Math 11, Fall 2007

Lecture 14

Scott Pauls [1]

[1] Department of Mathematics
Dartmouth College

10/26/07

Outline

Outline

1. Review and overview
   - Last class
2. Today's material
   - Integration in two variables
3. Group Work
4. Next class

Integration of a function of two variables

- Integration measure the volume under a portion of surface $z = f(x, y)$.
- The definite integral is defined via Riemann sums analogously to the one variable case
- Evaluation of an integral via the definition can be difficult.

Outline

Methods of integration

- In one variable calculus, the difficulty of evaluating integrals is overcome using the Fundamental Theorem of Calculus
- We can use the FTC in one variable by breaking up the two variable integral into two one variable integrals via Fubini's Theorem.

Methods of integration

- In one variable calculus, the difficulty of evaluating integrals is overcome using the Fundamental Theorem of Calculus
- We can use the FTC in one variable by breaking up the two variable integral into two one variable integrals via Fubini's Theorem.

Fubini's Theorem

Given $f(x, y)$, continuous on a rectangle $R = [a, b] \times [c, d]$,

$$\int \int_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy$$

This same theorem holds more generally: if f is bounded on R, f is discontinuous only on a finite number of smooth curves, and the iterated integrals converge.

Example

$$f(x,y) = x^2 + y^2,\ R = [0,1] \times [0,1]$$

$$\begin{aligned}
\int\int_R f(x,y)\, dA &= \int_0^1 \int_0^1 (x^2 + y^2)\, dx\, dy \\
&= \int_0^1 \left(\frac{x^3}{3} + y^2 x \right) \Big|_{x=0}^{x=1} dy \\
&= \int_0^1 \left(\frac{1}{3} + y^2 \right)\, dy \\
&= \left(\frac{y}{3} + \frac{y^3}{3} \right) \Big|_0^1 \\
&= \frac{2}{3}
\end{aligned}$$

Examples

1. $f(x, y) = x + \sqrt{y}$, $R = [1, 4] \times [0, 2]$
2. $f(x, y) = (x + y)^{-2}$, $R = [1, 2] \times [0, 1]$
3. Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin(y)$ and the planes $x = \pm 1, y = 0, y = \pi, z = 0$.

Work for next class

- Reading: 16.3
- f07hw14