1. Let $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & 1 \end{bmatrix}$.

(a) How many rows of A contain a pivot position?

(b) Is $A\mathbf{x} = \mathbf{b}$ consistent for each $\mathbf{b} \in \mathbb{R}^{\mathbf{4}}$? If not, for which $\mathbf{b} \in \mathbb{R}^{\mathbf{4}}$ is $A\mathbf{x} = \mathbf{b}$ consistent?

2. (a) If $\mathbf{v} = \mathbf{0}$, then is $\mathbf{v}$ linearly independent? Why or why not?

(b) $\mathbf{v} \neq \mathbf{0}$, is $\mathbf{v}$ linearly independent? Why or why not?

3. (a) Suppose that $\mathbf{v_1}$ is a multiple of $\mathbf{v_2}$ ($\mathbf{v_1}, \mathbf{v_2} \neq \mathbf{0}$). That is, there exists $c \in \mathbb{R}$ such that $\mathbf{v_1} = c\mathbf{v_2}$ ($c \neq 0$). Show that $\{\mathbf{v_1}, \mathbf{v_2}\}$ is a linearly dependent set.

   (b) If $\mathbf{v_1}$ and $\mathbf{v_2}$ are not multiples of each other, can $\{\mathbf{v_1}, \mathbf{v_2}\}$ be a linearly dependent set? Why or why not? [Hint: think about the Parallelogram Rule for vector addition.]

4. Consider the set $S = \{\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_p}\}$, where each $\mathbf{v_i} \in \mathbb{R}^{\mathbf{n}}$. Explain why if $p > n$, S must be a linearly dependent set. You may consider, if desired, the example where $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ with $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Discuss your final answer in the context of "free variables".

5. Consider $S = \{\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_p}\}$, where $\mathbf{v_p} = \mathbf{0}$. Why must S be a linearly dependent set? [Hint: Find a linear combination $\mathbf{y} = c_1\mathbf{v_1} + \cdots + c_p\mathbf{v_p}$ of the vectors in S such that this $\mathbf{y} = \mathbf{0}$ and not all weights c_i are zero.]