1. Consider the unit circle $x^2 + y^2 = 1$. The points $(-3/5, 4/5)$ and $(1, 0)$ lie on the circle and divide it into two parts. Find the arc length of the shorter part.

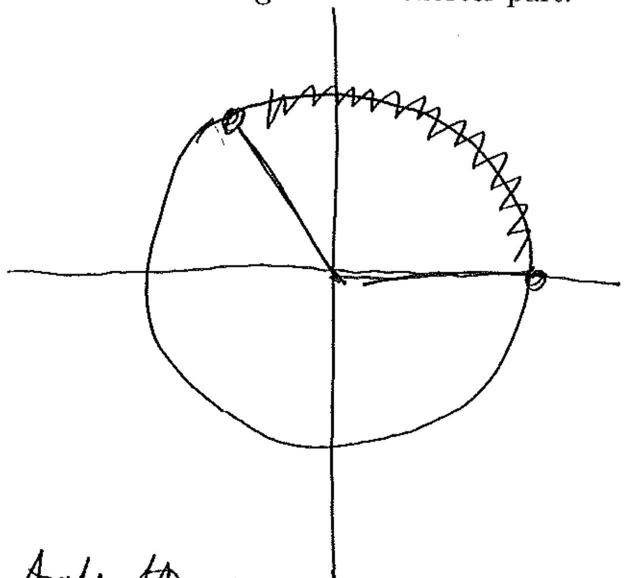
   (a) $\frac{6\pi}{5}$

   (b) $\frac{\pi}{2} + \arcsin\left\{\frac{3}{5}\right\}$

   (c) $\frac{3\pi}{2} + \arcsin\left\{\frac{3}{5}\right\}$

   (d) $\frac{8}{5}$

   (e) None of the above



$$y = \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{1-x^2+x^2}{1-x^2} = \frac{1}{1-x^2}$$

Arclength =

$$\int_{-3/5}^{1} \sqrt{\frac{1}{1-x^2}}\, dx = \sin^{-1}(x)\Big|_{-3/5}^{1}$$

$$= \pi/2 - \sin^{-1}(-3/5)$$

$$= \pi/2 + \sin^{-1}(3/5)$$

2. Find sin(arctan(2)).

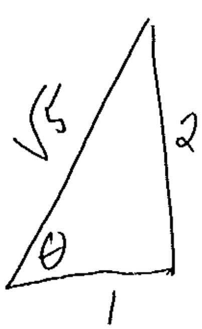
   (a) 1

   (b) 5

   (c) $2\sqrt{5}/5$

   (d) $\sqrt{5}$

   (e) None of the above



$$\sin\theta = \frac{2}{\sqrt{5}}$$

3. The length of the piece of the graph $y = \tan x$ from $x = 0$ to $x = \pi/4$ is:

(a) $\int_0^{\pi/4} (1 + \sec^2 x)\,dx$

(b) $\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\,dx$

(c) $\int_0^{\pi/4} (1 + \sec x)\,dx$

(d) $\int_0^{\pi/4} \sqrt{1 + \sec^2 x}\,dx$

(e) None of the above

$y' = \sec^2 x$

$1 + (y')^2 = 1 + \sec^4 x$

$\int_0^4 \sqrt{1+\sec^4 x}\,dx$

4. Find the area of the region bounded by $f(x) = x^3 + 3x^2 - 3$ and $g(x) = x^2 - 3$.

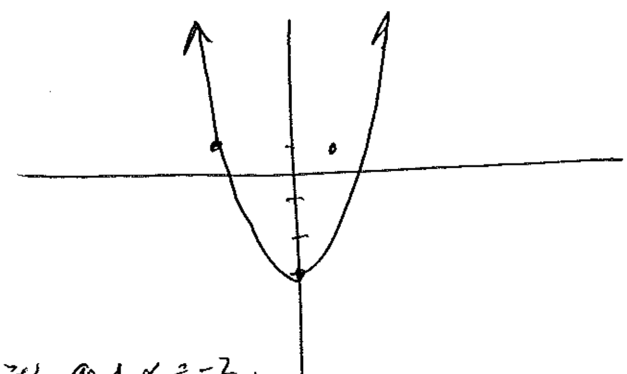
(a) 3

(b) 8/3

(c) 5/3

(d) 4/3

(e) None of the above



Curves intersect where

$x^3 + 3x^2 - 3 = x^2 - 3$

$x^3 + 2x^2 = 0$

$x^2(x+2) = 0$

$x = 0,\ x = -2$

between $x = 0$ and $x = -2$,

$x^2 - 3$ is ~~above~~ below

$x^3 + 3x^2 - 3$.

