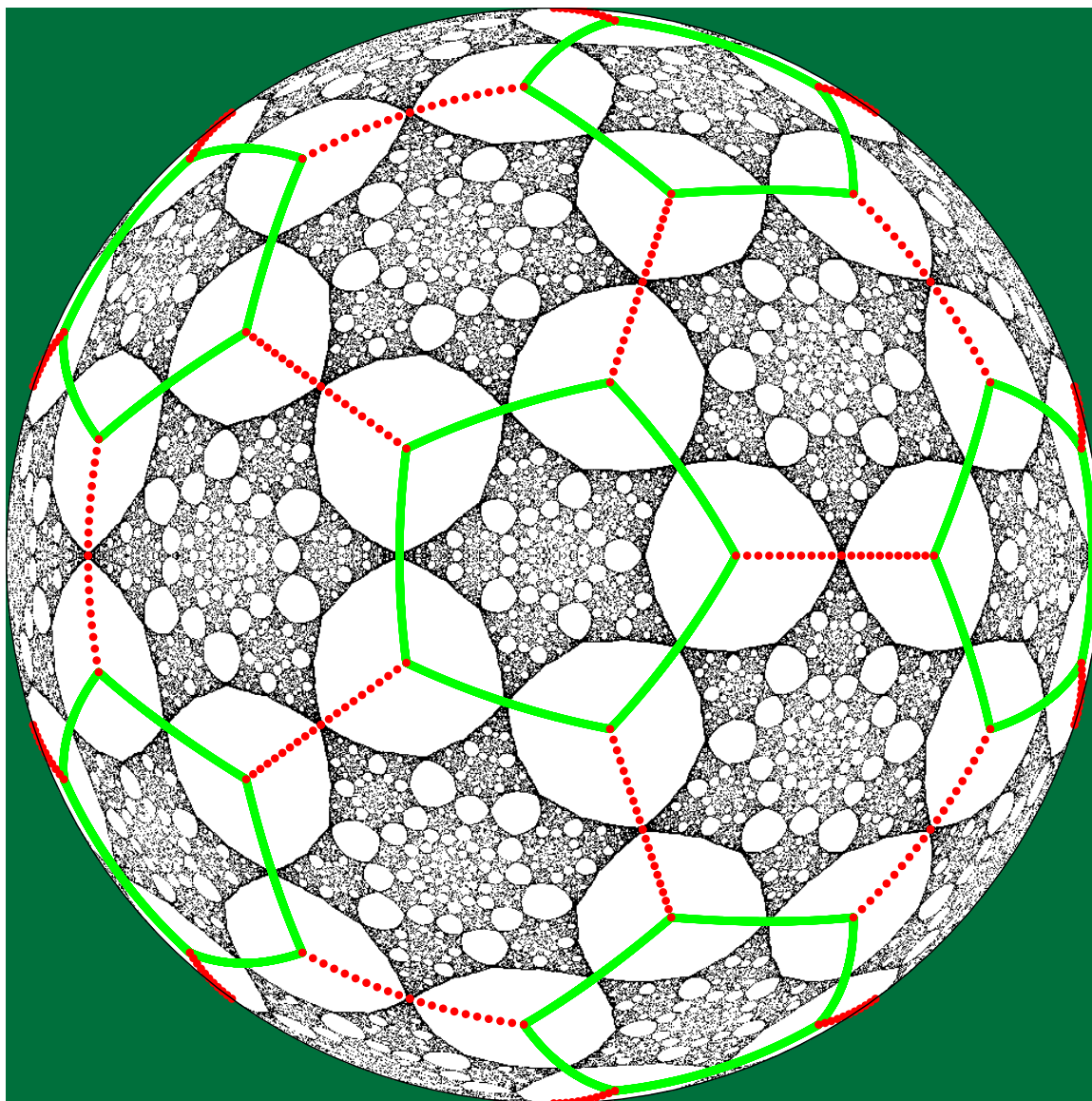
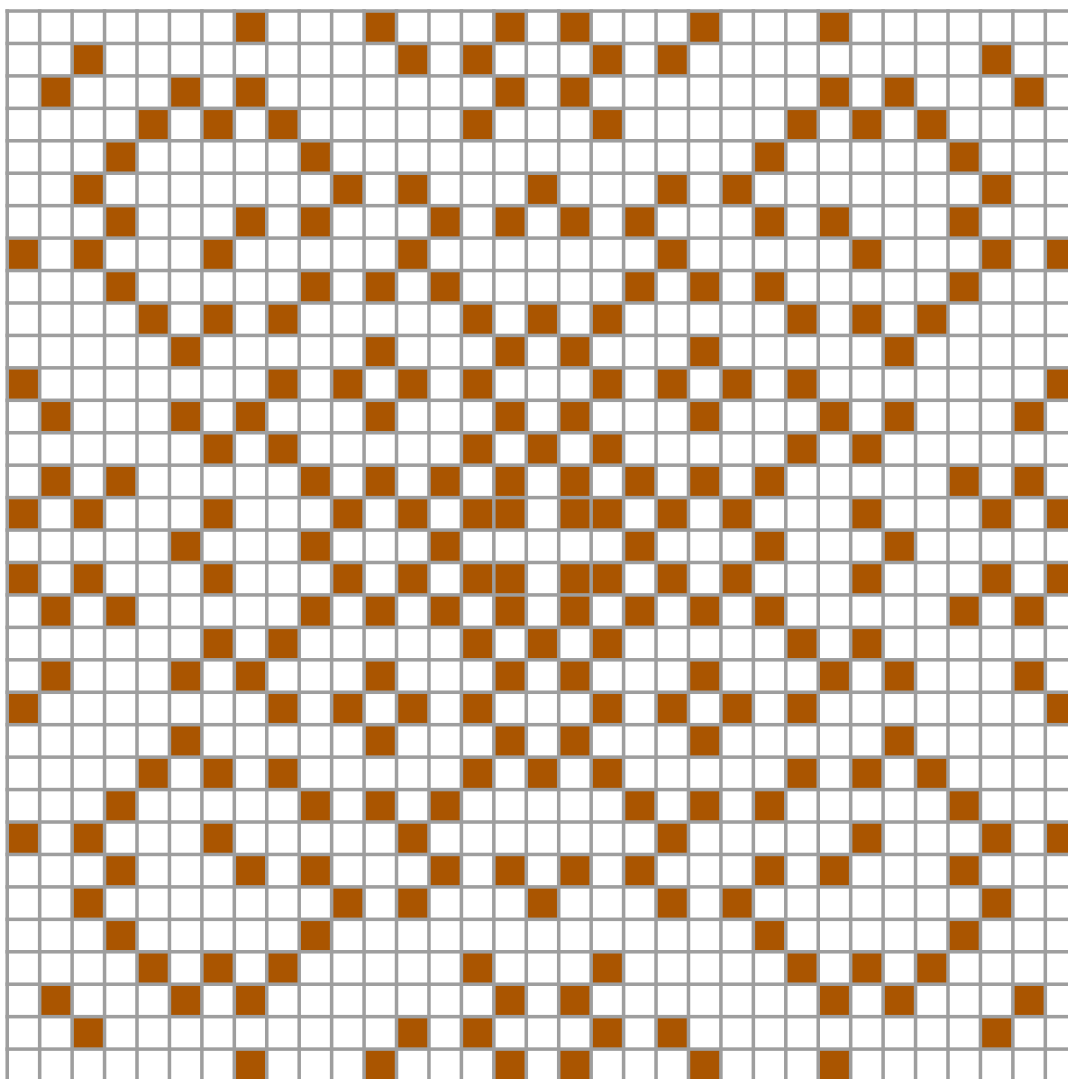


