Math 25 First Exam - Part a

October 20, 2009

Instructions: You should show all of your work on problems 1, 2, 3 with one exception: you may solve simple congruences or linear equations by inspection.

1. (15) Find the general solution to the linear Diophantine equation $12x + 28y = 20$.

2. (15) Show there are infinitely many primes $p \equiv 2 \pmod 3$.

3. (15) Find the solution to the system of linear congruences below; your answer will be in terms of a_1, a_2, a_3.

$$\begin{aligned} x &\equiv a_1 \pmod{7} \\ x &\equiv a_2 \pmod{8} \\ x &\equiv a_3 \pmod{9} \end{aligned}$$

4. (15) **True/False or Short Answer** No work need be shown; no partial credit.

(a) If a, b are positive integers, and $a^2 \mid b^2$, then $a \mid b$.

(b) If a, b, c are positive integers and $a \mid c$ and $b \mid c$, then $ab \mid c^2$.

(c) If m, a, b, c are positive integers and $a \not\equiv 0 \pmod{m}$ then $ab \equiv ac \pmod{m}$ implies $b \equiv c \pmod{m}$.

(d) If p is an odd prime, and a an integer with $p \nmid a$, then $a \not\equiv -a \pmod{p}$.

(e) If $p \neq q$ are primes, the congruence $(p^2q^3)x \equiv p^3q^2 \pmod{p^5q}$ has

____________________ incongruent solutions modulo p^5q.