Deformations of Chaotic Billiards and a New 'Wall Formula' for Heating Rate

Alex Barnett

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Collaborators:
Doron Cohen
Rick Heller

Lyman Laboratory of Physics, Harvard University

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Outline of today's talk

- Deforming billiards + motivation
- Key statements :
  1. Special class of deformations
  2. Vergini-Saraceno numerical method
  3. Improve 'wall formula'

- Theory of heating + 'wall formula' (classical)
- Explain 'special' deformations
- Quasi-orthogonality on the boundary (quantum)
- Improved 'wall formula' in action

SEE PAPERS: Alex Barnett, Doron Cohen and Eric J. Heller

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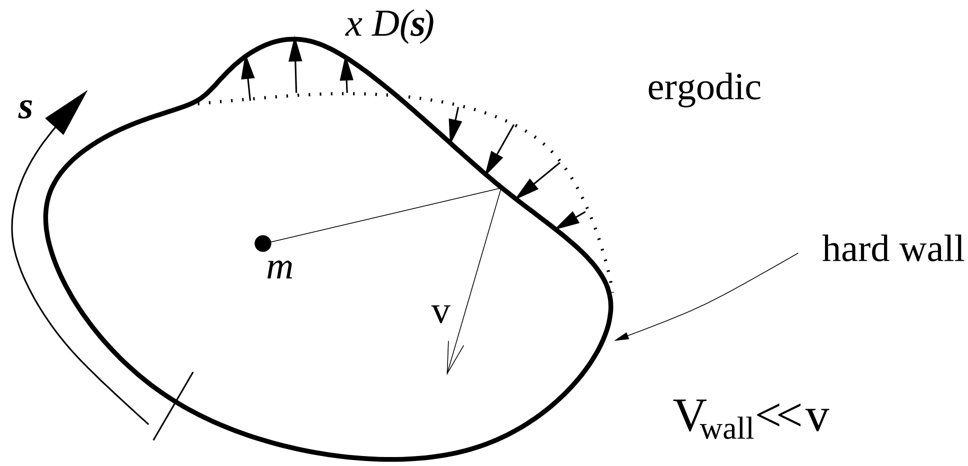
Deforming billiard (cavity) systems



$$D(\mathbf{s}) \;=\; \text{deformation shape function}$$
$$x(t) \;=\; A\sin\omega t \qquad \text{periodic ‘driving’}$$

Question: At what rate is the ‘gas’ particle heated up?

Motivations

- Dissipation rate of vibrations of nuclei (3D)
  - never considered ω-dependence
- Driven mesoscopic 2D quantum dots (*e.g.* $x =$ gate voltage)
  - find heating rate of electrons

1) Special class of deformations

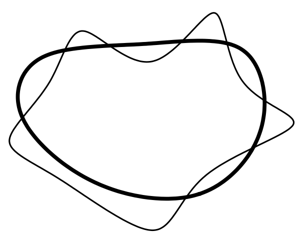
How does heating rate depend on deformation $D(\mathbf{s})$?

$$\text{heating} \quad \frac{d}{dt}\langle \mathcal{H} \rangle \; = \; \mu(\omega) \cdot \frac{1}{2}(A\omega)^2$$

$$\mu(\omega) \; = \; \text{friction coefficient}$$

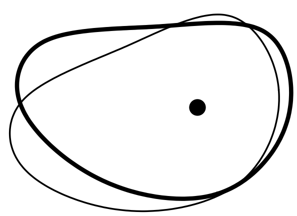
For low frequency $\omega \ll$ collision rate:

