SOLUTIONS

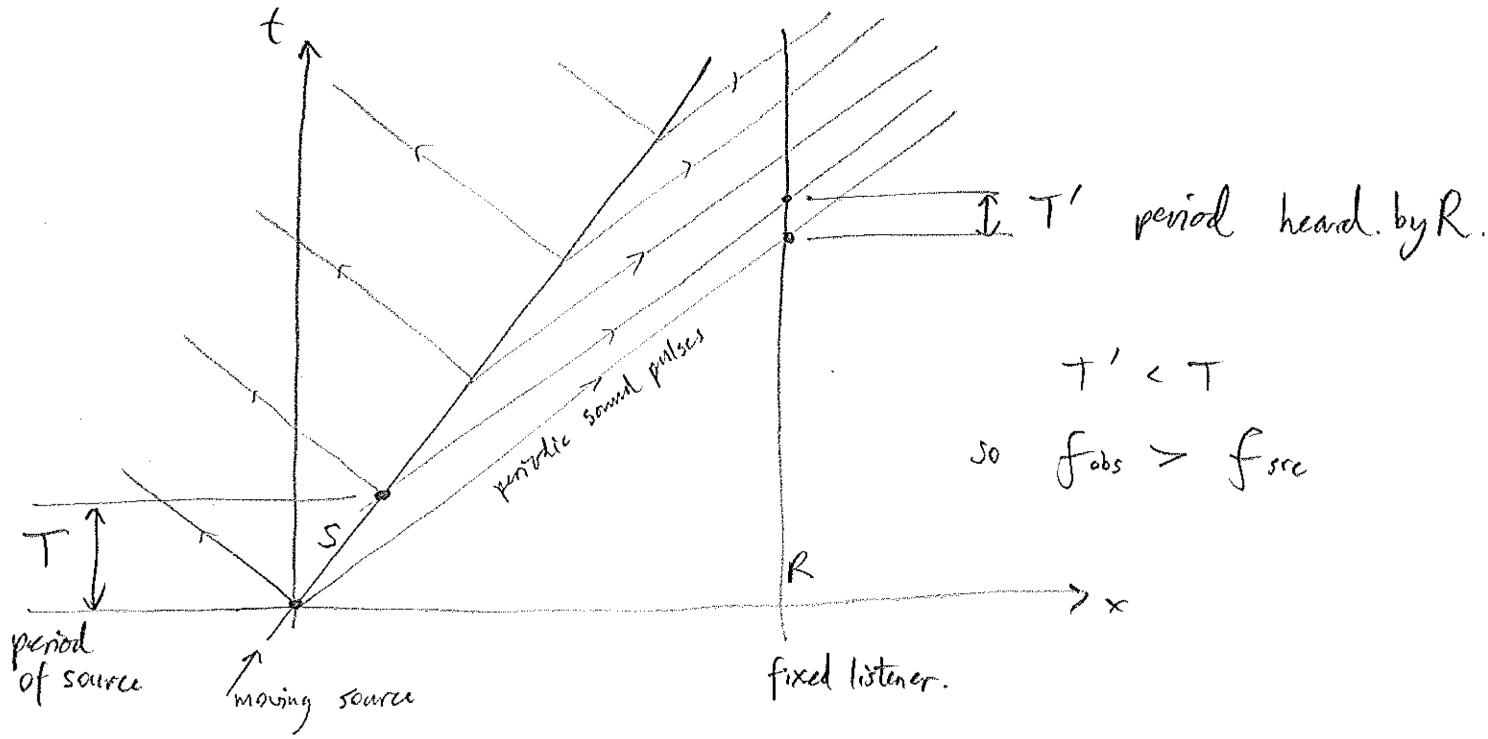
Math 5: Music and Sound FALL 2010: Final

3 hours, 9 questions, 80 points total

Try to show working. Heed the points available for each question. Do try the bonus once the rest is ok. The last page has useful information. Good luck, have fun, and it was great to have you in the course!

