

# Combinatorics of affine Hecke algebras of type C.

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(joint with Arun Ram)

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The **affine type C Hecke algebra**  $H_k$  is generated by invertible elements  $T_0, T_1, \dots, T_k$  with relations



$$(T_0 - t_0)(T_0 - t_0^{-1}) = 0 = (T_k - t_k)(T_k - t_k^{-1})$$

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### Goal today:

Tell you 3 descriptions of calibrated irreducible reps of  $H_k$ , where “calibrated” means  $\mathbb{C}[Y_1^{\pm 1}, \dots, Y_k^{\pm 1}]$  is simultaneously diagonalized.

## Central characters

The center of  $H_k$  is symmetric Laurent polynomials

$$Z(H_k) = \mathbb{C}[Y_1^{\pm 1}, \dots, Y_k^{\pm 1}]^{W_0}$$

w.r.t. the Weyl group  $W_0$  of type C.

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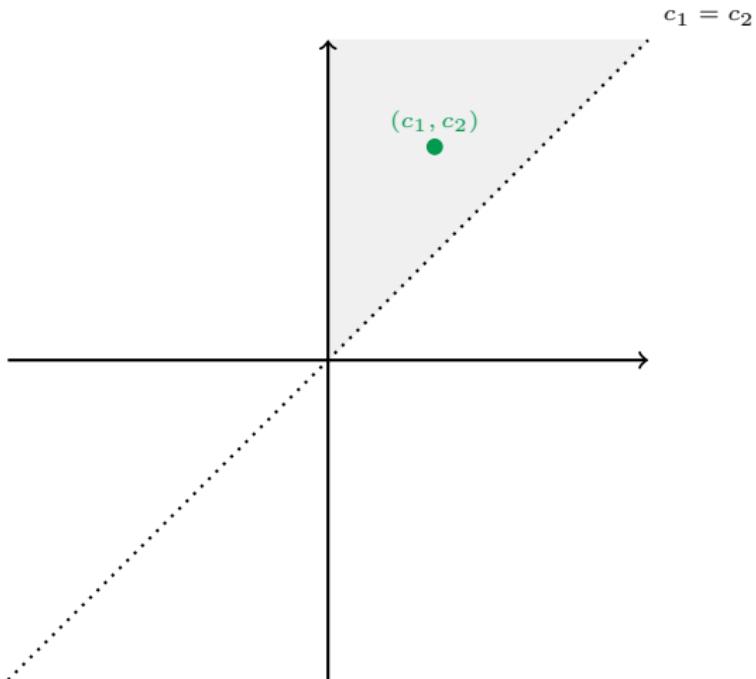
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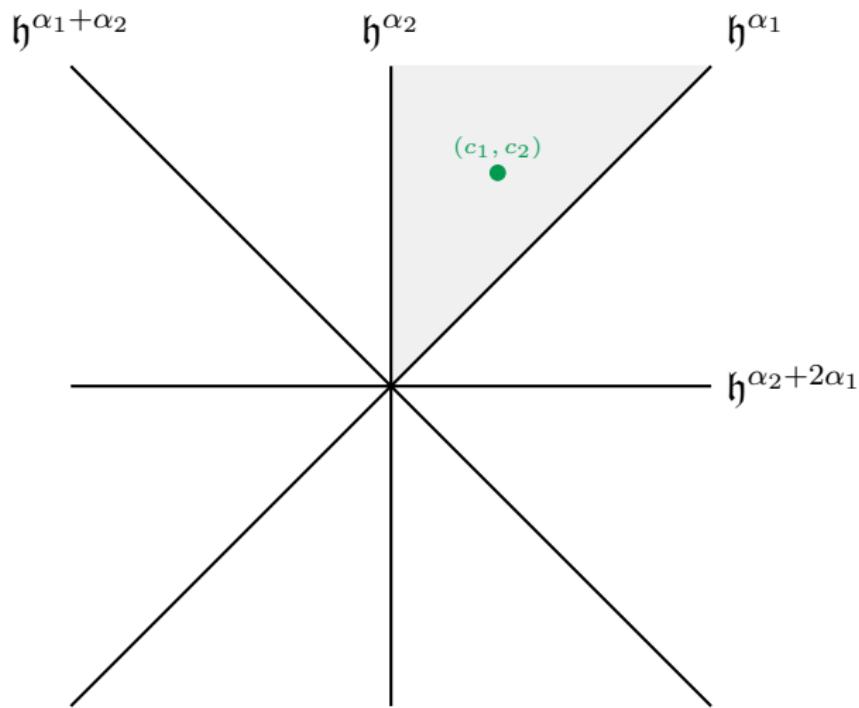
Fav equivalence class reps:  $0 \leq c_1 \leq \cdots \leq c_k$ . ( $W_0$  acts by signed permutations)  
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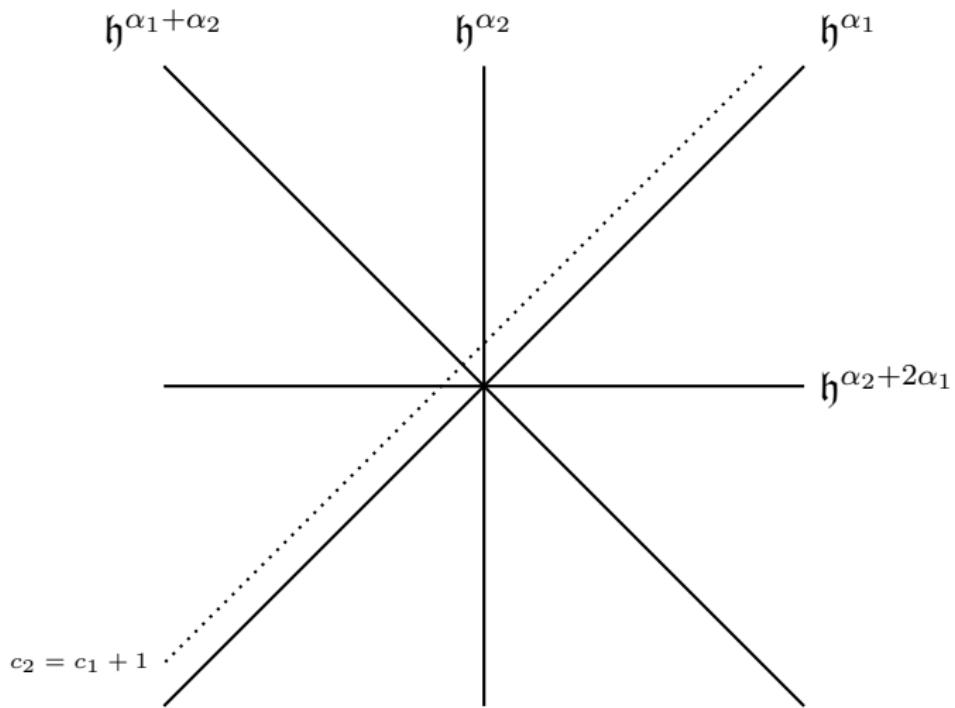
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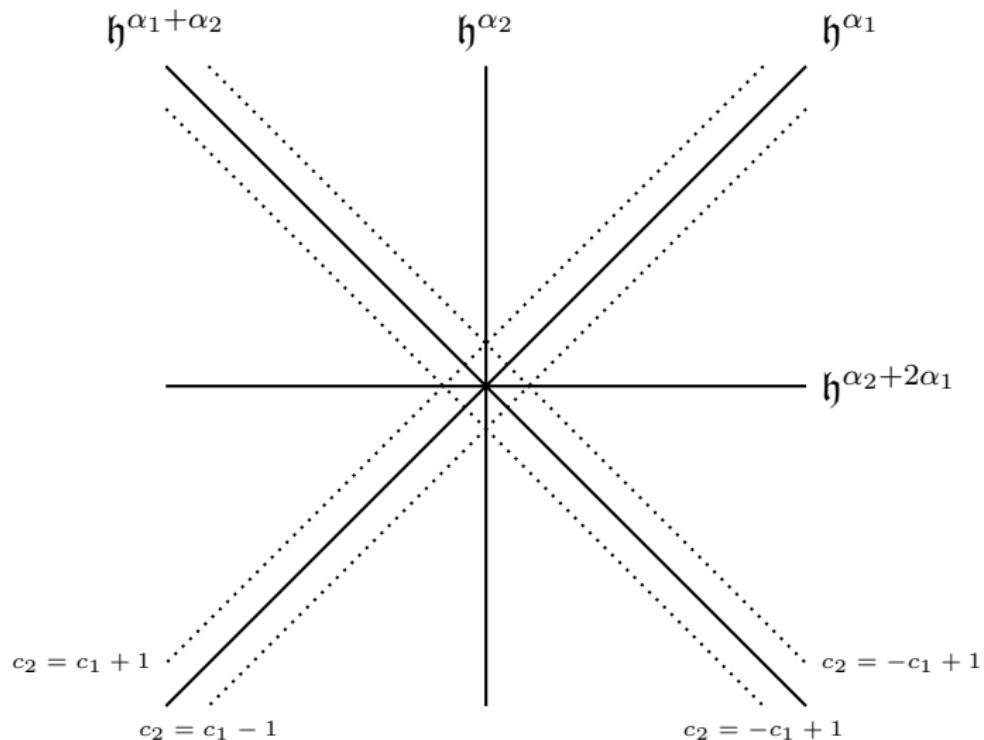
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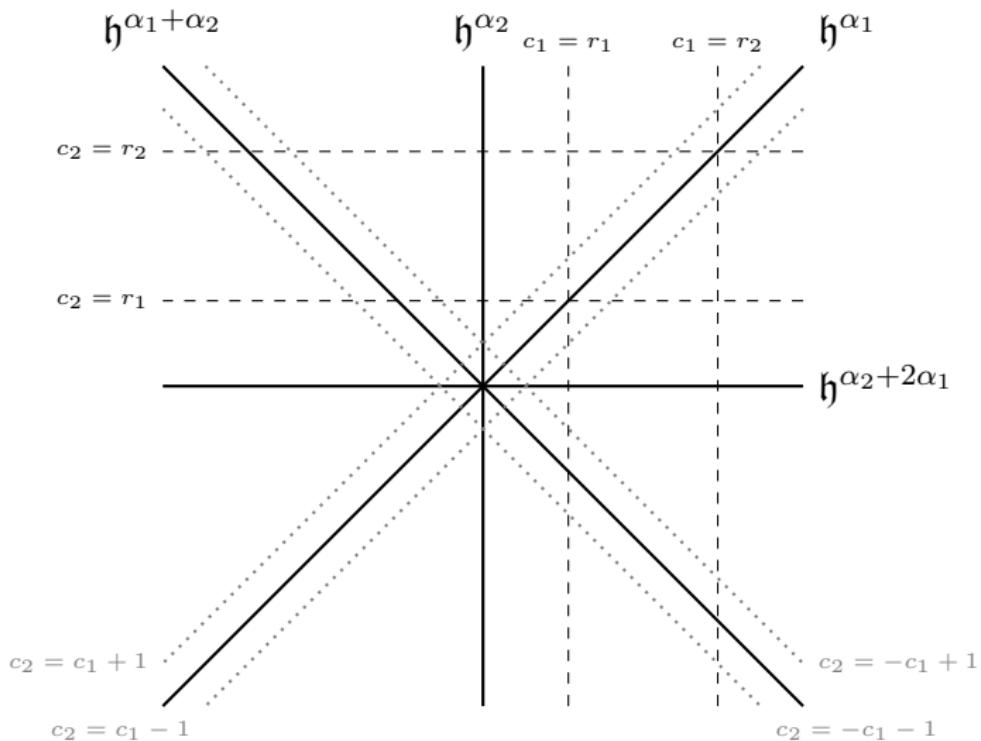
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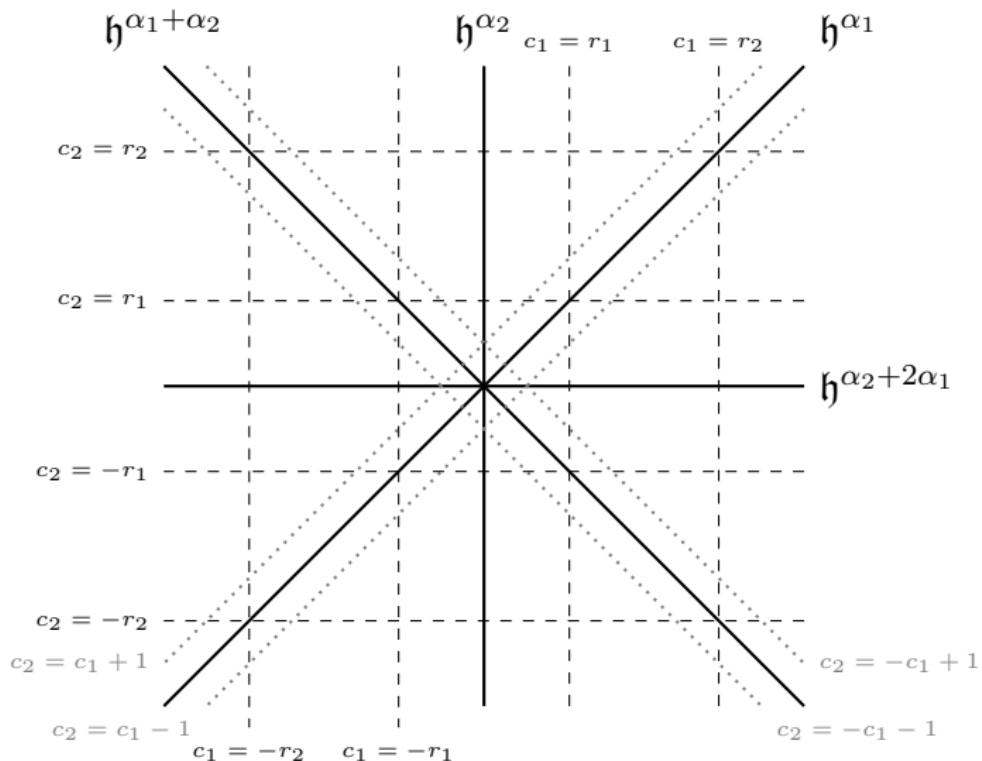


