x^2 - 1$

(e) $x^2 + 3$

4. The function $f(x) = \sin(x^3) - 4x + 1$ is:

(a) even.

(b) odd.

(c) neither even nor odd.

(d) both even and odd.

5. Let $f(x) = x^2 + 2x - 1$ and $g(x) = \sqrt{x+3}$. What is $(g \circ f)(x) = g(f(x))$?

(a) $x + 2 + 2\sqrt{x+3}$

(b) $(x^2 + 2x - 1)\sqrt{x+3}$

(c) $x + \sqrt{2x+2}$

(d) $\sqrt{x^2 + 2x + 2}$

(e) None of the above.

6. The derivative of $f(x) = \dfrac{1}{\sqrt{1+\cos(2x)}}$ is

(a) $-\dfrac{1}{2(1+\cos(2x))^{3/2}}$

(b) $\dfrac{\sin(2x)}{2(1+\cos(2x))^{3/2}}$

(c) $\dfrac{\sin(2x)}{(1+\cos(2x))^{3/2}}$

(d) $-\dfrac{\sin(2x)}{(1+\cos(2x))^{3/2}}$

(e) $\dfrac{\sin(2x)}{2(1+\cos(2x))^{1/2}}$

7. The derivative of $f(x) = \dfrac{\tan(x)}{x}$ is:

(a) $\dfrac{\sec^2(x)\,x - \tan(x)}{x^2}$

(b) $\dfrac{\sec^2(x)}{x}$

(c) $\sec^2(x)$

(d) $\sec^2(x)\,x + \tan(x)$

(e) $\tan(x)$

8. Assume that the position of an object which is moving in the x-axis is given by $x(t) = t^2 + 3t$. Let $av[1,3]$ be the average velocity between $t = 1$ and $t = 3$, and let $v(1)$ be the instantaneous velocity at $t = 1$. Then

(a) $av[1,3] = 14$ and $v(1) = 5$.

(b) $av[1,3] = 7$ and $v(1) = 5$.

(c) $av[1,3] = 7$ and $v(1) = 4$.

(d) $av[1,3] = 14$ and $v(1) = 4$.

(e) None of the above.

9. The function

$$f(x) = \frac{x^2 - 9}{x + 3}$$

(a) is not defined for $x = -3$.

(b) can be defined continuously at $x = -3$ (that is, it has a continuous extension).

(c) has a limit as $x \to -3$.

(d) is continuous on its domain.

(e) all of the above.

10. The equation $2^x - x^4 + 1 = 0$ has

(a) No real solutions.

(b) Only one solution, which is in the interval $[-2, 0]$.

(c) At least two solutions.

(d) Only one solution, which is in the interval $[0, 2]$.

NON-MULTIPLE CHOICE. PLEASE SHOW ALL YOUR WORK.

1. Let

$$f(x) = \begin{cases} x^2 & x < 1 \\ 2x - 1 & 1 \leq x \end{cases}$$

(a) (5 pts) Determine if $\lim_{x \to 1} f(x)$ exists.

(b) (5 pts) Using the definition of continuity explain whether the function is continuous at $x = 1$.

(c) (5 pts) Is f differentiable at $x = 1$? Explain your reasoning (no credit will be given without justification).

2. Let

$$f(x) = \begin{cases} kx & x < 1 \\ x^3 + 1 & 1 < x \end{cases}.$$

(a) (5 pts) Find the value of the constant k such that f can be defined continuously at $x = 1$.

(b) (5 pts) Is the continuous extension (from part (a)) differentiable at $x = 1$? Explain your reasoning (no credit will be given without justification).

3. (a) (7.5 pts) Use the limit definition of the derivative function to find the derivative $f'(x)$ of

$$f(x) = \frac{1}{\sqrt{x+1}}$$

No credit will be given if the definition is not used.

(b) (7.5 pts) Find an equation of the tangent line at $x = 3$.