9	Math 71 - Homework 4 - Partial Solutions
85/1	EaH => 4"(E) a G:
	let xEG, y6 4-1(5) Show xyx-16 4-1(E)
	4(xyx-1) = 4(x)4(4) (4(x)) [E E sence E a H and
	4(4) E : x4x' 64"(E)
85/5	gN = N € N smallest pasitive enteger such that GN =
	$N. (gN)^* = N \iff g^*N = N \iff g^* \in N$
	Example (one of many): (37 & Tiz Consider
	$\mathbb{Z}_{12}/(3)$ $ 2 =6$ (in \mathbb{Z}_{12}) $ 2+(3) =3$ in
	Z12/<37.
85/9	Im $\varphi = \mathbb{R}^{>0}$, Ker $\varphi = S'$ (unit and in \mathbb{C}^+)
	150 positive real Fiber of r circle of radius JF
86/10	Write 4(a+82) = a+42. Show 4 well-defined:
	quality a+87 = b+87 : 8/a-b :: 4/a-b ::
	a+47=6+42 3 4 well-defended. Ker4= {82,4+87}
	WIT IS THE TOTAL OF THE TOTAL O
	fiber ore 1+4 1 es {1+8 2,5+8 2 }.
88132	fiber ore 1+4 2 es {1+8 2,5+8 2}.
88132	fiber over 1+4 des {1+8 Z, 5+8 Z}. Of (i) has order 2: Isomorphie to Zz (same for
88(32	fiber over 1+4 Il is {1+8 I, 5+8 It. QE (i) has order 2: Isomorphic to II. (same for J, k) Qe/(±1) has order 4: Isomorphic to Ty or
	fiber over 1+4 des {1+8 Z, 5+8 Z}. Of (i) has order 2: Isomorphie to Zz (same for
	fiber ore 1+47 is {1+87,5+87}. Q= ((i) how order 2: Isomorphic to I2 (same for 1,k) Q= ({\frac{1}{2}} \frac{1}{2} hos order 4: Isomorphic to Ty or \textit{Z}_2 \text{Show it has no element of order 4:}
	fiber orce 1+47 is \$1+87,5+87. Qs ((i) how order 2: Isomorphic to I. () same for J, k) Qs (\{\frac{1}{2}\} hos order 4: Isomorphic to I.y or I2xI2. Show it has no element of order 4: Let ri be the rotations in Den and let sri be the reflections Show ri(rk)ri \(\int \chi r \) and sri \(\frac{1}{2} \chi r \chi r \chi r \chi \).
	fiber over 1+4 as \$1+8 5+8 \ \ Q_\varepsilon \left(i)\ has order 2: Isomorphic to \(I_\varepsilon\) (same for \\ \[\begin{align*} \left(k)\ Q_\varepsilon\right\) (\frac{1}{2} \text{ has order 4: Isomorphic to \(I_\varepsilon\) or \\ \(I_\varepsilon\varepsilon\) (\frac{1}{2} \text{ has no element of order 4: \\ \text{Let ri be the notations in Don and let soil be the reflections \\ \text{Show ri(rk)r-i \(\infty\) coul \(\text{Sri \(\text{rk}\) r-is \(\infty\) \(\text{cri \(\text{s}\)}\) (\text{ri \(\text{s}\)}\
	fiber over 1+41 is \$1+87,5+87. Qs ((i) how order 2: Isomorphic to I. (same for 1, k) Qs/(±1) has order 4: Isomorphic to Ty or I.x I.z. Show it has no element of order 4: Let i be the rotations in Dan and let sri be the reflections Show ri(rk)ri a (rk) and sri(rk)ris a (rk).
	fiber over 1+4 as {1+8 Z, 5+8 Z}. Qc /(i) how order 2 : Doomorphic to Ze (same for J, k) Qc/{±1} has order 4 : Esomorphic to Zy or Zx Zz. Show it has no element of order 4: Let ri be the rotations in Dzn and let sri be the reflections Show ri(rk)r-i & (rk) and sri(rk)r-is & (rk). Irow let pi he the rotations in Dzn and Sp1 the reflections on Dzh. Defene \$\phi(ri) = \phi^1, \phi(r^2) = \phi^2, \ldots partitions
88 34	fiber orce 1+4 % \$ \$1+8 Z, 5+8 Z \cdots Qs \((i)\) how order 2 : Isomorphic to \(Z_2\) (pame for J, k) Qs \((i)\) how order 4 : Esomorphic to \(Z_4\) or Z2 x Z2 : Show it has no element of order 4: Let ri be the notations in D2n and let sri be the reflections Show ri \((r^k)\) r-i \(\sigma\) \((r^k)\) and \(sri\) be the reflections Show let \(p^i\) he the notations in \(D_{2k}\) and \(sp^i\) the reflections so \(D_{2k}\) . Defense \(\phi(r^i) = p^i\), \(\phi(r^2) = p^n\), \(\phi\) \(\phi^n\), \(\phi(r^k) = p^n\), \(\phi(r
	fiber orce 1+4 % \$ \$1+8 Z, 5+8 Z \cdots Qs \((i)\) how order 2 : Isomorphic to \(Z_2\) (pame for J, k) Qs \((i)\) how order 4 : Esomorphic to \(Z_4\) or Z2 x Z2 : Show it has no element of order 4: Let ri be the notations in D2n and let sri be the reflections Show ri \((r^k)\) r-i \(\sigma\) \((r^k)\) and \(sri\) be the reflections Show let \(p^i\) he the notations in \(D_{2k}\) and \(sp^i\) the reflections so \(D_{2k}\) . Defense \(\phi(r^i) = p^i\), \(\phi(r^2) = p^n\), \(\phi\) \(\phi^n\), \(\phi(r^k) = p^n\), \(\phi(r

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ş	your theren.
89/38	Define \$: A x A -> A by \$(a,b) = a-b
	(a) Define O: H-gHg-1 by O(A) = ghg-1. Show O w an
and the second	isomorphism.
	(b) H and gHg-1 are subgroups of order M. Since these is
	exactly one subgroup of order m, H=gHg-1
95/8	let H =m, K =L. JA, D Am+BL=1
	Cet x G HAK. Then x = x' = 6xm)A(x" B = 1
96/11	Here is a pecture - you provide the proof.
	HEKEG Item are the cosets of K
	In G (K, ek, bK, ck)
	ak lik and the Gost of Hunk
	(unlabeled but call them H,
_	aH, BH, FH).
or and a second	Consider the function La: K - ak left mult by a. It is
AND PARTY.	a byection. The cosets of H which are in at are La lit),
	La (xH), La (pH), La (bH) = aH, axH, aBH, abH.
	Similarly for bk and ck. Nuff said.
96/15	G= Sn Fixi, 150 = m. Then G. are all permutations in Sn
=	which for i. Note Smy is isomorphic to Gn, all permutuhan
	en Sn which fex m. Therefore it sufferes to show G. ~ Go
	Giren T & G: Hen /1 2-1 2 2+1 m
	$T = \begin{pmatrix} \tau(i) & \cdots & \tau(i-1) & \epsilon(i+1) & \tau(n) \end{pmatrix}$
	assign to o a perm. PEGn as fallows: From the above array
	relate i leave the integer 1,, i-1 unchanged , replace the
	integers it1, on by i,, n-1 and add the column in on
	the right. Then PEGn. This defenes a function G> Gm.

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