Workshop 3
”$\mathcal{A}$ implies $\mathcal{B}$” Exercises II

Instructions:

Get into groups and work on the following exercises. Each group is expected to turn in *one neatly written copy* of their solutions at the end of the class period.

The exercises in this set ask you to prove or disprove various statements of the form ”$\mathcal{A}$ implies $\mathcal{B}$”. In each case, identify the hypothesis ($\mathcal{A}$) and the conclusion ($\mathcal{B}$) of the statement. Then prove or disprove the statement (whichever is indicated).

Exercise 1. [*The warm up*] Let A be an $m \times n$ matrix, let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ and let c be a scalar. Prove the following.

a. If $\mathbf{u}$ and $\mathbf{v}$ are solutions to $A\mathbf{x} = \mathbf{0}$ then so is $\mathbf{u} + \mathbf{v}$.

b. If $\mathbf{u}$ is a solution to $A\mathbf{x} = \mathbf{0}$ then so is $c\mathbf{u}$.

Exercise 2. [*The main event*] Let A be an $m \times n$ matrix. Show that if $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k$ are solutions to $A\mathbf{x} = \mathbf{0}$ and $\mathbf{v} \in \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ then $\mathbf{v}$ is also a solution to $A\mathbf{x} = \mathbf{0}$.

Exercise 3. Prove that the following statement is *false*: If the vectors $\mathbf{u}$ and $\mathbf{v}$ are solutions to the system $A\mathbf{x} = \mathbf{b}$ then so is $\mathbf{u} + \mathbf{v}$.

Exercise 4. Prove or disprove: If A and B are 2×2 matrices and $\mathbf{u} \in \mathbb{R}^2$ then $A(B\mathbf{u}) = B(A\mathbf{u})$.

Exercise 5. Let A be an $m \times n$ matrix and let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Show that if $\{\mathbf{u}, \mathbf{v}\}$ is a linearly dependent set then so is $\{A\mathbf{u}, A\mathbf{v}\}$. Can you generalize this statement to sets of more than two vectors?*