Math 46: Applied Math: Homework 9

due Wed June 1—half length one (to give you time to study)!

This covers the final topic: Fourier transform and its use in solving PDEs

p.395-398: #4. As a function of ξ this is called a Cauchy distribution. It comes up in statistics and has an infinite variance.

#5. b, c. Quick. These show that translation becomes multiplication in Fourier space.

#7. Once (or even before!) you've solved, answer this: how is the solution $u(x,t)$ at time t related to the solution for the case $c = 0$ at the same time t ? [Hint: the previous question is useful here]

A) Use the *sifting property*

$$\int_{-\infty}^{\infty} \delta(x-a) f(x) dx = f(a)$$

to find the Fourier transform of the delta distribution $\delta(x-a)$. Now write the inversion formula—this gives you a new and useful representation of the delta distribution. By interchanging the labels x and ξ, deduce the Fourier transform of the plane wave function e^{ikx}. Add your answer to Table 6.2.

#10. [Hint: write out $|\hat{u}(\xi)|^2 = \hat{u}(\xi)\overline{\hat{u}(\xi)}$ using a double integral, use the above, then simplify]. This is the continuous analogue of Parseval's equality on p. 213. The Fourier transform is a (continuous rather than countably infinite) orthogonal expansion.

#11.

#15. I suggest you don't use the hint until you have a convolution expression for $u(x,y)$ as in Example 6.35, of which you may piggyback off the final result. You may use the boundary condition $\lim_{y\to\infty} u(x,y)$ is bounded. The problem corresponds to injecting current density into the edge of a resisitive medium and solving for the voltage field—a useful medical imaging technique (Electrical Impedance Tomography).