Dartmouth College
Mathematics 25

Assignment 1
due Wednesday, September 30

1. Use Euclid's algorithm to compute the gcd of 12345 and 67890.

2. Using your work above, write the gcd you found as in Bezout's identity for 12345 and 67890.

3. Suppose that there are integers m, n, x, y so that $mx+ny = 1$. Show that $\gcd(m, n) = 1$.

4. Let n be a positive integer and E_n the set of integers which are relatively prime to n. Show that E_n is closed under multiplication, that is if $m, m' \in E_n$, so is their product mm'.

5. There is an alternate version of the definition of gcd than the one given in the text. Let a, b be integers, not both zero. We shall define a GCD of a, b, to be an integer D (not necessarily unique) such that

   - $D \mid a$, $D \mid b$, and
   - If $c \mid a$ and $c \mid b$, then $c \mid D$.

   Show that if D, D' are two GCDs of a, b, then $D' = \pm D$. Thus there is a unique positive GCD which we call $\mathrm{GCD}(a, b)$.

6. Now let a, b be integers, not both zero. Show that $\gcd(a, b) = \mathrm{GCD}(a, b)$. As a corollary, show that $c \mid a$ and $c \mid b$ if and only if $c \mid \gcd(a, b)$.