1. [7 points]

[4] (a) In the space below, draw a spacetime diagram (labeling axes) showing how it is possible for a fixed listener to hear a sound *higher* in frequency than emitted by a moving source.



$T' < T$

so $f_{obs} > f_{src}$

[3] (b) What speed does the source need to move, and in what direction, so that the observed pitch is a perfect 4th higher than the source pitch? (You may use *just intonation*, since it's simplest)

$$f_{obs} = \frac{f}{1 - v/c} = \frac{4}{3} f$$

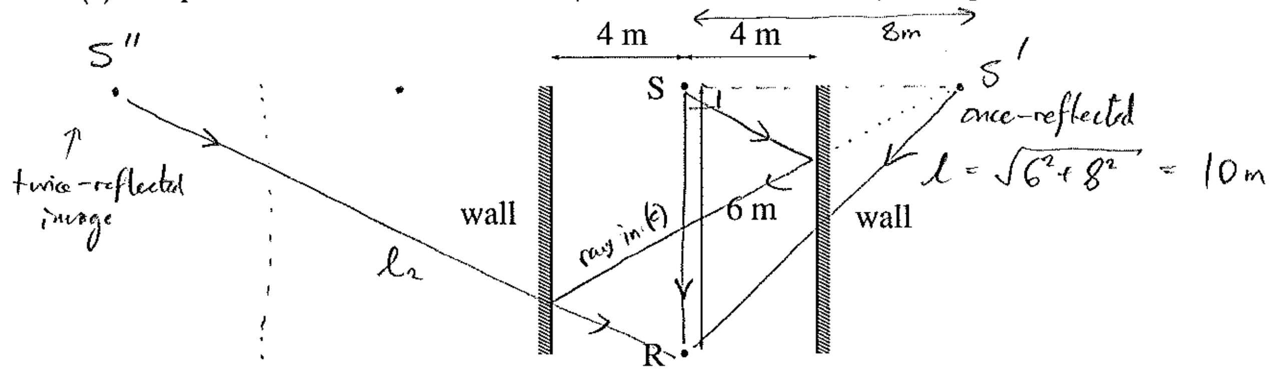
perfect 4th is 4:3

$$\text{so} \quad \frac{1}{1 - v/c} = 4/3 \quad \Rightarrow \quad 1 - v/c = \frac{1}{4/3} = 3/4$$

$$\Rightarrow \quad \frac{v}{c} = 1/4 \qquad v = c/4 = 85 \text{ m/s}. \quad \text{towards observer.}$$

2. [9 points] Steve and Rachel stand at opposite ends of a tunnel 6 m in length, as shown below in plan view.

(a) Compute the travel times of the direct, and of the once-reflected, sound paths from S to R.



[3] $t_{direct} = \frac{d}{c} = \frac{6}{340} = 0.01765\ldots$ s

$t_{once\text{-}refl} = \frac{l}{c} = \frac{10}{340} = 0.02941\ldots$ s

[3] (b) What is the *lowest* pure tone frequency emitted by Steve that would cause destructive interference of these two paths to Rachel?

Path length difference $= l - d = 10 - 6 = 4$ m.

For destructive, this must equal $n - \frac{1}{2}$ wavelengths. (lowest: choose $n=1$)

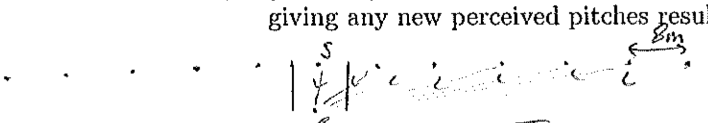
so $\frac{1}{2}\lambda = 4$m, $\lambda = 8$m $f = \frac{c}{\lambda} = 42.5$ Hz

[3] (c) Steve claps once. How long after the direct sound arrives does the *second* distinct echo occur? (not the first echo which you computed in a). On the diagram, construct a ray path from S to R corresponding to this second echo.

