The Comparison Tests

10/06/2006

Lecture 8

The Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

1. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

2. If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

Examples

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- $\sum_{n=1}^{\infty} \frac{1}{n!}$

The Limit Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c id a finite number and $c > 0$, then either both series converge or both diverge.

Examples

Which of the following series converge, and which diverge?

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- $\sum_{n=1}^{\infty} \frac{1+n \ln n}{n^2+5}$

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- $\sum_{n=1}^{\infty} \frac{2n+1}{n^2+2n+1}$
- $\sum_{n=1}^{\infty} \frac{1+n\ln n}{n^2+5}$
- $\sum_{n=1}^{\infty} \frac{\sin n\sqrt{n}}{4n+1}$

Estimate Sums

- If used the Comparison Test to show that a series $\sum a_n$ converges by comparison with a series $\sum b_n$, then we want to estimate the sum $\sum a_n$ by comparing remainders.

- Consider the remainders

$$R_n = s - s_n = a_{n+1} + a_{n+2} + \dots$$

and

$$T_n = t - t_n = b_{n+1} + b_{n+2} + \dots,$$

where $s = \sum a_n$ and $t = \sum b_n$.

Example

- Use the sum of the first 100 terms to approximate the sum of the series
$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}.$$