

Infinitesimal isospectral deformations of symmetric spaces

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Thursday, November 3, 2005
L01 Carson Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

Let (X, g) be an irreducible symmetric space of compact type. According to a result of Guillemin, the infinitesimal deformation corresponding to an isospectral deformation of the metric g belongs to the kernel of a certain Radon transform acting on the symmetric 2-forms on X . This is the motivation for defining the space $I(X)$ of infinitesimal isospectral deformations of X as a subspace of the kernel of this Radon transform. If $I(X)$ vanishes, an isospectral deformation of the metric g is trivial to first-order.

We shall give an overview of our results concerning the space $I(X)$:

- 1) A necessary condition for the vanishing of $I(X)$ is that it be reduced, i.e., it is not the cover of another symmetric space.
- 2) It was known that the space $I(X)$ vanishes when X is a projective space which is not equal a sphere (using work of Duistermaat-Guillemin, this leads to spectral rigidity results for these projective spaces). We generalize this result by showing that the space $I(X)$ vanishes when X is a Grassmannian which is reduced.
- 3) If X is the reduced space of the symmetric space $SU(n)/SO(n)$ or of the unitary group $SU(n)$, with $n \geq 3$, the space $I(X)$ does not vanish and we give explicit constructions of non-trivial infinitesimal deformations.