$l_2 = \sqrt{6^2 + 16^2} = 17.09$ m so $t_{twice\text{-}refl.} = \frac{l_2}{c} \approx 0.05026$ s

delay after direct $= t_{twice\text{-}refl} - t_{direct} \approx 0.0326$ s.

(d) [BONUS] Describe (or sketch) the *tail* (long time decaying part) of the echo signal Rachel hears, giving any new perceived pitches resulting from this acoustic environment.



There is an infinite line of image sources.

They give arrival times which tend to be separated by 8m.

$\Rightarrow$ decaying periodic signal, period $\frac{8m}{c}$, 2 or frequency $\frac{340}{8} \approx 42.5$ Hz perceived pitch.

signal:

3. [9 points] Two violinists are playing their E strings (with no fingers on the strings), which are supposed to sound the note E5.

[1] (a) Compute the frequency of the equal-tempered note E5. ← 7 semi's above A4

$A4 = 440$ Hz

$$f_{E5} = (440)\,2^{7/12} \approx 659.3\cdots \text{ Hz}$$

[3] (b) One of the violinists used equal-tempered tuning while the other tuned using Pythagorean tuning relative to their A4 string. How many cents different are they, and which one is sharp?

3:2 relative to A4.

Convert the Pythag. $f = \frac{3}{2}\,440 = 660$ Hz to cents:

$$\text{cents} = 1200\,\frac{\ln\frac{660}{440}}{\ln 2} = 1200\,\frac{\ln 3/2}{\ln 2} = 701.96\cdots$$

compare 700 for equal temp.

⇒ Pythag. violinist 1.96 ¢ sharp of equal-tempered.

[2] (c) Assuming each player produces something close to a pure tone, describe what you'll hear when they play their notes *together*. Give *all* new frequencies of phenomena perceived (but you don't need to give formulae).

Two pure tones of close freq. (< 15 Hz apart) ⇒ beats.

$$f_{\text{beat}} = |f_1 - f_2| = |659.26 - 660| \approx 0.74\cdots \text{ Hz}$$

(note, accuracy needed here!) perceived beating freq.

You will hear the average freq 659.6 Hz modulated in intensity at 0.74 Hz. (less than 1 beat per sec).

[2] (d) One of the violinists now wishes to tune (for some reason) their E string *up* an equal-tempered tritone. By what factor do they need to change the tension in the string?

6 semitones, $2^{6/12} = \sqrt{2}$ ratio.

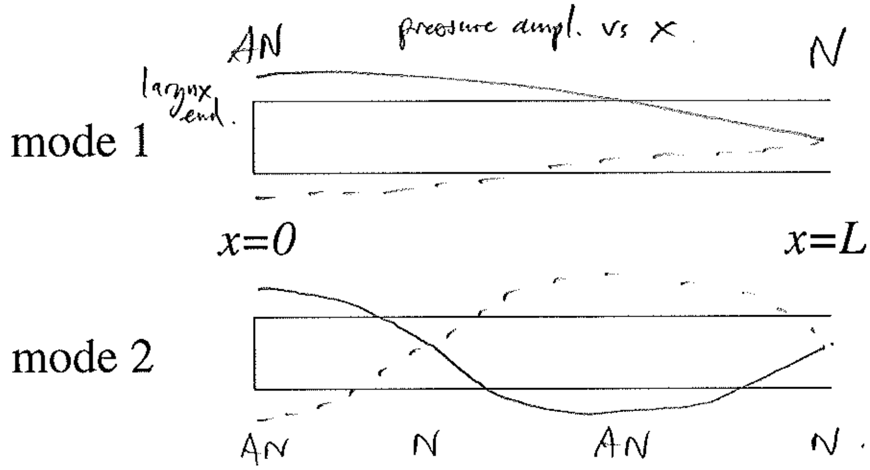
Use ratios: $f = \dfrac{c_{\text{string}}}{2L} = \dfrac{\sqrt{T/\mu}}{2L} \propto \sqrt{T}$ when μ, L fixed.

$$\Rightarrow \frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{ie} \quad \sqrt{2} = \sqrt{\frac{T_2}{T_1}} \quad \text{so} \quad \frac{T_2}{T_1} = 2$$

Double the tension.

