2.9: Derivatives of the Trigonometric Functions

Mathematics 3
Lecture 10
Dartmouth College

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Differentiation Formulas (cont'd)

Before we begin, let's work some examples using our knowledge of derivatives and limits so far...

Example 1: Find the tangent line to the graph of

$$y = \left(-1 + \sqrt{\frac{x+7}{4}} \right)^{10}$$

at the point on the graph where $x = 9$.

Differentiation Formulas (cont'd)

Example 2: Compute the following limit:

$$\lim_{x \to 8} \frac{\left(1 + \frac{x}{8}\right)^6 - 64}{x - 8}$$

Some Trigonometric Identities and Limits

To find the derivatives of the *sine* and *cosine* functions, we need:

Angle Sum Formulas

- $\sin(u + v) = \sin(u)\cos(v) + \sin(v)\cos(u)$

- $\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

Trigonometric Limits

- $\displaystyle\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

- $\displaystyle\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$

Example 3

Evaluate the following limit

$$\lim_{x \to 1} \frac{\sin 5(x-1)}{2x-2}$$

The Derivative of the Sine Function

$$\frac{d}{dx}\sin x = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \sin x \lim_{h \to 0} \frac{\cos h - 1}{h} + \cos x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \sin(x) \cdot 0 + 1 \cdot \cos(x)$$

$$= \cos(x)$$

Theorem

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x.$$

Example 4

- Let $y = \sin(2x)$. Show that this satisfies the Differential Equation

$$y'' + 4y = 0.$$

- Differentiate $g(w) = \cos\left(w^2 + \frac{1}{w}\right)$.

- Compare the derivatives $\frac{dy}{dx}$ of $y = \sin(x^2)$ and $y = \sin^2(x)$.

Example 5

- Differentiate

$$
\begin{aligned}
y &= \sin x \cos x \\
y &= \sin^2(\cos(x^2 + 2))
\end{aligned}
$$

Theorem

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Example 6

$$\frac{d}{dx}\cot^2 x$$

Example 7

a.) Find the derivative of the function

$$f(t) = (\sin^3(\tan^2(2t)))^4.$$

b.) Find $\frac{dy}{dx}$ for $y = \sqrt{\sec x^3}$.

Remember: The First Midterm Exam is next Monday!