$-\int_{-2}^{0} (x^2 - 3) - (x^3 + 3x^2 - 3)\,dx = \frac{4}{3}$.

5. Using 3 trapezoids, approximate $\int_0^6 (-x^2+3)dx$.

(a) -58
(b) -2
(c) -26
(d) -54
(e) None of the above

$h=2$

x	y
0	3
2	-1
4	-13
6	-33

$\frac{2}{2}\cdot(3+-33+2(-1-13)) =$

$3-33-2-26=-58.$

6. Evaluate: $\frac{d}{dx}\left\{\frac{1}{x}\int_0^{3x^2}\sin(t)\,dt\right\}$ $=$ sml

product rule for derivatives.

(a) $6\sin(3x^2) - \frac{1}{x^2}\int_0^{3x^2}\sin(t)\,dt$
(b) $-\frac{6}{x}\sin(3x^2)$
(c) $6\sin(x)$
(d) $-\frac{1}{x^2}\int_0^{3x^2}\sin(t)dt + \frac{6}{x}\sin(x)$
(e) None of the above

$\frac{1}{x}\cdot\sin(3x^2)\cdot 6x + \int_0^{3x^2}\sin t\,dt\cdot \cancel{\text{tox}}\ \frac{-1}{x^2}$

even ~~odd~~ odd odd

7. Simplify: $\int_{-3}^{3} (\sin(x^2) + 3x^3 + x^{11})dx$

(a) $2\int_{0}^{3} (\sin(x^2) + 3x^3 + x^{11})dx$

(b) $2\int_{0}^{3} \sin(x^2)dx$

(c) 0

(d) $\int_{-3}^{3} (\cos(x^2) + \frac{1}{4}x^4 + \frac{1}{12}x^{12})dx$

(e) None of the above

8. If a right Riemann sum yields

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n}\sec^2\left(\frac{5i}{n}\right) = \int_{0}^{b} f(x)\,dx\,,$$

what are b and $f(x)$?

$h = \frac{5}{n}$, so $b-a=5$.

function is $\sec^2 x$.

(a) $b = 5$, $f(x) = \tan(x)$

(b) $b = n$, $f(x) = \tan(x)\sec(x)$

(c) $b = 5$, $f(x) = \sec^2(x)$

(d) $b = \frac{5}{n}$, $f(x) = \tan(x)$

(e) None of the above

9. A cylindrical bucket with open top is designed to hold 8π cubic inches of water. What is the height of the bucket that minimizes the total surface area of the bucket (bottom & side)? (Note that if h and r are the height and radius of the bucket, the bottom is a circle with area πr^2, the side is a rectangle with area $2\pi rh$, and the volume of the bucket is $\pi r^2 h$.)

   (a) 1 inch
   (b) 2 inches
   (c) 4 inches
   (d) 8 inches
   (e) None of the above

$V = \pi r^2 h = 8\pi$

$r^2 h = 8$

$h = 8/r^2$

$S = \cancel{2}\cdot\pi r^2 + 2\pi rh$ minimize this.

$S = \pi r^2 + 2\pi r \cdot \left(\frac{8}{r^2}\right) = \pi r^2 + \frac{16\pi}{r}$

$S' = 2\pi r - \frac{16\pi}{r^2} = 0 \quad r = 2.$

10. Where is $f(x) = \frac{x}{x^2+1}$ increasing?

   (a) $x = 1$ and $x = -1$
   (b) $x = 0$, $x = \cup\overline{3}$ and $x = -\cup\overline{3}$
   (c) $(-\cup, -1] \cup [1, \cup)$
   (d) $(-\cup, -\cup\overline{3}] \cup (0, \cup\overline{3})$
   (e) None of the above

$f'(x) = \frac{(x^2+1)\cdot 1 - x\cdot 2x}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$

$f'(x) = 0 \Rightarrow x^2 = 1 \Rightarrow x = 1$ or -1.

$f'(x) \geq 0 \Rightarrow -1 \leq x \leq 1$

11. Solve: $\csc(x)y' = y$.

(a) $\sqrt{-\cos(x) + C}$
(b) $\sqrt{-\csc(x) + C}$
(c) $Ce^{\sin(x)}$
(d) $Ce^{-\cos(x)}$
(e) None of the above

$y' \cdot \frac{1}{y} = \sin(x)$

$\ln|y| = -\cos(x) + c$

$y = Ce^{-\cos(x)}$

12. Bacteria grow at a rate proportional to the amount present. If a bacteria population grows such that the population has doubled in 3 minutes, when will it triple?

