Final Exam

Math 3

Nov. 16, 2012

Name (Print): ____

Last First

On this the Math 3 final exam in Fall 2012, I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:____

Instructor (circle):

Lahr (Sec. 1, 8:45) Diesel (Sec. 2, 10:00)

Dorais (Sec. 3, 11:15) Dorais (Sec. 4, 12:30)

Wolff (Sec. 5, 1:45)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on page 1 of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 30 multiple-choice problems worth 5 points each for a total of 150 points. Check to see that you have 15 pages of questions plus the cover page and a blank page at the end for a total of 17 pages.

1. Consider the unit circle $x^2 + y^2 = 1$. The points $(-3/5, 4/5)$ and $(1, 0)$ lie on the circle and divide it into two parts. Find the arc length of the shorter part.

   (a) $\dfrac{6\pi}{5}$

   (b) $\dfrac{\pi}{2} + \arcsin\left(\dfrac{3}{5}\right)$

   (c) $\dfrac{3\pi}{2} + \arcsin\left(\dfrac{3}{5}\right)$

   (d) $\dfrac{8}{5}$

   (e) None of the above

2. Find $\sin(\arctan(2))$.

   (a) 1

   (b) 5

   (c) $2\sqrt{5}/5$

   (d) $\sqrt{5}$

   (e) None of the above

3. The length of the piece of the graph $y = \tan x$ from $x = 0$ to $x = \pi/4$ is:

(a) $\displaystyle\int_0^{\pi/4} (1 + \sec^2 x)\, dx$

(b) $\displaystyle\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$

(c) $\displaystyle\int_0^{\pi/4} (1 + \sec x)\, dx$

(d) $\displaystyle\int_0^{\pi/4} \sqrt{1 + \sec^2 x}\, dx$

(e) None of the above

4. Find the area of the region bounded by $f(x) = x^3 + 3x^2 - 3$ and $g(x) = x^2 - 3$.

(a) 3

(b) 8/3

(c) 5/3

(d) 4/3

(e) None of the above

5. Using 3 trapezoids, approximate $\int_0^6 (-x^2 + 3)dx$.

(a) -58
(b) -2
(c) -26
(d) -54
(e) None of the above

6. Evaluate: $\dfrac{d}{dx}\left(\dfrac{1}{x}\int_0^{3x^2} \sin(t)\,dt\right)$

(a) $6\sin(3x^2) - \dfrac{1}{x^2}\int_0^{3x^2} \sin(t)\,dt$
(b) $-\dfrac{6}{x}\sin(3x^2)$
(c) $6\sin(x)$
(d) $-\dfrac{1}{x^2}\int_0^{3x^2} \sin(t)dt + \dfrac{6}{x}\sin(x)$
(e) None of the above

7. Simplify: $\int_{-3}^{3}(\sin(x^2)+3x^3+x^{11})dx$

(a) $2\int_{0}^{3}(\sin(x^2)+3x^3+x^{11})dx$

(b) $2\int_{0}^{3}\sin(x^2)dx$

(c) 0

(d) $\int_{-3}^{3}(\cos(x^2)+\frac{1}{4}x^4+\frac{1}{12}x^{12})dx$

(e) None of the above

8. If a right Riemann sum yields

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{5}{n}\sec^2\left(\frac{5i}{n}\right)=\int_0^b f(x)\,dx\,,$$

what are b and $f(x)$?

(a) $b=5,\ f(x)=\tan(x)$

(b) $b=n,\ f(x)=\tan(x)\sec(x)$

(c) $b=5,\ f(x)=\sec^2(x)$

(d) $b=\frac{5}{n},\ f(x)=\tan(x)$

(e) None of the above

9. A cylindrical bucket with open top is designed to hold 8π cubic inches of water. What is the height of the bucket that minimizes the total surface area of the bucket (bottom & side)? (Note that if h and r are the height and radius of the bucket, the bottom is a circle with area πr^2, the side is a rectangle with area $2\pi rh$, and the volume of the bucket is $\pi r^2 h$.)

   (a) 1 inch
   (b) 2 inches
   (c) 4 inches
   (d) 8 inches
   (e) None of the above

10. Where is $f(x) = \dfrac{x}{x^2+1}$ increasing?

   (a) $x = 1$ and $x = -1$
   (b) $x = 0$, $x = \sqrt{3}$ and $x = -\sqrt{3}$
   (c) $(-\infty, -1] \cup [1, \infty)$
   (d) $(-\infty, -\sqrt{3}] \cup (0, \sqrt{3})$
   (e) None of the above

11. Solve: $\csc(x)y' = y$.

   (a) $\sqrt{-\cos(x) + C}$
   (b) $\sqrt{-\csc(x) + C}$
   (c) $Ce^{\sin(x)}$
   (d) $Ce^{-\cos(x)}$
   (e) None of the above

12. Bacteria grow at a rate proportional to the amount present. If a bacteria population grows such that the population has doubled in 3 minutes, when will it triple?

   (a) $t = \dfrac{\ln(2)}{3}$
   (b) $t = \dfrac{3\ln(3)}{\ln(2)}$
   (c) $t = \dfrac{\ln(3)}{3\ln(2)}$
   (d) $t = \dfrac{2\ln(2)}{\ln(3)}$
   (e) None of the above

13. Hooke's Law says that the acceleration of a mass suspended by a spring is proportional to the distance of the mass from the equilibrium position. If y denotes the position of the mass and 12 is the equilibrium position for the mass, which of the following differential equations best represents Hooke's Law?

