Methods of functional analysis and algebraic topology in the study of C*-algebras

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102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

Since the 1960's one of the fundamental ideas introduced in mathematics by algebraic topology is the use of various functors (homotopy, homology, cohomology, K-theory etc.) in the study of topological spaces and of the maps between them. In the 1970's, Atiyah, Brown-Douglas-Fillmore and Kasparov, among others, added methods of functional analysis into the picture.

In the first part of my talk I shall review two such functors associated to the category of separable C*-algebras, namely the KK-theory of Kasparov and the E-theory of Connes and Higson. Their motivation and the necessary definitions are based on a deep interplay of functional analysis and algebraic topology, and they proved to be extremely useful in studying both structural questions in C*-algebra theory and geometric phenomena across wide areas of mathematics. As part of the effort to better understand the relationship between these two theories, I was lead to introduce yet another bivariant K-theory, intermediate between the other two, called KE-theory. I shall motivate the construction of its cycles through the locality of first order elliptic operators on manifolds. A brief presentation of the product and of the main functorial properties of KE-theory will follow. The talk will end with an application: an index-theoretic computation relevant to the Baum-Connes conjecture for a-T-menable groups.

This talk should be accessible to general faculty.