

Workshop Problems 8

Problem 1. Let A be an $n \times n$ matrix. Show that A and A^T have the same characteristic polynomial and hence the same eigenvalues. [*Hint:* How are $A - \lambda I$ and $A^T - \lambda I$ related?]

Problem 2. Let A be an $n \times n$ matrix with the property that the row sums all equal the same number s . Show that s is an eigenvalue of A . [*Hint:* Find an eigenvector.]

Problem 3. Let A be an $n \times n$ matrix with the property that the column sums all equal the same number s . Show that s is an eigenvalue of A . [*Hint:* Use Problems 1 and 2 together.]

Problem 4. Let A be an $n \times n$ matrix. Show that if $A^2 = 0$ then the only eigenvalue of A is 0. [*Hint:* Multiply both sides of the equation $A\mathbf{v} = \lambda\mathbf{v}$ by A .]

Problem 5. Let A be an $n \times n$ matrix with eigenvalue λ and corresponding eigenvector \mathbf{v} . Use mathematical induction to show that λ^n is an eigenvalue of A^n with corresponding eigenvector \mathbf{v} for all integers $n \geq 1$. [*Hint:* Apply the hint in Problem 5 repeatedly.]