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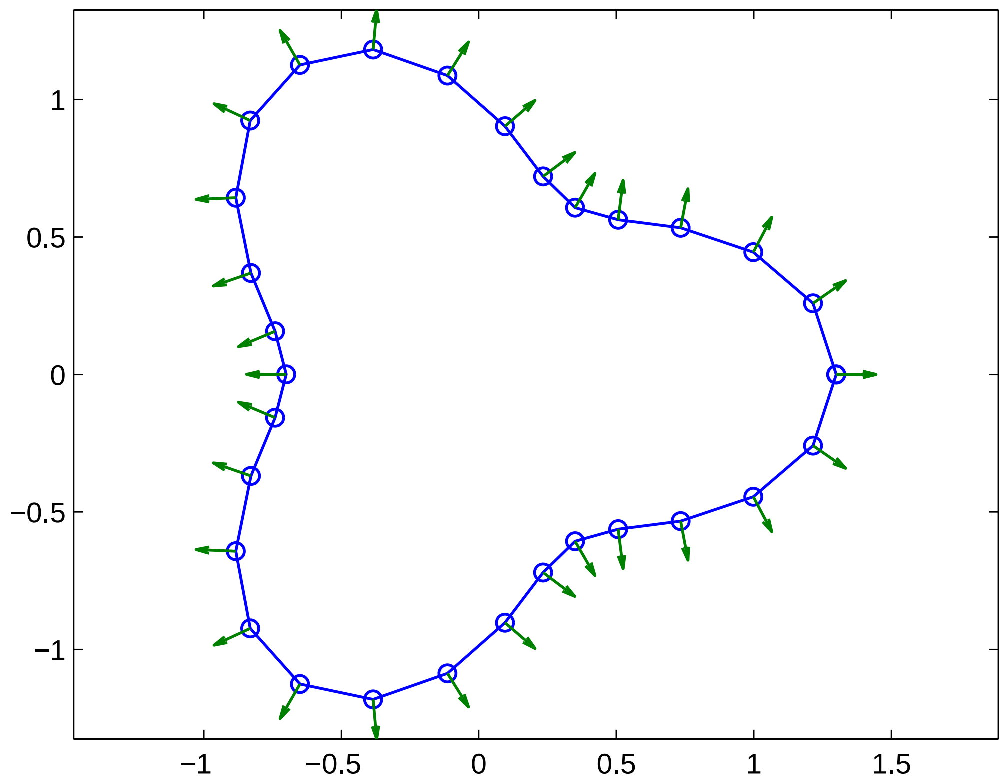
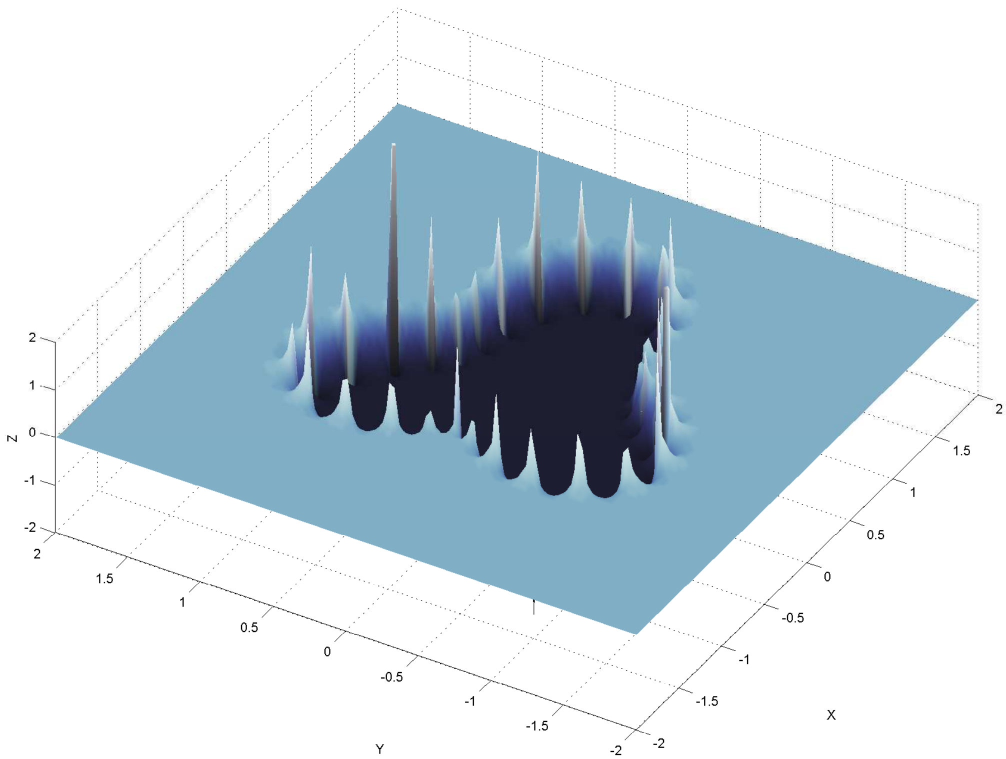
MATH 126 - Dr. Barnett
Homework 5
due February 13, 2012

