Math 14 Winter 2003 Final Exam

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PRINT NAME: ______________________________

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted. You must justify all of your answers to receive credit.**
You have two hours. Do any 10 out of 11 problems. Please do all your work on the paper provided.

The Honor Principle requires that you neither give nor receive any aid on this exam.

1. Let $f(x, y) = e^{3xy^2}$. Find the second-order Taylor polynomial around the point $(0, 1)$. Use this Taylor polynomial to approximate the value of $f(0.1, 0.9)$.

2. Let $\phi : \mathbb{R}^3 \to \mathbb{R}^3$ be a mapping such that $\phi(x, y, z) = (x^2, x + y^2, 3 + z)$. Put $D = \phi([0, 1] \times [0, 1] \times [0, 1])$. Calculate $\int_D (xz + y) dx dy dz$.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be a mapping such that $f(u, v) = (u^2, uv, v + 2u)$ and let $g : \mathbb{R}^3 \to \mathbb{R}^1$ be a mapping such that $Dg(x, y, z) = (xy, \frac{1}{2}x^2, z^2)$. Put $h = g \circ f$ and find $Dh(1, 2)$.

4. Evaluate the iterated integral $\int_0^1 \int_y^{y^{1/3}} e^{x^2} dxdy$.

5. Let $g(x, y, z) = x^2 - \frac{2}{3}y^3$ and let $\mathbf{F}(x, y, z) = y^2\mathbf{i} + x\mathbf{j} + \sin z\mathbf{k}$. Show that g is constant along the flow curves of $\mathbf{F}$.

6. Let $\mathbf{F}$ be a vector field such that for every path $c : [a, b] \to \mathbb{R}^3$ the integral $\int_c \mathbf{F} \cdot d\mathbf{s}$ depends only on the starting point $c(a)$ and on the ending point $c(b)$. Show that $\int_{\mathbf{d}} \mathbf{F} \cdot d\mathbf{s} = 0$, for every closed simple path $\mathbf{d}$.

7. Compute $\int_{\mathbf{c}} -yx^2\mathrm{dx} + xy^2\mathrm{dy}$ where $\mathbf{c}$ is the unit circle oriented in the **clockwise** direction.

8. Let $F = (z^3 \cos(xz), 2yz, 2z \sin(xz) + xz^2 \cos(xz) + y^2)$.

   (a) Show that F is a conservative vector field.

   (b) Compute $\int_{\mathbf{c}} F \cdot d\mathbf{S}$ along the path $\mathbf{c}(t) = (\sin(\pi t/2), 5t^3 + 2t^2 - 6t, e^{t^2 - t} + t^6 - t)$ with $0 \leq t \leq 1$.

9. Compute $\iint_S F \cdot d\mathbf{S}$ where $F = (2x^2y, z^2 - 3xy^2, 2z(xy+1))$ and S is the sphere of radius $\mathbf{R}$ centered at the origin with an outward orientation.

10. Let $\eta = e^z \mathrm{dxdy} + y\mathrm{dzdx}$. Compute $\int_{\partial\Omega} \eta$ where Ω is the region in $\mathbb{R}^3$ bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$. You may assume that $\partial\Omega$ is given the usual outward orientation.

11. Compute $\left| \iint_S \nabla \times F \cdot d\mathbf{S} \right|$ where $F = (yx^2 \cos(z+\pi), xy^2 \cos z, xe^z)$ and S is an oriented surface $z = 1 - x^2 - y^2$ where $z \geq 0$. (Hint: You may find it useful to refer back to one of the previous problems in order to compute this integral.)