(a) $t = \frac{\ln(2)}{3}$
(b) $t = \frac{3\ln(3)}{\ln(2)}$
(c) $t = \frac{\ln(3)}{3\ln(2)}$
(d) $t = \frac{2\ln(2)}{\ln(3)}$
(e) None of the above

$B(t)$ = population of bacteria

$\frac{dB}{dt} = kB$ so $B(t) = B_0 e^{kt}$

$B(3) = 2 \cdot B_0 = B_0 e^{3k}$

$2 = e^{3k}$

$\ln 2 = 3k$

$\frac{\ln 2}{3} = k$

$B(t) = B_0 e^{\frac{\ln 2}{3} \cdot t}$

$3B_0 = B_0 e^{\frac{\ln 2}{3} \cdot t}$

$3 = e^{\frac{\ln 2}{3} \cdot t}$

$\ln 3 = \frac{\ln 2}{3} \cdot t$

$t = \frac{3\ln 3}{\ln 2}$

13. Hooke's Law says that the acceleration of a mass suspended by a spring is proportional to the distance of the mass from the equilibrium position. If y denotes the position of the mass and 12 is the equilibrium position for the mass, which of the following differential equations best represents Hooke's Law?

(a) $y'' = -k(y - 12)$
(b) $y'' = y$
(c) $y'' = -y + 12$
(d) $y'' = ky - 12$
(e) $y'' = 12$

$\cancel{x}(t)$ = distance

$\cancel{x}''(t)$ = acceleration

$\cancel{x}''(t) = k(\cancel{x}(t) - 12)$

14. Dartmouth students run in a circle around a bonfire. As more spectators join, the radius of the circle decreases at a rate of 4 feet per minute. How fast is the area decreasing when the area is π square feet?

(a) There is not enough information
(b) 8π
(c) 16π
(d) 2π
(e) None of the above

$\frac{dr}{dt} = -4$

$A = \pi r^2$

Area $= \pi$ when $r = 1$.

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$= 2\pi \cdot 1 \cdot -4 = -8\pi$

So Area is decreasing @ 8π ft^2/min

15. If a ball rolling in a straight line decelerates at a rate of $4\,\mathrm{m/s^2}$ starting with velocity $8\,\mathrm{m/s}$ then how far has the ball traveled after $2\,\mathrm{s}$?

(a) 8 m
(b) 16 m
(c) 24 m
(d) 32 m
(e) None of the above

$f''(t) = -4$

$f'(t) = -4t + 8$

$f(t) = -2t^2 + 8t + c$

$f(2) - f(0) = -8 + 16 = 8.$

16. Use a linear approximation centered at 144 to estimate $\sqrt{145}$.

(a) $12\frac{1}{12}$
(b) $12\frac{1}{24}$
(c) 12
(d) $12\frac{1}{3}$
(e) None of the above

~~$x^2 - 145 = 0$~~

~~$f(x) = x^2 - 145$~~

~~$f'(x) = 2x$~~

~~$f(12) = -1$~~

~~$f'(12) = 24$~~

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f(144) = 12$

$f'(144) = \frac{1}{2 \cdot 12} = \frac{1}{24}$

$L(x) = 12 + \frac{1}{24}(x - 144)$

$L(145) = 12 + \frac{1}{24}$

17. Find the derivative of $f(x) = x^{3x^2}$

(a) x^{3x^2}
(b) $x^{3x^2}(6x)$
(c) $x^{3x^2}(6 + 3x^2 \ln(x))$
(d) $x^{3x^2}(6x \ln(x) + 3x)$
(e) None of the above

$\ln f = 3x^2 \ln x$

$\frac{1}{f} \cdot f' = 3x^2 \cdot \frac{1}{x} + \ln x \cdot 6x$

$f' = x^{3x^2} \cdot (3x + \ln x \cdot 6x)$

18. If an object moves with position function $s(t) = -\frac{1}{3}\sin(t) + 13$, from $t = 0$ to $t = 2\pi$ seconds, when is the object's velocity increasing?

(a) $[\pi, 2\pi]$
(b) $[0, \pi]$
(c) The velocity is always increasing
(d) The velocity is never increasing
(e) None of the above

$s'(t) = -\frac{1}{3} \cdot \cos t > 0$

~~when~~

$s''(t) = \frac{1}{3} \sin t > 0$ when $x \in [0, \pi]$.