Problem 1. See "hw5_1.m" for the code to generate the following figure.



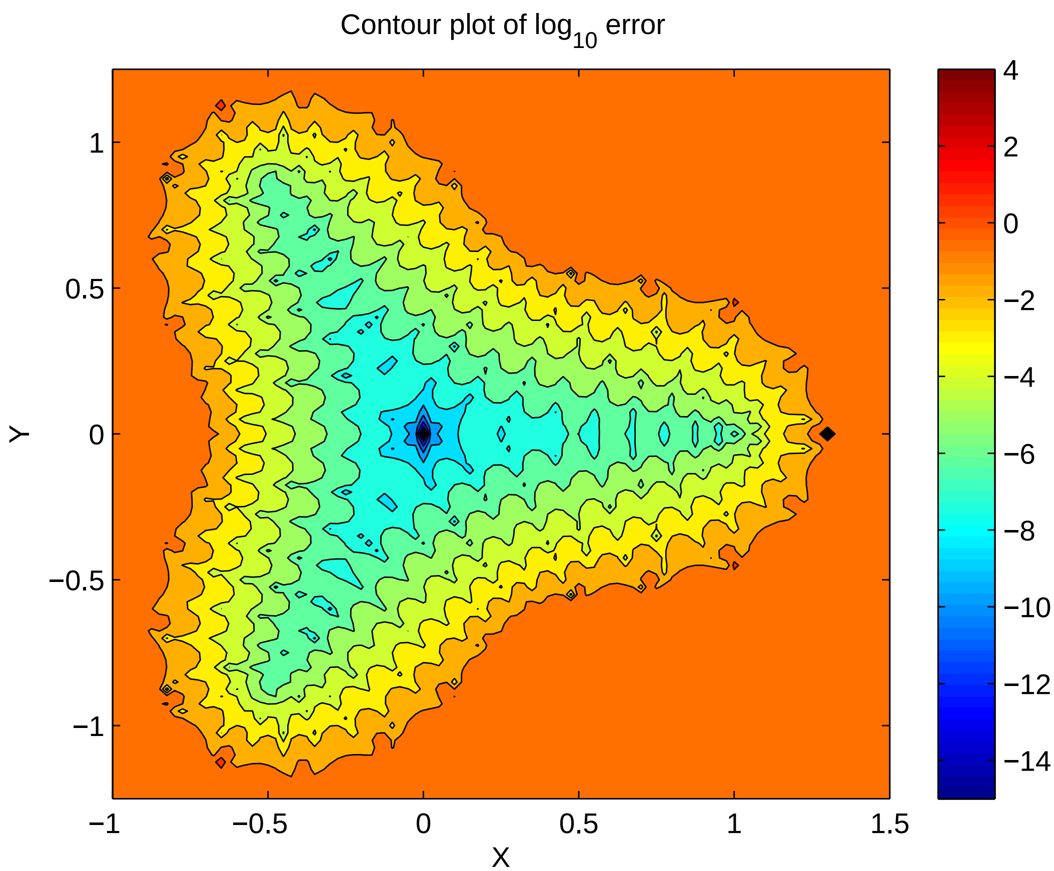
It does, indeed, look like a hairy amoeba; even more so when we take $N > 30$.

