

The Chain Rule

Lecture 23

February 23, 2007

The Chain Rule (case 1)

Definition

- Suppose that $z = f(x, y)$ is a differentiable function of x and y , where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t . Then z is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}.$$

Examples

- If $z = x^2y + xy^3$, where $x = \cos t$, $y = \sin t$, find dz/dx when $t = \pi/2$.
- Find dz/dt if $z = \sqrt{x^2 + y^2}$ and $x = e^{2t}$ and $y = e^{-2t}$.
- The pressure P (in kilopascals), volume V (in liters), and temperature T (in kelvins) of a mole of an ideal gas are related by the equation $PV = 8.31T$. Find the rate at which the pressure is changing when the temperature is $300K$ and increasing at a rate of $0.1K/s$ and the volume is $100 L$ and increasing at a rate of $0.2 L/s$.

The Chain Rule (Case 2)

Definition

- Suppose that $z = f(x, y)$ is a differentiable function of x and y , where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t . Then

$$\begin{aligned}\frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} \\ \frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}\end{aligned}$$

Examples

Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for the following examples:

- $z = e^{xy} \sin x$, where $x = 2s + 4t$, $y = \frac{2s}{3t}$.
- $z = \ln(x^2 + y^2)$, where $x = e^s \cos t$ and $y = e^s \sin t$.
- $w = xy + xz + yz$, where $x = st$, $y = e^{st}$, $z = x + t$.