# Improper Integrals

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Lecture 4

# Improper Integrals

- If the interval of integration of $f$ is infinite or $f$ has an infinite discontinuty in $[a, b]$, then the definite integral of $f$ is called an *improper integral*.

# Type I: Infinite intervals

- If $\int_a^t f(x)\mathrm{d}x$ exists for every number $t \geq a$, then

$$\int_a^\infty f(x)\mathrm{d}x = \lim_{t\to\infty} \int_a^t f(x)\mathrm{d}x,$$

provided this limit exists (as a finite number).

- If $\int_t^b f(x)\mathrm{d}x$ exists for every number $t \leq b$, then

$$\int_{-\infty}^{b} f(x)\mathrm{d}x = \lim_{t\to-\infty} \int_{t}^{b} f(x)\mathrm{d}x$$

  provided this limit exists (as a finite number).

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- If both $\int_a^\infty f(x)\mathrm{d}x$ and $\int_{-\infty}^b f(x)\mathrm{d}x$ are convergent, then we define

$$\int_{\infty}^{\infty} f(x)\mathrm{d}x = \int_{-\infty}^{a} f(x)\mathrm{d}x + \int_{a}^{\infty} f(x)\mathrm{d}x.$$

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- Is the area under the curve $y = \frac{\ln x}{x^2}$ from $x = 1$ to $x = \infty$ finite? If so, what is its value?

- For what values of $p$ is the integral

$$\int_1^\infty \frac{1}{x^p}\mathrm{d}x$$

convergent?