Math 22 Workshop II
6 April 2006

1. Let A be a $m \times n$ matrix, let $\mathbf{b}$ and $\mathbf{b}'$ be vectors in $\mathbf{R}^m$ and let c be a scalar. Prove the following statements.

   (a) If $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{b}'$ are both consistent, then $A\mathbf{x} = \mathbf{b} + \mathbf{b}'$ is consistent.

   (b) If $A\mathbf{x} = \mathbf{b}$ is consistent, then so is $A\mathbf{x} = c\mathbf{b}$.

2. Let A be a $m \times n$ matrix, let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbf{R}^n$ and let c be a scalar.

   (a) If $\mathbf{u}$ and $\mathbf{v}$ are solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$, then so is $\mathbf{u} + \mathbf{v}$.

   (b) If $\mathbf{u}$ is a solution to $A\mathbf{x} = \mathbf{0}$, then $c\mathbf{u}$ is too.

3. A variation on problem 2 (with the same hypotheses).

   (a) Is it true that $\mathbf{u}$ and $\mathbf{v}$ are solutions to $A\mathbf{x} = \mathbf{0}$ *if and only if* $\mathbf{u} + \mathbf{v}$ is?

   (b) Suppose that $c \neq 0$. Then is it true that $\mathbf{u}$ is a solution to $A\mathbf{x} = \mathbf{0}$ *if and only if* $c\mathbf{u}$ is?

4. Let A be a $m \times n$ matrix. Show that if $\mathbf{u}_1, \ldots, \mathbf{u}_p$ are all solutions to $A\mathbf{x} = \mathbf{0}$ and if $\mathbf{v} \in \operatorname{Span}\big(\{\, \mathbf{u}_1, \ldots, \mathbf{u}_p \,\}\big)$, then $\mathbf{v}$ is a solution to $A\mathbf{x} = \mathbf{0}$.

5. Prove or disprove the following statements.

   (a) If the vectors $\mathbf{u}$ and $\mathbf{v}$ are solutions to $A\mathbf{x} = \mathbf{b}$, then so is $\mathbf{u} + \mathbf{v}$.

   (b) If A and B are 2×2 matrices and if $\mathbf{u} \in \mathbf{R}^2$, then $A(B\mathbf{u}) = B(A\mathbf{u})$.