Math 11, Fall 2007

Lecture 8

Scott Pauls [1]

[1] Department of Mathematics
Dartmouth College

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Outline

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1. Review and overview
   - Last class
2. Today's material
   - Review of reading topics
3. Group Work
4. Summary
5. Next class

Differentiation

- Directional derivatives
- Partial derivatives
- Higher order partials

Outline

Concepts from reading
Tangent planes

- Tangent line to a spacecurve, $\vec{c}(t)$:

$$\vec{r}(s) = \vec{c}(t_0) + s\vec{T}(t_0)$$

- Problem: Since there are infinitely many tangent directions to a surface, there would be infinitely many tangent lines.
- Solution: Group them all together in a plane.

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Concepts from reading
Tangent plane

- Tangent vector in the x direction:

$$\vec{v}_1 = \langle 1, 0, f_x(x_0, y_0) \rangle$$

- Tangent vector in the y direction:

$$\vec{v}_2 = \langle 0, 1, f_y(x_0, y_0) \rangle$$

- Normal vector:

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \langle -f_x(x_0, y_0), -f_y(x_0, y_0), 1 \rangle$$

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Concepts from reading
Tangent plane

$\vec{n} = \langle -f_x(x_0, y_0), -f_y(x_0, y_0), 1 \rangle$, $\vec{r}_0 =< x + 0, y + 0, f(x_0, y_0) >$,
$\vec{r} =< x, y, z >$:

$$
\begin{aligned}
0 &= \vec{n} \cdot (\vec{r} - \vec{r}_0) \\
&= \langle -f_x(x_0, y_0), -f_y(x_0, y_0), 1 \rangle \cdot < x - x_0, y - y_0, z - z_0 > \\
&= -f_x(x_0, y_0)(x - x_0) - f_y(x_0, y_0)(y - y_0) + (z - z_0)
\end{aligned}
$$

Or,

$$z = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + z_0$$

Concepts from reading

Linear Approximation

The tangent plane provides an approximation of a function near the point. But, at problematic points, e.g.

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$$

at $(0,0)$

1. Does the limit as $(x,y) \to (0,0)$ exist?
2. What is the tangent plane at $(0,0)$?
3. What is the tangent plane at (ϵ, ϵ)?

Concepts from reading

Differentiable functions

Intuitive idea: we want the tangent planes to exist and vary continuously near the point. To formalize this, we introduce Δx and Δy, increments of x and y and

$$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b)$$

and define $f(x, y)$ to be differentiable at (a, b) if

$$\Delta z = f_x(a, b)\Delta x + f_y(a, b)\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

where ϵ_1, ϵ_2 tend to zero as $(\Delta x, \Delta y) \to (0, 0)$.

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Some computation

- Why is the intuitive idea equivalent to the definition?
- Use the definition to show that $x^2 + y^2$ is differentiable at $(0, 0)$.

Summary

- Tangent planes are the linear approxiamtion of a function at a point
- A function is differentiable at a point if the tangent planes vary continuously near that point
- Both of these notions follow from an examinination of the partial derivatives.

Work for next class

- Reading: 15.5-15.6
- f07hw9