Diff Eq FLOWCHART : Choosing right method, taxonomy.

Bennett 12/2/05.

START

1 indep var →

2 indep. vars. → Is it a PDE?

no conditions, or IC at single x or t

ODE — check order

conditions at 2 different x's? → **BVP** → find gen. ODE soln, then try to match BCs.

will be (near) separable

Heat Eqn — 1st order in t

2nd order in t or y.

u_{xx} & u_{yy} same sign?
- yes → **Wave Eqn** (y is t; t-hg v. sin. to heat)
- no, opposite → **Laplace Eqn** — sep. of var.

homog. system? steady state first.

BC-type fixes which eigenfns. / series to use. (sine or cos series). Find Fourier coeffs. of IC.

ODE — 1st order

linear?
- yes → write as $y' + p(t)y = g(t)$, Integrating factor — find $\phi(t,y)$.
- no → Can separate $f(y)\,dy = g(t)\,dt$?
  - yes → integrate both sides.
  - no → write as $M(t,y) + N(t,y)\frac{dy}{dt} = 0$, test if exact?
    - yes → Exact eqn, find $\phi(t,y)$.
    - no → stuck.

ODE — 2nd order

more than one dependent var.

2nd order ⇢ equivalent; could rewrite either way (N=2)

linear?
- yes → const. coeffs?
  - no → write as $y'' + p(t)y' + q(t)y = g(t)$, check if Euler Eqn?
    - yes → use $y = t^r$; look up.
    - no → Use power series about ordinary point.
  - yes → homogeneous? $g(t)=0$?
    - yes → Find roots of char. eqn, write down gen. soln.
    - no → First find homog. from char. eqn: $g = y_1, y_2$. Is $g(t) = e^{\alpha t}\sin\beta t\,(1 + ct + dt^2 \ldots)$ or sums of such?
      - yes → Method of Undetermined Coeffs. get $Y(t)$, careful to check for root-hitting (extra t factors if needed).
      - no → Variation of Params. use integral formula to get $Y(t)$. No tricks.
      → deduce stability. classify type.
- no → system of 1st order

System of 1st order

linear?
- yes → Write as $\vec{x}' = A\vec{x}$, Find eigenvalues of A → deduce stability → write gen. soln. → classify type.
- no → Find critical points, linearize about each, deduce stability from linearized matrix.

Eliminate t (write $\frac{dy}{dx} = \left(\frac{y'}{x'}\right)$), get 1st order ODE for $y(x)$. Find trajectories by solving it.