

Answer Key

1.

$$y = \frac{t}{3} - \frac{1}{9} + e^{-2t} + Ce^{-3t}.$$

2.

$$y = (2 - 5t)e^{2t}.$$

3.

$$-\frac{\pi}{4} + \sum_{n \text{ odd}} \frac{2}{n^2\pi} \cos(nx) + \sum_{n \geq 1} (-1)^n \frac{1}{n} \sin(nx).$$

4.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -\frac{3}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t} + \frac{7}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}.$$

5.

$$u(x, t) = \sum_{n \geq 1} c_n \sin(n\pi x) \cos\left(t\sqrt{\alpha^2 + a^2 n^2 \pi^2}\right)$$

where

$$c_n = 2 \int_0^1 f(x) \sin(n\pi x) dx.$$