Math 5 Name: ________________________________
Fall 2013 HW #8.

Motivating question: How far do you have to go in the decimal expansion of pi before you find a given d-digit number (let's say it is all 1s)?

(Sub) motivating question: How many times do you want to flip a coin before you flip "heads"? We will think about this based on counting the number of flips in which this **cannot** happen.

Part I: Tossing a regular coin.

From the warm-up we should be able to complete this table quickly.

Number of coin tosses	2	3	4	5	6	7	8	9	10	15	20
Total number of possible outcomes											
Ways to get zero heads											
Ratio of ways to get zero heads over # possible outcomes											
Ratio of ways to get at least one head over # possible outcomes											
Ratio for at least one head expressed as a percentage											

What is the formula (you have deduced) that tells us the percentage of tossing at least one head in **n** tosses?

Now, we can complete the following table and answer our motivating question, How many times do you want to flip a coin before you flip "heads"?

Probability for at least one head	>70%	>80%	>90%	>95%	>99%	>99.9%	>99.99%
Number of tosses necessary							

Part II: Tossing a "three sided" coin.

Now, suppose we have a "three sided" coin. (Its faces are "heads", "tails", "feet".) How does this change our thought process? Your formulas?

Draw out all the outcomes for tossing the coin: 2 times and 3 times. What is the total number of possible outcomes for each? How many ways can you toss "no heads" for each? (This will be a little different than before. There is space on the next page.)

This should give you a sense of how to deduce the formulas to complete the given tables.

Number of coin tosses	2	3	4	5	10	15	20
Total number of possible outcomes							
Ways to get zero heads							
Ratio of ways to get zero heads over # possible outcomes							
Ratio of ways to get at least one head over # possible outcomes							
Ratio for at least one head expressed as a percentage							

What is the formula (you have deduced) that tells us the percentage of tossing a three faced coin and obtaining at least one head in **n** tosses?

Now, we can complete the following table and answer an extension of our motivating question: how many times do you want to flip a (three-sided) coin before you flip "heads" with the given probabilities?

Probability for at least one head	>70%	>80%	>90%	>95%	>99%	>99.9%	>99.99%
Number of tosses necessary							

<u>Part III: Tossing a "TEN-sided" coin.</u>

Now, suppose we have a "ten sided" coin. (Its faces are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and we will let heads=1). How does this change our thought process? Your formulas?

You will not be able to draw out all the possibilities (they're just too many), so you want to rely on mathematics to deduce the formulas to complete the table. For the two-toss case, how many numbers between 00 and 99 include at least one 1? How many do not? For the three-toss case, how many numbers between 000 and 999 include at least one 1? How many do not?

Number of coin tosses	2	3	4	5	10	15	20
Total number of possible outcomes							
Ways to get zero 1s							
Ratio of ways to get zero 1s over # possible outcomes							
Ratio of ways to get at least one 1 over # possible outcomes							
Ratio for at least one head expressed as a percentage							

What is the formula (you have deduced) that tells us the percentage of tossing a ten faced coin and obtaining at least one 1 in **n** tosses?

Now, we can complete the following table and answer an extension of our motivating question, How many times do you want to flip a (ten-sided) coin before you flip the number 1?

Probability for at least one 1	>70%	>80%	>90%	>95%	>99%	>99.9%	>99.99%
Number of tosses necessary							

Part IV: Relating this to searching for digits of given lengths in pi.

Motivating question: How far do you have to go in the decimal expansion of pi before you find a given d-digit number (let's say it is all 1s)?

(a) Finding a 2-digit number in pi.

Let us suppose we want to find a 2-digit number in Pi. The 2-digit number can be arbitrary, but let's pretend the number we want is 11. Since there are a total of 100 2-digit numbers (00, 01, 02, …, 98, 99) we can think of it as having a 100-sided coin. We want to know how many tosses we need to do before we get the side 11 (with high enough probability).

Recall that the first 50 digits of pi (after the decimal) are: .1415926535897932384626433832795028841971693993751… For the purposes of a counting tosses of a 100-sided coin, we will consider the first two digits of pi, 14, to be the first toss. The second and third digits of pi (41) to be the second toss. The third and fourth digits (15) to be the third toss and so on and so forth. So, in the first 50 digits of pi, we have made 49 tosses. (As the number or digits of pi gets large, we can assume that the number of digits in pi = the number of tosses.)

So, using what we have done so far, we can deduce the formula that tells us the percentage of tossing a hundred-faced coin and obtaining at least one 11 in **n** tosses. Please write it below.

How many times do you want to flip a (hundred-sided) coin before you flip the number 11?

Probability for at least one 11	>70%	>80%	>90%	>95%	>99%	>99.9%	>99.99%
Number of tosses necessary							

(b) Finding a 3-digit number in pi.

Let us suppose we want to find a 3-digit number in Pi. The 3-digit number can be arbitrary, but let's pretend the number we want is 111. Since there are a total of 1000 3-digit numbers (000, 001, 002, …, 998, 999) we can think of it as having a 1000-sided coin. We want to know how many tosses we need to do before we get the side 111 (with high enough probability).

Recall that the first 50 digits of pi (after the decimal) are: .1415926535897932384626433832795028841971693993751… For the purposes of a counting tosses of a 100-sided coin, we will consider the first three digits of pi, 141, to be the first toss. The second through fourth digits of pi (415) to be the second toss. The third through fifth digits (592) to be the third toss and so on and so forth. So, in the first 50 digits of pi, we have made 48 tosses. (As the number or digits of pi gets large, we can assume that the number of digits in pi = the number of tosses.)

So, using what we have done so far, we can deduce the formula that tells us the percentage of tossing a thousand-faced coin and obtaining at least one 111 in **n** tosses? Please write it below

How many times do you want to flip a (thousand-sided) coin before you flip the number 111?

Probability for at least one 111	>70%	>80%	>90%	>95%	>99%	>99.9%	>99.99%
Number of tosses necessary							

(c) Finding a d-digit number in pi for d in the set {7, 8, 9, 10}.

Now, we have built up the "machinery" to answer the question we were discussing in class, that is, if we have 2 billion digits of pi, what probability do we have of obtaining "any" 7-digit number, or 8-digit number, or 9-digit number, or 10-digit number?

What is a general formula to find the probability of tossing a specific **d** digit number in **n** tosses (of a **10^d** sided coin)?

Given 2 billion digits of pi (so pretend we are making 2 billion tosses), what is the probability that we can find:

(i) any 7-digit number?

(ii) any 8-digit number?

(iii) any 9-digit number?

(iv) any 10-digit number?

Recall from class that we all seemed to have success finding 7 and 8 digit numbers; some yes, some no with 9-digit numbers; and few of us with 10-digit numbers. So the probability should match up.