Math 12, Fall 2007

Lecture 18

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Outline

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Integration of functions of more than one variable

- Double and triple integrals
- Fubini's theorem, iterated integrals, non-rectangular domains
- Change of coordinates: polar/cylindrical, spherical

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Multivariable functions

Functions from $\mathbb{R}^m$ to $\mathbb{R}^n$

- spacecurves: one input, many outputs e.g $\vec{r} : \mathbb{R} \to \mathbb{R}^3$
- Functions of more than one variable: many inputs, one output eg $f : \mathbb{R}^3 \to \mathbb{R}$
- many inputs/many outputs: $W : \mathbb{R}^m \to \mathbb{R}^n$
- Vector fields: $V : \mathbb{R}^3 \to \mathbb{R}^3$, $W : \mathbb{R}^2 \to \mathbb{R}^2$

Visualizing vector fields

$$V(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$$

The gradient vector field

A vector field $\vec{F}$ is called a conservative vector field if is the gradient of some scalar function, f (i.e. $\nabla f = \vec{F}$). f is called the potential function of $\vec{F}$. What is the relation to Clairaut's theorem?

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Integration along curves

- Integrate a function of more than one variable along a curve
  1. Given a curve C parametrized by $\vec{r}(t) =< x(t), y(t) >, a \leq t \leq b$
  2. Given $f(x, y)$, a function of two variables
  3. Integrate along the curve with respect to arclength, x or y:

$$\int_C f\, ds$$

  4. Substitute and evaluate

$$\int_C f\, ds = \int_a^b f(x(t), y(t))\sqrt{x'(t)^2 + y'(t)^2}\, dt$$

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Integration along curves

Variants:

- $$\int_C f\,dx, \quad \int_C f\,dy$$
- $$\int_C f\,dx = \int_a^b f(x(t),y(t))x'(t)\,dt$$
  $$\int_C f\,dy = \int_a^b f(x(t),y(t))y'(t)\,dt$$

Examples

- $$\int_C ye^x \, ds$$
  where C is the line segment joining $(1,2)$ to $(4,7)$
- $$\int_C z\, dx + x\, dy + y\, dz$$
  where C is given by $\vec{r}(t) =< t^2, t^3, t^2 >$, $0 \leq t \leq 1$.

Group work

- $$\int_C xz\, ds$$
  where C is given by $\vec{r}(t) = < \cos(t), \sin(t), t >$, $0 \leq t \leq 2\pi$.

Summary

- Vector fields and general functions
- Integration along curves

Work for next class

- Reading: 17.3
- Exam II on monday
- f07hw21 (due Wednesday)