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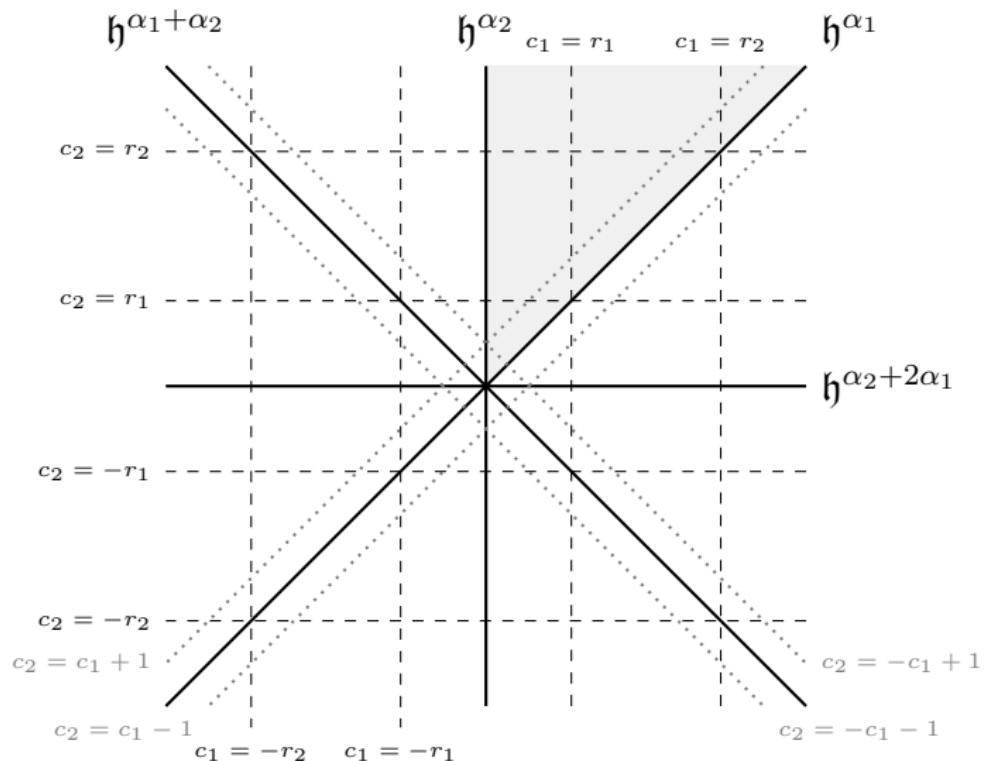