4. [10 points] The adult male human vocal tract can be modeled by a closed-open pipe $L = 0.17$ m long.

[3] (a) Sketch the first two modes showing graphs of pressure amplitude vs position, over the pipes below, labeling nodes and anti-nodes:



(dotted curve is pressure ½ period in time later).

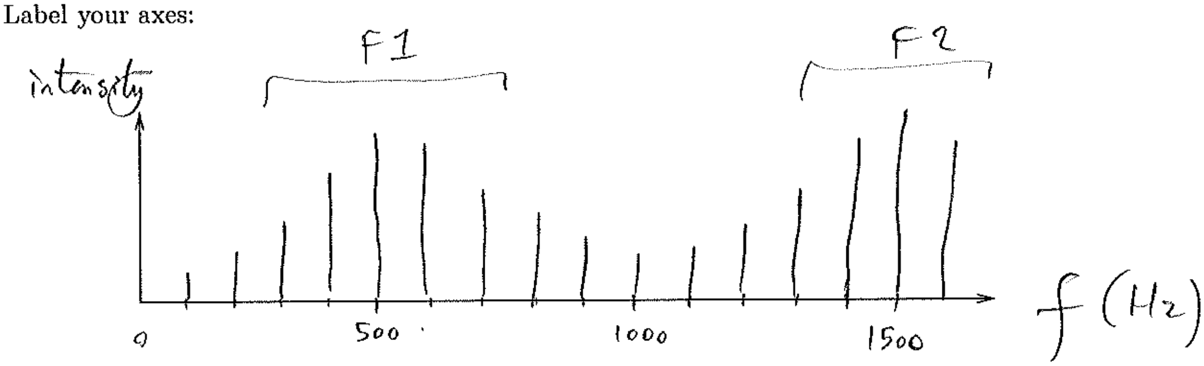
[2] (b) Compute the formant frequencies F1 and F2 using this model.

same as closed-open pipe: $f_n = \frac{c}{4L}, \frac{3c}{4L}, \frac{5c}{4L}, \ldots$

$F1 = f_1 = \frac{340}{4(0.17)} = 500\,Hz$

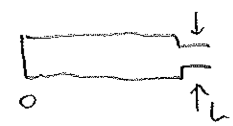
$F2 = f_2 = 3f_1 = 1500\,Hz$

[3] (c) Sketch a spectrum that could be produced by this male when singing a low note with pitch 100 Hz. Label your axes:



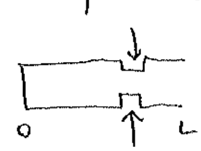
[2] (d) Give a location (e.g. $x = L/2$) where locally *constricting* the vocal tract would cause each of the following formant changes.

- lower both F1 and F2 together: $x = L$ ie at mouth



- lower F1 while raising F2: want nearest node for mode 1, AN for mode 2

$x = \frac{2}{3}L$



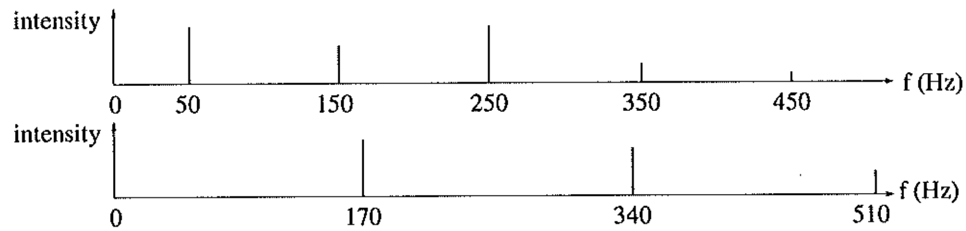
(is nearer node for mode 1)

5. [9 points] Consider two wind instruments A and B which are based upon pipes of uniform width.