    (a) $y'' = -k(y - 12)$
    (b) $y'' = y$
    (c) $y'' = -y + 12$
    (d) $y'' = ky - 12$
    (e) $y'' = 12$

14. Dartmouth students run in a circle around a bonfire. As more spectators join, the radius of the circle decreases at a rate of 4 feet per minute. How fast is the area decreasing when the area is π square feet?

    (a) There is not enough information
    (b) 8π
    (c) 16π
    (d) 2π
    (e) None of the above

15. If a ball rolling in a straight line decelerates at a rate of $4\,\mathrm{m/s^2}$ starting with velocity $8\,\mathrm{m/s}$ then how far has the ball traveled after $2\,\mathrm{s}$?

    (a) $8\,\mathrm{m}$
    (b) $16\,\mathrm{m}$
    (c) $24\,\mathrm{m}$
    (d) $32\,\mathrm{m}$
    (e) None of the above

16. Use a linear approximation centered at 144 to estimate $\sqrt{145}$.

    (a) $12\frac{1}{12}$
    (b) $12\frac{1}{24}$
    (c) 12
    (d) $12\frac{1}{3}$
    (e) None of the above

17. Find the derivative of $f(x) = x^{3x^2}$

(a) x^{3x^2}

(b) $x^{3x^2}(6x)$

(c) $x^{3x^2}(6 + 3x^2 \ln(x))$

(d) $x^{3x^2}(6x \ln(x) + 3x)$

(e) None of the above

18. If an object moves with position function $s(t) = -\frac{1}{3}\sin(t) + 13$, from $t = 0$ to $t = 2\pi$ seconds, when is the object's velocity increasing?

(a) $[\pi, 2\pi]$

(b) $[0, \pi]$

(c) The velocity is always increasing

(d) The velocity is never increasing

(e) None of the above

19. Find the derivative of $x^2 \sin(\sqrt{x})$.

(a) $x^2 \cos(\sqrt{x}) + 2x \sin(\sqrt{x})$

(b) $2x \cos(\sqrt{x})$

(c) $x^{3/2} \cos(\sqrt{x})/2 + 2x \sin(\sqrt{x})$

(d) $x^{3/2} \sin(\sqrt{x})/2 + 2x \cos(\sqrt{x})$

(e) None of the above

20. What is the derivative of e^2?

(a) $2e^2$

(b) $2e$

(c) e^2

(d) 0

(e) None of the above

21. $\lim_{h\to 0} \dfrac{(1+h)^4 - 1}{h}$ equals:

(a) 4

(b) 0

(c) The limit does not exist

(d) 1

(e) None of the above

22. Suppose $f(x) = \begin{cases} \dfrac{4-x^2}{2+x} & \text{if } x \neq -2, \\ 4 & \text{if } x = -2. \end{cases}$

Which of the following is true?

(a) $f(x)$ has a removable discontinuity at $x = -2$

(b) $\lim_{x\to -2} f(x)$ does not exist

(c) $f(x)$ is continuous on its domain

(d) $f(x)$ is not differentiable at $x = -2$

(e) None of the above

23. The slope of the tangent line to the curve $xy + 1 = x^3 + y^2$ at $(1, 1)$ is:

(a) -2

(b) 0

(c) 2

(d) The tangent line is vertical

(e) None of the above

24. $\displaystyle\lim_{x\to\infty} \frac{4 - 3x}{\sqrt{4x^2 - x + 3}}$ equals:

(a) $-3/2$

(b) $-1/2$

(c) $1/2$

(d) $3/2$

(e) None of the above

25. The derivative of $\dfrac{\sin(-x)}{\tan(x)}$ is:

(a) $\cos(x)$
(b) $\sin(x)$
(c) $-\cos(x)$
(d) $-\sin(x)$
(e) None of the above

26. Integrate: $\displaystyle\int (3\sqrt{x} + 3x^3 + 1)dx$

(a) $2x^{3/2} + \frac{3}{4}x^4 + x + C$
(b) $\frac{3}{2}x^{-1/2} + 9x + C$
(c) $\frac{3}{2}x^{-1/2} + \frac{3}{4}x^4 + x + C$
(d) $\frac{2}{3}x^{3/2} + \frac{3}{4}x^4 + x + C$
(e) None of the above

27. Integrate: $\displaystyle\int \frac{e^x}{4+e^x}dx$

(a) $\dfrac{e^{x+1}}{4+e^x} + C$

(b) $\ln(4+e^x) + C$

(c) $\ln(u) + C$

(d) $\dfrac{e^x}{4x+e^x} + C$

(e) None of the above

28. For what x value does $f(x) = \dfrac{5x+2}{3x}$ achieve an absolute maximum on $[-3,-1]$?

(a) No maximum value exists

(b) $x = -1.5$

(c) $x = -3$

(d) $x = -1$

(e) None of the above

29. Let $f(x) = \dfrac{x^2 + 6x + 9}{x^2 + x - 6}$. What is the domain of $f(x)$?

(a) $(-\infty, -3) \cup (-3, \infty)$

(b) $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

(c) $(-\infty, 2) \cup (2, \infty)$

(d) $(-\infty, \infty)$

(e) None of the above

30. For $f(x)$ as in problem 29, what are the horizontal asymptotes?

(a) $y = 2$

(b) $y = 3$

(c) $y = 2$ and $y = 3$

(d) $y = 1$

(e) None of the above

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