Hamiltonian dynamical systems with infinitely many periodic orbits and Floer homology

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007 Kemeny, 4:15PM
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Abstract

A general, but not universal, feature of Hamiltonian dynamical systems is that such systems tend to have numerous periodic orbits. In fact, as the Conley conjecture asserts, for a broad class of closed symplectic manifolds, every Hamiltonian diffeomorphism (the time-one map of a Hamiltonian flow) has infinitely many simple periodic orbits. On the other hand, it is easy to see that the conjecture fails for a general symplectic manifold such as the two-dimensional sphere or the Euclidean space. One variant of the Conley conjecture applicable to such manifolds, inspired by a celebrated theorem due to Franks, asserts that a Hamiltonian diffeomorphism with "more than necessary" fixed points has infinitely many simple periodic orbits. Here the threshold is usually interpreted as a lower bound arising from some version of the Arnold conjecture; for instance, it is n+1 for the n-dimensional complex projective space. For the two-sphere, the assertion is a special case of Franks' theorem: every area preserving homeomorphism of the sphere with more than two fixed points has infinitely many periodic points. In this talk we will discuss various aspects of the existence question for periodic orbits of Hamiltonian diffeomorphisms, focusing on recent results obtained by Floer homological methods in the realm of higher dimensional generalizations of Franks' theorem.

This talk should be accessible to graduate students.