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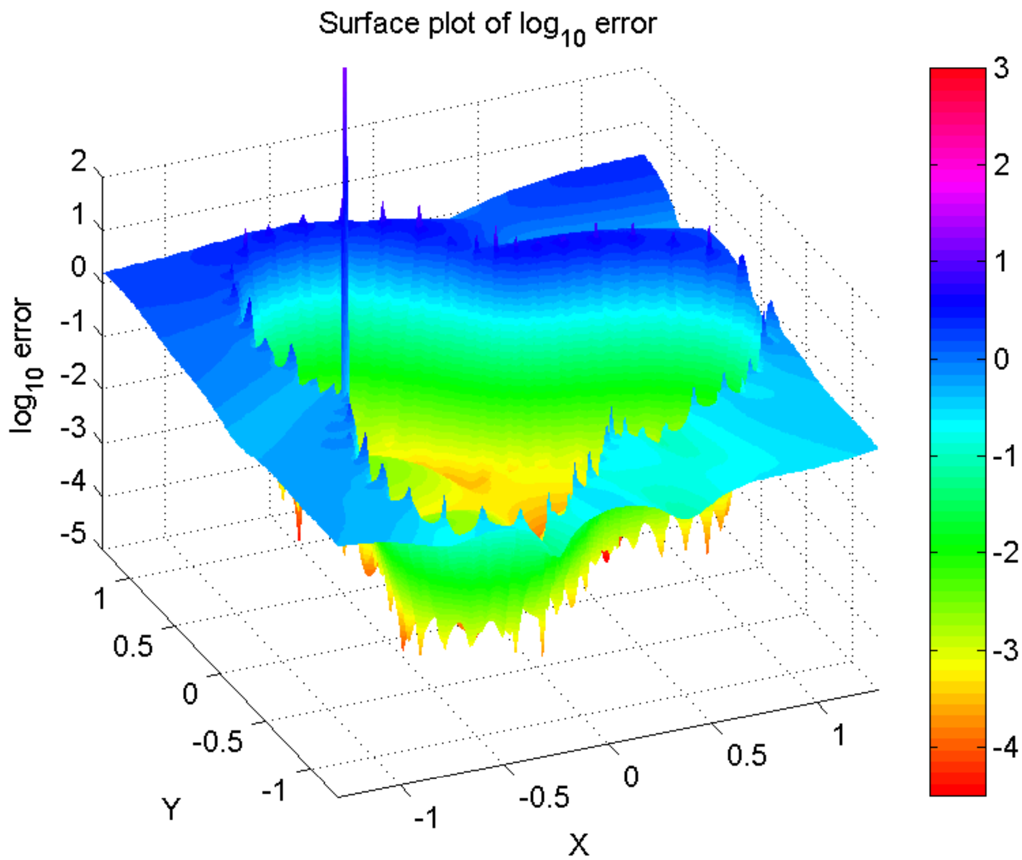
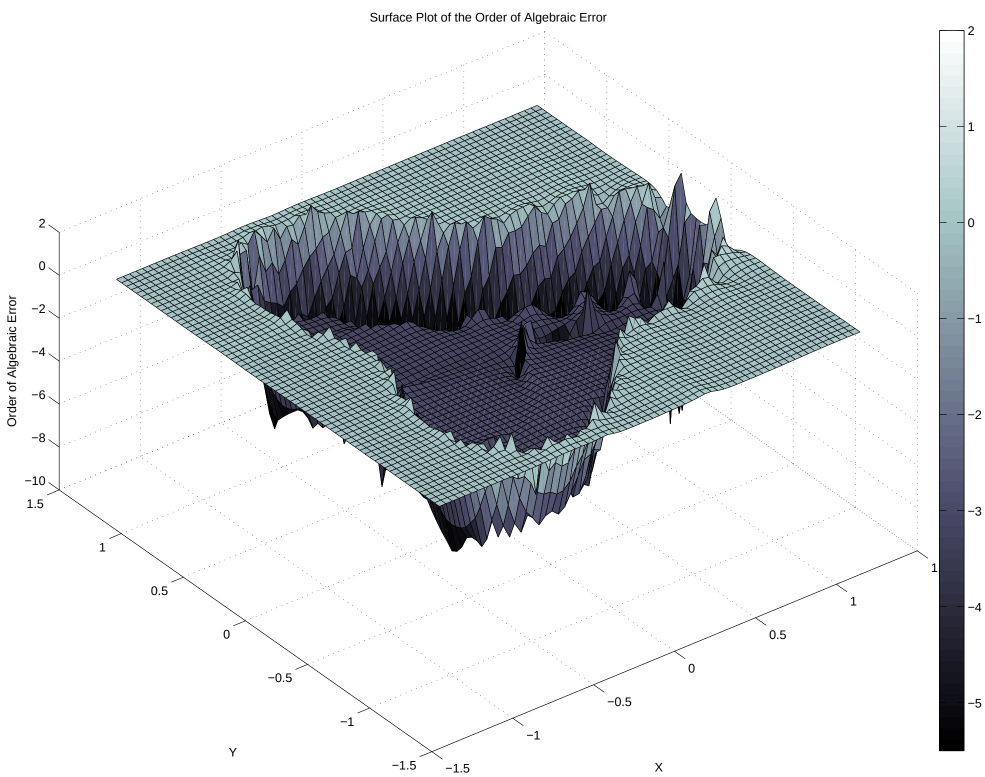
MATH 126 - Dr. Barnett
Homework 6
due February 20, 2012

