V63.0123-1 : Calculus III. Sample Final Solutions. Spring 2003

1. [10 points]

   (a) $e - e^{-1} + 2$.

   (b) $\sqrt{e}/(e+1)^2$.

2. [10 points]

   Use x as parameter since never bends back on itself in x. So $x(t) = t$, $y(t) = (t^2 - 1)/2$, $z(t) = (t^2 + 1)/2$.

3. [10 points]

   Ans: $\pi(e-1)/6$.

4. [10 points]

   (a) $\nabla \cdot \nabla f = \nabla \cdot [n(r^2)^{n/2-1}\mathbf{r}] = n(n+1)r^{n-2}$.

   (b) $n = 0$ and -1.

5. [10 points]

   (a) curl $\mathbf{F} = \mathbf{0}$ so it is conservative.

   (b) Find the scalar field $f(\mathbf{r}) = xz + y + K$ which gives $\mathbf{F} = \nabla f$. Start point $(1, 1, 1)$, end point $(2, 3, 4)$. Ans is difference in f is 9.

6. [10 points]

   (a) div $\mathbf{F} = 3$, and use Divergence Theorem.

   (b) Recognize $\mathbf{F} = 1.\hat{n}$ makes flux easy. Ans $4\pi/3$.

7. [10 points]

   $m = 1/3, \bar{x} = 3/5, \bar{y} = 3/4$.

8. [10 points]

   (a) $\sqrt{2}\pi$

   (b) $4\pi/15$.