[4] (a) For each instrument, using the spectra below, calculate the pipe length (ignore end corrections) and state whether the end conditions of the pipe are closest to open-open or closed-open.

Instrument A:

Instrument B:



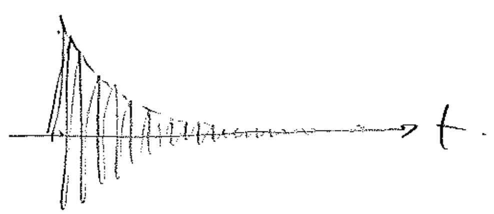
A has only <u>odd</u> harmonics $\Rightarrow$ closed-open pipe (clarinet, brass, etc.)

$$f_n = (2n-1)\frac{c}{4L}, \quad \text{ie } f_1 = \frac{c}{4L} \quad \text{so } L = \frac{c}{4f_1} = \frac{340}{4(50)} = \underline{1.7\text{m}}.$$

B has all harmonics $\Rightarrow$ open-open (flute), $f_1 = \frac{c}{2L}$ so $L = \frac{c}{2f_1} = \frac{340}{2(170)} = \underline{1\text{m}}$.

[3] (b) Using the spectrogram of slapping (impulsive excitation) of instrument A, the lowest 50 Hz mode amplitude is observed to decay 20 dB in 0.1 sec. Compute the Q factor of this mode.

slapped signal:



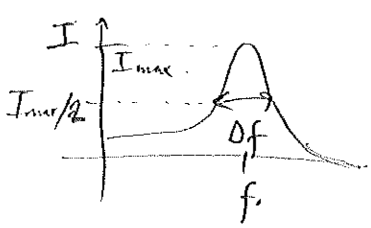
compute decay time since Q then known:

$$e^{-t/\tau} = \text{ampl. ratio} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10^{20/10}} = 10.$$

take logs: $-\frac{t}{\tau} = \ln 10$, ie $\tau = -\frac{0.1\text{s}}{\ln 10} = 0.043..\text{s}$.

$$Q = \pi f_0 \tau = \pi(50)\tau = 6.8 \quad \text{(not v. high)}.$$

(c) Within what range of pure tone frequencies does the mode discussed in b) get excited with at least half its maximum intensity?

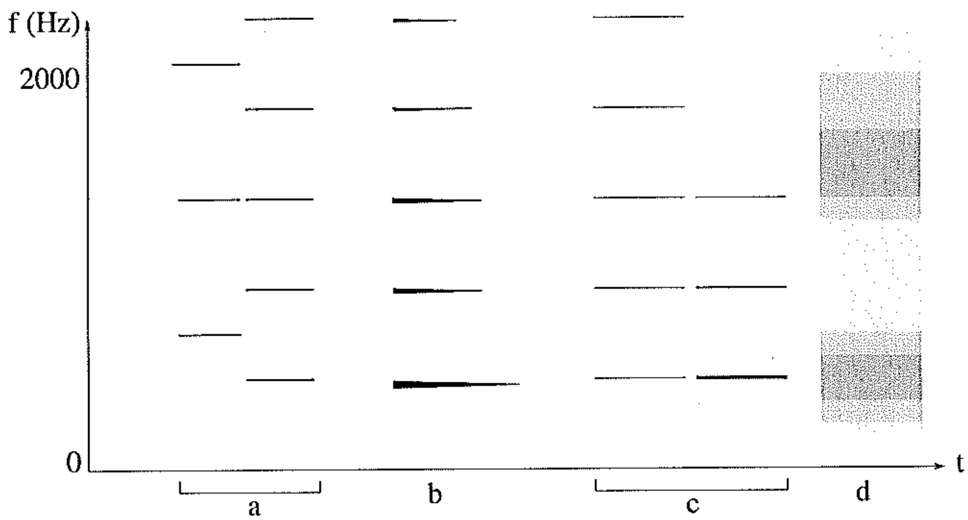


$$\Delta f = \frac{f_0}{Q} \approx 7.3\text{ Hz}$$

range is $[f_0 - \frac{\Delta f}{2}, f_0 + \frac{\Delta f}{2}] = [46.3, 53.7]\text{Hz}$,