Problem 2. For part (a), we produce the following surface plot, which is zero in the interior of the given contour described in Problem 1.



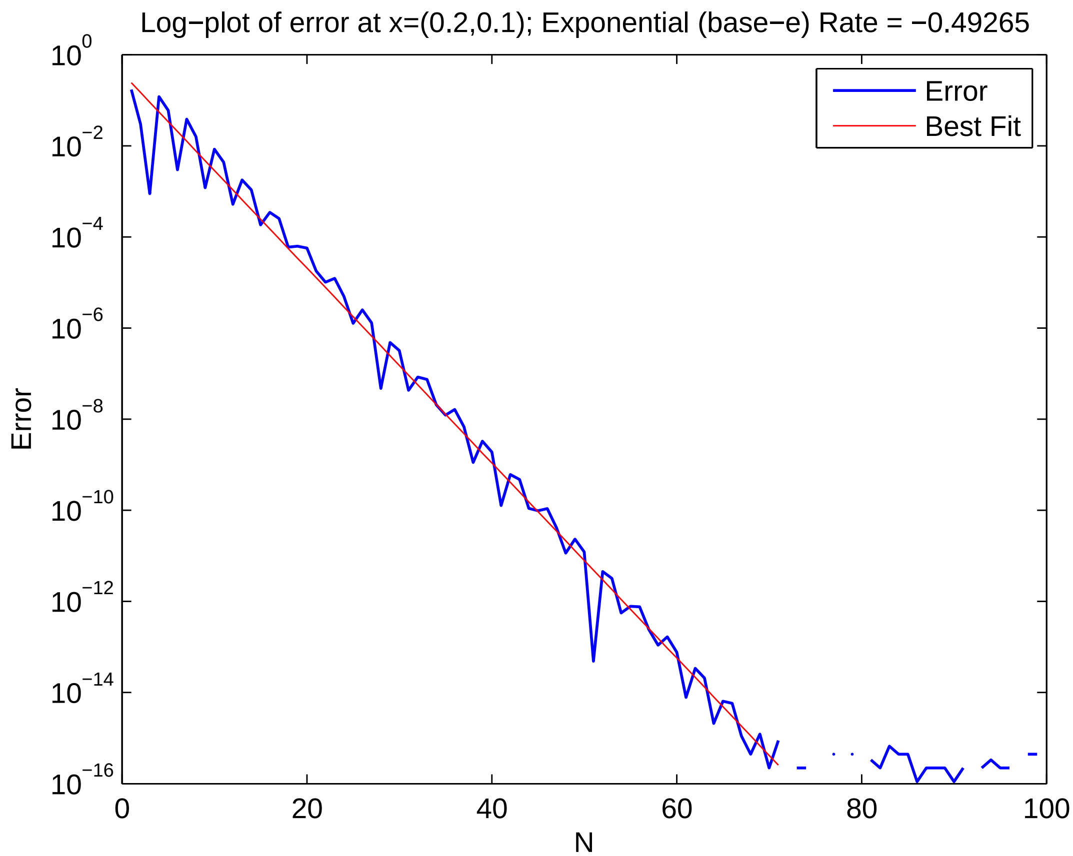
The dark blue region takes on values approximately equal to 1, while the light blue region on the exterior of our boundary is zero. The spikes correspond to the dipoles present at our quadrature nodes.

For part (b), we calculate the $\log_{10}$ error of our interior solution to the true analytic solution.



