Homework 4

February 5, 2009

1 Volumes

Instructions : You are encouraged to work out solutions to these prob- lems in groups! Discuss problems with your classmates, the tutors and/or the instructors. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper, showing all of your work (this part should be done on your own). When you are asked to give explanations, be sure to use complete sentences. You are welcome (and sometimes encouraged) to use calculators or computing devices.

Pepito has a blender in his house that is a circular cylinder of radius 2 inches and height 7 inches. Every morning he uses it to make banana milkshakes (very healthy stuff). He notices that after the blender starts spinning that the liquid forms a shape of a parabola. After careful analysis and measurements, he finds out the following: If we name ω the angular speed of the blender (which he can control because his blender is very expensive) and we name h the height of the liquid at it lowest point (lowest point of the parabola), then the parabola follows the equation

$$y = h + \frac{\omega^2 x^2}{2g}$$

where g is the acceleration due to gravity ($39.37 \frac{in}{s^2}$).

Assume we place the blender on a Cartesian plane such as in Figure 1, we would have $r = 2$, $L = 7$.

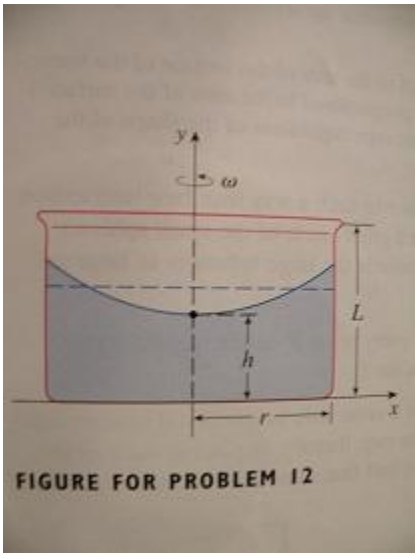

Figure 1: Blender of radius 2 inches and height 7 inches

1 Today, Pepito notices that at the speed the blender is going, the milkshake is 5 inches below the top of the blender and that the milkshake is 4 inches below the top if we look one inch away from the center of the blender. Pepito asks you to help him with the next questions:

a What is the angular speed of the blender?

b What is the volume of the milkshake? (Note, the volume would be rotating the parabola along the y-axis)

c How far below the top of the blender is the liquid at the wall of the blender?

d If he stops the blender, how high will the milkshake be in the blender?

e What is the average value of the parabola between $[0, 2]$?

f Is the answer to **d** the same as the answer to **e**? Why or why not?

2 Pepito has enough milk and bananas in his blender to make exactly 37.7 in^3 of banana shake. Can he turn the blender so fast that the liquid will touch the bottom in the middle? or will it spill on the sides first?