ie around just under 4 Hz either side of 50Hz

6. [10 points] A spectrogram is shown for a sequence of several sounds. For time periods a, b, c, and d below, describe what might be heard, stating any changes in pitch (up or down?), timbre, and overall amplitude, that occur within that time period:



[6]

a: sequence of two periodic signals, ie musical notes, both of constant amplitude (eg violin, flute)... Pitch goes down (by ratio 3:2, ie perfect fifth), timbre stays similar.

b: musical note of definite pitch decaying in amplitude.

c: two musical notes of the same pitch but different timbre. The second note is mellower (less harsh) than the first, since more amplitude of low coefficients c_1, c_2.

d: hissing noise sound with stronger frequency components in certain bands

[4] What instrument or action is most likely to produce sound b? Does it have a perceived pitch?

plucked string (since equally-spaced partials) or possibly slapped tube (open-open case). yes.

What instrument or action is most likely to produce sound d? Does it have a perceived pitch?

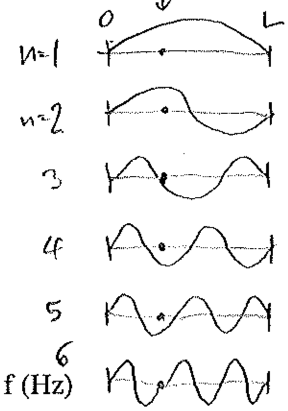
whispering a constant vowel sound, such as 'ee'. No. (no narrow partials) mode shapes $U_n(x) = \sin\frac{n\pi x}{L}$

7. [10 points] A guitar string of fundamental frequency 200 Hz is plucked 1/3 of the way up from the bridge.

$x = L/3$

[4] (a) By computing the excitation coefficients $\alpha_1, \alpha_2, \ldots, \alpha_6$, or otherwise, plot a spectrum that could be produced acoustically (go only up to 1200 Hz):

[we ignore $\frac{1}{n^2}$ factor for plucking in this basic model].



$\alpha_1 = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

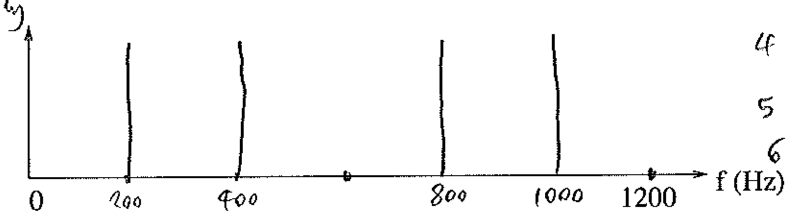
$\alpha_2 = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\alpha_3 = \sin\frac{3\pi}{3} = 0$

$\alpha_4 = \sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

$\alpha_5 = \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

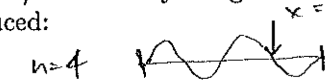
$\alpha_6 = \sin\frac{6\pi}{3} = 0$.



damping, as in what string players call 'harmonics'.

[2] (b) The player's finger now *lightly* touches the string 3/4 of the way up (i.e. 1/4 of the way along from the neck end), and plucks as before. Plot the acoustic spectrum produced:

$x = 3L/4$

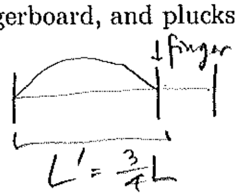


is the only mode in range $n=1$ to $n=6$ that has a node at $3L/4$.

