Math 11, Fall 2007

Lecture 24

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Outline

1. Review and overview
   - Last class
2. Today's material
   - Parametric surfaces
   - Surface area
3. Next class

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Divergence and Curl

- Curl: measures rotation
- Divergence: measures contraction/expansion
- Flux integrals: divergence form of Green's theorem
- Circulation integrals: curl version of Green's theorem

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Different representations

- Recall the defition of a spacecurve:

$$\vec{c}(t) = x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k}$$

- We can describe surfaces in a similar way:

$$\vec{r}(u, v) = x(u, v)\,\vec{i} + y(u, v)\,\vec{j} + z(u, v)\,\vec{k}$$

- Example: spherical coordinates

$$\vec{r}(\theta, \phi) = \sin(\phi)\cos(\theta)\,\vec{i} + \sin(\phi)\sin(\theta)\,\vec{j} + \cos(\phi)\,\vec{k}$$

Examples

- Graphs $z = f(x, y)$

$$\vec{r}(u, v) = \langle u, v, f(u, v) \rangle$$

- Surfaces of revolution: $z = f(x)$ rotated about the z-axis

$$\vec{r}(u, v) = \langle u \cos(v), u \sin(v), f(u) \rangle$$

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Surface Area

Idea: Approximate the area of a small patch of surface by the area of the corresponding piece of the tangent plane at that point.

1. Find tangent plane to $z = f(x, y)$
$$\vec{N} = \langle f_x, f_y, -1 \rangle$$
2. Area of parallelogram on the tangent plane is $|\vec{N}| = \sqrt{1 + |\nabla f|^2}$
3. Area=$\int_D |\vec{N}| \, dA$

Surface area

- For a surface given as $f(x, y) - z = 0$, for $(x, y) \in D$, the surface area is given as

$$\iint_D |\vec{N}| \, dA$$

- To use the same idea with a parameteric surface, we must compute the normal:

$$\vec{N} = \vec{r}_u \times \vec{r}_v$$

Examples

- Find the surface area of the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = h$.
- Find the surface area of a cone $z^2 = x^2 + y^2$ between $z = 0$ and $z = 1$.

Work for next class

- Reading: 17.7
- Webwork: f07hw26