First Midterm Practice Problems

DISCLAIMER: In no sense should this collection of problems be construed as representative of the actual exam. These are simply some problems left over from the preparation of the exam or from previous exams which should serve to indicate the general level of expectation.

Problem 1. Find a homogeneous second order linear ODE whose general solution (for $x > 0$) is given by

$$y = t^{1/2}(c_1 + c_2 \ln t).$$

[*Hint:* An Euler equation will work.]

Problem 2. Consider the functions y_1 and y_2 plotted together with their Wronskian W in Figure 1 over the interval (a, b).

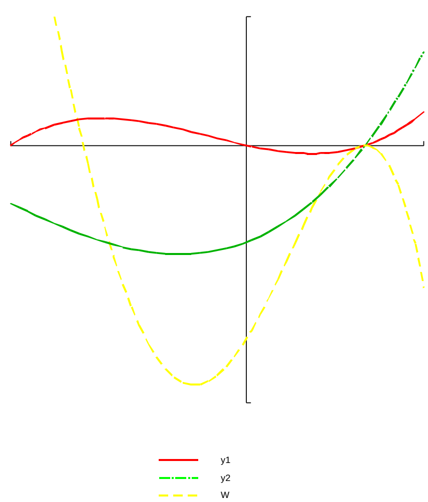

Figure 1: y_1, y_2 and $W(y_1, y_2)$.

a. Explain why there *cannot be* a first order linear equation

$$y' + p(t)y = g(t)$$

with $p(t), g(t)$ continuous on (a, b) which has *both* y_1 *and* y_2 as solutions.

b. Explain why there *cannot be* a homogeneous second order linear equation

$$y'' + p(t)y' + q(t)y = 0$$

with $p(t), q(t)$ continuous on (a, b) which has *both* y_1 *and* y_2 as solutions.

Problem 3. Solve the initial value problem

$$\begin{aligned} \frac{1}{2y} - \frac{dy/dx}{3x^2 - 6x + 2} &= 0 \\ y(1/2) &= \sqrt{3/8} \end{aligned}$$

and determine the largest interval on which the solution exists.

Problem 4. Consider the second order linear ODE

$$y'' + 2y' + 3y = e^{-t} g(t) \tag{1}$$

a. Find the general solution to (1) when $g(t) \equiv 0$.

b. Find the general solution to (1) when $g(t) = \cos(\sqrt{2}t)$.

c. Show that if $g(t)$ is continuous for all t and

$$\int_0^\infty |g(t)| dt < \infty$$

then then all solutions to (1) tend to 0 as $t \to \infty$.

Problem 5. Find the value of b for which the equation

$$(xy^2 + bx^2 y) + (x + y)x^2 \frac{dy}{dx} = 0$$

is exact then find the general solution using that value of b.