1. (30) (**Show all work**). A mass of 2 kg is suspended from a spring in a viscous fluid. The damping constant is 12 and spring constant is 68 newtons/meter. Let $y(t)$ be the position of the mass at time t with downward direction positive.

   (a) (5) Write down a differential equation which describes the motion of this mass.

   (b) (15) If the mass has an initial position 2 meters below the rest (equilibrium) position and is moving downward at 9 m/sec, find the solution $y(t)$ to the differential equation.

   (c) (10) In a similar situation with a frictionless spring (i.e., the spring is in the air and air resistance is neglected), the solution turns out to be $y(t) = \sqrt{3}\cos(5t) + \sin(5t)$. What is the value of t when the mass first reaches it maximum distance from equilibrium?

2. (20) (**Show all work**). Let $f(x) = \ln(1 + x)$.

   (a) (10) Find a power series in x about $a = 0$ (i.e. a Maclaurin series) for f. You need only write down the first four nonzero terms.

   (b) (5) Determine the radius of convergence of the above series.

   (c) (5) Write down a power series in x about $a = 0$ (i.e. a Maclaurin series) for the derivative, $f'(x)$. You need only write down the first four nonzero terms.

3. (32) **Multiple Choice** Circle the correct response. (No partial credit will be given)

   (a) $y = c_1e^{-3t} + c_2te^{-3t}$ is the general solution to which differential equation?

   A. $y'' - 6y' + 9y = 0$ **B**. $y'' - 9y = 0$

   C. $y'' + 6y' + 9y = 0$ **D**. $y'' + 9y = 0$ **E**. $y'' - 9y' = 0$

   (b) $y = e^{3t}(c_1 \sin(2t) + c_2 \cos(2t))$ is the general solution to which differential equation?

   A. $y'' + 6y' - 13y = 0$ **B**. $y'' - 13y' + 6y = 0$

   C. $y'' + 4y = 0$ **D**. $y'' - 6y' + 5y = 0$ **E**. $y'' - 6y' + 13y = 0$

   (c) Find the general solution to the differential equation $xy' + 7y = e^{x^7}$.

A. $y = \frac{e^{x^7}}{7x^7} + C$ **B.** $y = \frac{e^{x^7}}{7} + C$ **C.** $y = \frac{e^{x^7}}{x^7} + \frac{C}{x^7}$

D. $y = \frac{e^{x^7}}{7x^7} + \frac{C}{x^7}$ **E.** None of these

(d) If $y = c_1e^{3t} + c_2e^{2t}$ is the general solution to the differential equation $y'' - 5y' + 6y = 0$, solve the initial value problem with $y(0) = 4$ and $y'(0) = 5$.

A. $c_1 = 3, c_2 = 7$ **B.** $c_1 = -3, c_2 = 7$ **C.** $c_1 = 7, c_2 = 0$

D. $c_1 = 3, c_2 = -7$ **E.** $c_1 = -3, c_2 = -7$

(e) If $z = 3e^{2\pi i/7}$, then z^{16} is in which quadrant?

A. First **B.** Second **C.** Third **D.** Fourth

E. On an axis

(f) $(3 - 4i)^{-1} = \frac{1}{3 - 4i} =$

A. $\frac{3}{5} + i\frac{4}{5}$ **B.** $\frac{3}{5} - i\frac{4}{5}$ **C.** $\frac{3}{25} + i\frac{4}{25}$ **D.** $\frac{3}{25} - i\frac{4}{25}$

E. None of these

(g) If you invest money in a bank at 5% interest compounded continuously, how many years will it take to double?

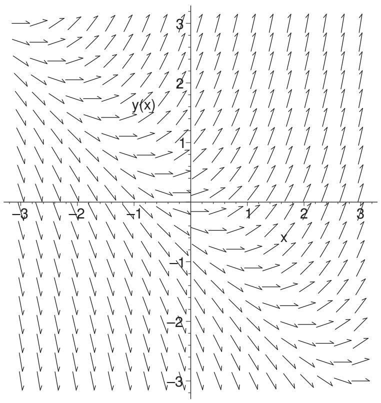
A. $\frac{\ln 2}{.05}$ **B.** $\frac{.05}{\ln 2}$ **C.** $5 \ln 2$ **D.** $2 \ln 5$

E. Depends on initial investment

(h) The direction field below corresponds to which differential equation?

A. $\frac{dy}{dx} = x - y$ **B.** $\frac{dy}{dx} = x + y$ **C.** $\frac{dy}{dx} = y - x$

D. $\frac{dy}{dx} = x^2 - y^2$ **E.** $\frac{dy}{dx} = 1$



4. (18) **True/False.** Circle the correct response.

T **F** Consider the infinite series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots$ and $\sum_{n=1000}^{\infty} a_n = a_{1000} + a_{1001} + \cdots$. Then $\sum_{n=1}^{\infty} a_n$ diverges if and only if $\sum_{n=1000}^{\infty} a_n$ diverges.

T **F** Let s_n be the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$. Then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\lim_{n\to\infty} s_n = 0$.

T **F** If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

T **F** If $a_n \leq b_n \leq c_n$ for all $n = 1, 2, 3, \ldots$, and $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} c_n = M$, then $\lim_{n\to\infty} b_n$ exists.

T **F** The sum of the first four nonzero terms of the Maclaurin series of e^{-x^2} is $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$.

T **F** Let a, b, c be constants, and $f(x)$ a function of x. If y_p is a solution of

(i) $ay'' + by' + cy = f(x)$,

and y_h is a solution of

(ii) $ay'' + by' + cy = 0$,

then for any constant A, $y_p + Ay_h$ is a solution of (i).