Math 13, Final Exam, March 11, 2002

Each problem is worth 7.5 points.

1. The arclength of the curve $\mathbf{c}(t) = (2\cos t, \sqrt{3}t, 2\sin t)$ on $1 \le t \le 7$ is:

   (a) $\sqrt{2}\pi$

   (b) $6\sqrt{7}$

   (c) 7π

   (d) 54

2. The equation of the plane that passes through point $(2, 4, -1)$ and is perpendicular to $l(t) = (-1, -2, 3)t + (0, 7, 1)$ is:

   (a) $7y + z + 1 = 28$

   (b) $-x + 2y + 3z = -11$

   (c) $7y - z + 1 = 22$

   (d) $x + 2y - 3z = 13$

3. Without regard to orientation, match the following geometric curves (left column) to their parametrization (right column).

(1) $9x^2 + 4y^2 = 4$	(i) $\mathbf{c}(t) = (2\cos t, 2\sin t, 2\sin t + 1)$
(2) $x^2 + y^2 = 4$	
intersect $y = z - 1$	(ii) $\mathbf{c}(t) = (\frac{3}{2}\cos t - \frac{1}{2}, \frac{3}{2}\sin t, \frac{3}{2}\cos t + \frac{3}{2})$
(3) $y = 4 - \sqrt{1 - x^2}$	(iii) $\mathbf{c}(t) = (\cos t, 4 - \sin t)$
(4) $z = 4 - x^2 - y^2$	
intersect $x = z - 2$	(iv) $\mathbf{c}(t) = (\frac{2}{3}\cos t, \sin t)$

   (a) 1-(ii), 2-(iv), 3-(iii), 4-(i)

   (b) 1-(iv), 2-(i), 3-(iii), 4-(ii)

   (c) 1-(ii), 2-(i), 3-(iv), 4-(iii)

   (d) 1-(iv), 2-(ii), 3-(iii), 4-(i)

4. Evaluate $\int_1^2 \int_{x^3}^{x} e^{y/x}\, dy\, dx$.

   (a) $e - \frac{e^4}{2}$

   (b) $4e - e^4$

   (c) Can't be evaluated without tables

   (d) $2e - \frac{e^4}{2}$

5. The rate of change of $f(x,y) = 2x^2y + e^x$ in direction parallel to vector $\mathbf{v} = (3,4)$ at point $(0, \ln 2)$ is:
   (a) $\frac{3}{5}$
   (b) 0
   (c) 3
   (d) $4\ln 2 + 2$

6. The volume inside the paraboloid $z = 4 - x^2 - y^2$ above $z = 2$ is:
   (a) $\frac{8\sqrt{2}}{3}\pi$
   (b) 4π
   (c) 2π
   (d) $\sqrt{2}\pi$

7. If $\nabla f = \mathbf{F}$, and $\mathbf{F} = (2y^2 - 1, 4xy + z, y)$, then f is:
   (a) $(2y^2 - 1)x + zy$
   (b) $(2xy + z)y$
   (c) $(z + 2xy)y$
   (d) $2xy^2 - x$

8. The work done by a force vector field $\mathbf{F} = \frac{1}{x}\mathbf{j}$ on a particle following the path given by $\frac{x^2}{2} + \frac{y^2}{4} = 1$ from point $(0,-2)$ to $(0,2)$ (counterclockwise) is:
   (a) 0
   (b) $-\sqrt{2}\pi$
   (c) π
   (d) $\sqrt{2}\pi$

9. If $3x^3y - 5x^2 + yz = 1$ is a level surface, then the direction of fastest change from the point $(1, 6, -2)$ is:
   (a) $(26, 3, 6)$
   (b) $(26, 1, 6)$
   (c) $(38, 3, 6)$
   (d) $(44, 1, 6)$

10. Let a surface be given parametrically by $\Phi(u,v) = (v, uv^2, u+v)$. The equation (in rectangular coordinates) of the plane tangent to the surface at point $(x,y,z) = (2,4,3)$ is:
   (a) $y - 4z = -8$
   (b) $-3x + y - 4z = -14$
   (c) $-12x + y + 16z = 28$
   (d) $y + 4z = 8$

