Math 12, Fall 2007

Lecture 24

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Outline

1. Review and overview
   - Last class
2. Today's material
   - Surface integrals
3. Next class

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Last class

- Parameterized surfaces
- Surface Area

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Integrals over surfaces

Let S be a parameterized surface with parameter domain D and $f : \mathbb{R}^3 \to \mathbb{R}$ be a function whose domain contains an open set which includes S.

$$\iint_S f(x, y, z)\, dS = \iint_D f(x, y, z) |\vec{N}|\, dA$$

Integrals over surfaces

A comparision of integrals

1.

$$\int_a^b ds = b - a$$

vs.

$$\int_a^b |\vec{r}'(t)|\, dt = \int_C ds = Length(C)$$

2.

$$\iint_D dA = Area(D),\, D \subset \mathbb{R}^2$$

vs.

$$\iint_S |\vec{N}|\, dA$$

S a parameterized surface

Integrals over surfaces
A comparision of integrals

1. $$\iiint_R dv = Volume(V)$$

vs

$$\iint_D f(x,y)\, dA$$

vs

$$\iint_S f(x,y,z)\, dS$$

Examples

- $$\iint_S yz\, dS$$

  S is given by
  $x = u^2, y = u\sin(v), z = u\cos(v), 0 \le u \le 1, 0 \le v \le \frac{\pi}{2}$.

- $$\iint_S \sqrt{1 + x^2 + y^2}\, dS$$

  where S is the helicoid given by

  $$\vec{r}(u, v) =< u\cos(v), u\sin(v), v >, 0 \le u \le 1, 0 \le v \le \pi$$

Integrals of vectors fields over surfaces

Recall for line integrals we had integrals of functions

$$\int_C f(x, y, z)\, ds$$

and integrals of vector fields

$$\int_C \vec{F} \cdot d\vec{r}$$

Orientation

An *orientation* for a surface S is a choice of continuous unit normal vector.

- Idea: let $\vec{N} = \vec{r}_u \times \vec{r}_v$ and check is $\vec{N}/|\vec{N}$ is continuous.
- Example of non-orientable surface: Möbius band
- A closed surface is *positively oriented* if it is equipped with its outward pointing normal.
- Example: Find positive and negative orientations on a sphere.

Integrals of vectors fields over surfaces

If $\vec{F}$ is a continuous vector field defined on an oriented surface S with unit normal vector $\vec{n}$, then

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$$

This integral is also called a flux integral.

Examples

- $\vec{F} = < x, -z, y >$, S is the part of the sphere of radius 2 in the first octant.
- $\vec{F} = < x, 2y, 3z >$ where S is the cube with vertices $(\pm 1, \pm 1, \pm 1)$

Work for next class

- Reading: 17.8
- Webwork: f07hw23