

# Hour Exam 2

## Math 3

Nov. 12, 2007

Name: \_\_\_\_\_

Instructor (circle):

Lahr (8:45)    Pomerance (11:15)    Treneer (12:30)

**Instructions:** You are not allowed to use calculators, books, or notes of any kind. All your answers to the multiple choice questions must be marked on the Scantron form provided, and your responses to the remaining questions must be written in this exam booklet. Take a moment now to print your name and section clearly on your Scantron form, and on pages 1 and 2 of your exam booklet. With regard to the multiple choice questions, you may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 10 multiple choice problems each worth 6 points, and there are 3 additional problems totaling 40 points. Check to see that you have 8 pages of questions plus cover and scoring pages.

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Problem	Points	Score
1 – 10 (MC)	60	
11	15	
12	10	
13	15	
Total	100	

1. For the function  $f(x) = x^2(x - 2)^2$ , which item correctly lists the intervals where the function is increasing?

- (a)  $(0, 2)$
- (b)  $(0, 1), (2, \infty)$
- (c)  $(1, \infty)$
- (d)  $(-\infty, \sqrt{2}), (4, \infty)$
- (e) none of the above

2. For the function  $f(x) = \frac{1}{x} - \frac{1}{x^2}$ , pick out the one statement that is correct.

- (a) This function has a local minimum at  $x = 0$ .
- (b) This function has an absolute minimum at  $x = 2$ .
- (c) This function has an absolute maximum at  $x = 2$ .
- (d) This function has a local minimum at  $x = -1$ .
- (e) This function has a local maximum at  $x = -1$ .

3. Suppose  $f(x)$  is continuous on the interval  $[0, 5]$  and differentiable on the interval  $(0, 5)$ , with  $f(0) = -1$  and  $f(5) = 9$ . Which one of the following statements holds?

- (a) There must be a number  $c$  with  $0 < c < 5$  with  $f(c) = 10$ .
- (b) There must be a number  $c$  with  $0 < c < 5$  with  $f'(c) = 2$ .
- (c) There must be a number  $c$  with  $0 < c < 5$  with  $f(c) = -10$ .
- (d) There must be a number  $c$  with  $0 < c < 5$  with  $f'(c) = 1/2$ .
- (e) There is not enough information given to force any of the above to hold.

4. The equation of the tangent line to the curve  $xy + x^2y^2 = 2$  at  $(1, 1)$  is

- (a)  $y = 3x - 2$
- (b)  $y = x$
- (c)  $y = 2 - x$
- (d)  $x = 1$
- (e) none of the above

5. Find  $\frac{dy}{dx}$  when  $y = x^{\sin(x)}$ . It is

(a)  $x^{\sin(x)-1} \cos(x)$

(b)  $x^{\sin(x)} \cos(x)$

(c)  $x^{\sin(x)} \ln(x)$

(d)  $x^{\sin(x)} \cos(x) \ln(x) + x^{\sin(x)} \frac{\sin(x)}{x}$

(e) none of the above

6. For this problem, the following information may be useful:  $\sqrt{2} \approx 1.41$ ,  $\sqrt{3} \approx 1.73$ ,  $e \approx 2.72$ ,  $\pi \approx 3.14$ . Suppose you use Newton's method to find a value of  $x$  where  $\cos(x) = x$  starting with an initial guess of  $x_0 = \pi/4$ . The value of the next iteration  $x_1$

(a) is bigger than  $\pi/4$

(b) is smaller than  $\pi/4$

(c) is equal to  $\pi/4$

(d) impossible to compute from the given information

(e) none of the above

7. Compute  $\int 2x^2 - 3x^3 - \frac{1}{x^2} dx$ . It is

- (a)  $4x - 9x^2 + \frac{2}{x^3}$
- (b)  $4x - 9x^2 + \frac{2}{x^3} + C$
- (c)  $x^3 - x^4 + \frac{1}{x} + C$
- (d)  $\frac{2}{3}x^3 - \frac{3}{4}x^4 + \frac{1}{x} + C$
- (e) none of the above

8. Find a solution  $y = y(x)$  to the equation  $\frac{dy}{dx} = y + 1$  and  $y(0) = 1$ . It is

- (a) 1
- (b)  $\frac{1}{2}y^2 + 1$
- (c)  $2e^x - 1$
- (d)  $\frac{1}{(x+1)^2}$
- (e) none of the above

9. A radioactive substance has a half life of 10 years. What fraction of this substance is still present after 15 years?

- (a)  $1/3$
- (b)  $3/8$
- (c)  $\sqrt{2}/4$
- (d)  $1/e$
- (e) none of the above

10. Suppose a particle is travelling along the  $x$ -axis so that its position at time  $t$  is given by  $x(t) = t^3 - 3t^2 - 24t + 5$ . Which one of the following statements is true?

- (a) The particle changes direction at time  $t = 1$ .
- (b) The particle is speeding up on the intervals  $(-2, 1)$  and  $(4, \infty)$ .
- (c) The particle is moving to the right on the interval  $(-2, 4)$ .
- (d) The particle is moving to the left on the interval  $(-\infty, 1)$ .
- (e) There is not enough information to determine any of the above.

NON-MULTIPLE CHOICE. PLEASE SHOW ALL YOUR WORK.

11. Suppose you use the linearization method to approximate  $\sqrt{10}$ .

- (a) What is the function  $f(x)$  that you will approximate using the linearization method?
  
  
  
  
  
  
  
  
  
  
- (b) What is the linearization  $L(x)$  of your function at a convenient point for the problem?
  
  
  
  
  
  
  
  
  
  
- (c) What is your numerical approximation of  $\sqrt{10}$  by this method?



12. Consider the differential equation  $\frac{dy}{dx} = y^2 + 2x$ .
- (a) Use Euler's method with initial point  $(-1, 0)$  and step size 1 to estimate  $y(1)$ .
- (b) Now use Euler's method with initial point  $(-1, 0)$  and step size  $1/2$  to estimate  $y(1)$ .

13. Consider the differential equation  $\frac{dy}{dx} = -x^2y^2$ .

(a) Find the general form of the solution  $y$  as a function of  $x$ .

(b) Find a particular solution to the equation using the initial value  $y(1) = \frac{1}{2}$ .