19. Find the derivative of $x^2 \sin(\sqrt{x})$.

(a) $x^2 \cos(\sqrt{x}) + 2x \sin(\sqrt{x})$
(b) $2x \cos(\sqrt{x})$
(c) $x^{3/2} \cos(\sqrt{x})/2 + 2x \sin(\sqrt{x})$
(d) $x^{3/2} \sin(\sqrt{x})/2 + 2x \cos(\sqrt{x})$
(e) None of the above

$x^2 \cdot \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}} + 2x \cdot \sin\sqrt{x}$

20. What is the derivative of e^2?

(a) $2e^2$
(b) $2e$
(c) e^2
(d) 0
(e) None of the above

21. $\lim_{h\infty 0} \frac{(1+h)^4 - 1}{h}$ equals:

(a) 4
(b) 0
(c) The limit does not exist
(d) 1
(e) None of the above

$f(x) = x^4$

$f'(x) = 4x^3$

$f'(1) = 4.$

22. Suppose $f(x) = \begin{cases} \frac{4 - x^2}{2 + x} & \text{if } x \neq -2, \\ 4 & \text{if } x = -2. \end{cases}$

Which of the following is true?

(a) $f(x)$ has a removable discontinuity at $x = -2$
(b) $\lim_{x\infty -2} f(x)$ does not exist
(c) $f(x)$ is continuous on its domain
(d) $f(x)$ is not differentiable at $x = -2$
(e) None of the above

23. The slope of the tangent line to the curve $xy + 1 = x^3 + y^2$ at $(1, 1)$ is:

(a) -2

(b) 0

(c) 2

(d) The tangent line is vertical

(e) None of the above

$$x \cdot y' + y = 3x^2 + 2y \cdot y'$$

$$\frac{3x^2 - y}{x - 2y} = y'$$

$$\frac{3-1}{1-2} = -2.$$

24. $\lim\limits_{x\infty\infty} \uplus \frac{4-3x}{4x^2 - x + 3}$ equals:

(a) $-3/2$

(b) $-1/2$

(c) $1/2$

(d) $3/2$

(e) None of the above

$$= \frac{4-3x}{\sqrt{(2x)^2\left(1 - \frac{1}{4x} + \frac{3}{4x^2}\right)}}$$

$$= \frac{4-3x}{2x\sqrt{1 - \frac{1}{4x} + \frac{3}{4x^2}}} = \frac{-3}{2}$$

25. The derivative of $\dfrac{\sin(-x)}{\tan(x)}$ is:

$= \dfrac{-\sin x}{\sin x/\cos x} = -\cos x.$

(a) $\cos(x)$
(b) $\sin(x)$
(c) $-\cos(x)$
(d) $-\sin(x)$
(e) None of the above

$\dfrac{d}{dx}(-\cos x) = \sin x.$

26. Integrate: $\int (3\sqrt{x} + 3x^3 + 1)dx$

(a) $2x^{3/2} + \frac{3}{4}x^4 + x + C$
(b) $\frac{3}{2}x^{-1/2} + 9x + C$
(c) $\frac{3}{2}x^{-1/2} + \frac{3}{4}x^4 + x + C$
(d) $\frac{2}{3}x^{3/2} + \frac{3}{4}x^4 + x + C$
(e) None of the above

$\dfrac{3x^{3/2}}{3/2} + \dfrac{3x^4}{4} + x + C$

27. Integrate: $\int \frac{e^x}{4+e^x}dx$

Let $u = 4+e^x$

$du = e^x dx$

(a) $\frac{e^{x+1}}{4+e^x} + C$

(b) $\ln(4+e^x) + C$

(c) $\ln(u) + C$

(d) $\frac{e^x}{4x+e^x} + C$

(e) None of the above

$\int \frac{1}{u} du = \ln|u| = \ln|4+e^x| + C$

$= \ln(4+e^x) + C.$

28. For what x value does $f(x) = \frac{5x+2}{3x}$ achieve an absolute maximum on $[-3, -1]$?

(a) No maximum value exists

(b) $x = -1.5$

(c) $x = -3$

(d) $x = -1$

(e) None of the above

$\frac{5x+2}{3x} = \frac{5}{3} + \frac{2}{3x}$

$f'(x) = \frac{-2}{3x^2}$ does not $= 0$ on $[-3,-1]$

x	$f(x)$
-3	$-2/27$
-1	$-2/3$ h-m

29. Let $f(x) = \dfrac{x^2+6x+9}{x^2+x-6}$. What is the domain of $f(x)$?

(a) $(-\cup, -3) \cup (-3, \cup)$
(b) $(-\cup, -3) \cup (-3, 2) \cup (2, \cup)$
(c) $(-\cup, 2) \cup (2, \cup)$
(d) $(-\cup, \cup)$
(e) None of the above

$$f(x) = \frac{(x+3)(x+3)}{(x+3)(x-2)}$$

-3 and 2 are <u>not</u> in the domain.

30. For $f(x)$ as in problem 29, what are the horizontal asymptotes?

(a) $y = 2$
(b) $y = 3$
(c) $y = 2$ and $y = 3$
(d) $y = 1$
(e) None of the above

$$\lim_{x \to \infty} f(x) = 1.$$