Homework 2

T1.14 level 0 at $x = 1/2$ = Break.

Breaks on level 1 of itinerary are where $G(x) = 1/2$.

Breaks on level 2 of itinerary are where $G^2(x) = 1/2$.

plus breaks from lower levels.

Pts where $G^2(x) = 1/2$ are

$x \approx 0.30866, 0.69134, 0.03806, 0.96194$.

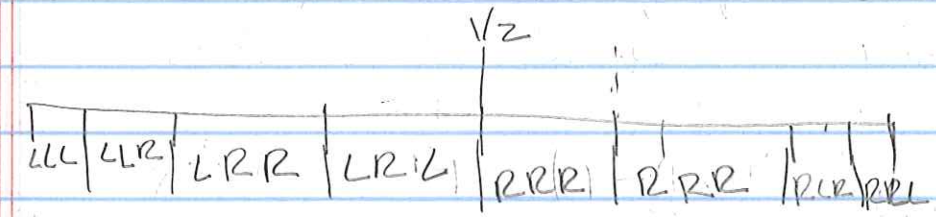
Pts where $G(x) = 1/2$ are $x \approx 0.14645, 0.85355$.

So $LL = (0, 0.0386)$ $LLR = (0.0386, 0.14645)$

(a) take $x_0 = 0.1$

& $RRL = (0.5, 0.69134)$

(b) take $x_0 = 0.6$.




T1.16

a) $x_0 > 0.5$

b) If $x_0 \gtrless 0.5$, then $G^6(x_0) \in LRLR$.

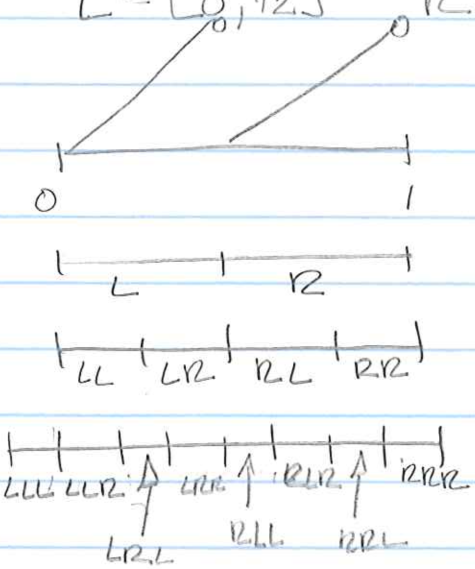
$x_0 < 1/2$.

NOTE: $x_0$ cannot equal $1/2$ since itinerary does not have an infinite repeat at end.

1.6 $f(x) = 2x \pmod 1$ on $[0,1]$

$L = [0, 1/2]$ $R = [1/2, 1]$

(a)




(b)


(c) each interval on level $k$ is $1/2^k$

let $J$ denote the 1st $k$ symbols ie $J = S_1 \cdots S_k$

Then pick $x_0 \in JLR$ then $y_0 \in JRL$ satisfies

$|x_0 - y_0| < 1/2^{k+1} = \varepsilon$.

but $|y_k - x_k| > 1/4$.

1.8 $f(x) = 4x(1-x)$

Prove that there are pts in $I = [0,1]$ that are not fixed pts, periodic pts, or eventually periodic pts of f.

Prove by contradiction. Using Itineraries.

Infact any infinite sequence where that does not have a repeating pattern will suffice.

$$\begin{array}{r} 3 \\ 18 \\ \times 4 \\ \hline 72 \\ +49 \\ \hline 131 \end{array}$$

2.1

(a) $\begin{vmatrix} 4-\lambda & 30 \\ 1 & 3-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda) - 30$

$$= 12 - 3\lambda - 4\lambda + \lambda^2 - 30$$

$$= \lambda^2 - 7\lambda - 18 = 0$$

$$\lambda = \frac{7 \pm \sqrt{49 - 4(-18)}}{2}$$

$$= 7/2 \pm \sqrt{49 + 72}$$

$$= 7/2 \pm \tfrac{1}{2}\sqrt{121} = 9, -2.$$

$|\lambda| > 1 \Rightarrow (0,0)$ is a source.

