

# Why would a homotopy theorist be interested in $R$ -localization?

Jeff Strom

Western Michigan University

Thursday, May 30, 2013

008 Kemeny (!), 4:00PM

(Tea: 300 Kemeny, 3:30 pm)

## Abstract

In the early 1950s, J.-P. Serre demonstrated that many computations in homotopy theory could be attacked “one prime at a time.” This allowed him to prove theorems about the  $p$ -part of the homotopy groups of spheres, for example, without troubling about the other primes.

In the late 1960s Dennis Sullivan showed that Serre’s innovation could be derived from a topological construction on spaces: the algebraic operation of  $R$ -localizing homotopy groups and homology groups (where  $R$  is a subring of the rationals) can be accomplished by first  $R$ -localizing the space in question, then computing the ordinary homotopy groups and homology groups of the resulting space. Thus, the powerful algebraic tool of localization was “lifted” to spaces.

But a pure homotopy theorist (let us say) is interested in algebra only insofar as it leads to useful information about cofibrations, fibrations, and so on. What reason, apart from simplifying computation, might drive a pure homotopy theorist to construct the  $R$ -localization functors? To answer this question, I will show (essentially) that the  $R$ -localization functors are precisely those functors from spaces to spaces which carry cofiber sequences to cofiber sequences.

This talk should be accessible to graduate students.