Disclaimer: The problems below comprise a recent Math 11 final exam. The problems are meant only to provide you with some practice, and are not meant necessarily to be representative of the style or content of this year's exam.

1. (20)Write your answer in the space provided. (*Show all work*).

   (a) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x-2)^n}{5^n}$.

   (b) Suppose that the power series $\sum_{n=0}^{\infty} \frac{a_n(x-3)^n}{7^n}$ has radius of convergence 7. Also, assume that $\sum_{n=0}^{\infty} a_n$ converges, and that $\sum_{n=0}^{\infty}(-1)^n a_n$ diverges. Determine the interval of convergence of the power series.

2. (20) Write your answer in the space provided. (*Show all work*).

   (a) (10)Find the Maclaurin series for $x^3 e^{-x/2}$.

   (b) (10)Find the point of intersection of the line $\mathbf{r} = \langle t, 2+3t, 5\rangle$ with the plane $x + 2y + 3z = 5$.

3. (20) (*Show all work*).

   (a) Find an equation of the plane which contains the two lines

   $$\mathbf{r} = \langle 3t, -3+2t, 5\rangle \qquad \text{and} \qquad \begin{cases} x = 6-3t \\ y = 7-5t \\ z = 3+t \end{cases}.$$

   (b) Determine whether the lines $\mathbf{r} = \langle 1+2t, 2+3t, 3+4t\rangle$ and $\mathbf{r} = \langle 5-2s, 3+2s, 7\rangle$ are parallel, skew, or intersecting.

4. (30) **(Short Answer)** Circle or box your answers! While it may not be necessary to show any work to receive full credit for this question, no partial credit can be assigned unless some (correct) work is shown.

   (a) Let $S = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^3}$. Estimate the error $\left| S - \sum_{k=1}^{123} \frac{(-1)^{k-1}}{k^3} \right|$.

   (b) Find the projection of the vector $\mathbf{u}$ in the direction of the vector $\mathbf{v}$ (i.e., $\text{Proj}_{\mathbf{v}}\mathbf{u}$) where $\mathbf{u} = \langle 3, 4, 5\rangle$ and $\mathbf{v} = \langle 1, 0, 1\rangle$.

(c) Suppose that $\mathbf{u}$ and $\mathbf{v}$ are vectors such that $\mathbf{u} \bullet \mathbf{v} = 2$, $\mathbf{u} \bullet \mathbf{u} = 8$, and $\mathbf{v} \bullet \mathbf{v} = 2$. Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

(d) If $a_n = \dfrac{(n!)^2}{(2n)!}$, find $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n}$.

(e) Find the general solution to the differential equation $3\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} - 2y = 0$.

5. (30) **(Multiple Choice): Circle the correct response.** No work need be shown.

(a) Consider the differential equation $\dfrac{dy}{dx} = f(x, y)$, and suppose that $f(2, 3) = 5$, and f is continuous. Using your knowledge of direction fields, if a solution to this differential equation passes through the point $(2, 3)$, through which other point is it most likely to pass?

A. $(1, 7.8)$ B. $(3, 7.8)$ C. $(1, -7.8)$ D. $(3, -7.8)$

(b) The (infinite) region in the first quadrant bounded above by $y = 1/x^{3/2}$, below by the x-axis, to the left by the y-axis, and to the right by $x = 1$ is revolved about the y-axis. The volume of the resulting (infinite) solid of revolution is:

A. infinite B. 4 C. 2 D. 2π E. 4π

(c) $\displaystyle\int \frac{x\,dx}{\sqrt{1-4x^2}} =$

A. $\dfrac{1}{2}\sin^{-1}(2x) + C$ B. $\dfrac{1}{8}\ln(\sqrt{1-4x^2}) + C$

C. $\dfrac{-1}{4}\sqrt{1-4x^2} + C$ D. $\dfrac{1}{2x}\sin^{-1}(2x) + C$

(d) If $f(x) = \dfrac{1}{1+x^6}$, then $f^{(12)}(0) =$

A. $\dfrac{1}{2!}$ B. 1 C. 0 D. 12! E. $\dfrac{1}{12!}$

(e) Consider the plane $3x - 4y + 5z = 4$ and the line $\mathbf{r} = \langle 5t - 1, 5t + 2, t + 3\rangle$. Which is true? (circle one)

A. The line lies in the plane B. The line intersects the plane orthogonally
C. The line is parallel to the plane, but does not lie in it
D. The line intersects the plane, but not orthogonally

6. (30)(Multiple Choice) Determine whether each of the series below is absolutely convergent (AC), conditionally convergent (CC), or divergent (D). Circle the correct response. No work need be shown.

(a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{7n-5}{2n+7}\right)^n$ AC CC D

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{5n^3+2n+7}}$ AC CC D

(c) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt[3]{n}}$ AC CC D

(d) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n e^{-n}}{\ln(n)}$ AC CC D

(e) $(1+\frac{1}{4}+\frac{1}{9})-(\frac{1}{16}+\frac{1}{25}+\frac{1}{36})+(\frac{1}{49}+\frac{1}{64}+\frac{1}{81})-(\frac{1}{100}+\frac{1}{121}+\frac{1}{144})+\cdots$

AC CC D