Problem 1. Here we see a logplot of the error of the interior boundary value problem over a grid $[-1.3, 1.3]^2$.



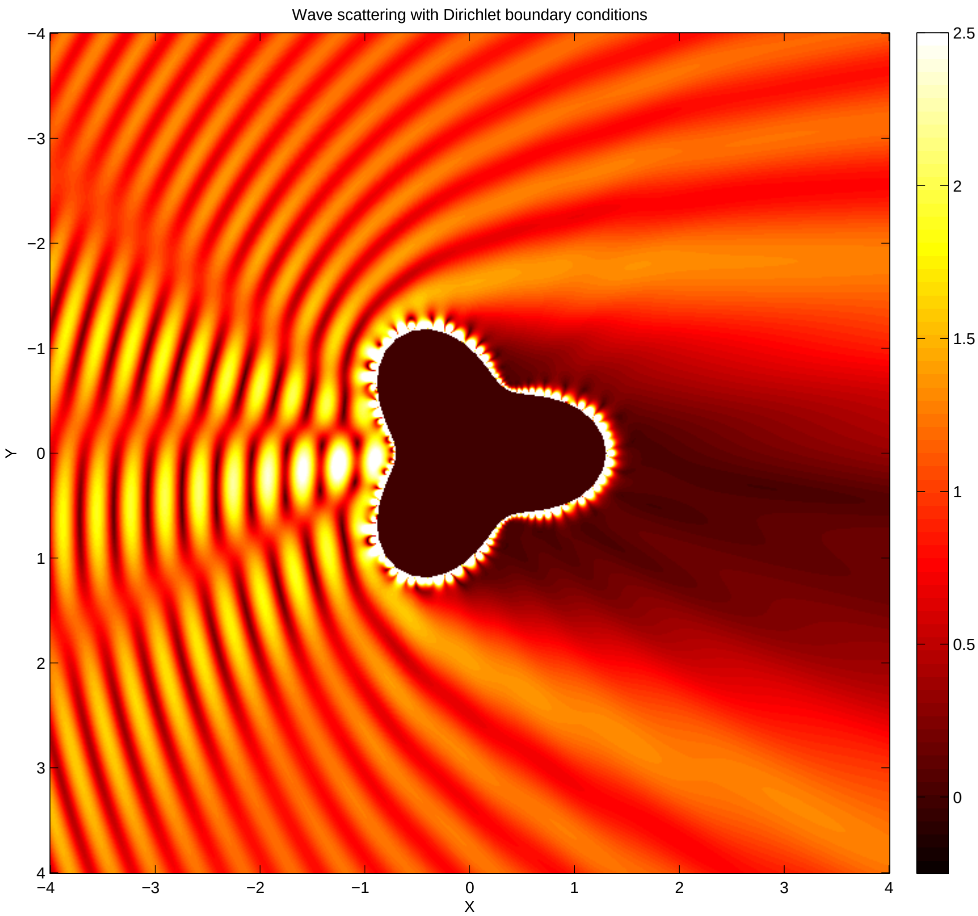
It appears that the error drops significantly better near the boundary than it did in the previous homework. However, the trade off is that the error, as we'll see in part (b), is algebraic, instead of exponential.