Generic



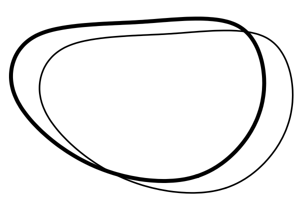
$\mu(\omega) \sim \text{const}$

Rotation



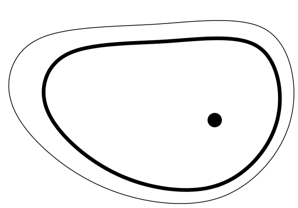
$\mu(\omega) \sim \omega^2$

Translation



$\mu(\omega) \sim \omega^4$

Dilation

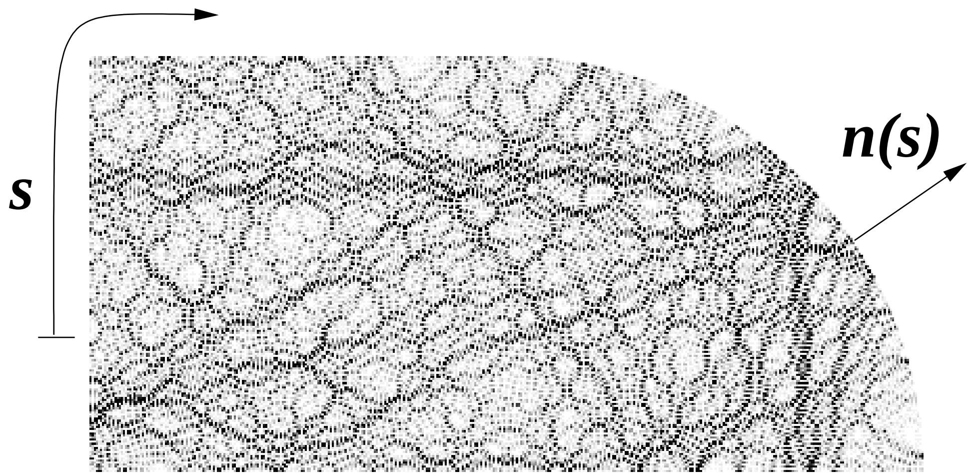


$\mu(\omega) \sim \omega^4$

Does **not** depend on billiard shape or chaoticity

Special class surprise: **friction vanishes at dc** $\mu(\omega \to 0) = 0$

2) Vergini-Saraceno numerical method



eigenstate ψ_n

boundary function $\varphi_n \equiv \mathbf{n} \cdot \nabla \psi_n$

Quasi-orthogonality on boundary:

$$\oint (\mathbf{r} \cdot \mathbf{n}) d\mathbf{s} \, \varphi_n(\mathbf{s}) \varphi_m(\mathbf{s}) \quad \propto \quad \delta_{nm} \; + \; \text{“\textit{error}”} \left(\frac{E_n - E_m}{\hbar} \right)$$

V-S numerical method for finding eigenstates ψ_n

- 10^3 times more efficient than any other known method!
- finds clusters of eigenstates simultaneously
- needs *“error”* small close to diagonal

BUT No-one has known size of *“error”*!

I have shown: mean square *“error”*$(\omega) \; = \; a\,\omega^4$

Due to $\mu(\omega) \sim \omega^4$ for **dilation** deformation

3) Improved ‘wall formula’ estimate for $\mu(0)$

Nuclear physics interest (last 25 years):

- seek analytic estimate of friction $\mu(0)$ given $D(\mathbf{s})$
- assumed uncorrelated collisions (strong chaos)
  $\rightarrow$ ‘wall formula’
- they knew $\mu(0) = 0$ for translations and rotations
  $\rightarrow$ *ad hoc* corrections

But now know special class of $D(\mathbf{s})$ for which $\mu(0) = 0$

(even for strong chaos)

We show: there is consistent way to *subtract* all special components of a general $D(\mathbf{s})$

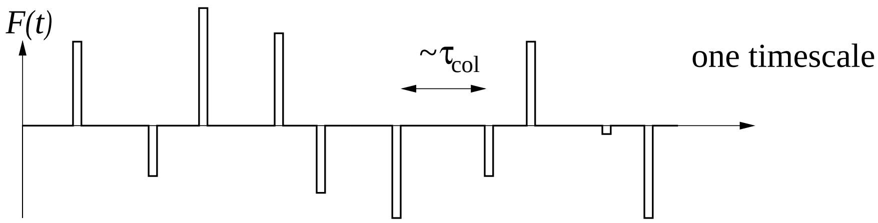
...**now** applying wall formula gives *improved* estimate of $\mu(0)$.

