Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

2. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\sqrt{n+1}+\sqrt{n-1})}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2 - (n-1)^2}{n^3}$ (Hint: Which is it? $\sim \sum \frac{1}{n}$ or $\sim \sum \frac{1}{n^2}$)

4. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n - 3^n}{4^n}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n + 4^n}{5^n}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n^2}$

7. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{4^n}\right)\left(\frac{1}{n}\right)$

8. Estimate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ with an error less than 10^{-6}.

9. $\sum_{n=1}^{\infty} \ln(1 + 1/n)^n$.

10. What is the value of $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{3^{2n}(2n)!}$

11. What is $\displaystyle\lim_{x\to 0} \frac{\sin(x^2) - x^2}{x^6}$?

12. Let $J(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$. What is $J^{(6)}(0)$?

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13. Consider the function $f(x) = \sin x \cos x$ (Hint: $\sin(2x) = 2\sin x \cos x$)

    (a) Determine T_4 for f about $a = \pi$

    (b) Find the Maclaurin series for f.

14. Evaluate $\int_0^{0.1} \ln(x^3 + 1)dx$ with error less than $\frac{1}{100}$.

15. Evaluate $e^{-\frac{1}{2}}$ with error less than $\frac{1}{10^3}$.

16. Find the radius and interval of convergence of the following power series:

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(ex - 2)^n}{3^n e^n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(4 - x)^n}{n!}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(23 - 6x)^{2n}}{\sqrt{n}}$

    (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(5 - 4x)^{2n+1}}{4^n}$

    (e) $\displaystyle\sum_{n=1}^{\infty} \frac{2x^n}{n^n}$