Math 12, Fall 2007

Lecture 25

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Outline

1. Review and overview
   - Last class
2. Today's material
   - Stokes' Theorem
3. Next class

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Surface integrals

Let S be a parameterized surface and $f : \mathbb{R}^3 \to \mathbb{R}$ be a function whose domain contains an open set which includes S.

$$\iint_S f(x,y,z)\, dS = \iint_S f(x,y,z) |\vec{N}|\, dA$$

If $\vec{F}$ is a vector field whose domain contains S then

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$$

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Green's Theorem

- Green's theorem swaps a line integral for a double integral over a region
- Exchanges functions for their derivatives

$$\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$$

Stokes' Theorem

Let S be an oriented piecewise-smooth surface that is bounded by a simple closed piecewise-smooth boundary curve C with **positive orientation.** Let $\vec{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ containing S. Then

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S curl\vec{F} \cdot d\vec{S}$$

Stokes' Theorem

- If S is a region in the plane then Stokes' Theorem reduces to Green's theorem.
- Similarly to Green's Theorem, Stokes' theorem swaps a line integral for an area integral and functions for their derivatives.

Examples

- Let
$$\vec{F} = \langle xy - xz, x^2/2 - yz, z^3 \rangle$$
Compute $\int_C \vec{F} \cdot d\vec{r}$ where C is the unit circle in the xy-plane thought of as the boundary of the disk.
- Use the same set up but now think of C as the boundary of the top half of the sphere of radius one.
- Let $\vec{F} = \langle y, -x, 0 \rangle$ and S be the cone $z^2 = x^2 + y^2$ for $0 \leq z \leq 1$. Find
$$\iint_S \vec{F} \cdot d\vec{S}$$

Bad Examples

- $\vec{F}(x,y,z) =< e^{xy}\cos(z), x^2z, xy >$. Integrate over the boundary C of S, the hemisphere $x = \sqrt{1-y^2-z^2}$ oriented in the direction of the positive x-axis. Assume C has positive orientation.
- $\vec{F}(z,y,z) =< x^2y^3z, \sin(xyz), xyz >$. Integrate over the boundary, C, of the surface S which is the part of the cone $y^2 = x^2 + z^2$ that lies between the planes $y = 0$ and $y = 3$ oriented in the direction of the positive y-axis.Assume C has positive orientation.

Work for next class

- Reading: 17.9
- f07hw24