Math 11, Fall 2007

Lecture 17

Scott Pauls [1]

[1] Department of Mathematics
Dartmouth College

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Outline

1. Review and overview
   - Last class
2. Today's material
   - Triple Integrals
3. Next class

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Changing coordinates

- Polar coords: $x = r\cos(\theta), y = r\sin(\theta)$

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Integrals of functions of three variables

1. Integrate a function of three variables over a region R in $\mathbb{R}^3$
2. Same derivation using Riemannian sums (just three sums instead of two)
3. If $B = [a, b] \times [c, d] \times [e, f]$ then

$$\iiint_D g(x, y, z)\, dV = \int_e^f \int_c^d \int_a^b g(x, y, z)\, dxdydz$$

Theorems

1. Fubini's theorem still holds: one can calculate iterated integrals and switch the order of integration.
2. Integrals over regions other than boxes: parameterize and change bounds of integration.
3. Triple integrals can be used to find volumes of regions:

$$\iiint_R dV = Volume(R)$$

Examples

1. Find the volume enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$,
2. Find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$.
3. Calculate
$$\iiint_E x^2 e^y \, dV$$
where E is bounded by $z = 1 - y^2$ and the plane $z = 0, x = 1, x = -1$.

Center of mass

If $\rho(x, y, z)$ is a density function of an object occupying the region E then the mass of the object is

$$m = \iiint_E \rho \, dV$$

The moments about the three coordinate planes are

$$M_{yz} = \iiint_E x\rho \, dV, \quad M_{xz} = \iiint_E y\rho \, dV, \quad M_{xy} = \iiint_E z\rho \, dV$$

and the center of mass is given by

$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$$

Center of mass

Example: Find the center of mass of a solid occupying the region E with density function $\rho(x, y, z) = x^2 + y^2 + z^2$ where E is the cube $[0, a] \times [0, a] \times [0, a]$

Work for next class

- Read 16.8
- f07hw18