Math 31 Lesson Plan

Day 13: Direct Products; Additive vs Multiplicative Notation

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Goals for students: Students will:

- Become more comfortable switching between additive and multiplicative notation

[**Lecture Notes: Write everything in blue, and every equation, on the board. [Square brackets] indicate anticipated student responses. *Italics* are instructions to myself.**]

Today I want to talk briefly about direct products. Then, I want to introduce Section 7. At the end I have some activities for you guys to work on switching between additive and multiplicative notation.

I wanted to start Section 7 today so that we'll be able to get started on Section 8, Symmetric Groups, on Friday. Again, my goal with this is to make you able to work on the homework over the weekend. On its own, Section 7 is a little boring; it's basically a preparation for Sections 8 and 12, which we'll be talking about next week. So the homework will mostly come from those sections this next week.

DEFINITION: The direct product of two groups $(G, *)$ and $H, \cdot)$ is denoted $G \times H$. As a set, $G \times H = \{(g, h) : g \in G, h \in H\}$, and the group operation on $G \times H$ is

$$(g_1, h_1)(g_2, h_2) = (g_1 * g_2, h_1 \cdot h_2).$$

Note that the order matters! Just like order matters in general group multiplication.

Your textbook checks that $G \times H$ is actually a group, so I'm going to skip that part, if you don't mind.

A lot of people had questions about Theorem 6.1(ii), so let's go over the proof of that.

THEOREM 6.1(II) *Let $G = G_1 \times G_2 \times \ldots \times G_n$ be a direct product such that each factor G_i is a cyclic group of finite order. Then G is cyclic iff $(|G_i|, |G_j|) = 1$ for all $i \neq j$.*

Proof: We have to prove two things:

- If G is cyclic, then $(|G_i|, |G_j|) = 1 \ \forall \ i \neq j$.

- If $(|G_i|, |G_j|) = 1 \ \forall \ i \neq j$, then G is cyclic.

For the first one, if G is cyclic, what can we do? [Write $G = \langle x \rangle$ for some x.] What will that x look like? let $g = (g_1, \dots, g_n)$ be a generator. Then every element in G is a power of g. That is, we can write

$$G = \{(g_1^r, \dots, g_n^r : r \in \mathbb{Z}.\}$$

It follows that $G_i = \langle g_i \rangle \ \forall \ i$. Who can tell me why? [Elements of G are n-tuples of elements from the G_i. If each such n-tuple has only powers of the g_i's, then the g_i's must generate.]

Now, what can we say about $o(g_i)$ in this case? [Hence, $o(g_i) = |G_i|$ by Corollary 4.6.] Using Part 1 of Theorem 6.1, what can we say? Therefore,

$$o(g) = lcm(|G_1|, \dots, |G_n|) = |G|.$$

But how many elements are in G? *think-pair-share*

Hence, $lcm(|G_1|, \dots, |G_n|) = |G_1| \dots |G_n|$, so the groups must be of pairwise relatively prime order. $\square$

Prove other direction!

Questions?

Let's see if any of the tips about proofwriting from yesterday stuck. *hand out Wksht 1* Grab a partner (or two) and work on the proofs of the following statements. *A & B groups, then switch papers?*

On the back there are some more computational problems about subgroups and finding orders in direct products. 1:10

Section 7: Functions Your book gives a formal definition of functions, but we all more or less know what they are, so I'd rather just talk about specific types of functions in the last few minutes of class today.

Definition: A function $f : S \to T$ is one-to-one or injective if what? [every element of S is mapped to a different element of T. Alternatively, if $f(s_1) = f(s_2)$ then $s_1 = s_2$, which is equivalent to the statement "If $s_1 \neq s_2$,, then $f(s_1) \neq f(s_2)$."]

Can someone give me an example of an injective function?

Definition: A function $f : S \to T$ is onto or surjective if what? [every element of T is the image of some element of S. In other words, for all $t \in T$, there exists $s \in S$ such that $f(s) = t$.]

Can someone give me an example of an onto function?

Note that to define the concept of injectivity and surjectivity, you need to specify the domain and range of the function (the sets S and T in the definitions). For example, the map $x \mapsto e^x$ is surjective from $\mathbb{R}$ to $\mathbb{R}^+$, but it's not surjective when considered as a map from $\mathbb{R}^+$ to $\mathbb{R}^+$.

Is there any relationship between injectivity and surjectivity in general? [no] Let's come up with examples:

- A function that is injective and surjective $f(x) = x$ *from* $\mathbb{R}$ *to* $\mathbb{R}$.
- A function that is injective but not surjective e^x *from* $\mathbb{R}$ *to* $\mathbb{R}$.
- A function that is not injective but is surjective $f(x) = x^2$ *from* $\mathbb{R}$ *to* $\mathbb{R}^+$
- A function that is not injective and not surjective $f(x) = x^2$ *from* $\mathbb{R}$ *to* $\mathbb{R}$.

Questions about this? We'll talk more about functions on Friday. 1:20

Let's talk about Additive vs Multiplicative Notation. When do you use additive notation? We use additive notation for abelian groups (sometimes). We use multiplicative notation for general groups. Questions?

Pass out Wksht 2. Grab a partner (or two) and work on these exercises to switch between additive and multiplicative notation. I'll be coming around if you have any questions.