Workshop 7
Coordinates

Instructions:

Get into groups and work on the following exercises. Each group is expected to turn in *one neatly written copy* of their solutions at the end of the class period.

Throughout these exercises $\mathcal{V}$ is a vector space with basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ and $[\mathbf{v}]_{\mathcal{B}}$ is the $\mathcal{B}$-coordinate vector of $\mathbf{v} \in \mathcal{V}$.

Exercise 1. Show that a subset $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ of $\mathcal{V}$ is linearly independent if and only if the set $\{[\mathbf{v}_1]_{\mathcal{B}}, [\mathbf{v}_2]_{\mathcal{B}}, \ldots, [\mathbf{v}_p]_{\mathcal{B}}\}$ of coordinate vectors is linearly independent in $\mathbb{R}^n$.

Exercise 2. Given vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p, \mathbf{w} \in \mathcal{V}$, show that $\mathbf{w}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ if and only if $[\mathbf{w}]_{\mathcal{B}}$ is a linear combination of $[\mathbf{v}_1]_{\mathcal{B}}, [\mathbf{v}_2]_{\mathcal{B}}, \ldots, [\mathbf{v}_p]_{\mathcal{B}}$.

Exercise 3. Suppose that $\mathcal{V} = \mathbb{R}^n$. Let $\mathcal{E}$ denote the standard basis for $\mathbb{R}^n$. Consider the linear transformation $I : \mathbb{R}^n \to \mathbb{R}^n$ given by $I(\mathbf{x}) = \mathbf{x}$. Find $[I]_{\mathcal{B}}^{\mathcal{E}}$. Do you recognize this matrix?

Exercise 4. Let $\mathcal{C} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ be a set of vectors in $\mathcal{V}$ with the following property: given any vector $\mathbf{w} \in \mathcal{V}$, the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p = \mathbf{w}$ has *at most* one solution. Show that $\mathcal{C}$ must be a linearly independent set.