Maxima and Minima

Lecture 28

March 5, 2007

Second Derivative Test

Fact

Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2.$$

1. *If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.*
2. *If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.*
3. *If $D < 0$, then $f(a, b)$ is not a local maximum or minimum. In this case the point (a, b) is called a* ***saddle point*** *of f.*

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- Find the points on the surface $x^2y^2z = 1$ that are closest to the origin.
- Find three positive numbers x, y, and z whose sum is 100 and whose product is maximum.
- A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.
- Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid

$$9x^2 + 36y^2 + 4z^2 = 36.$$

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- A **bounded set** in $\mathbb{R}^2$ is one that is contained within some disk.

Extreme Value Theorem for Functions of Two Variables

Theorem

If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.

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1. *Find the values of f at the critical points of f in D.*
2. *Find the extreme values of f on the boundary of D.*
3. *The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.*

Examples

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- Find the absolute maximum and minimum values of the function $f(x.y) = x^2 - 2xy + 2y$ on the rectangle

$$D = \{(x,y)|0 \leq x \leq 3\ ,\ 0 \leq y \leq 2\}.$$

Examples

- Find the absolute maximum and minimum values of the function $f(x.y) = x^2 - 2xy + 2y$ on the rectangle
$$D = \{(x, y)|0 \leq x \leq 3 \ , \ 0 \leq y \leq 2\}.$$
- Find the absolute maximum and minimum values of the function $f(x, y) = 3 + xy - x - 2y$ on the closed triangular region with vertices $(1, 0)$, $(5, 0)$, and $(1, 4)$.