And here we see a surface plot of the algebraic order of convergence over our entire grid.



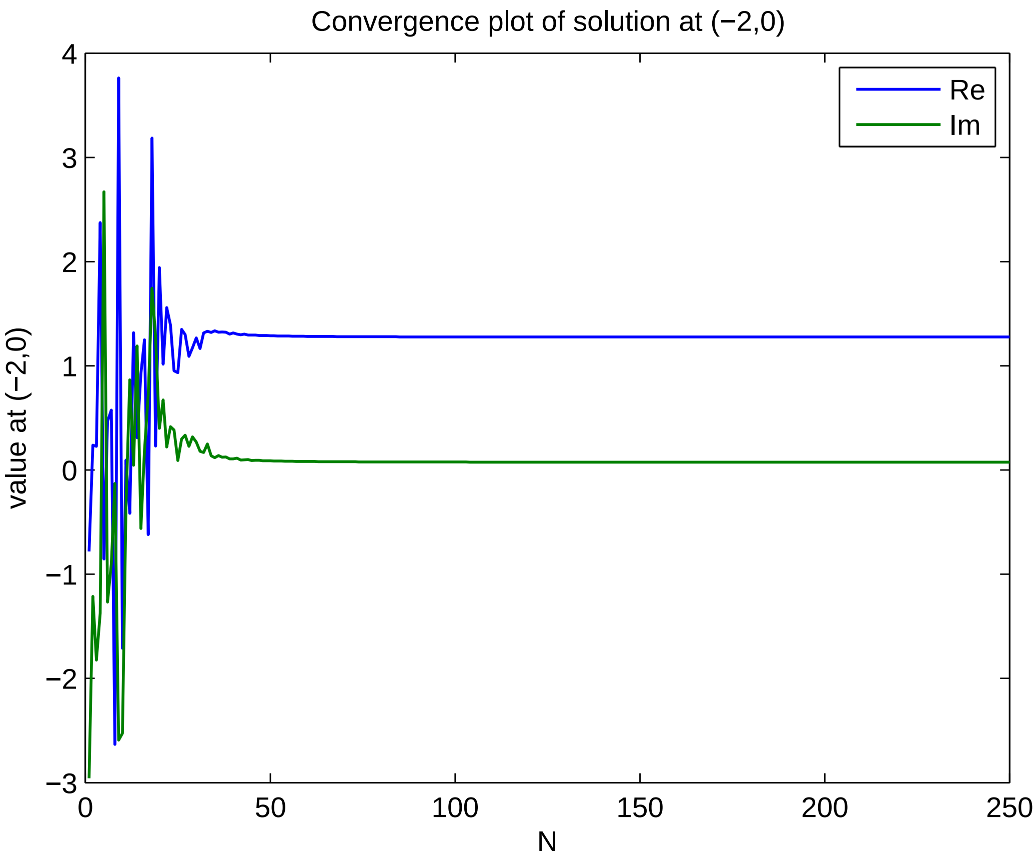
The algebraic order of convergence at $(0.2, 0.1)$ is ≈ -3.111. This convergence is not nearly as good as the exponential convergence we obtain with Laplace. Since we obtain algebraic convergence, this says that the Helmholtz double-layer potential is not analytic; otherwise, we would have exponential convergence.

Problem 2. The following plot is a color plot of the magnitude of the waves bouncing off of our boundary. The values in this plot are a superposition of the incident and scatter waves.



As you can see, the magnitude of these waves approach zero at the boundary; this is most readily visible on the back side of our boundary. Note that the values interior to our region were artificially set to zero.