What is the perceived pitch? (leave this as a frequency)

Higher $(n>6)$ modes that are undamped are $n = 8, 12, \ldots$ multiples of 4, so freqs are multiple of 800Hz. $\Rightarrow$ perceive 800 Hz.

[2] (c) The player now instead *heavily* presses the string at the same point as in b), pressing it against the fingerboard, and plucks as before. What now is the perceived pitch? (leave this as a frequency)

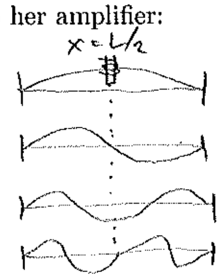


The length L of string now changes to $L' = \frac{3}{4}L$

Assuming tension constant, $\frac{f_2}{f_1} = \frac{c/2L'}{c/2L} = 4/3$ so $f_2 = \frac{4}{3}f_1$ ≈ 267 Hz

original 200Hz $\rightarrow f_1$

[2] (d) Removing the finger as in a), the guitarist switches on an electric pickup 1/2 way along the string, and plucks as before. Plot a spectrum of the electrical signal now sent to her amplifier:



detection of pickup:

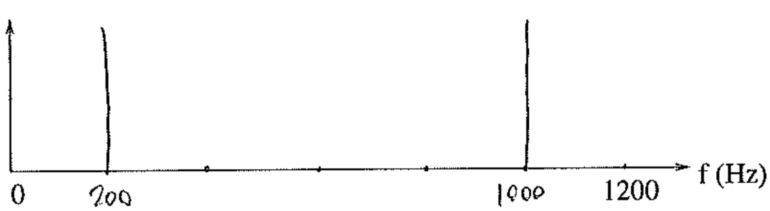
$\sin\frac{\pi}{2} = 1$

$\sin\pi = 0$

$\sin\frac{3\pi}{2} = -1$

$\sin 2\pi = 0$

⋮

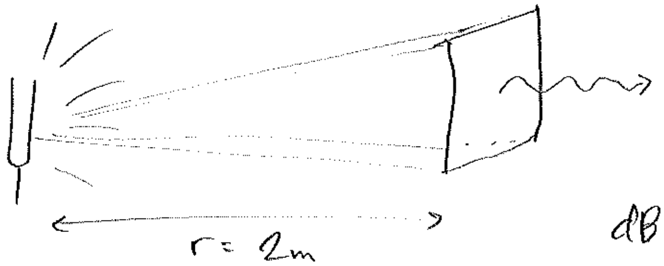


[kill the even multiple of 200Hz from part a).

etc: odd harmonics picked up maximally, even ones absent.

8. [8 points] A tuning fork is struck and produces a pure sinusoid at 300 Hz. A listener is a distance 2 m from the tuning fork.

[3] (a) Initially the tuning fork radiates 0.005 W acoustic power in all directions. What intensity in dB does the listener hear?



$$I = \frac{P}{4\pi r^2} = 9.95\times10^{-5}\ W/m^2$$

$$dB = 10\log_{10}\frac{I}{I_r} = 10\log_{10}\frac{9.95\times10^{-5}}{10^{-12}}$$

$$= 79.98..\ dB \approx 80.0\ dB.$$

[2] (b) The Q-factor of the tuning fork is 1000. What is the decay time?

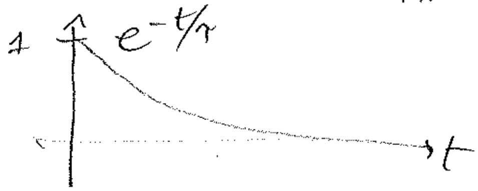
$$Q = \pi f_0 \tau \quad \text{so} \quad \tau = \frac{Q}{\pi f_0} = \frac{10^3}{\pi\, 300} \approx 1.06\ sec$$

[3] (c) How long since it was struck with the above initial strength does it take until the intensity at the listener reaches the lower threshold of human hearing which is about 10^{-10} W/m^2 at 300 Hz? (careful, not 10^{-12} W/m^2)

(Note: travel time to listener = 0.0058 s, negligible!)