11. Evaluate $\int_{\mathbf{c}} -2ydx + 5xdy$ if $\mathbf{c}$ is the counter-clockwise path around the triangle in the x-y plane with vertices $(0,0)$, $(2,1)$, and $(5,0)$.
    (a) 21
    (b) $\frac{-21}{2}$
    (c) $\frac{35}{2}$
    (d) -7

12. The surface area of the portion of $z = 1 + x^2 + y^2$ where $1 \leq z \leq 2$ is:
    (a) $\frac{\pi}{3} 5\sqrt{5}$
    (b) 4π
    (c) $\frac{\pi}{6}(5\sqrt{5} - 1)$
    (d) $\frac{\pi}{4}(\sqrt{5} - 1)$

13. If $f(x, y, z) = (3x^2 + y, zx, 4x^2 + 1)$ and $g(u, v, w) = (5uv, w + 3)$, then the size of the derivative matrix of the composition of f and g is:
    (a) 3×3
    (b) 3×2
    (c) 2×3
    (d) 3×1

14. Using the Fundamental Theorem of Line Integrals, evaluate $\int_{\mathbf{c}} 2ydx + 2xdy$ for $\mathbf{c}(t) = \left(\frac{1}{9-t^3}, \sqrt{2+t}\right)$ and $1 \leq t \leq 2$.
    (a) $2 - \frac{\sqrt{3}}{8}$
    (b) $2\sqrt{3} - 4$
    (c) $\sqrt{3} + \frac{1}{8}$
    (d) $4 - \frac{\sqrt{3}}{4}$

15. Which of the following does NOT represent a sphere? (Consider all variables defined as cylindrical, spherical, or rectangular coordinates as usual).
    (a) $z = \pm\sqrt{1 - 3r^2}$
    (b) $\Phi(\theta, \phi) = (2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi)$
    (c) $x^2 + y^2 - 8y + z^2 + 15 = 0$
    (d) All of the above represent spheres

16. The flux through the surface $z = 2xy + x^2$, in the positive z direction, of the vector field $\mathbf{F} = -\mathbf{i} + y^2\mathbf{j} + 2xy^2\mathbf{k}$ over the region $D : [0,4] \times [0,2]$ is

(a) 48

(b) $56\frac{1}{3}$

(c) $82\frac{2}{3}$

(d) 36

17. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j}$. Let D be a closed region in the xy plane such that $\partial D = C$. Let C be parametrized by $\mathbf{c}(t)$ for $a \leq t \leq b$. Which of the following is NOT equal to $\int_C \mathbf{F} \cdot d\mathbf{s}$?

(a) $\int\int_D (\nabla \times \mathbf{F}) \cdot \mathbf{k} dxdy$

(b) $\int_a^b F_1 dx + F_2 dy$

(c) $\int_C \mathbf{F} \cdot \mathbf{n} ds$

(d) $\int\int_D \frac{\delta F_2}{\delta x} - \frac{\delta F_1}{\delta y} dxdy$

18. The curl of $\mathbf{F} = (3z^2x, 2xy, -6y\cos z)$ is:

(a) $(-6\cos z, -3z^2, 6y)$

(b) $(-6\cos z, 6zx, 2y)$

(c) $(3z^2, 2x, 6y\sin z)$

(d) $(-6zx, 2y, 6y\sin z)$

19. Find $\int\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ if S is the surface $z = x^2 + y^2$ where $z \leq 4$, oriented so that the normal points into the paraboloid, and $\mathbf{F} = -zy\mathbf{i} + zx\mathbf{j} + z^3\mathbf{k}$.

(a) 0

(b) 32π

(c) 64π

(d) -64π

20. Find the flux out of the solid half unit sphere above the xy plane, if $\mathbf{F} = 4z^3y\mathbf{i} + 2y\mathbf{j} + (z^2 - 2z)\mathbf{k}$.

(a) $\frac{2\pi}{3}(\pi + 2)$

(b) $\frac{1}{4}$

(c) $\frac{4}{3}\pi$

(d) $\frac{\pi}{2}$