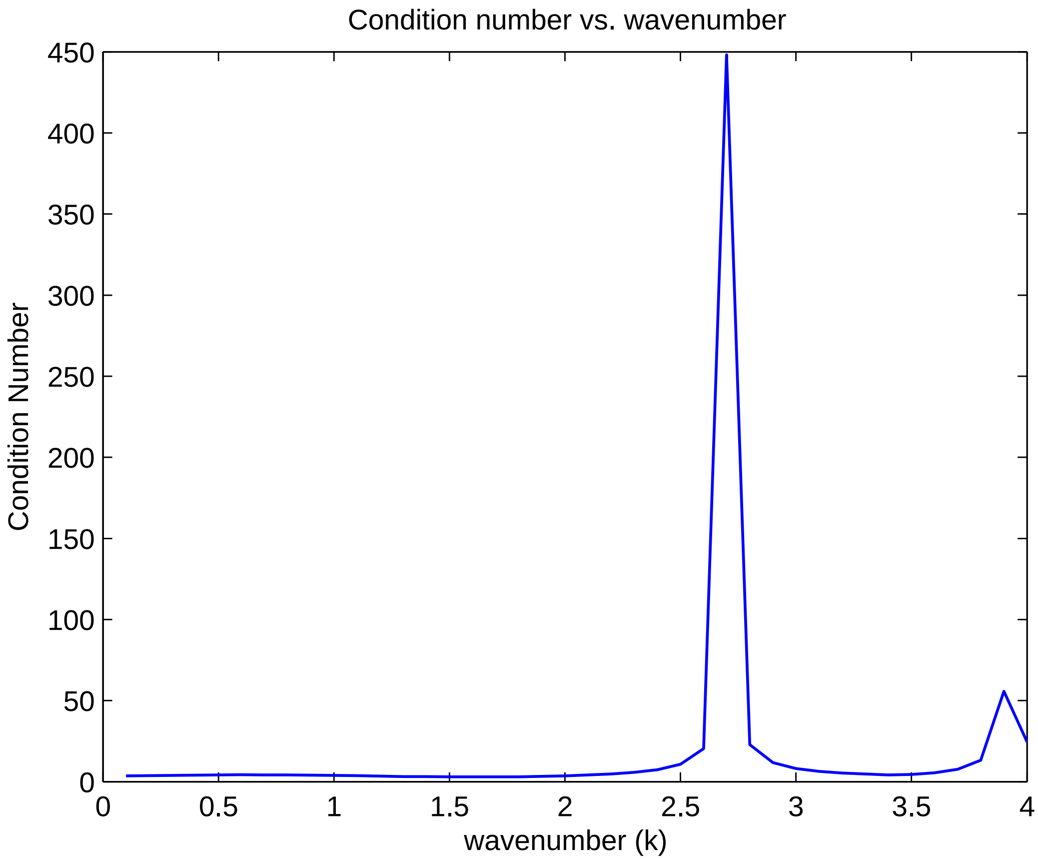
The plot below shows convergence of the real and imaginary parts of our solution at $(-2, 0)$.



The solution to this scattering problem at the point $(-2, 0)$ was found to be

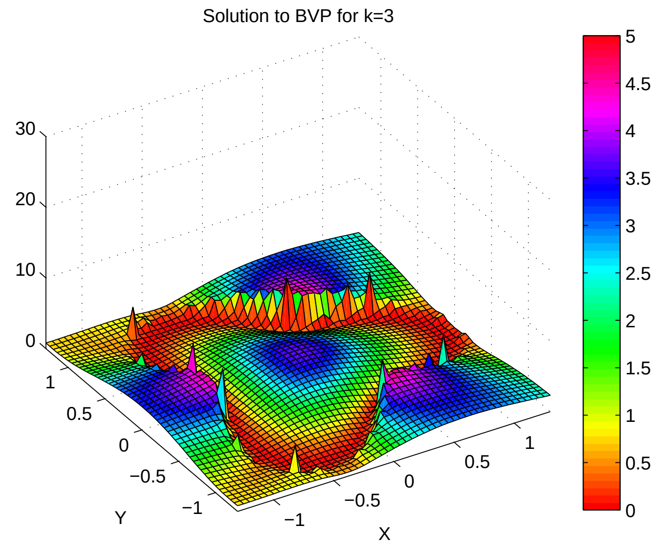
$$\approx 1.2762605 + 0.0752339i.$$

Problem 3. For part (a), the smallest wavenumber k such that the kernel $I - 2D$ blows up is ≈ 2.70473440137. This wavenumber was approximated by "hw6_3a.eps" which adaptively refines the wavenumber vs. condition number plot, centering on the wavenumber which maximizes the condition number. To see the general trend of how the condition number changes with the wavenumber, see the plot below.