Imaginary numbers are real



Math 17, Spring 2013

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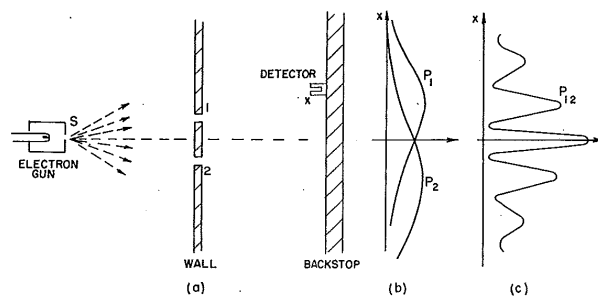


Fig 3-1. Interference experiment with electrons.

We will begin in this chapter by dealing with some general quantum mechanical ideas. Some of the statements will be quite precise, others only partially precise. It will be hard to tell you as we go along which is which, but by the time you have finished the rest of the book, you will understand in looking back which parts hold up and which parts were only explained roughly. The chapters which follow this one will not be so imprecise. In fact, one of the reasons we have tried carefully to be precise in the succeeding chapters is so that we can show you one of the most beautiful things about quantum mechanics—how much can be deduced from so little.

We begin by discussing again the superposition of *probability amplitudes*. As an example we will refer to the experiment described in Chapter 1, and shown again here in Fig. 3-1. There is a source s of particles, say electrons; then there is a wall with two slits in it; after the wall, there is a detector located at some position x . We ask for the probability that a particle will be found at x . Our *first general principle* in quantum mechanics is that the *probability* that a particle will arrive at x , when let out at the source s , can be represented quantitatively by the absolute square of a complex number called a *probability amplitude*—in this case, the “amplitude that a particle from s will arrive at x .” We will use such amplitudes so frequently that we will use a shorthand notation—invented by Dirac and generally used in quantum mechanics—to represent this idea. We write the probability amplitude this way:

$$\langle \text{Particle arrives at } x \mid \text{particle leaves } s \rangle. \quad (3.1)$$

In other words, the two brackets $\langle \rangle$ are a sign equivalent to “the amplitude that”; the expression at the *right* of the vertical line always gives the *starting* condition, and the one at the *left*, the *final* condition. Sometimes it will also be convenient to abbreviate still more and describe the initial and final conditions by single letters. For example, we may on occasion write the amplitude (3.1) as

$$\langle x \mid s \rangle. \quad (3.2)$$

We want to emphasize that such an amplitude is, of course, just a single number—a *complex* number.

We have already seen in the discussion of Chapter 1 that when there are two ways for the particle to reach the detector, the resulting probability is not the sum of the two probabilities, but must be written as the absolute square of the sum of two amplitudes. We had that the probability that an electron arrives at the detector when both paths are open is

$$P_{12} = |\phi_1 + \phi_2|^2. \quad (3.3)$$