Series (cont'd)

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Series

Definition. *Given a series* $\sum_{i=1}^{\infty} a_n$*, let* s_n *denote its* n*th partial sum:*

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n.$$

If the sequence $\lim_{n\to\infty} s_n = s$ *exists as a real number, then the series* $\sum a_n$ *is called* **convergent** *and we write*

$$\sum a_n = s.$$

Otherwise the series is called **divergent**.

The Geometric Series

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- Suppose that $a_n = ar^{n-1}$ for some $a \neq 0$. r is called the ratio.
- The geometric series is convergent if $|r| < 1$ and is sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}.$$

- If $|r| \geq 1$, the geometric series is divergent.

Examples

- $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

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- $\sum_{n=1}^{\infty} \frac{2^n}{7^{2n+1}}$

Other Examples

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- $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
- $\sum_{n=1}^{\infty} \ln(n+1) - \ln(n)$

- If the series $\sum a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

- If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$ then the series $\sum a_n$ is divergent.

- Example: Determine whether the series

$$\sum_{n=1}^{\infty} \frac{3n^2}{5n^2+2}$$

is convergent.

Series Laws

If $\sum a_n$ and $\sum b_n$ are convergent then so are the following series

- $\sum(a_n \pm b_n) = \sum a_n \pm \sum b_n$;
- $\sum c a_n = c \sum a_n$.

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- $\sum(a_n \pm b_n) = \sum a_n \pm \sum b_n$;
- $\sum c a_n = c \sum a_n$.
- Example: Find the sum of the following series

$$\sum_{n=1}^{\infty} \frac{4}{n(n+1)} + \frac{2^n}{3^n}.$$

The Integral Test

Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$.

1. If $\int_1^\infty f(x)\mathrm{d}x$ is convergent, then $\sum_{n=1}^\infty a_n$ is convergent.

2. If $\int_1^\infty f(x)\mathrm{d}x$ is divergent, then $\sum_{n=1}^\infty a_n$ is divergent.

Examples

- Determine whether

$$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

is a convergent series.

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- Determine whether
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- For what values of p is the series
$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$
convergent?

- Determine whether

$$\sum_{n=1}^{\infty} n e^{-2n}$$

is a convergent series.