Math 12, Fall 2007

Lecture 27

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Outline

1. Review and overview
   - Last class
2. Today's material
   - Orientation
   - The Divergence Theorem
3. Next class

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Stokes' Theorem

Let S be an oriented piecewise-smooth surface that is bounded by a simple closed piecewise-smooth boundary curve C with positive orientation. Let $\vec{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ containing S. Then

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S curl \vec{F} \cdot d\vec{S}$$

Examples

- Let
$$\vec{F} = \langle xy - xz, x^2/2 - yz, z^3 \rangle$$
Compute $\int_C \vec{F} \cdot d\vec{r}$ where C is the unit circle in the xy-plane thought of as the boundary of the disk.
- Use the same set up but now think of C as the boundary of the top half of the sphere of radius one.
- Let $\vec{F} = \langle y, -x, 0 \rangle$ and S be the cone $z^2 = x^2 + y^2$ for $0 \leq z \leq 1$. Find
$$\iint_S \vec{F} \cdot d\vec{S}$$

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Orientation of a manifold

Recall the various orientations we already know:

- Positive orientation of a closed plane curve
- Positive orientation of a closed surface
- Orientation of a curve induced by a parametrization $\vec{r}(t)$
- Orientation of a surface induced by a parametrization.

Orientation in Stokes' Theorem

Given an oriented surface S bounded by a curve C, how do we assign a positive orientation?

1. Same idea as positive orientation for a plane curve. If we walk around the curve with our head pointing in the direction of the normal, the region of the surface should be to the left.
2. If $\vec{N}$ is the normal vector, and $\vec{r}'(t)$ is the tangent vector to the curve, $\vec{N} \times \vec{r}'(t)$ should point into the region.

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The Divergence Theorem

Let E be a simple solid region and let S be the boundary surface of E, given with positive (outward) orientation. Let $\vec{F}$ be a vector field whose component functions have continuous partial derivatives on an open region containing E. Then,

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E div\vec{F}\, dV$$

Examples

In each example, compute $\iint_S \vec{F} \cdot d\vec{S}$

- $\vec{F} =< x^4, -x^3z^2, 4xy^2z >$, S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 2, z = 0$.
- $\vec{F} =< x^3y, -x^2y^2, -x^2yz >$, S is the surface of the solid bounded by the hyperboloid $x^2 + y^2 - z^2 = 1$ and the planes $z = -2, z = 2$.

Work for next class

- Review reading, finish webwork and start studying for the exam.