MATH 22 PRACTICE PROBLEMS

NOTE: In no sense should this collection of problems be construed as representative of the actual exam. These are simply some problems left over from the preparation of the exam or from previous exams which should serve to indicate the general level of expectation.

Problem 1. Let

$$H = \left\{ \begin{pmatrix} a + c \\ a + b \\ a + 4b - 3c \\ a + 3b - 2c \end{pmatrix} : a, b, c \text{ are real} \right\}.$$

a. Show that H is a subspace of $\mathbb{R}^4$.

b. Find a basis for H.

Problem 2. Let $\mathcal{B} = \{1, t - 1, (t - 1)^2\}$ and let $\mathcal{C} = \{1, t, t^2\}$

a. Show that $\mathcal{B}$ is a basis for $\mathbb{P}_2$.

c. Let $p = 1 + t + t^2$. Compute $[p]_{\mathcal{B}}$.

Problem 3. Let $T : V \to W$ be a linear transformation between two vector spaces. Let $T(V)$ denote the range of T, which is a subspace of W. If $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ is a basis for V, show that $\{T(\mathbf{b}_1), T(\mathbf{b}_2), \ldots, T(\mathbf{b}_n)\}$ spans $T(V)$.

Problem 4. Let

$$A = \begin{pmatrix} -3 & 11 & 9 & 4 & -1 \\ -4 & 3 & -2 & -2 & 0 \\ -2 & 4 & 2 & 2 & 4 \end{pmatrix}.$$

a. Find bases for $\operatorname{Col} A$ and $\operatorname{Nul} A$.

b. Use your results from part (a) to determine if the map $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one or onto.

Problem 5. Suppose that

$$A = \begin{pmatrix} 1 & 2 & b \\ 2 & a & c \\ 1 & 2 & d \\ 4 & 8 & e \end{pmatrix}$$

and that A can be row reduced to

$$U = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

    a. How are b, c, d, e related to the other entries in A?

    b. What value(s) can a have?

    c. Find a basis for Nul A.

Problem 6. Show that the matrix

$$\begin{pmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{pmatrix}$$

is *not* invertible.

Problem 7. Show that

$$N = \left\{ \begin{pmatrix} a & 0 & c \\ 0 & b & 0 \\ c & 0 & a \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$$

is a subspace of $M_{3\times3}$.

Problem 8. Let $\mathcal{B} = \{1, t, t^3\}$ and let $\mathcal{C} = \{5, 2 - t, t^3 - 7t + 18\}$. Use the spanning set theorem to show that $\mathcal{B}$ and $\mathcal{C}$ span the same subspace of $\mathbb{P}_3$.