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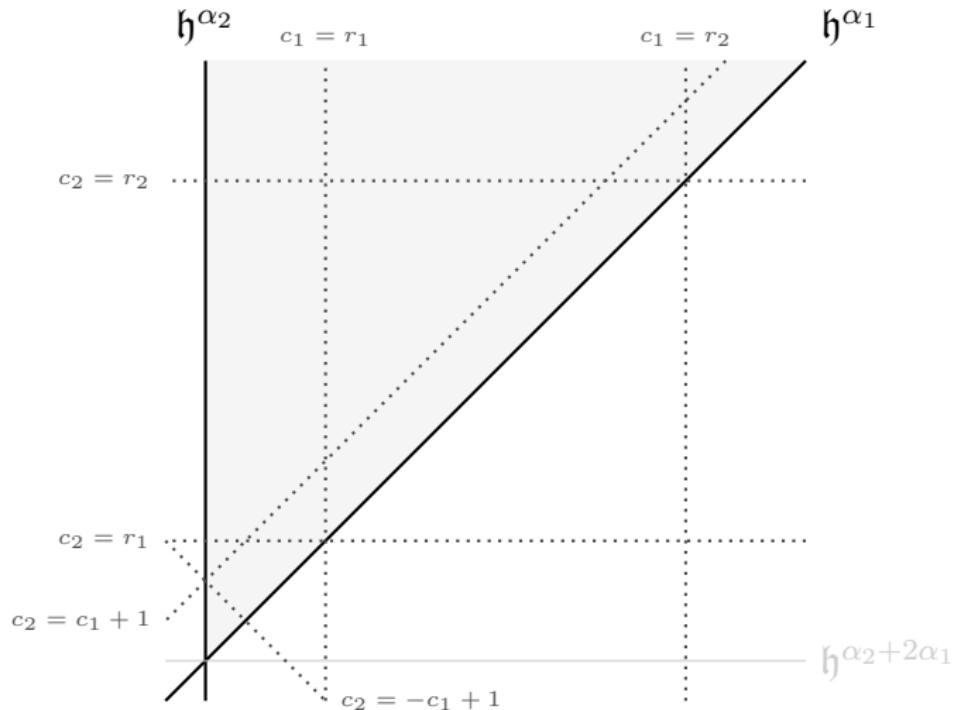
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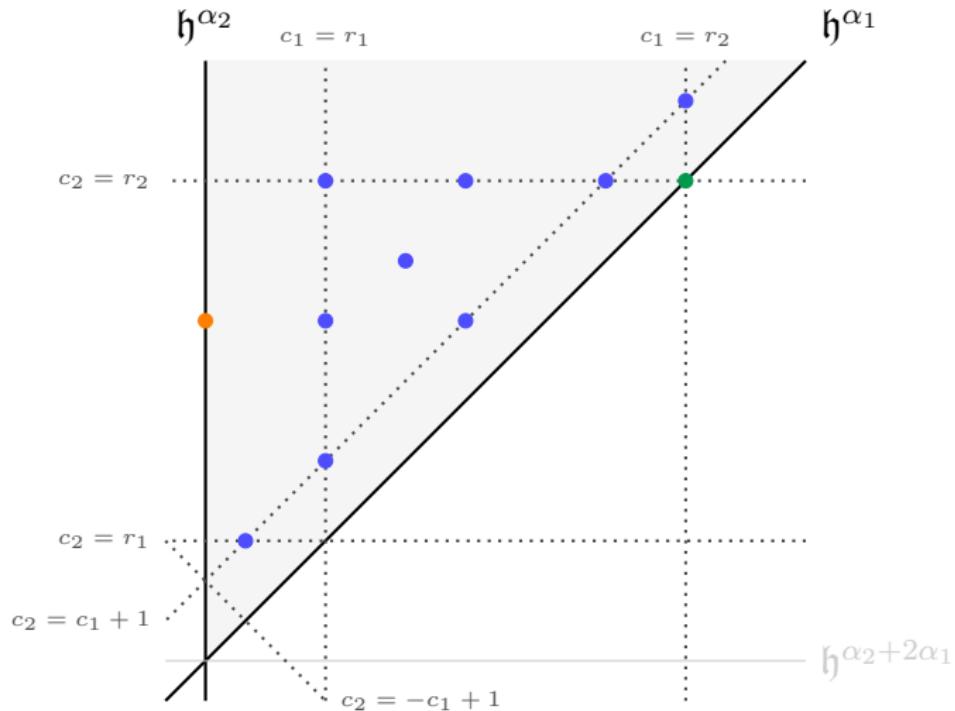
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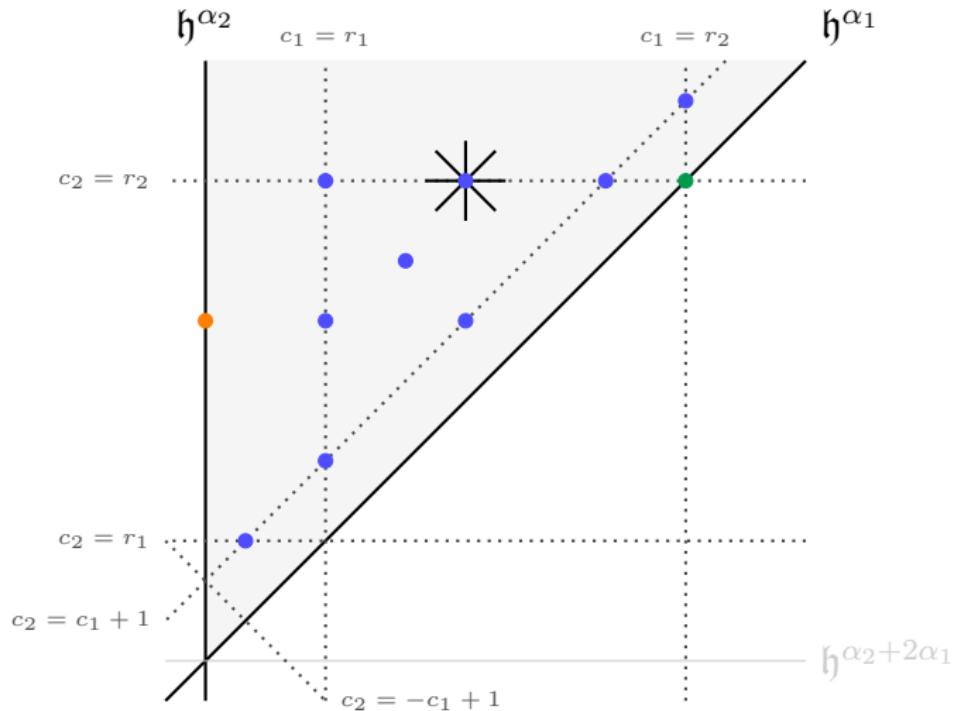
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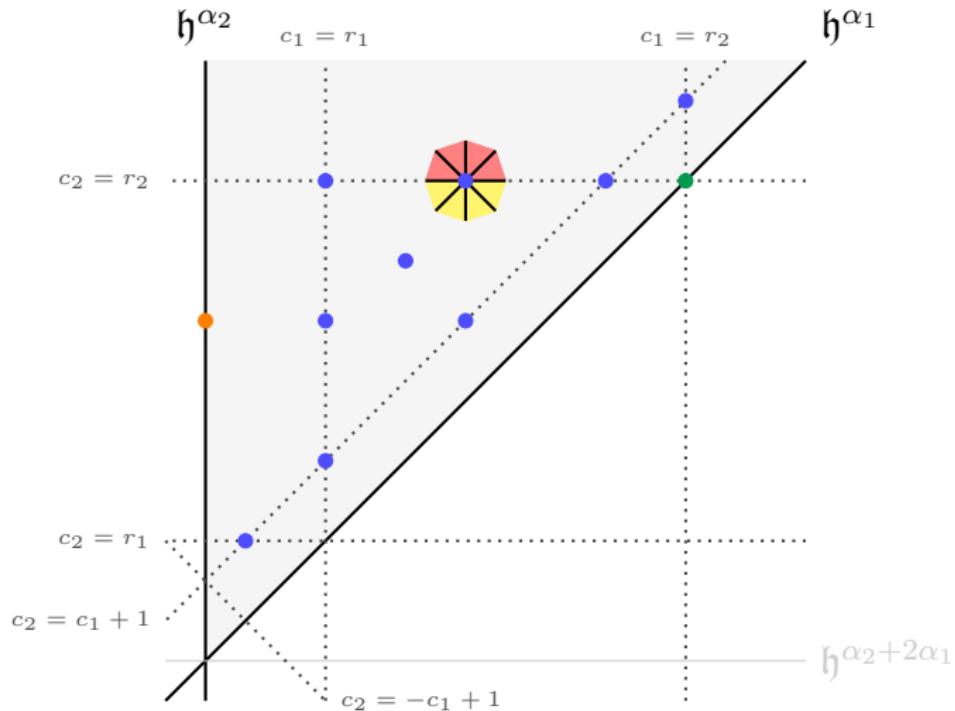
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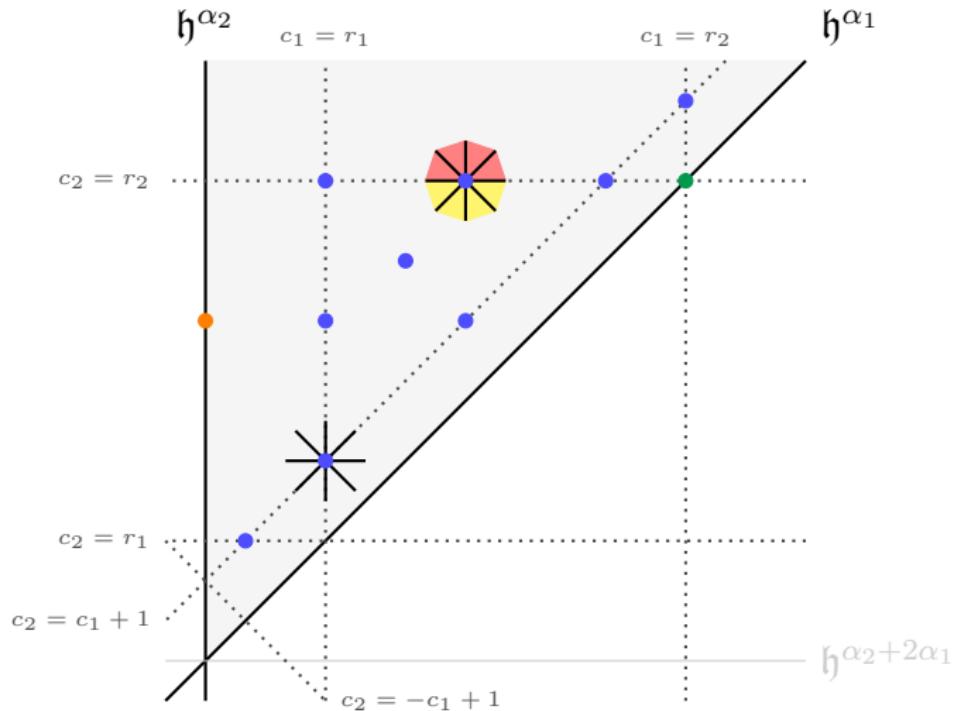
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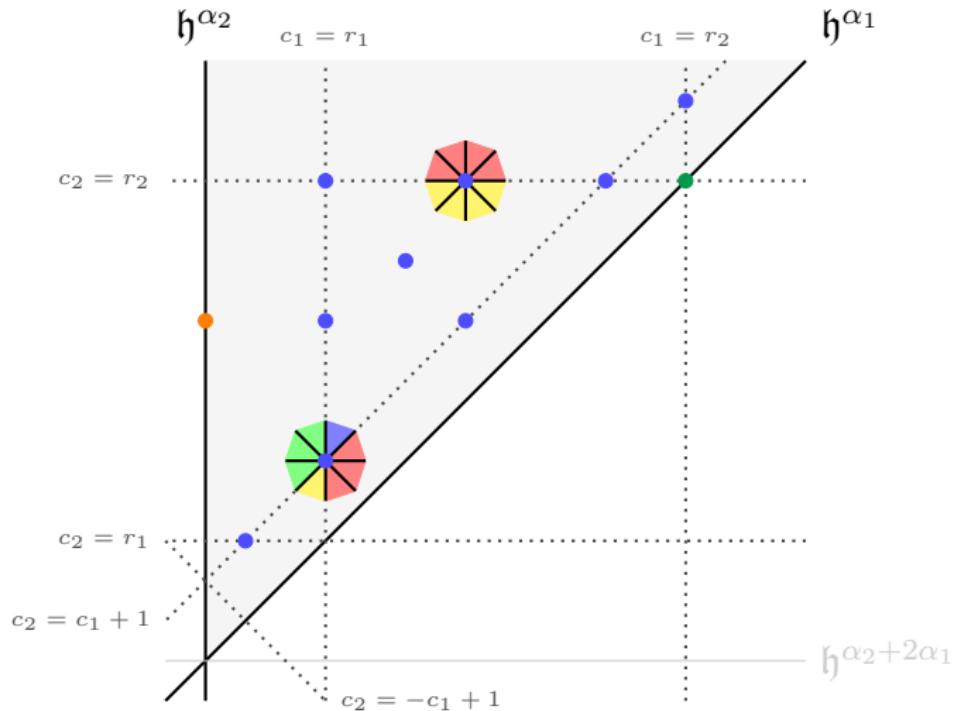
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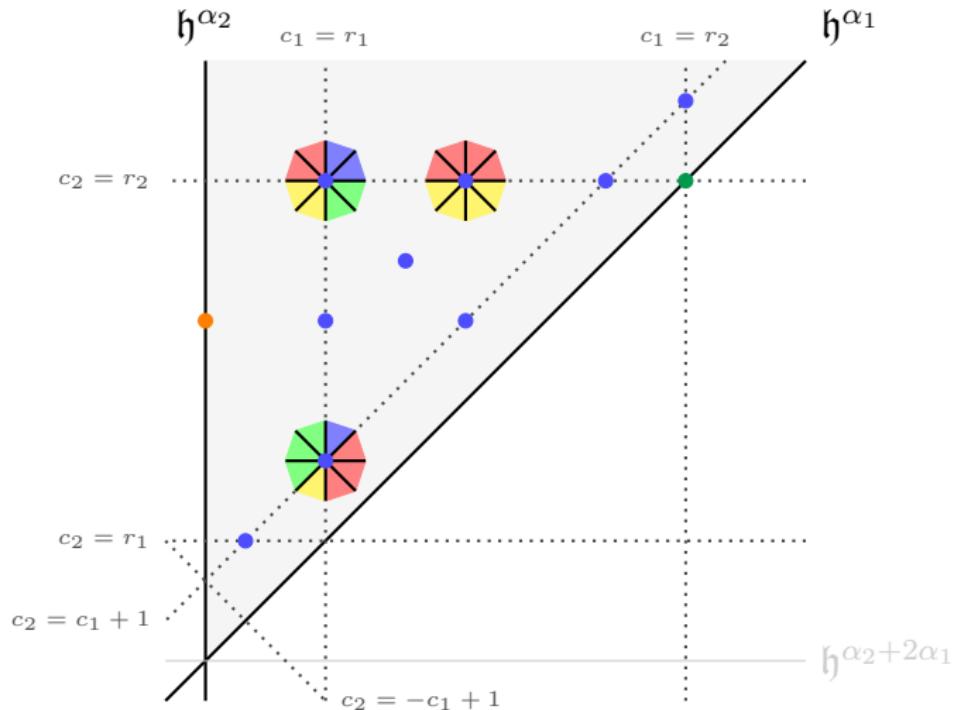
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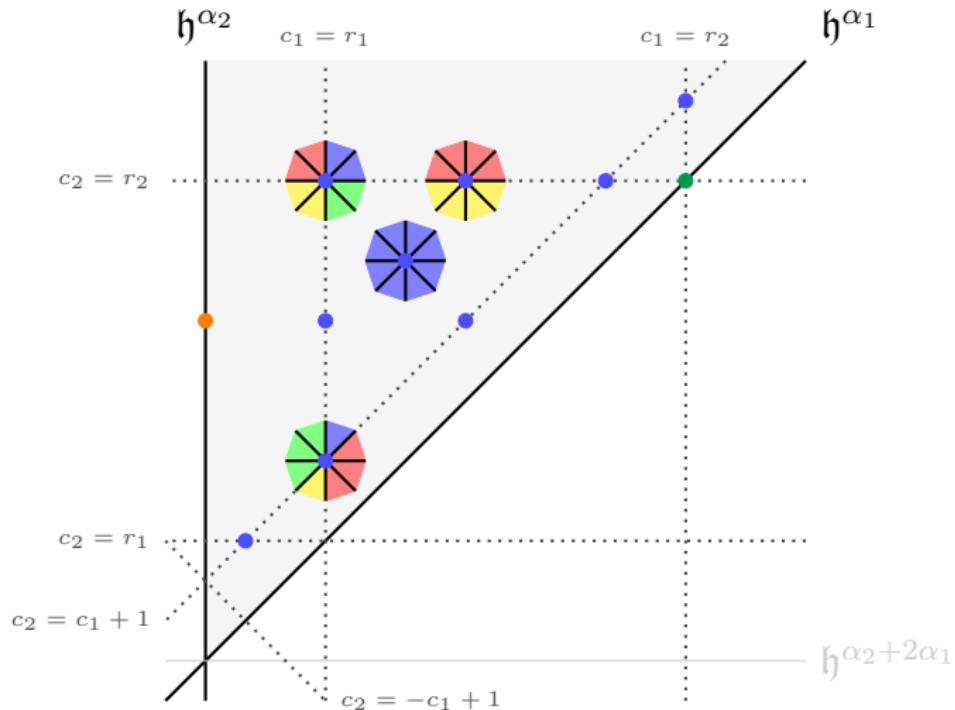
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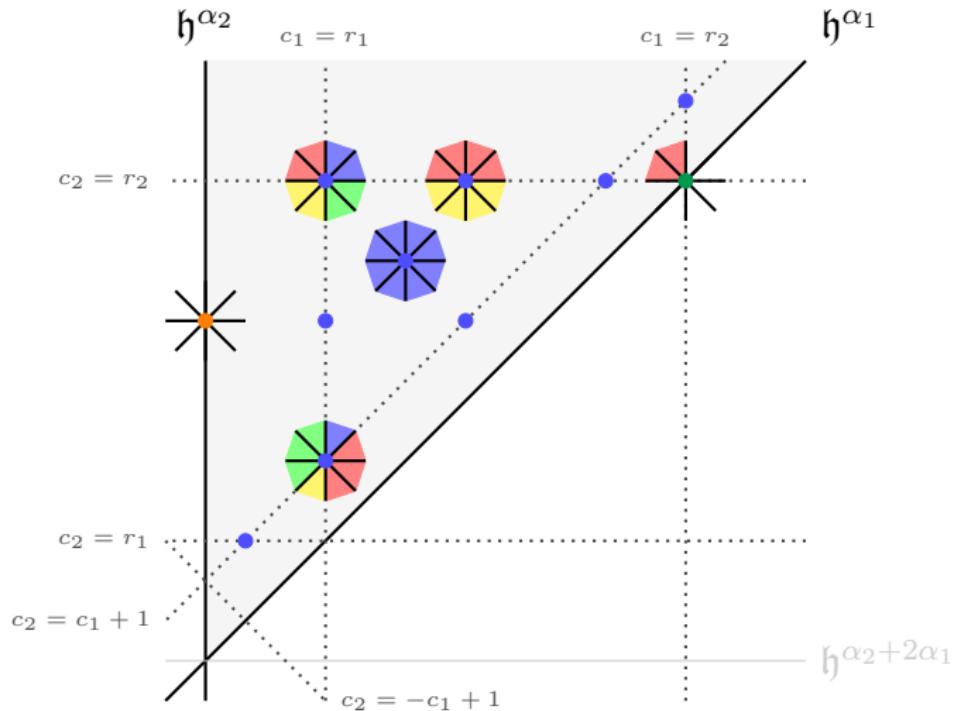
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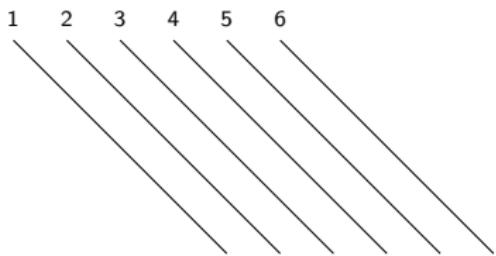
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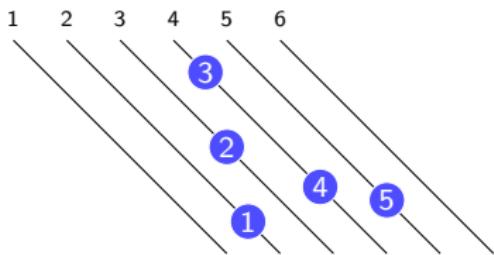