This replaces all *ad hoc* corrections

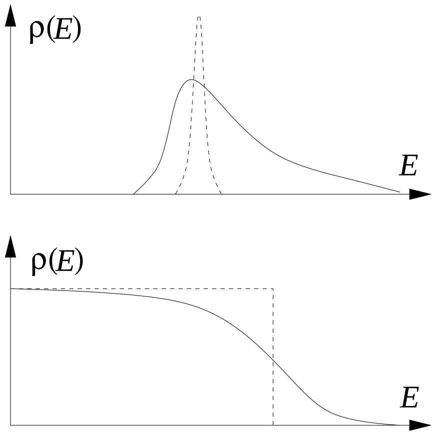
Theory of heating rate: energy spreading

Particle energy gets random 'kicks': $\dot{\mathcal{H}} = -\dot{x}(t)\mathcal{F}(t)$

where generalized 'force' on parameter $\mathcal{F}(t) \equiv -\dfrac{\partial \mathcal{H}}{\partial x}(t)$



Energy diffusion rate $D_{\mathrm{E}} \propto \tilde{C}_{\mathrm{E}}(\omega) \equiv$ **power spectrum** of $\mathcal{F}(t)$



Causes irreversible energy growth
(Jarzynski, Cohen)

Why? D_{E} increases with E

Friction coefficient $\mu(\omega) \propto \tilde{C}_{\mathrm{E}}(\omega)$
…relation depends on $\rho(E)$

The 'wall formula': white noise approximation (WNA)

Assume $\mathcal{F}(t) \approx$ white noise $\rightarrow$ $\tilde{C}_{\mathrm{E}}(\omega) =$ const (flat spectrum)

$$\tilde{C}_{\mathrm{E}}(0) \xrightarrow{\text{ergodicity}} b_{\mathrm{E}} \cdot \oint [D(\mathbf{s})]^2 d\mathbf{s}, \qquad \text{'wall formula' (Swiatecki)}$$

Some $D(\mathbf{s})$ obey WNA well, others badly...

Explanation of ‘special’ deformations

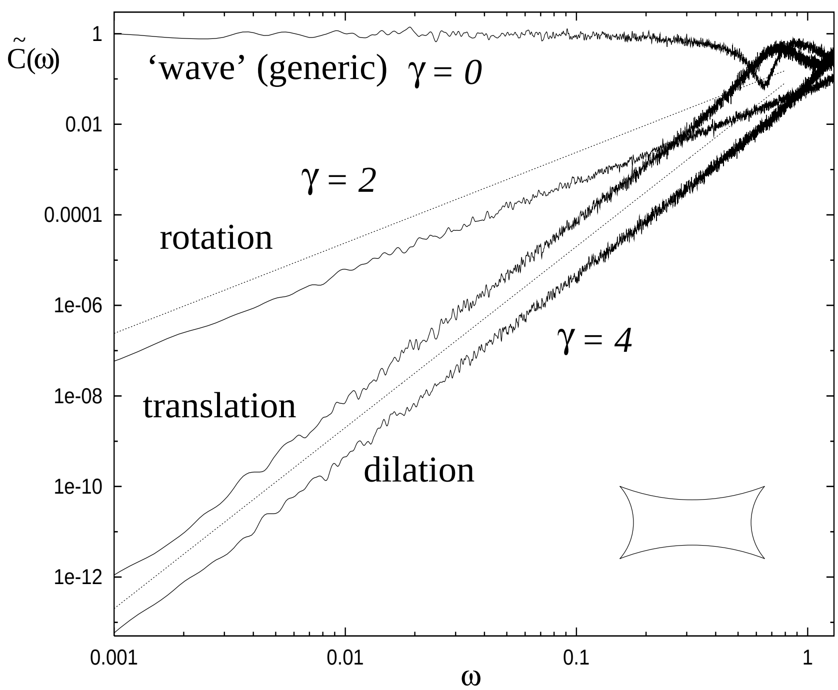
Some $D(\mathbf{s})$, WNA **fails**: $\tilde{C}_{\mathrm{E}}(0) = 0$ (even in strong chaos)

Could always write $\mathcal{F}(t) \;=\; \left(\frac{d}{dt}\right)^n \mathcal{G}(t)$

$\Rightarrow$ power spectra $\tilde{C}_{\mathrm{E}}(\omega) \;=\; \omega^{2n}\tilde{C}_{\mathcal{G}}(\omega)$ $\quad (\, d/dt \xrightarrow{\mathrm{FT}} i\omega \,)$

