

Supplementary Homework for Math 43
Due Monday, May 13, 2002

S1: Suppose that f has an essential singularity at z_0 . Then prove that $h(z) := \exp(f(z))$ also has an essential singularity at z_0 .

S2: Suppose that f is analytic everywhere except $z = 0$ and that for all $z \neq 0$, we have

$$0 \leq |f(z)| \leq \sqrt{|z|}.$$

Prove that f is identically zero.

S3: Suppose that f has an isolated singularity at z_0 and

$$\lim_{z \rightarrow z_0} (z - z_0)f(z) = L$$

for some $L \in \mathbf{C}$. Show that f has a simple pole at z_0 and that $\text{Res}(f; z_0) = L$.