MATH 2   PROBLEM SET #8   SOLUTIONS

(1.) $\int e^{-x} dx$

$u = -x \qquad \dfrac{du}{dx} = -1 \qquad du = -dx \qquad -du = dx$

$\int e^{-x} dx = \int -e^{u} du = -e^{u} + C = \boxed{-e^{-x} + C}.$

(2.) $\int x^3 (2+x^4)^5 dx$

$u = 2 + x^4 \qquad \dfrac{du}{dx} = 4x^3 \qquad du = 4x^3 dx \qquad x^3 dx = \dfrac{1}{4} du$

$\int x^3 (2+x^4)^5 dx = \int \dfrac{1}{4} u^5 du = \dfrac{1}{24} u^6 + C = \boxed{\dfrac{1}{24}(2+x^4)^6 + C}$

(5.) $\int \cos^3 \theta \sin \theta \, d\theta$

$u = \cos \theta \qquad \dfrac{du}{d\theta} = -\sin \theta \qquad du = -\sin \theta \, d\theta \qquad -du = \sin \theta \, d\theta$

$\int \cos^3 \theta \sin \theta \, d\theta = \int -u^3 du = -\dfrac{u^4}{4} + C = \boxed{-\dfrac{\cos^4 \theta}{4} + C}.$

(7.) $\int x \sin(x^2) \, dx$

$u = x^2 \qquad \dfrac{du}{dx} = 2x \qquad du = 2x \, dx \qquad \dfrac{1}{2} du = x \, dx$

$\int x \sin(x^2) \, dx = \int \dfrac{1}{2} \sin(u) \, du = -\dfrac{1}{2} \cos(u) + C$

$= \boxed{-\dfrac{1}{2} \cos(x^2) + C}.$

(53.) $\int_0^1 x^2 (1+2x^3)^5 dx \qquad u = 1 + 2x^3 \qquad du/dx = 6x^2 \qquad \dfrac{1}{6} du = x^2 dx$

$\int_0^1 x^2 (1+2x^3)^5 dx = \int_1^3 \dfrac{1}{6} u^5 du = \left[\dfrac{1}{36} u^6 \right]_1^3 = \boxed{\dfrac{1}{36}(3^6 - 1) = \dfrac{182}{9}}.$

(54.) $\int_0^{\sqrt{\pi}} x \cos(x^2) \, dx \qquad u = x^2 \qquad du/dx = 2x \qquad \dfrac{1}{2} du = x \, dx$

$\int_0^{\sqrt{\pi}} x \cos(x^2) \, dx = \int_0^{\pi} \dfrac{1}{2} \cos(u) \, du = \left[\dfrac{1}{2} \sin u \right]_0^{\pi} = 0.$