The pretentious view of analytic number theory

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Abstract

Analytic number theory was born out of the desire to understand the distribution of the primes. In particular, much of modern analytic number theory can trace its origins to Riemann's seminal 1859 memoir, in which he outlines an attack on the prime number theorem by means of the study of the zeros of what we now call the Riemann zeta function. This approach has been fruitful, and has given rise to a lot of beautiful mathematics, but this hides the dirty truth of analytic number theory: what we can prove, and what we believe to be true, are incredibly far apart. What's more, the best known zero-free region is more than fifty years old. In other words, analytic number theory is in dire need of new ideas. The pretentious view of analytic number theory, as put forward by Granville and Soundararajan, is an attempt at doing this; it is analytic number theory without zeros of L-functions and without analytic continuation. As a substitute, Granville and Soundararajan propose a general study of multiplicative functions, with the goal being to obtain deep structure theorems from which arithmetic results arise as corollaries; as such, it can be seen as finally establishing the context for the elementary proof of the prime number theorem developed by Erdos and Selberg. In this talk, we ask about the structure of functions exhibiting more cancellation than they have a right to. We are able to completely classify such functions in a natural setting, and we establish the right context to consider this question pretentiously. Parts of this are joint work with Junehyuk Jung.

This talk should be accessible to graduate students.