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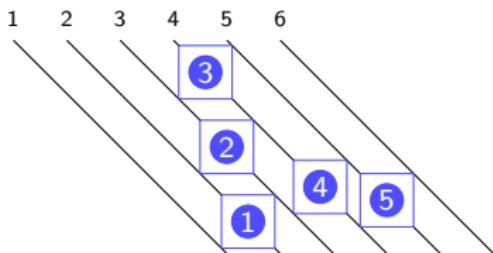
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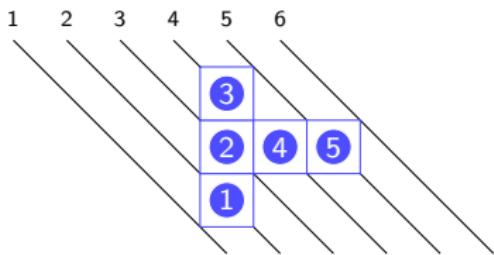
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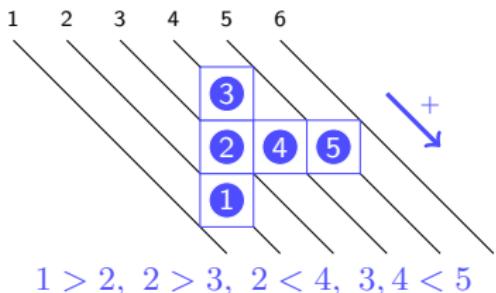
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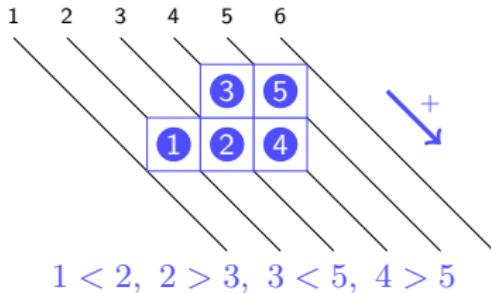
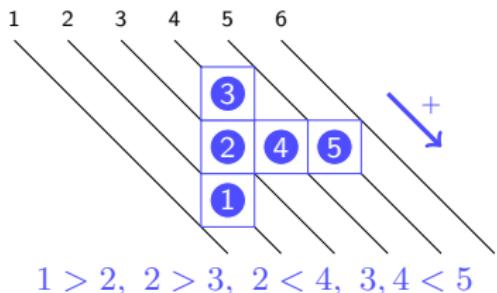
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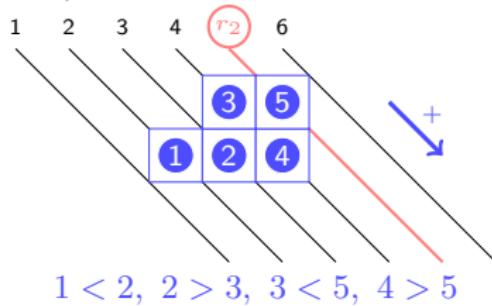
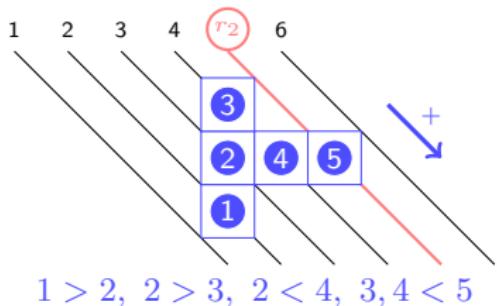
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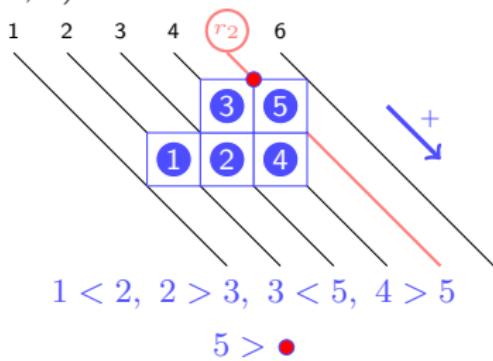
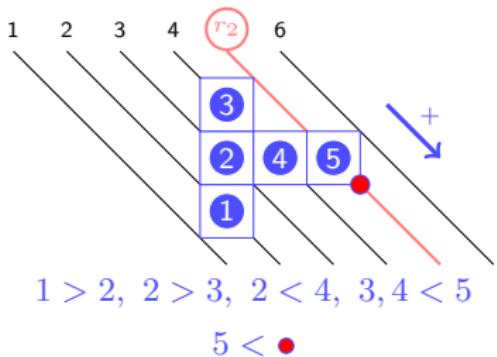
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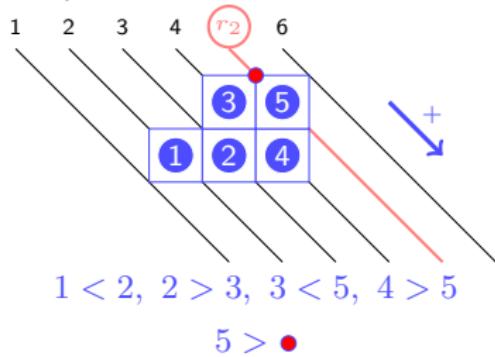
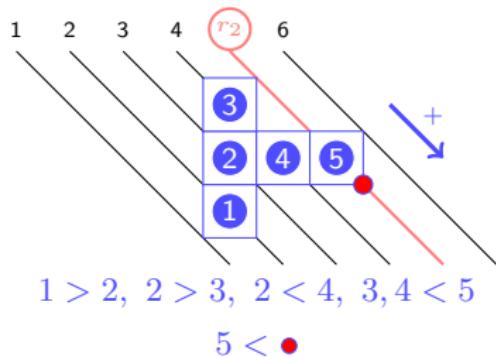
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Basis indexed by standard fillings with  $\{\pm 1, \dots, \pm k\}$  with restrictions:

(1) Exactly one of  $i$  or  $-i$  appears.

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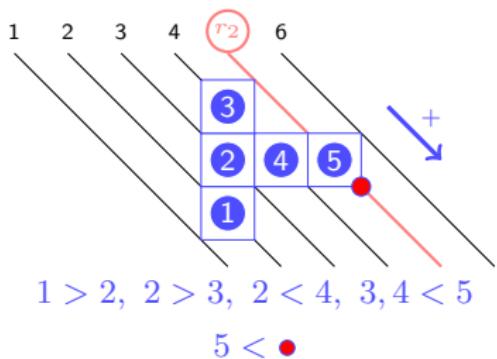
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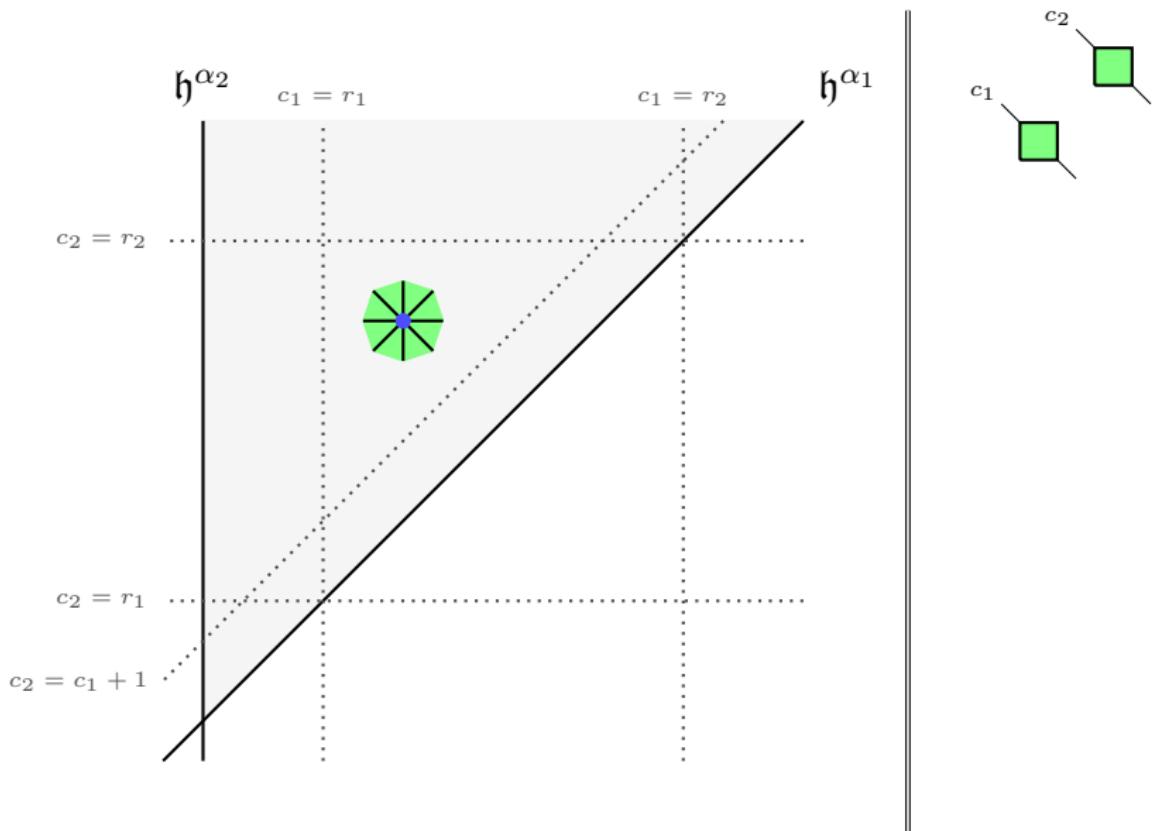
|    |    |    |      |
|----|----|----|------|
| -5 |    |    |      |
| -4 | -2 | -1 | ●    |
| -3 |    |    | ok!  |
| -5 |    |    |      |
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| 1  |    |    | bad! |
| 5  |    |    |      |

