Math 11, Fall 2007

Lecture 11

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Outline

1. Review and overview
   - Last class
2. Today's material
   - Extremal values
   - Canonical examples
   - More examples
3. Next class

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Differentiation
The gradient vector field

- $$\nabla f = \langle f_x, f_y \rangle$$
- The gradient encodes all the derivative information:
  1. $D_{\vec{u}} f = \nabla f \cdot \vec{u}$
  2. The gradient points in the direction of maximal ascent
  3. Extremal values must occur when $|\nabla f| = 0$
  4. For $f : \mathbb{R}^3 \to \mathbb{R}$, ∇f gives a normal vector to the level surface, $f(x, y, z) = 0$.

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Locating Max/Min values

If $z = f(x, y)$ has a maximum or minimum value at (x_0, y_0) then $\nabla f(x_0, y_0) = \vec{0}$.

Example: $f(x, y) = x^3y + 12x^2 - 8y$

$$f_x = 3x^2y + 24x, \; f_y = x^3 - 8$$

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Second derivative test

Suppose the second partial derivatives of f are continuous on a disk with center (x_0, y_0) and that $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$. Let

$$D = f_{xx}(x_0, y_0)f_{yy}(x_0, y_0) - (f_{xy}(x_0, y_0))^2$$

- If $D > 0$ and $f_{xx}(x_0, y_0) > 0$ then $f(x_0, y_0)$ is a local minimum.
- If $D > 0$ and $f_{xx}(x_0, y_0) < 0$ then $f(x_0, y_0)$ is a local maximum.
- If $D < 0$ then $f(x_0, y_0)$ is a not a local minimum or maximum.
- If $D = 0$ then the test is inconclusive

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Simple examples

- $f(x, y) = x^2 + y^2$
- $f(x, y) = xy$
- $f(x, y) = -x^2 - y^2$

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Examples

- $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$
- $f(x,y) = \sin(x^2 + y^2)$

Work for next class

- Reading: 15.7
- Exam 1!
- f07hw12