

Answer ALL questions. Unless instructed otherwise, you should show ALL your work and simplify your final answer as much as possible. Please box your final answer to each part.

Problem 1: [8 pts] Find the following indefinite integral

$$\int \frac{9}{x^2 \sqrt{9 + 4x^2}} dx$$

Problem 2: [9 pts] Find the area of the region bounded by the curves $y = \arctan x$ and $y = x \arctan x$.

Problem 3: [8 pts] The unbounded region R is bounded above by the curve $y = \frac{1}{\sqrt{x^2+3x+2}}$, below by the x -axis and to the left by $x = 0$. This region R is rotated about the x -axis. Is the volume of the resulting solid finite or infinite? If it is finite, evaluate it.

Problem 4: [10 pts] (This problem did not originally appear on this quiz, but for math 9 you should expect to see a question of this sort of type)

Suppose $f(x)$ is a continuous function on $(0, \infty)$ such that

- $f(x) > 0$ for all x ,
- $\int_1^\infty f(x) \, dx$ converges.

For each of the following statements, decide if they must always be true. If so, explain why. If not, give an example of an $f(x)$ satisfying the above conditions such that the statement is false.

(a) $\int_1^\infty f'(x) \, dx$ converges.

(b) $\int_1^\infty f(\sqrt{x}) \, dx$ converges.

(c) $\int_1^\infty \frac{f(x)}{1+f(x)} \, dx$ converges.

(d) $\int_1^\infty e^{-f(x)} \, dx$ diverges.

(e) $\int_1^\infty (\int_1^x f(t) \, dt) \, dx$ converges.