| m8s05 | Midterm | Exam Time: Wed. May 4th, 6:00 - 8:00 |
|---|---|---|

| Name: | Student No.: |
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**Instructions:**

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT seperate the pages of your exam.

| Problem | Points | Score |
|---|---|---|
| A1 | 10 | |
| A2 | 7 | |
| A3 | 12 | |
| A4 | 15 | |
| A5 | 6 | |
| Total | 50 | |

Name:

**Section A:** Answer ALL questions.

**Problem A1:** [10 pts] Are the following series absolutely convergent, conditionally convergent or divergent. Fully explain your answers.

**(a)** $\sum_{n=2}^{\infty} \frac{n^3 - n}{(1+n^2)^2}$.

**(b)** $\sum_{n=0}^{\infty} (-1)^n \frac{n3^n}{(n+1)!}$.

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**Problem A2:** [7=5+2 pts] Consider the following power series

$$\sum_{n=0}^{\infty}(-1)^n\frac{5n}{4^n}(x-2)^{2n}.$$

(**a**) What is the radius of convergence of the power series?

(**b**) Using your answer to part (a), determine whether the following series are convergent or divergent. (Explain how you use your answer to part (a). You will not receive credit for directly testing the series for convergence.)

$$(I)\quad \sum_{n=0}^{\infty}(-1)^n\frac{5n}{4^n}5^n, \qquad\qquad (II)\quad \sum_{n=0}^{\infty}(-1)^n\frac{5n}{4^n}.$$

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**Problem A3:** [12=4+4+4 pts]

(**a**) Does the sequence $a_n = e^n \sin\left(\frac{1}{n}\right)$ converge of diverge. If it converges, what does it converge to?

(**b**) Find the sum of the series

$$\sum_{n=2}^{\infty} 2\frac{2^{2n}}{5^{n-1}}.$$

(**c**) Evaluate the following integral

$$\int \cos^3 x \sin^2 x \, dx.$$

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**Problem A4:** [15=7+8 pts]

(**a**) Find a power series expansion centered at $x = 0$ for the function

$$\frac{1}{2+x}.$$

What is the radius of convergence of your series?

(**b**) Use the fact that

$$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad \text{for } |x| < 1$$

and the integration properties of power series to explicitly find the value of

$$\sum_{n=0}^{\infty} (-1)^n \frac{(1/\sqrt{3})^{2n+2}}{(2n+1)(2n+2)}.$$

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**Problem A5:** [6 pts]

Use the midpoint rule estimation theorem, to find how many intervals are required to guarantee that the midpoint rule estimates

$$\int_0^2 \frac{1}{10}x^3 \, dx$$

to within $\frac{1}{100}$. (You will not receive any credit for a solution based upon explicitly computing the integral.)