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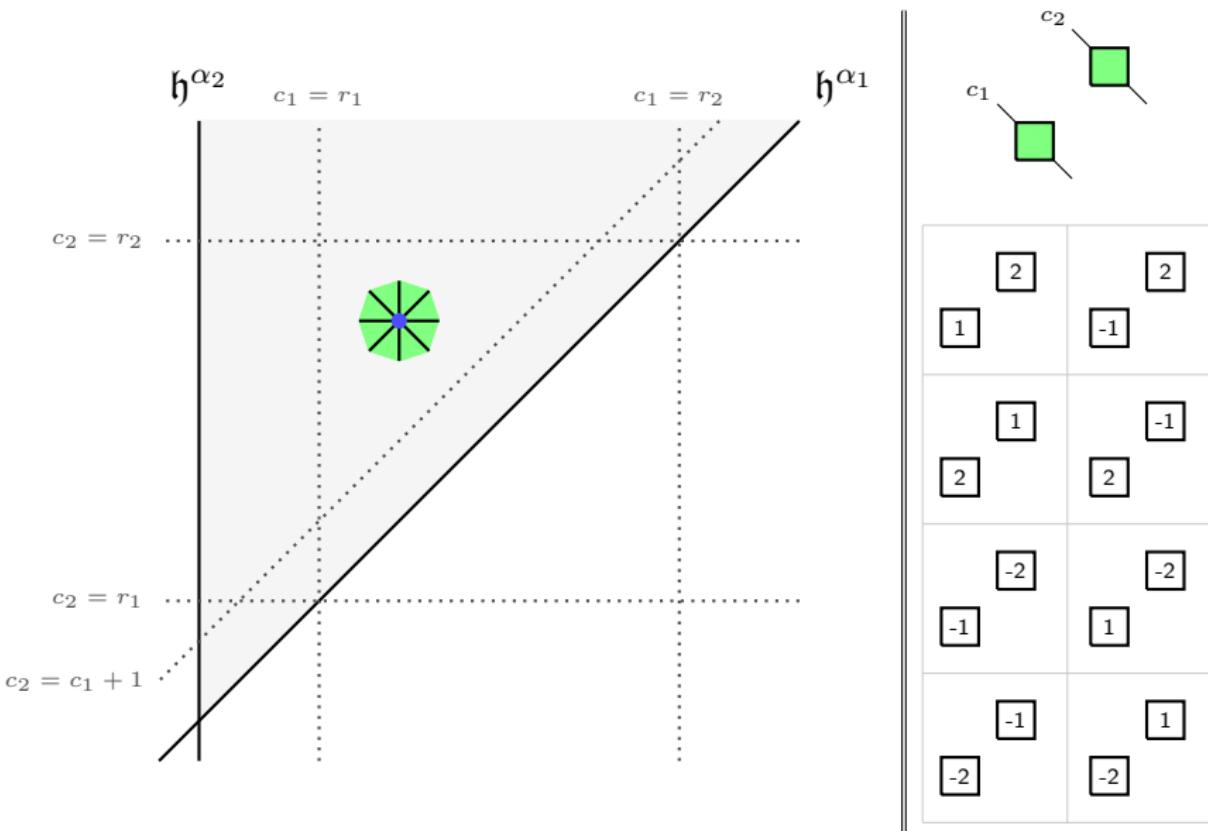
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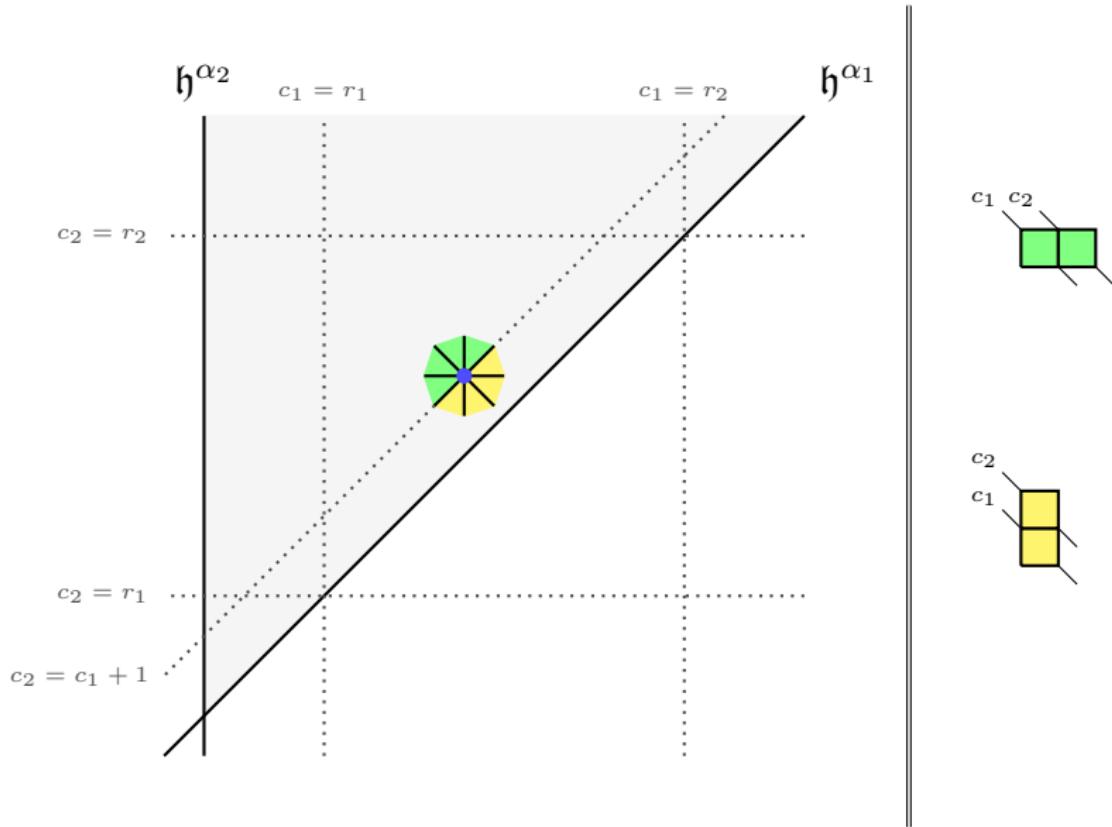
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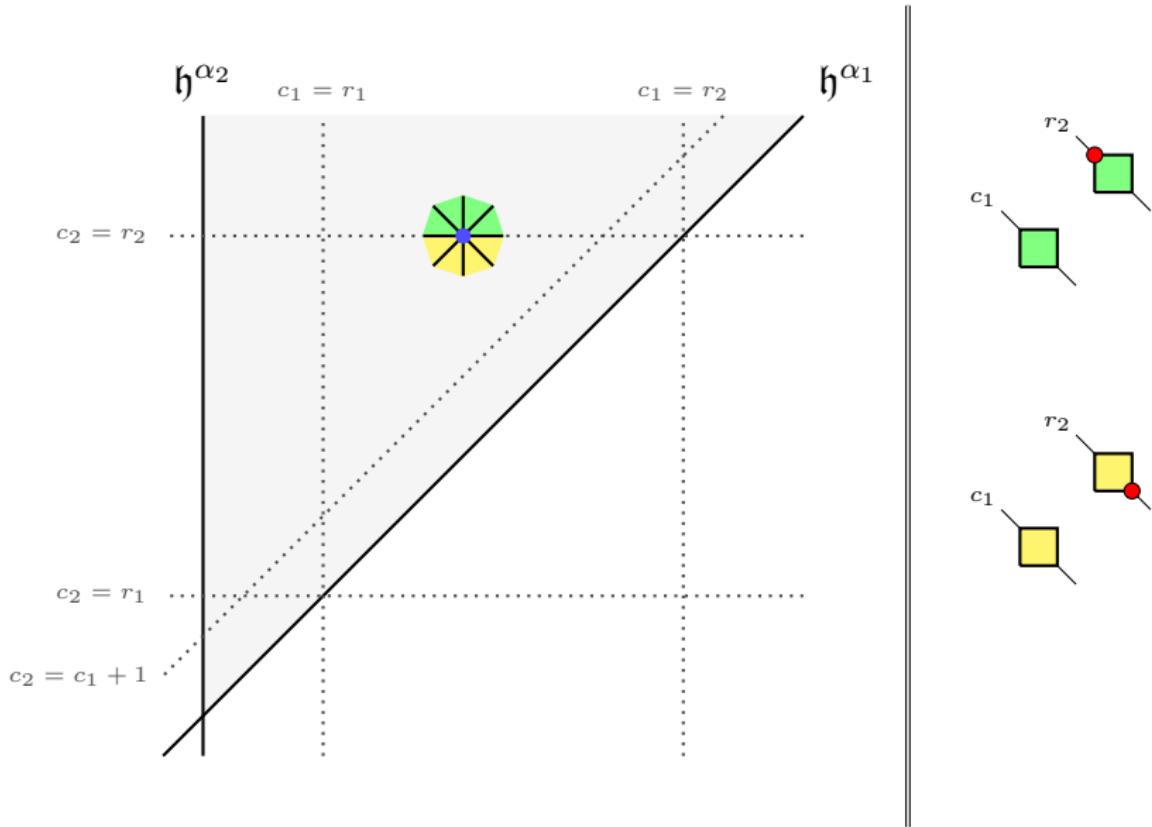
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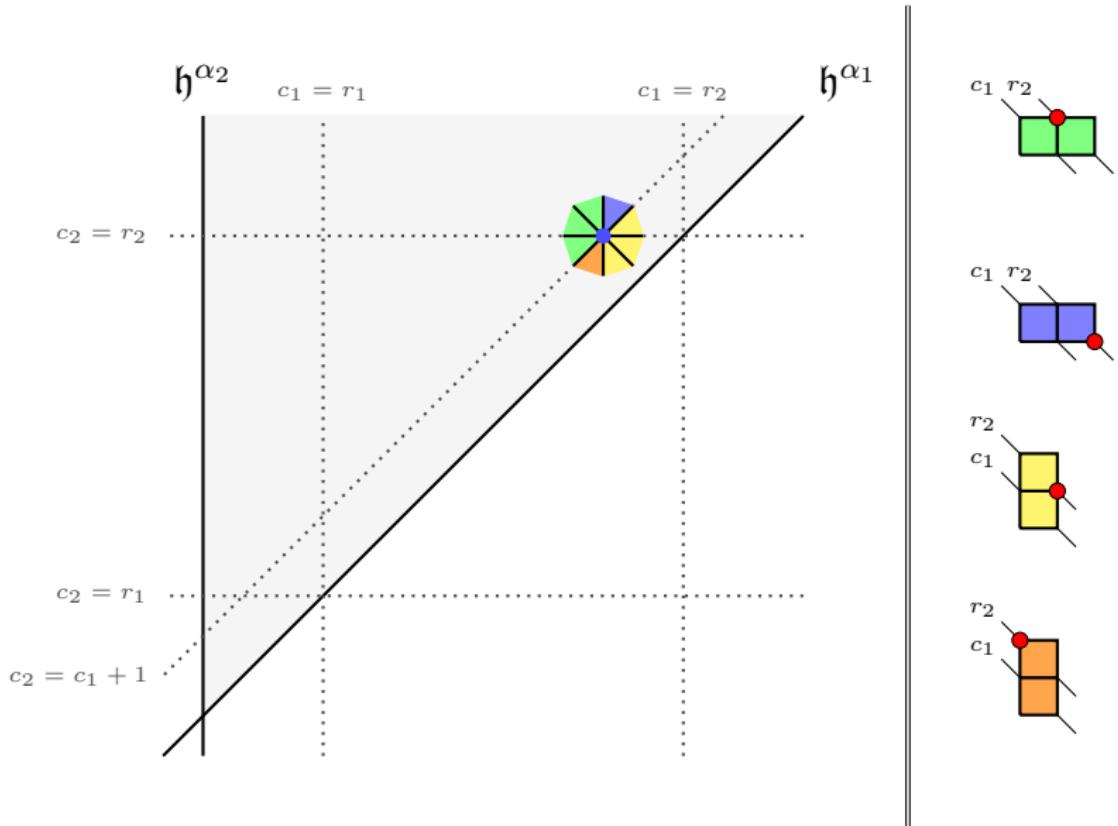
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Basis indexed by chambers.

**Description 2:** Marked box arrangements.  
Basis indexed by good fillings.

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**Description 2:** Marked box arrangements.  
Basis indexed by good fillings.

**Description 3:** Partitions.  
Representation arise in Schur-Weyl duality with certain  $U_q\mathfrak{gl}_n$  reps.

## Centralizer properties

Let  $U = U_q \mathfrak{gl}_n$  be the quantum group for  $\mathfrak{gl}_n(\mathbb{C})$ . We're interested in certain finite dimensional simple  $U$ -modules  $L(\lambda)$  indexed by partitions:

$$\lambda = \begin{array}{c} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & & & \end{array}$$

(drawn as a collection of boxes piled up and to the left)

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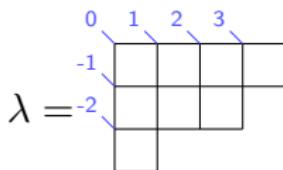
In particular, rectangular partitions:

$$(a^c) = c \begin{array}{|c|c|c|c|} \hline a & & & \\ \hline \end{array}$$

$H_k$  has a commuting action with  $U$  on the space  
 $L((a^c)) \otimes L((b^d)) \otimes (L(\square))^{\otimes k}$ .

