Algebra Syllabus

Last modified: January 2008

The student should easily be able to give all relevant definitions, provide standard examples, state major theorems, and provide the ideas behind their proofs. Specific topics which will be covered include:

Groups: Homomorphism theorems; group actions, Cayley's theorem, class equation; elementary permutation group theory; the Jordan-Hölder theorem; Sylow theorems; free, abelian, and simple groups; direct and semi-direct products; structure of finitely generated abelian groups; basic category theory, especially the notions of universal objects such as free objects and (co)products, as well as contravariant and covariant functors between categories.

Rings: Unique factorization domains, principal ideal domains, polynomial rings, irreducibility criteria; matrix rings; Noetherian rings and the Hilbert basis theorem; ideal theory; rings of quotients and localization.

Fields: Prime fields; characteristic of a field; finite, algebraic, transcendental, normal, separable and inseparable extensions; finite fields; algebraic closure of a field; Galois theory; detailed examples of the Galois correspondence.

Modules and Linear Algebra: Modules; free and projective modules, tensor products of modules and algebras, structure theory for finitely generated modules over PIDs; Finite dimensional vector spaces; linear dependence and independence; linear transformations; dual space; determinants; characteristic values and canonical forms.

REFERENCES

Artin, *Algebra*

Dennis & Farb, *Noncommutative Algebra*

Dummit & Foote, *Abstract Algebra*

Hoffman and Kunze, *Linear Algebra*

Jacobson, *Basic Algebra I, II*

Knapp, *Basic & Advanced Algebra*

Lang, *Algebra*

Rotman, *The Theory of Groups*