m33s06	Sample Midterm	Exam Time: , 6:00 - 8:00

Name:	Student No.:

Instructions:

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT seperate the pages of your exam.

Problem	Points	Score
A1	10	
A2	10	
A3	10	
A4	10	
A5	10	
Total	50	

Name:

Section A: Answer ALL questions.

Problem A1: [10 pts]

(**a**) Find the fundamental solution for the operator $D^2 + 4D + 5$.

(**b**) An unforced spring-mass naturally obeys the ODE

$$y'' + 4y' + 5y = 0.$$

The spring-mass is initially at equilibrium (zero position and velocity) but is caused to vibrate by an impulse force of $+1$ being applied at $t = 0$ (i.e. the vibration is just the fundamental solution). What impulse force applied at $t = \pi$ will cause the spring-mass to instantly revert to equilibrium?

Name:

Problem A2: [10 pts]

(**a**) Solve the advection equation

$$\begin{cases} u_t + \frac{1}{1+t^2} u_t = 0 \\ u(x,0) = e^{-x^2} \end{cases}.$$

(**b**) Sketch the characteristic curves.

(**c**) What is the long term behavior of the solution? How could you have predicted this by looking at the characteristic curves?

Name:

Problem A3: [10 pts] Use the method of Laplace and Fourier transforms to solve the advection-diffusion equation

$$\begin{cases} u_t - cu_{xx} + au_x = 0 \\ u(x,0) = \delta(x) \end{cases}$$

where $c > 0$.

(**a**) Suppose $a = c = 1$ and $u(x,t)$ is constrained to have at most polynomial growth in x. What are the steady state solutions of the advection-diffusion equation?

(**b**) If $u(x,0) = e^{-x^2}$, what is $\lim_{t\to\infty} u(0,t)$?

Name:

Problem A4: [10 pts] A string vibrates according to the IVP

$$\begin{cases} u_{tt} - 4u_{xx} = 0 \\ u(x,0) = \chi_{[-1,1]}(x) \\ u_t(x,0) = xe^{-x^2} \end{cases}$$

(**a**) Plot the graph of $u(0,t)$ for $0 \le t \le 2$. Justify your plot.

(**b**) What is the eventual position (as $t \to \infty$) of the portion of string $-10 \le x \le 10$?

Name:

Problem A5: [10 pts]   Solve the general IVP

$$\begin{cases} u_{tt} + 2u_{tx} - u_{xx} + \lambda^2 u = 0 \\ u(x,0) = g(x) \\ u_t(x,0) = 0 \end{cases}$$

using the Laplace-Fourier method. Suppose the IVP is used to model the position of a string. Describe what is happening to the string. Pick a sensible g and sketch a few time snap-shots.