HW 6

13.3

P 821

#40 $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \vec{k}$

Scalar projection of $\vec{b}$ onto $\vec{a}$

$$\mathrm{Comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|} = \frac{\langle 1, 1, 1 \rangle \cdot \langle 1, -1, 1 \rangle}{\sqrt{1+1+1}}$$

$$= \frac{1 - 1 + 1}{\sqrt{3}} = \frac{1}{\sqrt{3}} .$$

Vector projection of $\vec{b}$ onto $\vec{a}$

$$\mathrm{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \vec{a}$$

$$= \frac{1}{\sqrt{3}} \left(\frac{\vec{a}}{\|\vec{a}\|} \right)$$

$$= \frac{1}{\sqrt{3}} \left(\frac{\langle 1, 1, 1 \rangle}{\sqrt{3}} \right)$$

$$= \left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle$$

$$\left[= \frac{1}{3} \vec{i} + \frac{1}{3} \vec{j} + \frac{1}{3} \vec{k} \right]$$

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$\|\vec{a}\| = 3, \quad \|\vec{b}\| = 2$

(a) $\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\| \sin\theta = 3 * 2 \sin \pi/2$

$= 6$

(b) $\vec{a}\times\vec{b}$ is orthogonal to both $\vec{a}$ & $\vec{b}$ (imp)

i.e. $\vec{a}\times\vec{b} \perp \vec{k}$

Hence $\vec{a}\times\vec{b}$ is in xy plane

So z-component of $\vec{a}\times\vec{b}$ is 0.

~~x = 0~~ By right hand rule, x component of $\vec{a}\times\vec{b}$ is +ve & y-component of $\vec{a}\times\vec{b}$ is -ve.

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$\vec{a} = \langle 3, 1, 2\rangle, \quad \vec{b} = \langle -1, 1, 0\rangle, \quad \vec{c} = \langle 0, 0, -4\rangle$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 0 & -4 \end{vmatrix} = -4\vec{i} - (4)\vec{j}$$

$$= \langle -4, -4, 0\rangle.$$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 2 \\ -4 & -4 & 0 \end{vmatrix} = 8\vec{i} - 8\vec{j} - 8\vec{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 2 \\ -1 & 1 & 0 \end{vmatrix} = -2\vec{i} \quad 2\vec{j} + 4\vec{k}$$

$$(\vec{a} \times \vec{b}) \times \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -2 & 4 \\ 0 & 0 & -4 \end{vmatrix} = +8\vec{i} - 8\vec{j}$$

Hence $\quad \vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$.

#32

$P(-1,3,1)$, $Q(0,5,2)$, $R(4,3,-1)$

$\overrightarrow{PQ} = \langle 1, 2, 1 \rangle$

$\overrightarrow{PR} = \langle 5, 0, -2 \rangle$.

The vector orthogonal to the plane thr P, Q & R

is $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 5 & 0 & -2 \end{vmatrix} = -4\vec{i} + 7\vec{j} - 10\vec{k}$

$(= \langle -4, 7, -10 \rangle)$

The area of the parallelogram determined by $\overrightarrow{PQ}$ & $\overrightarrow{PR}$ is $\|\overrightarrow{PQ} \times \overrightarrow{PR}\|$

$$= \| \langle -4, 7, -10 \rangle \|$$

$$= \sqrt{16 + 49 + 100}$$

$$= \sqrt{165}$$

So the area of $\Delta PQR = \frac{1}{2}\sqrt{165}$.

~~13.8.5~~

13.5. #32 Plane thd the origin & pts P(2, −4, 6) Q(5, 1, 3)

$$\overrightarrow{OP} = \langle 2, -4, 6 \rangle$$

$$\overrightarrow{OQ} = \langle 5, 1, 3 \rangle$$

The normal vector to the plane $\vec{n}$

$$= \overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -4 & 6 \\ 5 & 1 & 3 \end{vmatrix}$$

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$$= -18\vec{i} + 24\vec{j} + 22\vec{k}$$

−12

$$(= \langle -18, 24, 22 \rangle$$

So the eq^n of the plane is

$$-18(x-2)+24(y+4)+22(z-6)=0$$

(or: origin is on the plane

so eq^n $-18(x-0)+24(y-0)+22(z-0)=0$

i.e. $-18x+24y+22z=0$).

#62

(a) For the lines to intersect:

$$1+t = 2-s$$
$$1-t = s$$
$$2t = 2 \Rightarrow t=1$$

& $1-1=s \Rightarrow s=0$

$s=0$ & $t=1$ satisfy $1+t=2-s$

Hence the lines intersect at the pt

$(1+1,\ 1-1,\ 0+2) = (2,0,2)$.

(b) Direction of the lines are given by the vectors $\vec{a}=\langle 1,-1,2\rangle$ & $\vec{b}=\langle -1,1,0\rangle$ ($\parallel^{lel}$ to the lines)

Hence a normal vector for the plane is

$$\langle 1,-1,2\rangle \times \langle -1,1,0\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= -2\vec{i} - 2\vec{j} + 0\vec{k}$$

$$(= \langle -2, -2, 0\rangle)$$

The eq^n of the plane is: (pt $(2,0,2)$ is on the plane)

$$-2(x-2) - 2(y-0) = 0$$

(i.e. $2x + 2y = 4$

i.e. $x + y = 2$)

\# 70.

the dist $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$

Here $(x_1, y_1, z_1) = (-6, 3, 5)$

& $\langle a, b, c\rangle = \langle 1, -2, 4\rangle$

& $d = -8$

$$D = \frac{|-6-6-20-8|}{\sqrt{1+4+16}} = \frac{40}{\sqrt{21}}.$$