Hour Exam 2

Math 3

November 9, 2005

Name:____

Instructor (circle): Lahr (8:45) Elizalde (9:00) Ionescu (11:15)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All your answers to the multiple choice questions must be marked on the Scantron form provided, and your responses to the remaining questions must be written in this exam booklet. Take a moment now to **print your name and section clearly on your Scantron form, and on your exam booklet**. With regard to the multiple choice questions, you may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 10 multiple choice problems each worth 6 points, and there are 3 additional problems totaling 40 points. Check to see that you have 8 pages of questions plus this cover page.

Non-multliple choice questions:

Problem	Points	Score
1	15	
2	13	
3	12	
Total	40	

MULTIPLE-CHOICE

1. Consider the function $f(x) = \frac{1}{x} - 2$ on the interval $[1, 3]$. Using the Mean Value Theorem we can conclude:

(a) The graph of the function has a tangent line between 1 and 3 with slope $\frac{8}{3}$.

(b) The graph of the function has a tangent line between 1 and 3 with slope $-\frac{1}{3}$.

(c) The function has a zero in the interval $[1, 3]$.

(d) The Mean Value Theorem does not apply because this function is not continuous in $[1, 3]$.

(e) The Mean Value Theorem does not apply because this function is not differentiable in $[1, 3]$.

2. The slope of the tangent line to the curve $x^2y^3 + y = 2$ through the point $(1, 1)$ is:

(a) 0

(b) 1

(c) -1

(d) $-\dfrac{1}{2}$

(e) None of the above.

3. Suppose that we apply Newton's method to approximate the root of the equation $x^3 - 2x^2 - 1 = 0$. If we start at $x_0 = 2$, then, after one iteration of the method, x_1 is:

(a) 6

(b) 2.25

(c) 0

(d) 2

(e) None of the above.

4. Let $y = f(x)$ be the solution to the initial value problem $y' = 2x - y$, $y(1) = 3$. What is the slope of the tangent line to the graph of f at $x = 1$?

(a) -1

(b) 3

(c) $-\frac{7}{2}$

(d) 2

(e) f is not differentiable at $x = 1$.

5. A ball is dropped from an air baloon at a height of 490 meters. What is the speed of the ball right before it hits the ground?
[Note: The acceleration due to gravity is 32.2 ft/s^2 or 9.8 m/s^2.]

(a) 980 m/s

(b) 98 m/s

(c) 4.9 m/s

(d) 49 m/s

(e) 10 m/s

6. The solution of the initial value problem $\frac{dy}{dx} = (y+1)\cos x$, $y(0) = 1$ is:

(a) $2e^{\sin x} - 1$

(b) $e^{\sin x}$

(c) $e^{\sin x} - 1$

(d) $\sin x + C$, where C is a constant.

(e) None of the above.

7. The integral $\int \frac{\sin^3 x+\cos x}{\sin^2 x}\,\mathrm{d}\,x$ is:

(a) $-\cos x - \frac{1}{\sin x} + C$, where C is a constant.

(b) $-\cos x + \sin x + C$, where C is a constant.

(c) $\frac{\frac{1}{4}\sin^4 x+\sin x}{\frac{1}{3}\sin^3 x} + C$, where C is a constant.

(d) $\frac{3\sin^2 x\cos x-\sin x}{2\sin x\cos x} + C$, where C is a constant.

(e) $\cos x + \frac{\cos x}{\sin x} + C$, where C is a constant.

8. Using the linear approximation of $f(x) = \sqrt[3]{x}$ at $x_0 = 27$, we can approximate $\sqrt[3]{26}$ to be:

(a) $3 + \frac{1}{27}$

(b) $3 - \frac{1}{9}$

(c) 2

(d) $3 - \frac{1}{27}$

(e) 2.999.

9. The derivative of $3^x + \ln 3x + e^{\ln 3x}$ is:

(a) $(\ln 3)3^x + \frac{1}{x} + 3$.

(b) $3^x + \frac{1}{3x} + 3$.

(c) $3^x + \frac{1}{x} + 3x$.

(d) $(\ln 3)3^x + \frac{1}{x} + 3e^{\ln 3x}$.

(e) None of the above.

10. The solution of the initial value problem $\frac{dy}{dx} = x^3 + \sin x$, $y(0) = 0$ is:

(a) $\frac{x^4}{4} - \cos x$.

(b) $x^4 + \sin x^2$.

(c) $4x^4 - \cos x + 1$.

(d) $\frac{x^4}{4} - \cos x + 1$.

(e) None of the above.

NON-MULTIPLE-CHOICE
You can earn partial credit on the following three problems. **However, you must show all your work.**

1. Let $f(x) = 2x^3 - 3x^2 + 1$.

   (a) (5 pts) At what values of x does f has a local maximum?

   (b) (5 pts) What is the maximum value of $f(x)$ on the interval $[-1, 2]$?

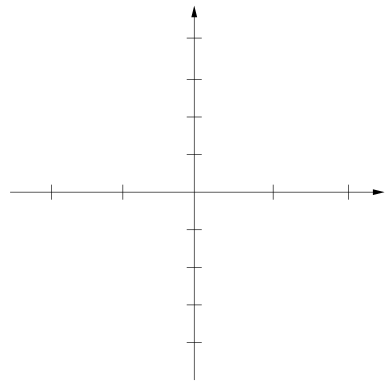
   (c) (5 pts) Find the intervals of increase and decrease of f.

2. Consider the initial value problem $y' = xy$, $y(0) = 1$.

(a) (5 pts) Use Euler's method to find an approximation of the solution curve that starts at $(0, 1)$ using steps of size 1, and ends with an approximation of $y(2)$. What is the approximate value of $y(2)$?

(b) (5 pts) Solve the initial value problem.

(c) (3 pts) On the interval $[0, 2]$, sketch on a single set of xy-axes the solution found in (b) and the approximation found in (a). Use the axes below and add your own numbers to the tick marks.



3. Uranium is a radioactive substance that decays according to an exponential model. Let $y(t)$ be the amount of uranium present at time t, and let y_0 be the original amount.

(a) (4 pts) Write the differential equation that describes this model.

(b) (4 pts) Write the solution to this differential equation.

(c) (4 pts) Suppose that the half-life of the substance is 700 milion years (recall that the half-life is the length of time required for the amount of substance to be reduced to half its size). Starting with 100 grams, how much will be left after 1 million years? [Note: Leave your answer in terms of exponentials and logs.]