Math 13 - Second Hour Exam, Winter 2002

Part I: Multiple choice. Each problem is worth 5 points.

1. The integral $\displaystyle\int\int_R \frac{\sqrt{y}}{x}\, dx\, dy$, where $R = [1, e] \times [1, 4]$ is equal to
   (a) $\frac{2}{3} \ln 4(e^{3/2} - 1)$
   (b) 2
   (c) $\frac{14}{3}$
   (d) $\frac{2}{3}(e^{-2} - 1)$

2. Suppose that the integral $\displaystyle\int\int_D f(x, y)\, dx\, dy$ is equal to $\displaystyle\int_0^{\pi}\int_2^3 dr\, d\theta$. Find $f(x, y)$:
   (a) $f(x, y) = (x^2 + y^2)^{-1/2}$
   (b) $f(x, y) = 1$
   (c) $f(x, y) = x^2 + y^2$
   (d) $f(x, y) = (x^2 + y^2)^{-1}$

3. Which of the following integrals is NOT equal to the volume under the paraboloid $z = 5 - x^2 - y^2$ and above the xy plane?
   (a) $\displaystyle\int_{-\sqrt{5}}^{\sqrt{5}}\int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} 5 - x^2 - y^2\, dy\, dx$
   (b) $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{5}}\int_0^{5-r^2} r\, dz\, dr\, d\theta$
   (c) $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{5}} 5 - r^2\, dr\, d\theta$
   (d) $\displaystyle\int_{-\sqrt{5}}^{\sqrt{5}}\int_{-\sqrt{5-y^2}}^{\sqrt{5-y^2}}\int_0^{5-x^2-y^2} dz\, dx\, dy$

4. Using the Fundamental Theorem of Line Integrals, calculate the work done by the force field $\mathbf{F} = (y + 1, x)$ on a particle moving on a path $\mathbf{c}(t) = \left(\frac{1}{(\ln t)^{3/2}}, (\ln t) + 3\right)$ as t goes from e to e^4.
   (a) $-4\frac{7}{8}$
   (b) integral cannot be evaluated without tables or computer
   (c) -4
   (d) $2\frac{7}{8}$

5. Given the point $(-5, 0, 2)$ in Cartesian (rectangular) coordinates, its cylindrical coordinates are

   (a) $(5, \frac{\pi}{2}, 2)$

   (b) $(2, \frac{3\pi}{2}, 5)$

   (c) $(5, \pi, 2)$

   (d) $(-5, \frac{3\pi}{2}, 2)$

6. Classify the following three statements as True (T) or False (F) in the order (1), (2), (3).

   (1) If C is a circle, and $\mathbf{F}$ is any vector field, then $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = 0$ is always true.

   (2) If $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} < 0$, then the force field $\mathbf{F}$ is hindering the progress of a particle on path C.

   (3) If $\mathbf{c}(t)$ is a flow line of $\mathbf{F}$, then $\displaystyle\int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\ dt = 0$

   (a) FFF

   (b) TTF

   (c) TFT

   (d) FTF

7. Given the point $(\sqrt{2}, 0, \sqrt{2})$ in Cartesian (rectangular) coordinates, its spherical coordinates are

   (a) $(\frac{1}{2}, 0, \pi)$

   (b) $(2, \frac{\pi}{2}, \frac{\pi}{2})$

   (c) $(\frac{1}{2}, \frac{\pi}{4}, \pi)$

   (d) $(2, 0, \frac{\pi}{4})$

8. The following integral represents the line integral along the geometric curve $y = -x^2 + 4x$ from point $(4, 0)$ to point $(2, 4)$ of vector field $\mathbf{F}$ (note, you are looking specifically at the parametrization of the curve):

   (a) $\displaystyle\int_0^2 \mathbf{F}(-t, -t^2 - 4t) \cdot (-1, -2t - 4)\ dt$

   (b) $\displaystyle\int_0^2 \mathbf{F}(4 - t, -(4 - t)^2 + 4(4 - t)) \cdot (-1, 4 - 2t)\ dt$

   (c) $\displaystyle\int_2^4 \mathbf{F}(t, -t^2 + 4t) \cdot (1, -2t + 4)\ dt$

   (d) None of the above integrals represent the described line integral.

9. Match the integrals with the volume that they represent.

(1) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$

(2) $\displaystyle\int_0^{2\pi}\int_0^{4} 6r \; dr \; d\theta$

(3) $\displaystyle\int_0^{2}\int_{\frac{-(y-2)}{2}}^{\frac{-(y-8)}{2}} 3 \; dx \; dy$

(4) $\displaystyle\int_0^{2\pi}\int_0^{5}\int_r^{5} r \; dz \; dr \; d\theta$

(i) volume of a parallelogram box (ii) volume of a sphere

(iii) volume of a cone (iv) volume of a cylinder

(a) 1 – ii, 2 – iv, 3 – i, 4 – iii

(b) 1 – iv, 2 – iii, 3 – ii, 4 – i

(c) 1 – iii, 2 – i, 3 –i, 4 – iv

(d) 1 - ii, 2 – iii, 3 – iv, 4 – i

10. Consider the integral $\displaystyle\int_1^{2}\int_{4x^2}^{16} f(x,y) \; dy \; dx$. Changing the order of integration makes it equal to:

(a) $\displaystyle\int_0^{16}\int_4^{\frac{\sqrt{y}}{2}} f(x,y) \; dx \; dy$

(b) $\displaystyle\int_4^{16}\int_{\frac{\sqrt{y}}{2}}^{16} f(x,y) \; dx \; dy$

(c) $\displaystyle\int_4^{16}\int_1^{\frac{\sqrt{y}}{2}} f(x,y) \; dx \; dy$

(d) $\displaystyle\int_4^{16}\int_1^{4y^2} f(x,y) \; dx \; dy$

Part II: You can earn partial credit on the next four problems.

11. (12 points) Set up the following integral using the most computationally convenient coordinates (you do not have to solve): $\int\int\int_W \sqrt{x^2+y^2+z^2}\, dV$ if W is the portion of the sphere centered at $(0,0,0)$ with radius 4 that is bounded by the planes $x \geq 0$, $y \geq 0$, and $z \geq 0$.

12. (12 points) Evaluate the integral: $\displaystyle\int_0^{\ln 10} \int_{e^x}^{10} \frac{1}{\ln y}\, dy\, dx$

13. (13 points) Set up the integral that represents the volume of the region that lies inside of both the cone $z = 10 - \sqrt{x^2+y^2}$ and the cylinder $x^2+y^2 = 4$, and is bounded by the xy plane. Use the most computationally convenient coordinates; you do not have to solve.

14. (13 points) Find the work done by a force field $\mathbf{F} = y^2\mathbf{i} + (y-x)\mathbf{j}$ on a particle moving around the triangle in the plane given by the line $y = \frac{x}{4}$, and the points (in order of movement) $(0,0), (4,1)$, and $(4,0)$.