Review and overview
Today's material
Next class

# Math 12, Fall 2007
## Lecture 23

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## Outline

1. Review and overview
   - Last class

2. Today's material
   - Parametric surfaces
   - Surface area

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## Divergence and Curl

- Curl: measures rotation
- Divergence: measures contraction/expansion
- Flux integrals: divergence form of Green's theorem
- Circulation integrals: curl version of Green's theorem

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## Different representations

- Recall the defition of a spacecurve:

$$\vec{c}(t) = x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k}$$

- We can describe surfaces in a similar way:

$$\vec{r}(u,v) = x(u,v)\,\vec{i} + y(u,v)\,\vec{j} + z(u,v)\,\vec{k}$$

- Example: spherical coordinates

$$\vec{r}(\theta,\phi) = \sin(\phi)\cos(\theta)\,\vec{i} + \sin(\phi)\sin(\theta)\,\vec{j} + \cos(\phi)\,\vec{k}$$

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## Examples

- Graphs $z = f(x, y)$

$$\vec{r}(u, v) = \langle u, v, f(u, v) \rangle$$

- Surfaces of revolution: $z = f(x)$ rotated about the $z$-axis

$$\vec{r}(u, v) = \langle u \cos(v), u \sin(v), f(u) \rangle$$

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## Surface Area

Idea: Approximate the area of a small patch of surface by the area of the corresponding piece of the tangent plane at that point.

1. Find tangent plane to $z = f(x, y)$

$$\vec{N} = \langle f_x, f_y, -1 \rangle$$

2. Area of parallelogram on the tangent plane is
$|\vec{N}| = \sqrt{1 + |\nabla f|^2}$

3. Area=$\int_D |\vec{N}| \, dA$

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## Surface area

- For a surface given as $f(x, y) - z = 0$, for $(x, y) \in D$, the surface area is given as

$$\iint_D |\vec{N}| \, dA$$

- To use the same idea with a parameteric surface, we must compute the normal:

$$\vec{N} = \vec{r}_u \times \vec{r}_v$$

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## Examples

- Find the surface area of the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = h$.
- Find the surface area of a cone $z^2 = x^2 + y^2$ between $z = 0$ and $z = 1$.

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## Work for next class

- Reading: 17.7
- Webwork: f07hw22