Intensity must decay from 9.95×10^{-5} W/m^2 to 10^{-10} W/m^2

ie $\frac{I_2}{I_1} = \frac{10^{-10}}{9.95\times10^{-5}} = 1.01\times10^{-6}$



amplitude ratio $= e^{-t/\tau}$ in exponential decay. $= \sqrt{\frac{I_2}{I_1}}$

$$\Rightarrow e^{-t/\tau} = \sqrt{1.01\times10^{-6}} = 1.00\times10^{-3}$$

$$\Rightarrow t = -\tau\ln(10^{-3}) = -(1.06)\ln(10^{-3}) \approx \underline{7.3\ s}.$$

due to ambiguity in question, this could also mean: what is travel delay to a distant listener for whom I is initially 10^{-10} W/m^2. Ans: $\underline{5.87\ s}$.

ratios: 2.586, 1.496 $\approx 3/2$, 1.336 $\approx 4/3$, 1.900

9. [8 points] Short unrelated questions.

[3] (a) A bell produces the following partials all at roughly equal amplitudes: 302, 781, 1168, 1560, 2964 Hz. What 'strike tone' (perceived pitch) frequency is perceived, and why?

ratios $781 : 1168 : 1560$ are very close (within <1%)

to $2 : 3 : 4$

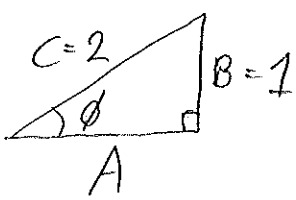
with the (missing) fundamental f being $\frac{781}{2} \approx \frac{1168}{3} \approx \frac{1560}{4}$

$\uparrow$ 390.5 $\quad\uparrow$ 389.3 $\quad\uparrow$ 390

$f \approx 390$ Hz (within ± 1 Hz)

Note: 302 & 2964 Hz don't fall into the harmonic series, so don't contribute to perceived pitch.

[3] (b) Find A such that the pure tone signal $A \sin \omega t + \cos \omega t$ has an amplitude of 2.



$A \sin \omega t + B \cos \omega t = C \sin(\omega t + \phi)$

A, B, C, ϕ related by right triangle shown.

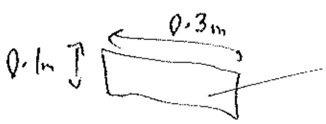
C & B are known; seek A & don't care about ϕ.

Use Pythagoras: $C^2 = A^2 + B^2$ so $A = \sqrt{C^2 - B^2}$

$= \sqrt{2^2 - 1^2} = \sqrt{3}$

(strictly, $A = -\sqrt{3}$ also a solution).

[2] (c) When you open a window in a moving car, a Helmholtz resonance may be excited (as when blowing over a bottle). If the window opening is a rectangle 10 cm by 30 cm, the effective neck length is 20 cm, and the resonant frequency is 12.1 Hz, compute the volume of the car cabin in m^3. [BONUS: Comment on its audibility.]



$a = 0.1(0.3) = 0.03 \text{ m}^2$ $\qquad \ell = 0.2\text{m}$.

$f_{Helm} = \frac{c}{2\pi}\sqrt{\frac{a}{V\ell}}$ $\qquad$ solve for V

$\frac{a}{V\ell} = \left(\frac{2\pi f_{Helm}}{c}\right)^2$, or $V = \frac{a}{\ell}\left(\frac{c}{2\pi f_{Helm}}\right)^2$

$V = \frac{0.03}{0.2}\left(\frac{340}{2\pi(12.1)}\right)^2 \approx \underline{3.00 \text{ m}^3}$ (about right for ≈ 100 cu ft car cabin)

Bonus: 12 Hz is subsonic (below human hearing range), will feel as pressure in chest, eardrums, vibration.