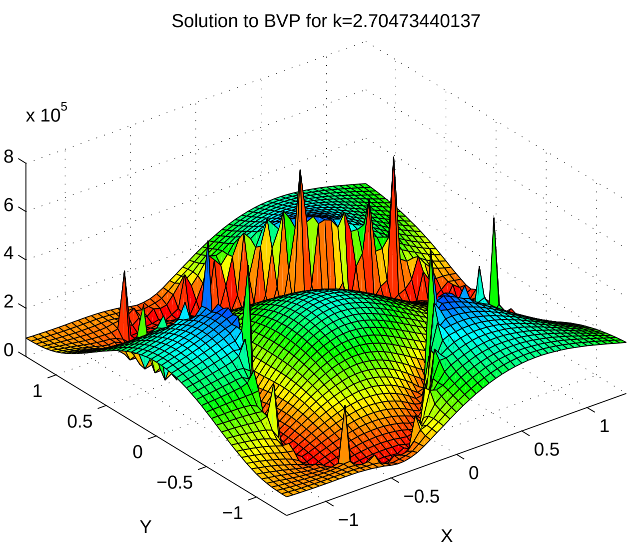


To calculate these condition numbers, we used a fixed number of quadrature nodes, yielding a 50×50 Nyström kernel.

To answer part (b), we compare two surface plots of the BVP problem with $\exp(-(x^2+y^2))$ as our boundary data.

