The Cross Product

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The Cross Product

- If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of $\mathbf{a}$ and $\mathbf{b}$ is the vector

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

Examples

- Find the crossed product of the vectors $\langle -1, 2, 1\rangle$ and $\langle 1, -2, 2\rangle$.

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- Find the crossed product of the vectors $\langle \sqrt{2}, -\sqrt{2}, 1\rangle$ and $\langle 1/2, 1, 1\rangle$.

- The vector $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

- If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ then

$$|a \times b| = |a||b| \sin \theta$$

- Two nonzero vectors are parallel if and only if $\mathbf{a} \times \mathbf{b} = 0$

- The length of the cross product $\mathbf{a} \times \mathbf{b}$ is equal to the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$.

Examples

- Find a vector perpendicular to both $\langle -2, 2, 0 \rangle$ and $\langle 0, 1, 2 \rangle$ of the form $\langle 1, \underline{\hspace{2cm}}, \underline{\hspace{2cm}} \rangle$

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- Find a vector perpendicular to both $\langle -2, 2, 0 \rangle$ and $\langle 0, 1, 2 \rangle$ of the form $\langle 1, \underline{\qquad}, \underline{\qquad} \rangle$

- Find the area of the triangle with vertices $P(0, 0, 0)$, $Q(-2, 2, 5)$, $R(0, 3, -3)$.

Properties of the Crossed Product

- If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$.
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$.
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$.
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$.
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$.
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.

The volume of a parallelepiped

- The volume of a parallelepiped determined by the vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ is the magnitude of their scalar triple product:
$$V = |a \cdot (b \times c)|$$

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- Find the volume of the parallelepiped with adjacent edges PQ, PR, PS where $P(1,4,-3)$, $Q(3,7,0)$, $R(0,3,-4)$, $S(7,2,-1)$.