Variants of a problem of Erdős and Romanov

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Romanov proved that there is a positive proportion of odd positive integers of the form $2^n + p$ for some positive integer n and prime p. Erdős proved that the odd integers not representable in this way also account for a positive proportion of all the integers. Note that if m is a positive integer latter kind, then $m - 1$ is even and not representable under the form $2^n + (p - 1) = 2^n + \phi(p)$ for any positive integer n and prime p, where ϕ is the Euler function. Similarly, $m + 1$ is even and not representable under the form $2^n + (p + 1) = 2^n + \sigma(p)$ for any positive integer n and prime p, where σ is the sum of divisors function. It makes sense to ask if we can remove the assumption that p is prime in the above statements and prove that there are infinitely many positive integers not of the form $2^n + \phi(m)$, or not of the form $2^n + \sigma(m)$ for any positive integers n and m, respectively. In my talk, I will prove that the answer to the above questions is yes. In fact, each of the above two sets of positive integers has a positive lower density. We shall also discuss some related problems and pose some open questions.
This talk is based on joint work with V. J. Mejía Huguet (Mexico) and F. Nicolae (Germany).