# Series

January 16, 2007

# Series

- An **infinite series** is an expression obtained by adding the terms of an infinite sequence $\{a_n\}$. It is denoted

$$\sum_{n=1}^{\infty} a_n \quad \text{or} \quad \sum a_n.$$

- **Partial sums** are expressions

$$s_n = a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^{n} a_i.$$

**Definition.** *Given a series* $\sum_{i=1}^{\infty} a_n$*, let* $s_n$ *denote its* $n$*th partial sum:*

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n.$$

*If the sequence* $\lim_{n\to\infty} s_n = s$ *exists as a real number, then the series* $\sum a_n$ *is called* **convergent** *and we write*

$$\sum a_n = s.$$

*Otherwise the series is called* **divergent**.

# The Geometric Series

- Suppose that $a_n = ar^{n-1}$ for some $a \neq 0$. $r$ is called the ratio.

# The Geometric Series

- Suppose that $a_n = ar^{n-1}$ for some $a \neq 0$. $r$ is called the ratio.

- The geometric series is convergent if $|r| < 1$ and is sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}.$$

- If $|r| \geq 1$, the geometric series is divergent.

# Examples

- $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

# Examples

- $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

- $\sum_{n=1}^{\infty} \frac{5^n}{4^n}$

# Examples

- $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$
- $\sum_{n=1}^{\infty} \frac{5^n}{4^n}$
- $\sum_{n=1}^{\infty} \frac{2^n}{7^{2n+1}}$

# Other Examples

- $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

# Other Examples

- $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

- $\sum_{n=1}^{\infty} \ln(n+1) - \ln(n)$

- If the series $\sum a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

- If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$ then the series $\sum a_n$ is divergent.

- Example: Determine whether the series

$$\sum_{n=1}^{\infty} \frac{3n^2}{5n^2+2}$$

is convergent.

# Series Laws

If $\sum a_n$ and $\sum b_n$ are convergent then so are the following series

- $\sum(a_n \pm b_n) = \sum a_n \pm \sum b_n$;
- $\sum ca_n = c \sum a_n$.

# Series Laws

If $\sum a_n$ and $\sum b_n$ are convergent then so are the following series

- $\sum(a_n \pm b_n) = \sum a_n \pm \sum b_n$;
- $\sum ca_n = c \sum a_n$.
- Example: Find the sum of the following series

$$\sum_{n=1}^{\infty} \frac{4}{n(n+1)} + \frac{2^n}{3^n}.$$