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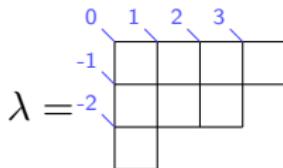
A Young diagram representing a rectangular partition with 12 boxes arranged in 4 rows. Row 1 has 3 boxes. Row 2 has 3 boxes. Row 3 has 3 boxes. Row 4 has 3 boxes. The letter 'a' is written above the first column.

$H_k$  has a commuting action with  $U$  on the space  
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The content of a box is its diagonal number.

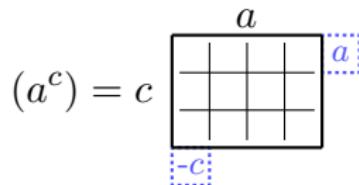
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The eigenvalues of  $T_0$  and  $T_k$  are controlled by the contents of addable boxes to  $(a^c)$  and  $(b^d)$ .

# Exploring $L((a^c)) \otimes L((b^d)) \otimes (L(\square))^{\otimes k}$

Products of rectangles:

$$L((a^c)) \otimes L((b^d)) = \bigoplus_{\lambda \in \Lambda} L(\lambda) \quad (\text{multiplicity one!})$$

where  $\Lambda$  is the following set of partitions:

(Littlewood-Richardson, Okada)

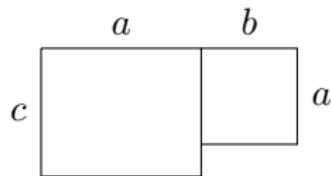
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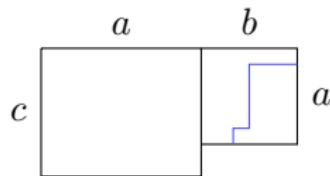
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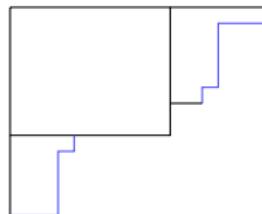
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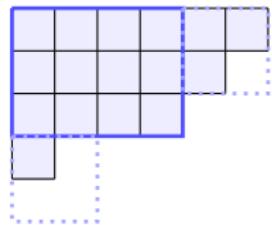
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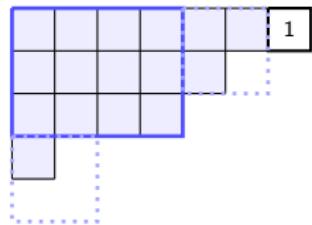
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$$\begin{array}{c} \boxed{(a^c)} \otimes \boxed{\begin{smallmatrix} & & \\ & & \\ & & \end{smallmatrix}} = \boxed{\begin{smallmatrix} & & \\ & & \\ & & \end{smallmatrix}} \oplus \boxed{\begin{smallmatrix} & & \\ & & \\ & & \end{smallmatrix}} \oplus \boxed{\begin{smallmatrix} & & \\ & & \\ & & \end{smallmatrix}} \\ \oplus \boxed{\begin{smallmatrix} & & \\ & & \\ & & \end{smallmatrix}} \oplus \boxed{\begin{smallmatrix} & & \\ & & \\ & & \end{smallmatrix}} \oplus \boxed{\begin{smallmatrix} & & \\ & & \\ & & \end{smallmatrix}} \end{array}$$

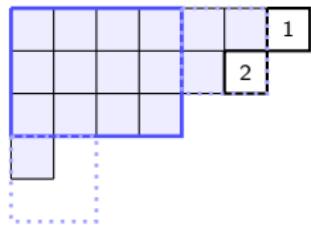
$$L \left( \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} \right) \otimes L \left( \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \right)$$



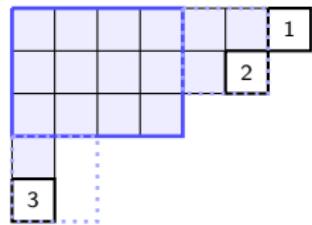
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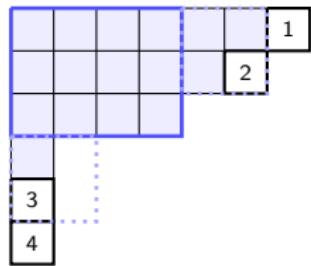
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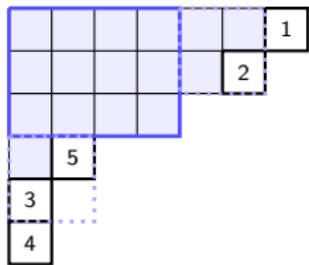
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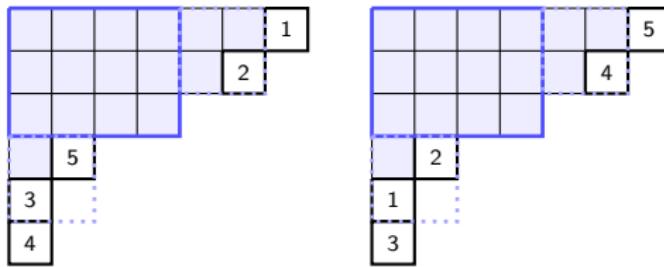


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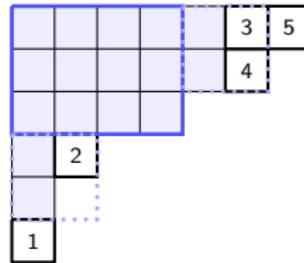
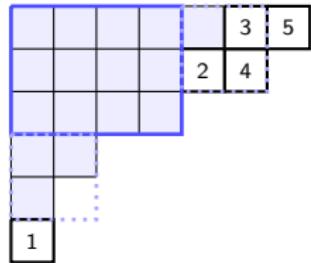
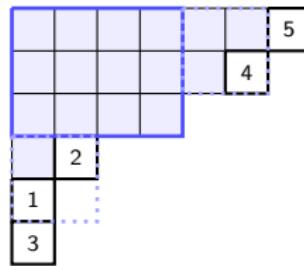
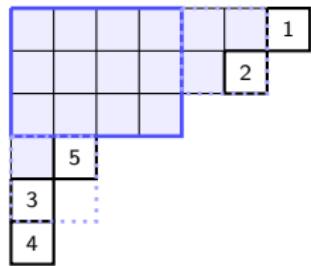
(\*)  $H_k$  representations in tensor space are labeled by certain partitions  $\lambda$ .

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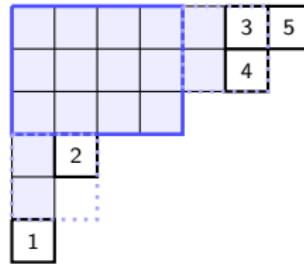
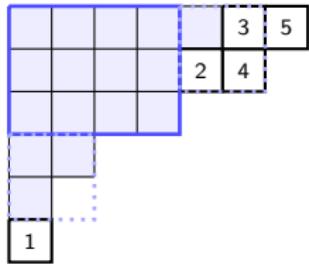
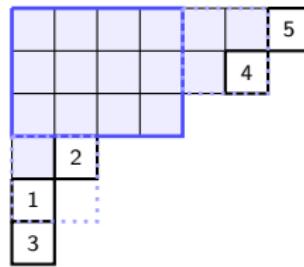
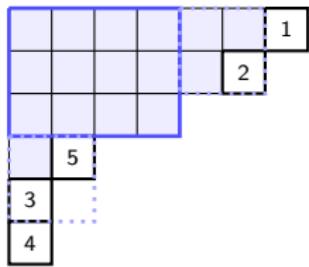
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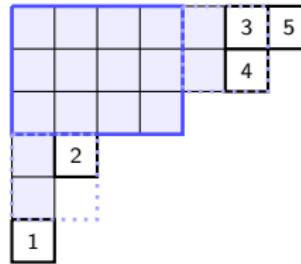
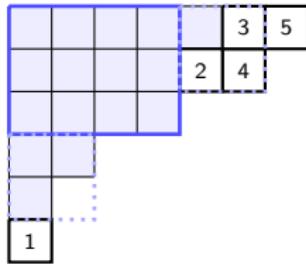
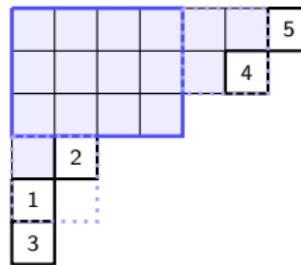
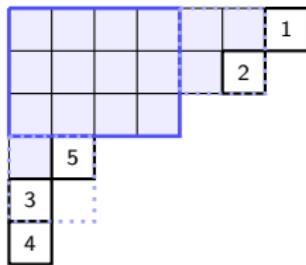
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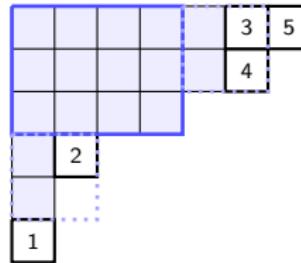
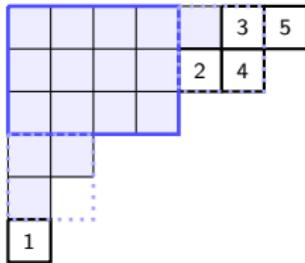
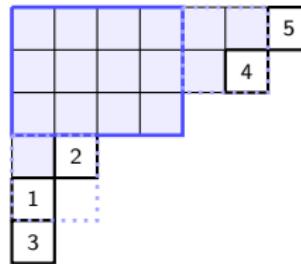
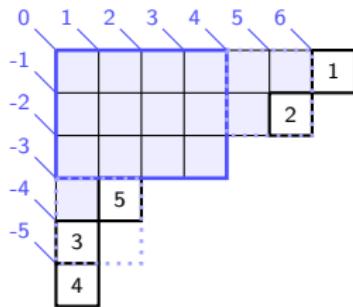
- (\*)  $H_k$  representations in tensor space are labeled by certain partitions  $\lambda$ .
- (\*) Basis labeled by tableaux from some partition  $\mu$  in  $(a^c) \otimes (b^d)$  to  $\lambda$ .

$$L \left( \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} \right) \otimes L \left( \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \right) \otimes L \left( \square \right)$$