(b) $\begin{vmatrix} 1-\lambda & 1/2 \\ 1/4 & 3/4-\lambda \end{vmatrix} = (1-\lambda)(3/4-\lambda) - 1/8$

$$= \lambda^2 - 3/4\lambda - \lambda - 1/8 + 3/4$$

$$= \lambda^2 - 7/4\lambda - 1/8 + 3/4$$

$\lambda = 1\,1/4,\ 1/4$

$\Rightarrow (0,0)$ is a saddle pt.

(c) $\begin{vmatrix} -0.4-\lambda & 2.4 \\ -0.4 & 1.6-\lambda \end{vmatrix} = (-0.4-\lambda)(1.6-\lambda) + 0.4(2.4)$

$$= \lambda^2 + 0.4\lambda - 1.6\lambda + (-0.4)(1.6) + 0.4(2.4)$$

$\lambda = 0.4, 0.8.$

$\Rightarrow (0,0)$ is a sink.

2.3 $g(x,y) = \begin{pmatrix} x^2 - 5x + y \\ x^2 \end{pmatrix}$

1st Find the fixed pts. ie solve

$\left\{ \begin{array}{l} x = x^2 - 5x + y \\ y = x^2 \end{array} \right\}$ at the same time.

$x = x^2 - 5x + x^2 \rightarrow 0 = 2x^2 - 6x$

$x = 0, 3$

$\Rightarrow y = 0, 3$

$(0,0)$, $(3, 9)$ are fixed pts.

2nd We need to clasify

Need to look at Df evaluated at fixed pts.

$Df(\bar{x}) = \begin{pmatrix} 2x - 5 & y \\ 2x & 0 \end{pmatrix}$

$Df\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & 0 \end{pmatrix}$ eigenvalues $-5, 0$.

$\rightarrow$ origin is a saddle pt

$Df\begin{pmatrix} 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 6-5 & 9 \\ 6 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 9 \\ 6 & 0 \end{pmatrix}$

find eigenvalues. $\begin{vmatrix} 1-\lambda & 9 \\ 6 & -\lambda \end{vmatrix} = -\lambda(1-\lambda) - 54$

$= \lambda^2 - \lambda - 54 = 0$

$\lambda = \dfrac{1 \pm \sqrt{1 + 4(54)}}{2} \Rightarrow |\lambda| > 1$ $\} \rightarrow$ source.

T2.5 Goal: Prove Hénon map has a period 2 orbit iff $4a > 3(1-b)^2$

$$f(x,y) = \begin{pmatrix} a - x^2 + by \\ x \end{pmatrix}$$

$$f^2(x,y) = f(f(x,y)) = \begin{pmatrix} a - (a-x^2+by)^2 + bx \\ a - x^2 + by \end{pmatrix}$$

We are looking for fixed pts of $f^2$ so solve

① $x = a - (a-x^2+by)^2 + bx$
② $y = a - x^2 + by$ } at same time

Solve ② for $y$ & plug into ①.

we get $(x-a)^2 + (1-b)^3 x - (1-b)^2 a = 0$

$$(x^2 - (1-b)x - a + (1-b)^2)\underbrace{(x^2 + (1-b)x - a)}_{\text{P-1 fixed pts.}} = 0.$$

so period-2 fixed pts come from

$$x^2 - (1-b)x - a + (1-b)^2 = 0.$$

$$x = \frac{(1-b) \pm \sqrt{(1-b)^2 - 4(-a + (1-b)^2)}}{2}$$

$$= \frac{(1-b) \pm \sqrt{4a - 3(1-b)^2}}{2}$$

The only way we get a real # is if

$$4a > 3(1-b)^2$$ //