Math 8 Practice Exam Problems: This was the first hour exam from Fall 2000. Our exam will have a slightly different format (50% multiple choice), but the content is roughly the same.

1. Find the general solution to the differential equation $(1+x^2)y' + 2xy = 3\sqrt{x}$.

2. Solve the following differential equation with initial conditions: $y'' - 4y' + 13y = 0$, $y(0) = 0$, $y'(0) = 6$.

3. Find the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1+x}{xy}$ where $x > 0$ and $y(1) = -4$.

4. Compute the Taylor polynomial of degree three (that is the first four terms of the Taylor series) for the function $f(x) = \sqrt{x}$ at $a = 4$.

5. (a) Express the complex number $-1 + i$ in polar form.
   (b) Express $(\sqrt{3} - i)^{12}$ in the form $a + bi$.
   (c) For two complex numbers z, w, prove that $\overline{zw} = \overline{z}\,\overline{w}$.

6. A mass of 2 kilograms is suspended from a spring whose spring constant is 50 Newtons/meter. The initial position of the mass is $\sqrt{3}$ meters below the rest position, and the initial velocity is 5 meters/second directed away from the rest position. Let the origin be the rest position of the mass, and let $y(t)$ be the position of the function of the mass at time t.
   (a) Find the function $y(t)$.
   (b) Find the maximum distance of the mass from the rest position during the motion; that is, find the maximum value of the function $y(t)$.

7. True/False
   (a) Every increasing sequence converges
   (b) The infinite repeating decimal $.43014301\ldots$ can be expressed as a geometric series.
   (c) $\displaystyle\lim_{n\to\infty}\left(e - 2 - \frac{1}{2!} - \frac{1}{3!} - \cdots - \frac{1}{n!}\right) = 0.$
   (d) If $\displaystyle\lim_{n\to\infty} a_n = 0$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.
   (e) If the function f is defined by a Maclaurin series $f(x) = \sum_{n=0}^{\infty} c_n x^n$, then $f^{(99)}(0) = 99!\, c_{99}$.
   (f) $\displaystyle\lim_{x\to 0}\left(\frac{\sin x - x}{x^3}\right) = -\frac{1}{6}.$