Addendum to Using Maple in Math 23

1 Introduction

This handout is meant to supplement "An Introduction to Using Maple in Math 23." It assumes that you have read the previous handout and are familiar with some of the basics of Maple that were introduced there.

2 Phase Portraits

The `phaseportrait` command allows you to draw phase portraits of systems of differential equations. It will simultaneously draw many solution curves (trajectories) and will include a direction field if the system is autonomous. In order to use it, the `DEtools` package must first be loaded by typing `with(DEtools):` and pressing "enter." As with the other commands we've talked about, it's easiest to understand how to use `phaseportrait` by looking at an example. Suppose we would like to draw a phase portrait for the system

$$\begin{aligned} \frac{dx}{dt} &= x - x^2 - xy \\ \frac{dy}{dt} &= \frac{1}{2}y - \frac{1}{4}y^2 - \frac{3}{4}xy. \end{aligned}$$

This system has critical points at $(0,0), (0,2), (1/2, 1/2), (1,0)$, as is easily verified. Accordingly, we want to plot the system in a window that doesn't include too much more than the critical points, say $-1 \leq x \leq 3/2$, $-1 \leq y \leq 3$. We would type in the following:

```
phaseportrait([diff(x(t),t) = x(t) - x(t)^2 - x(t)*y(t),
diff(y(t),t) = 1/2*y(t) - 1/4*y(t)^2 - 3/4*x(t)*y(t)],
[x(t),y(t)], t=-10..40, [[x(0)=-.2, y(0)=.5],
[x(0)=.25, y(0)=.25]], x=-1..3/2, y=-1..3, arrows = slim,
stepsize = .05, linecolor = blue);
```

Let's try and decipher this command. The first thing we see inside the parentheses is `[diff(x(t),t) = x(t) - x(t)^2 - x(t)*y(t), diff(y(t),t) = 1/2*y(t) - 1/4*y(t)^2 - 3/4*x(t)*y(t)]`. This is just the equations of the system.

Notice two things: the equations are enclosed in a pair of square brackets and instead of `x` and `y` we have written `x(t)` and `y(t)`, specifying explicitly that x and y are dependent and t is independent. The notation for dx/dt is `diff(x(t),t)`, as was used before.

The next thing we see is `[x(t),y(t)]`. This just tells Maple that x and y are functions of the independent variable t. Notice that, as above, this part is enclosed in square brackets. Next we come to `t=-10..40`. This is the t range that Maple uses when plotting the solution curves. It is usually a good idea to have the t range include both positive and negative values (here we have $-10 \leq t \leq 40$). You should try various choices for the range and see what produces the nicest phase portrait for the system at hand. For example, if your solution curves don't seem to be getting very close to what you know is an asymptotically stable point, try extending the t range. You'll see what I mean if you change the range above to `t=-10..10`.

Now we come to what will probably become the longest single part of the command

```
[[x(0)=-.2, y(0)=.5], [x(0)=.25, y(0)=.25]]
```

This is a list of initial conditions. Here we are asking Maple to draw two solution curves, one with the initial conditions $x(0) = -.2$, $y(0) = .5$ and another with $x(0) = .25$, $y(0) = .25$. Notice that each pair of initial conditions is enclosed in square brackets, each pair is separated by a comma and the entire list is enclosed in another set of square brackets. More often than not, two solution curves don't give a very complete phase portrait. Therefore, it is usually necessary to specify (many!) more initial conditions. A list of initial conditions such as

```
[[x(0)=.5, y(0)=.1],[x(0)=0,y(0)=3],[x(0)=.25, y(0)=2.5],
[x(0)=-.2,y(0)=.5],[x(0)=-.5, y(0)=-.25],[x(0)=.25, y(0)=.25],
[x(0)=-.5, y(0)=-.25],[x(0)=.5, y(0)=.49],[x(0)=.25, y(0)=.25],
[x(0)=.5, y(0)=.51],[x(0)=.5, y(0)=1],[x(0)=-.25, y(0)=1],
[x(0)=-.75, y(0)=1.5],[x(0)=.25, y(0)=.5],[x(0)=1, y(0)=1],
[x(0)=0, y(0)=.1],[x(0)=1, y(0)=.4],[x(0)=.5,y(0)=-.5],
[x(0)=.5, y(0)=2],[x(0)=.5, y(0)=1.4],[x(0)=1.5, y(0)=0],
[x(0)=.5, y(0)=0],[x(0)=-.1, y(0)=0],[x(0)=0,y(0)=-.5]]
```

would not be inappropriate for this system. How should you come up with such a list? By looking at the phase plane and direction field! One technique is to proceed as follows. Use `phaseportrait` to plot the phase plane and direction field with one or two "randomly chosen" initial conditions. Then look at the picture you get and decide where you'd like to see more curves. For example, if you felt like you wanted a solution curve through the point $(1, 1)$ then you would add `[x(0)=1, y(0)=1]` to the list of initial conditions. If you wanted a solution curve through the point $(-.5, .25)$ then you'd add `[x(0)=-.5, y(0)=.25]` to the list. It's likely that you'll need to do this several times until you get exactly the picture you want. It may be tedious to do so, but it certainly isn't difficult.

The rest of the stuff in the `phaseportrait` command is pretty straightforward. `x=-1..3/2` and `y=-1..3` specify the x and y ranges of the plotting window, respectively. `arrows = slim` just tells Maple to draw the direction field using "slim" arrows. You can leave this option off entirely if you want to (the arrows just look a little different) or you can replace it with `arrows = none` if you don't want the phase portrait to include a direction field. Without getting into the details of how Maple draws the solution curves, let's just say that `stepsize = .05` controls the "smoothness" of the curves it draws. If you find yourself getting jagged curves, try decreasing this value. In most cases .05 should suffice. Finally, `linecolor = blue` is the (previously elusive!) color change command. Here we are specifying that the curves should be drawn in blue. Any of the colors mentioned earlier would work in place of `blue`. If you want the arrows in the direction field to be a different color (red is the default) then use the `color` option as above, e.g. add `color=green` for green arrows.

As a final comment, let me say that if you have a long list of initial conditions then don't be surprised if it takes Maple a little while to draw the phase portrait. To give you some frame of reference, it takes Maple roughly 15 seconds (on my office G5 iMac) to draw the phase portrait for the system we've been discussing with the long list of initial conditions above.