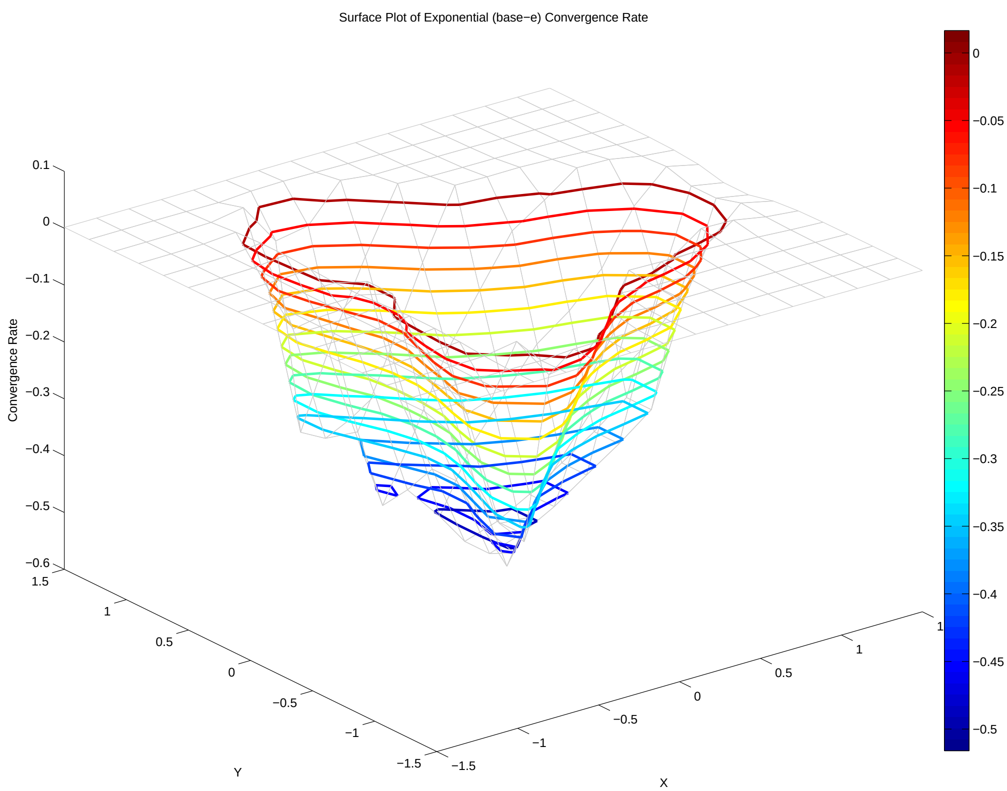
As is clearly visible in the above contour plot, the error depends on its distance from the boundary. Points close to the boundary yield larger error than those which are furthest away.

In the following plot, we calculated the error with respect to the quadrature nodes used to calculate the interior values.



Since the error is linear on a logplot, this tells us that the error converges exponentially to the true value at the point $(0.2, 0.1)$. In fact, we can calculate the approximate rate of convergence by looking at the slope of the best fit line (in red). The slope of this line tells us that the rate of convergence is ≈ -0.49265. Furthermore, in order to reach machine precision accuracy at this point in the interior, we require $N = 70$.

Lastly, the following 3D-contour plot gives us a way of visualizing how the convergence rate varies over our interior.



The rates of convergence improve as we get deeper inside the interior of our bounded region. Similar to our previous contour plot, which plotted the $\log_{10}$ error in our interior, we likewise find that the convergence rate obeys a similar rule, i.e. it is worst at the boundary and improves the further away we get from the boundary.

Problem 3. Sorry, but I did not complete Problem 3.

Problem 4. See "hw5_4.m" for the code pertaining to this problem. The solution at the point $(0.2, 0.1)$ as calculated by this script was determined to be 1.0801, which is approximately equal to $\cos(0.2)\exp(0.1)$. The error between this solution and the true value was calculated to be ≈ 0.0030673, which is not quite the best approximately, but still quite reasonable.