Quiz 6: Integration by Parts

February 15, 2012

Name: _Solutions_ Section:_____

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. This quiz has two sides.

1. Find $\int (x+1)\cos(x)\,dx$.

$u = x+1 \qquad dv = \cos(x)\,dx$

$du = dx \qquad\quad v = \sin(x)$

$= (x+1)\sin(x) - \int \sin(x)\,dx$

$= \boxed{(x+1)\sin(x) + \cos(x) + C}$

this could be done 3 ways:

2. Find $\int_1^4 \ln(\sqrt{x})\,dx.$

using substitution:

$y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}}\,dx$

$= \int_1^2 2y \ln(y)\,dy$

IBP: $u = \ln(y)$ $dv = 2y\,dy$

$du = \frac{1}{y}\,dy$ $v = y^2$

$= y^2 \ln(y)\Big|_1^2 - \int_1^2 \frac{y^2}{y}\,dy$

$= \left(y^2 \ln(y) - \frac{y^2}{2}\right)_1^2$

$= 4\ln(y) - 2 + \frac{1}{2} = \boxed{4\ln(y) - \frac{3}{2}}$

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using just IBP:

$u = \ln(\sqrt{x})$

$du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2x}\,dx$

$dv = dx$

$v = x$

$= x\ln(\sqrt{x})\Big|_1^4 - \int_1^4 \frac{x}{2x}\,dx$

$= 4\ln(2) - \left(\frac{x}{2}\right)\Big|_1^4$

$= 4\ln(2) - 2 + \frac{1}{2}$

$= \boxed{4\ln(2) - \frac{3}{2}}$

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using log-rules.

$\int_1^4 \ln(\sqrt{x})\,dx$

$= \int_1^4 \frac{1}{2}\ln(x)\,dx$

$u = \ln(x)$ $dv = \frac{1}{2}dx$

$du = \frac{1}{x}dx$ $v = \frac{1}{2}x$

$= \frac{1}{2}x\ln(x)\Big|_1^4 - \int_1^4 \frac{1}{2}\cdot\frac{x}{x}\,dx$

$= \frac{1}{2}(4)\ln(4) - \left(\frac{1}{2}x\right)\Big|_1^4$

$= 2\ln(4) - 2 + \frac{1}{2}$

$= \boxed{4\ln(2) - \frac{3}{2}}$

Extra Credit: Find the integral. Your answer should be in terms of a.

$$\int x^a \ln(x)\,dx$$

$u = \ln(x)$ $dv = x^a dx$

$du = \frac{1}{x}dx$ $v = \frac{x^{a+1}}{a+1}$

$= \frac{x^{a+1}}{a+1}\ln(x) - \int \frac{x^{a+1}}{a+1}\cdot\frac{1}{x}\,dx = \frac{x^{a+1}}{a+1}\ln(x) - \frac{1}{a+1}\int x^a\,dx$

$= \boxed{\frac{x^{a+1}}{a+1}\ln(x) - \frac{1}{(a+1)^2}x^{a+1} + C}$