Math 105
Homework Problems 1

January 10, 2006

Exercise 1. Let K be a field and let $p(x) \in K[x]$ be an irreducible polynomial (monic, if you like). Since the ring $K[x]$ is a Dedekind ring, its localization R at the prime ideal $(p(x))$ is a DVR. This gives rise to a valuation $|\cdot|_p$ on the quotient field $K(x)$ (as discussed in class) whose valuation ring is precisely R. Let $\mathfrak{p}$ denote the maximal ideal of R.

a. Identify the residue field $L = R/\mathfrak{p}$.

b. Show that, when $p(x)$ is linear, the completion of $K(x)$ relative to $|\cdot|_p$ is isomorphic to the field $K((z))$ of Laurent series over the field K in the indeterminate z. [*Note:* For general nonarchimedean valuations this is *not* true.]

Exercise 2. Exercise 4 on page 99.

Exercise 3. Exercise 5 on page 99. You can use Hensel's lemma to do this exercise if you like, but it's not absolutely necessary.

Exercise 4. Exercise 6 on page 99. Hensel's lemma doesn't help here. Why not?

Exercise 5. Exercise 7 on page 99.