Name:_____ KEY _____

Section:_____

Math 1 Fall 2005 Exam 2

Instructions

This exam is being given under the Dartmouth College Honor Principle. It is a closed book, closed note exam and calculators are not permitted. All work is to be your own. If work is not shown on problems where we view it as necessary, we reserve the right to give you no credit. Please box your answers. Good luck!

Show your work.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others. I acknowledge that I understand that my current course grade will be included when this exam is returned and this may also be visible to others.

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For questions 1-4, find the derivative of the given function.

1.

$$x^5 + 5x^2 + 7\sqrt{x}$$

$$5x^4 + 10x + \frac{7}{2} x^{-1/2}$$

2.

$$\frac{x^2 + 3}{\sin(x)}$$

$$\frac{(2x) \sin(x) - (x^2 + 3)(\cos(x))}{(\sin(x))^2}$$

3.

$$3\left(\sin(x)\right)^4$$

$$12\left(\sin(x)\right)^3 \cos(x)$$

4.

$$\frac{5}{x} + \left(x^3 - 27\right)^4$$

$$-5x^{-2} + 4\left(x^3 - 27\right)^3 \cdot 3x^2$$

For problems 5-11, refer to the function

$$f(x) = -3x^2 + 6x + 10$$

5. What is the derivative of $f(x)$?

$-6x + 6$

6. What is the second derivative of $f(x)$?

-6

7. What are the critical points of $f(x)$?

$-6x + 6 = 0$

$x = 1$

8. What are the local extrema of $f(x)$ on the entire real line? List all of them by x and y-coordinate and say whether they are local minima or local maxima.

$f(1) = 13$ increase to 13
                      decrease after 13
$f'(0) = +6$
$f'(2) = -6$ $f(1) = 13$ is a local maximum
                      no other local extrema.

9. Where is $f(x)$ increasing? Where is it decreasing?

$f(x)$ is increasing on $(-\infty, 1)$
$f(x)$ is decreasing on $(1, \infty)$

10. Where is $f(x)$ concave up? Where is it $f(x)$ concave down?

$f''(x) = -6$
$f(x)$ is always concave down.

11. Find the minimum and maximum values of $f(x)$ on $[0, 2]$.

$f(0) = 10$
$f(2) = 10$
$f(1) = 13$

minimum value 10
at $x = 0$ and $x = 2$
maximum value 13
at $x = 1$.

For problems 12-16, refer to the function

$$g(x) = \left(-5x^2 + 7\right)^5$$

12. What is the derivative of $g(x)$?

$$5\left(-5x^2+7\right)^4 (-10x)$$

13. What is the second derivative of $g(x)$?

$$5\left(-5x^2+7\right)^4 (-10) + 20\left(-5x^2+7\right)^3 (-10x)^2$$

14. What are the critical points of $g(x)$?

$$5\left(-5x^2+7\right)^4 (-10x) = 0$$

$$-5x^2+7=0 \qquad\qquad -10x = 0$$

$$x^2 = \cancel{} \; 7/5 \qquad\qquad x = 0$$

$$x = \pm\sqrt{7/5}$$

15. Where is $g(x)$ increasing? Where is it decreasing?

$$g'(-2) = 5 \cdot (\overset{neg}{(-pos)})^4 (pos) \geqslant 0 \qquad \text{increasing}$$

$$g'(-1) = 5 \cdot (pos)^4 (pos) > 0 \qquad \text{increasing}$$

$$g'(1) = 5 \cdot (pos)^4 (neg) < 0 \qquad \text{decreasing}$$

$$g'(2) = 5 \cdot (neg)^4 (neg) < 0 \qquad \text{decreasing}$$

$g(x)$ is increasing on $(-\infty, 0)$
$g(x)$ is decreasing on $(0, \infty)$

16. Find the minimum and maximum values of $g(x)$ on $[-1, 2]$.

$$g(-1) = (+2)^5$$

$$g(0) = (+7)^5$$

$$g(\sqrt{7/5}) = 0$$

$$g(2) = (-13)^5$$

minimum: $(-13)^5$ at $x = 2$

maximum: $\cancel{0 \quad at \quad x = \sqrt{7/5}}$

7^5 at $x = 0$

For problems 17-20, refer to the function

$$h(x) = x^3 - 3x^2 + 5$$

17. What is the derivative of $h(x)$?

$$3x^2 - 6x$$

18. What is the second derivative of $h(x)$?

$$6x - 6$$

19. What are the local extrema of $h(x)$ on the entire real line? List all of them by x and y-coordinate and say whether they are local minima or local maxima.

crit pt: $3x^2 - 6x = 0$ $x = 0, 2$
            $3x(x - 2) = 0$

$h'(-1) > 0$ increasing $h(x)$ is increasing on $(-\infty, 0)$
$h'(1) < 0$ decreasing   and $(2, \infty)$
$h'(4) > 0$ increasing $h(x)$ is decreasing on $(0, 2)$

$h(0) = 5$ is a local max
$h(2) = 1$ is a local min

20. Where is $h(x)$ concave up? Where is it concave down?

$6x - 6 = 0$ $x = 1$
$h''(0) = -6$ concave down
$h''(2) = 6$ concave up
$h(x)$ is concave down on $(-\infty, 1)$
$h(x)$ is concave up on $(1, \infty)$

Scratch Paper