# Math 31 Lesson Plan

## Day 4: What is a Good Proof? (x-hour)

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**Supplies needed:**

- Sign in sheet

**Goals for students:** Students will:

- Improve the clarity of their proof-writing.
- Gain confidence in their proof-writing abilities.
- Understand the proofs of the selected propositions (which we had mentioned in class but hadn't discussed the proofs)

[**Lecture Notes: Write everything in blue, and every equation, on the board. [Square brackets] indicate anticipated student responses. *Italics* are instructions to myself.**]

*pass around sign-in sheet*

Today we're going to talk a little bit about characteristics of a good proof. Then you guys are going to prove, in groups, some theorems that we've discussed in class but haven't proved, and we'll see how your proofs dovetail with these characteristics – and we'll modify our definition of a good proof if necessary!

If we're going to try and figure out what makes a proof good or bad, we need to understand first what a proof is supposed to do.

What's the point of a proof? *Ask for their thoughts; the below are mine.*

- Establish that a statement is true.
- Convince someone (yourself, or another person) that a statement is true.
- Explain why a statement is true.

*Write Characteristics of a good proof on one board. Ask the class for input.*

A good proof should:

- Have no mistakes (everything on the page should be true)
- Include words and symbols
- Consist of full sentences
- Be easy to read
- Not include irrelevant information; explain how each line contributes to the proof of the statement

- Prove the statement
- Explain the connection between the statement of the theorem and the last line of the proof.
- Explain what must be proved; prove it; explain what was proved.

1:15

Now that we've discussed some characteristics of good proofs, I'd like you to divide into groups of 3 or 4. Try to get someone new in your group, that you haven't worked with before. *Make sure there are an even number of groups. Label each group A or B* "A" groups, I would like you to prove the last problem from the worksheet on Friday:

PROP A: If $A_1, \ldots, A_n$ and $B_1, \ldots, B_n$ are sets such that $A_i \subseteq B_i$ for each $1 \leq i \leq n$, then $\bigcap_{i=1}^n A_i \subseteq \bigcap_{i=1}^n B_i$.

"B" groups, I would like you to prove the second form of Mathematical Induction:

PROP B (THEOREM 0.3) : (Mathematical Induction, second form) *Suppose $P(n)$ is a statement about positive integers, and suppose we know two things:*

- *$P(1)$ is true;*
- *For every positive integer $m$, if $P(k)$ is true for every $k < m$, then $P(m)$ is true.*

*Then it follows that $P(n)$ is true for every integer $n$.*

**Proof:**

Remember, the book said that the proof of this is a lot like the proof of the first form of induction.

Once you finish, each A group should pair up with a B group and switch papers. When you're reading the other group's proof, Think about:

- Which of the characteristics on the board does the proof exhibit?
- Are there other characteristics that you want to add to the list, after reading this proof?

*make sure everyone has switched papers by 1:35.*

*If time, have the groups join together to give each other their feedback directly.*

*Bring class back together* Now that you've read your classmates' proofs, do you want to make any changes to that list on the board? 1:40