Math 11, Fall 2007

Lecture 9

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Outline

1. Review and overview
   - Last classes
2. Today's material
   - The chain rule
3. Group Work
4. Next class

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Differentiation

Reducing to the one variable case

- Derivatives of space curves, $\vec{r}(t) =< x(t), y(t), z(t) >$,

$$\vec{r}'(t) =< x'(t), y'(t), z'(t) >$$

- Derivatives of $f(x, y)$ in specific directions
  1. Directional derivatives, $D_{\vec{v}}f$
  2. Partial derivatives, f_x, f_y
  3. Higher order partials

Differentiation

- Tangent plane: local approximation of a function if the function is *differentiable*
- Differentiability $\iff$ tangent planes vary continuously $\iff$ f_x, f_y exist and are continuous

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The Chain Rule

- On Variable Chain rule:

$$\frac{d}{dx}f(g(x)) = \frac{df}{dx}(g(x))\frac{dg}{dx}$$

- In more than one variable, the chain rule is more complicated: $f(x, y)$ where $x = g(s), y = h(s)$. What is $\frac{\partial f}{\partial s}$?
- Idea: changes in s produce changes in both x and y so,

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{dx}{ds} + \frac{\partial f}{\partial y}\frac{dy}{ds}$$

- We've really seen this already: restrict $f(x, y)$ to a curve to produce directional derivatives.

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Chain Rule

The general chain rule: Let f be a function of variables $x_1, \ldots, x_n$ and each x_i is a function of variables $s_1, \ldots, s_m$. To find $\frac{\partial f}{\partial s_i}$:

- Differentiate f with respect to each x_j
- Differenitate each x_j with respect to s_i
- Put everything together
- Helpful to draw "tree diagram"

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Implicit Differentiation

If we consider the curve $x^2 + y^2 = 1$, what is $\frac{dy}{dx}$?

- $$\frac{\partial f}{\partial x}\frac{dx}{dx} + \frac{\partial f}{\partial y}\frac{dy}{dx} = 0$$
- Also works with a surface $F(x, y, z) = 0$

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Examples

1. $f(x, y) = ln(u^2 + v^2 + w^2), u = x + 2y, v = 2x - y, w = 2xy$
2. $z = \sin(a)\tan(b), a = 3s + t, b = s - t$
3. Consider the sphere $x^2 + y^2 + z^2 - r^2 = 0$ as a function of four variables which implicitly defines z as a function of x, y, r. What is $\frac{\partial z}{\partial r}$?
4. $z = f(x, y), x = r\cos(\theta), y = r\sin(\theta)$. Find $z_r, z_\theta, z_{r\theta}$

Work for next class

- Reading: 15.6
- f07hw10