Special deformations: $\mathcal{G}(t) \;=\;$ some function of $(\mathbf{r}(t), \mathbf{p}(t))$

$\Rightarrow$ $\tilde{C}_{\mathcal{G}}(0)$ finite $\quad \Rightarrow$ $\tilde{C}_{\mathrm{E}}(0)$ vanishes



Generic $\tilde{C}_{\mathcal{G}}(\omega) \sim \omega^0 \;\rightarrow$ power laws $\tilde{C}_{\mathrm{E}}(\omega) \sim \omega^{\gamma}, \quad \gamma = 2n$

Improved estimate for $\tilde{C}_{\mathrm{E}}(0)$ in action

- COMPONENTS OF GENERAL DEFORMATION:

Linear subspaces of $D(\mathbf{s})$: 'special' $\perp$ WNA-good

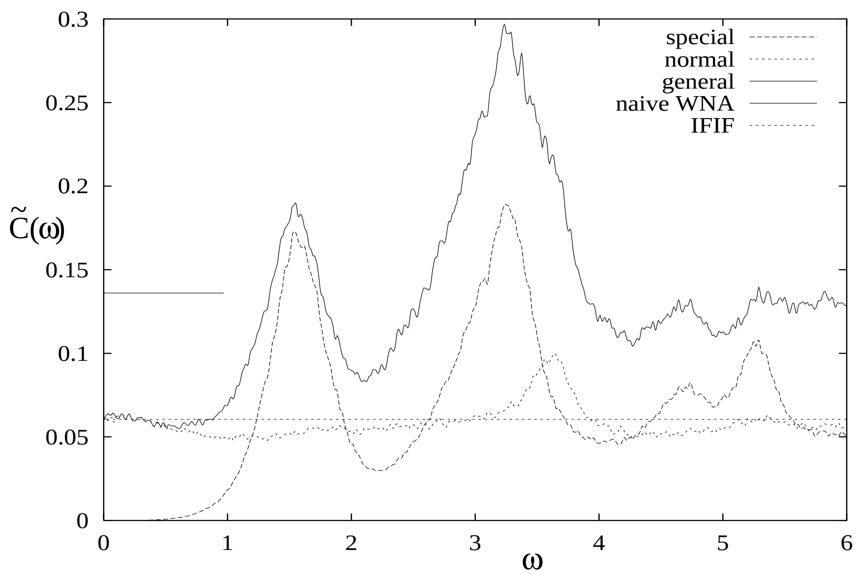
$$\text{Orthogonality}: \quad 1 \perp 2 \Leftrightarrow \oint d\mathbf{s} D_1(\mathbf{s}) D_2(\mathbf{s}) = 0$$

- SUBTRACT SPECIAL COMPONENT:

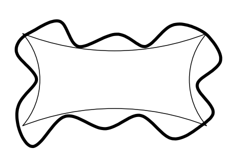
Make orthonormal set $\{D_{\mathrm{i}}(\mathbf{s})\}$ of special defs, $i = 1 \cdots 1 + \frac{1}{2}d(d+1)$

$$D_{\perp}(\mathbf{s}) \;=\; D(\mathbf{s}) - \sum_{\mathrm{i}} \alpha_{\mathrm{i}}\, D_{\mathrm{i}}(\mathbf{s}), \quad \text{components } \alpha_{\mathrm{i}} = \oint d\mathbf{s} D_{\mathrm{i}}(\mathbf{s}) D(\mathbf{s})$$

- NOW APPLY WNA TO $D_{\perp}(\mathbf{s})$:



'wave + const'



Conclusions

1. Classical & quantum dissipation rates computed in 2D billiards

   - first study of frequency-dependence in billiards
   - semiclassical correspondence found
   - applications: driven quantum dots, nuclei…

2. Class of 'special' deformations

   - friction coeff μ vanishes at dc
   - predicts new power laws $\mu(\omega) \sim \omega^{\gamma}$
   - dilation (*new*) $\rightarrow$ eigenstates quasi-orthogonal on boundary (*semiclassical* reason for Vergini-Saraceno method success)

3. Systematic subtraction of 'special' components of general $D(\mathbf{s})$

   - improved upon 25-year-old 'wall formula'