Math 13. Multivariable Calculus. Written Homework 7.

Due on Wednesday, 5/15/13.

You may leave this homework in the boxes outside of Kemeny 108 by 1:45 pm on Wednesday. Please write problems 1-3 on separate pages from problems 4-6 and turn them in in the corresponding columns.

1. (Chapter 16.4, #22) Let D be a region bounded by a simple closed path C in the xy-plane. Use Green's Theorem to prove that the coordinates $(\bar{x}, \bar{y})$ of the centroid (the centroid is the center of mass of D, if we assume that D is a lamina of uniform density) of D are

$$\bar{x} = \frac{1}{2A} \int_C x^2 \, dy, \quad \bar{y} = -\frac{1}{2A} \int_C y^2 \, dx.$$

2. (Chapter 16.5, #20) Is there a smooth vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\nabla \times \mathbf{G} = \langle xyz, -y^2 z, yz^2 \rangle$? Explain.

3. Prove the following statements assuming that the appropriate partial derivatives exist and are continuous.

   (a) $\operatorname{div}(f\mathbf{F}) = f \operatorname{div} \mathbf{F} + \mathbf{F} \cdot \nabla f$.

   (b) If $\mathbf{c}$ is a constant vector and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, then $\operatorname{curl} \frac{1}{2}(\mathbf{c} \times \mathbf{r}) = \mathbf{c}$.

4. (Chapter 16.6, #24) Find a parametric representation for the surface which is the part of the sphere $x^2 + y^2 + z^2 = 16$ which lies between the planes $z = -2$ and $z = 2$.

5. (Chapter 16.6, #26) Find a parametric representation for the part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$.

6. (Chapter 16.6, #36) Let $\mathbf{r}(u, v) = \langle \sin u, \cos u \sin v, \sin v \rangle$. Find an equation for the tangent plane to this surface at $u = \pi/6$, $v = \pi/6$.