Representations of Functions as Power Series

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- $\frac{x^4}{x+3}$

Differeniatiation and Integration of Power Series

If the power series $\sum c_n(x-a)^n$ has radius of convergence $R > 0$, then the function f defined by

$$f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$$

is differentiable (and continuous) on the interval $(a-R, a+R)$ and

1. $f^{'}(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \cdots$ (term by term differentiation)

2. $\int f(x)\mathrm{d}x = C + c_0(x-a) + c_1\frac{(x-a)^2}{2} + c_2\frac{(x-a)^3}{3} + \cdots$ (term by term integration)

The radius of convergence of the power series in these equations are both R.

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- $f(x) = \frac{x^2}{(1+x)^2}$.

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