

Answer ALL questions. Unless instructed otherwise, you should show ALL your work and simplify your final answer as much as possible. Please box your final answer to each part.

Problem 1: [8 pts] Find the total mass of the metal laminate that is given by the domain $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$ that has density $\rho(x, y) = x^2 + y^2$.

Problem 2: [8 pts] The solid D fills the region $1 \leq x^2 + y^2 + z^2 \leq 4$, $z \geq 0$ has charge density $q(x, y, z) = 4z$. What force does this solid exert on a charged particle at $(0, 0, 0)$ with charge $+1$ coulombs?
Hint: 2 components of the force will vanish by symmetry

Problem 3: [9 pts] The vector field

$$\vec{F} = \begin{pmatrix} \frac{z^2 x}{\sqrt{1+x^2+y^2}} - z - 1 \\ \frac{z^2 y}{\sqrt{1+x^2+y^2}} \\ 2z\sqrt{1+x^2+y^2} - x \end{pmatrix}$$

is conservative. Let C be the curve consisting of straight-line segments that connects the following points in order: $(0, 0, 0)$, $(2, 1, -3)$, $(3, 6, 1)$, $(4, -10, 8)$, $(12, 12, 12)$, $(2, -7, 4)$, $(1, 0, 0)$. Find

$$\int_C \vec{F} \cdot d\vec{r}.$$