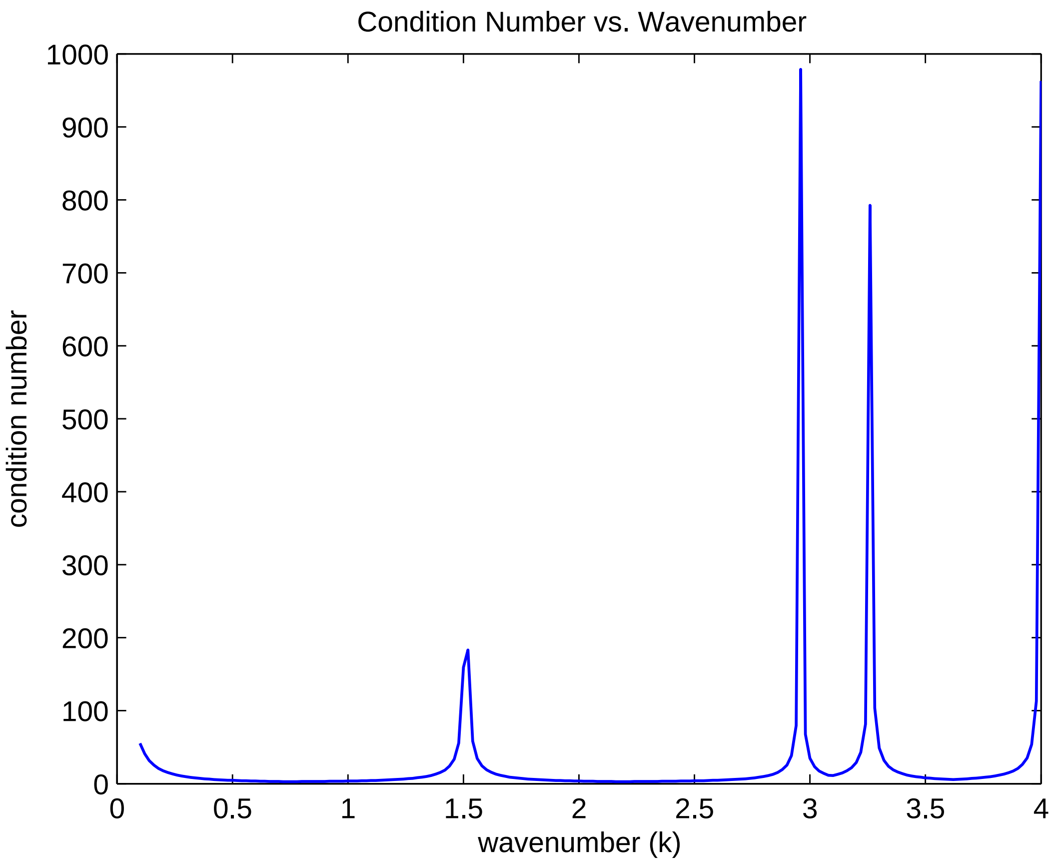


As you can see in the figure above, when we fix $k = 3$, we have a relatively well-behaved interior. However, when we plot the data with k equal to the value we found in part (a), we obtain the following figure:

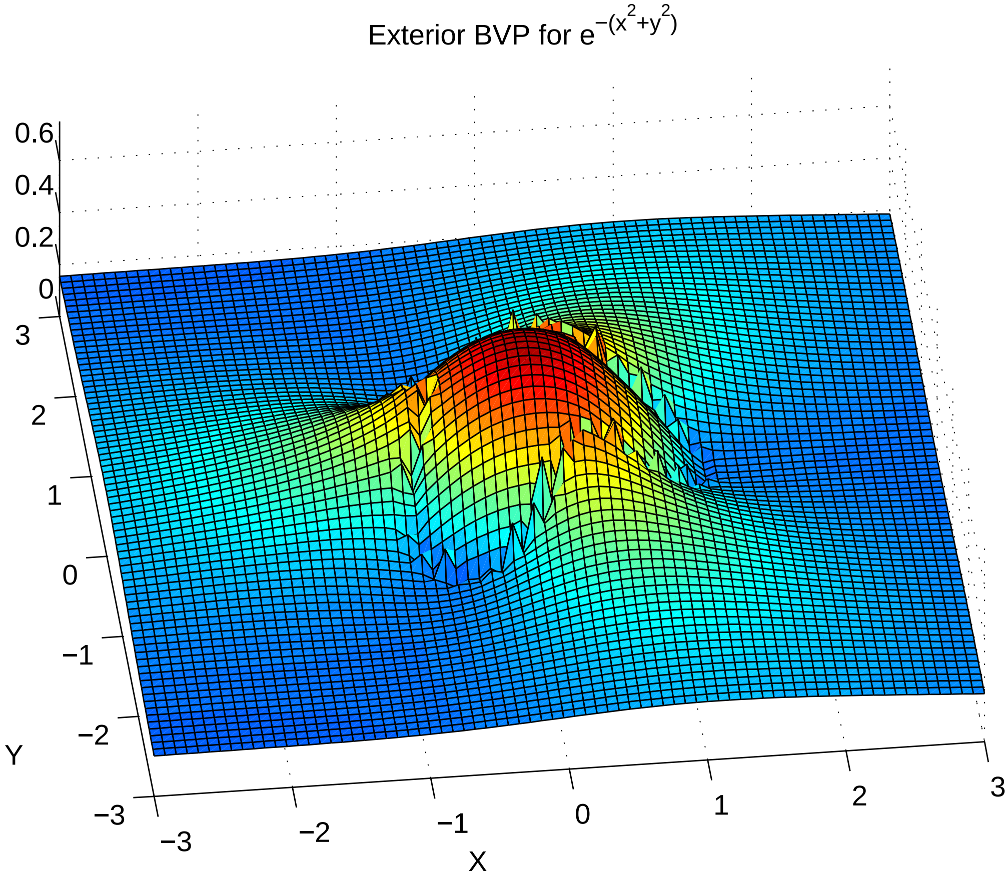


Noting the vertical scaling (which is on the order of 10^5!), we can safely say that the solution likewise "blows up" as the condition number of our Nyström kernel blows up.

For part (c), the first value of k for which the kernel blows up is ≈ 1.51069720040. Again, an adaptive method was used to obtain this value, see "hw6_3d_acc.m" for the code to determine this value. For a general trend of how the condition number changes with the wavenumber over the interval $[0.1, 4]$, see the figure below.



Lastly, we consider the following figure.



This figure was generated with the same wavenumber found in part (c). As is evident from this image, the exterior solution is well-behaved and *does not* blow up as it did for the interior problem.