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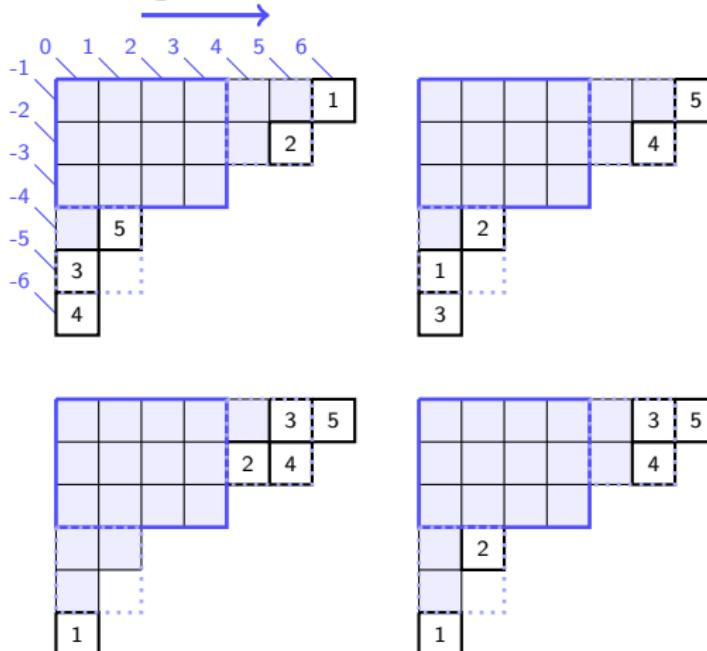
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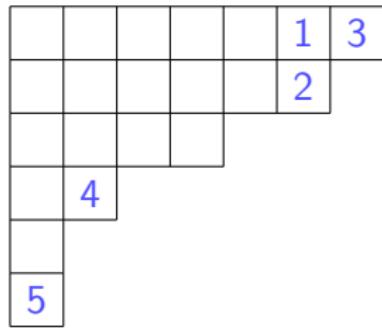
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Shift by  $\frac{1}{2}(a - c + b - d)$

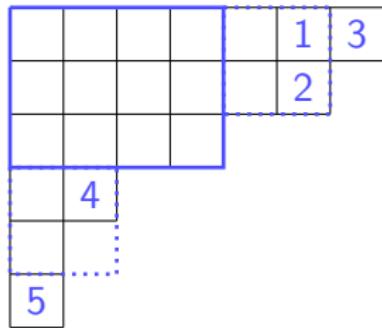


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- (\*) Basis labeled by tableaux from some partition  $\mu$  in  $(a^c) \otimes (b^d)$  to  $\lambda$ .
- (\*) Calibrated:  $Y_i$  acts by  $t$  to the shifted content of box <sub>$i$</sub> .

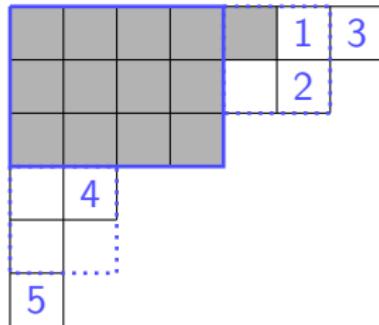
From  $\{\text{partitions in tensor space}\}$  to  $\{\text{box arrangements}\}$



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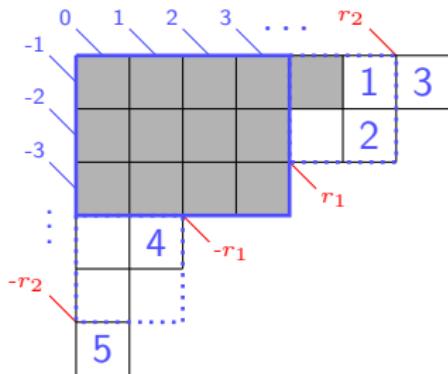


From  $\{\text{partitions in tensor space}\}$  to  $\{\text{box arrangements}\}$



■ = boxes that must appear in the partition at level 0.

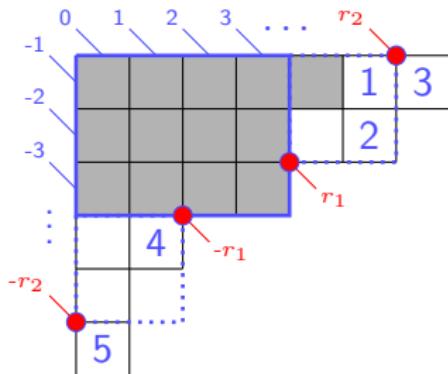
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$$\gamma(Y_1) = t^{4.5}, \gamma(Y_2) = t^{3.5}, \gamma(Y_3) = t^{r_2}, \gamma(Y_4) = t^{-2.5}, \gamma(Y_5) = t^{-r_2}.$$

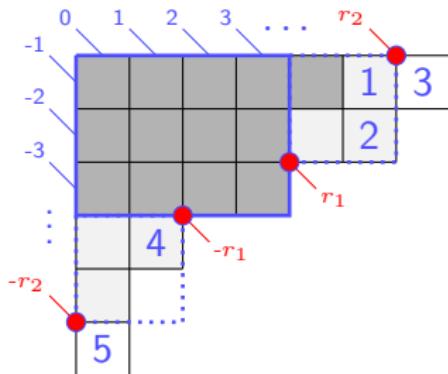
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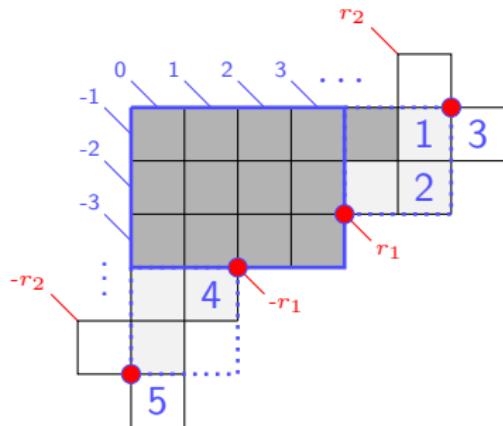
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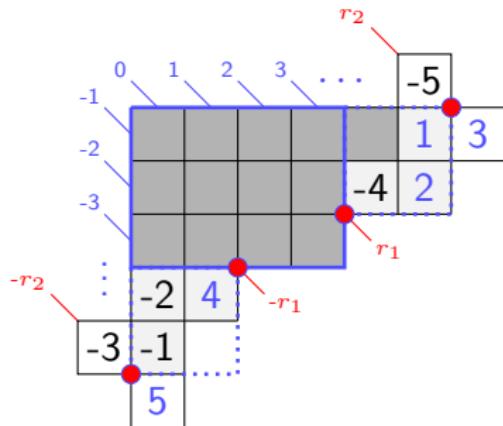
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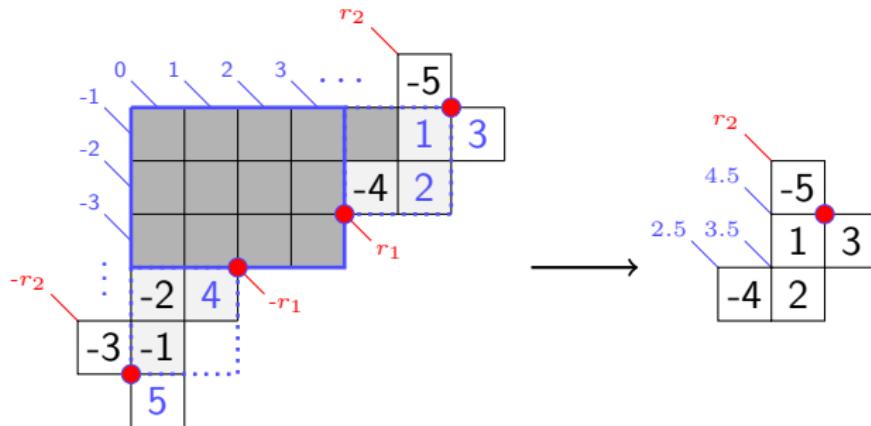
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From  $\{\text{partitions in tensor space}\}$  to  $\{\text{box arrangements}\}$



$\blacksquare = \text{boxes that must appear in the partition at level 0.}$

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versus

$$\gamma(Y_1) = t^{4.5}, \gamma(Y_2) = t^{3.5}, \gamma(Y_3) = t^{r_2}, \gamma(Y_4^{-1}) = t^{2.5}, \gamma(Y_5^{-1}) = t^{r_2}.$$

# Thanks!

$\Phi(c) = \{ \xi_i | \xi_i c \in J \} \cup \{ \xi_i | c - \xi_i \in J \}$   
 $= \{-81, -80, -77, -78, -79, 86, 81, 82, 83, -c_1 - c_2\}$   
 Box 1 and Box 2 are in adj. diag's?  
 $\frac{1}{2}(a-c+b-d) = \frac{1}{2}(8-6+6-5) = \frac{3}{2} = 0$   
 $\frac{1}{2}(a+c+b+d) = \frac{1}{2}(8+6+6+5) = 12$   
 central character =  $(-\frac{1}{2}\xi_1 - 24\xi_2, -14\xi_3)$   
 $Z(c) = \{ \xi_i | c_i = 0 \} \cup \{ \xi_i | c_i = \xi_j \}$   
 $R = \{ \xi_i | c_i = 0 \} \cup \{ \xi_i - \xi_j | c_i = \xi_j \}$   
 If  $\xi_i \in J$  and  $c_i \neq \xi_j$ , then  $w(-80) \geq 0$   
 $15\xi = (c_{1,1}, \dots, c_{n,n})$   
 $b \leftarrow \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$   
 $a \leftarrow \{ \alpha_i \}$   
 $\text{"standard tableau"} = \begin{pmatrix} -87 & -86 & -85 & -84 & -83 & -82 \\ 87 & -6 & 82 & & & \\ & & & & & \end{pmatrix} = w$   
 solid & dashed regions  
 location must be on pos. side of solid hypers  
 pos. is defined by which side each dashed region  
 dashed hypers loc. region is on neg. side of  
 standard tableau  $\Leftrightarrow w(c, J) \neq R(w, \eta_Z(c)) = \emptyset$ ,  $R(w, \eta_Z(c)) = J$   
 pos. side solid dashed