Supplementary Homework for Math 43

S1: Consider Cardano's cubic equation $x^3 - 3px + 2q = 0$ mentioned in lecture. Let $D := q^2 - p^3$.

1. Verify the claim made in lecture that if $D = 0$, then Cardano's equation has exactly two distinct real roots. Find the values of those roots. (Hint: Cardano's solution is $-2\sqrt[3]{q}$.)

2. Use Cardano's solution to find the real root of $y^3 + 3y^2 + 6y + 6 = 0$.