

Dartmouth College
Mathematics 25

Assignment 8
due Wednesday, November 18

1. Write out a table of indices modulo 23 with respect to the primitive root 5.
2. Use the table above to find all solutions to $3x^{10} \equiv 1 \pmod{23}$.
3. Use the table above to find all solutions to $13^x \equiv 5 \pmod{23}$.
4. If p is an odd prime, and g a primitive root modulo p , show that $\text{ind}_g(p-1) = \frac{p-1}{2}$.
5. Compute $\left(\frac{7}{11}\right)$ using Gauss' lemma.
6. What is the analog of Corollary 7.10 for $\left(\frac{-2}{p}\right)$? That is, determine congruence conditions for an odd prime p for when $-2 \in Q_p$. Make sure you use the answer for $\left(\frac{2}{p}\right)$ as part of your solution.
7. Evaluate $\left(\frac{317}{11}\right)$.
8. Observe that $89 \equiv -2 \pmod{13}$ to compute $\left(\frac{89}{13}\right)$.
9. Suppose that p is an odd prime and a is an integer with $p \nmid a$. Determine the most general conditions on a (and p) so that $x^2 \equiv \left(\frac{a}{p}\right) \pmod{p}$ is always solvable.